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A Declarative Language for Advanced Analytics and its Scalable Implementation

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Computer Science

by

Alexander Philip Shkapsky

2016
A Declarative Language for Advanced Analytics and its Scalable Implementation

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Alexander Philip Shkapsky

Doctor of Philosophy in Computer Science

University of California, Los Angeles, 2016

Professor Carlo Zaniolo, Chair

Advanced analytics are used to discover hidden patterns and trends in massive datasets. Great strides have been made by researchers to provide computational models, systems and accompanying languages for analytics. However, there is still a dire need for highly expressive declarative languages that enable the compilation, optimization and evaluation of advanced analytics over massive datasets. Specifically, a language for analytics needs (i) to support the expression of analytics over multiple data models (ii) to provide high-level declarative constructs enabling system optimizations, and (iii) be conducive for iterative or recursive evaluation.

In this dissertation, we propose an expressive Datalog language for advanced analytics, and compilation and optimization techniques for its efficient evaluation on systems designed for iterative execution. Specifically, this dissertation makes two main contributions:

(i) We develop and demonstrate a next generation Datalog System - the Deductive Application Language System (DeALS). To extend the range of analytics supported in DeALS, we add support for aggregation in recursion into our logic-based language. We propose the design and implementation of several monotonic aggregates that can be used in recursive Datalog rules and evaluated efficiently using our novel optimization techniques. We demonstrate the effectiveness of these aggregates and conduct an experimental comparison with other Datalog systems and determine that DeALS combines superior generality with superior performance.
(ii) We design and implement BigDatalog, a Datalog system on Apache Spark, for large-scale advanced analytics. We implement BigDatalog for efficient distributed evaluation and to utilize communication-reduction techniques during evaluation. We propose compilation and optimization techniques, as well as job scheduling techniques, to support efficiently the evaluation of DeAL programs on Spark. We conduct an experimental comparison with other state-of-the-art large-scale Datalog systems and demonstrate the efficacy of our techniques and effectiveness of our Spark extensions in supporting Datalog-based analytics.
The dissertation of Alexander Philip Shkapsky is approved.

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For my wife
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CHAPTER 1

Introduction

The high velocity of the modern world requires accurate, timely and sophisticated analytics to support informed decision making, better allocation of resources and identification of new opportunities. In fact, analytics is responsible for many of the advanced applications we use daily. Analytics is a broad term and includes tasks from data analysis, data mining and machine learning. Using web analytics to track user visits and page views, websites can better allocate resources and balance load across their clusters of web servers. Network intrusion detection systems need to quickly process network and server logs to identify unwanted visitors. Algorithms such as PageRank [PBM99] are used to identify the most influential members in a social network graph or important websites in a web graph.

The development of advanced analytics is an iterative process that requires tuning a pipeline of subtasks and can involve a variety of systems and tools. Common subtasks include i) filtering, joining and aggregating multiple datasets, ii) designing, implementing and testing complex learning models and iii) ensuring the application can properly scale. Moreover, a single analytics application can require elements of multiple data models, such as both relational and graph datasets.

Big Data is a rich application area for analytics due in part to the massive and continually growing datasets and the variety of data available. Both industry and academia have proposed many scalable systems and computational models for Big Data analytics. Google’s MapReduce [DG04], and its open source version Hadoop MapReduce [had15], have led the charge by providing practitioners with a powerful framework for distributed parallel processing of large-scale batch jobs. Platforms supporting a larger or more general operator set than MapReduce [BEH10, IBY07]
or extending the MapReduce framework with iterative constructs [BHB12, ELZ10, pro15, RPB13, SKH12, ZGG12, ZGG11] have also been presented. Parallel RDBMS [CDD09, FPC09, LFV12], designs based on parallel database systems [BCG11] and hybrid MapReduce/RDBMS models [ABH10, YYT10] have also emerged. Distributed in-memory designs for low-latency data analysis and iterative machine learning [PL10, ZCD12] have appeared. For large-scale graph analytics and graph machine learning, systems for both parallel [KBG12, LGK10] and distributed [gir15, GLG12, LBG12, MAB10, SWL13, TBC13] processing have been proposed.

However, even with these powerful systems available, advanced analytics still requires better support from powerful high-level programming languages with advanced built-in constructs backed by declarative semantics that are amenable to parallelization on multiple platforms. We identify three language challenges for advanced analytics:

First, as the needs for analytics applications can vary greatly from acyclic batch processing over relational data to long-running iterative and recursive workflows on graphs, a language should be expressive enough to support both relational and graph data models. For example, using a MapReduce system for many types of graphs processing is inefficient, whereas the vertex-centric models used by some graph-based systems are not suitable for many other types of analytics; it is also difficult to express graph queries using SQL. Often programmers are plagued with unnecessary code complexity from having to compensate for a computational model that quickly becomes suboptimal in terms of execution performance and scalability. Furthermore, this results in the programmer having to stitch together application subcomponents written in different APIs or executed by different systems. Supporting multiple data models at the language level increases logical data independence, reduces portability concerns and helps mitigate the need to migrate from one language or low-level API to another, which is often a major engineering effort.

Second, a language should provide high-level declarative constructs enabling system optimization. A noticeable construct lacking from most languages proposed for analytics is recursion. Recent work has proposed moving analytics into the RDBMS using built-in or user-defined function for analytics [HRS12, FKR12], but these approaches inherit the RDBMS’s poor support for recursion. Although domain specific languages (DSL) such as ScalOps [WCR11] are a step in the
right direction, other language integrated APIs [YIF08, ZCD12] embedded in another high-level language inherit support for iteration through the language’s control flow (e.g., C# or Scala). By enabling the user to specify what they want to do and tasking the system with how to do it, the system is given the best opportunity to provide optimized execution plans. We see this in declarative languages, such as SQL and Datalog, as optimizers can more easily identify declarative constructs and apply well-known optimization techniques.

Lastly, to enable the efficient support of sophisticated analytics, which include data mining and machine learning algorithms, the language must be evaluated by a system that efficiently supports iteration. One approach has been to port existing languages, such as SQL, to large-scale platforms for batch processing. However, the ported language is limited to efficiently supporting only the applications the platform can support efficiently. For example, high-level languages [BEG11, GNC09, TSJ09] and frameworks [cas15a, cas15b, sca15] for Hadoop efficiently support pipelines of acyclic batch jobs but poorly support iterative jobs. Another example is how RDBMS and their compiler optimizers targeting acyclic executions poorly support recursive queries. A system designed for iteration and/or recursion can more naturally and efficiently support a wide-range of complex analytics tasks.

1.1 Overview

In this dissertation, we made the following contributions towards the challenges outlined above.

1.1.1 Deductive Application Language System

We have developed the Deductive Application Language System (DeALS) for analytics. DeALS builds on earlier Deductive Database technology [AOT03] to efficiently support DeAL, a Datalog language that supports recursion, aggregation, negation and advanced constructs including user-defined aggregates and non-monotonic aggregation in recursion via XY-stratified Datalog [ZAO93]. DeALS supports the expression and evaluation of a wide range of complex applications including relational, aggregate, graph and recursive queries – all within a single unified declarative
1.1.2 Aggregation in Recursion

Aggregates in recursive queries are essential in many important applications including shortest paths computations, link and graph structure analysis and bill of materials queries. To enable the expression of these and many other analytics programs in our language, we design and efficiently implement a set of monotonic aggregates that are based on recent theoretical results [MSZ13a, MSZ13b] that use the set-containment semantics of standard DATALOG, and therefore can be used in recursive rules and evaluated efficiently. We provide novel optimization techniques to evaluate recursive rules with monotonic aggregates efficiently. We conduct an experimental comparison with other DATALOG systems which showed that our implementation in DeALS indeed provides superior performance, especially on sparse graphs.

1.1.3 BigDatalog

To support complex analytics on a scalable runtime over large datasets, we design and implement BigDatalog, a Datalog system on Apache Spark. We propose efficient compilation and optimization techniques specifically for Spark to enable the efficient evaluation of BigDatalog programs, including recursive programs with monotonic aggregates. We propose system-level optimizations in Spark to increase the efficiency of recursive query evaluation. We conduct an experimental comparison with (i) other large-scale state-of-the-art DATALOG systems, (ii) native Spark programs, and (iii) Spark’s graph processing module GraphX. The results show that BigDatalog outperforms the other systems on classical recursive queries such as transitive closure, and also outperforms on many important analytics queries on large datasets.

1.1.4 Outline

The remainder of this dissertation is organized as follows. Chapter 2 reviews Datalog preliminaries. Chapter 3 presents our contributions toward DeAL and its system (DeALS). Chapter 4 presents
the design and implementation of monotonic aggregates that can be used in recursive Datalog programs. Chapter 5 presents BigDatalog. We conclude and present future work in Chapter 6.
CHAPTER 2

Background: Datalog

A DATALOG program is a finite set of rules. A rule $r$ has the form $h \leftarrow b_1, \ldots, b_n$, where $h$ is the head of $r$ and $b_1, \ldots, b_n$ is the body. $h$ and each $b_i$ are literals with the form $p_i(t_1, \ldots, t_j)$ where $p_i$ is a predicate and $t_1, \ldots, t_j$ are terms which can be constants, variables or functions. We say $r$ is a rule of predicate $h$, unless it has an empty body, and then it is a fact. The comma separating literals in a body is a logical conjunction (AND). A successful assignment of all variables in the body produces a fact for the head’s predicate. Predicates are also considered relations and w.l.o.g. throughout this dissertation we will use the terms predicate and relation interchangeably. A query indicates the desired predicate to evaluate. As a convention, predicate and function names begin with lower case letters, and variable names begin with upper case letters.

Datalog by Example. Program 1 is Transitive Closure (TC), the quintessential DATALOG program. TC recursively produces all pairs of vertices that are connected by some path in a graph.

Program 1. Transitive Closure

\[
\begin{align*}
\text{r1.} & \ tc(X,Y) \leftarrow \ arc(X,Y). \\
\text{r2.} & \ tc(X,Y) \leftarrow tc(X,Z), \ arc(Z,Y).
\end{align*}
\]

Program 1 is explained as follows. $r1$ is an exit rule because it serves as a base case of the recursion. In $r1$, the arc predicate represents the edges of the graph – arc is a base relation (i.e., a table in RDBMS). $r1$ produces a $tc$ fact for each arc fact. $r2$ is a recursive rule since it has the $tc$ predicate in both its head and body. $r2$ will recursively produce $tc$ facts from the conjunction (i.e., equi-join in relational terminology) of previously produced $tc$ facts and arc facts. The query to evaluate TC is of the form $tc(X,Y)$. Lastly, this program uses a linear recursion in $r2$, since there is a single recursive predicate literal, whereas a non-linear recursion will have multiple
recursive literals in its body. The number of iterations required to evaluate Program 1 is equal to the longest simple path in the graph.

2.1 Datalog Evaluation

**DATALOG** is a declarative language and therefore rules are independent of the operators used to implement them (e.g., type of join used). Furthermore, rules are independent of the particular evaluation order and technique used as long as the monotonic \textit{w.r.t.} set-containment\(^1\), and least fixpoint, semantics of **DATALOG** is maintained. Lastly, the order of literals in a rule body provides no semantic meaning and most implementations, including this work, evaluate literals in a left-to-right fashion.

A naïve evaluation of Program 1 will execute \(r1\) and then repeatedly evaluate \(r2\), joining arc facts with already discovered \(tc\) facts in each iteration, until no new facts are produced – a fixpoint has been reached. This approach will inefficiently re-produce known facts in every iteration. We can instead use the well-known Semi-Naïve Evaluation (SN) [Ban86] which is efficient and produces no duplicates. Both the naïve evaluation and SN are bottom-up evaluation techniques, which start from the initial database and perform a repeated application of the rules until a fixpoint is reached.

Algorithm 1 is SN. \(M\) is the initial model (database), \(S\) contains all facts obtained thus far, \(\delta S\) and \(\delta S'\) contain facts obtained during the previous and current iteration, respectively, and \(T_E\) and \(T_R\) are the Immediate Consequence Operator (ICO) for the exit rule(s) and the recursive rule(s), respectively. The algorithm evaluates as follows. Firstly, \(T_E\) (i.e. the exit rules) is applied to \(M\) to derive the first set of new \(\delta\) facts \(\delta S\) (line 2). Then, until no new facts are derived during an iteration, \(T_R\) is evaluated on \(\delta S\) to derive new facts to be used in the next iteration. The new set of \(\delta\) facts (\(\delta S'\)) is produced only after the removal of facts found in previous steps (line 5). Then, \(\delta S'\) is merged into \(S\) (line 6) and becomes the set of facts used in the next iteration (line 7).

Next, we walk through an application of SN using Program 1 as our target. To enable SN,\(^1\) With set-containment monotonicity, evaluation only grows a predicate’s set of facts.
Algorithm 1 Semi-Naïve Evaluation

1: \( S := M; \)
2: \( \delta S := T_E(M); \)
3: \( S := S \cup \delta S; \)
4: \( \textbf{do} \)
5: \( \delta S' := T_R(\delta S) - S; \)
6: \( S := S \cup \delta S'; \)
7: \( \delta S := \delta S'; \)
8: \( \textbf{while} (\delta S \neq \emptyset) \)
9: \( \textbf{return} S; \)

Algorithm 2 Semi-Naïve Evaluation of Program 1

1: \( \delta \text{tc} := \text{arc}(X, Y) \)
2: \( \text{tc} := \delta \text{tc} \)
3: \( \textbf{do} \)
4: \( \delta \text{tc}' := \pi_{X,Y}(\delta \text{tc}(X, Z) \land \text{arc}(Z, Y)) - \text{tc} \)
5: \( \text{tc} := \text{tc} \cup \delta \text{tc}' \)
6: \( \delta \text{tc} := \delta \text{tc}' \)
7: \( \textbf{while} (\delta \text{tc} \neq \emptyset) \)
8: \( \textbf{return} \text{tc} \)

Algorithm 2 is the SN framework applied to Program 1. Note that the rules have been converted to a relational operator form (lines 1,4). In Algorithm 2 \( \text{tc} \) is the set of all facts produced for the recursive predicate and \( \delta \text{tc} (\delta \text{tc}') \) is the set of facts produced for \( \text{tc} \) during the previous (current) iteration. The exit rules are evaluated first. The facts of \( \text{arc} \) become the initial set of facts for
both $\delta tc$ (line 1) and $tc$ (line 2). Then, SN iterates until a fixpoint is reached (line 7). Each iteration begins by joining $\delta tc$ with arc and projecting $X,Y$ terms to produce candidate $tc$ facts (line 4). These facts are then set-differenced with $tc$ to eliminate duplicates and produce $\delta tc'$ (line 4), which is unioned into $tc$ (line 5) and becomes $\delta tc$ (line 6) for the next iteration.
CHAPTER 3

The Deductive Application Language System

In this dissertation we describe our contributions toward the next generation of Datalog systems demonstrated by the Deductive Application Language (DeAL) and the DeAL System\(^1\) (DeALS) we have developed at UCLA. Unlike other recent systems that target DATALOG at a specific domain [SGL13, EF10], DeALS has been designed as a general system seeking to satisfy many needs. In fact, DeALS builds off the lessons learned in the course of a long experience with logic-based data languages, and the LDL [CGK90] and LDL++ [AOT03] experiences in particular, and is based on this earlier Deductive Database technology [AOT03]. Specifically, DeAL is a DATALOG language that supports recursion, non-monotonic aggregates, negation and several advanced language constructs. These include user-defined aggregates, which enabled important knowledge discovery applications [GMT04], XY-stratified DATALOG [ZAO93], a form of local stratification that supports non-monotonic aggregation in recursion, choice [GZG92], which is backed by formal stable-model semantics and was found quite useful in program analysis [Hel10] and monotonic aggregates that can be used in recursive rules, which are the topic of Chapter 4.

In addition to a rich set of constructs, DeALS was also designed to support a roster of optimization techniques including magic sets, supplementary magic sets and existential quantification. With support for many powerful constructs, a wide range of complex applications can be declaratively expressed and efficiently executed in our system. DeAL’s support for graph queries was demonstrated at VLDB [SZZ13].

\(^1\)http://wis.cs.ucla.edu/deals
3.1 System Overview

In this section, after we briefly discuss the configurations for running DeALS, we review the type system and present a high level overview of the system architecture.

Using DeALS. DeALS can be run as a standalone process and service requests from the DeALS driver, which is an external API for DeALS. Using the driver, programmers can build client-server DeALS applications, and in fact, this is how the DeALS command line client is implemented. DeALS can also be used as a library and embedded within an application. With this option, the DeALS system API is programmed directly against and DeALS runs within the application’s process. DeALS is implemented in Java and uses several third party libraries including ANTLR [ant15], Log4J [log15] and JUnit [jun15].

Type System. DeALS supports a rich type system consisting of the following types: byte, \{2, 4, 8, 16, 32\} byte integers, double precision floating point decimal, string, list, complex (i.e., composite objects), and datetime. As a Datalog system, fast equality comparisons are essential and therefore variable-length types are encoded as a fixed-length value (e.g., as integer). For example, strings are variable-length and are dictionary encoded as integers. During compilation, data types for variables will be inferred starting from the definitions of the base relations up through the program rules.

Next, we discuss the three main components of the system – the compiler, the interpreter, and the storage manager.

Compiler. Given a database declaration, which consists of schema definitions for base relation(s), a program and a query form, the DeAL compiler performs the following steps: First, the database declaration and program rules are parsed and the base relation definitions are loaded into the storage manager. An example database declaration for Program 1 is database\({\{\text{arc}(X: \text{integer}, Y: \text{integer})}\}}. Second, rules with aggregates are expanded and rewritten into two rules, which is necessary to support the head aggregate syntax used by DeAL. An example of this w.r.t. monotonic aggregate rules can be found in Section 4.12. Next, the programs rules are analyzed and
grouped by predicate. Then, rules are formed into a Predicate Connection Graph (PCG), which is a type of AND/OR tree where OR nodes represent predicate occurrences in rule bodies and AND nodes represent rule heads [AOT03]. The PCG is analyzed and a binding passing analysis will be conducted to determine if any optimizations such as magic set rewriting (e.g., if the query form was presented with bound arguments) can be applied. Lastly, the Program Generator will use the PCG to produce an execution object graph of DeAL interpreter framework objects, which we refer to as an Interpreter Program (IP). IP will be evaluated to produce the result set for the program.

**Interpreter.** The interpreter evaluates programs in main-memory and uses tuple-at-a-time pipelining. The interpreter is a bottom-up evaluator and evaluates nodes of IP in a left-to-right manner with intelligent backtracking [CGK90]. During evaluation, IP nodes retrieve and store facts in the storage manager.

![Interpreter Plan for Program 1](image)

Figure 3.1: Interpreter Plan for Program 1.

Figure 3.1 is the IP for Program 1 and can be understood as follows. OR nodes are rectangles and AND nodes are circles. The compiler has identified the exit rule (r1) and recursive rule (r2) and placed their subtrees under the Semi-Naïve Recursion (OR) node. The exit rule subtree retrieves facts from the arc base relation and loads them into tc’s relation via the Semi-Naïve node. The recursive rule subtree representing r2 will join the facts produced by the recursive literal and the arc base relation on Z and project facts with (X, Y) which will be added to tc’s relation.
**Storage Manager.** The *DeALS* storage manager provides support for main memory storage and indexing for predicate relations. *DeALS* supports several B+Tree data structure variants for tuple storage and indexing. A B+Tree stores fixed-size keys in internal and leaf nodes and non-key attributes in leaf nodes. Leaf nodes have pointers to their right neighbors to enable fast scanning of the tree. Through testing we determined our implementations perform best on average using a linear key search at both internal and leaf nodes with 256 bytes allocated for keys (e.g., 32 64-bit `long` keys) in each node, which results in shallow trees. *DeALS* supports B+Tree TupleStores, which store tuples in a B+Tree. *DeALS* also supports an Unordered Heap TupleStore (*UHT*) where tuples are stored as fixed-size entries in insertion order in large byte arrays. *UHT* can be given multiple indexes (e.g., B+Tree), which they remain synchronized with at all times. *UHT* enable a highwatermark approach for SN where each iteration is a contiguous range of tuples. Lastly, the interpreter uses nested loops joins and will automatically create an index and use an index nested loops join if the argument binding passing analysis identifies a bound join argument. For instance, for *r2* in Program 1, *arc* will be indexed on *Z* (first argument) which is bound by *tc*.

### 3.1.1 Declarative Optimizer

Although the Program Generator can be made to construct an optimized IP, our experience taught us that encoding the logic to optimize IP into the same procedure that construct IP can be difficult to implement and expensive to maintain. Continuing, rather than write the optimizer in Java, we instead added a declarative optimizer framework to *DeALS*. Using our framework, the logic to determine if an optimization should be applied is encoded as a *DeAL* program. Procedural “hooks” are still needed, both to trigger the evaluation of the optimizer program and to apply the necessary changes to an IP based on the output of the optimizer program. Now, after the Program Generator produces an IP, the optimizer applies a set of optimizations to produce the final IP for the interpreter to evaluate.

To enable *DeAL* programs to evaluate an IP, facts describing it are created and loaded into base relations. The schema for the optimizer base relations is displayed as Figure 3.2. Facts for base relations are produced by recursively traversing an IP.
Program 2 is an example DeAL optimizer program to determine if a recursive predicate’s node should be materialized instead of pipelined if there are multiple occurrences of the predicate in the program (i.e., the recursion will be computed multiple times otherwise).

**Program 2. DeAL Optimizer Program to Materialize Recursion Nodes**

1. `nodesToMaterialize(ID) ← predicate(ID, Name, D, 'RecursiveOrNode'),
   predicateCount(Name, 'RecursiveOrNode', N), N > 1,
   predicateDependency(ID, ChildID),
   predicate(ChildID, D + 1, 'SemiaiveNode'),
   firstOccurrence(ID, Name), hasAllFreeBindings(ID).

2. `predicateCount(Name, Type, count(Name)) ← predicate(_, Name, _, Type).

3. `firstOccurrence(ID, Name) ← predicate(ID, Name, _, _),
   ~ earlierOccurrence(ID, Name).

4. `earlierOccurrence(ID, Name) ← predicate(ID2, Name, _, _), ID2 < ID.

5. `hasAllFreeBindings(ID) ← allBindingCount(ID, N), freeBindingCount(ID, N).

6. `freeBindingCount(ID, count(ID)) ← binding(_, ID, 'f').

7. `allBindingCount(ID, count(ID)) ← binding(_, ID, _, _).

Program 2 is executed with the evaluation of the `nodesToMaterialize` predicate (r1). This predicate identifies a RecursiveOrNode, the node type that manages a recursive predicate, that has been added to the IP multiple times and has a child Semi-Naive recursion node type that performs the actual recursion (DeALS supports several types of recursive nodes). r1 only produces the first occurrence of the node, since it will be evaluated first by the interpreter, and it must not have any bound arguments. Note, in r3, ~ is the symbol used for negation in DeAL.

There are currently several optimizer programs and this list is expected to grow as new opti-
mization techniques are developed and as the system is extended to support new constructs. Examples of optimizer programs include:

- The counterpart of Program 2 - identify nodes to replace with read-only nodes that will read from the materialized node.
- Determining if a base relation is only used once and can have its secondary index removed and the relation converted to a more efficient B+tree storage structure.
- Identifying aggregate nodes to materialize because the same aggregate is used multiple times in the program.
- Identifying if a monotonic aggregate program (c.f. Chapter 4) that compares the produced aggregate value can be optimized for short-circuit evaluations.
CHAPTER 4

Optimizing Recursive Queries With Monotonic Aggregates in DeALS

4.1 Introduction

The growing demand for analytics has placed renewed focus on improving support for aggregation in recursion. Aggregates in recursive queries are essential in many important applications and are increasingly being applied in areas such as computer networking [LCG09] and social networks [SGL13]. Many significant applications require iterating over counts or probability computations, including machine learning algorithms for Markov chains and hidden Markov models, and data mining algorithms such as Apriori. Besides these new applications, we can mention a long list of traditional ones such as Bill of Materials (BOM), a.k.a. subparts explosion: this classical recursive query for DBMS requires aggregating the various parts in the part-subpart hierarchy. Finally, we have problems such as computing the shortest paths or counting the number of paths between vertices in a graph, which are now covered as foundations by most CS101 textbooks.

Although aggregates were not covered in E.F. Codd’s definition of the relational calculi [Cod72], it did not take long before early versions of relational languages such as SQL included support for aggregate functions, namely count, sum, avg, min and max, along with associated constructs such as group by. However, a general extension of recursive query theory and implementation techniques to allow for aggregates proved an elusive goal, and even recent versions of SQL that provide strong support for OLAP and other advanced aggregates disallow the use of aggregates in recursion and only support queries that are stratified \(w.r.t\). to aggregates.

The desirability of extending aggregates to recursive queries was widely recognized early and
many partial solutions were proposed over the years for Datalog languages [GZ01, Kol91, CM90, MPR90, MS95, RS92, ZCF97]. The fact that, in general, aggregates are non-monotonic w.r.t. set-containment led to proposals based on non-monotonic theories, such as locally stratified programs and perfect models [ZAO93, LLM98], well-founded models [Gel92] and stable models [FPL11]. An alternative approach was proposed by Ross and Sagiv [RS92], who observed that particular aggregates, such as continuous count, are monotonic in lattices other than set-containment and thus can be used in non-stratified programs. However practical difficulties with this approach were soon pointed out, namely that determining the correct lattices by programmers and compilers would be quite difficult [Van93], and this prevented their utilization in practical query languages for a long time. Fortunately, we recently witnessed some important developments change the situation. Firstly, Hellerstein et al., after announcing a resurgence of Datalog, showed that monotonicity in special lattices can be very useful in proving formal properties such as eventual consistency [CMA12]. Secondly, we see monotonic aggregates making a strong comeback in practical query languages thanks to the results published in [MSZ13a, MSZ13b] and in [SGL13], summarized next.

The formalization of monotonic aggregates proposed in [MSZ13a, MSZ13b] preserves monotonicity w.r.t. set-containment, and it is thus conducive to simplicity and performance that follow respectively from the facts that (i) users no longer have to deal with lattices, and (ii) the query optimization techniques, such as SN and magic sets remain applicable [MSZ13a]. SociaLite [SGL13] also made an important contribution by showing that shortest path queries and other algorithms using aggregates in recursion, can be implemented very efficiently so that in many situations using DATALOG becomes preferable to that of hand-coding Big Data analytics in some procedural language.

In this chapter, we describe how we introduced powerful monotonic aggregates and their accompanying efficient evaluation techniques into DeALS. We show how we retrofitted DeALS, which already supported a rich set of constructs and optimizations, to also support these new optimization techniques for monotonic aggregates. We demonstrate how DeALS now achieves both performance and generality, and we will underscore this by comparing not only with SociaLite
but also with systems such as DLV[FPL08] and LogicBlox[GAK12] that realize different performance/generality tradeoffs.

Overview. The first of two main parts of this chapter begins with Section 4.2 which presents the syntax and semantics for the min (mmin) and max (mmax) monotonic aggregates. Section 4.3 discusses the evaluation and optimization of monotonic aggregate programs. Section 4.4 presents implementation details for mmin and mmax and the DeALs storage manager. Section 4.5 presents experimental results for Sections 4.2-4.4. The second part of this chapter begins with Section 4.6 discussing the count (mcount) and sum (msum) monotonic aggregates, followed by their implementation in Section 4.7 and experimental validation in Section 4.8. Section 4.9 presents the formal semantics on which our aggregates are based. Section 4.10 describes how DeAL programs are mapped into DatalogFS programs. Section 4.11 presents additional optimizations for programs with monotonic aggregates. Section 4.12 presents rule rewriting techniques used to support monotonic aggregates. Section 4.13 provides additional DeAL program examples. Additional related works are reviewed in Section 4.15 and we conclude in Section 4.16.

4.2 MMIN and MMAX Monotonic Aggregates

An mmin or mmax monotonic aggregate rule has the form:

\[ p(K_1, \ldots, K_m, \text{aggr}(T)) \leftarrow \text{Rule Body}. \]

In the rule head, \( K_1, \ldots, K_m \) are the zero or more group-by arguments we also refer to as \( \overline{K} \), \( \text{aggr} \in \{\text{mmax}, \text{mmin}\} \) is the monotonic aggregate, and \( T \), the aggregate term, is a variable.

The aggregate functions mmin and mmax map an input set or multiset, we will call \( G \), to an output set, we will call \( D \). Then, given \( G \), for each element \( g \in G \) mmin will put \( g \) into output set \( D \) if \( g \) is less than the least value mmin has previously computed (observed) for \( G \). Similarly, given an input set \( G \), for each element \( g \in G \) mmax will put \( g \) in output set \( D \) if \( g \) is greater than the greatest value mmax has previously computed for \( G \). The mmin and mmax aggregates are monotonic w.r.t. set-containment and can be used in recursive rules, and \( G \) should be viewed
as a set containing the union of all values for a single group (group-by key) across all iterations. These aggregates memorize the most recently computed value and thus require a single pass\(^1\) over \(G\). When viewed as a sequence, the values produced by \(\text{mmin}\) and \(\text{mmax}\) are monotonic.

### 4.2.1 Running Example

The All-Pairs Shortest Paths (APSP) program has received much attention in the literature [CM90, RS92, Gel92, GGZ91, SR91]. APSP calculates the length of the shortest path between each pair of connected vertices in a weighted directed graph.

**Program 3.** *APSP with \(\text{mmin}\)*

\[
\begin{align*}
&\text{r1.} \text{spaths}(X, Y, \text{mmin}(D)) \leftarrow \text{edge}(X, Y, D). \\
&\text{r2.} \text{spaths}(X, Y, \text{mmin}(D)) \leftarrow \text{spaths}(X, Z, D_1), \text{edge}(Z, Y, D_2), D = D_1 + D_2. \\
&\text{r3.} \text{shortestpaths}(X, Y, \text{min}(D)) \leftarrow \text{spaths}(X, Y, D).
\end{align*}
\]

Program 3 is the *DeAL* APSP program with the \(\text{mmin}\) aggregate. The edge predicate denotes the edges of the graph. The intuition for this program is as follows. In the recursion \((r1, r2)\), an \text{spaths} fact will be derived if a path from \(X\) to \(Y\) is either i) new or ii) has length shorter than the currently known length from \(X\) to \(Y\). \(r1\) finds the shortest path for each edge. \(r2\) is the left-linear recursive rule that computes new shortest paths for \text{spaths} by extending previously derived paths in \text{spaths} with an edge. Logically, this approach can result in many facts \text{spaths} for \(X, Y\), each with a different length. Therefore, the program is stratified using a traditional (non-monotonic) \(\text{min}\) aggregate \((r3)\) to select the shortest path for each \(X, Y\).

**APSP By Example.** Next, we walk through an evaluation using SN for Program 3.

\[
\begin{align*}
&\text{edge}(a, b, 1). \text{edge}(a, c, 3). \text{edge}(a, d, 4). \text{edge}(b, c, 1). \text{edge}(b, d, 4). \text{edge}(c, d, 1).
\end{align*}
\]

**Figure 4.1:** edge Facts for Program 3.

First, \(r1\) in Program 3, the exit rule, is evaluated on the edge facts in Figure 4.1. In the rule head in \(r1\), \(X\) and \(Y\), the non-aggregate arguments, are the group-by arguments. The \(\text{mmin}\) aggregate is

\(^1\)SQL 2003 max, min, count and sum aggregates on the unlimited preceding window are similar to *DeAL*’s monotonic aggregates.
applied to each of the six edge facts and six spaths facts are successfully derived (not displayed to conserve space) because no aggregate values had been previously computed (memorized) and each group (i.e. \((a, b)\)) was represented among the facts only once. For the spaths predicate, \(m_{\text{min}}\) is now initialized with a value for each group.

\[
\text{spaths}(a, c, 2) \leftarrow \text{spaths}(a, b, 1), \text{edge}(b, c, 1), 2 = 1 + 1.
\]

\[
\text{FAIL} \leftarrow \text{spaths}(a, b, 1), \text{edge}(b, d, 4), 5 = 1 + 4. \quad [i]
\]

\[
\text{FAIL} \leftarrow \text{spaths}(a, c, 3), \text{edge}(c, d, 1), 4 = 3 + 1. \quad [ii]
\]

\[
\text{spaths}(b, d, 2) \leftarrow \text{spaths}(b, c, 1), \text{edge}(c, d, 1), 2 = 1 + 1.
\]

**Figure 4.2: Derivations of Program 3, \(r2\) - Iteration 1.**

SN evaluates the recursive \(r2\) rule in Program 3 using the six spaths facts derived by \(r1\). Figure 4.2 displays four derivations attempted by \(r2\) in its first iteration. Derivations not displayed failed to join spaths and edge facts. The first attempt results in a new spaths fact because \(\text{spaths}(a, c, 2)\) has an aggregate value less than the previous value for \((a, c)\), which was 3 (from \(r1\)). The failures denoted [i] and [ii] occurred because the facts to be derived would have aggregate values not less than the previous value for \((a, d)\), which is 4. Finally, \(\text{spaths}(b, d, 2)\) is derived (2 < 4 for \((b, d)\)).

\[
\text{spaths}(a, d, 3) \leftarrow \text{spaths}(a, c, 2), \text{edge}(c, d, 1), 3 = 2 + 1.
\]

**Figure 4.3: Derivations of Program 3, \(r2\) - Iteration 2.**

Using the two facts derived in Figure 4.2, SN performs a second iteration using \(r2\). As displayed in Figure 4.3, \(\text{spaths}(a, d, 3)\) is derived because (3 < 4) for \((a, d)\). Now, no new facts can be derived and a fixpoint is reached.

\[
\text{shortestpaths}(a, c, 2) \leftarrow \{\text{spaths}(a, c, 3), \text{spaths}(a, c, 2)\}
\]

\[
\text{shortestpaths}(a, d, 3) \leftarrow \{\text{spaths}(a, d, 4), \text{spaths}(a, d, 3)\}
\]

\[
\text{shortestpaths}(b, d, 2) \leftarrow \{\text{spaths}(b, d, 4), \text{spaths}(b, d, 2)\}
\]

**Figure 4.4: Derivations of Program 3, \(r3\).**

20
Lastly, \( r3 \) is evaluated over the \( \text{spaths} \) facts derived during recursion and uses a stratified \( \min \) aggregate to derive only the fact with the shortest path for each group. Figure 4.4 displays derivations of \( r3 \) on groups that had multiple facts derived in recursion showing why rules like \( r3 \) are necessary with our semantics. In Section 4.4, we will discuss optimizations so rules such as \( r3 \) do not have to be evaluated.

### 4.3 Monotonic Aggregate Evaluation

In this section, we present optimized evaluation techniques for programs with monotonic aggregates.

SN (Chapter 2) can be used to evaluate DeAL programs with monotonic aggregates. Symbolic differentiation rules [ZCF97] are applied to monotonic aggregate rules in a straightforward manner to produce rules for SN. However, even though SN efficiently evaluates general Datalog programs, monotonic aggregate programs can be evaluated with even greater efficiency than SN provides. The \( \text{max-based optimization} \) [MSZ13b] identified that counting only needs to be performed on maximum (\( \max \)) values if only monotonic arithmetic and boolean functions are used. In this work, we expand this observation which we refer to as the \textit{Monotonic Optimization}. The intuition behind the \textit{Monotonic Optimization} is that with our monotonic aggregates, monotonicity is preserved and values other than the \( \max \) (\( \text{mmax} \)) or \( \min \) (\( \text{mmin} \)) will add no new results and thus can be ignored. Only the \( \max \) (\( \min \)) intermediate values need to be used in derivations to produce the final \( \max \) (\( \min \)) value. In fact, the last fact produced by the aggregate for a group contains the greatest (\( \text{mmax} \)) or least (\( \text{mmin} \)) aggregate value, making this fact the only fact for the group that we need to produce for the next iteration.

#### 4.3.1 Monotonic Aggregate Semi-Naïve Evaluation

The Monotonic Optimization enables an optimized SN for monotonic aggregates we call \textit{Monotonic Aggregate Semi-Naïve Evaluation} (MASN).
Algorithm 3 Monotonic Aggregate Semi-Naïve Evaluation (MASN)

1: $S := M$;
2: $\delta S := \text{getLast}(T_E(M))$;
3: $S := S \cup \delta S$;
4: do
5: $\delta S' := \text{getLast}(T_R(\delta S)) - S$;
6: $S := S \cup \delta S'$;
7: $\delta S := \delta S'$;
8: while ($\delta S \neq \emptyset$)
9: return $S$;

Algorithm 3 is MASN, which closely resembles SN. The main difference from SN is that for MASN we use $\text{getLast}(\cdot)^2$ to produce, from the input set, a set containing (i) all facts from predicates that do not have monotonic aggregates and (ii) the last derived fact for each group from monotonic aggregate predicates. Now after the $T_E$ or $T_R$ produces a set of facts, $\text{getLast}$ will be applied to produce the actual new $\delta S'$.

4.3.2 Eager Monotonic Aggregate Semi-naive Evaluation

MASN employs a level-by-level iteration boundary of a breadth-first search (BFS) algorithm where $\delta$ facts derived during the current iteration will be held for use until the next iteration. However, facts produced from monotonic aggregate rules can be used immediately upon derivation. Looking at the derivations in the walk-through evaluation of APSP in Section 4.2.1 one can see a case where SN, and in this case MASN as it would have evaluated the same as SN, did not capitalize on this property of monotonic aggregates.

Figure 4.5 shows the derivations of interest extracted from Figure 4.2. We see the second derivation performed using $\text{spaths}(a, c, 3)$ (from $\delta S$) and resulting in failure because the value for $(a, d)$ was 4. However, at the time the derivation is attempted, $\text{spaths}(a, c, 2)$, the result of the

---

2\textit{DeALS} supports MASN by maintaining a single fact per group in $\delta S'$.
Program 3 r2 evaluation with SN or MASN

| spaths(a, c, 2) ← spaths(a, b, 1), edge(b, c, 1), 2 = 1 + 1. |
| FAIL ← spaths(a, c, 3), edge(c, d, 1), 4 = 3 + 1. |

Figure 4.5: Example of Iteration Boundary of MASN.

immediately previous derivation, existed. Had spaths(a, c, 2) been used, spaths(a, d, 3) would have been derived here, instead of requiring another iteration (Figure 4.3).

To further capitalize on the Monotonic Optimization, we propose Eager Monotonic Aggregate Semi-Naïve Evaluation (EMSN). With EMSN, facts produced from monotonic aggregate rules are immediately available to be used in derivations. EMSN evaluates recursive rules with monotonic aggregates in a fact-oriented (fact-at-a-time) manner and the facts to use in an iteration are determined by the set of groups (keys) that had aggregate facts derived during the previous iteration. With EMSN, derivations with monotonic aggregates are always performed with the current aggregate value for the group.

Algorithm 4 is EMSN. In EMSN, recursive rules with monotonic aggregates are evaluated fact-at-a-time and all other recursive rules are evaluated using SN. Rules are partitioned into two sets, each with its own ICOs – $T_{EA}$ and $T_{RA}$ are the ICO for the monotonic aggregate exit and recursive rules, respectively, and $T_{EN}$ and $T_{RN}$ are the ICO for the remaining exit and recursive rules, respectively. $T_{RA}$ will be applied on one fact at a time. $\delta S_A$ and $\delta S'_A$ are the sets of facts obtained during the previous and current iteration, respectively, for the monotonic aggregate rules, while $\delta S$ and $\delta S'$ are the sets of facts obtained during the previous and current iteration, respectively, for the remaining rules. $\delta S_{A,keys}$ is the set of keys for the aggregate groups that had facts derived during the previous iteration. $M$ is the initial model, $S$ contains all facts obtained thus far. newfact is a fact derived from a single application of $T_{RA}$.

Important points of Figure 4 are as follows. We use $getKeys()$ to project out the aggregate value from the aggregate facts to produce the set of facts representing groups (keys) to use in derivations in the next iteration. For example, $getKeys\{spaths(a, b, 1)\}$ would produce $\{spaths(a, b)\}$. $getKeys()$ is applied to the set produced by $T_{EA}(M)$ to produce the initial
Algorithm 4 Eager Monotonic Aggregate Semi-Naïve Evaluation (EMSN)

1: \( S := M; \)
2: \( \delta S_A := TE_A(M); \)
3: \( \delta S_{A_{Keys}} := getKeys(\delta S_A); \)
4: \( \delta S := TE_N(M); \)
5: \( S := S \cup \delta S \cup \delta S_A; \)
6: do
7: for all \( key \in \delta S_{A_{Keys}} \) do
8: \( \text{while} (newfact := TR_A(getFact(key))) \) do
9: \( \delta S'_A := \delta S'_A \cup \{newfact\}; \)
10: \( \delta S' := TR_N(\delta S) - S; \)
11: \( S := S \cup \delta S' \cup \delta S'_A; \)
12: \( \delta S_{A_{Keys}} := getKeys(\delta S'_A); \)
13: \( \delta S := \delta S'; \)
14: \( \delta S'_A := \emptyset; \)
15: \( \text{while} (\delta S \neq \emptyset \text{ and } \delta S_{A_{Keys}} \neq \emptyset) \)
16: return \( S; \)

\( \delta S_{A_{Keys}} \) (line 3). \( TE_N \) (remaining rules) is applied to \( M \) to produce the initial \( \delta S \) (line 4). Once in the recursion, individually, each key in \( \delta S_{A_{Keys}} \) is used to retrieve its group’s current fact from the aggregate relation \( (getFact(key)) \), which \( TR_A \) is then applied to (line 8). Successful derivations result in \( newfact \) being added to \( \delta S'_A \) (line 9). Then, \( TR_N \) (remaining rules) is applied to \( \delta S \) and duplicates are eliminated producing \( \delta S' \), the set of non-monotonic-aggregate facts to be used in the next iteration (line 10). \( getKeys(\delta S'_A) \) produces the set of keys to be used in derivations in the next iteration (line 12). This process repeats until no new facts are produced during an iteration.

<table>
<thead>
<tr>
<th>Program 3 r2 evaluation with EMSN</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{spaths}(a, c, 2) \leftarrow \text{spaths}(a, b, 1), \text{edge}(b, c, 1), 2 = 1+1. )</td>
</tr>
<tr>
<td>( \text{spaths}(a, d, 3) \leftarrow \text{spaths}(a, c, 2), \text{edge}(c, d, 1), 3 = 2+1. )</td>
</tr>
</tbody>
</table>

Figure 4.6: EMSN Fact-at-a-time Efficiency.
Now, consider the same scenario from Figure 4.5, but this time using EMSN. As shown in Figure 4.6, now after $\text{spaths}(a, c, 2)$ is produced, it is immediately used in the next derivation resulting in $\text{spaths}(a, d, 3)$ being derived an iteration earlier than with SN or MASN. Moreover, $\text{spaths}(a, c, 3)$ will not be used in any further derivations as it would result in the derivations of facts that will not lead to a final answer now with the existence of $\text{spaths}(a, c, 2)$.

**Discussion.** With the application of the ICO for recursive monotonic aggregate rules ($T_{RA}$) on an individual fact, rather than on a set of facts, EMSN can use facts immediately upon derivation. Although EMSN is based on SN, and therefore BFS, EMSN has depth-first search (DFS) characteristics. Like BFS, EMSN still uses a level-at-a-time (iteration) approach guided by facts in $\delta S$ and $\delta S_A$ that were derived during the previous iteration. However, because EMSN uses the most recent aggregate value for the group, regardless of when the value was computed, EMSN can evaluate deeper than a single level of the search space during an iteration of evaluation. The result is higher ($m_{\text{max}}$) or lower ($m_{\text{min}}$) aggregate values being derived earlier in evaluation, which in turn prunes the search space to avoid derivation of facts that will not result in final values.

We considered an alternative approach for EMSN that instead maintains the set of aggregate facts derived during an iteration ($\delta S'_A$) where modification to the aggregate relation results in either an update or insert to $\delta S'_A$. However, with each iteration $\delta S'_A$ would become $\delta S_A$ and a new $\delta S'_A$ would be started, therefore every modification to the aggregate relation would require both $\delta S_A$ and the new $\delta S'_A$ to be searched to be updated with the new value, even if the aggregate group is not present in $\delta S_A$. Although $\delta S_A$ and $\delta S'_A$ are generally smaller than the aggregate relation, if an aggregate group has many new results during an iteration, efficiency gained from searching smaller sets instead of searching a larger aggregate relation to retrieve the aggregate value when needed (line 8 in Figure 4) would be offset by searching these sets many times. Furthermore, this requires a more complicated implementation to properly synchronize facts in multiple data structures.
4.4 MMIN and MMAX Implementation

This section contains details of our system implementation for supporting the mmin and mmax aggregates.

4.4.1 Storage Manager Extension

Early experimentation found aggregation using either i) UHT (Chapter 3) with B+Tree or Linear Hashing-based secondary indexes or ii) B+Tree TupleStores lacking in execution time performance. The B+Tree Aggregator TupleStore (B+AT) is a B+Tree TupleStore optimized for pipelined aggregation in recursive queries that provides both good read and write performance. B+AT store fixed-size keys in internal and leaf nodes and fixed-size aggregate values in leaf nodes. Keys are unique and only one aggregate value per key is maintained. Leaf nodes have pointers to their right neighbors and linear search is used in both internal and leaf nodes. In a B+AT, aggregation is performed in the leaves, therefore only one search of the tree is needed to retrieve the previous value, compare it with the new value and perform the update.

Unlike with UHT, facts in B+AT are not easily tracked by reference or range because of node splitting. Therefore, during evaluation of recursive queries with EMSN, after an aggregate value is modified, the B+AT inserts the modified entry’s key into the set of keys, which is also a B+Tree, to process for the next iteration ($\delta S_{A_{Keys}}$ in Figure 4). This approach requires two tree searches with an aggregate value modification – one B+AT search which results in a modified aggregate value and one $\delta S_{A_{Keys}}$ search to record the key. Should no modification occur, then only the B+AT is searched. To retrieve aggregate facts to use in derivations (line 8 in Figure 4), a specialized cursor scans $\delta S_{A_{Keys}}$, using each key to retrieve the key’s aggregate value from the B+AT.

Hash table approaches can be an appealing alternative to B+Trees. In Section 4.5 we present experimental results comparing DeALS with a system that utilizes a hash table approach and highlight some of key differences between using B+Trees and hash tables.

---

3Since we scan the set, we use a B+Tree which stores the keys in order and can benefit EMSN.
4.4.2 $m_{\text{min}}$ and $m_{\text{max}}$ Implementation

The $m_{\text{min}}$ and $m_{\text{max}}$ implementation tracks the least ($m_{\text{min}}$) or greatest ($m_{\text{max}}$) value computed for each group where each group has one tuple in the TupleStore. We use a single relation schema with one column for each of the predicate’s group-by argument and a column for the aggregate value. Specifically, $B+AT$ keys are the group-by arguments with the aggregate value stored in the leaf. $UHT$ are given indexes with the group-by arguments as keys. For instance, $\text{spaths}$ in Program 3 uses $B+AT$ with keys $(X, Y)$ and each $X, Y$ is stored with its current value ($D$) in a leaf node.

4.4.3 Operational Optimizations

Non-Redundant Relation Storage. Due to the Monotonic Optimization, we only need to maintain a single fact per group and when a new value for the group is successfully derived, we overwrite the previous value. If the recursive predicate and monotonic aggregate use separate stores, with EMSN and pipelining, the result is the recursive relation store is merely being synchronized with the aggregate relation store. Therefore, we do not allocate the recursive predicate a store, and instead have it read from the monotonic aggregate store.

Final Results via Monotonic Aggregate. Since the monotonic aggregate maintains the value for each group in its TupleStore, when a fixpoint is reached, its TupleStore contains the final results. For instance, instead of evaluating $r3$ in Program 3 the recursion is materialized by the system, as it would have been by the stratified aggregate, and the final values are retrieved from the monotonic aggregate’s TupleStore.

4.5 MMIN & MMAX Performance Analysis

All experiments on synthetic graphs were run on a machine with an i7-4770 CPU and 32GB memory running Ubuntu 14.04 LTS 64-bit. The experiments on real-life graphs were run on a machine with four AMD Opteron 6376 CPUs and 256GB memory running Ubuntu 12.04 LTS 64-
Memory utilization is collected by the Linux `time` command. Execution time and memory utilization are calculated by performing the same experiment five times, discarding the highest and lowest values, and taking the average of the remaining three values. All experiments on systems written in Java were run using Java 1.8.0 except for SociaLite (0.8.1) which did not support Java 1.8.0. Experiments for SociaLite were run using Java 1.7.0.

Datasets. An $n$-vertex graph used in experiments has integer vertex labels ranging from 0 to $n - 1$. We used three kinds of synthetic graphs — 1) directed acyclic graphs (DAGs), generated by connecting each pair of vertices $i$ and $j$ ($i < j$) with (edge) probability $p$; 2) random graphs, generated by connecting each pair of vertices with (edge) probability $p$; 3) scale-free graphs, generated using GTgraph\(^4\). The graphs are shuffled after generation where one random permutation is applied to the vertex labels and another random permutation is applied to the edges. The real-life graphs are not shuffled but we relabeled graphs whose vertex labels are beyond the range of $[0, n - 1]$ while

\(^4\)GTgraph, [http://www.cse.psu.edu/~madduri/software/GTgraph/](http://www.cse.psu.edu/~madduri/software/GTgraph/).
maintaining the original edge order. A text description such as “10K/20K” indicates the graph has 10,000 vertices and 20,000 edges.

**Configuration.** B+AT and B+Tree indexes for UHT were configured with 256 bytes allocated for keys in each node (internal and leaf). Other than experiments in Section 4.5.2, DeALS used EMSN with B+AT.

### 4.5.1 Datalog Implementation Comparison

*DeALS* is a sequential, interpreted, main memory Java implementation. We compare *DeALS* execution time and memory utilization performance against three Datalog implementations supporting aggregates in recursion — 1) the DLV system\(^5\), the state-of-the-art implementation of disjunctive logic programming; 2) the commercial LogicBlox system, which supports aggregates in recursion using *staged partial fixpoint* semantics\(^6\). From log files produced during execution of recursive queries with aggregates, we determined LogicBlox indeed uses an approach akin to SN, which uses only new facts found in the current iteration in derivations in the next iteration; 3) the SociaLite\(^7\) graph query language, which is compiled into code-generated Java, efficiently evaluates single-source shortest paths (SSSP) queries using an approach with Dijkstra’s algorithm-like efficiency [SGL13] and supports a left-linear recursive APSP which it evaluates using SN. We used SociaLite 0.8.1 as it had the best sequential execution time performance of SociaLite versions available to us.

**APSP on Synthetic Graphs.** Figure 4.7 shows the results of APSP on synthetic graphs with random integer edge costs between 1-100 executed with *DeALS*, SociaLite and DLV. We experimented with two versions of LogicBlox — 3.10.15 and 4.1.3. The former does not support aggregates in recursion and although we can express APSP in a stratified program, it only terminates on DAGs, and only G1(0.286s) and G2(18.328s) finish within 24 hours. The latter supports aggregates

---

\(^5\)DLV with recursive aggregates support, [http://www.dbai.tuwien.ac.at/proj/dlv/dlvRecAggr/](http://www.dbai.tuwien.ac.at/proj/dlv/dlvRecAggr/).


\(^7\)[https://sites.google.com/site/socialitelang/](https://sites.google.com/site/socialitelang/)
in recursion however, only G1(4.809s), G2(6.697m), G7(4.774h) and G13(3.977h) finish within 24 hours. We do not report LogicBlox results in Figure 4.7 nor for the remaining experiments.

Among the 18 graphs described in Figure 4.7, we found SociaLite has the fastest execution time on three graphs and DeALS has the fastest execution time on the remaining 15 graphs. DeALS is more than two times faster than SociaLite on sparse graphs where the average degree of each vertex is only two (e.g., G7, G10 and G16). This advantage decreases as the average degree increases from two to ten. The main reason for this change is due to the different designs used by DeALS and SociaLite. SociaLite uses an array of hash tables with an initial capacity of around 1,000 entries to maintain the delta relations, whereas DeALS uses a B+Tree. The initialization cost of a hash table is higher than that of a B+Tree, while the cost of accessing a hash table is lower than that of a B+Tree. For graphs with small average degree, the initialization cost may account for a large percentage of the execution time, thus DeALS is faster than SociaLite. The impact of the initialization cost reduces as the average degree increases, and thus SociaLite is faster than DeALS on denser graphs. However, this faster execution time comes at the expense of higher memory utilization. SociaLite uses more than two times the memory as DeALS on all 18 graphs. Although the C-based DLV has significantly lower memory utilization than both Java-based DeALS and SociaLite, DLV is extremely slow compared with both DeALS and SociaLite on DAGs. These results suggest that DeALS achieves the best execution time versus memory utilization trade-off on sparse graphs among the three compared systems.

APSP on Real-life Graphs. Table 4.1 shows the results of APSP on three real-life graphs from the Stanford Large Network Dataset Collection\(^8\). The provided graphs do not have edge costs, therefore we assigned unit cost to each edge. The results are similar to that of synthetic graphs — DeALS executes fastest while DLV has the lowest memory utilization. These results suggest that on real-life workloads the B+Tree-based design (low initialization cost) adopted by DeALS is more favorable than the hash table-based design used by SociaLite.

SSSP on Real-Life Graphs. Figure 4.8 shows the results of SSSP on five real-life graphs from the

Table 4.1: Execution time and memory utilization of APSP on real-life graphs

<table>
<thead>
<tr>
<th></th>
<th>HepTh</th>
<th>Gnutella</th>
<th>Slashdot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
</tr>
<tr>
<td>Time(h)</td>
<td>0.98</td>
<td>17.72</td>
<td>0.39</td>
</tr>
<tr>
<td>Mem(GB)</td>
<td>12.76</td>
<td>0.99</td>
<td>7.19</td>
</tr>
</tbody>
</table>

S1, S2, S3 represent SociaLite, DLV and DeALS respectively.

HepTh(28K/353K): High-energy physics theory citation network.
Gnutella(63K/148K): Gnutella peer-to-peer network.
Slashdot(82K/549K): Slashdot social network.

USA road networks datasets\(^9\). For each graph, we evaluate SSSP on ten randomly selected vertices, and we report the min / (geometric) average / max execution time in the form of error bars. Since execution time captured for DLV includes time for loading the graph, evaluating the query and outputting the result, and query evaluation only accounts for a small percentage of overall time observed, timing for DLV is less informative for this experiment and thus we only report results for SociaLite and DeALS. SociaLite generates a Java program that evaluates the query using the Dijkstra’s algorithm. The generated code achieves more than one order of magnitude speedup comparing to LogicBlox [SGL13]. However, our interpreted DeALS is faster than the code-generated SociaLite for SSSP on the road network graphs as shown in Figure 4.8. This result is not surprising in the sense that EMSN optimizes SN, and the Bellman-Ford algorithm (equivalent to SN) usually yields comparable performance with the Dijkstra’s algorithm on large sparse graphs.

4.5.2 DeALS Storage Manager Evaluation

This experiment shows i) how EMSN performs relative to MASN and ii) how B+AT perform relative to UHT. We evaluated APSP on synthetic graphs G1, G2, G7, G8, G13 and G14 from Figure 4.7. The (geometric) average execution time and memory utilization over the six graphs are displayed in Table 4.2. Using UHT with B+Tree indexes, EMSN has a lower average execution time than MASN by 13%. A noticeable difference in performance is observed when using B+AT vs. UHT for EMSN. B+AT is 2.6 times faster than UHT and requires only approximately 25% of memory needed by UHT.

<table>
<thead>
<tr>
<th>Evaluation Type</th>
<th>Storage Configuration</th>
<th>Time (s)</th>
<th>Memory (GB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MASN</td>
<td>UHT w/ B+Tree index</td>
<td>77.743</td>
<td>4.087</td>
</tr>
<tr>
<td>EMSN</td>
<td>UHT w/ B+Tree index</td>
<td>68.927</td>
<td>4.131</td>
</tr>
<tr>
<td>EMSN</td>
<td>B+AT</td>
<td>26.450</td>
<td>1.048</td>
</tr>
</tbody>
</table>

4.5.3 Statistical Analysis of Evaluation Methods

To provide a characterization of the relative performance of EMSN compared to SN, we perform an analysis over sets of graphs using the statistical estimation technique from [GKS91]. Statistics calculated\(^{10}\) are 1) total number of facts derived by the recursive predicate (derived facts) and 2) total aggregate size of \(\delta S\) across all iterations (\(\delta\) facts). These two statistics help to quantify the amount of computation the evaluation method must perform. For each statistic and each vertex number/edge probability combination, after each run of the program, the statistic is included in the average \(\bar{m}\) until a statistically significant number (30) of different graphs has been used AND \(\bar{m}\) is within 5% error with 95% confidence\(^{11}\).

\(^{10}\)For these statistics, we assume a normal distribution \(N(\mu, \sigma^2)\), with mean \(\mu\) and variance \(\sigma^2\).

\(^{11}\)As in [GKS91], \(\bar{m}\) is accepted when \(\epsilon < (0.05 \times \bar{m})\), where \(\epsilon = (1.96 + \sigma)/\sqrt{k}\). \(\sigma\) is standard deviation. \(k\) is the number of graphs used. 1.96, from the tables for standard normal distribution for 0.975, gives the 95% confidence coefficient.
We perform the analysis comparing APSP evaluated using EMSN to APSP evaluated using SN. We use randomly generated DAGs and random graphs with edge probability between 0.1 and 0.9 (increments of 0.1) and random integer edge cost between 1-50. EMSN and SN use the same sequence of graphs. On DAGs, SN requires 3-11% more derived and $\delta$ facts and on random graphs requires 13-18% more derived and $\delta$ facts than EMSN, respectively.

4.6 MCOUNT and MSUM Monotonic Aggregates

With efficient support for monotonic count and sum aggregates, DeALS supports many exciting applications.

An mcount or msun monotonic aggregate rule has the form:

$$p(K_1, \ldots, K_m, \text{aggr}((T, P_T))) \leftarrow \text{Rule Body}.$$ 

In the rule head, $K_1, \ldots, K_m$ are the zero or more group-by arguments ($\bar{K}$), $\text{aggr} \in \{\text{mcount}, \text{msun}\}$ is the monotonic aggregate, and $(T, P_T)$ is the aggregate term pair passed from the body where $T$ is a variable and $P_T$ is a constant or a variable indicating the partial count/sum contributed by $T$.

As with mmin and mmax, the mcount and msun aggregates are monotonic w.r.t. set-containment and can be used in recursive rules. When viewed as a sequence, the values produced by mcount and msun are monotonic. The mcount and msun aggregate functions map an input set or multiset, we will call $G$, to an output set, we will call $D$. Elements $g \in G$ have the form $(J, N_J)$, where $N_J$ indicates the partial count/sum contributed by $J$. Note, $J$ maps to $T$ and $N_J$ maps to $P_T$ in the definition of the aggregate term above. Now, given $G$, for each element $g \in G$, if $N_J > N_{J_{\text{prev}}}$, where $N_{J_{\text{prev}}}$ is the memorized previous count (sum) for $J$ or 0 if no previous count (sum) for $J$ exists, mcount (msun) computes the count (sum) for $G$ by summing the maximum partial count (sum) $N_J$ for all $J$. Since only the maximum $N_J$ for each $J$ is summed, no double counting (summing) occurs. Lastly, msun only computes with positive numbers, thereby ensuring its monotonicity.
4.6.1 Running Example

Program 4 is the DeAL program to count the paths between pairs of vertices in an acyclic graph. This program is not expressible with Datalog with stratified aggregation [MS95]. We will use Program 4 as our running example for mcount to explain monotonic counting in DeAL.

**Program 4. Counting Paths in a DAG**

\[
\begin{align*}
    r1. & \text{cpaths}(X, Y, \text{mcount}((X, 1))) \leftarrow \text{edge}(X, Y). \\
    r2. & \text{cpaths}(X, Y, \text{mcount}((Z, C))) \leftarrow \text{cpaths}(X, Z, C), \text{edge}(Z, Y). \\
    r3. & \text{countpaths}(X, Y, \text{max}(C)) \leftarrow \text{cpaths}(X, Y, C).
\end{align*}
\]

In Program 4, \(r1\) counts each edge as one path between its vertices. In \(r2\), any \text{edge}(Z, Y) that extends from a computed path count \text{cpath}(X, Z, C) establishes there are \(C\) distinct paths from \(X\) to \(Y\) through \(Z\). The \text{mcount}((Z, C)) aggregate in the head sums the count of paths from \(X\) to \(Y\) through every \(Z\) to produce the count from \(X\) to \(Y\). Lastly, \(r3\) indicates only the maximum count for each path \(X, Y\) in \text{cpaths} is desired. As explained in Section 4.4.3, \(r3\) does not have to be evaluated.

**Counting Paths By Example.** Next, we walk through an evaluation of Counting Paths in Program 4 using EMSN to further explain mcount; this explanation is easily generalizable to msum. The edge facts in Figure 4.9 are the example dataset.

First, \(r1\) in Program 4 is evaluated and results in the six \text{cpaths} derivations as shown in the

<table>
<thead>
<tr>
<th>Facts</th>
<th>(r1) Successful Derivations</th>
<th>Partial Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>edge(a, b).</td>
<td>\text{cpaths}(a, b, 1) \leftarrow \text{edge}(a, b).</td>
<td>(a, 1) for (a, b)</td>
</tr>
<tr>
<td>edge(a, c).</td>
<td>\text{cpaths}(a, c, 1) \leftarrow \text{edge}(a, c).</td>
<td>(a, 1) for (a, c)</td>
</tr>
<tr>
<td>edge(a, d).</td>
<td>\text{cpaths}(a, d, 1) \leftarrow \text{edge}(a, d).</td>
<td>(a, 1) for (a, d)*</td>
</tr>
<tr>
<td>edge(b, c).</td>
<td>\text{cpaths}(b, c, 1) \leftarrow \text{edge}(b, c).</td>
<td>(b, 1) for (b, c)</td>
</tr>
<tr>
<td>edge(b, d).</td>
<td>\text{cpaths}(b, d, 1) \leftarrow \text{edge}(b, d).</td>
<td>(b, 1) for (b, d)</td>
</tr>
<tr>
<td>edge(c, d).</td>
<td>\text{cpaths}(c, d, 1) \leftarrow \text{edge}(c, d).</td>
<td>(c, 1) for (c, d)</td>
</tr>
</tbody>
</table>

**Figure 4.9: edge Facts.** **Figure 4.10: Derivations of Program 4, \(r1\) evaluation.**
Figure 4.10. Each cpaths fact has a count of 1 indicating one path between each pair of vertices connected by edge facts. Displayed in the right column of Figure 4.10 is the memorized partial count (recall \((J, N_J)\)) for each group. For example, in the first derivation, \(J=a, N_J=1, (a, 1)\) is memorized for group \((a, b)\).

<table>
<thead>
<tr>
<th>r2 Successful Derivations</th>
<th>Partial Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>cpaths(a, c, 2) ← cpaths(a, b, 1), edge(b, c).</td>
<td>((b, 1)) for ((a, c))</td>
</tr>
<tr>
<td>cpaths(a, d, 2) ← cpaths(a, b, 1), edge(b, d).</td>
<td>((b, 1)) for ((a, d)^*)</td>
</tr>
<tr>
<td>cpaths(a, d, 4) ← cpaths(a, c, 2), edge(c, d).</td>
<td>((c, 2)) for ((a, d)^*)</td>
</tr>
<tr>
<td>cpaths(b, d, 2) ← cpaths(b, c, 1), edge(c, d).</td>
<td>((c, 1)) for ((b, d))</td>
</tr>
</tbody>
</table>

Figure 4.11: Derivations of Program 4, r2 - Iteration 1.

EMSN evaluates the recursive r2 rule from Program 4 using the cpaths derived by r1. Figure 4.11 shows the successful derivations performed by r2. As each cpaths fact is derived, it replaces the previous fact for the group (i.e. cpaths(a, c, 2) replaces cpaths(a, c, 1)). Note the derivation of cpaths(a, d, 2) from joining cpaths(a, b, 1) and edge(b, d). It represents a count of two for \((a, d)\), even though the rule body contributed only one path count. However, looking at the *-ed entry in Figure 4.10, we see a partial count of \((a, 1)\) towards \((a, d)\) was accrued during evaluation of r1. Therefore, when computing the new count for \((a, d)\), \((a, 1)\) and the newly derived \((b, 1)\) are summed to result in cpaths(a, d, 2). Next, we observe the benefits of using EMSN with the derivation of cpaths(a, d, 4). Since cpaths(a, c, 2) existed even though it was derived this iteration, it was used and successfully joined with edge(c, d). Then, the partial counts for \((a, d)\), which are \((a, 1), (b, 1),\) and \((c, 2)\), are summed to produce cpaths(a, d, 4). Finally, with no new facts produced after those in Figure 4.11, a fixpoint is reached, and since there is no need to evaluate r3 (Section 4.4.3) we have our result.
4.7 MCOUNT and MSUM Implementation

In this section, we present implementation details for the \texttt{mcount} and \texttt{msum} aggregates. We use definitions from Section 4.6 (e.g., $G$). Note, $G$ is a single group produced from the implicit \texttt{group-by} for a distinct assignment of $\bar{K}$, the zero or more \texttt{group-by} arguments. We will also refer to the TupleStore descriptions from Section 4.4.1. Lastly, although we use \texttt{mcount} to present our efficient count/sum technique, this discussion is generalizable to \texttt{msum}.

For \texttt{mcount} and \texttt{msum}, we use an approach based on delta-maintenance ($\Delta$-\textit{Maintenance}) techniques. Recalling our explanation for \texttt{mcount} in Section 4.6, given a new partial count $N_J > N_{J\text{prev}}$, \texttt{mcount} will sum all maximum partial counts to compute the new total count for $G$. However, rather than recompute the total count, we can instead use $\Delta$-\textit{Maintenance} to increase $N$ (the current total count for $G$) by $N_J - N_{J\text{prev}}$ and put the updated count, now the total current count for $G$, into output set $D$. This produces the same result as if the maximum partial count $N_J$ for all $J$ are summed to produce the total count $N$ for $G$, however avoids the re-summation of all $N_J$ with each greater $N_J$. This requires memorizing both $N$ for $G$ and $N_J$ for all $J$.

4.7.1 Storage Designs

Table 4.3 displays storage designs we investigated for \texttt{mcount} and \texttt{msum}. Here we use $N$ to indicate the current count/sum for the group-by arguments ($\bar{R}$). As in Section 4.6, each $T$ contributes a partial count/sum $P_T$ towards a distinct assignment of $\bar{R}$ (group).

<table>
<thead>
<tr>
<th>Name</th>
<th>Schema</th>
<th>Indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double</td>
<td>$\langle \bar{R}, N \rangle</td>
<td>\langle \bar{R}, T, P_T \rangle$</td>
</tr>
<tr>
<td>List</td>
<td>$\langle \bar{R}, N, List[\langle T, P_T \rangle] \rangle$</td>
<td>$\bar{R}$</td>
</tr>
<tr>
<td>B+Tree</td>
<td>$\langle \bar{R}, N, B+Tree[\langle T, P_T \rangle] \rangle$</td>
<td>$\bar{R}$</td>
</tr>
<tr>
<td>Hashtable</td>
<td>$\langle \bar{R}, N, Hashtable[\langle T, P_T \rangle] \rangle$</td>
<td>$\bar{R}$</td>
</tr>
</tbody>
</table>
Double uses two relations, one relation \((R, N)\) indexed on \(R\) to store tuples containing the group’s total aggregate value and a second relation \((R, T, P_T)\) indexed on \((R, T)\) to store the partial count \(P_T\) for each distinct assignment of \((R, T)\). Early testing showed Double using UHT without \(\Delta\)-Maintenance taking 2-5 times longer to execute than with \(\Delta\)-Maintenance.

We investigated designs using KeyValue type columns as a more efficient way of managing \((T, P_T)\) pairs. We developed three single relation designs \((R, N, KeyValue[(T, P_T)])\), where \(N\) is the total count for \(R\) and \(KeyValue[(T, P_T)]\) is a reference to the tuple’s own KeyValue-type data structure. The relation is indexed on \(R\) and each group has a single tuple. The KeyValue-types each represent a different retrieval time complexity; a List \((O(n))\) type, a B+Tree \((O(\log(n)))\) type, and a Hashtable \((O(1))\) type. Hashtable is based on Linear Hashing and stores the hashed key in the bucket to avoid rehashing. B+Tree stores keys \(T\) in internal and leaf nodes and non-key attributes \(P_T\) in leaf nodes, and uses linear search. Lastly, List stores \((T, P_T)\) pairs ordered by \(T\) and uses a linear search. These are main memory structures, so designs attempt to limit the number of objects (e.g., List uses byte arrays). KeyValue designs use \(\Delta\)-Maintenance.

For the designs shown in Table 4.3, DeALS supports List, HashTable and B+Tree with B+AT and all designs with UHT indexed as shown. For example, using \(r1, r2\) from Program 4, with B+Tree, the B+AT would have \(X, Y\) as keys and each entry in a leaf would have the current total count \(N\) and a reference to a B+Tree KeyValue-type to store \((T, P_T)\) pairs. This design is essentially a B+Tree of B+Trees.

### 4.8 MCOUNT and MSUM Performance Analysis

**Configuration.** B+Tree TupleStores and indexes, B+AT and the B+Tree KeyValue design were configured with 256 bytes allocated for keys in each node (internal and leaf). The Hashtable KeyValue design used a directory and segment size of 256, 16 initial buckets and load factor of 10.
4.8.1 Statistical Analysis of Evaluation Methods

We also perform the statistical analysis described in Section 4.5.3 comparing the evaluation of Counting Paths (Program 4) using EMSN to Counting Paths using SN. The experiment uses randomly generated DAGs of 100-250 vertices (increments of 50) and edge probability between 0.1 and 0.9 (increments of 0.1). EMSN and SN use the same sequence of graphs.

Figure 4.12a and Figure 4.12b show the results of the analysis. Each point on a line represents the ratio of SN to EMSN for number of derived facts (Figure 4.12a) or number of \( \delta \) facts (Figure 4.12b) for the size of the graph indicated by the line and edge probability indicated by the x-axis. For example, in Figure 4.12b, SN produces more than three times as many \( \delta \) facts as EMSN for graphs of 200 and 250 vertices starting at 0.2 (20%) edge probability. Figure 4.12a and Figure 4.12b show that for the test graphs, Counting Paths using SN derives 1.94 to 3.48 times more facts than EMSN. As edge probability increases, so does the ratio between SN and EMSN because the higher edge probability allows EMSN to derive and use facts earlier, which prunes the search space faster.
4.8.2 Storage Design Evaluation

This experiment tests how each of the storage designs presented in Section 4.7.1 performed on DAGs. Figure 4.13 shows the (geometric) average execution time and memory utilization, along with minimum and maximum values, on 45 random 250-vertex DAGs (5 graphs for each edge probability from 0.1 to 0.9) for each design. In Figure 4.13 results are shown in left-to-right order from worst to best average execution time performance.

<table>
<thead>
<tr>
<th>Design</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>Double (B+Tree)</td>
</tr>
<tr>
<td>D2</td>
<td>Double (UHT)</td>
</tr>
<tr>
<td>D3</td>
<td>List (UHT)</td>
</tr>
<tr>
<td>D4</td>
<td>Hashable (UHT)</td>
</tr>
<tr>
<td>D5</td>
<td>B+Tree (UHT)</td>
</tr>
<tr>
<td>D6</td>
<td>HashTable (B+AT)</td>
</tr>
<tr>
<td>D7</td>
<td>List (B+AT)</td>
</tr>
<tr>
<td>D8</td>
<td>B+Tree (B+AT)</td>
</tr>
</tbody>
</table>

![Figure 4.13: mcount and msum Storage Design Performance.]

Recall the TupleStore descriptions from Section 4.4.1 and storage designs from Section 4.7.1. D2 is the Double design as described in Table 4.3 executed using UHT with B+Tree indexes. D1 is Double using a B+Tree TupleStore for the \((\bar{R}, T, P_T)\) relation\(^{12}\). D3-D5 and D6-D8 are KeyValue designs executed using UHT and B+AT, respectively. In Figure 4.13 we see D6-D8, the three B+AT KeyValue designs, have the best execution time performance. D7 and D8 have the lowest average execution time performance with B+Tree having better maximum (51s vs. 62s) execution time. Compared with D7, D8 has slightly better memory average utilization (969MB vs. 989MB) but lower maximum memory utilization (1.4GB vs. 2GB). Note, D1 and D2 have lowest average execution time performance.

\(^{12}\)In Double, the \((\bar{R}, T, P_T)\) relation will contain many times more tuples than the \((\bar{R}, n)\) relation (still UHT), so we focused on optimizing the larger.
memory utilization but their average execution times are nearly twice that of D7 and D8. Of the designs, D8, the $B+Tree$ design using $B+AT$, best balances good average execution time performance with good average memory utilization.

4.8.3 Discussion

Finding other systems to perform an experimental comparison with $mcount$ and $msum$ proved challenging. Support in Datalog implementations for count and sum aggregates that can be used in recursion is not as mature as that of min and max aggregates. Using LogicBlox version 4, we were able to execute the Counting Paths program but experienced similar slow performance as with APSP (Section 4.5.1). Using SociaLite, we were unable to execute the Counting Paths program, and BOM queries such as subparts explosion, produced results different from ground truth. Lastly, we were able to execute a version of the Counting Paths program using DLV, but again the results were different from ground truth.

4.9 Formal Semantics

So far we have worked with the operational semantics of our monotonic aggregates and shown how this is conducive to the expression of algorithms by programmers. While most users only need to work at this level, it is important that we also show how this coincides with the formal semantics discussed in those two Datalog$^{FS}$ papers [MSZ13a, MSZ13b], inasmuch as properties such as least fixpoint and stable models will follow from it.

We start with the example inspired by [RS92] (Party Invitations) for determining who will come to a party. In the $DeAL$ Program 5, some people will come to the party for sure, whereas others only join when at least three of their friends are coming. The idea is that with cntComing each person watches the number of their friends that are coming grow, and once that number reaches three, the person will then come to the party too. To count the number of friends, rather than the final count used in [RS92], we can use the $mcount$ continuous count aggregate that enumerates all the integers until the actual maximum, i.e. it returns $I$, the actual maximum, representing the
integer interval $[1, I]$.  

**Program 5.** *Who will come to the party?*

$$r_1. \text{willcome}(X) \leftarrow \text{sure}(X).$$

$$r_2. \text{willcome}(X) \leftarrow \text{cntComing}(X, N), \, N \geq 3.$$  

$$r_3. \text{cntComing}(Y, \text{mcount}(X)) \leftarrow \text{friend}(Y, X), \, \text{willcome}(X).$$

Here the use of \text{mcount} over \text{count} is justified on the grounds of performance, since it is inefficient to count all friends of people if only three are required. More importantly though, while \text{count} is non-monotonic (unless we use the special lattices suggested by [RS92]), \text{mcount} is monotonic in the lattice of set containment used by the standard Datalog. So no ad hoc semantic extension is needed and concepts and techniques such as magic sets, perfect models and stable models can be immediately generalized to programs with \text{mcount}.

### 4.9.1 DeAL Interval Semantics

The lessons learned with \text{mcount} tell us that we can derive the monotonic counterpart of an aggregate by simply assuming that it produces an interval of integer values, rather than just one value. In the following we (i) apply this idea to max and min to obtain \text{mmax} and \text{mmin}, and then (ii) generalize these monotonic aggregates to arbitrary numbers, and show that the least fixpoint computation under this formal interval-based semantics can be implemented using the SN semantics used in Section 4.2 under general conditions that hold for all our examples. Due to space limitations the discussion is kept at an informal level: formal proofs are given in [MSZ13a, MSZ13b].

![Figure 4.14: Counting Interval Semantics.](image)

Before we introduce DeAL's interval semantics, consider the following example of interval semantics for counting, based on the Datalog$^F_S$ interval semantics, depicted in Figure 4.14. If
r2 in Program 4 produced \( \text{cpaths}(a, b, 3) \) and then \( \text{cpaths}(a, b, 4) \), we cannot sum the aggregate values to get a new count for group \( (a, b) \). Instead, with the counts for \( \text{cpaths}(a, b, 3) \) and \( \text{cpaths}(a, b, 4) \) represented by \([1, 3]\) and \([1, 4]\), respectively, \([1, 3] \cup [1, 4] = max(3, 4) = 4\). Thus \( \text{cpaths}(a, b, 4) \) represents \((a, b)\)’s count.

Figure 4.15: +/- Rational Numbers Interval Semantics.

+/- Rational Numbers. Computations on numbers in mantissa * exponent representation is tantamount to integer arithmetic on their numerators over a fixed large denominator. For instance, for floating point representations, the smallest value of exponent supported could be \(-95\), whereby every number can be viewed as the integer numerator of a rational number with denominator \(10^{95}\). However, because of the limited length of the mantissa, this numerator will often be rounded-off — a monotonic operation that preserves the existence of a least-fixpoint semantics for our programs [MSZ13a]. Thus floating-point-type representation in programming languages can be used without violating monotonicity [MSZ13a, MSZ13b].

Now, say \( Lb \) represents the lower bound for all negative numbers supported by the system architecture. We can use the interval \([Lb, N]\) to represent any number regardless of its sign. The result of unioning the sets representing these numbers is a set representing the max, independent of whether this is positive or negative. Using Figure 4.15 as an example, with \( N_1 \) and \( N_2 \) represented by \([Lb, N_1]\) and \([Lb, N_2]\), respectively, then \([Lb, N_1] \cup [Lb, N_2]\) represents the larger of the two, i.e. \( max(N_1, N_2) = N_2 \). Thus, we can support negative numbers.

Minimum. Now, say \( Ub \) represents the upper bound for all positive numbers supported by the system architecture. We can represent the set of all numbers between \( N \) and \( Ub \), i.e. the interval \([N, Ub]\), as the number \( N \). Observe that a number smaller than \( N \) is represented by an interval that contains the interval \([N, Ub]\). As before, if we take a union of two or more such representations,
the result is the largest interval. Using Figure 4.16 as an example, with $N_1$ and $N_2$ represented by 
$[N_1, Ub]$ and $[N_2, Ub]$, respectively, then $[N_1, Ub] \cup [N_2, Ub]$ represents the smaller of the two, i.e.,
$min(N_1, N_2) = N_2$.

As another example of these semantics, consider the first derivation in Figure 4.2. Here
$\text{spaths}(a, c, 2)$ was derived because the previous value for $(a, c)$ was 3. In the interval semantics,
$\text{spaths}(a, c, 2)$ would be represented as $[2, Ub]$, and 3 as $[3, Ub]$, thus we have $[2, Ub] \cup [3, Ub]$ is
$min(2, 3) = 2$.

4.9.2 Normal Programs

DeAL programs that only use monotonic arithmetic and monotonic boolean (comparison) functions on values produced by monotonic aggregates will be called normal\textsuperscript{13}. All practical algorithms we have considered only require the use of normal DeAL programs. Two classes of normal programs exist.

Class 1. This class of normal programs uses monotonic aggregates to compute the max (min) values for use outside the recursion (e.g., to return the facts with the final max (min) values). Programs in this class include Programs 3 - 19 which are expressed using a stratified max (min) aggregate to select the max (min) value produced by the recursive rules when a fixpoint is reached. Examining the intermediate results of Class 1 programs: at each step of the fixpoint iteration, we have (i) the max (min) value and (ii) values less than the max value (greater than the min value). However, we do not need (ii), as long as the values in the head are computed from those in the

\textsuperscript{13}The compiler can easily check if a program is normal when the program contains only arithmetic and simple functions (e.g. addition, multiplication).
body via an arithmetic function that is monotonic.

Class 2. Values produced by monotonic aggregates in Class 2 normal programs are not passed to rule heads, but are tested against conditions in the rule body. Here too, as long as the functions applied to the values are monotonic, rules are satisfied if and only if they are satisfied for the max (min) values. Program 5 is a Class 2 normal program.

4.9.3 Normal Program Evaluation

Recall the algorithm for MASN in Figure 3. Let us call $L$ the set produced by $T_E(M)$ or $T_R(\delta S)$ and $F$ the set produced from applying $getLast()$ to $L$. For SN, $\delta S$ will be $L$, whereas for MASN, $\delta S$ will be $F$. Let $W = L - F$. $W = \emptyset$ when, for facts of monotonic aggregate predicates, each group with facts derived during the iteration has only one fact. For an iteration, if $W = \emptyset$, MASN evaluates the program the same as SN. Otherwise, $W$ contains facts that will not lead to final answers for Class 1 normal programs and for Class 2 normal programs, any condition satisfied (in a rule body) by a fact in $W$ will also be satisfied by the fact of the same group in $F$. Thus, MASN does not need to evaluate $W$. In the next iteration, SN will derive all of the same facts as MASN, but also derive facts evaluating $W$. We have already established that these facts, because they were derived from $W$, will not lead to final answers, and thus MASN does not need to derive facts with these either.

Theorem 1. A normal program with monotonic aggregates evaluated using MASN will produce the same result as that program evaluated using SN.

Recall the algorithm for EMSN in Figure 4. Assume we are evaluating a normal program with SN. Let us call $K_{SN}$ the set of facts used in derivations ($\cup$ of all $\delta S$) by SN. Now assume we are evaluating the same normal program with EMSN. Let us call $K_{EMSN}$ the set of all facts used in derivations by EMSN, which means $K_{EMSN}$ contains facts that were retrieved from the aggregate’s relation, meaning they had the current value for the group at the time the fact was used in a derivation. Now, let $C = K_{SN} - K_{EMSN}$. If $C = \emptyset$, EMSN evaluates the program the same as SN. Otherwise, $C$ contains facts that were not used by EMSN because at the time
of derivation, the values in the aggregate’s relation for those facts’ groups were greater \((m_{\text{max}}, m_{\text{count}}, m_{\text{sum}})\) or lesser \((m_{\text{min}})\) than the aggregate value in the fact. We know \(K_{SN} \cap K_{EMSN} = K_{EMSN}\) and \(K_{EMSN} \subset K_{SN}\) because EMSN will ignore facts to use in derivations that SN will use in derivations, but EMSN will not use different facts than SN. Stated another way, SN will attempt derivations with all facts EMSN attempts derivations with. Therefore, for Class 1 normal programs, facts in \(C\) are not going to lead to final answers. For Class 2 normal programs, any condition satisfied (in a rule body) by a fact in \(C\) would have also been satisfied by the value that was instead used. EMSN does not need to evaluate \(C\). We have:

**Theorem 2.** A normal program with monotonic aggregates evaluated using EMSN will produce the same result as that program evaluated using SN.

### 4.10 Mapping DeAL to Datalog\(^{FS}\)

In this section, we show how DeAL programs can be mapped into Datalog\(^{FS}\) programs. Before presenting the transformation rules, we review Datalog\(^{FS}\) syntax and relevant language constructs.

#### 4.10.1 Datalog\(^{FS}\)

Here we provide additional background on Datalog\(^{FS}\) syntax and constructs.

**Syntax** Like a Datalog program, a Datalog\(^{FS}\) program is a set of rules. Rule heads can either be an atom or an \(FS-\text{assert}\) statement. An \(FS-\text{assert}\) statement describes multi-occurring predicates and is an atom followed by : \(K\), where \(K\) is either a constant or a variable. Body literals can be atoms, negated atoms or \(FS\) Goals. \(FS\) Goals can be either:

- **Running-\(FS\) Goal** with the form \(K : [b\text{-expression}]\)
- **Final-\(FS\) Goal** with the form \(K = ![b\text{-expression}]\)

A \(b\text{-expression}\) is shorthand for bracket expression, which is either an individual or a conjunction of positive atoms. \(K :\), the **Running-\(FS\) term**, is either a constant or a variable not contained
in the $b$-expression. This syntax allows for very concise program expression, especially when the program would otherwise require specifying a large number of conjunctions. For a Running-FS Goal, the head of the rule must belong to a stratum that is not lower than that of every predicate in the $b$-expression. For a Final-FS Goal, the head of the rule must belong to a stratum that is strictly higher than that of every predicate in the $b$-expression.

**Running-FS Goals** We review the Running-FS Goal construct using Program 6, the Datalog$^F_S$ version of the DeAL program in Program 5 (also Program 8 from [MSZ13b]). This program finds friends who will go to a party if at least three of their friends will go. The predicate $\text{friend}(Y, X)$ denotes person $Y$ views person $X$ as a friend. The predicate $\text{sure}(X)$ denotes $X$ will go to the party.

**Program 6.** *Who will come to the party? in Datalog$^F_S$*

\[
\begin{align*}
\text{r1. willcome}(X) & \leftarrow \text{sure}(X), \\
\text{r2. willcome}(Y) & \leftarrow 3 : [\text{friend}(Y, X), \text{willcome}(X)].
\end{align*}
\]

In $r2$, the goal is a Running-FS Goal, with the Running-FS term of $3 :$ and the $b$-expression of $\text{friend}(Y, X), \text{willcome}(X)$). The two kinds of variables in Datalog$^F_S$ are present in $r2$. $Y$, a *Global* variable, appears in the head of the rule (outside of the $b$-expression) and has rule-level scope. *Local* variable $X$, appears only within the $b$-expression, also its scope. In $r2$, with the Running-FS term of $3 :$, we view $X$ as an existential variable enforcing the constraint that there must exist at least three distinct occurrences of $\text{friend}(Y, X), \text{willcome}(X)$.

To ensure the proper semantics of a program with a Running-FS Goal, *at least* $K$ is the correct comparison. Because Running-FS Goals are monotonic and therefore, only an increasing number of new instances of distinct variable assignments will be found, using *equals* $K$ or *at most* $K$ could result in incorrect or unexpected program behavior. Final-FS Goals allow for the use of an equality comparison.

**Final-FS Goals** A Final-FS Goal, with the form $K \neq ![b$-expression$,]$ is used to find the total count of satisfying predicates from $b$-expression. This construct does not increase expressive power, but allows the programmer to express queries in a compact manner.

**Program 7.** *Counting Paths in a DAG in Datalog$^F_S$*
In Program 7, the Datalog$^F_S$ version of the DeAL program in Program 4, (also Program 17 in [MSZ13b]), K = ![path(X,Y)] finds exactly the number K for each distinct variable assignments of X, Y that make ![path(X,Y)] true.

At this point, the reader has sufficient background on the syntax and language constructs of Datalog$^F_S$ so we can introduce the transformation rules to map DeAL programs into Datalog$^F_S$ programs. Note, because Datalog$^F_S$ is based on monotonic w.r.t. set-containment semantics, we are only concerned with mapping DeAL rules with $m_{\text{max}}$ and $m_{\text{count}}$ into Datalog$^F_S$ programs. We refer the reader to [MSZ13b] for more details on Datalog$^F_S$.

4.10.2 Transformation Rules

Monotonic aggregates are found in three types of DeAL program rules. If a monotonic aggregate is an argument in a rule head or an MA-Value is used in a rule goal, we say the head and/or the body is $F_S$, otherwise, it is Normal. By typing DeAL rules as such, we can identify which transformation rule to apply. Table 4.4 contains the rule types.

<table>
<thead>
<tr>
<th>Rule Head</th>
<th>Rule Body</th>
<th>Rule Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS</td>
<td>Normal</td>
<td>Normal-to-FS</td>
</tr>
<tr>
<td>FS</td>
<td>FS</td>
<td>FS-to-FS</td>
</tr>
<tr>
<td>Normal</td>
<td>FS</td>
<td>FS-to-Normal</td>
</tr>
</tbody>
</table>

Table 4.4: Types of DeAL Rules with Monotonic Aggregates

We have identified five transformation rules. Before we delve into the details of each rule, the following are some general rules of thumb. When transforming rule heads, monotonic aggregates are removed and corresponding $F_S$-assert statements are added. When transforming rule bodies, first MA-Values are removed, then the goal is added to the $b$-expression of a Running or Final-FS
Goal. The variable removed from the goal will be assigned from the FS goal. Normal Datalog goals are left unchanged.

Our transformation rules are presented with rule bodies with singular goals. Cases for rule bodies with multiple goals can be easily extrapolated.

**TR-N2FS.1 - Transformation Rule Normal-to-FS #1**
In rule type Normal-to-FS with an mmax aggregate in the DeAL rule head, the mmax aggregates are removed and the head is made into an FS-assert statement.

```
head(Y_1, ..., Y_m, mmax(N))
```

to

```
head(Y_1, ..., Y_m) : N
```

**TR-N2FS.2 - Transformation Rule Normal-to-FS #2**
Rule type Normal-to-FS with an mcount aggregate in the head requires removing the mcount aggregates from the head, making the head into an FS-assert statement, and adding a Running-FS Goal to the body to generate the multi-occurring predicate. The goals required to maintain the semantics of the original rule are added to the Running-FS Goal's b-expression. The Running-FS term variable is the same as the variable in the FS-assert statement.

```
head(Y_1, ..., Y_m, mcount(X))
```

**where X is an argument Y_1, ..., Y_m from the body**

```
to

head(Y_1, ..., Y_m) : 1
```

However, if X is not a variable in the head, i.e. not a grouping variable, then body is treated as FS and TR-FS2FS is applied under the case for mcount((Y_k, X)). This ensures all eligible distinct assignments within the body will get counted and properly returned from the rule.
**TR-FS2FS - Transformation Rule FS-to-FS**

In rule type **FS-to-FS**, for the rule head, we do the same as in **TR-N2FS**, but the variable is renamed.

\[
\text{head}(Y_1, \ldots, Y_m, \text{mmax}(X))
\]

or

\[
\text{head}(Y_1, \ldots, Y_m, \text{mcount}((Y_k, X)))
\]

*where \( Y_k \) is an argument from the body*

\[ \text{to} \]

\[
\text{head}(Y_1, \ldots, Y_m) : K
\]

For the rule body, for goals with an \( \text{MA-Value} \), the argument is first removed from the goal, then the goal is added to the \textit{b-expression} of the Running-FS Goal. Lastly, the variable from the \textit{FS-assert} statement is set as the Running-FS term.

\[
\text{goal}(Y_1, \ldots, Y_m, \text{N})
\]

*where \( \text{N} \) is an MA-Value*

\[ \text{to} \]

\[
K : [\text{goal}(Y_1, \ldots, Y_m)]
\]

**TR-FS2N.1 - Transformation Rule FS-to-Normal #1**

In rule type **FS-to-Normal**, if the \textit{DeAL} rule head has a stratified \textit{max} aggregate, it will be transformed into a Datalog\(^FS\) rule head as follows:

\[
\text{head}(Y_1, \ldots, Y_m, \text{max}(\text{N}))
\]

\[ \text{to} \]

\[
\text{head}(Y_1, \ldots, Y_m, K)
\]

*where \( K \) is the result of a Final-FS Goal.*

The rule body will be transformed to have a Final-FS Goal with the goal added to the \textit{b-expression} as follows:
However, if we used the standard \( \text{goal}(Y_1, \ldots, Y_m, N) \)

\[ \text{where } N \text{ is an MA-Value} \]

to
\[ K = !\left[ \text{goal}(Y_1, \ldots, Y_m) \right] \]

**TR-FS2N.2 - Transformation Rule FS-to-Normal #2**

A second case applies to \textbf{FS-to-Normal} rules if a goal’s argument is an \textit{MA-Value} which is evaluated inside the body and not passed to the head. Then, the head is left unchanged and the rule body will be transformed from:

\[ \text{goal}(Y_1, \ldots, Y_m, N), N \geq C \]

\[ \text{where } N \text{ is an MA-Value and } C \text{ is a constant} \]

to
\[ C : \left[ \text{goal}(Y_1, \ldots, Y_m) \right] \]

**4.10.3 Transformation Rules Examples**

Using the transformation rules from Section 4.10.2, we now show step-by-step examples of \textit{DeAL} programs being transformed into Datalog\textsuperscript{FS} programs. The first example is Program 8, a Bill of Materials (BOM) example.

**Program 8. How many days until delivery?**

\[
\begin{align*}
\text{r1.} & \text{delivery(Part, mmax(Days)) } \leftarrow \text{ basic(Part, Days).} \\
\text{r2.} & \text{delivery(Part, mmax(Days)) } \leftarrow \text{ assbl(Part, Sub, ), delivery(Sub, Days).} \\
\text{r3.} & \text{actualDays(Part, max(Days)) } \leftarrow \text{ delivery(Part, Days).} 
\end{align*}
\]

\textit{r1} is a \textbf{Normal-to-FS} rule with an \texttt{mmax} aggregate therefore, **TR-N2FS.1** is applied. The monotonic aggregate in the head (\texttt{mmax(Days)}) is removed and the rule head becomes an \textit{FS-assert term} using the variable Days from the aggregate. \textit{r1} from Program 8 is converted from

\[ \text{delivery(Part, mmax(Days)) } \leftarrow \text{ basic(Part, Days).} \]

to

50
delivery(Part): Days ← basic(Part, Days).

Applying TR-FS2FS to r2, mmax(Days) is removed and Days is used in the FS-assert term. The delivery(Sub, Days) goal is added to the b-expression of a Running-FS Goal, but only after the Days argument is removed. Days is set as the Running-FS term. r2 from Program 8 is converted from

delivery(Part, mmax(Days)) ← assbl(Part, Sub, _), delivery(Sub, Days).

to

delivery(Part): Days ← assbl(Part, Sub, _), Days : [delivery(Part)].

Lastly, TR-FS2N.1 is applied to r3 because of the stratified max aggregate in the head. max(Days) is exchanged with Days in the head. Before the delivery(Part, Days) goal is added to the b-expression of a Final-FS Goal, Days is removed and is assigned the result of the Final-FS Goal.

actualDays(Part, max(Days)) ← delivery(Part, Days).

to

actualDays(Part, Days) ← Days = ![delivery(Part)].

Program 9. Program 8 transformed to DatalogFS

r1. delivery(Part): Days ← basic(Part, Days).

r2. delivery(Part): Days ← assbl(Part, Sub, _), Days : [delivery(Part)].

r3. actualDays(Part, Days) ← Days = ![delivery(Part)].

To show how TR-FS2N.2 is applied, we walk through the transformation of the DeAL Program 5, also displayed as Program 10 for the reader’s convenience. TR-FS2N.2 is needed for programs such as Program 10 that evaluate the result of an FS Goal inside the body, and do not pass it to the head, and therefore do not have an aggregate in the head requiring transformation.

Program 10. Who will come to the party?

r1. willcome(X) ← sure(X).

r2. willcome(X) ← cntComing(X, N), N ≥ 3.

r3. cntComing(Y, mcount(X)) ← friend(Y, X), willcome(X).
In Program 10, \textit{r2} is an \textbf{FS-to-Normal} rule and \textbf{TR-FS2N.2} is applied. Since \textit{N} in \texttt{cntComing(Y, N)} is an \textit{MA-Value}, \textit{N} is removed and \texttt{cntComing(Y)} is added to the \textit{b-expression} of a \textit{Running-FS Goal}. 3 is set as the \textit{Running-FS term} and the comparison predicate is removed.

\[
\text{willcome}(Y) \leftarrow \text{cntComing}(Y, N), N \geq 3.
\]

to

\[
\text{willcome}(Y) \leftarrow 3 : [\text{cntComming}(Y)].
\]

\textit{r3} is a \textbf{Normal-to-FS} rule, therefore \textbf{TR-N2FS} is applied. However, this is the special case where \textit{X} is not in the head, therefore \textbf{TR-FS2FS} is applied. \texttt{mcount\langle X\rangle} is removed from the head and an \textit{FS-assert statement} is added with variable \textit{N}. A \textit{Running-FS Goal} is added to the body and since both goals are required to count the friends coming, both are added to the \textit{b-expression}.

\[
\text{countwillcome}(Y, \text{mcount}(X)) \leftarrow \text{friend}(Y, X), \text{willcome}(X).
\]

to

\[
\text{countwillcome}(Y) : N \leftarrow N : [\text{friend}(Y, X), \text{willcome}(X)].
\]

After transforming \textit{r2} and \textit{r3}, we have an equivalent Datalog\textsuperscript{FS} program as shown in Program 11.

**Program 11.** \textit{Program 5/10 transformed to Datalog\textsuperscript{FS}}

\[
\begin{align*}
\textbf{r1.} & \quad \text{willcome}(X) \leftarrow \text{sure}(X). \\
\textbf{r2.} & \quad \text{willcome}(Y) \leftarrow 3 : [\text{cntComming}(Y)]. \\
\textbf{r3.} & \quad \text{cntComming}(Y) : N \leftarrow N : [\text{friend}(Y, X), \text{willcome}(X)].
\end{align*}
\]

**Reducing Datalog\textsuperscript{FS} rules** Although they are equivalent programs, the Program 11 has three rules, whereas the Program 6 has two rules. This is because the \textit{DeAL} program it originated from required an additional rule (\textit{r3}) to compute the aggregation. While Datalog\textsuperscript{FS} rules can perform an aggregation, capture the result of the aggregation and perform a comparison against the result in one rule, such as \textit{r2} in Program 6, \textit{DeAL}’s head aggregate syntax doesn’t allow for this. Instead, rules such as \textit{r3} in Program 10 must be written to perform the aggregation, and a second rule, such as \textit{r2} in Program 10, must receive the result and perform the comparison.
Fortunately, in Program 11 we can reduce the program to two rules without changing the meaning of the program. Because both rule bodies are a single *Running-FS Goal*, we substitute the *b-expression* of the *Running-FS Goal* in \( r_2 \) with the *b-expression* of the *Running-FS Goal* of \( r_3 \) and eliminate \( r_3 \). Finally, the resulting program is the Datalog\(^{FS} \) Program 6.

This concludes our presentation of how to map *DeAL* programs into Datalog\(^{FS} \) programs. Through syntactic transformation rules, we have shown how *DeAL* programs are indeed equivalent to Datalog\(^{FS} \) programs. Combined with the semantics of *DeAL*’s monotonic aggregates from Section 4.9, the reader should recognize that *DeAL* does indeed support Datalog\(^{FS} \).

### 4.11 Additional Optimizations

In this section, we present optimizations for programs with monotonic aggregates.

#### 4.11.1 Magic Sets

The preservation of the standard Datalog semantics via the interval semantics of our aggregates have made it possible for *DeALS* to preserve the powerful optimization techniques of Datalog, one being the Magic Sets \([BMS86]\) optimization. Furthermore, the use of head notation for monotonic aggregates simplifies the compile-time rewriting used to implement the Magic Set method. For example, with Program 8 to find out how long part p22 will require for delivery, we use the following query: \(?actualDays(p22, AD)\). After a binding passing analysis, the compiler derives the following rules from the query form and \( r_2 \) respectively:

\[
\text{m.delivery}(p22).
\]

\[
r_2m. \text{m.delivery}(Sub) \leftarrow \text{assbl}(Part, Sub), \text{m.delivery}(Part).
\]

These Magic Set rules will generate all the subparts of p22, so that the final computation of \( r_3 \), will now be filtered by an \text{m.delivery} predicate:

\[
r_1. \text{delivery}(Part, mmax(Days)) \leftarrow \text{basic}(Part, Days), \text{m.delivery}(Part).
\]

\[
r_2. \text{delivery}(Part, mmax(Days)) \leftarrow \text{assbl}(Part, Sub, ), \text{delivery}(Sub, Days), \text{m.delivery}(Part).
\]
As an additional example, using Program 4 we can find the number of paths from node7 with the query `countpaths(node7, Y, C)`. The compiler will generate the following:

\[
\begin{align*}
\text{m.cpaths(node7).} \\
\text{r3.countpaths(node7, Y, max(C)) ← cpaths(node7, Y, C).}
\end{align*}
\]

The first argument of \( r3 \) is bound to node7 and the magic predicate containing node7 is generated and added to the body of both \( r1 \) and \( r2 \) as follows:

\[
\begin{align*}
\text{r1.cpaths(X, Y, mcount(X)) ← m.cpaths(X), arc(X, Y).} \\
\text{r2.cpaths(X, Z, mcount((Y, C))) ← m.cpaths(X), cpaths(X, Y, C), arc(Y, Z).}
\end{align*}
\]

As the magic set contains only the constant node7, the original program can now be specialized as follows:

\[
\begin{align*}
\text{r1.cpaths(node7, Y, mcount(X)) ← arc(node7, Y).} \\
\text{r2.cpaths(node7, Z, mcount((Y, C))) ← cpaths(node7, Y, C), arc(Y, Z).}
\end{align*}
\]

### 4.11.2 Comparison-Only Monotonic Aggregation

In cases where an MA-Value is only evaluated inside the body of a rule, and not passed to the head, there is an opportunity for optimization. For example, Program 12 is the hindex query to find all researchers with at least 13 publications that have been cited 13 times. The \textit{refer} relation contains the citations from \( \text{PnFro} \) to \( \text{PnTo} \) and the \textit{author} relation contains the author(s) of each paper \( \text{Pno} \). \( r1 \) counts the number of citations paper number \( \text{PnTo} \) has received from other papers. \( r2 \), counts the number of papers cited at least 13 times for each author. Finally, \( r3 \) determines if an author has at least 13 papers cited 13 times.

**Program 12. Find researchers with hindex of at least 13**

\[
\begin{align*}
\text{r1.numCite(PnTo, mcount(PnFro)) ← refer(PnFro, PnTo).} \\
\text{r2.numCite13X(Auth, mcount(Pno)) ← author(Auth, Pno), numCite(Pno, N), N >= 13.} \\
\text{r3.hindex13(Auth) ← numCite13X(Auth, K), K >= 13.}
\end{align*}
\]

Notice in \( r3 \), \( K \) is not passed to the head, but only evaluated inside the body (\( K >= 13 \)). After \( K \) reaches 13 for an assignment of \( \text{Auth} \), it will stay at 13 or increase, and \( >= \) is a monotonic
boolean function such that once it is true, it will stay true forever. These stable properties ensure that no further evaluation need be performed for this author. However, the aggregate is unaware of this as it is simply executing the underlying \texttt{author} predicate, which will assign Auth to the next author only after it has exhausted all Pno for the current Auth\textsuperscript{14}. A change in the assignment of Auth must be forced, which can be achieved using rewriting as follows:

\begin{align*}
\text{r3. } & \text{hindex13(Auth) } \leftarrow \text{hindex13}\_\text{author(Auth), numCite13X(Auth, N), N } \geq 13. \\
\text{r3c. } & \text{hindex13}\_\text{author(Auth) } \leftarrow \text{author(Auth, _).}
\end{align*}

Now \texttt{hindex13}\_\texttt{author}, which is the set of all authors, is responsible for assigning Auth. And, once the predicate $N \geq 13$ in \texttt{r3} is satisfied, the next execution of \texttt{r3} will result in an assignment of Auth to the next author. The execution trade off here is that although a scan of \texttt{author} is still being performed, now by \texttt{hindex13}\_\texttt{author}, an index lookup is now used on \texttt{author} and the program only aggregates the minimum count necessary to satisfy the $N \geq 13$ condition.

We conducted an experiment of this approach using a citation network\textsuperscript{15}. After a best-effort extraction of the authors from the abstracts, we computed hindexes up to 45 on the graph. We found this technique to have better performance up until hindex 9 ($< 1\%$ of authors had 9), at which point it became more costly in terms of query execution time by on average 10\%.

4.12 Monotonic Aggregate Rule Rewriting

We utilize compiler rewriting techniques to provide support for monotonic aggregate functions in \textit{DeALS}. Our rewriting approach relies on a combination of straightforward rule rewriting techniques paired with specialized built-in predicates. After rewriting monotonic aggregate rules, no changes are required to rules invoking monotonic aggregate rules because the original signature (predicate name and arity) is left unchanged by the rewriting. All rewriting is done during compilation before program execution begins.

\textsuperscript{14}base relations are sorted
\textsuperscript{15}http://snap.stanford.edu/data/cit-HepTh.html
4.12.1 **Rewriting** \texttt{mmax} Rules

After the compiler validates the aggregate rules, the compiler rewrites an aggregate rule into two rules. We will explain our approach by applying the rewriting to \texttt{r1} from Program 8, also below for the reader’s convenience.

\[ r1.\text{delivery}(\text{Part}, \text{mmax}(\text{Days})) \leftarrow \text{basic}(\text{Part}, \text{Days}). \]

From \texttt{r1}, the compiler produces the rules in Program 13.

**Program 13. Rules produced from rewriting \texttt{r1}**

\[
\begin{align*}
\texttt{r1}_1.\text{fs\_aggr\_delivery}(\text{Part}, \text{Days}, \text{nil}) & \leftarrow \text{basic}(\text{Part}, \text{Days}). \\
\texttt{r1}_2.\text{delivery}(\text{Part}, \text{AggrVal}) & \leftarrow \\
& g1.\text{fs\_aggr\_delivery}(\text{Part}, \text{Days}, \text{OldMaxVal}), \\
& g2.\text{fs\_max}(\text{Days}, \text{OldMaxVal}, \text{NewMaxVal}), \\
& g3.\text{fs\_aggr\_delivery}(\text{Part}, \text{NewMaxVal}, \text{AggrVal}).
\end{align*}
\]

In Program 13, \texttt{r1}_1 is the Inner rule, which is given the body of the original rule. \texttt{r1}_2 is the Outer rule. The first goal of the Outer rule, \texttt{g1}, is a special built-in predicate\footnote{Built-in predicates enable rapid prototyping of different logical and physical implementations. The rule rewriting processes are insulated from changes with each new approach - especially helpful for prototyping \texttt{mcount}.}, called a Read-Aggregate predicate. For a \texttt{K} (\texttt{Part}) this predicate will 1) compute the next value to aggregate by executing the Inner rule (\texttt{r1}_1) and 2) retrieve the previous value from the aggregate’s storage, if a value exists.

A Read-Aggregate predicate is true when a new value is found.

If \texttt{g1} is true, Days will be bound to a new value and \texttt{OldMaxVal} will be bound to the previous value for the assignment of Part or to \texttt{nil} if there was no previous value. \texttt{g2}, the \texttt{fs\_max} built-in predicate, compares Days and \texttt{OldMaxVal} to determine if Days is indeed a new maximum value. If \texttt{OldMaxVal} is \texttt{nil} or Days \textgreater \texttt{OldMaxVal}, then \texttt{g2} is true and \texttt{NewMaxVal} is bound to the value assigned Days, otherwise, \texttt{g2} is false. \texttt{g3} is also a special built-in predicate called a Write-Aggregate predicate, which writes the value assigned to \texttt{NewMaxVal} to the aggregate’s storage as the value for the assignment of Part. This predicate will also bind \texttt{AggrVal} to the value assigned
to NewMaxVal\textsuperscript{17}. Invocations of the *Outer rule* return true when a new maximum value is found for the assignment of Part.

### 4.12.2 Rewriting \texttt{mmin} Rules

The rewriting of aggregate rules with \texttt{mmin} is similar to that of rules with \texttt{mmax}. Looking at Program 13, for \texttt{g2}, in place of the \texttt{mmax} built-in predicate, the \texttt{mmin} built-in predicate with the same form will be used. The \texttt{mmin} built-in predicate compares its first and second arguments and will be true when the second argument is \texttt{nil} or when the first argument is less than the second argument. When true, the third argument will be bound to the value assigned to the first argument. Variables will be named accordingly (\texttt{OldMinVal} instead of \texttt{OldMaxVal}, \texttt{NewMinVal} instead of \texttt{NewMaxVal}).

### 4.12.3 Rewriting \texttt{mcount} Rules

Aggregate rules with \texttt{mcount} are also rewritten in a way similar to rules with \texttt{mmax}. The semantics of \texttt{mcount} require producing only the maximum value, therefore \texttt{mcount} rewritten rules also utilize the \texttt{fsmax} built-in predicate. The noticeable difference in the rules produced for \texttt{mcount} is that because the \texttt{mcount} aggregate term is a pair (Section 4.6), the pair is also written into the predicate arguments in the new rules.

**Program 14. Form of aggregate rule with \texttt{mcount}**

\[
p(K_1, \ldots, K_m, \texttt{mcount}((J,N))) \leftarrow \text{Rule Body}
\]

Program 14 is the form of an aggregate rule with \texttt{mcount}. In the aggregate term \((J,N)\), \texttt{J} is an extra grouping argument. \texttt{mcount} utilizes different built-in *Read-Aggregate* and *Write-Aggregate* predicates than \texttt{mmax} and \texttt{mmin}. The \texttt{mcount Read-Aggregate} predicate retrieves the value stored for the combined \texttt{K} and \texttt{J}. The \texttt{mcount Write-Aggregate} predicate writes the new value for the combined \texttt{K} and \texttt{J} to the aggregate’s storage and then binds the \texttt{AggrVal} argument to the

\textsuperscript{17}Using \texttt{AggrVal} and \texttt{NewMaxVal} instead of having \texttt{NewMaxVal} in the head keeps \texttt{mmax} and \texttt{mcount} rule rewriting consistent.
aggregated value for only the $K$.

As a concrete example of $mcount$ rewriting, consider the rules produced by rewriting $r2$ from Program 4 (also below) in Program 15.

$$r2.\text{cpaths}(X, Z, mcount((Y, C))) \leftarrow \text{cpaths}(X, Y, C), \text{arc}(Y, Z).$$

**Program 15. Rewritten rules from $r2$ in Program 4**

$$r2_1. \text{fs\_aggr\_cpaths\_2}(X, Z, (Y, C), \text{nil}) \leftarrow \text{cpaths}(X, Y, C), \text{arc}(Y, Z).$$

$$r2_2. \text{cpaths}(X, Z, \text{AggrValue}) \leftarrow$$

$$\quad g1. \text{fs\_aggr\_cpaths\_2}(X, Z, (Y, C), \text{OldMaxVal}),$$

$$\quad g2. \text{fsmax}(C, \text{OldMaxVal}, \text{NewMaxVal}),$$

$$\quad g3. \text{fs\_aggr\_cpaths\_2}(X, Z, (Y, \text{NewMaxVal}), \text{AggrVal}).$$

There are several things to take note of in Program 15. First, the original rule $r2$ was a recursive rule, so $r2_1$ and $r2_2$ are mutually recursive. Next, the rewriting has separated $r2$ into an aggregate relation ($\text{fs\_aggr\_cpaths\_2}$) and a recursive relation ($\text{cpaths}$). Third, the complete aggregate term ($Y, C$) was copied to the head of the Inner rule and to $g1$, the Read-Aggregate predicate, which will assign these variables. The term is kept as a pair so the compiler can more easily identify it as a value instead of as a key variable. Lastly, because $r2$ is the second rule in its relation, the rule head for the Inner rule $r2_1$, as well as the goals for the Read-Aggregate ($g1$) and Write-Aggregate ($g3$) predicates, have had a .2 suffix added to ensure that Outer rules only call Inner rules created from rewriting the same original rule.

### 4.12.4 Rewriting $msum$ Rules

Aggregate rules with $msum$ are rewritten similarly to $mcount$ rules. As reviewed in Section 4.6, $msum$ is operationally the same as $mcount$ with the exception that $msum$ operates on positive numbers only. Therefore, for rules with $msum$, a predicate is added to the body to ensure only positive numbers are being aggregated, thereby maintaining $msum$'s monotonicity.
4.13 Additional DeAL Programs

This section includes additional programs to show DeAL’s expressiveness and support for a variety of applications.

Program 16. Company Control

\[
\begin{align*}
    r1. & cshares(A, B, \text{dirct}, \text{mmax}(P)) \leftarrow \text{ownedshares}(A, B, P). \\
    r2. & cshares(A, C, \text{indrc}, \text{mcount}((B, P))) \leftarrow \text{bought}(A, B), \text{cshares}(B, C, \_, P). \\
    r3. & \text{bought}(A, B) \leftarrow \text{cshares}(A, B, \_, P), P > 50, A \neq B.
\end{align*}
\]

Traditional Database Management System recursive queries are efficiently implemented using DeAL. In the Company Control program proposed in [MPR90], companies can purchase ownership shares of other companies. In addition to the shares that company A owns directly, company A also controls the shares controlled by company B when A has a controlling majority (> 50%) of B’s shares.

Program 17. What is the maximum cost of a part?

\[
\begin{align*}
    r1. & \text{cost}(\text{Part}, \text{msum}((\text{Part}, \text{Cost}))) \leftarrow \text{basic}(\text{Part}, \_, \text{Cost}). \\
    r2. & \text{cost}(\text{Part}, \text{msum}((\text{Sub}, \text{Cost}))) \leftarrow \text{assb}(\text{Part}, \text{Sub}, \text{Num}), \text{cost}(\text{Sub}, \text{Scost}), \\
        & \qquad \text{Cost} = \text{Scost} \times \text{Num}. \\
    r3. & \text{totalCost}(\text{Part}, \text{max}(\text{Cost})) \leftarrow \text{cost}(\text{Part}, \text{Cost}).
\end{align*}
\]

Program 17 is the Bill of Materials (BOM) program for finding the days required to deliver a part and the program for computing the max cost of a part from the cost of its subparts, respectively. The \text{assb} predicate denotes each part’s required subparts and number required and \text{basic} denotes the number of days for a part to be received and the part’s cost.

Program 18. Viterbi Algorithm

\[
\begin{align*}
    r1. & \text{calcV}(0, X, \text{mmax}(L)) \leftarrow s(0, EX), p(X, EX, L1), \pi(X, L2), L = L1 \times L2. \\
    r2. & \text{calcV}(T, Y, \text{mmax}(L)) \leftarrow s(T, EY), p(Y, EY, L1), T1 = T - 1, t(X, Y, L2), \\
        & \quad \text{calcV}(T1, X, L3), L = L1 \times L2 \times L3. \\
    r3. & \text{viterbi}(T, Y, \text{max}(L)) \leftarrow \text{calcV}(T, Y, L).
\end{align*}
\]
Program 18 is the Viterbi algorithm for hidden Markov models. Four base predicates are used — \( \tau \) denotes the transition probability \( L_2 \) from state \( X \) to \( Y \); \( s \) denotes the observed sequence of length \( L+1 \); \( \pi \) denotes the likelihood \( L_2 \) that \( X \) is the initial state; \( p \) denotes the likelihood \( L_1 \) that state \( X (Y) \) emitted \( E_X (E_Y) \). \( r1 \) finds the most likely initial observation for each \( X \). \( r2 \) finds the most likely transition for observation \( T \) for each \( Y \). \( r3 \) finds the max likelihood for \( T, Y \).

**Program 19. Max Probability Path**

\[
\begin{align*}
\text{r1.} & \quad \text{reach}(X, Y, \text{mmax}(P)) \leftarrow \text{net}(X, Y, P). \\
\text{r2.} & \quad \text{reach}(X, Y, \text{mmax}(P)) \leftarrow \text{reach}(X, Z, P1), \text{reach}(Z, Y, P2), P = P1 \ast P2. \\
\text{r3.} & \quad \text{maxP}(X, Y, \text{max}(P)) \leftarrow \text{reach}(X, Y, P).
\end{align*}
\]

Program 19 is the non-linear program for computing the max probability path between two nodes in a network. The \text{net} predicate denotes the probability \( P \) of reaching \( Y \) from \( X \).

Many Data Mining and Machine Learning applications use graphs as the underlying model, such as a Markov chain; and many graph analytics queries use a probability model. A Markov chain is represented by the transition matrix \( W \) of \( s \times s \) components where \( w_{ij} \) is the probability to go from state \( i \) to state \( j \) in one step. A Markov chain is called \textit{irreducible} if for each pair of states \( i, j \), the probabilities to go from \( i \) to \( j \) and from \( j \) to \( i \) in one or more steps is greater than zero.

Computing stabilized probabilities of a Markov chain has many real-world applications, such as estimating the distribution of population in a region, and determining the PageRank of web nodes.

Let \( P \) be a vector of stabilized probabilities of cardinality \( s \), the equilibrium condition in terms of matrices is: \( P = W \cdot P \). Although computing this fixpoint is far from trivial, irreducible chains can be modeled quite naturally in \textit{DeAL}. If \texttt{p.state}(\( X, K \)) denotes that \( K \) is the rank of node \( X, 1 \leq X \leq s \), and \texttt{w.matrix}(\( Y, X, W \)) denotes that there is an arc from \( Y \) to \( X \) with weight \( W \). Then, we compute the fixpoint using Program 20.

**Program 20. Markov Chains**
Note that each fixpoint of such a program is an equilibrium \( P = W \cdot P \) of the Markov Chain represented by matrix \( W \). In order to find a non trivial fixpoint (≠ 0) for program \( P \), we start with facts such as \( \text{p\_state\_init}\(1\),\(0\),\(1\))., \( \text{p\_state\_init}\(2\),\(0\),\(1\)).,... , \( \text{p\_state\_init}\(s\),\(0\),\(1\))., which provide the baseline set facts of the least fixpoint \( T_{Pbl} \). For each irreducible Markov chain there exists a non trivial fixpoint, therefore \( T_{P} \) has one that is not null at every node, and there exists a finite fixpoint for \( T_{Pbl} \). We compute this fixpoint using \( r1 \) and \( r2 \). \( r3 \) produces only the maximum probability for each node \( X \). Then rules \( r4 \) and \( r5 \) normalize the probabilities to produce the final result.

### 4.14 Syntax Comparison With Other Languages

Here we present examples showing the syntactic differences between using \( DeAL \)'s monotonic aggregates and other implemented approaches. The example programs shown compute the shortest paths on a directed graph. Recall, the \( DeAL \) program using monotonic aggregates for Shortest Paths is Program 3. In the interest of space, we omit structural declarations and rules to establish the 0 cost path between a vertex and itself. With the exception of Program 21, the programs will terminate on cyclic graphs with positive cost edge weights.

**Program 21. Stratified Shortest Paths**

\[
\begin{align*}
  r1. & \text{spaths}(X, Y, D) \leftarrow \text{edge}(X, Y, D). \\
  r2. & \text{spaths}(X, Z, D) \leftarrow \text{spaths}(X, Y, D1), \text{edge}(Y, Z, D2), D = D1 + D2. \\
  r3. & \text{shortestpaths}(X, Z, \min(D)) \leftarrow \text{spaths}(X, Z, D).
\end{align*}
\]

Program 21 is a \( DeAL \) program that first computes the transitive closure of the graph with the cost of each path (\( r1, r2 \)) and then uses a stratified \texttt{min} aggregate to find the shortest path between two vertices (\( r3 \)).
**Program 22. XY-Stratified Shortest Paths**

`r1. delta_sp(0, X, Y, C) ← edge(X, Y, C).
r2. delta_sp(J+1, X, Z, min(C)) ← delta_sp(J, X, Y, C), edge(Y, Z, C), C=C1+C2,
   if(all_sp(J, X, Z, C) then C3 > C).
r3. all_sp(J+1, X, Z, C) ← all_sp(J, X, Z, C), ¬delta_sp(J+1, X, Z, _).
r4. all_sp(J, X, Z, C) ← delta_sp(J, X, Z, C).
r5. lastsp(I) ← delta_sp(I, _, _, _), ¬delta_sp(I+1, _, _, _).
r6. shortestpaths(X, Y, C) ← lastsp(I), all_sp(I, X, Y, C).

Program 22 is the XY-Stratified DeAL program. XY-Stratified programs [AOT03, ZAO93] are executed using an iterative state machinery that processes facts across two states (J, J+1) and can therefore support a wide range of algorithms, but can suffer from inefficient performance. The intuition for Program 22 is after each iteration, a stratified min aggregate is used to select the new shortest paths (delta_sp) and the non-shortest paths from earlier iterations are discarded. all_sp represents all shortest paths found and when no new shortest paths are found, the program terminates.

In Program 22, `r1` copies all edge facts into the initial state of delta_sp. `r2` joins the paths in delta_sp with edge to find new paths shorter than paths already in all_sp\(^{18}\). `r2` also applies a stratified min aggregate to find the shortest paths between two vertices discovered at state J to bring to the next state J+1. `r3` says that paths in all_sp at state J+1 consist of paths from all_sp at state J except for new shortest paths just found (delta_sp) because these are shorter, and the longer paths already in all_sp must be deleted. The expressive power of XY-Stratified rules comes at the expense of a larger program and having to specify procedural-like rules such as `r4`, which copies facts between relations, `r5` which finds the final iteration at which a fixpoint is reached, and `r6` which retrieves the results from the final state.

**Program 23. Shortest Paths in LogiQL**

`r1. path(x, y, d) ← edge(x, y, d);
   edge(x, z, d1), spath[z, y] = d2, d = d1 + d2.
r2. spath[x, y] = dist ← agg(⟨dist = min(d)⟩) path(x, y, d).

\(^{18}\)The conditional `if (then)` predicate is the logical equivalent of two rules, one with and one without negation.
Program 23 is the LogicBlox’s LogiQL program to compute the shortest path in a directed graph. LogicBlox has recently included support for aggregation in recursion\(^{19}\). This program uses what they term a *staged partial fixpoint semantics* and syntactically exhibits a mutual recursion between \(r1\), which generates the paths, and \(r2\) which applies the \(\min\) aggregate to find the shortest path between two vertices.

**Program 24. Shortest Paths in Bloom\(^L\)**

\[
\begin{align*}
r1.\text{path} & \leq \text{edge}\{\langle e.\text{from}, e.\text{to}, \text{MinLattice.new}(e.\text{c}) \rangle \}\]
\end{align*}
\]

\[
\begin{align*}
r2.\text{path} & \leq (\text{path} * \text{edge}).\text{pairs}(\text{to} => \text{from}) \text{ do } \langle p, e \rangle \\
& \quad \{p.\text{from}, e.\text{to}, p.\text{c} + e.\text{c} \}
\end{align*}
\]

\[
\begin{align*}
r3.\text{shortestpaths} & \leq \text{path}\{\langle p \rangle \} \{p.\text{from}, p.\text{to}, p.\text{c} \}
\end{align*}
\]

Program 24 is the Bloom\(^L\) [CMA12] program for shortest paths which utilizes the MinLattice lattice type to find the minimum cost path between two vertices. \(r1\) identifies \text{edge} facts as \text{path} facts using the MinLattice to track the minimum cost for each pair of vertices. \(r2\) the generates new shortest paths by concatenating \text{edge} facts with existing \text{path} facts. Although designed for distributed programming, the semantics of the lattice types used in Bloom\(^L\) share many similarities with the semantics proposed in [RS92]. We refer the reader to [CMA12] for more details on lattice types in Bloom\(^L\).

**Discussion.** From the examples in Section 4.14, the reader should observe that Program 3, the DeAL program using monotonic aggregates, pushes the aggregation inside the recursion. This is particularly noticeable when comparing Program 3 with Example 21. The monotonic aggregate is more concise than the XY-Stratified program (Program 22). From a syntactic point of view, the LogiQL program is concise but iterates over building a set of paths and then pruning suboptimal paths from the set. This is more similar to the XY-stratified program than to the monotonic aggregate approach as the monotonic aggregates only build the set of paths and refine the costs. The

\(^{19}\)Found in LogicBlox 4.0 migration guide https://download.logicblox.com/content/docs4/release-notes/4-0-Migration/pdf/LB-MigrationGuide-40.pdf. We renamed predicate \(e\) to \text{edge} to match the other programs.
Bloom\textsuperscript{L} program is concise and the syntax requires only specifying the type of the cost argument in the base case of the program.

4.15 Additional Related Work

We have previously discussed the contributions of many works on supporting aggregation in recursion including [SGL13, RS92, FPL08]. For extrema aggregates, [GGZ91] proposes rewriting programs by pushing the aggregate into the recursion and Greedy Fixpoint evaluation to select the next min/max value to execute. Another approach proposes using aggregate selections to identify irrelevant facts that are to be discarded by an extended version of SN [SR91]. Nondeterministic constructs and stable models have been proposed to define monotone aggregates [ZW99]. The DRed [GMS93] algorithm incrementally maintains recursive views that can include negation and aggregation. The Bloom\textsuperscript{L} distributed programming language [CMA12] uses logical monotonicity via built-in and user-defined lattice types to support eventual consistency.

4.16 Monotonic Aggregates Summary

In this chapter, we have shown how monotonicity can be extended to the aggregate involved in recursive computations while preserving the syntax, semantics, and optimization techniques of traditional Datalog. The significance of this result follows from the fact that this problem had remained long unsolved, and that many new applications can be expressed with the proposed extensions that make them amenable to parallel execution on multiprocessor and distributed systems.
CHAPTER 5

BigDatalog - DeAL on Apache Spark

In this chapter, we extend DeALS for distributed evaluation to support advanced analytics over massive datasets. To support efficient distributed DATALOG evaluation, we must utilize a distributed engine designed for iteration. Towards this end, we select Apache Spark [apa15b] as our execution engine. Spark is attracting a great deal of interest as a general platform for large-scale analytics, particularly because of its support for in-memory iterative analytics. By utilizing Spark we are attempting to maximize the potential usage of DeAL and the impact of the lessons we learned.

One could expect that as a system designed for iterative applications, Apache Spark would also be well suited for recursive applications such as shortest paths computations, and link and graph structure analysis. However this ignores three decades of recursive query evaluation and optimization techniques. Spark’s support of recursion through iteration is inefficient: in an iterative Spark application, a new job is submitted for every iteration and thus the system has only limited visibility over an application’s entire execution. From a programming perspective, the development of efficient recursive applications in Spark requires the programmer to have a deep understanding of (1) the algorithm being implemented, (2) the Spark API, and (3) Spark internals. Nevertheless, Spark is a promising system for recursive applications because it provides many features essential for recursive evaluation, including dataset caching and low task startup costs. Along those lines, as we implement BigDatalog, we examine how Spark can be made to efficiently support recursive applications.

BigDatalog is a full DATALOG language implementation on Spark. BigDatalog provides logical abstraction and enables the concise and declarative expression of relational and recursive queries.
while maintaining an efficient execution. BigDatalog enables a host of declarative language optimization techniques and it exploits semantic extensions for programs with aggregation in recursion [MSZ13a, MSZ13b] and programs that can be evaluated without communication [SL91]. Furthermore, since BigDatalog supports relational algebra, aggregation and recursion, the Spark programmer can now implement complex analytics pipelines of relational, graph and machine learning tasks in a single language instead of having to stitch together programs written in different APIs of Spark subsystems such as SparkSQL [AXL15] and GraphX [GXD14].

**Motivating Example.** As an example of the performance improvement that BigDatalog achieves for the evaluation of recursive queries consider Figure 5.1. This figure shows the execution time required to compute a 100 million vertex pair transitive closure of a graph using a highly optimized handwritten Spark program versus the BigDatalog version (cluster specs. in Section 5.5). This example shows how BigDatalog is both considerably better than its host framework and also performs quite well when compared with large-scale DATALOG systems, namely Myria [WBH15] and SociaLite [SPS13] in our test environment. This order of magnitude speed-up is achieved by employing the efficient evaluation techniques and optimizations of DATALOG in Spark.

![Figure 5.1: Example Recursive Query Performance.](image)

**Contributions and Outline.** In this chapter we make the following contributions:

- We design and implement the BigDatalog compiler. We show how BigDatalog programs are compiled into recursive physical plans for Spark.
We present a parallel evaluation technique for DATALOG evaluation on Spark. We introduce recursion operators and data structures to efficiently implement the technique on Spark.

We propose physical planning and scheduler optimizations for recursive queries on Spark.

We present distributed monotonic aggregates, an accompanying evaluation technique, and data structures to support DATALOG programs with aggregates on Spark.

Outline. In Section 5.1, we review Spark. Section 5.2 introduces BigDatalog and the distributed evaluation technique used to evaluate BigDatalog programs, and shows how BigDatalog programs are compiled into physical plans for execution on Spark. Section 5.3 presents evaluation, physical plan and job scheduling optimizations. Section 5.4, describes our aggregate design and implementation. Section 5.5 presents experimental results including comparisons of BigDatalog with other large-scale DATALOG systems. Section 5.6 reviews related works. Section 5.7 presents a summary for BigDatalog and plans for future work.

5.1 Preliminaries

In this section, we review Spark and discuss the challenges of using Spark as a DATALOG runtime.

5.1.1 Apache Spark

Spark provides a language-integrated Scala API enabling the expression of programs as dataflows of transformations (e.g., map, filter) on Resilient Distributed Datasets (RDD) [ZCD12]. An RDD is a distributed shared memory abstraction representing a partitioned dataset; RDDs are immutable, and transformations are coarse-grained and thus apply to all items in the RDD to produce a new RDD. Spark executes transformations lazily: a job is submitted for execution only when actions such as count or reduce are called by the user’s program. Once a job is submitted, the scheduler groups transformations that can be pipelined (e.g., map over a join) into a single stage. The stages composing a dataflow are executed synchronously in a topological order: a stage will not be scheduled until all stages it is dependent upon have finished successfully. Between
stages, Spark *shuffles* the dataset to *repartition* it among the nodes of the cluster. When a stage can be run, the scheduler creates a set of *tasks* (i.e., execution units) consisting of one task for each input *partition*, and launches the tasks on *worker nodes*.

RDDs can be explicitly cached by the programmer in memory or on disk at workers. RDDs provide fault tolerance by recomputing the sequence of transformations for the missing partition(s). Spark has libraries for structured data processing (SparkSQL), streaming [ZDL13], machine learning (MLlib), and graph processing (GraphX).

**SparkSQL.** Spark’s structured data and relational processing module, supports a subset of SQL. SparkSQL provides logical and physical relational operators. SparkSQL physical operators use a pipelined iterator model and are implemented as functions applied over the iterator from an upstream operator. The Catalyst framework [AXL15] supports the compilation and optimization of SparkSQL programs into physical plans. In this work, we use and extend SparkSQL operators. We also propose BigDatalog operators that are implemented using the Catalyst framework so BigDatalog can use Catalyst planning features on recursive plans.

**Iterative Spark Programs.** Spark iterative applications are implemented by having a *driver program* iterate over a sequence of transformations terminated by an action. Each iteration is a new job that operates on cached RDD(s) produced by the previous iteration. Iteration terminates after a user-defined number of iterations or based on a user-defined predicate that determines when convergence has been reached. Examples of algorithms supported by this approach include PageRank, logistic regression, and the Semi-Naïve TC shown in Figure 5.2.

Figure 5.2 is explained as follows. After some initial setup including distributing the graph among nodes of the cluster (line 1) and preparing the edges of the graph for joins (line 2), the program enters a *do-while* loop. It will iterate by executing a new job for each *count* action, until an iteration produces no new results (line 9). In each iteration, the program will join facts from the previous iteration (*deltaTC*) with *arcs* (line 5), project the pair of vertices (line 6), and eliminate duplicates (line 7). The set of all previously produced pairs is then combined with the newly produced pairs (line 8). Reused RDDs are cached (lines 2, 7, 8).
Note the simplicity of the DATALOG program in Program 1 compared to the Spark program in Figure 5.2. Spark requires the programmer (1) be familiar with semi-naïve evaluation, (2) directly express a dataflow’s physical plan composed of properly ordered operations and (3) handle memory management (RDD caching) to obtain better performance. Instead, BigDatalog enables high-level specification amenable to optimizations and rescues the programmer from extensive coding, debugging and maintenance effort.

5.1.2 Challenges for Datalog on Spark

Now that we have provided background on Spark, we can discuss the three main challenges we face with implementing DATALOG in Spark.

1. Acyclic Plans: Supporting compilation, optimization and evaluation of DATALOG programs on Spark requires features not currently supported. A recursive, rule-based syntax requires a different compiler front-end than SparkSQL language queries. SparkSQL lacks recursion operators, operators are designed for acyclic use, and the Catalyst optimizer is targeted for non-recursive plans.

2. Scheduling: Spark uses a synchronous stage-based scheduler that issues tasks for a stage only
after all tasks of the previous stages have completed. For (monotonic) DATALOG programs, like the ones studied in this chapter, this can be seen as unnecessary coordination because monotonic DATALOG programs are eventually consistent [ANB11].

3. **RDD Immutability & Memory Utilization**: Each iteration of recursion will produce a new RDD representing the facts obtained thus far during evaluation of the recursive predicate. This RDD will contain both new facts and all the facts produced in earlier iterations, which are already contained in earlier RDDs. If poorly managed, recursive applications on Spark can experience memory utilization problems.

5.2 **BigDatalog**

BigDatalog is a distributed DATALOG language implementation for analytics. With BigDatalog, programs are expressed as DATALOG rules, then compiled, optimized and executed on Spark. BigDatalog will manage the persistence of datasets and make partitioning decisions. BigDatalog uses set-semantics and supports recursion, non-monotonic aggregation (\(\text{count, sum, min, max, avg}\)) and aggregation in recursion with monotonic aggregates (cf. Section 5.4).

5.2.1 **Benchmark Programs**

In this chapter, we focus on monotonic (positive) programs which include classical recursive queries from the literature as well as aggregate queries, some of which are long studied (e.g., shortest paths) and others studied more recently (connected components) [SPS13, WBH15].

**Classical Recursive Queries**

- **Transitive Closure** (TC) (Program 1)

- **Same Generation** (SG) identifies pairs of vertices where both are the same number of hops from a common ancestor.

- **Reachability** (REACH) produces all nodes connected by some path to a given source node.

**Aggregation in Recursion Queries**
• **Single-Source Shortest Paths** (SSSP) computes the length of the shortest path from a source vertex to each vertex it is connected to.

• **Connected Components** (CC) identifies connected components in the graph.

The programs selected can be separated into two categories. The first category includes programs such as TC and SG which produce large result sets, even from small graphs. For TC and SG we will use graphs that are relatively small in terms of number of vertices or edges. The second category includes programs such as REACH, CC, and SSSP that produce result sets smaller than the input graphs, and so for this category we experiment with large graphs. Next, we show how Program 1 is executed on Spark using the BigDatalog API.

### 5.2.2 BigDatalog API By Example

The program snippet shown in Figure 5.3 computes the size of the transitive closure of the graph using the BigDatalog API for Spark. In a driver program, the user first gets a `BigDatalogContext` (line 1), which wraps the `SparkContext (sc)` – the entry point for writing and executing Spark programs. The user then specifies a database schema definition for base relations and program rules (lines 2-4). Lines 3-4 implement TC from Program 1. The database definition and rules are given to the BigDatalog compiler which loads the database schema into a relation catalog (line 5). Next, the data source path (e.g., local or HDFS file path, or RDD) for the `arc` relation is provided (line 6). Then, the query to evaluate is given to the `BigDatalogContext` (line 7) which compiles it and returns an execution plan used to evaluate the query. As with other Spark programs, evaluation is lazy – the query is evaluated when `count` is executed (line 8).

### 5.2.3 Parallel Semi-Naïve Evaluation on Spark

BigDatalog programs are evaluated using a parallel version of SN we call *Parallel Semi-Naïve Evaluation* (PSN). PSN is an execution framework for a recursive predicate and it is implemented using RDD transformations. Since Spark evaluates synchronously, PSN will evaluate one iteration at a time, where an iteration will not begin until all tasks from the previous iteration have
Figure 5.3: BigDatalog Program for Spark.

The two types of rules for a recursive predicate – the exit rules and recursive rules – are compiled into separate physical plans (plans) which are then used in the PSN framework. Plans are composed of SparkSQL and BigDatalog operators. A plan produces an RDD representing the plan’s transformations. The RDD for the exit rules plan is first evaluated once. Then, each iteration until a fixpoint is reached, the recursive rules plan is used to produce a new RDD, which is evaluated to produce the facts for the iteration. This design allows BigDatalog to apply the same sequence of transformations over different RDDs each iteration without the cost of re-compilation, logical planning or physical planning. Lastly, note that as with SN, PSN will also evaluate symbolically rewritten rules (e.g., \( tc(X, Y) \leftarrow \delta tc(X, Z), \ arc(Z, Y). \)).

Algorithm 5 is the pseudocode for the PSN using RDDs and is explained as follows. The exitRulesPlan (line 1) and recursiveRulesPlan (line 5) are plans for the exit rules and recursive rules, respectively. We use toRDD() (lines 1, 5) to produce the RDD for the plan. Each iteration produces two new RDDs – an RDD for the new results produced during the iteration (delta) and an RDD for all results produced thus far for the predicate (all). The updateCatalog (lines 3, 8) stores new all and delta RDDs into a catalog for plans to access. The exit rule plan is evaluated first. The result is de-duplicated (distinct) (line 1) to produce the initial delta and all RDDs (line 2), which are used to evaluate the first iteration of the recursion. Each iteration is a new job.
Algorithm 5 PSN Framework with RDDs - Psuedocode

1: delta = exitRulesPlan.toRDD().distinct()
2: all = delta
3: updateCatalog(all, delta)
4: do
5:   delta = recursiveRulesPlan.toRDD().subtract(all).distinct()
6: all = all.union(delta)
7: updateCatalog(all, delta)
8: while (delta.count() > 0)
9:   return all

executed by count (line 9). First, the recursiveRulesPlan is evaluated using the delta RDD from the previous iteration. This will produce an RDD that is set-differenced (subtract) with the all RDD (line 6) and de-duplicated to produce a new delta RDD. With lazy evaluation, the union of all and delta (line 7) from the previous iteration is evaluated prior to its use in subtract (line 6).

We have implemented PSN to cache RDDs that will be reused, namely all and delta, but we omit this from Algorithm 5 to simplify its presentation. Lastly, in cases of mutual recursion, when two or more rules belonging to different predicates reference each other (e.g., A ← B, B ← A), one predicate will “drive” the recursion with PSN and the other recursive predicate(s) will be an operator in the driver’s recursive rules plan.

5.2.4 Compilation and Planning

The BigDatalog compiler supports the parallel, distributed bottom-up evaluation of DATALOG programs. Figure 5.4 depicts the compilation workflow for BigDatalog programs. The input for the compiler is a database schema definition, a set of rules and a query. From this, the compiler creates a logical plan for the program, which is optimized using database techniques such as projection
pruning. The logical plan for a non-recursive BigDatalog query is mapped into a SparkSQL plan and executed accordingly. However, logical plans for recursive queries are converted to BigDatalog physical plans.

Figure 5.4: BigDatalog Compilation Workflow.

5.2.4.1 Logical Plans

Here, we use Program 1 (TC) to describe how the compiler produces a logical plan. Given the query $\text{tc}(X,Y)$, the program is first compiled into a Predicate Connection Graph (PCG) (Section 3) to identify the exit rules ($r_1$) and recursive rules ($r_2$) of the $\text{tc}$ recursive predicate. From the PCG, the logical query plan is produced by mapping it into a tree of relational and recursion (e.g., SN) operators. A recursion operator has two child logical (sub)plans: one plan for the predicate’s exit rules and the other for the predicate’s recursive rules. Figure 5.5(a) is the logical plan produced by the BigDatalog compiler for Program 1. The left side of Figure 5.5(a) is the exit rules plan with only the $\text{arc}$ relation, representing $r_1$. The right side of Figure 5.5(a) is the recursive rules plan made up of relational operators to produce one iteration of $r_2$.

Program 25. Same Generation

\begin{align*}
  r_1 &. \text{sg}(X,Y) \leftarrow \text{arc}(P,X), \text{arc}(P,Y), \text{arc}(P,Y), X \neq Y. \\
  r_2 &. \text{sg}(X,Y) \leftarrow \text{arc}(A,X), \delta \text{sg}(A,B), \text{arc}(B,Y).
\end{align*}

Consider Program 25, the Same Generation (SG) program. The exit rule $r_1$ produces all $X,Y$ pairs with the same parents (i.e. siblings) and the recursive rule $r_2$ produces new $X,Y$ pairs where both $X$ and $Y$ have parents of the same generation. For PSN, $r_2$ is (symbolically) rewritten as $\text{sg}(X,Y) \leftarrow \text{arc}(A,X), \delta \text{sg}(A,B), \text{arc}(B,Y)$. The left side of Figure 5.5(b) is the exit rules plan
with a self-join of arc to find siblings. The right side of Figure 5.5(b) is the recursive rules plan which includes a three-way join of $\delta_{sg}$ and arc.

![Diagram](image)

(a) TC  (b) SG

Figure 5.5: BigDatalog Logical Plans.

5.2.4.2 Physical Plans

BigDatalog translates logical plans into physical plans comprised of SparkSQL and BigDatalog physical operators. Like SparkSQL operators, BigDatalog operators use the SparkSQL Row type. Most logical-to-physical operator mapping is straightforward, however recursion, join and shuffle operators require discussion.

**Recursion Operators.** The Recursion Operator (RO) is a special driver operator that runs on the master and executes PSN, i.e., the psuedocode from Algorithm 5. An RO has two child physical (sub)plans, the Exit Rules Plan (ERP) and the Recursive Rules Plan (RRP). A Recursive Relation operator represents a recursive predicate in the RRP and produces the recursive relation when evaluated (i.e., the plan version of a recursive predicate body literal).

**Join Operators.** BigDatalog uses binary hash join operators where one input is loaded into a lookup hashtable and tuples of the other input are streamed; the hash of the tuple’s join key attributes is used to probe the lookup table for matches. Since rules, PCG and logical plans all

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support multi-way joins, but physical join operators are binary operators, we convert a multi-way logical join operator into a hierarchy of binary physical join operators, in a left-to-right fashion.

In a linear recursion, where only one join input is a recursive relation, the non-recursive input is loaded into lookup tables and the recursive relation is streamed. For instance, from the logical plan for TC in Figure 5.5(a), the RRP will have a join where \( \delta_{tc} \) is streamed and \( \text{arc} \) is loaded into lookup tables. To help explain our approach for non-linear recursions, we use the following non-linear program. In this program, \( r_2 \) creates new \( tc \) facts of the form \((X, Y)\) by joining \( tc \) facts of the form \((X, Z)\) with \( tc \) facts of the form \((Z, Y)\).

**Program 26.** Non-Linear Transitive Closure

\[
r_1 . \text{tc}(X, Y) \leftarrow \text{arc}(X, Y).
\]

\[
r_2 . \text{tc}(X, Y) \leftarrow \text{tc}(X, Z), \text{tc}(Z, Y).
\]

In Program 26, \( r_2 \) will be (symbolically) rewritten for SN, as:

\[
\text{tc}(X, Y) \leftarrow \delta_{tc}(X, Z), \text{tc}(Z, Y).
\]

Since both inputs to the join are recursive relations, \( \delta_{tc} \) will be loaded into lookup tables and \( tc \) will be streamed. We choose this approach because loading the smaller of the two into lookup tables is less expensive and after a few iterations, \( tc \) is likely to be much larger than \( \delta_{tc} \). Program 26 performs the logarithm of the number of iterations of the linear TC Program 1.

**Shuffle Operators.** After mapping the logical operators into physical operators, the last step to produce a physical plan for execution is to add *shuffle operators* for distributed evaluation. Shuffle operators are used to repartition the dataset when there is a mismatch between an operator’s required input partitioning and a child operator’s output partitioning. For example, a shuffle operator is needed to repartition an input to a join if the input is not partitioned on the join keys. We use a Catalyst feature to analyze the physical plan and add shuffle operators where needed. We use hash partitioning and a static number of partitions throughout evaluation. Future work is to investigate dynamically adjusting the number of partitions during evaluation.

**Example Plans.** The physical plans produced for Program 1 (TC) and Program 25 (SG) are displayed in Figure 5.6(a) and 5.6(b), respectively. Using Figure 5.6(a) as our point of reference,
we explain how our operators from above are used in plans. The root of the plan is the RO for the tc recursive predicate. In the RRP, $\delta tc$ is a recursive relation and when evaluated will produce tc’s facts from the previous iteration. Both inputs to the hash join are shuffled. The subscript $Z, [N]$ indicates the partitioning key is the Z argument (from the program rule), and there will be N partitions. Here Z is the join argument so that tuples of arc and $\delta tc$ having the same key will be co-located on the same worker.

Figure 5.7 depicts the RDD lineage graph produced when evaluating the physical plan shown in Figure 5.6(a) up through two iterations of PSN. Mapping the evaluation of Figure 5.6(a) with PSN into RDDs is straightforward, however some important points are the following. The reader should notice the box with ERP (RRP) in the corner encloses the ERP (RRP) RDDs and transformations. RDDs (and transformations) not enclosed in boxes are produced by the PSN framework. We use a dashed line to indicate $tc_0$ is the same RDD as $\delta tc_0$. Evaluation begins by reading in an input file and partitioning it via `coalesce` into a user-defined number of partitions (e.g., number of cpu cores in the cluster) to create the arc RDD. Lastly, note that in addition to the shuffle operations prior to the join of arc and $\delta tc$, the `subtract` and `distinct` transformations applied by PSN also shuffle.
5.3 Optimizations

This section presents optimizations to improve the performance of BigDatalog programs. Details on the datasets used in experiments in this section can be found in Section 5.5.

5.3.1 Optimizing PSN

As shown with Algorithm 5, PSN can be implemented with RDDs and transformations such as `subtract`, `distinct` and `union`. However, using standard RDD transformations is inefficient because each iteration the results of the recursive rules are set-differenced with the entire recursive relation (line 6 in Algorithm 5), which is growing each iteration. Since an RDD stores facts in an array, expensive data structures must be created each iteration. We propose instead to use the `SetRDD`, a specialized RDD for storing distinct `Rows` and tailored for set operations needed for
PSN. Each partition of SetRDD is a single set data structure. By storing facts in a set, SetRDD efficiently supports set-difference and de-duplication operations. By caching SetRDD, the need to rebuild expensive data structures each iteration is eliminated. SetRDD methods include `union`, which unions two SetRDDs to produce a new SetRDD, and `diff`, which produces a new SetRDD containing rows from the input RDD not already in the SetRDD. Converting PSN to use SetRDD is straightforward.

```scala
class SetRDD(partitionsRDD: RDD[SetRDDPartition[Row]])
  extends RDD[Row] {
    def diff(other: RDD[Row]): SetRDD
    override def union(other: RDD[Row]): SetRDD
    ...
}
abstract class SetRDDPartition[T] {
    def diff(iter: Iterator[T]): SetRDDPartition[T]
    def union(other: SetRDDPartition[T]): SetRDDPartition[T]
    ...
}
object SetRDD {
  def apply(rdd: RDD[Row], schema: Schema): SetRDD = {
    new SetRDD(rdd.mapPartitions[SetRDDPartition[Row]] (
      iter => Iterator(SetRDDHashSetPartition(iter, schema)),
      true))
  }
}
```

Figure 5.8: SetRDD Interface.

In SetRDD, facts are efficiently stored using type-specific data structures to store primitive types sufficient to capture the schema or, in the case of larger schema, `Row` objects themselves. For instance, the recursive relation of either TC or SG will store pairs of vertices with a schema of two 4-byte integers. We combine the two 4-byte integers into one 8-byte integer (`long`) and use a

---

1. This design is inspired by GraphX’s specialized RDD for vertices (`VertexRDDImpl`) which stores multiple data structures (e.g., bitset, array) in each RDD partition.
2. This is a set union i.e., it removes duplicates.
HashSet from fastutil [fas15] optimized for long. Storing primitives saves memory and garbage collections as compared to storing more expensive wrapper types (e.g., java.lang.Long) and provides fast existential checks for de-duplication. SetRDD can be configured to also utilize specialized B+trees for int, long, and byte arrays (for larger schema). Row objects are produced from the primitive types by a schema-aware iterator.

Figure 5.8 displays the interfaces for the SetRDD and SetRDDPartition classes and Figure 5.9 displays the implementation of SetRDDHashSetPartition. Note the HashSet-typed argument in the constructor to SetRDDHashSetPartition - this is a type-specific HashSet structure and has methods size, insert(Row) and contains(Row). The design of SetRDBPlusTreePartition is similar and uses a B+Tree instead of HashSet.

**Monotonicity and RDD Immutability.** We can apply an optimization enabled by DATALOG set-containment semantics to efficiently produce a new SetRDD for the union transformation. Although an RDD is intended to be immutable, we make SetRDD mutable under the union operation. From union, a new SetRDD will be produced with partitions referencing the same HashSets as its creator. For example, in Figure 5.9 both union methods add facts to the partition’s existing HashSet. In the event that a task performing the union fails and must be re-executed, it will not lead to incorrect results because union is monotonic and facts can be added only once to a relation. This design saves system memory because only one HashSet exists per partition across all iterations.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>TC</th>
<th>SG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tree17</td>
<td>Grid150</td>
</tr>
<tr>
<td>PSN</td>
<td>244</td>
<td>OOM</td>
</tr>
<tr>
<td>PSN with SetRDD</td>
<td>41</td>
<td>134</td>
</tr>
</tbody>
</table>

Table 5.1: PSN vs. PSN with SetRDD Performance

Table 5.1 displays the results of evaluating TC and SG with both PSN and PSN with SetRDD. PSN with SetRDD outperforms PSN significantly in all cases.
class SetRDDHashSetPartition(set: HashSet, schema: Schema) 
extends SetRDDPartition[Row] {

override def union(otherPartition: SetRDDPartition[Row]):
    SetRDDHashSetPartition = {

    otherPartition match {
    case otherPartition: SetRDDHashSetPartition => {
        new SetRDDHashSetPartition(
            set.union(otherPartition.set), schema)
    }
    case other => union(otherPartition.iterator)
    }
}

override def union(iter: Iterator[Row]):
    SetRDDHashSetPartition = {

    while (iter.hasNext)
        set.insert(iter.next())

    new SetRDDHashSetPartition(set, schema)
    }

...}

Figure 5.9: SetRDDHashSetPartition Implementation.

5.3.2 Partitioning

The initial version of PSN used RDD transformations (e.g., distinct, subtract) that performed the necessary shuffling operations. That approach was sufficient to produce a correct result, but could be inefficient to evaluate. Now, SetRDD’s diff and union transformations are designed to require properly partitioned input (i.e., they will not shuffle). Therefore, none of the transformations used in PSN will repartition inputs so shuffle operators need to be placed into ERP and RRP to produce properly partitioned output for PSN transformations. This approach allows for a simplified and generalized PSN framework and brings the insertion of shuffle operators to
the workflow under the control of the BigDatalog compiler. With full control over shuffle operator placement, (i.e., communication decisions), BigDatalog can produce very efficient evaluations.

Earlier DATALOG research showed a good partitioning strategy (i.e., the arguments on which to partition) for a recursive predicate was important for efficient parallel evaluation [CW89, GST90, GST92, WO90]. In general, we seek a partitioning strategy that limits shuffling. The default partitioning strategy employed by BigDatalog is to partition the recursive predicate on the first argument. Now we can produce plans for PSN that will terminate with a shuffle operator if the output of the plan does not match the partitioning strategy of the predicate. Figure 5.10(a) is the plan for Program 1 for PSN with SetRDD. With the recursive predicate (tc) partitioned on the first argument notice how both the ERP and RRP terminate with a shuffle operator.

**User-Defined Partitioning.** In the plan in Figure 5.10(a) \( \delta tc \) requires shuffling prior to the join since it is not partitioned on the join key (Z) because the default partitioning is the first argument (X). However, if the second argument were instead made the default, the inefficiency with Figure 5.10(a) would be resolved but then other programs such as SG in Figure 5.10(b) would suffer (\( \delta sg \) would require a shuffle prior to the join). Therefore, to support programs where the default partitioning will lead to inefficient execution, BigDatalog allows the user to define a recursive
predicate’s partitioning via a configuration option. For example, by overriding the default partitioning and making \( \text{tc} \)'s second argument the partitioning strategy, the shuffle for \( \delta \text{tc} \) before the join in Figure 5.10(a) will not be inserted to the plan and the plan shown in Figure 5.11 would be produced.

![Figure 5.11: Plan for TC partitioned on 2nd argument.](image)

Table 5.2 shows the results of TC evaluated with the plan in Figure 5.10(a) versus the same plan, but using the second argument as \( \text{tc} \)'s partitioning strategy. In fact, on all graphs from Table 5.6, the plan using the second argument matched or outperformed the other.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Tree17</th>
<th>Grid250</th>
<th>G10K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Argument</td>
<td>41</td>
<td>370</td>
<td>20</td>
</tr>
<tr>
<td>2nd Argument</td>
<td>26</td>
<td>265</td>
<td>19</td>
</tr>
</tbody>
</table>

### 5.3.3 Join Optimizations for Linear Recursion

**Input Caching.** Since we use a static number of partitions and because non-recursive inputs do not change during evaluation, for a join implementing a linear recursion, the non-recursive join input can be cached. This can lead to significant performance improvements since input partitions
no longer have to be shuffled and loaded into lookup tables prior to a join each iteration. Table 5.3 shows the improved performance of caching. For TC on Tree17, shuffling and loading lookup tables for join each iteration is a significant part of the execution time, even though there are only 17 iterations. Grid250 benefits because it requires 500 iterations and now only needs to load the lookup tables once.

![Diagram of linear recursion joins](image)

(a) Shuffle Join  
(b) Broadcast Join

Figure 5.12: Linear Recursion Joins. Numbers 1 and 2 indicate partition ids of the respective dataset.

**Broadcast Joins.** Figure 5.12 depicts two approaches where the non-recursive input is a base relation and each input dataset has two partitions. For linear recursions, instead of joining partitions as described earlier and depicted in Figure 5.12(a), each partition of a recursive relation can be joined with an entire relation as depicted in Figure 5.12(b). We now refer to the former type of join as a *shuffle join* and the latter as a *broadcast join*. In both scenarios, the non-recursive input is loaded into a lookup table. For a broadcast join, the cost of loading the entire relation into a lookup table is amortized over the recursion because the lookup table is cached and then reused every iteration. However, to execute a broadcast join the relation must fit in memory. With large memories currently available, we believe broadcast joins are acceptable for many DATALOG programs.

Figure 5.13 shows an RRP for Program 25 (SG) where the three-way join from the logical plan (Figure 5.5(b)) has been converted to a two-level broadcast join. In the event that a broadcast

---

3Broadcast joins are supported with Spark’s broadcast variable infrastructure.
relation is used multiple times in a plan, as in Figure 5.13, BigDatalog will broadcast it once and all broadcast join operators joining the relation will use it.⁴

Table 5.3 shows the results of using broadcast joins compared to shuffle joins on tree and grid graphs for TC and SG. SG greatly benefits on both graphs from using broadcast joins because three shuffles are eliminated from the plan and these graphs require minimal broadcast time. However, broadcast joins proved inefficient for Tree17 for TC because the job to load the lookup table and broadcast it to workers takes half of the execution time. Nevertheless, broadcast join is the default join operator for linear recursion, but shuffle join can be selected via configuration setting.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>TC</th>
<th>SG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tree17 Grid250</td>
<td>Tree11 Grid250</td>
</tr>
<tr>
<td>Shuffle join no caching</td>
<td>26 265</td>
<td>59 107</td>
</tr>
<tr>
<td>Shuffle join caching</td>
<td>17 196</td>
<td>56 81</td>
</tr>
<tr>
<td>Broadcast join</td>
<td>53 197</td>
<td>45 54</td>
</tr>
</tbody>
</table>

⁴We extend SparkSQL’s broadcast join operator with support for sharing a broadcast relation between join operators.
5.3.4 Decomposable Programs

Previous research on parallel evaluation of DATALOG programs determined some programs are *decomposable* and thus evaluable in parallel without redundancy (a fact is only produced once) and without processor communication or synchronization [WS88]. Techniques for evaluating decomposable programs are appealing for BigDatalog because data-parallel systems like Spark can scale to utilize many machines and large numbers of cpu cores. Furthermore, mitigating the cost of synchronization and shuffling can lead to significant execution time speedup.

Program 1 (linear TC) is a decomposable program [WS88] and the following example demonstrates this. This example is based on the example in [WS88]. Using the facts from Figure 5.14, we make the evaluation of the exit rules result in two partitions. Figure 5.15 shows the derivations of the \( r1 \) exit rule. One task creates a partition only with odd numbers for the first argument (left side of Figure 5.15) and a second task creates a partition only with even numbers for the first argument (right side of Figure 5.15). In Figure 5.15 we use an expression with a modulo operator in the exit rule to identify the partition number, however BigDatalog will actually use a shuffle operator to achieve this. Now entering the recursive part of the program, the work is partitioned for the evaluation of two independent TC evaluations, as long as \( \alpha \gamma c \) is available for each evaluation. This is illustrated in Figure 5.16. One task produces the TC for vertex pairs starting at even numbered vertices, the other produces the TC for vertex pairs starting at odd numbered vertices. It should be clear how this approach can be extended to any number of tasks (i.e., use \( \text{mod} N \) where \( N \) is the number of tasks).

However, even if a program is decomposable, the system still needs to be able to produce physical plans to evaluate it as such. We consider a BigDatalog physical plan decomposable if RRP has no shuffle operators. The physical plan for Program 1 shown in Figure 5.10(a) has shuffle operators in RRP. Instead, BigDatalog can produce a decomposable physical plan for Program 1 by partitioning \( \text{tc} \) on the first argument and using a broadcast join. The partitioning strategy (first argument) divides the recursive relation so each partition can be evaluated independently and without shuffling, and the broadcast join allows each partition of the recursive relation to join with
Facts: arc(0, 2). arc(1, 2). arc(2, 3). arc(4, 2).

Figure 5.14: arc Facts for Example Decomposable Evaluation.

<table>
<thead>
<tr>
<th>r1 Derivations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partition 0 (even)</strong></td>
</tr>
<tr>
<td>tc(0, 2) ← arc(0, 2), 0 mod 2 = 0.</td>
</tr>
<tr>
<td>FAIL ← arc(1, 2), 1 mod 2 ≠ 0.</td>
</tr>
<tr>
<td>tc(2, 3) ← arc(2, 3), 2 mod 2 = 0.</td>
</tr>
<tr>
<td>tc(4, 2) ← arc(4, 2), 4 mod 2 = 0.</td>
</tr>
<tr>
<td>δtc₀ : {tc(0, 2). tc(2, 3). tc(4, 2).}</td>
</tr>
</tbody>
</table>

Figure 5.15: Example r1 Derivations for Decomposable TC Evaluation.

<table>
<thead>
<tr>
<th>r2 Successful Derivations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Even Partition</strong></td>
</tr>
<tr>
<td>1st Iteration</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2nd Iteration</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Figure 5.16: Example r2 Derivations for Decomposable TC Evaluation.

the *entire* arc base relation. Figure 5.17 is the decomposable physical plan for Program 1. Since we do not pre-partition base relations, the ERP has a shuffle operator to repartition the arc base relation into N partitions by arc’s first argument X.

Figure 5.18 depicts the RDD lineage graph produced when evaluating the physical plan shown in Figure 5.17 up through two iterations of PSN. Mapping the evaluation of Figure 5.17 with PSN into RDDs is straightforward, however there are some important points to take note of. Firstly, the shuffle in the ERP is the only shuffle in this RDD lineage graph. Secondly, the broadcast *arc*
Figure 5.17: Decomposable TC Plan.

relation is copied to each worker where it is cached and reused each iteration.

Figure 5.18: RDD Lineage Graph for Decomposable TC Physical Plan.

Table 5.4 displays the execution times using the shuffle join plan and the decomposable plan in Figure 5.17. With the exception of Tree17, the decomposable plan greatly outperforms the plan.
using shuffle joins.

Table 5.4: Comparison of Shuffle vs. Decomposable TC Plans

<table>
<thead>
<tr>
<th></th>
<th>Tree17</th>
<th>Grid250</th>
<th>G10K</th>
<th>G10K-0.01</th>
<th>G20K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shuffle</td>
<td>26</td>
<td>265</td>
<td>19</td>
<td>121</td>
<td>101</td>
</tr>
<tr>
<td>Decomposable</td>
<td>49</td>
<td>55</td>
<td>7</td>
<td>22</td>
<td>19</td>
</tr>
</tbody>
</table>

Although linear TC is decomposable, not all linear recursions are. Unary (single argument) predicates such as \texttt{reach} in Program 27 are not decomposable. The Reachability (REACH) program identifies all nodes reachable from the given source node. \texttt{r1} initializes the recursion from 0. Then, in \texttt{r2} previously computed \texttt{reach} facts are joined to \texttt{arc} to find new vertices reachable from 0.

Program 27. Reachability

\begin{align*}
\texttt{r1} &. \texttt{reach}(Y) \leftarrow Y = 0. \\
\texttt{r2} &. \texttt{reach}(Y) \leftarrow \texttt{reach}(X), \texttt{arc}(X, Y).
\end{align*}

Identifying Decomposable Programs. BigDatalog identifies decomposable programs via syntactic analysis of program rules using techniques presented in the \textit{generalized pivoting} work [SL91]. The authors of [SL91] show that the existence of a \textit{generalized pivot set} (GPS) for a program is a sufficient condition for decomposability and present techniques to identify GPS in arbitrary DATALOG programs. We have implemented the techniques described in [SL91] to determine the GPS for BigDatalog programs. When a BigDatalog program is submitted to the compiler, the compiler will apply the generalized pivoting solver to determine if the program’s recursive predicates have GPS. If they indeed do, we now have a partitioning strategy and, in conjunction with broadcast joins, we can efficiently evaluate the program with these settings. For example, Program 1 has a GPS which says to partition the \texttt{tc} predicate on its first argument.

Note that this technique is enabled by using DATALOG which allows BigDatalog to analyze the program at the logical level. The Spark API is unable to provide this type of support alone because programs are written in terms of physical operations.
5.3.5 Job Optimizations

Lineage. Since RDDs produced during an iteration are input for the next iteration, RDD lineage can grow long for recursive programs. Since lineage is inspected frequently during execution, for long running recursions we found this can result in a stack overflow. The standard solution is to checkpoint the RDD which clears the lineage after being written to disk, however a disk write is not always acceptable. To address this problem we implement a technique for cached RDDs that will clear lineage, but does not require checkpointing.\(^5\) We sacrifice some degree of fault tolerance in favor of execution time performance, although this technique can still utilize cache replication and disk backup. Otherwise, we leverage the standard fault tolerance mechanisms provided by Spark.

![Figure 5.19: Multi-Job scheduling of Program 1 (TC). ShuffleMapStages are orange. ResultStages, which produce output for a job, are gray. Job 0 is a broadcast of the arc base relation.](image)

Scheduler-Aware Recursion. With PSN as shown in Algorithm 5, the scheduler is unaware that subsequent iterations could be required and therefore is unable to optimize recursive execution. We now refer to this approach as Multi-Job PSN, which is depicted in Figure 5.19 for evaluating Program 1. To address the inefficiency with Multi-Job PSN, we investigate pushing the recursion into the scheduler so recursive queries are supported as Single-Job PSN. We extend the Spark scheduler to use a special type of stage for recursion (FixpointStage) and support a fixpoint job, which is different from normal jobs in that 1) each iteration, the scheduler evaluates a new RDD over the previous iteration’s results and 2) the scheduler will issue iterations until evaluation of the RDD results in an empty RDD indicating a fixpoint has been reached. Figure 5.20 depicts

\(^5\)A solution to this problem in Spark 1.5.0 is to use Local Checkpointing, i.e., checkpointing on the local worker.
evaluating Program 1 in this manner. Lastly, checkpointing is applied at the end of a job, so previously each iteration of Multi-Job PSN could be checkpointed. To support checkpointing an iteration in Single-Job PSN, checkpointing is also pushed into the scheduler.

Figure 5.20: Single-Job scheduling of Program 1 (TC). ShuffleMapStages are orange. ResultStages, which produce output for a job, are gray. FixpointResultStages are blue. Job 0 is a broadcast of the arc base relation.

Optimizing Single-Job PSN. With Single-Job PSN, the scheduler is now aware that multiple iterations could be required. If a program is partitioned such that it does not require shuffling in the recursion, the scheduler will not create stages with shuffle operators. When the scheduler detects this situation, it configures the stage’s tasks to iterate on workers and execute the same RDD until a fixpoint is reached. To support reusing the same RDD, the RDD partitions in the local cache from the previous iteration are overwritten with the RDD partitions produced during the current iteration. We call this Single-Job PSN Reuse. This approach eliminates the cost of scheduling and task creation for subsequent iterations. Figure 5.21 depicts this approach for Program 1 (TC) evaluated with the plan from Figure 5.17.

Table 5.5 displays results of the execution times of TC and SG using the Multi-Job PSN, Single-Job PSN and Single-Job PSN Reuse. For datasets that require many iterations, such as Grid250, the performance improvement is substantial.

Note that this scheduler optimization is used on decomposable programs. Being able to identify a decomposable program (i.e., generalized pivoting) is independent from this optimization and thus this can be used in general to evaluate a decomposable plan, not just by BigDatalog.

91
Figure 5.21: Single-Job Reuse scheduling of Program 1 (TC). ShuffleMapStages are orange. ResultStages, which produce output for a job, are gray. FixpointResultStages are blue. Job 0 is a broadcast of the arc base relation.

Table 5.5: Comparison of PSN Job Strategies

<table>
<thead>
<tr>
<th></th>
<th>TC</th>
<th></th>
<th>SG</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tree17</td>
<td>Grid250</td>
<td>Tree11</td>
<td>Grid250</td>
</tr>
<tr>
<td>Multi-Job PSN</td>
<td>51</td>
<td>111</td>
<td>53</td>
<td>75</td>
</tr>
<tr>
<td>Single-Job PSN</td>
<td>49</td>
<td>55</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>Single-Job PSN Reuse</td>
<td>45</td>
<td>26</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

5.4 Aggregates

Traditional SQL aggregates (\(\min, \max, \sum, \text{count}, \text{avg}\)) are non-monotonic and thus cannot be used in recursion. Researchers have recently proposed aggregates that are monotonic w.r.t. set containment, the same monotonicity used by standard Datalog, meaning these aggregates can be used in recursive rules and evaluated using techniques such as SN and magic sets [MSZ13a, MSZ13b]. A sequential version of these aggregates was presented in Chapter 4. In this chapter we present a distributed version of the aggregates.

BigDatalog supports the same four monotonic aggregates as \(\text{DeALS - mmin, mmax, mcount, msum}\). The declarative semantics allows the aggregates inside the recursion so long as monotonicity w.r.t. set containment is maintained. Therefore, during evaluation the monotonic aggregates can produce new higher (mmax, mcount, msum) or lower (mmin) values with each input fact and thus an outer non-monotonic aggregate (min or max) is necessary to produce only the final value.
An example of this can be seen in Program 28, the single-source shortest paths program (SSSP). Note, BigDatalog uses aggregates functions in rule heads with the non-aggregate arguments as the grouping arguments.

**Program 28. Single-Source Shortest Paths**

\[
\begin{align*}
    r1 & . \text{sssp2}(Y, \text{mmin}(D)) \leftarrow Y = 1, D = 0. \\
    r2 & . \text{sssp2}(Y, \text{mmin}(D)) \leftarrow \text{sssp2}(X, D1), \text{arc}(X, Y, D2), D = D1 + D2. \\
    r3 & . \text{sssp}(X, \text{min}(D)) \leftarrow \text{sssp2}(X, D).
\end{align*}
\]

The SSSP program computes the length of the shortest path from a source vertex to all vertices it is connected to. This program uses a \text{mmin} monotonic aggregate. Here the \text{arc} predicate in \(r2\) denotes edges of the graph \((X, Y)\) with edge cost \(D2\). \(r1\) seeds the recursion with starting vertex 1. Then, \(r2\) will recursively produce all new minimum cost paths to a node \(Y\) though node \(X\). Lastly, \(r3\) produces only the minimum cost path for each node \(X\), however in our actual implementation, we do not have to evaluate \(r3\) since at the completion of the recursion, \text{sssp2}'s relation will contain the shortest path from 1 to each vertex.

**Evaluation and Implementation.** Programs with monotonic aggregates in recursive rules are evaluated with an aggregate version of PSN we call \textit{Parallel Semi-Na"ive - Aggregate} (PSN-A). Compared with PSN, PSN-A is a simpler framework. Since new facts are only produced when a greater (\text{mmax}, \text{mcount}, \text{msum}) or lesser (\text{mmin}) value than the previous value for the (aggregate) group is produced, de-duplication is unnecessary. Furthermore, the union operation performed in PSN is unnecessary in PSN-A because new results are added to the aggregate relation during aggregate evaluation. We implement PSN-A in an aggregate version of an RO. Also, we use a specialized RDD called an \textit{AggregateSetRDD}, in which each partition is a key/value map where each entry represents a unique group and its current value. We use B+trees as the data structures for the maps and cache the AggregateRDD to avoid the expense of reloading the maps each iteration. Additionally, since the aggregates functions are monotonic, as with SetRDD’s \texttt{union} operation, AggregateSetRDD is mutable under aggregate evaluation. Furthermore, an AggregateSetRDD will reference the same maps as its creator. Should a task fail during evaluation, any changes to the
RDD partition will not result in incorrect results since a value can only be updated if it is higher \((m_{\text{max}}, m_{\text{count}}, m_{\text{sum}})\) or lower \((m_{\text{min}})\) than previously computed values.

For efficient distributed evaluation, monotonic aggregates require a partitioning strategy such that a group exists in only one partition. If a group is allowed in multiple partitions, a task could produce a value even though a better value exists. If the input partitioning does not match the aggregate’s grouping arguments, the aggregate’s input is shuffled. At the end of an iteration, the aggregate will produce a set containing the last entry made by any group updated during the iteration. This set is then used in the next iteration (i.e., \(\delta\)).

**Program 29. Connected Components**

\[
\begin{align*}
r_1 \cdot cc2(X, m_{\text{min}}(X)) & \leftarrow \text{arc}(X, \_). \\
r_2 \cdot cc2(Y, m_{\text{min}}(Z)) & \leftarrow cc2(X, Z), \text{arc}(X, Y). \\
r_3 \cdot cc(X, \text{min}(Y)) & \leftarrow cc2(X, Y). 
\end{align*}
\]

The connected components (CC) program is for identifying the connected components of a graph. This program works by initially assigning the node’s id to itself \((r1)\) and then propagating a new lower node id for any edge the node is connected to. \(r3\) is necessary to select only the minimum node id \(Y\) for each \(X\) found in \(cc2\).

Lastly, in addition to the aggregate programs presented in this chapter, BigDatalog also supports programs such as Programs 4 and 5 presented in Chapter 4.

### 5.5 Experiments

In this section, we present experimental results comparing BigDatalog against other large-scale DATALOG systems. Additionally, we perform experiments to understand how BigDatalog scales as either cluster size or dataset size increases.
5.5.1 Experimental Setup

Our experiments are conducted on a 16 node cluster. Each node runs Ubuntu 14.04 LTS and has an Intel i7-4770 CPU (3.40GHz, 4 core/8 thread), 32GB memory and a 1 TB 7200 RPM hard drive. Nodes of the cluster are connected with 1Gbit network. BigDatalog is implemented using Spark 1.4.0 and Hadoop 1.0.4.

Table 5.6: TC and SG Synthetic Test Graphs

<table>
<thead>
<tr>
<th>Name</th>
<th>Vertices</th>
<th>Edges</th>
<th>TC</th>
<th>SG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree11*</td>
<td>71,391</td>
<td>71,390</td>
<td>805,001</td>
<td>2,086,271,974</td>
</tr>
<tr>
<td>Tree17*</td>
<td>13,766,856</td>
<td>13,766,855</td>
<td>237,977,708</td>
<td></td>
</tr>
<tr>
<td>Grid150*</td>
<td>22,801</td>
<td>45,300</td>
<td>131,675,775</td>
<td>2,295,050</td>
</tr>
<tr>
<td>Grid250*</td>
<td>63,001</td>
<td>125,500</td>
<td>1,000,140,875</td>
<td>10,541,750</td>
</tr>
<tr>
<td>G5K</td>
<td>5,000</td>
<td>24,973</td>
<td>24,606,562</td>
<td>24,611,547</td>
</tr>
<tr>
<td>G10K*</td>
<td>10,000</td>
<td>100,185</td>
<td>100,000,000</td>
<td>100,000,000</td>
</tr>
<tr>
<td>G10K-0.01*</td>
<td>10,000</td>
<td>999,720</td>
<td>100,000,000</td>
<td>100,000,000</td>
</tr>
<tr>
<td>G10K-0.1*</td>
<td>10,000</td>
<td>9,999,550</td>
<td>100,000,000</td>
<td>100,000,000</td>
</tr>
<tr>
<td>G20K</td>
<td>20,000</td>
<td>399,810</td>
<td>400,000,000</td>
<td>400,000,000</td>
</tr>
<tr>
<td>G40K</td>
<td>40,000</td>
<td>1,598,714</td>
<td>1,600,000,000</td>
<td>1,600,000,000</td>
</tr>
<tr>
<td>G80K</td>
<td>80,000</td>
<td>6,399,376</td>
<td>6,400,000,000</td>
<td>6,400,000,000</td>
</tr>
</tbody>
</table>

Datasets. Table 5.6 displays the synthetic graphs used for TC and SG experiments. We use these graphs to understand how BigDatalog evaluates TC and SG on graphs exhibiting specific structural properties. Tree11 and Tree17 are trees of height 11 and 17 respectively, and the degree of a non-leaf vertex is a random number between 2 and 6. Grid150 is a 151 by 151 grid while Grid250 is a 251 by 251 grid. The G\(n\)-\(p\) graphs are \(n\)-vertex random graphs (Erdős-Rényi model) generated by randomly connecting vertices so that each pair is connected with probability \(p\). G\(n\)-\(p\) graph names omitting \(p\) use default probability 0.001. Graphs in Table 5.6 with an asterisk were taken from [YZ14].

Experiments on REACH (Program 27), CC (Program 29) and SSSP (Program 28) use the RMAT graphs RMAT-\(n\) for \(n \in \{1M, 2M, 4M, 8M, 16M, 32M, 64M, 128M\}\) generated by the RMAT graph generator [gtg15] with parameters \((a, b, c) = (0.45, 0.25, 0.15)\). RMAT-\(n\) has \(n\) vertices and 10\(n\) directed edges with uniform integer weights range from \([0, 100)\).
5.5.2 Datalog Systems Comparison

In this section, we report experimental results comparing BigDatalog against other large-scale DATALOG systems. The purpose of this comparison is to show that with the enhancements and optimizations proposed in this dissertation Spark is indeed an efficient runtime for DATALOG and recursive applications. Next, we present the systems we compared with.

*Myria* has been extended to support asynchronous DATALOG evaluation [WBH15]. Its runtime is a pipelined, parallel, distributed execution engine that evaluates a graph of operators which is broken down into fragments, where each fragment is executed in a separate thread on worker nodes. Datasets are sharded and stored in PostgreSQL instances at worker nodes and read entirely into memory during evaluation. Myria uses hash partitioning.

*Distributed SociaLite* is a DATALOG language implementation for social network analysis [SPS13]. SociaLite programs are code generated and evaluated with a parallel engine on each worker. SociaLite supports range and hash partitioning and uses message passing to transfer data between workers.

For our experiments we also compare with native Spark and use two approaches. For TC and SG, we use optimized Semi-Naïve programs written in the Spark API. These programs attempt to reduce any unnecessary shuffling. For the experiments on REACH, CC and SSSP, we use programs for GraphX, Spark’s graph processing module that implements Pregel [MAB10] on top of Spark. [GXD14] showed GraphX outperforms native Spark on these types of programs, which we validated in our experimental environment.

For each system, one machine was dedicated as the master and each of the 15 worker nodes was allowed 30 GB RAM and 8 CPU cores (120 total cores). Myria was configured with one instance of Myria and PostgreSQL per node, since each node has one disk, which was confirmed as appropriate by the author of [WBH15]. For Spark programs and BigDatalog, we evaluate with one partition per available CPU core. BigDatalog uses Single-Job PSN with SetRDD.
5.5.2.1 TC and SG Experiments

We perform experiments comparing the execution time of BigDatalog for TC and SG programs with Myria, SociaLite and handwritten Spark programs on graphs listed in Table 5.6. Although these graphs appear small in terms of number of vertices and edges, TC and SG are capable of producing result sets many orders of magnitude larger than the input dataset. BigDatalog is the only system that finishes the evaluation for TC and SG on all graphs in Table 5.6, except SG on Tree17 since the size of the result is larger than the total disk space of the cluster. Figure 5.22 and Figure 5.23 display the evaluation times for TC and SG, respectively, for all four systems. Note, larger graphs are used in experiments later in this section, albeit they are only evaluated using BigDatalog, and we do not display those results here.

Aside from Grid150, BigDatalog has the best execution time on all graphs for TC, and on four of the remaining six graphs outperforms the other systems by an order of magnitude. BigDatalog uses the decomposed plan from Figure 5.17 for TC, which only performs an initial shuffle of the dataset. This design shows to be effective for evaluating TC on these graphs. However, if the evaluation requires many iterations while very little work is performed in each iteration, the overhead of scheduling takes a significant portion of execution time, and thus BigDatalog performs slightly slower compared to Myria on Grid150. If BigDatalog instead evaluates Grid150 with Single-Job PSN Reuse (Section 5.3.5), the execution time drops to thirteen seconds. The handwritten Spark programs are also affected by the overhead of scheduling on Grid150 and Grid250, which require 300 and 500 iterations, respectively, but also suffer memory utilization issues related to dataset caching and run out of memory. For the remaining five graphs, the native Spark programs are slower compared with BigDatalog due to the overhead of shuffling. The same amount
of data is also transmitted via shuffling or message passing for both Myria and SociaLite. But
their performance is less stable compared with Spark, e.g., Myria runs out of memory on G20K
and SociaLite is always more than 10X slower, since the implementation of their communication
subsystem is less robust compared to Spark’s.

Figure 5.23: SG System Comparison.

SG is a “harder” program than TC, because shuffling is required and two joins are evaluated
each iteration. BigDatalog uses the plan with broadcast joins whose RRP is shown in Figure
5.13. BigDatalog performs the best on graphs other than Grid150 and Grid250. BigDatalog
is slower than Myria and SociaLite on both graphs partly due to the overhead of scheduling, but
the evaluation of these two graphs produces the smallest result sets as well as the least amount
of intermediate results. Unlike with TC on Grid150, the handwritten SG Spark program can
evaluate Grid150 because there is half as many iterations with SG than with TC and therefore
less caching is necessary. However, SG with native Spark is over 50X slower than BigDatalog
on Grid150 since BigDatalog only requires a single shuffle per iteration due to the broadcast
joins, whereas the Spark program has to shuffle between two nested-loop joins. Additionally,
the handwritten SG Spark program runs out of memory on three graphs due to caching. Lastly,
BigDatalog is faster than Myria and SociaLite on Tree11, G10K and G10K−0.01 partly due to
the more robust shuffling implementation Spark provides.

Map-side Distinct. Recall the RRP for SG in Figure 5.13. Within a task, two joins will be
evaluated and the projected output will be input to a shuffle. The joins can generate a massive
amount of intermediate duplicate results, however de-duplication occurs after the shuffle (in PSN).
Therefore, to prevent a large amount of data being written to disk for the shuffle, we place a
distinct operator into the plan immediately before the shuffle, much like a map-side combiner.
This has no effect on program correctness because of DATALOG’s set semantics. This has a minor
affect on execution time performance on the smaller graphs (i.e., Grid150) but allows BigDatalog
to better support larger graphs.

5.5.2.2 REACH, CC and SSSP Experiments

We perform experiments comparing the execution time of BigDatalog for REACH, CC and SSSP
programs with Myria, SociaLite and GraphX programs on RMAT graphs. These results also help
us understand how BigDatalog scales on different programs as the graph sizes increase compared
to other systems. Figure 5.24 shows the experimental results in which the x-axis represents test
graphs from RMAT-1M to RMAT-128M. For each system, we report the total time of evaluation
starting from loading the data from persistent storage, i.e., from PostgreSQL for Myria and from
HDFS for the remaining three systems, until the evaluation completes. For Figure 5.24(a) and
Figure 5.24(c) each point represents the average time to evaluate the program on the test graph
over a set of randomly selected vertices. A point is not reported in a figure if a system runs out of
memory for the experiment.

We use three graph queries in this comparison. REACH (Program 27) finds all vertices con-
nected by some path to a given source vertex with a simple linear recursion; CC (Program 29)
uses a label propagation approach to determining the lowest vertex id a vertex is connected to, thus
establishing membership in a component, and this program uses a monotonic m\text{min} aggregate in
the recursion. SSSP (Program 28) also uses a m\text{min} aggregate and a linear recursion. All three
programs require shuffling during evaluation. Let \( n, m \) and \( d \) be the number of vertices, number
of edges, and diameter of a graph, respectively. The number of intermediate results produced dur-
ing evaluation is \( O(m) \), \( O(dm) \) and \( O(nm) \) for REACH, CC and SSSP, respectively. Figure 5.24
displays the results for the three programs by increasing order of the amount of communication as
\( 1 \leq d \leq n \).

Figure 5.24(a) shows that Myria performs the best on all test instances for REACH. BigDatalog
narrow the execution time gap with Myria as the size of the graph increases due to the increased
amount of communication. As the amount of communication further increases in CC and SSSP, BigDatalog outperforms Myria starting from RMAT-8M for CC and RMAT-4M for SSSP as shown in Figure 5.24(b) and Figure 5.24(c), respectively. SociaLite’s performance on all three experiments is impacted because of loading and initialization of base relations and SociaLite takes significantly longer at this task than the other systems. As explained to us by an author of [SPS13], the cause of this is that SociaLite is implemented for a much faster network connection than we have available. However, none of the other systems suffer from these same inefficiencies. In general, as with TC and SG, BigDatalog benefits from Spark’s efficient shuffling implementation compared to the less efficient message passing implementations used by Myria and SociaLite. Finally, BigDatalog always outperforms GraphX on these experiments. BigDatalog has less scheduling overhead by using a single job versus a job for each iteration in GraphX. Additionally, BigDatalog uses more efficient data structures, w.r.t. size in memory, compared to GraphX’s vertex and edge RDDs, which each require maintaining more data structures than the SetRDD/AggregateSetRDD design used in BigDatalog.

### 5.5.3 Additional Scaling Experiments

We have shown how BigDatalog scales for REACH, CC and SSSP on RMAT graphs. In this section, we report additional experimental results of how BigDatalog scales over different cluster
and dataset sizes for TC and SG.

![Scaling-out Cluster Size](image.png)

**Figure 5.25: Scaling-out Cluster Size.**

**Scaling-out.** We investigate how BigDatalog scales over different cluster sizes. We use the largest $G_n$-$p$ graphs that could be evaluated on all cluster sizes. Figure 5.25(a) shows the speedup for TC on $G_{20K}$ as the number of workers increases from one to 15 (all with one master) w.r.t. using only one worker, and Figure 5.25(b) shows the same experiment run for SG with $G_{10K}$. Both figures show a linear speedup. Using 15 workers, TC and SG are 12X and 14X faster, respectively, than with a single worker.

![Scaling-up TC on Random Graphs](image.png)

**Figure 5.26: Scaling-up TC on Random Graphs.**

**Scaling-up.** We use the full cluster to see how BigDatalog scales over random graphs from Table 5.6 of increasing sizes for TC and SG. Broadcasting the arc relation requires between one second for $G_{5K}$ to twelve seconds for $G_{80K}$. In this experiment, with each successively larger graph size (e.g., $G_{5K}$ to $G_{10K}$), the size of the transitive closure quadruples, but we do not observe a quadrupling of the evaluation time. Instead, evaluation time increases first less than 1.5X ($G_{5K}$
to G10K), then 3X (G10K to G20K), 6X (G20K to G40K) and 9X (G40K to G80K). Rather than focus on the size of the TC \(w.r.t.\) execution time, the reason for the increase in execution time is explained by examining the results in Table 5.7.

Table 5.7: TC Scaling Experiments Result Details

<table>
<thead>
<tr>
<th>Graph</th>
<th>Time - broadcast(^a) (s)</th>
<th>TC</th>
<th>Generated Facts</th>
<th>Generated Facts / TC</th>
<th>Generated Facts / Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>G5K</td>
<td>4</td>
<td>24,606,562</td>
<td>122,849,424</td>
<td>4.99</td>
<td>30,712,356</td>
</tr>
<tr>
<td>G10K</td>
<td>6</td>
<td>100,000,000</td>
<td>1,001,943,756</td>
<td>10.02</td>
<td>166,990,626</td>
</tr>
<tr>
<td>G20K</td>
<td>17</td>
<td>400,000,000</td>
<td>7,976,284,603</td>
<td>19.94</td>
<td>469,193,212</td>
</tr>
<tr>
<td>G40K</td>
<td>119</td>
<td>1,600,000,000</td>
<td>50,681,485,537</td>
<td>31.68</td>
<td>425,894,836</td>
</tr>
<tr>
<td>G80K</td>
<td>1112</td>
<td>6,400,000,000</td>
<td>510,697,190,536</td>
<td>79.80</td>
<td>459,673,439</td>
</tr>
</tbody>
</table>

\(^a\) - execution time not including the time to broadcast arc.

Among the items displayed in Table 5.7 is the execution time minus the time to broadcast arc, which is the total time the program required to actually evaluate TC. Table 5.7 also shows the number of generated facts, which is the number of total facts produced prior to de-duplication and is representative of the actual work the system must perform to produce the TC (i.e., HashSet lookups), the ratio between TC size and generated facts, and the number of generated facts per second (time - broadcast time), which should be viewed as the evaluation throughput. These details help to explain why the execution times increase at a rate greater than the increase in TC size – the number of generated facts is increasing at a rate much greater than the increase in TC size. The last column shows that even with the increase in number of generated facts, BigDatalog still maintains good throughput throughout. Continuing, the first two graphs are too small to stress the system, but once the graph is large enough (e.g., G20K) the system exhibits a much greater throughput, which is stable across the larger graphs.

![Figure 5.27: Scaling-up SG on Random Graphs.](image-url)
Figure 5.27 displays the scaling up experimental results for SG. Since we double the size of the graph starting from $G_{5K}$, random graphs larger than $G_{20K}$ exceed our cluster’s resources. For these experiments, the broadcast of the $arc$ relation was a less significant part of the overall execution time requiring between one second for $G_{5K}$ and two seconds for $G_{20K}$.

Table 5.8: SG Scaling Experiments Result Details

<table>
<thead>
<tr>
<th>Graph</th>
<th>Time - broadcast (s)</th>
<th>SG Generated Facts</th>
<th>Generated Facts / SG</th>
<th>Generated Facts / Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{5K}$</td>
<td>11</td>
<td>24,611,547</td>
<td>612,891,161</td>
<td>24.90</td>
</tr>
<tr>
<td>$G_{10K}$</td>
<td>73</td>
<td>100,000,000</td>
<td>10,037,915,957</td>
<td>100.38</td>
</tr>
<tr>
<td>$G_{20K}$</td>
<td>905</td>
<td>400,000,000</td>
<td>159,342,570,063</td>
<td>398.36</td>
</tr>
</tbody>
</table>

Table 5.8 displays the same details as Table 5.7 but for SG. Table 5.8 displays the execution time-minus the broadcast time of $arc$, the result set size, the number of generated facts as well as statistics for the ratio of generated facts for each SG fact and generated fact per second of evaluation (throughput). With SG, the number of generated facts is much higher than we observe with TC, reflecting the greater amount of work SG requires. For example, on $G_{10K}$ and $G_{20K}$ SG produces 10X and 20X the number of generated facts, respectively, than TC produces. We also observe a much greater rate of increase in generated facts between graph sizes for SG compared to TC. For example, from $G_{10K}$ to $G_{20K}$ we see a 16X increase in generated facts for SG versus only an 8X increase for TC. For SG, we do not achieve as high a throughput as with TC, which is explained in part by the fact that SG requires shuffling, whereas our TC program evaluates purely in main memory after an initial shuffle.

Lastly, to evaluate these graphs for SG, we used the map-side distinct optimization described in Section 5.5.2.1. Table 5.9 shows the impact of using this optimization. The trade off here is that the distinct operator will filter out duplicates prior to being shuffled and de-duplicated in PSN. The trade off here is the memory required for the distinct operator versus the additional disk space required to shuffle the un-deduplicated results. From Table 5.9 we see the effect on execution time is minimal on the smallest graph tested (one second). However as the graph size increases and the number of facts shuffled greatly increases, the effect on execution time is substantial.
Table 5.9: Impact of Map-side Distinct on SG Scaling Experiments

<table>
<thead>
<tr>
<th>Graph</th>
<th>Time (s)</th>
<th>Facts Shuffled</th>
<th>Time (s)</th>
<th>Facts Shuffled</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>G5K</td>
<td>12</td>
<td>226,350,156</td>
<td>13</td>
<td>612,891,161</td>
<td>2.71</td>
</tr>
<tr>
<td>G10K</td>
<td>74</td>
<td>1,982,954,558</td>
<td>121</td>
<td>10,037,915,957</td>
<td>5.06</td>
</tr>
<tr>
<td>G20K</td>
<td>907</td>
<td>9,894,980,670</td>
<td>6645</td>
<td>159,342,570,063</td>
<td>16.10</td>
</tr>
</tbody>
</table>

5.6 Related Works

We first review works on parallel DATALOG evaluation and then discuss related systems for large-scale data analysis.

**Parallel Datalog Evaluation.** We have previously discussed the contributions of [WS88] with decomposable programs and the work of generalized pivoting [SL91]. Early work on parallelization of bottom-up evaluation of DATALOG programs was largely of a theoretical nature for instance [Van86] proposed a message passing framework for parallel evaluation of logic programs. Techniques to partition program evaluation efficiently among processors [WO90], the tradeoff between redundant evaluation and communication [GST90, GST92] and classifying how certain types of DATALOG programs can be evaluated [CW89] were also studied. A parallel Semi-Naïve fixpoint has been proposed for a message passing design [WO90] that includes a step for sending and receiving tuples from other processors during computation. The PSN used in this work applies the same program over different partitions of the database at each worker and uses shuffle operators in place of processor communication.

[BBC12] showed how to use XY-stratified DATALOG to support computational models for large-scale machine learning, although no full DATALOG language implementation on a large-scale system was provided. Recent theoretical work on recursive query evaluation showed a version of non-linear transitive closure that is more efficient than linear versions for distributed settings [AU12], however this work never addresses the problem of how to convert arbitrary programs to this desirable form. The Bloom$^t$ [CMA12] distributed programming language uses various
monotonic lattices to identify program elements not requiring coordination.

**Systems for Large Scale Data Analysis.** We have previously discussed [AXL15, SPS13, WBH15, ZCD12] in earlier sections. DryadLINQ [YIF08] provides a high level imperative/declarative language that relies on control flow in its host language to support iteration. Extended MapReduce system designs providing API support include Haloop [BHB12] (loop-aware scheduling and caching), PrIter [ZGG11] (prioritized execution of high priority nodes), and Twister [ELZ10] (in-memory iterative MapReduce with cachable tasks). Incremental iterations were integrated into Stratosphere (now Apache Flink ([apa15a])) to support iterative algorithms with sparse computational dependencies [ETK12]. ScalOps [WCR11], a Scala DSL designed for machine learning, supports a loop construct to include iteration in a recursive query plan executed on Hyracks [BCG11], a distributed dataflow engine. Naiad [MMI13] uses a time-based dataflow computational model with a vertex-based programming model to support iterative workflows and incremental updates. The SCOPE [ZBW12] system includes a SQL-like language that provides map and reduce-style operators and allows user defined aggregates (UDA) to be recursive. REX [MIG12], a distributed parallel engine, provides a SQL-based language and supports incremental updates and recursive queries via a UDA framework. [OKH13] presents an extension of SQL that includes constructs to specify incrementally computable views for more efficient MapReduce execution. Lastly, systems for both parallel [KBG12, LGK10] and distributed [gir15, GLG12, LBG12, MAB10, TBC13] graph analytics have been proposed however, these systems only support graph workloads and require programming against a low-level API.

### 5.7 Summary of BigDatalog

In this chapter, we presented BigDatalog, a DATALOG language implementation on Apache Spark. Using our system Spark programmers can now benefit from using a declarative, recursive language to implement their distributed algorithms, while maintaining the efficiency of highly optimized programs. On our large test graph instances BigDatalog outperforms other state-of-the-art DATALOG systems on the majority of our tests. Moreover, our experimental results confirmed that
among Spark-based systems BigDatalog outperforms both GraphX and native Spark for recursive queries.

**Addressing our Challenges.** We addressed the challenges for using Spark as a DATALOG runtime as outlined in Section 5.1.1 as follows: now with BigDatalog, recursive queries are compiled and optimized for efficient evaluation on Spark, which was verified with our experimental results in Section 5.5 (Challenge 1). We implemented BigDatalog to identify and produce physical plans for efficiently evaluating decomposable programs. In addition, we propose a new type of job for recursive programs to allow the scheduler greater control over iterations (Challenge 2). Lastly, we propose the SetRDD and AggregateSetRDD, specialized RDDs that utilize DATALOG semantics to support memory-efficient recursive evaluation in Spark (Challenge 3).
CHAPTER 6

Conclusion and Future Work

In this dissertation, we have presented our contributions towards the language challenges for advanced analytics as outlined in our introduction.

We have developed DeALS, a DATALOG system for analytics. We have designed and implemented a set of monotonic aggregates in DeALS that can be used in recursion. These aggregates expand the range of programs DeALS can efficiently support, which includes many graph queries and graph analytics that were otherwise difficult to express or inefficient to evaluate using previous constructs. We have demonstrated the efficiency of DeALS through an experimental comparison with other DATALOG systems and found that DeALS shows it is possible to provide a general DATALOG system capable of supporting a wide range of programs, that also provides superior performance.

We have designed and implemented BigDatalog – DeALS on Apache Spark. We presented compiler and optimizer techniques to efficiently evaluate BigDatalog programs on Spark. We proposed job scheduling optimizations and show how certain DATALOG programs can be evaluated without communication during recursion in Spark. We conducted an experimental evaluation and compared BigDatalog with other large-scale DATALOG systems on both classical recursive queries, such as transitive closure, as well as aggregate queries. Compared with the other systems, BigDatalog outperformed them on many types of programs, including the classical recursive queries on larger graphs. Compared with native Spark programs, BigDatalog exhibited at least an order of magnitude improvement for almost every graph tested, and also outperformed Spark’s GraphX module. Lastly, the experiments verified that our approach in general is quite effective in supporting DATALOG-based analytics on Spark.
In the course of conducting the research for this dissertation, several opportunities for exciting new research has been identified. In the area of distributed DATALOG evaluation, one first direction is to extend BigDatalog to support XY-DATALOG and realize the vision of [BBC12] to use DATALOG to support complex machine learning analytics such as logistic regression over a massively parallel system. Another direction is to investigate system extensions for provenance and fault tolerance enabled by efficient monotonic DATALOG constructs. Lastly, an exciting area for future research that builds on this dissertation’s work on supporting declarative languages over multiple runtimes is to provide (i) a rule-based or cost-based optimizer to determine the appropriate runtime (e.g., sequential, distributed, etc.) and (ii) an accompanying scheduler to provide an optimized evaluation for DeAL users.
REFERENCES


