Macroeconomic Implications of the Zero Lower Bound

by

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Abstract

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What policies are effective at combatting recessions when the zero lower bound (ZLB) binds? This dissertation contributes to this question in at least three ways. First, it examines several such policies in a standard macroeconomic framework. Second, it uses extensive robustness checks as well as macroeconomic and financial data to validate or reject the key mechanisms that are at work in these models. Third, in the case of rejection, the standard framework is modified to match the data and this improved framework is used to re-evaluate the policies in question. This produces new insights relative to existing literature that has largely remained within the standard macroeconomic framework.

This dissertation first analyzes whether central banks should raise their inflation targets in light of the ZLB. It explicitly incorporates positive steady-state (or “trend”) inflation in standard macroeconomic models as well as the ZLB on nominal interest rates. For plausible calibrations with costly but infrequent episodes at the zero-lower bound, the optimal inflation rate is low, typically less than two percent, even after considering a variety of extensions, including endogenous and state-dependent price stickiness and downward nominal wage rigidities. The key intuition behind this result is that the unconditional cost of the zero lower bound is small even though each individual ZLB event is quite costly. In short, raising the inflation target is too blunt an instrument to efficiently reduce the severe costs of zero-bound episodes.

Second, this dissertation considers whether fiscal policy be effective in an open economy with flexible exchange rates. Standard open economy models suggest that the open economy fiscal multiplier is small when exchange rates are flexible. This premise is reassessed by explicitly incorporating the ZLB on nominal interest rates in a small open economy New Keynesian model. It finds (1) when the ZLB binds and uncovered interest rate parity (UIP) holds, then the open economy fiscal multiplier is larger than 1 and bigger than the closed economy fiscal multiplier, (2) these conclusions can be reversed given significant violations of UIP, and (3) for estimated departures from UIP, the open economy fiscal multiplier at the ZLB is above 1 but smaller than the closed economy fiscal multiplier.
Third, this dissertation tests for a key propagation mechanism in standard macroeconomic models — the inflation expectations channel. Accordingly, government spending multipliers are large and negative supply shocks are expansionary at the ZLB because they lower expected real interest rates, which stimulates consumption. The second prediction is tested with oil supply shocks, an earthquake, and inflation risk premia, demonstrating that negative supply shocks are contractionary at the ZLB despite also lowering expected real interest rates. These facts are rationalized in a model with financial frictions. In this model demand-side policies, such as fiscal stimulus through government spending, are substantially less effective at the ZLB than in standard sticky-price models, because raising inflation expectations by raising production costs is no longer a source of stimulus.
To my parents.
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Chapter 1
Introduction

1.1 Research Question

The zero lower bound (ZLB) on nominal interest rates has been a key constraint in some of the largest downturns in economic history — the Great Depression, the “Lost Decade” in Japan, and the current economic crisis. With standard monetary policy running out of ammunition, different policies are required to mitigate the economic slump. But there is wide disagreement among policy makers and academics about what those policies should be and how effective they are. For instance, some policy makers and economists argue in favor of fiscal stimulus (e.g., Christiano, Eichenbaum, and Rebelo (2011), Eggertsson and Krugman (2011), Woodford (2012)), forward guidance (e.g., Eggertsson and Woodford (2003, 2004), Bernanke (2012)), restricting potential output (Eggertsson (2011, 2012)), and allowing for permanently higher inflation targets (Blanchard, Dell’Ariccia, and Mauro (2010); Ball (2013)). However, others disagree that such discretionary policies are effective (e.g., Cochrane (2009)) or desirable (Taylor (2012)). In this dissertation I attempt to contribute to this debate.

1.2 Methodology

This dissertation examines the macroeconomic implications of the ZLB both theoretically and empirically. Similar to existing theoretical treatments of the ZLB I largely follow the New Keynesian framework of Eggertsson and Woodford (2003). Much of the aforementioned policy recommendations are based on models in this framework, which I now briefly introduce. The very basic New Keynesian model with government consists of four equations. First, an Euler equation

\[ c(t) = c(\infty) - \int_0^\infty (i(t+s) - \pi(t+s) - \bar{\pi} - \rho)ds + v(t), \]  

(1.1)
where \( c(t) \) is today’s consumption, \( c(\infty) \) is long-run consumption, \( i(t) \) the nominal interest rate, \( \pi(t) \) is inflation, \( \bar{\pi} \) is the central bank’s inflation target, \( \rho \) is the discount rate, and \( v(t) \) is a disturbance and this demand equation. Accordingly, when real interest rates lie above the discount rate then consumption today is low relative to long-run consumption and vice-versa.

Second, a Phillips Curve,

\[
\pi(t) = \bar{\pi} + \int_0^\infty e^{-\rho t} \kappa [y(t) - y^*(t)],
\]

(1.2)

where \( y(t) \) is output and \( y^*(t) \) is the natural rate of output. Thus, when output is above potential, i.e. the output gap \([y(t) - y^*(t)]\) is positive, then inflation is also above its long-run level \( \bar{\pi} \).

Third, an interest rate rule,

\[
i(t) = \max\{\rho + \bar{\pi} + \phi_\pi [\pi(t) - \bar{\pi}] + \phi_y [y(t) - y^*(t)] + \varepsilon(t), 0\},
\]

(1.3)

where the central bank responds to deviations of inflation and output from their targets subject to the ZLB constraint.

Fourth, output produced has to equal output consumed by private agents and the government,

\[
y(t) = c(t) + g(t).
\]

(1.4)

When the central bank is unconstrained it can offset the demand disturbances \( v(t) \) through its interest rate policy, and thereby keep output close to potential \( y(t) \approx y^*(t) \). However if there is a sufficiently large negative demand shock, \( v(t) \ll 0 \) then the ZLB will bind as the central bank has lowered nominal interest rates all the way to zero. In that case interest rates will be too high, which depresses consumption as agents will want to save.

Aforementioned policies can be understood as means to deal with this distortion. For instance, higher government spending \( g(t) \) will raise output, which induces higher inflation in Equation (1.2), lowers expected real interest rates at the ZLB and stimulates consumption. Forward guidance promises a lower future path of nominal interest rates when the ZLB ceases to bind (lower \( \varepsilon(t+s) \)), which directly stimulates consumption by reducing the incentives to save. Reducing potential output is also stimulative at the ZLB, since a lower \( y^*(t) \) in Equation (1.2) raises inflation, which induces higher consumption through lower real interest rates. Thus, in New Keynesian models these policies work on the same mechanism – lowering expected real interest rates by raising inflation expectations. I call this the “inflation expectations channel.” In contrast, the main benefit of a higher inflation target is that steady-state nominal interest rates \( \bar{i} = \rho + \bar{\pi} \) are higher, which implies that it will take a larger shock to \( v(t) \) for the ZLB to bind.
1.3. CONTRIBUTIONS

Much research has been devoted to extending the basic structure of Equations (1.1)–(1.4) (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007)) for use in policy analysis. In present context, there is a presumption that such models, designed and estimated to match data in normal times, will do a reasonably good job at the ZLB as well.

However, we have very little empirical evidence about how well these models characterize the ZLB, because of the rarity of such episodes. Systematic national accounts did not exist during the Great Depression. Even Japan has been at the ZLB for less than 20 years, which is typically too short to use standard macroeconometric methods. Nevertheless, while a systematic test of the model is currently not feasible, in this dissertation I make use of various real and financial data to test key implications of the New Keynesian framework. This allows me whether mechanisms such as the “inflation expectations channel” that underlie many policy recommendations are born out in the data.

1.3 Contributions

This dissertation is organized into three chapters that deal with separate aspects of the ZLB and their policy implications. In the following paragraphs I outline the contributions of each chapter and relate it to the existing literature.

1.3.1 Should Central Banks Raise their Inflation Target?

In Chapter 2, Olivier Coibion, Yuriy Gorodnichenko, and I contribute to this question by explicitly incorporating positive steady-state (or “trend”) inflation into New Keynesian models as well as the ZLB on nominal interest rates. We derive the effects of non-zero steady-state inflation on the loss function, thereby laying the groundwork for welfare analysis. The main trade-off we capture is that higher trend inflation imposes costs through greater price dispersion and inflation volatility but reduces the incidence of the ZLB by raising steady-state nominal interest rates. While hitting the ZLB is very costly in the model, our baseline finding is that the optimal rate of inflation is low, typically less than two percent a year, even when we allow for features that lower the costs or raise the benefits of positive steady-state inflation.

The key intuition for this result is that the unconditional cost of the zero lower bound is small even though each individual ZLB event is quite costly. In our baseline calibration, an 8-quarter ZLB event at 2% trend inflation has a cost equivalent to a 6.2% permanent reduction in consumption, above and beyond the usual business cycle cost. However, in the model such an event is also rare, occurring about once every 20 years assuming that ZLB events always last 8 quarters, so that the uncon-
1.3. CONTRIBUTIONS

ditional cost of the ZLB at 2% trend inflation is equivalent to a 0.08% permanent reduction in consumption. This leaves little room for further improvements in welfare by raising the long-run inflation rate and reducing the incidence of the ZLB. Thus, even modest costs of trend inflation, which must be borne every period, will imply an optimal inflation rate below 2%, despite reasonable values for both the frequency and cost of the ZLB. This explains why our results are robust to a variety of settings, such as downward-nominal-wage-rigidity and state-dependent price stickiness, and suggests that our results are not particular to the New Keynesian model. In short, raising the inflation target is too blunt an instrument to efficiently reduce the severe costs of zero-bound episodes.

This chapter is closely related to recent work that has also emphasized the effects of the zero bound on interest rates for the optimal inflation rate, such as Walsh (2009), Billi (2011), and Williams (2009). A key difference between the approach taken in this paper and such previous work is that we explicitly model the effects of positive trend inflation on the steady-state, dynamics, and loss function of the model. In Schmitt-Grohé and Uribe (2010) the chance of hitting the ZLB is practically zero and therefore does not quantitatively affect the optimal rate of inflation, whereas we focus on a setting where costly ZLB events occur at their historic frequency. In fact, our results go some way in resolving the “puzzle” pointed out by Schmitt-Grohé and Uribe (2010) that existing monetary theories routinely imply negative optimal inflation rates, and thus cannot explain the size of observed inflation targets.

1.3.2 How Large are Fiscal Multipliers?

Since permanently higher inflation targets are not well-suited to deal with temporary ZLB episodes, policies conditional on hitting the ZLB may hold promise. In this chapter, I show that open economy fiscal multipliers can be large in New Keynesian models when the economy is at the ZLB. I build a small open economy model following Gali and Monacelli (2005) and Clarida and Gertler (2001), and derive fiscal multipliers in and outside the liquidity trap, assuming that uncovered interest rate parity (UIP) holds. I find that at the ZLB the fiscal multiplier is above one and increasing in the import share. Thus, once the zero bound binds, the fiscal multiplier in a closed economy (which has a zero import share) is smaller than the fiscal multiplier in an open economy. Intuitively, the inflation generated by government spending shocks lowers real interest rates at the ZLB, which makes domestic real bonds less attractive. Thus, the real exchange rate depreciates, which raises net exports and the fiscal multiplier, particularly if the import share is large.

Next, I derive fiscal multipliers when UIP is violated. Departures from UIP are rationalized through a wedge in the Backus-Smith condition, which is assumed to be an increasing function of excess real returns of domestic bonds over foreign bonds. I find that for moderately sized friction in UIP, the open economy fiscal multiplier will
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be decreasing in the import share, and for even larger frictions the multiplier will be below 1, thus overturning the results from the baseline model.

The chapter proceeds to estimate the size of the friction and thus determine the likely properties of the open economy fiscal multiplier at the ZLB. The friction is derived by comparing model-implied nominal exchange rate responses to generic inflation surprises with their empirical counterpart. Contrary to the large depreciation predicted by the New Keynesian model, I estimate that the nominal exchange rate appreciation by 0.021% after a 1% positive inflation surprise at the ZLB. This implies that the friction to UIP is quantitatively important for fiscal multipliers at the ZLB – in a calibrated model the fiscal multiplier at the ZLB is 2.5 in the frictionless baseline model, but “only” 1.5 in the model with UIP friction. Furthermore, unlike in the baseline model, the open economy fiscal multiplier in the friction model is significantly smaller than in the closed economy.

However, even though exchange rate crowding out can be quantitatively significant in the data-consistent model, the fiscal multipliers remain large by the standards of the open economy literature. In existing accounts, which ignore the ZLB, fiscal multipliers tend to be close to zero, because a fiscal expansion is usually associated with an appreciation in the real exchange rate and thus crowding out of net exports (Dornbusch (1976)). Nevertheless the fiscal multipliers I derive are substantially smaller than closed-economy fiscal multipliers at the ZLB that have been shown to be as large as three or four (e.g., Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2006), Eggertsson (2009), and Woodford (2011a)). Thus, my results suggest that policy makers should be cautious in expecting such large positive outcome from fiscal policy.

1.3.3 Are Negative Supply Shocks Expansionary at the ZLB?

The preceding chapter has highlighted that government spending at the ZLB can be very effective. This is because it increases marginal costs of production, which raises inflation expectations through the Phillips curve (1.2), lowers real interest rates and stimulates consumption. Thus, implicit in this argument is that negative supply shocks, i.e. shocks that raise marginal costs and inflation expectations, are expansionary through the inflation expectations channel. In this chapter, I test this robust prediction of New Keynesian models at the ZLB, and derive implications for the effectiveness of demand-side policies.

First, I determine the macroeconomic impact of two negative supply shocks at the ZLB: oil supply shocks, and the Japanese earthquake in 2011. My results show that while inflation expectations rise and expected real interest rates fall as predicted by the theory, these negative supply shocks are still contractionary overall. I also
provide evidence against a weaker interpretation of the expectations channel. Because expected future nominal rates rise less at the ZLB, supply shocks should be less contractionary than in normal times; however, I document that oil supply shocks appear to be, if anything, more contractionary at the ZLB.

Second, I argue that inflation risk premia can signal if generic negative supply shocks are also contractionary at the ZLB. In standard models, higher expected inflation raises consumption at the ZLB irrespective of its source, so nominal assets become a hedge — they gain value in deflationary states when consumption is low. Conditional on the ZLB, the inflation risk premium should therefore be negative. However, empirically the one-year inflation risk premium at the ZLB is typically positive, suggesting that investors want to insure against shocks that raise inflation and lower consumption. This indicates that generic negative supply shocks are not only contractionary at the ZLB, but also a significant contributor to inflation risk over this horizon.

I then show that incorporating financial frictions in the Euler equation reconciles the theory with the data. My model features a balance sheet constraint on financial intermediaries that generates an endogenous spread between the borrowing and the deposit rate. Because a negative supply shock reduces profits and share values, the net worth of financial intermediaries falls, tightening their balance sheet constraints. In turn, banks contract loan supply, the borrowing spread rises, and borrowers reduce consumption such that negative supply shocks are contractionary at the ZLB. While my model is more successful at matching the data at the ZLB, in normal times it behaves similarly to a standard new Keynesian model because the central bank dampens the financial accelerator through its interest rate policy. This suggests that distinguishing between models is difficult in normal times because the central bank attenuates differences between models. In contrast, the ZLB provides a unique testing ground, and the evidence favors the model with credit friction.

Because the inflation expectations channel is rejected in the data, demand-side policies are much less effective in models that match these facts. Consequently, in the calibrated model with credit frictions, demand-side policies are up to 50% less effective than in a standard new Keynesian model. This suggests that policy makers should be cautious in expecting large positive outcomes from such policies at the ZLB.

My empirical results suggest that the “Paradox of Toil” (Egbertsson (2010b, 2011, 2012)), whereby negative supply shocks are expansionary at the ZLB, is not borne out in the data. This chapter also relates to an ongoing debate whether fiscal multipliers are large at the ZLB (e.g., Christiano, Eichenbaum, and Rebelo (2011); Woodford (2011a)) or small (e.g., Cogan, Cwik, Taylor, and Wieland (2010); Drautzburg and Uhlig (2011)). I show that when negative supply shocks are contractionary at the ZLB, then the inflation expectations channel cannot be a source of large multipliers. To the extent that multipliers may be large at the ZLB, my results suggest that
these are due to other mechanisms such as consumption-labor complementarities (Nakamura and Steinsson (2011)), low capacity utilization (Christiano, Eichenbaum, and Rebelo (2011)), or high unemployment (Michaillat (2012)).

1.4 Overview of the dissertation

The dissertation proceeds as follows. Chapter 2 considers whether central banks should raise their inflation targets in light of the ZLB. It finds little support for this notion in a wide range of macroeconomic models, because a permanent policy is a blunt tool to combat these very costly but also very rare events. Hence, Chapter 3 focuses on a conditional policy: fiscal stimulus at the ZLB in open economies. It shows that for empirically reasonable exchange rate behavior open-economy fiscal multipliers are typically smaller at the ZLB than have been found in closed economy models. However, they are still “large” because the expectations channel applies to domestic consumption. Chapter 4 tests for the presence of the inflation expectations channel by examining whether negative supply shocks are expansionary at the ZLB.
Chapter 2

The Optimal Inflation Rate in New Keynesian Models\textsuperscript{1}

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“The crisis has shown that interest rates can actually hit the zero level, and when this happens it is a severe constraint on monetary policy that ties your hands during times of trouble. As a matter of logic, higher average inflation and thus higher average nominal interest rates before the crisis would have given more room for monetary policy to be eased during the crisis and would have resulted in less deterioration of fiscal positions. What we need to think about now is whether this could justify setting a higher inflation target in the future.”

Olivier Blanchard, February 12th, 2010.

2.1 Introduction

Despite the importance of quantifying the optimal inflation rate for policy-makers, modern monetary models of the business cycle, namely the New Keynesian framework, have been strikingly ill-suited to address this question because of their near exclusive reliance on the assumption of zero steady-state inflation, particularly in welfare analysis. Our first contribution is to address the implications of positive steady-state inflation for welfare analysis by solving for the micro-founded loss function in an otherwise standard New Keynesian model with labor as the only factor of

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production. We identify three distinct costs of positive trend inflation. The first is the steady-state effect: with staggered price setting, higher inflation leads to greater price dispersion which causes an inefficient allocation of resources among firms, thereby lowering aggregate welfare. The second is that positive steady-state inflation raises the welfare cost of a given amount of inflation volatility. This cost reflects the fact that inflation variations create distortions in relative prices given staggered price setting. Since positive trend inflation already generates some inefficient price dispersion, the additional distortion in relative prices from an inflation shock becomes more costly as firms have to compensate workers for the increasingly high marginal disutility of sector-specific labor. Thus, the increased distortion in relative prices due to an inflation shock becomes costlier as we increase the initial price dispersion which makes the variance of inflation costlier for welfare as the steady-state level of inflation rises. In addition to the two costs from relative price dispersion, a third cost of inflation in our model comes from the dynamic effect of positive inflation on pricing decisions. Greater steady-state inflation induces more forward-looking behavior when sticky-price firms are able to reset their prices because the gradual depreciation of the relative reset price can lead to larger losses than under zero inflation. As a result, inflation becomes more volatile which lowers aggregate welfare. This cost of inflation arising from the positive relationship between the level and volatility of inflation has been well-documented empirically but is commonly ignored in quantitative analyses because of questions as to the source of the relationship.\(^2\) As with the price-dispersion costs of inflation, this cost arises endogenously in the New Keynesian model when one incorporates positive steady-state inflation.

The key benefit of positive inflation in our model is a reduced frequency of hitting the zero bound on nominal interest rates. As emphasized in Christiano, Eichenbaum, and Rebelo (n.d.), hitting the zero bound induces a deflationary mechanism which leads to increased volatility and hence large welfare costs. Because a higher steady-state level of inflation implies a higher level of nominal interest rates, raising the inflation target can reduce the incidence of zero-bound episodes, as suggested by Blanchard in the opening quote. Our approach for modeling the zero bound follows Bodenstein, Erceg, and Guerrieri (2009) by solving for the duration of the zero bound endogenously, unlike in Christiano et al. (n.d.) or Eggertsson and Woodford (2004). This is important because the welfare costs of inflation are a function of the variance of inflation and output, which themselves depend on the frequency at which the zero bound is reached as well as the duration of zero bound episodes.

After calibrating the model to broadly match the moments of macroeconomic series and the historical incidence of hitting the zero lower bound in the U.S., we

\(^2\)For example, Mankiw (2007) undergraduate Macroeconomics textbook notes that “in thinking about the costs of inflation, it is important to note a widely documented but little understood fact: high inflation is variable inflation.”
then solve for the rate of inflation that maximizes welfare. While the ZLB ensures that the optimal inflation rate is positive, for plausible calibrations of the structural parameters of the model and the properties of the shocks driving the economy, the optimal inflation rate is quite low: typically less than two percent per year. This result is remarkably robust to changes in parameter values, as long as these do not dramatically increase the implied frequency of being at the zero lower bound. In addition, we show that our results are robust if the central bank follows optimal stabilization policy, rather than the baseline Taylor rule. In particular, if the central bank cannot commit to a policy rule, then the optimal inflation rate remains within the range of inflation rates targeted by central banks and is of qualitatively similar magnitude as in our baseline calibration. Furthermore, we show that all three costs of inflation—"the steady state effect, the increasing cost of inflation volatility, and the positive link between the level and volatility of inflation"—are quantitatively important: each cost is individually large enough to bring the optimal inflation rate down to 3.6% or lower when the ZLB is present.

The key intuition behind the low optimal inflation rate is that the unconditional cost of the zero lower bound is small even though each individual ZLB event is quite costly. In our baseline calibration, an 8-quarter ZLB event at 2% trend inflation has a cost equivalent to a 6.2% permanent reduction in consumption, above and beyond the usual business cycle cost. This is, for example, significantly higher than Williams (2009) estimate of the costs of hitting the ZLB during the current recession. However, in the model such an event is also rare, occurring about once every 20 years assuming that ZLB events always last 8 quarters, so that the unconditional cost of the ZLB at 2% trend inflation is equivalent to a 0.08% permanent reduction in consumption. This leaves little room for further improvements in welfare by raising the long-run inflation rate. Thus, even modest costs of trend inflation, which must be borne every period, will imply an optimal inflation rate below 2%, despite reasonable values for both the frequency and cost of the ZLB. This explains why our results are robust to a variety of settings that we further discuss below and suggests that our results are not particular to the New Keynesian model.

Furthermore, while the New Keynesian model implies that the optimal weight on the variance of the output gap in the welfare loss function is small, we show that increasing the weight on the output gap to be more than ten times as large as that on the annualized inflation variance would still leave the optimal inflation rate at less than 2.5%. Thus, it is unlikely that augmenting the baseline model with mechanisms which could raise the welfare cost of output fluctuations (such as involuntary unemployment or income disparities across agents) would significantly raise the optimal target rate of inflation. Finally, while we use historical U.S. data to calibrate the frequency of hitting the ZLB, an approach which can be problematic when applied to rare events, we show in robustness analysis that even a tripling of the frequency of being at the ZLB (such that the economy would spend 15% of the
2.1. INTRODUCTION

time at the ZLB for an inflation rate of 3% would raise the optimal inflation rate only to 3% which is the upper bound of most central banks’ inflation targets.

To further investigate the robustness of this result, we extend our baseline model to consider several mechanisms which might raise the optimal rate of inflation. First, in the presence of uncertainty about underlying parameter values, policy-makers might want to choose a higher target inflation rate as a buffer against the possibility that the true parameters imply more frequent and costly incidence of the zero bound. Incorporating this uncertainty only raises the optimal inflation rate from 1.5% to 1.9% per year. Second, one might be concerned that our findings hinge on modeling price stickiness as in Calvo (1983). Since this approach implies that some firms do not change prices for extended periods of time, it could overstate the cost of price dispersion and therefore underestimate the optimal inflation rate. To address this possibility, we reproduce our analysis using Taylor (1979) staggered price setting of fixed durations. The latter reduces price dispersion relative to the Calvo assumption but raises the optimal inflation rate to only 2.2% when prices are changed every three quarters. Another limitation of the Calvo assumption is that the rate at which prices are changed is commonly treated as a structural parameter, yet the frequency of price setting may depend on the inflation rate, even for low inflation rates like those experienced in the U.S. As a result, we consider two modifications that allow for price flexibility to vary with the trend rate of inflation. In the first specification, we let the degree of price rigidity vary systematically with the trend level of inflation. In the second specification, we employ the Dotsey, King, and Wolman (1999) model of state-dependent pricing, which allows the degree of price stickiness to vary endogenously both in the short-run and in the long-run, and thus we address one of the major criticisms of the previous literature on the optimal inflation rate. Both modifications yield optimal inflation rates of less than two percent per year. Finally, we incorporate downward nominal wage rigidity, which Tobin (1972) suggests might push the optimal inflation rate higher by facilitating the downward adjustment of real wages. This “greasing the wheels” effect, however, significantly lowers the optimal inflation rate by lowering the volatility of marginal costs and hence of inflation.

Our analysis abstracts from several other factors which might affect the optimal inflation rate. For example, Friedman (1969) argued that the optimal rate of inflation must be negative to equalize the marginal cost and benefit of holding money. Because our model is that of a cashless economy, this cost of inflation is absent, but would tend to lower the optimal rate of inflation even further, as emphasized by Khan, King, and Wolman (2003), Schmitt-Grohe and Uribe (2007); Schmitt-Grohé and Uribe (2010) and Aruoba and Schorfheide (2011). Similarly, a long literature has studied the costs and benefits of the seigniorage revenue to policymakers associated with positive inflation, a feature which we also abstract from since seigniorage revenues for countries like the U.S. are quite small, as are the deadweight losses associated with it (Cooley and Hansen (1991), Summers (1991)). Feldstein (1997)
emphasizes an additional cost of inflation arising from fixed nominal tax brackets, which would again lower the optimal inflation rate. Finally, because we do not model the possibility of endogenous countercyclical fiscal policy nor do we incorporate the possibility of nonstandard monetary policy actions during ZLB episodes, it is likely that we overstate the costs of hitting the ZLB and therefore again overstate the optimal rate of inflation. Nevertheless, our finding that the threat of the ZLB coupled with limited commitment on the part of the central bank implies positive but low optimal inflation rates, goes some way in resolving the “puzzle” pointed out by Schmitt-Grohé and Uribe (2010) that existing monetary theories routinely imply negative optimal inflation rates, and thus cannot explain the size of observed inflation targets.

This chapter is closely related to recent work that has also emphasized the effects of the zero bound on interest rates for the optimal inflation rate, such as Walsh (2009), Billi (2011), and Williams (2009). A key difference between the approach taken in this chapter and such previous work is that we explicitly model the effects of positive trend inflation on the steady-state, dynamics, and loss function of the model. Billi (2011) and Walsh (2009), for example, use a New Keynesian model log-linearized around zero steady-state inflation and therefore do not explicitly incorporate the positive relationship between the level and volatility of inflation, while Williams (2009) relies on a non-microfounded model. In addition, these papers do not take into account the effects of positive steady-state inflation on the approximation to the utility function and thus do not fully incorporate the costs of inflation arising from price dispersion. Schmitt-Grohé and Uribe (2010) provide an authoritative treatment of many of the costs and benefits of trend inflation in the context of New Keynesian models. However, their calibration implies that the chance of hitting the ZLB is practically zero and therefore does not quantitatively affect the optimal rate of inflation, whereas we focus on a setting where costly ZLB events occur at their historic frequency. Furthermore, none of these papers consider the endogenous nature of price rigidity with respect to trend inflation.

An advantage of working with a micro-founded model and its implied welfare function is the ability to engage in normative analysis. In our baseline model, the endogenous response of monetary policy-makers to macroeconomic conditions is captured by a Taylor rule. Thus, we are also able to study the welfare effects of altering the systematic response of policy-makers to endogenous fluctuations (i.e. the coefficients of the Taylor rule) and determine the new optimal steady-state rate of inflation. The most striking finding from this analysis is that even modest price-level targeting (PLT) would raise welfare by non-trivial amounts for any steady-state inflation rate and come close to the Ramsey-optimal policy, consistent with the finding of Eggertsson and Woodford (2003) and Wolman (2005). In short, the optimal policy rule for the model can be closely characterized by the name of “price stability” as typically stated in the legal mandates of most central banks.
2.2. A NEW KEYNESIAN MODEL WITH POSITIVE STEAD-STATE INFLATION

Given our results, we conclude that raising the target rate of inflation is likely too blunt an instrument to reduce the incidence and severity of zero-bound episodes. In all of the New Keynesian models we consider, even the small costs associated with higher trend inflation rates, which must be borne every period, more than offset the welfare benefits of fewer and less severe ZLB events. Instead, changes in the policy rule, such as PLT, may be more effective both in avoiding and minimizing the costs associated with these crises. In the absence of such changes to the interest rate rule, our results suggest that addressing the large welfare losses associated with the ZLB is likely to best be pursued through policies targeted specifically to these episodes, such as countercyclical fiscal policy or the use of non-standard monetary policy tools.

Section 2 presents the baseline New Keynesian model and derivations when allowing for positive steady-state inflation, including the associated loss function. Section 3 includes our calibration of the model as well as the results for the optimal rate of inflation while section 4 investigates the robustness of our results to parameter values. Section 5 then considers extensions of the baseline model which could potentially lead to higher estimates of the optimal inflation target. Section 6 considers additional normative implications of the model, including optimal stabilization policy and price level targeting. Section 7 concludes.

2.2 A New Keynesian Model with Positive Stead-State Inflation

We consider a standard New Keynesian model with a representative consumer, a continuum of monopolistic producers of intermediate goods, a fiscal authority and a central bank.

2.2.1 Model

The representative consumer maximizes the present discounted value of the utility stream from consumption and leisure

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left\{ \log(C_{t+j} - hGA_{t+j}C_{t+j}) - \frac{\eta}{\eta + 1} \int_0^1 N_{t+j}(i)^{1+\frac{1}{\eta}} \, di \right\}$$  \(2.1\)

where $C$ is consumption of the final good, $N(i)$ is labor supplied to individual industry $i$, $GA$ is the gross growth rate of technology, $\eta$ is the Frisch labor supply elasticity, $h$ the internal habit parameter and $\beta$ is the discount factor.\(^3\) The budget

\(^3\)We use internal habits rather than external habits because they more closely match the (lack of) persistence in consumption growth in the data. The gross growth rate of technology enters the habit term to simplify derivations.
constraint each period $t$ is given by

$$\xi_t : C_t + \frac{S_t}{P_t} \leq \int_0^1 (N_t(i) \frac{W_t(i)}{P_t})di + \frac{S_{t-1}}{P_{t-1}}q_{t-1}R_{t-1} + T_t \quad (2.2)$$

where $S$ is the stock of one-period bonds held by the consumer, $R$ is the gross nominal interest rate, $P$ is the price of the final good, $W(i)$ is the nominal wage earned from labor in industry $i$, $T$ is real transfers and profits from ownership of firms, $q$ is a risk premium shock, and $\xi$ is the shadow value of wealth. As discussed in Smets and Wouters (2007), a positive shock to $q$, which is the wedge between the interest rate controlled by the central bank and the return on assets held by the households, increases the required return on assets and reduces current consumption. The shock $q$ has similar effects as net-worth shocks in models with financial accelerators. Such financial shocks have arguably played a major role in causing the zero lower bound to bind in practice. Amano and Shukayev (2012) also document that shocks like $q$ are essential for generating a binding zero lower bound in the New Keynesian model.

The first order conditions from this utility-maximization problem are then:

$$(C_t - hGA_tC_{t-1})^{-1} - \beta hE_tGA_{t+1}(C_{t+1} - hGA_{t+1}C_t)^{-1} = \xi_t \quad (2.3)$$

$$N_t(i)^{1/\theta} = \xi_t \frac{W_t(i)}{P_t} \quad (2.4)$$

$$\frac{\xi_t}{P_t} = \beta E_t \left[ \frac{\xi_{t+1}}{P_{t+1}} q_t R_t \right] \quad (2.5)$$

Production of the final good is done by a perfectly competitive sector which combines a continuum of intermediate goods into a final good per the following aggregator

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\theta}{\theta-1}} \right]^\frac{\theta}{\theta-1} \quad (2.6)$$

where $Y$ is the final good and $Y(i)$ is intermediate good $i$, while $\theta$ denotes the elasticity of substitution across intermediate goods, yielding the following demand curve for goods of intermediate sector $i$

$$Y_t(i) = Y_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \quad (2.7)$$

and the following expression for the aggregate price level

$$P_t = \left[ \int_0^1 P_t(i)^{1-\theta} \right]^\frac{1}{1-\theta} \quad (2.8)$$
2.2. A NEW KEYNESIAN MODEL WITH POSITIVE STEADY-STATE INFLATION

The production of each intermediate good is done by a monopolist facing a production function linear in labor

\[ Y_t(i) = A_t N_t(i) \tag{2.9} \]

where \( A \) denotes the level of technology, common across firms. Each intermediate good producer has sticky prices, modeled as in Calvo (1983) where \( 1 - \lambda \) is the probability that each firm will be able to reoptimize its price each period. We allow for indexation of prices to steady-state inflation by firms who do not reoptimize their prices each period, with \( \omega \) representing the degree of indexation (0 for no indexation to 1 for full indexation). Denoting the optimal reset price of firm \( i \) by \( B_t(i) \), re-optimizing firms solve the following profit-maximization problem

\[
\max E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} \left[ Y_{t+j}(i) B_t(i) \bar{\Pi}^j - W_{t+j}(i) N_{t+j}(i) \right] \tag{2.10}
\]

where \( Q \) is the stochastic discount factor and \( \bar{\Pi} \) is the gross steady-state level of inflation. The optimal relative reset price is then given by

\[
\frac{B_t(i)}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} Y_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{(\theta + 1)} \bar{\Pi}^{-\omega \theta} \left( \frac{MC_{t,j}(i)}{P_{t+j}} \right)}{E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} Y_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{(\theta)} \bar{\Pi}^{-\omega/(\theta - 1)}} \tag{2.11}
\]

where firm-specific marginal costs can be related to aggregate variables using

\[
\frac{MC_{t+j}(i)}{P_{t+j}} = \left( \frac{A_{t+j}}{A_{t+j}} \right) \left( \frac{1}{\theta} \right)^{1/\eta} \left( \frac{B_t(i)}{P_t} \right)^{-\frac{\theta}{\eta}} \left( \frac{P_{t+j}}{P_t} \right)^{\frac{\theta}{\eta}} \tag{2.12}
\]

Given these price-setting assumptions, the dynamics of the price level are governed by

\[ P_t^{1-\theta} = (1 - \lambda) B_t^{1-\theta} + \lambda P_{t-1}^{1-\theta} \bar{\Pi}^{\omega(1-\theta)}. \tag{2.13} \]

We allow for government consumption of final goods (G), so the goods market clearing condition for the economy is

\[ Y_t = C_t + G_t. \tag{2.14} \]

We define the aggregate labor input as

\[ N_t = \left[ \int_0^1 N_t(i) \frac{\theta}{\theta - 1} \right]^{\theta-1} \tag{2.15} \]
2.2. A NEW KEYNESIAN MODEL WITH POSITIVE STEAD-STATE INFLATION

2.2.2 Steady-state and log-linearization

Following Coibion and Gorodnichenko (2011b), we log-linearize the model around the steady-state in which inflation need not be zero. Since positive trend inflation may imply that the steady state and the flexible price level of output differ, we adopt the following notational convention. Variables with a bar denote steady state values, e.g. \( \bar{Y}_t \) is the steady state level of output. Lower-case letters denote the log of a variable, e.g. \( y_t = \log Y_t \) is the log of current output. We assume that technology is a random walk and hence we normalize all non-stationary real variables by the level of technology. We let hats on lower case letters denote log deviations from steady state, e.g. \( \hat{y}_t = y_t - \bar{y}_t \) is the approximate percentage deviation of output from steady state. Since we define the steady state as embodying the current level of technology, deviations from the steady state are stationary. Finally, we denote deviations from the flexible price level steady state with a tilde, e.g. \( \tilde{y}_t = y_t - \bar{y}_t^F \) is the approximate percentage deviation of output from its flexible price steady state, where the superscript \( F \) denotes a flexible price level quantity. Define the net steady-state level of inflation as \( \bar{\pi} = \log(\bar{\Pi}) \). The log-linearized consumption Euler equation is

\[
\hat{\xi}_t = \mathbb{E}_t[\hat{\xi}_{t+1} + \hat{\pi}_t + \hat{\xi}_{t+1} + \hat{\pi}_t]
\]  

(2.16)

where the marginal utility of consumption is given by

\[
\hat{\xi}_t = \frac{h}{(1-h)(1-\beta h)} \hat{c}_{t-1} - \frac{1 - \beta h^2}{(1-h)(1-\beta h)} \hat{c}_t + \frac{\beta h}{(1-h)(1-\beta h)} \mathbb{E}_t \hat{c}_{t+1}
\]  

(2.17)

and the goods market clearing condition becomes

\[
\hat{y}_t = \bar{c}_y \hat{c}_t + \bar{g}_y \hat{g}_t
\]  

(2.18)

where \( \bar{c}_y \) and \( \bar{g}_y \) are the steady-state ratios of consumption and government to output respectively. Also, integrating over firm-specific production functions and log-linearizing yields

\[
\hat{y}_t = \hat{n}_t
\]  

(2.19)

Allowing for positive steady-state inflation (i.e., \( \bar{\pi} > 0 \)) primarily affects the steady-state and price-setting components of the model. For example, the steady-state level of the output gap (which is defined as the deviation of steady state output from its flexible price level counterpart \( \bar{X}_t = \frac{\bar{Y}_t}{\bar{Y}_{t}} \)) is given by

\[
\bar{X}^n_{\eta+1} = \frac{1 - \lambda \bar{\Pi}^{(1-\omega)^{\theta(\theta+1)\eta^{-1}}} \left( 1 - \lambda \right) \bar{\Pi}^{(1-\omega)(\theta-1)}}{1 - \lambda \bar{\Pi}^{(1-\omega)(\theta-1)}} \left( \frac{1 - \lambda \bar{\Pi}^{(1-\omega)\eta(\theta-1)}}{1 - \lambda \bar{\Pi}^{(1-\omega)(\theta-1)}} \right)^{\frac{\eta+\theta}{\eta(\theta-1)}}
\]  

(2.20)

Note that the steady-state level of the gap is equal to one when steady-state inflation is zero (i.e., \( \bar{\Pi} = 1 \)) or when the degree of price indexation is exactly equal to one. As
2.2. A NEW KEYNESIAN MODEL WITH POSITIVE STEADY-STATE INFLATION

emphasized by Ascari and Ropele (2009), there is a non-linear relationship between the steady-state levels of inflation and output. For very low but positive trend inflation, $\bar{X}$ is increasing in trend inflation but the sign is quickly reversed so that $\bar{X}$ is falling with trend inflation for most positive levels of trend inflation. Secondly, positive steady-state inflation affects the relationship between aggregate inflation and the re-optimizing price. Specifically, the relationship between the two in the steady state is now given by

$$
\bar{B} = \left(1 - \frac{\lambda}{\lambda \Pi (1-\omega)(\theta-1)}\right)^{\frac{1}{\sigma-\tau}}
$$

and the log-linearized equation is described by

$$
\hat{\pi}_t = \left(1 - \frac{\lambda \Pi (1-\omega)(\theta-1)}{\lambda \Pi (1-\omega)(\theta-1)}\right) \hat{b}_t \quad \Rightarrow \quad \hat{b}_t = M \hat{\pi}_t
$$

so that inflation is less sensitive to changes in the re-optimizing price as steady-state inflation rises because goods with high relative prices receive a smaller share of expenditures. Similarly, positive steady-state inflation has important effects on the log-linearized optimal reset price equation, which is given by

$$
\left(1 + \frac{\theta}{\eta}\right) \hat{b}_t = (1 - \gamma_2) \sum_{j=0}^{\infty} \gamma_2^j \left(\frac{1}{\eta} \mathbb{E}_t \hat{g}_{t+j} - \mathbb{E}_t \hat{\xi}_{t+j}\right)
+ \mathbb{E}_t \sum_{j=1}^{\infty} (\gamma_2^j - \gamma_1^j) \left(\hat{g}_{t+j} + \hat{\xi}_{t+j-1}\right)
+ \sum_{j=1}^{\infty} \left[\gamma_2^j \left(1 + \frac{\theta(\eta + 1)}{\eta}\right) - \gamma_1^j \theta\right] \mathbb{E}_t \hat{\pi}_{t+j} + \hat{m}_t
$$

where $\hat{m}_t$ is a cost-push shock, $\gamma_1 = \lambda \beta \Pi (1-\omega)(\theta-1)$ and $\gamma_2 = \gamma_1 \Pi (1-\omega)(1+\theta\eta^{-1})$ so that without steady-state inflation or full indexation we have $\gamma_1 = \gamma_2$. When $\omega < 1$, a higher $\bar{\pi}$ increases the coefficients on future output and inflation but also leads to the inclusion of a new term composed of future differences between output growth and interest rates. Each of these effects makes price-setting decisions more forward-looking. The increased coefficient on expectations of future inflation, which reflects the expected future depreciation of the reset price and the losses associated with it, plays a particularly important role. In response to an inflationary shock, a firm which can reset its price will expect higher inflation today and in the future as other firms update their prices in response to the shock. Given this expectation, the more forward looking a firm is (the higher $\bar{\pi}$), the greater the optimal reset price must be in anticipation of other firms raising their prices in the future. Thus, reset prices become more responsive to current shocks with higher $\bar{\pi}$. We confirm numerically...
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that this effect dominates the reduced sensitivity of inflation to the reset price in equation (2.22), thereby endogenously generating a positive relationship between the level and the volatility of inflation.

To close the model, we assume that the log deviation of the desired gross interest rate from its steady state value \( \hat{r}_t^* \) follows a Taylor rule

\[
\hat{r}_t^* = \rho_1 \hat{r}_{t-1}^* + \rho_2 \hat{r}(t-2)^* + (1 - \rho_1 - \rho_2)[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y^*) + \phi_{gy}(gy_t - gy^*) + \phi_p(p_t - p_t^*)] + \varepsilon_t^r
\]

where \( \phi_\pi, \phi_y, \phi_{gy}, \phi_p \) capture the strength of the policy response to deviations of inflation, the output gap, the output growth rate and the price level from their respective targets, parameters \( \rho_1 \) and \( \rho_2 \) reflect interest rate smoothing, while \( \varepsilon_t^r \) is a policy shock. We set \( \pi^* = \bar{\pi}, p_t^* = \pi_t^* = \bar{\pi}, y^* = \bar{y} \) and \( gy^* = \bar{gy} \) so that the central bank has no inflationary or output bias. The growth rate of output is related to the output gap by

\[
\hat{gy}_t = \hat{y}_t - \hat{y}_{t-1}
\]

Since the actual level of the net interest rate is bounded by zero, the log deviation of the gross interest rate is bounded by \( \hat{r}_t = \log(R_t) - \log(\bar{R}) \geq -\log(\bar{R}) = -\bar{r} \) with the dynamics of the actual interest rate given by

\[
\hat{r}_t = \max\{\hat{r}_t^*, -\bar{r}\}.
\]

We consider the Taylor rule a reasonable benchmark, because it is likely to be the closest description of the current policy process, and because suggestions to raise the optimal inflation rate are not commonly associated with simultaneous changes in the way that stabilization policy is conducted. However, in section 6.1, we also derive the optimal \( \bar{\pi} \) given optimal stabilization policy under discretion and commitment.

2.2.3 Shocks

We assume that technology follows a random walk process with drift:

\[
a_t = a_{t-1} + \mu + \varepsilon_t^a.
\]

Each of the risk premium, government, and Phillips Curve shocks follow AR(1) processes

\[
\hat{q}_t = \rho_q \hat{q}_{t-1} + \varepsilon_t^q,
\]

\[
\hat{g}_t = \rho_g \hat{g}_{t-1} + \hat{g}_{t-1}^{-1} \varepsilon_t^g,
\]

\[
\hat{m}_t = \rho_m \hat{m}_{t-1} + \varepsilon_t^m
\]

We assume that \( \varepsilon_t^a, \varepsilon_t^q, \varepsilon_t^g, \varepsilon_t^m, \varepsilon_t^r \) are mutually and serially uncorrelated.
2.3 Welfare function

To quantify welfare for different levels of steady-state inflation, we use a second-order approximation to the household utility function as in Woodford (2003).\(^4\) The main result can be summarized by the following proposition, with all proofs in online appendix A.

**Proposition 1** The 2nd order approximation to expected per period utility in eq. (2.1) is\(^5\)

\[
\Theta_0 + \Theta_1 V(\hat{y}_t) + \Theta_2 V(\hat{\pi}_t) + \Theta_3 V(\hat{c}_t)
\]  

(2.31)

where parameters \(\Theta_i, i = 0, 1, 2\) depend on the steady state inflation \(\bar{\pi}\) and are given by

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\(^4\)In our welfare calculations, we use the 2nd order approximation to the consumer utility function while the structural relationships in the economy are approximated to first order. As discussed in Woodford (2011b), this approach is valid if distortions to the steady state are small so that the first order terms in the utility approximation are premultiplied by coefficients that can also be treated as first order terms. Since given our parameterization the distortions from imperfect competition and inflation are small (Woodford, 2003, as in), this condition is satisfied in our analysis. Furthermore, we show in online appendix F that the log-linear solution closely approximates the nonlinear solution, which implies that second order effects on the moments of inflation and output are small and can be ignored in the welfare calculations.

\(^5\)The complete approximation also contains three linear terms, the expected output gap, expected consumption and expected inflation. Since the distortions to the steady state are small for the levels of trend inflation we consider, the coefficients that multiply these terms can be considered as first order so we can evaluate these terms using the first order approximation to the laws of motion as in Woodford (2003). We confirmed in numeric simulations that they can be ignored. Furthermore, second order effects on the expected output gap and expected inflation are likely to be quantitatively small since the linear solution closely approximates the nonlinear solution to the model (see online appendix F).
2.3. WELFARE FUNCTION

\[ \Theta_0 = \left[ 1 - \frac{(1 - \Phi)(1 - \beta h)}{(1 - \bar{g}_y)(1 - h)}(1 - (1 + \eta^{-1})Q^0_y) \right] \log \bar{X} \]
\[ - \frac{(1 - \Phi)(1 + \eta^{-1})(1 - \beta h)}{2(1 - \bar{g}_y)(1 - h)} \log \bar{X}^2 \]
\[ - \frac{(1 - \Phi)(1 - \beta h)}{(1 - \bar{g}_y)(1 - h)} \left\{ (1 + \eta^{-1})[Q^0_y - Q^0_y + \frac{Q^0_y}{2} (1 + \eta^{-1}) \Delta] \right\} \]
\[ \Theta_1 = - \frac{(1 + \eta^{-1})(1 - \beta h)}{2(1 - \bar{g}_y)(1 - h)}; \]
\[ \Theta_2 = - \frac{\theta^2}{2(1 - \bar{g}_y)(1 - h)} \Gamma_3 \{ [Q^1_y(\theta^{-1} - 1) \]
\[ + (1 + \eta^{-1})(1 + \frac{\theta - 1}{\theta} Q^0_y Q^1_y)] - (1 + \eta^{-1}) \frac{\theta - 1}{\theta} Q^1_y \log \bar{X} \}, \]
\[ \Theta_3 = - \frac{h(1 - \rho_c)}{(1 - h)^2} \]
\[ \Gamma_0 = \{ 1 + (\theta - 1)Q^1_p[(1 - \lambda)(\bar{b} + Q^0_p) - \lambda(1 - \omega)\bar{\pi}] \}^{-1}, \]
\[ \Gamma_1 = \{ 1 - (\theta - 1)(1 - \omega)\bar{\pi}Q^1_p \} \Gamma_0, \Gamma_3 = \frac{\Gamma_0}{1 - \lambda \Gamma_1} \{ (1 - \lambda)M^2 + \lambda \}, \]
\[ Q^0_y = \frac{\theta - 1}{2\theta} \bar{Y} [1 + \frac{1}{2} \left( \frac{1 - \theta}{\theta} \right)^2 \bar{Y}]^{-2}, \]
\[ Q^1_y = \left[ 1 - \frac{1}{2} \left( \frac{1 - \theta}{\theta} \right)^2 \bar{Y} \right] /[1 + \frac{1}{2} \left( \frac{1 - \theta}{\theta} \right)^2 \bar{Y}]^{-3}, \]
\[ Q^0_p = \frac{1 - \theta}{2} \Delta /[1 + \frac{1}{2} (1 - \theta)^2 \Delta]^{-2}, \]
\[ Q^1_p = \left[ 1 - \frac{1}{2} (1 - \theta)^2 \Delta \right] /[1 + \frac{1}{2} (1 - \theta)^2 \Delta]^{-3}, \]
\[ \Delta = \pi^2 \lambda (1 - \omega)^2 \left( \frac{1 - \lambda}{1 - \lambda} \right)^2, \Phi = - \log \left( \frac{\theta - 1}{\theta} \right), \text{ and } \rho_c = \text{Cor}(\hat{c}_t, \hat{c}_{t-1}). \]

This approximation of the household utility places no restrictions on the path of nominal interest rates and thus is invariant to stabilization policies chosen by the central bank. The loss function in Proposition 1 illustrates the three mechanisms via which trend inflation affects welfare: the steady-state effects, the effects on the coefficients of the utility-function approximation, and the dynamics of the economy via the second moments of macroeconomic variables.\(^6\) First, the term \( \Theta_0 \) captures the steady-state effects from positive trend inflation, which hinge on the increase

\(^6\)When \( \bar{\pi} = 0 \), equation (2.31) reduces to the standard second-order approximation of the utility function as in Proposition 6.4 of Woodford (2003). There is a slight difference between our approx-
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in the cross-sectional steady-state dispersion in prices (and therefore in inefficient allocations of resources across sectors) associated with positive trend inflation. Note that as $\bar{\pi}$ approaches zero, $\Theta_0$ converges to zero. As shown by Ascari and Ropele (2009), when $\bar{\pi} = 0$, $\frac{\partial \Theta_0}{\partial \bar{\pi}} > 0$, but the sign of the slope quickly reverses at marginally positive inflation rates. In our baseline calibration, $\Theta_0$ is strictly negative and $\frac{\partial \Theta_0}{\partial \bar{\pi}} < 0$ when trend inflation exceeds 0.12% per annum. Thus for quantitatively relevant inflation rates, the welfare loss from steady-state effects is increasing in the steady-state level of inflation. This is intuitive since, except for very small levels of inflation, the steady state level of output declines with higher $\bar{\pi}$ because the steady state cross-sectional price dispersion rises. The steady-state cost of inflation from price dispersion is one of the best-known costs of inflation and arises naturally from the integration of positive trend inflation into the New Keynesian model. Consistent with this effect being driven by the increase in dispersion, one can show that the steady-state effect is eliminated with full indexation of prices and mitigated with partial indexation.

Second, the coefficient on the variance of output around its steady state $\Theta_1 < 0$ does not depend on trend inflation. This term is directly related to the increasing disutility of labor supply. With a convex cost of labor supply, the expected disutility rises with the variance of output around its steady state. However, even though $\Theta_1$ is independent of $\bar{\pi}$, this does not imply that a positive $\bar{\pi}$ does not impose any output cost. Rather, trend inflation reduces the steady state level of output, which is already captured by $\Theta_0$. Once this is taken into account, then log utility implies that a given level of output variance around the (new) steady state is as costly as it was before. Furthermore, the variance of output around its steady state depends on the dynamic properties of the model which are affected by the level of trend inflation.

The coefficient on the variance of inflation $\Theta_2 < 0$ captures the sensitivity of the welfare loss due to the cross-sectional dispersion of prices. One can also show analytically that for $\bar{\pi} \approx 0$, $\frac{\partial \Theta_2}{\partial \bar{\pi}} < 0$ so that the cross-sectional dispersion of prices becomes ceteris paribus costlier in terms of welfare. Because an inflationary shock creates distortions in relative prices and positive trend inflation already generates some price dispersion and an inefficient allocation of resources, firms operating at an inefficient level have to compensate workers for the increasingly high marginal disutility of sector-specific labor. With this rising marginal disutility, the increased distortion in relative prices due to an inflation shock becomes costlier due to the higher initial price dispersion making the variance of inflation costlier for welfare as the trend level of inflation rises. This is a second, and to the best of our knowledge previously unidentified, channel through which the price dispersion from staggered

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7The parameter $\Phi$ measures the deviation of the flexible-price level of output from the flexible-price perfect-competition level of output. See Woodford (2003) for derivation.
price setting under positive inflation reduces welfare.

Finally, the coefficient on the variance of consumption $\Theta_3 < 0$ captures the desire of habit-driven consumers to smooth consumption. The greater the degree of habit formation, the more costly a given amount of consumption volatility becomes. Conversely, the greater the autocorrelation of consumption, the smaller are period-by-period changes in consumption, and the less costly consumption volatility becomes. Trend inflation changes this coefficient only by affecting the persistence of consumption.

2.4 Calibration and Optimal Inflation

Having derived the approximation to the utility function, we now turn to solving for the optimal inflation rate. Because utility depends on the volatility of macroeconomic variables, this will be a function of the structural parameters and shock processes. Therefore, we first discuss our parameter selection and then consider the implications for the optimal inflation rate in the model.

2.4.1 Parameters

Our baseline parameter values are illustrated in Table 1. For the utility function, we set $\eta$, the Frisch labor supply elasticity, equal to one. The steady-state discount factor $\beta$ is set to 0.998 to match the real rate of 2.3% per year on 6-month commercial paper or assets with similar short-term maturities given that we set the steady-state growth rate of real GDP per capita to be 1.5% per year ($\bar{G}_Y = 1.015^{0.25}$), as in Coibion and Gorodnichenko (2011b). We set the elasticity of substitution across intermediate goods $\theta$ to 7, so that steady-state markups are equal to 17%. This size of the markup is consistent with estimates presented in Burnside (1996) and Basu and Fernald (1997). The degree of price stickiness ($\lambda$) is set to 0.55, which amounts to firms resetting prices approximately every 7 months on average. This is midway between the micro estimates of Bils and Klenow (2004), who find that firms change prices every 4 to 5 months, and those of Nakamura and Steinsson (2008), who find that firms change prices every 9 to 11 months. The implied sensitivity of inflation to marginal costs is 0.11, consistent with estimates from Altig, Christiano, Eichenbaum, and Linde (2011).

The degree of price indexation is assumed to be zero in the baseline for three reasons. First, the workhorse New Keynesian model is based only on price stickiness, making this the most natural benchmark (Woodford, 2003). Second, any price indexation implies that firms are constantly changing prices, a feature strongly at odds with the empirical findings of Bils and Klenow (2004) and more recently Nakamura and Steinsson (2008), among many others. Third, while indexation is often included
to replicate the apparent role for lagged inflation in empirical estimates of the New Keynesian Phillips Curve (NKPC; see Gali and Gertler, 1999), Cogley and Sbordone (2008) show that once one controls for steady-state inflation, estimates of the NKPC reject the presence of indexation in price setting decisions. However, we relax the assumption of no indexation in the robustness checks.

The coefficients for the Taylor rule are taken from Coibion and Gorodnichenko (2011b). These estimates point to strong long-run responses by the central bank to inflation and output growth (2.5 and 1.5 respectively) and a moderate response to the output gap (0.43).\(^8\) The steady-state share of consumption is set to 0.80 so that the share of government spending is twenty percent. The calibration of the shocks is primarily taken from the estimated DSGE model of Smets and Wouters (2007) with the exception of the persistence of the risk premium shocks for which we consider a larger value calibrated at 0.947 to match the historical frequency of hitting the ZLB and the routinely high persistence of risk premia in financial time series.\(^9\)

In our baseline model, positive trend inflation is costly because it leads to more price dispersion and therefore less efficient allocations, more volatile inflation, and a greater welfare cost for a given amount of inflation volatility. On the other hand, positive trend inflation gives policy-makers more room to avoid the ZLB on interest rates. Therefore, a key determinant of the tradeoff between the two depends on how frequently the ZLB is binding for different levels of trend inflation. To illustrate the implications of our parameter calibration for how often we hit the ZLB, Figure 1 plots the fraction of time spent at the ZLB from simulating our model for different steady-state levels of the inflation rate. In addition, we plot the steady-state level of the nominal interest rate associated with each inflation rate, where the steady-state nominal rate in the model is determined by \(\bar{R} = \bar{\Pi} \bar{G} Y^{-1}\). Our calibration implies that with a steady-state inflation rate of approximately 3.5%, the average rate for the U.S. since the early 1950’s, the economy should be at the ZLB approximately 5 percent of the time. This is consistent with the post-WWII experience of the U.S.: with U.S. interest rates at the ZLB since late 2008 and expected to remain so by the end of 2011, this yields a historical frequency at the ZLB of 5 percent (i.e. around 3 years out of 60), although we also consider much higher frequencies in section 4.2.\(^10\) For example, we show that our results are qualitatively

\(^8\)Because empirical Taylor rules are estimated using annualized rates while the Taylor rule in the model is expressed at quarterly rates, we rescale the coefficient on the output gap in the model such that \(\phi_y = 0.43/4 = 0.11\).

\(^9\)This calibration is, e.g., consistent with the persistence of the spread between Baa and Aaa bonds which we estimate to be 0.945 between 1920:1 and 2009:2 and 0.941 between 1950:1 and 2009:2 at the quarterly frequency.

\(^10\)Of possible concern may be that this calculation includes the high-inflation environment from 1970-85. Excluding those years generates a historical frequency at the ZLB of 3/45=6.66% but now at a lower trend inflation rate of 3% per year. Our baseline calibration generates approximately that frequency at 3% trend inflation.
robust to assuming that the current ZLB episode can last until 2017 (i.e. a ZLB frequency of 15% at 3% trend inflation), thereby corresponding to a full “lost decade” far exceeding in length the current Fed commitment to sustain “exceptionally low” interest rates until at least mid-2013.

In addition, our baseline calibration agrees with the historical changes in interest rates associated with post-WWII U.S. recessions. For example, starting with the 1958 recession and excluding the current recession, the average decline in the Federal Funds Rate during a recession has been 4.76 percentage points. The model predicts that the average nominal interest rate with 3.5% steady-state inflation is around 6%, so the ZLB would not have been binding during the average recession, consistent with the historical experience. Only the 1981-82 recession led to a decline in nominal interest rates that would have been sufficiently large to reach the ZLB (8.66% drop in interest rates), but did not because nominal interest rates and estimates of trend inflation over this period were much higher than their average values. Thus, with 3-3.5% inflation, our calibration (dotted line in Figure 1) implies that it would take unusually large recessions for the ZLB to become binding. In addition, our calibration indicates that at much lower levels of $\bar{\pi}$, the ZLB would be binding much more frequently: e.g. at $\bar{\pi} = 0$, the ZLB would be binding 27% of the time. This seems conservative since it exceeds the historical frequency of U.S. recessions. The model predicts a steady-state level of interest rates of less than 2.5% when $\bar{\pi} = 0$, and six out the last eight recessions (again excluding the current episode) were associated with decreases in interest rates that exceeded this value (specifically the 1969, 1973, 1980, 1981, 1990 and 2001 recessions). Our calibration is also largely in line with the frequency of the ZLB we would have observed given historical declines in nominal interest rates during recessions and counterfactual levels of trend inflation (broken line in Figure 1). Thus, our parameterization provides a reasonable representation of the likelihood of hitting the ZLB for different inflation rates given the historical experience of the U.S.

Our calibration also accounts for the key moments of output, inflation, interest rates and consumption. Table 2 presents the variance and autocorrelation of each HP-filtered variable in the model and U.S. data from 1947Q1 to 2011Q1. The model reproduces both the absolute and relative volatilities of these variables as well as their persistence, although the persistence of consumption and output are slightly lower in the model than the data. The model also replicates the strong comovement of consumption with output and the much lower comovement of inflation and interest rates with output.

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111This value is calculated by taking the average level of the Federal Funds rate (FFR) over the last 6 months prior to the start of each NBER recession and subtracting the minimum level of the FFR in the aftermath of that recession.
2.4. CALIBRATION AND OPTIMAL INFLATION

2.4.2 Optimal Inflation

Having derived the dynamics of the model, parameterized the shocks, and obtained the second-order approximation to the utility function, we now simulate the model for different levels of trend inflation $\bar{\pi}$ and compute the expected utility for each $\bar{\pi}$. We use the Bodenstein, Erceg, and Guerrieri (2009) algorithm to solve the non-linear model and verify in online appendix F that this algorithm has very high accuracy, even after large shocks leading to a binding ZLB. The results taking into account the ZLB and in the case when we ignore the ZLB are plotted in Panel A of Figure 2. When the ZLB is not taken into account, the optimal rate of inflation is zero because there are only costs to inflation and no benefits. Figure 2 also plots the other extreme when we include the ZLB but do not take into account the effects of positive steady-state inflation on the loss function or the dynamics of the model. In this case, there are no costs to inflation so utility is strictly increasing as steady-state inflation rises and the frequency of the ZLB diminishes. Our key result is the specification which incorporates both the costs and benefits of inflation. As a result of the ZLB constraint, we find that utility is increasing at very low levels of inflation so that zero inflation is not optimal when the zero bound is present. Second, the peak level of utility is reached when the inflation rate is 1.5% at an annualized rate. This magnitude is close to the bottom end of the target range of most central banks, which are commonly between 1% and 3%. Thus, our baseline results imply that taking into account the zero bound on interest rates raises the optimal level of inflation, but with no additional benefits to inflation included in the model, the optimal inflation rate is within the standard range of inflation targets. Third, the costs of even moderate inflation can be nontrivial: a 5% inflation rate would lower utility by approximately 1% relative to the optimal level, which given log utility in consumption is equivalent to a permanent 1% decrease in the level of consumption. As we show later, the magnitude of the welfare costs of inflation varies with the calibration and price setting assumptions, but the optimal rate of inflation is remarkably insensitive to these modifications.

Panel B of Figure 2 quantifies the importance of each of the three costs of inflation – the steady state effect, the increasing cost of inflation volatility, and the positive link between the level and volatility of inflation – by calculating the optimal inflation rate subject to the ZLB when only one of these costs, in turn, is included. The first finding to note is that allowing for any of the three inflation costs is sufficient to bring the optimal inflation rate to 3.6% or below. Thus, all three inflation costs incorporated in the model are individually large enough to prevent the ZLB from pushing the optimal inflation rate much above the current target range of most central banks. Second, the steady-state cost is the largest cost of inflation out of the three, bringing the optimal inflation rate down to 1.6% by itself. However, even if we omit steady-state costs and include only the other two channels, the optimal
inflation rate would be less than 3%.

To get a sense of which factors drive these results, the top row of Figure 3 plots the coefficients of the second-order approximation to the utility function from Proposition 1. First, higher $\bar{\pi}$ has important negative steady-state effects on utility, as the increasing price dispersion inefficiently lowers the steady-state level of production and consumption. Second, the coefficient on the variance of consumption becomes slightly smaller in absolute value for low levels of inflation then rises moderately at higher levels of inflation. Third, the coefficient on inflation variance is decreasing in $\bar{\pi}$, i.e., holding the inflation variance constant, higher $\bar{\pi}$ raises the utility cost of the variance in inflation. This reflects the fact that when the steady state level of price dispersion is already high then a temporary increase in price dispersion due to an inflation shock is even more costly. Moving from zero inflation to six percent inflation raises the coefficient on the inflation variance by almost 30% in absolute value. Thus, as $\bar{\pi}$ rises, policy-makers should place an increasing weight on the variance of inflation relative to the variance of the output gap.

The middle row of Figure 3 plots the effects of $\bar{\pi}$ on the variance of inflation, consumption and the output gap, i.e. the dynamic effects of steady-state inflation and the ZLB. In addition, we plot the corresponding moments in the absence of the zero-bound on interest rates to characterize the contribution of the zero-bound on macroeconomic dynamics. A notable feature of the figure is how rapidly consumption, output and inflation volatility rise as $\bar{\pi}$ falls when the ZLB is present. Intuitively, the ZLB is hit more often at a low $\bar{\pi}$. With the nominal rate fixed at zero, the central bank cannot stabilize the economy by cutting interest rates further and thus macroeconomic volatility increases. As we increase $\bar{\pi}$, macroeconomic volatility diminishes. This is the benefit of higher $\bar{\pi}$ in the model. The effect of changes in $\bar{\pi}$, however, is non-linear for the variance of inflation when we take into account the zero-bound on interest rates. At low levels of inflation, increasing $\bar{\pi}$ reduces the volatility of inflation for the same reason as for output: the reduced frequency of hitting the zero bound. On the other hand, higher $\bar{\pi}$ also tends to make pricing decisions more forward-looking, so that, absent the zero bound, inflation volatility is consistently rising with $\bar{\pi}$, a feature emphasized in Kiley (2011) and consistent with a long literature documenting a positive relationship between the level and variance of inflation (Okun, 1971; Taylor, 1981). When $\bar{\pi}$ rises past a specific value, the latter effect dominates and the variance of inflation rises with $\bar{\pi}$. Given our baseline values, this switch occurs at an annualized trend inflation rate of approximately 3.5%. These results show the importance of modeling both the ZLB and the effects of $\bar{\pi}$ on the dynamics of the model.

The bottom row of Figure 3 then plots the contribution of these different effects on the welfare costs of inflation, i.e. each of the terms in Proposition 1. These include the steady-state effects of $\bar{\pi}$ as well as the interaction of the effects of $\bar{\pi}$ on the coefficients of the utility function approximation and the dynamics of the
2.4. CALIBRATION AND OPTIMAL INFLATION

The most striking result is that the welfare costs and benefits of positive $\bar{\pi}$ are essentially driven by only two components: the steady-state effect and the contribution of inflation variance to utility. In particular, the U-shape pattern of the inflation variance combined with decreasing $\Theta_2$ plays the key role in delivering a positive level of the optimal inflation rate, while the effects of the ZLB on the contribution of the output gap and consumption variability are an order of magnitude smaller and therefore play a limited role in determining the optimal inflation rate.

2.4.3 Are the costs of business cycles and the ZLB too small in the model?

The minor contribution of output gap volatility to the optimal inflation rate might be interpreted as an indication that the model understates the costs of business cycles in general and the ZLB in particular. For the former, the implied welfare costs of business cycles in our model are approximately 0.5% of steady-state consumption at the historical trend inflation rate, in line with many of the estimates surveyed in Barlevy (2004) and much larger than in Lucas (1987). To assess the cost of hitting the ZLB, we compute the average welfare loss net of steady-state effects from simulating the model under different inflation rates both with and without the zero bound. The difference between the two provides a measure of the additional welfare cost of business cycles due to the presence of the ZLB. We can then divide this cost by the average frequency of being at the zero bound from our simulations, for each level of steady-state inflation, to get a per-quarter average welfare loss measure conditional on being at the ZLB which is plotted in Panel A of Figure 4. As $\bar{\pi}$ rises, this per-period cost declines because the average duration of ZLB episodes gets shorter and the output losses during the ZLB are increasing non-linearly with the duration of the ZLB (Christiano et al., n.d., see). For example, the average cost of a quarter spent at the ZLB is approximately equivalent to a permanent 1.4% reduction in consumption when inflation is 1% but declines to 0.4% at a 3.5% rate of inflation. The latter implies that the additional cost of being restrained by the zero bound for 8 quarters is equivalent to a 3.2% permanent reduction in consumption, or approximately $320 billion per year based on 2008 consumption data. For comparison, Williams (2009) uses the Federal Reserve’s FRB/US model to estimate that the ZLB between 2009 and 2010 cost $1.8 trillion in lost output over four years, or roughly $300 billion per year in lost consumption over four years if one assumes that the decline in consumption was proportional to the decline in output. Thus, the costs of both business cycles and the ZLB in the model cannot be described as being uncharacteristically small.

However, while the conditional costs of long ZLB events are quite large, they also occur relatively infrequently. For example, if we assume that all ZLB episodes are 8

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quarters long, then at 3.5% trend inflation an 8-quarter episode at the ZLB occurs with probability 0.007 each quarter, or about 3 times every 100 years. This implies that the expected cost of the ZLB is a 0.02% permanent reduction of consumption. Similar calculations for 2% trend inflation reveal that while the conditional cost of an 8-quarter ZLB event is about a 6.2% permanent reduction of consumption, the unconditional cost of the ZLB is only a 0.08% permanent reduction in consumption. Thus, while the model implies that a higher inflation target can significantly reduce the cost of a given ZLB event, as suggested by Blanchard, taken over a long horizon the expected gain in mitigating the ZLB from such a policy is small. As a result, even modest steady-state costs of inflation, because they must be borne every period, are sufficient to push the optimal inflation rate below 2%.

2.4.4 How does optimal inflation depend on the coefficient on the variance of the output gap?

Even though the costs of business cycles are significant and ZLB episodes are both very costly and occurring with reasonable probability, one may be concerned that these costs are incorrectly measured due to the small relative weight assigned to output gap fluctuations in the utility function. At $\bar{\pi} = 0$, the coefficient on the output gap variance in the loss function is less than one-hundredth that on the quarterly inflation variance (or one-tenth for the annualized inflation variance), and this difference becomes even more pronounced as $\bar{\pi}$ rises. The low weight on output gap volatility is standard in New Keynesian models and could reflect the lack of involuntary unemployment, which inflicts substantial hardship to a fraction of the population and whose welfare effects may be poorly approximated by changes in aggregate consumption and employment, or the absence of distribution effects, as business cycles disproportionately impact low income/wealth agents with higher marginal utilities of consumption than the average consumer.

To assess how sensitive the optimal inflation rate is to the coefficient on the output gap variance, we increase this coefficient by a factor ranging from 1 to 100 and reproduce our results for the optimal $\bar{\pi}$ for each factor (see Panel B of Figure 4). Raising the coefficient on the variance of the output gap pushes the optimal inflation rate higher, but the coefficient on the output gap variance needs to be very large to qualitatively affect our findings. For example, much of the traditional literature on optimal monetary policy assumed equal weights on output and annualized inflation variances in the loss function. With inflation being measured at an annualized rate, this equal weighting obtains at zero steady-state inflation when $\Theta_1$ is multiplied by a factor of approximately 10. Yet this weighting would push the optimal inflation rate up only modestly to 1.6% per year. Even if one increased the coefficient on output gap volatility by a factor of 100, the optimal inflation rate would rise only to 2.4%.
2.5. ROBUSTNESS OF THE OPTIMAL INFLATION RATE TO ALTERNATIVE PARAMETER VALUES

Placing such weight on output volatility would raise the implied per-quarter cost of having the ZLB bind to an equivalent of a 3% permanent reduction in consumption, such that an episode of 8 consecutive quarters at the ZLB would deliver welfare losses equivalent to roughly 24% of steady-state consumption, above and beyond the costs of the shock in the absence of the ZLB. Thus, while one can mechanically raise the optimal inflation rate via larger weights on output fluctuations than implied by the model, weighting schemes which meaningfully raise the optimal inflation rate point to welfare costs of business cycles, and particularly episodes at the ZLB, that depart from the conventional wisdom.

2.5 Robustness of the Optimal Inflation Rate to Alternative Parameter Values

In this section, we investigate the robustness of the optimal inflation rate to our parameterization of the model. We focus particularly on pricing and utility parameters, the discount factor, and the risk premium shock.

2.5.1 Pricing and Utility Parameters

Figure 5 plots the optimal inflation rates and associated welfare losses for different levels of $\bar{\pi}$ for alternative pricing and utility parameters. First, we consider the role of the elasticity of substitution $\theta$ (Panel A), which plays a critical role in determining the cost of price dispersion, and therefore costs of inflation, in the model. Note that the welfare costs of inflation are larger when $\theta$ is high. This result captures the fact that a higher elasticity of substitution generates more steady-state output dispersion and, through greater real rigidity, higher welfare cost of fluctuations for any $\bar{\pi}$. However, the effects of this parameter on the optimal $\bar{\pi}$ are relatively small: even a value of $\theta = 3$ yields an optimal $\bar{\pi}$ of less than 2%. Thus, the optimal inflation rate is robust to the range of values of $\theta$ commonly considered in the macroeconomics literature.

However, microeconomic estimates of demand elasticities commonly find much lower values than those employed in macroeconomic models. Hausman, Leonard, and Zona (1994), for example, document that while the elasticity of demand for individual beer brands may be around 5, the overall elasticity of demand for beer is closer to 1.5. To the extent that goods across industries may be less substitutable than goods within industries, our baseline model could overstate the amount of price and output dispersion and therefore the steady-state costs of inflation. This effect would likely be particularly pronounced if price changes within industries were synchronized so that the relevant amount of price dispersion for welfare arises primarily from cross-industry substitution at low elasticities.

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We can assess the implications of these possibilities by introducing a few modifications to our baseline model. Specifically, we can consider a two-tier model in which the elasticity of substitution within industries is given by $\theta$ and the elasticity of substitution across industries is given by $\mu$ where we assume $\theta > \mu$. Prices are sticky at the firm level but not at the industry level – industries simply combine individual goods and sell them off to consumers. In the absence of industry-specific shocks and price-synchronization within industries, average price levels across industries must be equal since industries are symmetric and Calvo price-stickiness washes out at the industry level. As a result, there would be no price dispersion across industries. In addition, if we assume that there is a continuum of firms that aggregate for a given industry $i$, then there is no industry mark-up. Thus the steady state mark-up on goods would be $\frac{\theta}{(\theta - 1)}$ and this model would have exactly the same implications as our baseline model with all of the results being driven by $\theta$ rather than $\mu$. This suggests that, absent price synchronization, it is the elasticity of substitution within industries which is most relevant for our results. Intuitively, since all industries have the same price index and the same intra-industry price dispersion, we can combine them into a “representative” industry, thereby reducing this model to our baseline setup.

An important potential caveat to this result, however, is if price changes are synchronized within industries, as in Bhaskar (2002). Price synchronization within industries could significantly reduce price dispersion within industries while the remaining price dispersion across industries would matter little for welfare under low cross-industry elasticities of substitution. To quantify this possibility, we used a model with Taylor pricing (as in section 5.4) with four industries, each of which had staggered pricing over four quarters, but in which disproportionate shares of firms within each industry reset their prices in the same quarter. We allowed for the elasticity of substitution within industries ($\theta = 7$) to exceed that across industries ($\mu = 3$). Even high levels of synchronization (i.e. 70% of firms in an industry resetting their prices in the same quarter) had only modest effects on steady-state levels of price dispersion and thus on our welfare calculations.\(^{12}\) While these modifications suggest that our results are robust to a number of alternative assumptions about industrial structure and pricing assumptions, one should nonetheless bear in mind that there is significant uncertainty about how best to model the substitution of and

\[^{12}\text{These results are available upon request. The empirical evidence on price synchronization is mixed. Dhyne and Konieczny (2007), for example, document that price synchronization within industries is larger than across industries but also document a remarkable degree of staggering of price changes within industries. For example, they observe (p. 11), “[...]price changes at the individual product category level are neither perfectly staggered nor synchronized, but their behavior is much closer to perfect staggering.” Likewise, a more recent study, Klenow and Malin (2010) use data from U.S. Bureau of Labor Statistics and find that the timing of price changes is little synchronized across sellers even within very narrow product categories.}\]
2.5. ROBUSTNESS OF THE OPTIMAL INFLATION RATE TO ALTERNATIVE PARAMETER VALUES

pricing goods across industries and that alternative specifications may quantitatively affect the predictions about the costs of price dispersion.

Second, we also investigate the role of price indexation. In our baseline, we assumed $\omega = 0$, based on the fact that firms do not change prices every period in the data, as documented by Bils and Klenow (2004) and Nakamura and Steinsson (2008), as well as the results of Cogley and Sbordone (2008) who argue that once one controls for time-varying trend inflation, we cannot reject the null that $\omega = 0$ for the US. However, because price-indexation is such a common component of New Keynesian models, we consider the effects of price indexation on our results. Panel B of Figure 5 indicates that higher levels of indexation lead to higher optimal rates of inflation because indexation reduces the dispersion of prices. Yet with $\omega = 0.5$, which is most likely an upper bound for an empirically plausible degree of indexation in low-inflation economies like the U.S., the optimal $\bar{\pi}$ remains less than 2.4%.\(^{13}\)

Third, we examine the effects of price stickiness. Our baseline calibration, $\lambda = 0.55$, is midway between the findings of Bils and Klenow (2004) of median price durations of 4-5 months and those of Nakamura and Steinsson (2008) of median price durations of 9-11 months. We now consider values of $\lambda$ ranging from 0.50 to 0.75 (Panel C), which imply inflation sensitivity to marginal cost ranging from 0.14 to 0.02 (at zero trend inflation). With more price stickiness, price dispersion is greater, and this effect is amplified at higher levels of steady-state inflation, thereby generating much larger welfare losses. In addition, it limits the severity of deflationary spirals at the ZLB, which reduces the benefits of higher trend inflation. Nonetheless, this has only minor effects on the optimal inflation rate.

Finally, we consider variation in the levels of habit formation and the Frisch labor supply elasticity in Panels D and E. The specific values applied to both parameters can significantly affect the level of welfare losses at the optimal inflation rate but neither qualitatively alters the optimal inflation rate in the model.

2.5.2 Discount Factor and Risk Premium Shocks

We also consider the sensitivity of our results to the discount factor and the parameters governing the risk premium shocks (Figure 6). First, we reproduce our baseline welfare figure for different levels of the persistence to risk premium shocks. The results are quite sensitive to this parameter, which reflects the fact that these shocks play a crucial role in hitting the zero lower bound. For example, Figure 6 illustrates that when we raise the persistence of the shock from 0.947 to 0.96, the optimal inflation rate rises from 1.5% to 3% because this increase in the persistence

\(^{13}\)Standard estimates of the Phillips Curve without trend inflation (e.g., Galì and Gertler, 1999; Levin, Onatski, Williams, and Williams, 2006) suggest that the fraction of indexing firms is at most 0.5 and more likely between 0.25-0.35. We also solved the model with dynamic indexation and found nearly identical results.
of the shock has a large effect on the frequency and duration of being at the ZLB. At 3.5% inflation, this frequency more than doubles relative to our baseline scenario, thereby raising the benefit of higher steady-state inflation. The reverse occurs with lower persistence of risk premium shocks: the frequency of being at the ZLB declines sharply as does the optimal inflation rate. Second, similar results obtain when we vary the volatility of the risk premium shock. When we increase the standard deviation of these shocks to 0.0035 from our baseline of 0.0024, the optimal inflation rate again rises to slightly over 3 percent. As with the persistence of the shocks, this is driven by a higher frequency of being at the ZLB: at 3.5% inflation, this alternative shock volatility implies the economy would be at the ZLB three times as often as under our baseline calibration.

Third, we consider the sensitivity of our results to the steady-state level of the discount factor $\beta$. This parameter is also important in determining the frequency at which the economy is at the ZLB since it affects the steady-state level of nominal interest rates. As with the risk premium shock variables, a higher value of $\beta$ is associated with a lower steady-state level of nominal interest rates, so that the ZLB will be binding more frequently. For example, with $\beta = 0.9999$ (which corresponds to a real rate of 1.54% per year), the ZLB is binding approximately 7% of the time when steady-state inflation is 3.5%. At the maximum, however, the optimal $\bar{\pi}$ is only 0.6% higher than implied by our baseline results. These robustness checks clearly illustrate how important the frequency at which the economy hits the ZLB is for our results. Naturally, parameter changes which make the ZLB binding more often raise the optimal rate of inflation because a higher $\bar{\pi}$ lowers the frequency of hitting the ZLB. Thus, the key point is not the specific values chosen for these parameters but rather having a combination of them that closely reproduces the historical frequency of hitting the ZLB for the U.S. Nonetheless, even if we consider parameter values that double or even triple the frequency of hitting the ZLB at the historical average rate of inflation for the U.S., the optimal inflation rate rises only to about 3%. This suggests that the evidence for an inflation target in the neighborhood of 2% is robust to a wide range of plausible calibrations of hitting the ZLB.

### 2.5.3 Summary

These results indicate that the optimal inflation rate in the baseline model is robust to reasonable variations in the parameters as calibrated to the U.S. economy. However, the fact that more persistent and volatile shocks could potentially raise the frequency of the ZLB suggests that the optimal inflation rate is likely to vary across countries. For example, smaller open economies are typically more subject to volatile terms of trade shocks than the U.S. and this increased volatility could justify higher target rates of inflation if it increases the incidence of hitting the ZLB. Similarly, economies such as Japan in which the real interest rate has historically been very
low might also find it optimal to pursue higher inflation targets. More broadly, our results highlight the importance of carefully calibrating the model parameters to the specifics of the economy before drawing general conclusions about optimal policies.

2.6 What could raise the Optimal Inflation Rate?

While our baseline model emphasizes the tradeoff between higher $\bar{\pi}$ to insure against the zero-bound on nominal interest rates versus the utility costs associated with higher trend inflation, previous research has identified additional factors beyond the lower-bound on nominal interest rates which might lead to higher levels of optimal inflation. In this section, we extend our analysis to assess their quantitative importance. First, we include capital formation in the model. Second, we allow for uncertainty about parameter values on the part of policy-makers. Third, we integrate downward nominal wage rigidity, i.e. “greasing the wheels,” into the model. Fourth, we explore whether our results are sensitive to using Taylor pricing. Finally, we consider the possibility that the degree of price stickiness varies with $\bar{\pi}$.

2.6.1 Capital

First, we consider how sensitive our results are to the introduction of capital. We present a detailed model in online appendix B and only provide a verbal description in this section. In this model, firms produce output with a Cobb-Douglas technology (capital share $\alpha = 0.33$). All capital goods are homogeneous and can be equally well employed by all firms. Capital is accumulated by the representative consumer subject to a quadratic adjustment cost to capital ($\psi = 3$ as in Woodford, 2003) and rented out in a perfectly competitive rental market. The aggregate capital stock depreciates at rate $\delta = 0.02$ per quarter. We calculate the new steady state level of output relative to the flexible price level output and derive the analogue of Proposition 1 in online appendix B with proofs in online appendix C.

By allowing capital to freely move between firms we reduce the steady state welfare cost from trend inflation since firms that have a relatively low price can now hire additional capital rather than sector-specific workers to boost their output. Capital also increases the likelihood of hitting the ZLB and therefore the benefits of inflation because including capital permits disinvestment when agents prefer storing wealth in safe bonds rather than capital, so we are more likely to be in a situation where an increase in $q_t$ pushes interest rates to zero. We isolate the first channel by setting $\rho_q = 0.943$ to match the historical frequency at the ZLB. As shown in Panel A of Figure 7, utility peaks at a trend inflation rate of 2.1% suggesting that capital does not lower the cost of inflation substantially.
2.6. WHAT COULD RAISE THE OPTIMAL INFLATION RATE?

2.6.2 Model Uncertainty

An additional feature that could potentially lead to higher rates of optimal inflation is uncertainty about the model on the part of policy-makers. If some plausible parameter values lead to much higher frequencies of hitting the ZLB or raise the output costs of being at the ZLB, then policy-makers might want to insure against these outcomes by allowing for a higher $\pi$. We quantify this uncertainty via the variance-covariance matrix of the estimated parameters from Smets and Wouters (2007), placing an upper bound on parameter values to eliminate draws where the ZLB binds unrealistically often, in excess of 10% at the historical average of annual trend inflation.

To assess the optimal inflation rate given uncertainty about parameter values, we compute the expected utility associated with each level of steady-state inflation by repeatedly drawing from the distribution of parameter values. Panel B of Figure 7 plots the implied levels of expected utility associated with each steady-state level of inflation. Maximum utility is achieved with an inflation rate of 1.9% per year. As expected, this is higher than our baseline result, which reflects the fact that some parameter draws lead to much larger costs of being at the zero-bound, a feature which also leads to a much more pronounced inverted U-shape of the welfare losses from steady-state inflation. Nonetheless, this optimal rate of inflation remains well within the bounds of current inflation targets of modern central banks.

We also consider another exercise in which we repetitively draw from the parameter space and solve for the optimal inflation rate associated with each draw. The distribution of optimal inflation rates has a 90% confidence interval ranging from 0.3% to 2.9% per year, which again is very close to the target range for inflation of most central banks.

2.6.3 Downward Wage Rigidity

A common motivation for positive trend inflation, aside from the zero-lower bound, is the “greasing the wheels” effect raised by Tobin (1972). If wages are downwardly rigid, as usually found in the data (e.g., Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward, 2007), then positive trend inflation will facilitate the downward-adjustment of real wages required to adjust to negative shocks. To quantify the effects of downward nominal wage rigidity in our model, we integrate it in a manner analogous to the zero-bound on interest rates by imposing that changes in the aggregate nominal wage index be above a minimum bound $\Delta \hat{w}_t = \max\{\Delta \hat{w}_t^m, \Delta \hat{w}_t^*\}$ where $\Delta \hat{w}_t^*$ is the change in wages that would occur in the absence of the zero-bound on nominal wages and $\Delta \hat{w}_t^m$ is the lower-bound on nominal wage changes. Note that even with zero steady-state inflation, steady-state nominal wages grow at the rate of technological progress. Thus, we set $\Delta \hat{w}_t^m$ to be
2.6. WHAT COULD RAISE THE OPTIMAL INFLATION RATE?

equal to minus the sum of the growth rate of technology and the steady state rate of inflation.

Panels C and D of Figure 7 present the utility associated with different $\bar{\pi}$ under both the zero-bound on interest rates and downward-wage rigidity. The result is striking: the optimal inflation rate falls to 0.3% per year with downward wage rigidity. With downward wage rigidity, marginal costs are less volatile, so the variance of inflation is reduced relative to the case with flexible wages. In addition, the fact that marginal costs are downwardly rigid means that, in the face of a negative demand shock, inflation will decline by less and therefore interest rates will fall less, reducing the frequency of the ZLB. With $\bar{\pi} = 0$, the ZLB binds less than 20 percent of the time with downward wage rigidity but over 27 percent of the time with flexible wages. This decrease in the frequency of binding ZLB at low $\bar{\pi}$ reduces the benefit of higher trend inflation and leads to lower estimates of the optimal inflation rate.\footnote{Kim and Ruge-Murcia (2009) similarly find that downward wage rigidity, by itself, has little positive effect on the optimal inflation rate in an estimated DSGE model.}

2.6.4 Taylor pricing

Our baseline model relies on the Calvo (1983) price-setting framework. While analytically convenient, this approach suffers from several drawbacks that could affect our estimates of the optimal inflation rate. One such factor is that with exogenous probabilities of changing prices each period, there is always a fraction of firms using very outdated prices. With positive steady-state inflation, these prices will be very low in relative terms, leading to high estimates of the cost of price dispersion even at moderate inflation rates. To assess how important this factor is for the optimal rate of inflation, we consider an alternative approach to Calvo pricing, namely the staggered contracts approach of Taylor (1979), in which firms set prices for a pre-determined duration of time. With fixed durations of price stickiness, price dispersion should be smaller than under Calvo pricing for sufficiently high inflation rates. The derivation of the utility approximation as well as the structural log-linearized equations of the model is similar to Calvo pricing (online appendix D contains details for the utility approximation when the duration of price spells is equal to three quarters).

Panels E and F of Figure 7 compare the results under Taylor pricing (using durations of price contracts equal to 3 and 4 quarters) with the results from Calvo pricing. The optimal inflation rates for the Taylor model are 2.2 and 1.8 percent per year for price durations of 3 and 4 quarters respectively, which is close to the 1.5 percent per year found for the baseline Calvo model. The models yield similar volatilities of inflation and the output gap and frequencies of hitting ZLB (see Coibion, Gorodnichenko, and Wieland, 2011 for more details) are approximately the same in all models. Instead, the main difference is that the Taylor model has smaller

\footnote{Kim and Ruge-Murcia (2009) similarly find that downward wage rigidity, by itself, has little positive effect on the optimal inflation rate in an estimated DSGE model.}
2.6. WHAT COULD RAISE THE OPTIMAL INFLATION RATE?

welfare losses as $\bar{\pi}$ increases above 2 percent because of the reduced steady-state price dispersion. Intuitively, since firms under Calvo pricing may be stuck with a suboptimal price for a long time, the cost of positive steady state inflation is larger than in the Taylor model where firms are guaranteed to change prices in a fixed number of periods. The quantitative implications for the optimal inflation rate, however, are quite small.

2.6.5 Endogenous and State-Dependent Price Stickiness

Because theory implies that the cost to firms of not changing prices should increase as $\bar{\pi}$ rises (Romer, 1990), higher levels of inflation should be associated with lower levels of price stickiness, which would tend to lower the welfare costs of positive trend inflation. Thus, by ignoring this endogeneity, we might be overstating the costs of positive inflation and thereby underestimating the optimal rate of inflation. To address this possibility, we consider the sensitivity of our baseline results to a possible systematic link between the inflation rate and the degree of price stickiness in two ways. As a first approach, we follow Nakamura and Steinsson’s empirical approach and posit a linear relationship between the (monthly) frequency of price changes and the steady state annual rate of inflation, with the coefficient on inflation denoted by $\beta_\pi$. The average estimate of Nakamura and Steinsson across price measures and time periods is approximately $\beta_\pi = 0.5$, and the upper bound of their confidence intervals is approximately $\beta_\pi = 1$. We calibrate the degree of price rigidity such that $\lambda = 0.55$ (our baseline value) at a steady-state level of annual inflation of 3.5%. In this setting, the degree of price stickiness varies with the steady-state rate of inflation but not with business cycle conditions.

As a second approach, we consider state-dependent pricing in the spirit of Dotsey et al. (1999). Using the same model as in section II, we replace the exogenous probability of changing prices with an explicit optimizing decision based on comparison of menu costs and the gains from price adjustment. Specifically, we assume that each period, firms draw from a uniform distribution of costs to changing prices and, conditional on their draw, decide whether or not to reset their price. Because all firms are identical, every firm that chooses to reset its price picks the same price. As in the Taylor (1979) staggered contracts model, the distribution of prices in the economy depends only on past reset prices and the share of firms which changed their price in previous quarters, but, in contrast to time-dependent models, these shares are time-varying. Derivations of the model and the associated welfare loss function are provided in online appendix E. Using the same parameter values as the baseline model, we calibrate the average size of menu costs to yield the same degree of price stickiness at a steady-state inflation rate of 3.5% as in the baseline case. Because this calibration implies strong strategic complementarity in price setting, large menu costs (approximately 7% of output) are necessary to match the duration
of price spells in the data.\footnote{The size of the menu costs is in line with those reported in Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) of approximately 6\% of operating expenses. However, Zbaracki et al. (2004) emphasize that these costs appear to be largely associated with multiproduct pricing decisions, whereas in our model firms produce a single good. Unfortunately, there is little other direct empirical evidence on the size of menu costs.} We therefore quantify the sensitivity of our results to this parameterization by varying the elasticity of substitution across intermediate goods from $\theta = 10$ to $\theta = 5$ while simultaneously varying the menu costs from 11\% to 4\% of revenue to maintain the same average price duration at 3.5\% inflation as in our baseline calibration.

Panels A and D in Figure 8 illustrate the implied variation in the degree of price stickiness for both methods. In each case, higher steady state levels of inflation lead to more frequent updating of prices. Panels B and E plot the welfare associated with different $\bar{\pi}$ and different parameters for each approach. In the first approach, more endogeneity in the degree of price stickiness leads to slightly lower optimal levels of inflation. The fact that the optimal inflation rate declines when the sensitivity of price stickiness to inflation rises may seem counterintuitive: higher inflation should lead to faster updating of prices and therefore less price dispersion. But the endogenous rate of price stickiness also implies that inflation volatility rises more rapidly with average inflation than in the baseline case which raises the cost of inflation. Furthermore, a higher degree of price stickiness at low inflation rates reduces the severity of deflationary spirals and thus the benefits of higher trend inflation. Given our parameter values and the range of inflation rates that we consider, the latter two effects approximately offset the former at low levels of inflation so endogenous price stickiness actually leads to slightly lower optimal inflation rates than the standard Calvo model. Second, the welfare costs of inflation at the optimal rate are rising with endogenous price setting. This reflects the fact that, given the same low optimal rate of inflation, more endogeneity is associated with higher degrees of price stickiness and therefore a higher cost of inflation when $\bar{\pi} < 3.5\%$. Third, despite the fact that the optimal rate of inflation varies little with endogenous price stickiness, the costs of much higher $\bar{\pi}$ are significantly lower relative to our baseline, because higher inflation leads to more frequent price changes and therefore price dispersion rises less rapidly with steady state inflation than under constant price stickiness.

Similarly, allowing for state-dependent pricing lowers the optimal inflation rate relative to our baseline model as well as the welfare losses from higher inflation rates, with the latter reflecting the reduced sensitivity of price dispersion to inflation due to endogenous pricing decisions, as in Burstein and Hellwig (2008).\footnote{For easier comparison with the other models, we present welfare results net of menu costs. Including the menu costs pushes the optimal inflation rate down by approximately 0.2\%-0.5\% per year.} Reducing the size of menu costs raises the optimal inflation rate, but the effects are quantitatively
small. In short, the state-dependent model confirms the results reached using the endogenous Calvo price durations: allowing for price stickiness to fall with inflation lowers the optimal inflation rate relative our baseline model, even as the costs of higher inflation are substantially reduced.

2.7 Normative Implications

We have so far been treating the question of the optimal inflation rate independently of the systematic response of the central bank to macroeconomic fluctuations. In this section, we investigate the implications of optimal stabilization policy, both under commitment and discretion, for the optimal rate of inflation. We then consider whether altering the parameters of the Taylor rule can lead to outcomes that approach those achieved using optimal stabilization policy with commitment.

2.7.1 Optimal Stabilization Policy

In the baseline model, stabilization policy follows a Taylor rule calibrated to match the historical behavior of the Federal Reserve. Following Giannoni and Woodford (2010) and Woodford (2011b), we derive the optimal policy rules under commitment and discretion, and simulate the model with our baseline parameter values. Panel A of Figure 9 plots expected utility under both policies. When the central bank can commit to a particular policy rule then the optimal inflation rate is practically zero (0.2%). This is not because the ZLB binds less frequently—it actually binds much more frequently at the optimal inflation rate—but rather because the cost of the ZLB with commitment is negligible. This reflects the fact that in the event of a large shock, the central bank will promise to keep interest rates low for an extended period, which significantly reduces the impact of the shock and thus the cost of the ZLB. Under discretionary policy, the reverse is true. The inability of the central bank to promise to keep interest rates low after the ZLB constraint ceases to bind yields very large costs of the ZLB. As a result, optimal policy entails a higher level of trend inflation, reaching 2.7% in our baseline calibration, which is still within the target range of most central banks today. Thus, a positive but low optimal inflation rate does not hinge on the assumption that the central bank follows a Taylor rule, but

17We assume that the central bank can always commit to a long-run inflation target, even if it cannot commit to a particular stabilization policy rule. Therefore, there is no inflation bias under this policy.

18Billi (2011) reports an optimal inflation rate of 13% under discretion in the presence of the ZLB. The main differences between his approach and ours are that a) he assumes higher levels of indexation (0.90) and b) he log-linearizes the New Keynesian model and performs welfare approximations under the assumption of zero steady-state inflation so none of the three costs of inflation which we emphasize here are included in his model.
2.7. NORMATIVE IMPLICATIONS

also obtains with optimal policy under discretion. The commitment case suggests that an improved monetary policy could deliver important welfare gains by reducing the costs of the ZLB when they occur rather than trying to avoid these episodes.

2.7.2 Taylor Rule Parameters, Price-Level Targeting and the Optimal Inflation Rate

Given the size of the welfare differential between optimal policy with discretion/Taylor rule versus commitment, we investigate to what extent alternative monetary policy rules can improve outcomes in the face of the ZLB. Thus, we consider the implications of alternative parameter values in the Taylor rule, illustrated in Panels C-F of Figure 9. Varying the long-run response to inflation from 2 to 5 has unambiguous effects on welfare: stronger long-run responses to inflation \( \phi_\pi \) raise welfare in the model for all inflation rates. Intuitively, this stronger systematic response reduces inflation and output volatility, thereby leading to a lower frequency of being at the ZLB and higher utility. However, this has little effect on the optimal inflation rate, which ranges from 1.7% when \( \phi_\pi = 2 \) to 1.2% when \( \phi_\pi = 5 \). In terms of responses to the real side of the economy which are captured by coefficients \( \phi_y \) and \( \phi_yy \), stronger responses to output growth generally lower welfare while higher responses to the output gap are welfare-improving. However, the quantitative changes in welfare are small and neither measure plays a significant role in determining the optimal inflation rate within the determinacy region of the parameter space.

While our baseline specification of the Taylor rule restricts the endogenous response of the central bank to inflation and the real side of the economy, an additional factor sometimes considered is price-level targeting (PLT). While the evidence for central banks actually following PLT remains scarce, PLT has nonetheless received substantial attention in the literature for several reasons. First, as emphasized in Woodford (2003), PLT guarantees determinacy under zero trend inflation for any positive response to the price level gap. Second, Coibion and Gorodnichenko (2011b) show that PLT ensures determinacy for \( \bar{\pi} > 0 \) as well, and is not subject to the deterioration of the Taylor principle as a result of positive trend inflation which occurs when the central bank responds only to inflation. Third, Gorodnichenko and Shapiro (2007) show that PLT robustly helps stabilize inflation expectations, thereby yielding smaller inflation and output volatility than would occur in inflation-targeting regimes.

We extend our baseline model to include PLT in the central bank’s reaction function (\( \phi_p > 0 \)). Panel F of Figure 9 shows the effects of PLT on welfare for different \( \bar{\pi} \) as well as its implications for the optimal rate of inflation. First, PLT strictly increases welfare for any \( \bar{\pi} \), especially at low levels of inflation. Second, PLT leads to much lower levels of optimal inflation than inflation-targeting regimes. Even
for moderate responses to the price level gap, the optimal level of inflation is less than 0.3 percent per year, which is close to the optimal 0.2 percent per year inflation rate in the Ramsey problem under commitment. This magnitude practically implies price level stability (rather than inflation stability) which is, in fact, the mandated objective for most central banks.

The intuition for why PLT delivers such a small optimal inflation rate is straightforward. First, as observed in Gorodnichenko and Shapiro (2007), PLT stabilizes expectations and has a profound effect on output and inflation volatility. In our simulations, the reduction in inflation and output volatility is so substantial that the welfare costs of inflation are almost exclusively driven by the steady-state effects. As a result of reduced volatility, the ZLB binds less frequently. For example, with \( \phi_p = 0.3 \), the ZLB binds less than 1.4 percent of the time at a steady-state level of inflation of 3.5%. Second, even if the nominal rate hits zero, the policy rule remains a potent factor in stimulating the economy despite the ZLB because agents know that the deflationary pressures during the ZLB will have to be offset by above-average inflation in the future. This limits the downward movement in inflationary expectations and therefore the associated increase in real interest rates. In short, PLT limits the extent of deflationary spirals so that the exit from a ZLB episode occurs more rapidly and the welfare costs of the ZLB are substantially reduced. To give a sense of the magnitude of the associated welfare change, we note that by increasing \( \phi_p \) from zero to 0.25 (combined with the appropriate change in the optimal rate of inflation), a policymaker could raise welfare by the equivalent to a permanent increase in consumption of 0.5 percent and approach the welfare gains for the optimal policy in the Ramsey problem under commitment. Thus, these results provide a new justification for the consideration of PLT by monetary policymakers.

2.8 Concluding Remarks

If nothing else, the Great Recession has taught monetary economists one lesson: the zero lower bound (ZLB) is not a theoretical curiosity of interest only to historians of the Great Depression or as a precautionary tale against overly cautious policy-makers such as the Japanese monetary and fiscal authorities in the early 1990s. Instead, the pervasiveness of the zero bound constraint among major industrial countries has demonstrated the necessity of incorporating this issue into modern macroeconomic models. Indeed, the recent interest in raising the inflation targets of central banks has resurrected a basic question for macroeconomists: what is the optimal inflation rate? Strikingly, New Keynesian models, with their pervasive reliance on the assumption of zero steady-state inflation, have been ill-equipped to answer this key question for central bankers.

We provide an integrated treatment of the effects of non-zero steady-state infla-
tion in New Keynesian models. Most importantly, we derive an approximation to the utility function of the representative agent which incorporates the various dimensions along which steady-state inflation matters: the steady-state, the dynamics of the model, and the coefficients of the utility-function approximation. This allows us to study the optimal rate of inflation using a welfare criterion derived explicitly from the microfoundations of the model. Combining this with the zero-bound on nominal interest rates, we are then able to study the costs and benefits of steady-state inflation and quantify the optimal rate of inflation in models with time and state-dependent pricing. Our baseline result is that this optimal rate of inflation is fairly low: typically less than two percent a year. We show that this result is robust to a variety of parameter specifications and modifications of the model.

Given that most central banks are targeting inflation rates between 1% and 3% a year, our results can be interpreted as supporting the current regimes, while providing little evidence in favor of raising these targets to provide additional insurance against the zero-bound constraint on interest rates. Furthermore, our results go some way in resolving the apparent disconnect between observed inflation targets and prescriptions from standard monetary models. From a normative point of view, we also show that welfare could be substantially improved by introducing price-level targeting. The latter helps to stabilize economic fluctuations and reduces the probability of hitting the zero-lower bound. As a result, the optimal inflation rate under a price level targeting regime would be close to zero. In other words, optimal monetary policy, characterized as a combination of a low inflation target and a systematic response of nominal interest rates to deviations of the price-level from its target, can be interpreted as being very close to the “price stability” enshrined in the legal mandates of most central banks.

In the absence of such a change in the interest rate rule, our results suggest that higher inflation targets are likely too blunt an instrument to address the ZLB in a way that significantly increases aggregate welfare. This is not because ZLB episodes are not costly, but rather because the perpetual costs of higher inflation outweigh the benefits of more infrequent ZLB episodes. Addressing the very large costs associated with ZLB episodes is therefore likely to require alternative policies more explicitly focused on these specific episodes, such as countercyclical fiscal policies or the use of non-standard monetary policy tools. The lack of consensus on the efficacy of these policy tools, however, suggests that they should be high on the research agenda of macroeconomists.
## Table 2.1: Baseline parameter values

<table>
<thead>
<tr>
<th>Parameters of Utility Function</th>
<th>Steady-State Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta ): Frisch Labor Elasticity</td>
<td>1 ( \bar{g}_y ): Growth Rate of RGDP/cap</td>
</tr>
<tr>
<td>( \beta ): Discount factor</td>
<td>0.998 ( \bar{c}_y ): Consumption Share of GDP</td>
</tr>
<tr>
<td>( h ): Habit in consumption</td>
<td>0.7 ( \bar{g}_y ): Government Share of GDP</td>
</tr>
</tbody>
</table>

### Pricing Parameters

| \( \theta \): Elasticity of substitution | 7 \( \rho_g \): Government Spending Shocks | 0.97 |
| \( \lambda \): Degree of Price Stickiness | 0.55 \( \rho_m \): Cost-Push Shocks | 0.9 |
| \( \omega \): Price indexation | 0 \( \rho_q \): Risk Premium Shocks | 0.947 |

### Taylor Rule Parameters

| \( \phi_\pi \): Long run response to inflation | 2.5 \( \sigma_g \): Government Spending Shocks | 0.0052 |
| \( \phi_{gy} \): Long run response to output growth | 1.5 \( \sigma_m \): Cost-Push Shocks | 0.0014 |
| \( \phi_y \): Long run response to output gap | 0.11 \( \sigma_q \): Risk Premium Shocks | 0.0024 |
| \( \rho_1 \): Interest smoothing | 1.05 \( \sigma_a \): Technology Shocks | 0.009 |
| \( \rho_2 \): Interest smoothing | -0.13 \( \sigma_r \): Monetary Policy Shocks | 0.0024 |

Notes: The table presents the baseline parameter values assigned to the model in section 3.1 and used for solving for the optimal inflation rate in section 3.2. “p.a.” means per annum.
### Table 2.2: Model fit at the baseline parameterization

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th></th>
<th>AR(1) coefficient</th>
<th></th>
<th>Corr(x,output)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0121</td>
<td>0.0128</td>
<td>0.75</td>
<td>0.81</td>
<td>0.82</td>
<td>0.78</td>
</tr>
<tr>
<td>Output</td>
<td>0.0124</td>
<td>0.0167</td>
<td>0.71</td>
<td>0.85</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0067</td>
<td>0.0065</td>
<td>0.77</td>
<td>0.76</td>
<td>0.1</td>
<td>0.19</td>
</tr>
<tr>
<td>Interest Rates</td>
<td>0.0081</td>
<td>0.007</td>
<td>0.96</td>
<td>0.97</td>
<td>0.1</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Notes: The table presents moments of the data and simulated series from the model parameterized at baseline values. Consistent with Gorodnichenko and Ng (2010), both model and data series are detrended with the Hodrick-Prescott ($\lambda = 1600$) filter to remove the trending component in the series.
Figure 2.1: Frequency of being in the zero lower bound and steady-state nominal interest rate.

Notes: The figure plots the steady-state level of the annualized nominal interest rate (right axis) implied by the baseline model of section 3 for different steady-state inflation rates. In addition, the figure plots the frequency of hitting the zero bound on nominal interest rates (left axis) from simulating the baseline model at different steady-state inflation rates as well as historical frequencies of ZLB for the U.S. for counterfactual inflation rates. See section 3.1 for details.
2.8. CONCLUDING REMARKS

(a) Effect of ZLB

(b) Effects of positive trend inflation

Figure 2.2: Utility at different levels of steady-state inflation

Notes: The figures plot the approximation to the utility function in Proposition 1 from simulating the model for different levels of steady-state inflation. Panel A includes results for the baseline model, the baseline model without the ZLB, as well as the model with the ZLB but omitting the three cost channels of inflation: steady-state effects, the changing coefficient on inflation variance in utility and the dynamic effects. Panel B reproduces our baseline with ZLB, then presents results when we restrict the model to include only one cost of inflation and the ZLB. “Dynamic cost only” includes only the dynamic effects of positive inflation and keeps the rest of the model being approximated around zero trend inflation, “Steady-state cost only” includes only the steady-state cost of inflation and keeps the rest of the model being approximated around zero trend inflation, while “Changing inflation weight only” includes only the changing coefficients on inflation variance in the loss function and keeps the rest of the model being approximated around zero trend inflation. See section 3.2 for details.
2.8. CONCLUDING REMARKS

Figure 2.3: The sources of utility costs of inflation

Notes: The first row of the figure plots the coefficients of the approximation to the utility function from Proposition 1 for different levels of trend inflation. The second row plots the variance of macroeconomic variables that enter the approximation to the utility function in Proposition 1 from simulating the model subject to the zero bound on nominal interest rates for different levels of steady-state inflation using the baseline parameter values of the model. The dashed black lines are the corresponding moments without the zero-bound on nominal interest rates, while the dotted lines correspond to the moments of the model when the dynamics are approximated at zero trend inflation. The third row plots the contribution of the different components of the approximation to the utility function in Proposition 1. See section 3.2 for details.
Figure 2.4: The costs of business cycles

(a) The costs of the ZLB per episode
(b) The costs of the ZLB per hit
(d) Output gap volatility weight

Notes: The top panel plots the average duration of ZLB episodes in the baseline calibration and the implied average welfare cost per quarter of being at the ZLB for different levels of trend inflation. The bottom panel plots the effects of changing the coefficient on the variance of the output gap in the utility function approximation of Proposition 1 on the optimal inflation rate (left graph), the welfare costs of business cycle fluctuations (middle graph), and the average welfare costs of hitting the ZLB (right graph). The latter two are measured using Proposition 1 net of the steady-state effects of trend inflation. See sections 3.3 and 3.4 for details.
2.8. CONCLUDING REMARKS

Figure 2.5: Sensitivity: pricing parameters, habit, and labor supply elasticity

Notes: Figures plot the optimal level of steady-state inflation and the welfare loss at the optimal steady state level of inflation as a function of a structural parameter. See section 4.1 for details.
Notes: Figures in the left column plot the welfare loss as a function of steady state inflation for alternative values of structural parameters. The solid thick back line corresponds to the baseline parameterization. Figures in the middle column plot the frequency of hitting the ZLB for different parameter values. Figures in the right column plot the optimal level of steady-state inflation and the welfare loss at the optimal steady state level of inflation as a function of a structural parameter. See section 4.2 for details.
2.8. CONCLUDING REMARKS

Notes: Panel A plots welfare in the baseline model and the model with capital. See section 5.1 for details. Panel B plots the expected utility in the baseline model and model-uncertainty. See section 5.2 for details. Panel C plots the utility associated with different steady-state inflation rates under the baseline model as well as the model with downward nominal wage rigidity. Panel D plots the variance of inflation for different steady-state inflation rates using our baseline model and the model with downward nominal wage rigidity. See section 5.3 for details. Panels E and F plot the implications of Calvo vs. Taylor price setting for welfare. Taylor, X quarters corresponds to the duration of price contracts equal to X quarters. See section 5.4 for details.

Figure 2.7: Robustness
### 2.8. CONCLUDING REMARKS

(e) Endogenous calvo parameter  \[ \beta \pi \] is the effect of steady state inflation on the frequency of price changes.  \[ 1 - \lambda \]. \[ \beta \pi = 0 \] is our baseline case of exogenous price stickiness. See section 5.5 for details.

(f) State-dependent pricing (Dotsey et al. (1999))

Figure 2.8: Endogenous and state-dependent price stickiness

Notes: The figures plot the implications of endogenous and state-dependent price stickiness on the model. \( \beta \pi \) is the effect of steady state inflation on the frequency of price changes. \( 1 - \lambda \). \( \beta \pi = 0 \) is our baseline case of exogenous price stickiness. See section 5.5 for details.
2.8. CONCLUDING REMARKS

Figure 2.9: Positive implications: optimal policy

Notes: Panels A and B plot outcomes for three scenarios: optimal policy with commitment (Ramsey); optimal policy without commitment (discretion); the baseline Taylor rule described in section 3.1. Optimal policies with and without discretion are described in section 6.1. Panels C-F plot the welfare loss as a function of $\bar{\pi}$ for alternative values of the monetary policy rule parameters. The solid thick blue line corresponds to the baseline parameterization. $\phi_\pi$ is the long-run response of interest rates to inflation, $\phi_g$ is the response to output growth, $\phi_y$ is the response to the output gap, and $\phi_p$ is the response to the price-level gap. See section 6.2 for details.
Chapter 3

Fiscal Multipliers at the ZLB: International Theory and Evidence

3.1 Introduction

In the preceding chapter, Olivier Coibion, Yuriy Gorodnichenko, and I have shown that raising the target inflation rate is a blunt tool to deal with temporary, but very costly ZLB episodes. This argument does not apply to temporary fiscal stimulus, whose efficacy I investigate in this section. In particular, I determine whether fiscal stimulus generates crowding out through the real exchange rate.

In recent work Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2006), Eggertsson (2009), and Woodford (2011a) have shown that fiscal multipliers – the increase in output for each unit of government spending are generally above 1 in New Keynesian models at the ZLB, and may even be as large as 3 or 4.\(^1\) However, these may be overestimates of the fiscal multipliers because they are derived in closed economy models. Fiscal multipliers tend to be smaller in open economy models, because a fiscal expansion is usually associated with an appreciation in the real exchange rate and thus crowding out of net exports.\(^2\)

In my first contribution, I show that open economy fiscal multipliers can be large

\(^1\)This contrasts with typical estimates from time series data when monetary policy is unconstrained, which tend to be below 1 (see e.g. Barro (1981), Blanchard and Perotti (2002), Ramey (2011), Hall (2010a)). There has also been significant advances in measuring fiscal multipliers at the state and local level (Nakamura and Steinsson (2011); Serrato and Wingender (2010)). However, the latter ultimately require a theoretical model to determine the aggregate fiscal multipliers consistent with these estimates.

\(^2\)For example, in the (Dornbusch (1976)) model the decline in net exports is as large as the expansion in government spending so that the fiscal multiplier is zero. Ilzetzki, Mendoza, and Vegh (2010) provide empirical evidence that the fiscal multiplier in open economies with flexible exchange rates is in fact statistically indistinguishable from zero. However, their dataset does not cover ZLB episodes.
when the economy is in a liquidity trap.\textsuperscript{3} I build a small open economy model following Gali and Monacelli (2005) and Clarida and Gertler (2001), and derive fiscal multipliers in and outside the liquidity trap, assuming that uncovered interest rate parity (UIP) holds. The resulting fiscal multiplier in normal times behaves as expected: it is less than one and decreasing in the openness (import share) of the economy. However, at the ZLB, the fiscal multiplier is above one and increasing in the import share. Thus, once the zero bound binds, the fiscal multiplier in a closed economy (which has a zero import share) is smaller than the fiscal multiplier in an open economy.\textsuperscript{4}

The intuition behind these results is as follows. A fiscal stimulus generates inflation, which, in normal times, precipitates the central bank to raise nominal interest rates such that real interest rates rise (the Taylor principle). Since the interest rate elasticity of output is strictly negative, i.e. the sum of consumption and net exports decline with rising real interest rates, the resulting fiscal multiplier is less than 1. For standard parameterizations, the closed economy will (in absolute value) have a lower interest rate elasticity of output than the open economy, so that it will also enjoy a larger multiplier.

However, if the economy finds itself in a liquidity trap then the Taylor principle is violated. The inflationary effect of government expenditures will not be met by higher nominal rates, so that real interest rates fall. Given the strictly negative interest rate elasticity of output, the sum of consumption and net exports will rise, and the fiscal multiplier will now exceed 1. Because the open economy has (in absolute value) a higher interest rate elasticity of output than the closed economy, its fiscal multiplier will be larger. Furthermore, assuming standard parameterizations, net exports rise following a fiscal expansion in the liquidity trap, so that fiscal policy becomes a beggar-thy-neighbour policy.

The second contribution of this paper is to derive fiscal multipliers when UIP is violated. Departures from UIP are rationalized through a wedge in the Backus-Smith condition, which is assumed to be an increasing function of excess real returns of domestic bonds over foreign bonds. This friction limits movements of the terms of trade following a government spending shock, and can even change the sign of the terms of trade response. Depending on the size of the friction, the net export response can be very different compared to the baseline model. In fact, for moderately sized frictions, the open economy fiscal multiplier will be decreasing in the import share, and for even larger frictions the multiplier will be below 1, thus overturning the results from the baseline model.

\textsuperscript{3}I use the terms “zero lower bound” and “liquidity trap” interchangeably.

\textsuperscript{4}In independent work, Fujiwara and Ueda (2010) have found a qualitatively similar result under more stringent conditions on the parameter space. Furthermore, they do not consider how sensitive the results are to violations of UIP and thus do not provide results for empirically estimated departures from UIP.
3.2. AN OPEN ECONOMY MODEL

The third contribution of this paper is to estimate the size of the friction and thus determine the likely properties of the open economy fiscal multiplier at the ZLB. To the best of my knowledge this is the first attempt to empirically test one aspect of the large ZLB multipliers in the New Keynesian model. The friction is derived by comparing model-implied nominal exchange rate responses to generic inflation surprises with their empirical counterpart. I use generic inflation surprises to identify the friction, because both demand and supply shocks are subject to the same friction in the model. However, I also show that these inflation surprises behave like demand shocks, and are thus likely to be subject to similar departures from UIP as a government spending shock.

Given the high frequency exchange rate response to these inflation surprises, I find that the friction to UIP is quantitatively significant at the ZLB. While the frictionless baseline model predicts that the nominal exchange rate depreciates by at least 1% for each 1% point of surprise inflation, I estimate that the nominal exchange rate appreciates by 0.021% after a 1% positive inflation surprise.

A calibrated model illustrates that this estimated friction can significantly lower the fiscal multiplier at the ZLB, even at moderate import shares. For example, at an import share of $\gamma = 0.15$ - typical of the US - the fiscal multiplier at the ZLB is 2.5 in the frictionless baseline model, but “only” 1.5 in the model with friction. Furthermore, the open economy fiscal multiplier in the friction model is significantly smaller than in the closed economy. For example, for import shares typical of European countries, the open economy fiscal multiplier is 30% smaller than the closed economy fiscal multiplier. However, even though exchange rate crowding out can be quantitatively significant, the fiscal multipliers remain large by the standards of the open economy literature.

The rest of this paper proceeds as follows. In section 3.2, I develop a small open economy model that allows for government spending shocks and the zero lower bound. I derive the frictionless fiscal multiplier in normal times and at the ZLB in section 3.3, and I investigate its sensitivity to frictions to international asset markets in section 3.4. In section 3.5 I test for the exchange rate responses following inflationary shocks to estimate the friction to the UIP equation. In section 3.6 I calibrate the model with the estimated friction to explore quantitative implications for the fiscal multiplier at the ZLB. Section 3.7 concludes.

3.2 An open economy model

I begin with a variant of the open economy models developed by Gali and Monacelli (2005) and Clarida and Gertler (2001), which were designed for the purpose of analyzing monetary policy in the open economy. They lend themselves equally well to the analysis of open economy fiscal policy at the ZLB. In this section, I only report
the log-linearized equations with a detailed description relegated to Appendix B.1. There are two countries in this model: the home country denoted $H$ and the foreign country denoted $F$. The home agent consumes both a domestically produced good $c^H_t$ and a foreignly produced good $c^F_t$,

$$\dot{c}_t = (1 - \gamma)\dot{c}^H_t + \gamma\dot{c}^F_t.$$ 

The weight on the foreign good in the consumption basket is $\gamma$, which is equal to the import share in the steady state. Since economies with higher $\gamma$ import (and export) a greater fraction of their consumption, I interpret this parameter as a measure of “openness.” The price index corresponding to the domestic basket is $\dot{p}_t = (1 - \gamma)\dot{p}^H_t + \gamma\dot{p}^F_t$. I abstract from nontradable goods as they significantly complicate derivations but provide little additional insight.

The foreign representative agent purchases a basket with import share $\psi$,

$$\dot{c}^*_t = \psi\dot{c}^{H*}_t + (1 - \psi)\dot{c}^{F*}_t,$$

I let $\psi = \gamma/n$ where $n$ is the relative size of the foreign economy. I will typically consider the limit where $n \to \infty$, so that the home economy is small. The price of the foreign consumption basket is $\dot{p}^*_t = \psi\dot{p}^{H*}_t + (1 - \psi)\dot{p}^{F*}_t$.

To satisfy the resource constraint, log domestic output $\dot{y}_t$ is the sum of home and foreign consumption of the domestic good as well as the home government’s demand $\dot{g}_t$,

$$\dot{y}_t = s_g\dot{g}_t + (1 - s_g) \left[ (1 - \gamma)\dot{c}_t^H + \gamma\dot{c}_t^{H*} \right],$$

where $s_g$ is the steady-state share of government in domestic output. A similar equation holds for the foreign country with the appropriate weights on the domestic and foreign good.

Given the Dixit-Stiglitz structure, the relative demand between home and foreign produced goods depends only on the terms of trade $s_t$,

$$\dot{c}_t^H - \dot{c}_t^F = \eta\dot{s}_t, \quad \dot{c}_t^{H*} - \dot{c}_t^{F*} = \eta\dot{s}_t,$$

where $\eta$ is the elasticity of substitution between home and foreign goods. The terms of trade are here defined as the ratio of foreign to domestic prices, $\dot{s}_t = \dot{p}^F_t - \dot{p}^H_t = \dot{p}^{F*}_t - \dot{p}^{H*}_t$, where the last equality follows from the Law of One Price.

It is typical in the literature to assume that financial markets are complete, which implies that the relative marginal utility levels of foreign and home residents must

---

5Intuitively, adding nontradable goods reduces the difference between open economy fiscal multipliers and closed economy fiscal multipliers, both in normal times and at the ZLB. However, the terms of trade response in a model with tradables is still given by equation (3.3). Thus, the empirical test in section 3.5 remains valid in a model with tradables.

6The foreign disutility of work is decreasing in $n$ to generate this result. See appendix B.1 for details.
be proportional to the real exchange rate (the Backus and Smith (1993) condition). I consider a more general formulation which includes a friction $\hat{f}_t$ between the real exchange rate and relative marginal utilities. With the wedge I can accommodate departures from uncovered interest rate parity (UIP), which will ultimately be important to determine the size of the fiscal multiplier. I keep the friction in reduced form, because there is little agreement in the literature as to what mechanism accounts for deviations from UIP. However, in Appendix B.2 I also provide an example following Bodenstein (2008), where the Backus-Smith condition takes precisely this form. In summary, the modified Backus-Smith condition of the model is,

$$
\sigma(\hat{c}_t - \hat{c}_t^*) = \hat{\lambda}_t - \hat{f}_t,
$$

(3.1)

where $\hat{\lambda}_t = (1 - \gamma - \psi)\hat{s}_t$ is the real exchange rate and $\sigma$ is the inverse of the intertemporal elasticity of substitution (IES).

Intertemporal optimization by domestic residents yields an Euler equation

$$
\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}E_t[\hat{r}_{t+1} - \hat{\pi}^H_{t+1} - \gamma \Delta \hat{s}_{t+1} + \hat{\beta}_{t+1}],
$$

where $\hat{r}_{t+1}$ is the current domestic nominal interest rate, $\hat{\pi}^H_{t+1}$ the inflation rate of domestically produced goods, and $\hat{\beta}_{t+1}$ a shock to the discount factor. An increase in the discount factor raises the desire to save, which lowers consumption today relative to tomorrow. If this shock is sufficiently large, then the only nominal interest rate that is consistent with market clearing in the savings-investment market is zero.\(^8\)

The foreign residents also satisfy an Euler equation akin to (3.2), with the obvious substitutions. Combining the two Euler equations with the Backus-Smith condition also generates an equation similar to UIP, but for the terms of trade

$$
E_t\Delta \hat{s}_{t+1} = (\hat{r}_{t+1} - E_t\hat{\pi}^H_{t+1}) - (\hat{r}_{t+1}^* - E_t\hat{\pi}^F_{t+1}) - (\hat{f}_t - E_t\hat{f}_{t+1}).
$$

(3.2)

Of course, this is equivalent to the typical UIP condition for the nominal exchange rate $\hat{e}_t$,

$$
E_t\Delta \hat{e}_{t+1} = \hat{r}_{t+1} - \hat{r}_{t+1}^* - (\hat{f}_t - E_t\hat{f}_{t+1}).
$$

(3.3)

When there is no friction, $\hat{f}_t = 0 \forall t$, the above equation reduces exactly to UIP. Departures from UIP arise only if the friction is expected to change over time.

Calvo pricing of domestic and foreign varieties with Calvo probability $\theta$, gives rise to a New Keynesian Phillips Curve for each country. The domestic Phillips Curve is given by,

$$
\hat{\pi}^H_t = \beta E_t\hat{\pi}^H_{t+1} + \kappa[\nu\hat{y}_t + \sigma\hat{c}_t + \gamma \hat{s}_t].
$$

---

\(^7\)See Lustig and Verdelhan (2007) for a risk-based explanation and Burnside, Eichenbaum, and Rebelo (2011) for a liquidity-based explanation.

\(^8\)Technically, this market will be cleared by the real interest rate. However, with sticky prices, some of the adjustment of the real interest rate has to be brought about by changes in the nominal rate.
where \( \kappa = (1 - \theta)(1 - \beta \theta)/\theta \) and \( \nu \) is the inverse frisch elasticity of labor supply. Real marginal cost for firms are also a function of the terms of trade, because workers care about the CPI-deflated wage rate, which is affected by the price of foreign goods.

The final piece of the log-linear model is the Taylor rule. I first log-linearize the Taylor rule without the zero bound constraint and then impose this constraint on the log-linear approximation. The lower bound on the log-linearized nominal interest rate is given by 

\[
\hat{r}_{t+1} = \max\{\phi \hat{\pi}^H_t, -\bar{r}\},
\]

with the foreign central bank following a similar rule.

### 3.3 Fiscal Multipliers in the frictionless open economy

To simplify derivations and build intuition, I consider the small open economy without friction, where \( n \to \infty, \psi \to 0 \) and \( f_t = 0 \forall t \). The analysis of the large open economy is relegated to Appendix B.4 and the analysis of the economy with friction is deferred to Section 3.4. In the frictionless model, I can then express output exclusively in terms of government spending, the terms of trade, and now exogenous foreign consumption,

\[
\hat{y}_t = s g \hat{g}_t + \frac{1 - s g}{\sigma} [1 + \gamma (2 - \gamma)(\sigma \eta - 1)] \hat{\pi}_t + (1 - s g) \hat{c}^*_t,
\]

where \( \varepsilon_{ys} > 0 \) is the elasticity of output with respect to the terms of trade. This equation tells us that a deterioration in the terms of trade (increase in \( \hat{s}_t \)) is associated with an increase in domestic output. Intuitively, the deterioration in the terms of trade captures both the behavior of net exports, and, through its association with declining real interest rates, the increase in domestic consumption (by the Euler equation).

More precisely, solving the UIP condition (3.2) for the terms of trade forward and setting \( f_t = 0 \) yields,

\[
\hat{s}_t = -E_t \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j}) - (\hat{r}^*_t - \hat{\pi}^*_t)].
\]

This equation emerges as a no-arbitrage condition in the friction-less model.\(^9\) In-
tuitively, an increase in domestic real interest rates relative to foreign real interest rates makes domestic real bonds more attractive. As arbitrageurs purchase domestic real bonds, the real exchange rate appreciates, which is reflected as an improvement in the terms of trade.

Substituting the solution for $s_t$ condition into equation (3.4) yields,

$$\hat{y}_t = s_g \hat{g}_t + \varepsilon_{yr} E_t \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j}) - (\hat{r}^*_{t+j} - \hat{\pi}^*_{t+j})] + (1 - g) \hat{c}^*_t,$$

(3.6)

where $\varepsilon_{yr} = -\varepsilon_{ys} < 0$ is the real interest rate elasticity of output. According to equation (3.6), the effect of fiscal policy on output in this small open economy depends on the reaction of real interest rates. Differentiating (3.6) with respect to domestic government spending (taking foreign consumption and real interest rates as exogenous) yields the fiscal multiplier,

$$\frac{\partial Y_t}{\partial G_t} = 1 + \varepsilon_{ys} \frac{s_g}{s} \frac{\partial \hat{s}_t}{\partial \hat{g}_t} = 1 + \varepsilon_{yr} \frac{s_g}{s} \frac{\partial \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j})]}{\partial \hat{g}_t}.$$

(3.7)

According to equation (3.7), if real interest rates do not change with fiscal policy, then the fiscal multiplier is always equal to 1. This is independent of whether the economy is open ($\gamma > 0$) or closed ($\gamma = 0$). This result was proven by Woodford (2011a) for the closed economy without capital, and it also extends to the frictionless small open economy.\(^{10}\) The fiscal multiplier will be less than 1 if the sum of expected future interest rates increase with fiscal policy ($\partial E_t \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j})]/\partial \hat{g}_t > 0$), and greater than 1 if the sum of expected future interest rates fall with fiscal policy ($\partial E_t \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j})]/\partial \hat{g}_t < 0$).\(^{11}\)

The sum of expected future interest rate changes (and thus the terms of trade) are a sufficient statistic for the fiscal multiplier, because they determine the behavior of both consumption and net exports. Solving the Euler equation forward, yields

$$\hat{c}_t = -\frac{1 - \gamma}{\sigma} E_t \sum_{j=1}^{\infty} (\hat{r}_{t+j} - \hat{\pi}^H_{t+j}) - \frac{\gamma}{\sigma} E_t \sum_{j=1}^{\infty} (\hat{r}^*_{t+j} - \hat{\pi}^*_{t+j}).$$

(3.8)

Thus, higher expected domestic real interest rates are associated with lower consumption and vice versa, through standard intertemporal substitution. However,

\(^{10}\)Similar isomorphisms between a closed economy and a complete markets small-open economy also extend to monetary policy, as stressed by Clarida and Gertler (2001).

\(^{11}\)Note that equation (3.7) yields a multiplier for each period. Thus a sequence of $\{\partial E_t \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j})]/\partial \hat{g}_t\}_{t=0}^{\infty}$ will deliver a (potentially different) fiscal multiplier for each time period, $\{\partial Y_t/\partial G_t\}_{t=0}^{\infty}$. However, this does not imply that the contemporaneous fiscal multiplier is independent of the expected path of future fiscal policy. Rather, these expectations are already reflected in the current realization of $\partial E_t \sum_{j=1}^{\infty} [(\hat{r}_{t+j} - \hat{\pi}^H_{t+j})]/\partial \hat{g}_t$.  

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the interest rate elasticity of consumption $\varepsilon_{cr}$ is smaller (in absolute value) in the open economy, because the domestic consumer can substitute towards foreign goods when higher real interest rates generate an improvement in the terms of trade. Similarly, domestic net exports, defined as $n\hat{x}_t = \gamma(c^H_t - c^F_t)$, are equal to

$$n\hat{x}_t = -\left[ (2 - \gamma)\gamma\eta - \frac{\gamma(1 - \gamma)}{\sigma} \right] E_t \sum_{j=1}^{\infty} \left[ (\hat{r}_{t+j} - \hat{\pi}^H_{t+j}) - (\hat{r}^*_t - \hat{\pi}^*_t) \right].$$

(3.9)

Depending on the parameters, higher expected domestic real interest can be associated with either increased or decreased net exports as there are two competing effects. The first term in the square brackets of equation (3.9) captures the substitution effect - higher domestic real interest rates generate an appreciation in the terms of trade and thus a substitution away from domestic goods in both the foreign and home consumption bundle. The second term in the square brackets reflects the risk sharing condition, which demands that aggregate domestic consumption falls more than aggregate foreign consumption, because the cost of domestic consumption has risen more. This will increase net exports because domestic consumption demand for the foreign good declines more than the foreign consumption demand for the home good. However, even though the sign of the interest rate elasticity of net exports is ambiguous, the interest rate elasticity of the sum of consumption and net exports (equal to $\varepsilon_{cr} + \varepsilon_{nxr}$) is strictly negative. Therefore the behavior of real interest rates alone can tell us what is happening to the private components of output following a fiscal shock.

While one may be skeptical about such a close connection between the fiscal multiplier, real interest rates, and the terms of trade, recent results by Ilzetzki, Mendoza, and Vegh (2010) are at least suggestive of such a relationship. Specifically, they show that a government spending shock in a flexible exchange rate regime is associated with an immediate rise in nominal interest rates, an appreciation in the real exchange rate, and a long-run fiscal multiplier below 1. On the other hand, with fixed exchange rates, nominal interest rates fall, the real exchange rate depreciates and the long-run fiscal multiplier is above 1.\(^{12}\) Since the real exchange rate in this model, $\hat{\lambda}_t$, is proportional to the terms of trade, $\hat{\lambda}_t = (1 - \gamma)\hat{s}_t$, the estimates by Ilzetzki, Mendoza, and Vegh (2010) suggest that a relationship like equation (3.7) is not implausible.

In order to calculate $\partial E_t \sum_{j=1}^{\infty} [\hat{r}_{t+j} - \hat{\pi}^H_{t+j}] / \partial g_t$ and the associated fiscal multiplier in the model, I need to specify the nature of the exogenous stochastic process

\(^{12}\)In the baseline model there is no distinction between long-run and short-run multipliers because it has no internal persistence. Therefore the mapping from the Ilzetzki, Mendoza, and Vegh (2010) should be interpreted somewhat cautiously. However, since the real exchange rate is a forward looking variable, it presumably contains information about the long-run effect of fiscal policy. Therefore I take the long-run fiscal multiplier as the natural benchmark.
3.3. FISCAL MULTIPLIERS IN THE FRICTIONLESS OPEN ECONOMY

$v_t = (\hat{\beta}_t \hat{c}_t)'$. I assume that the exogenous shocks $v_t$ follows a stochastic process as in Christiano, Eichenbaum, and Rebelo (2011). At time $t$, $v_t$ unexpectedly takes on a positive value, $v_t > 0$ (some elements of $v_t$ may still be zeros). At time $t+1$, and all future dates, I assume the following transition probabilities: If at $t+j > t$, $v_{t+j} > 0$, then it remains at this level with probability $p$, $\text{Prob}[v_{t+j+1} = v_{t+j} | v_{t+j} = v_t] = p$. With probability $1 - p$, $v_{t+j+1} = 0$, which is an absorbing state. The transition probabilities imply that $E_t v_{t+j} = p^j v_t$.

I outline the solution procedure with this shock process in Appendix B.3. This stochastic process is particularly useful because it allows me to derive analytic solutions to the fiscal multiplier in and outside the liquidity trap. In particular, the fiscal multiplier in normal times is given by,

$$\mu_G = \frac{(1 - p)(1 - \beta p) + \kappa(\phi - p)}{(1 - p)(1 - \beta p) + \kappa(\phi - p) \{1 + (1 - s_g)\frac{\nu}{\sigma} [1 + \gamma(2 - \gamma)(\sigma \eta - 1)]\}}$$

(3.10)

and satisfies $0 < \mu_G < 1$. The fiscal multiplier in normal times is decreasing in openness, $\frac{\partial \mu_G}{\partial \gamma} < 0$, if and only if $\sigma \eta > 1$.

If the duration of the liquidity trap remains unchanged, then the fiscal multiplier in the liquidity trap is given by,

$$\mu_{LT}^G = \frac{(1 - p)(1 - \beta p) - \kappa p}{(1 - p)(1 - \beta p) - \kappa p \{1 + (1 - s_g)\frac{\nu}{\sigma} [1 + \gamma(2 - \gamma)(\sigma \eta - 1)]\}}$$

(3.11)

and satisfies $\mu_{LT}^G > 1$. The fiscal multiplier in the liquidity trap is increasing in openness, $\frac{\partial \mu_{LT}^G}{\partial \gamma} > 0$, if and only if $\sigma \eta > 1$.

The intuition behind these results is as follows. An increase in government spending will initially cause an equal increase in output. As output increases, firms have to pay higher wages because of increasing disutility of labor. Since firms’ marginal cost increase, they will pass these costs onto consumers in form of higher prices. Thus, an increase in government spending raises inflation. In normal times, the central bank will raise nominal interest rates to fight this inflation. In fact, because of the Taylor principle, real interest rates will rise in response to inflation.

According to equation (3.8), higher domestic real interest rates will generate a fall in consumption, whereas the effect on net exports (equation (3.9)) depends on the interest rate elasticity of the terms of trade. However, the sum of domestic consumption and net exports unambiguously falls. This offsets some of the increase in output due to fiscal policy and explains why the multiplier is less than one.

The condition on $\sigma \cdot \eta$ determines how the interest rate elasticity of output changes

---

13 It is easy to show that in a liquidity trap the denominator must be positive. See also Christiano, Eichenbaum, and Rebelo (2011) and Eggertsson (2009).
3.4 FISCAL MULTIPLIERS IN FRICTION ECONOMY

with openness \((\gamma)\).\(^{14}\) In standard calibrations\(^{15}\) \(\sigma \eta > 1\), which implies that the (absolute) interest rate elasticity of net exports rises sufficiently with \(\gamma\), to compensate for the reduced interest rate elasticity of domestic consumption. Thus, if \(\sigma \eta > 1\), then a higher value of \(\gamma\) implies that a given rise in real interest rates will have a greater negative impact on output. Since real interest rates increase following a fiscal shock in normal times, a greater interest rate elasticity of output will imply a lower fiscal multiplier. Therefore, in normal times, the fiscal multiplier in an open economy \((\gamma > 0)\) is smaller than in a closed economy \((\gamma = 0)\).

However, if the economy is in a liquidity trap then nominal interest rates remain fixed at zero. The inflation generated by government spending will then reduce real interest rates. According to equations (3.8) and (3.9), the sum of domestic consumption and net exports now expands, which amplifies the output response to the fiscal policy shock and generates a fiscal multiplier above 1. If \(\sigma \eta > 1\), then the liquidity trap fiscal multiplier in an open economy \((\gamma > 0)\) is larger than in a closed economy \((\gamma = 0)\), because its interest rate elasticity of output is greater in absolute value.

In this example there need not be crowding out of net exports in a liquidity trap. Indeed, if \(\sigma \eta > 1\), then the interest rate elasticity of net exports is negative and there can be crowding in if real interest rates fall. Thus, at the ZLB, the open economy dimension may actually support fiscal efforts to lift the economy. Of course, if domestic net exports expand then foreign net exports must decline, so that a domestic fiscal expansion may actually reduce foreign output. This would make fiscal expansion in the liquidity trap a beggar-thy-neighbour policy, which will require a different policy coordination than fiscal expansion in normal times (which is a free-rider problem). In particular, with uncoordinated fiscal policy, countries may expand government spending too much, as they do not internalize the beggar-thy-neighbour effects of fiscal expansion.

3.4 Fiscal Multipliers in friction economy

The results in the preceding section have been derived assuming that UIP holds in the data, at least conditional on a government spending shock. To see how the results may be sensitive to departures from UIP, consider the exchange rate movements

\(^{14}\)This condition is weaker than in Fujiwara and Ueda (2010) who assume \(\eta = 1\).

\(^{15}\)For example, Ferrero, Gertler, and Svensson (2008) set \(\sigma = 1\) and \(\eta = 2\), while Bodenstein, Erceg, and Guerrieri (2010a) set \(\sigma = 1\) and \(\eta = 1.5\). Obstfeld and Rogoff (2005) argue that \(\eta = 2\) is a reasonable calibration balancing micro and macro estimates. However, they suggest that micro estimates (which imply higher elasticities) are likely less biased, and thus also experiment with higher values of \(\eta\).
3.4. FISCAL MULTIPLIERS IN FRICTION ECONOMY

implied by the model: In normal times the nominal exchange rate appreciates by

$$\frac{\partial \hat{e}_t}{\partial g_t} = -\frac{\phi - 1}{(1 - p)(1 - \gamma)} \frac{\partial \hat{\pi}_t}{\partial g_t} < 0.$$  \hspace{1cm} (3.12)

However, even without any exchange rate response the terms of trade would improve in normal times, and there would have been some crowding out of domestic net exports (for reasonable parameters). This suggests that the qualitative results for the fiscal multiplier in normal times would still hold, even if the nominal exchange rate fails to appreciate due to limits to arbitrage.

The results are more sensitive in the liquidity trap. In this case the nominal exchange rate depreciates by,

$$\frac{\partial \hat{e}_t}{\partial g_t} = \frac{1}{(1 - p)(1 - \gamma)} \frac{\partial \hat{\pi}_t}{\partial g_t} > 0,$$  \hspace{1cm} (3.13)

which generates a deterioration in the terms of trade even though domestic prices rise. Suppose that limits to arbitrage prevent the nominal exchange rate from adjusting at all. Then the terms of trade would improve and we would likely see a crowding out of net exports. This suggests that the fiscal multiplier in the liquidity trap may be particularly sensitive to departures from UIP.

To investigate this issue more formally, I determine the fiscal multiplier for various degrees of friction in the UIP condition. I assume that the friction is proportional to the excess return on domestic real bonds,

$$\hat{f}_t = \frac{\tau}{1 - p} [(\hat{r}_{t+1} - E_t \hat{\pi}_{t+1}^H) - (\hat{r}_{t+1}^* - E_t \hat{\pi}_{t+1}^{F*})],$$  \hspace{1cm} (3.14)

where \(\tau\) captures the size of the friction, which is scaled by \((1 - p)\) since the expected friction also enters the UIP condition. Typically in this model, \(E_t \hat{f}_{t+1} = p \hat{f}_t\), and the UIP condition becomes

$$E_t \Delta \hat{s}_{t+1} = (1 - \tau)[(\hat{r}_{t+1} - E_t \hat{\pi}_{t+1}^H) - (\hat{r}_{t+1}^* - E_t \hat{\pi}_{t+1}^{F*})].$$  \hspace{1cm} (3.15)

When \(\tau > 0\) the friction will limit the movement of the terms of trade and thus the nominal exchange rate relative to the baseline model. For example, \(\tau = (\phi - 1)/(\phi - p) > 0\) corresponds to the case where the nominal exchange rate fails to appreciate in normal times, whereas \(\tau = 1/p > 0\) implies that the nominal exchange rate does not depreciate at the ZLB. The functional form of \(f_t\) may appear somewhat arbitrary, but since \(\tau\) will be estimated conditional on it, it is more akin to a convenient normalization.

The friction \(\tau\) will affect the fiscal multiplier in two ways. First, by limiting the movement in the nominal exchange rate, it can reduce the effect of fiscal shock on
net exports, and even switch the sign of the net export response. For example, at the ZLB when \( \tau = 1/p \), the nominal exchange rate is unchanged, so that the terms of trade improve. This will crowd out net exports, whereas in the baseline model (\( \tau = 0 \)) net exports were crowded in. Second, for a given terms of trade response the friction now allows for a non-proportional consumption response, unlike the baseline model. Suppose that the friction is such that the terms of trade response is zero following a government spending shock (\( \tau = 1 \)). If consumption falls (as it does in normal times), then a fraction of it will fall on foreign production, so the multiplier in normal times will be higher. Vice-versa, if consumption rises (as at the ZLB), then the increase in consumption will benefit foreign producers and the domestic fiscal multiplier is smaller. This behavior was absent in the baseline model, because a zero terms of trade response implied a zero consumption response.

We can see both effects in action by calculating the fiscal multipliers as in section 3.3. Incorporating the friction, fiscal multiplier in normal times is now given by,

\[
\mu^F_G = \frac{(1 - p)(1 - \beta p) + \kappa(\phi - p)}{(1 - p)(1 - \beta p) + \kappa(\phi_{\pi} - p) \left\{ 1 + (1 - s_g)^\kappa \left[ (1 - \tau \gamma) + (1 - \tau) \gamma(2 - \gamma)(\sigma \eta - 1) \right] \right\}}.
\]

The friction has only affected the last two terms in the denominator: The weight \((1 - \tau)\) on the second term reflects the first effect discussed above - by limiting the change in the terms of trade, net exports become less sensitive to government spending. The first term, \((1 - \tau \gamma)\), captures the second effect noted in the preceding paragraph: When domestic consumption falls, for a given terms of trade response, then this is partly absorbed by net exports. This allows for a higher multiplier in normal times, so long as the import share \(\gamma\) is positive. Note that when \(\gamma = 0\), we obtain the closed economy fiscal multiplier for all values of \(\tau\). Intuitively, net exports are always zero independent of the terms of trade, and consumption is entirely determined by domestic real interest rates, for which the friction is irrelevant.

For small to moderate frictions, the results derived in the preceding section also hold in normal times. In particular, if \(\tau < 1\), then the multiplier is less than 1, \(0 < \mu^F_G < 1\). In fact, for realistic calibrations of \(\sigma \eta\) and the import share \(\gamma\), much larger frictions can also satisfy this inequality. Thus, the upper bound on the fiscal multiplier in normal times is quite robust to deviations from UIP. On the other hand, the comparative static with respect to the import share can be quite sensitive to \(\tau\). In the friction economy, the fiscal multiplier in normal times is decreasing in openness, \(\frac{\partial \mu^F_G}{\partial \gamma} < 0\), if \(\sigma \eta > 1\) and \(\tau < \tilde{\tau} = 2(1 - \gamma)(\sigma \eta - 1)/(1 + 2(1 - \gamma)(\sigma \eta - 1)) < 1\). Depending on parameter values this expression can be quite small, although standard calibrations will put \(\tilde{\tau}\) in the upper half of the unit interval.
3.5 EMPIRICAL STRATEGY & RESULTS

The friction affects the multiplier at the ZLB symmetrically,

\[
\mu_{\text{LT,F}}^G = \frac{(1 - p)(1 - \beta p) - \kappa p}{(1 - p)(1 - \beta p) - \kappa p \{ 1 + (1 - s_g)\frac{\epsilon}{\sigma} [(1 - \tau \gamma) + (1 - \tau)(2 - \gamma)(\sigma \eta - 1)\}}.
\]

(3.16)

where once again \(\mu_{\text{LT,F}}^G > 1\) if \(\tau < 1\). However, even for larger \(\tau\) the multiplier will be above 1 for standard parameterizations. The fiscal multiplier in the liquidity trap is increasing in openness, \(\frac{\partial \mu_{\text{LT,F}}^G}{\partial \gamma} > 0\), if \(\sigma \eta > 1\) and \(\tau < \bar{\tau} < 1\). Thus, the friction is more likely to affect the relative size of open and closed economy fiscal multipliers, rather than the relative size of the fiscal multiplier in normal times and at the ZLB.

3.5 Empirical Strategy & Results

Ultimately, the degree to which UIP fails and its influence on the size and properties of the fiscal multiplier at the ZLB is an empirical question. While ideally one would like to side track this issue and obtain direct estimates of fiscal multipliers there is not enough data to use standard empirical tools such as SVARs or estimated DSGE models. The aim of this paper is thus more modest: obtain estimates of the friction in UIP and, through the lens of the model, ask if the results obtained in the frictionless case are robust. In particular, given empirical departures from UIP,

1. is the fiscal multiplier at the ZLB above 1?
2. is the open economy fiscal multiplier at the ZLB larger than the closed economy fiscal multiplier?

To the best of my knowledge this is the first approach that tries to empirically test an aspect of the large fiscal multipliers that obtain in New Keynesian models at the ZLB.

There is a long literature that has estimated unconditional departures from UIP by directly estimating equation (3.3) with realized exchange rate data (see e.g. Engel (1996)). However, determining the size of the fiscal multiplier requires knowledge of the conditional departures from UIP. In other words, we need to know the size of the friction conditional on a government spending shock. Typically conditional deviations tend to be smaller than unconditional departures (e.g. Faust, Rogers, Wang, and Wright (2007)), so this distinction can be quantitatively important.

To determine the conditional departures from UIP I examine the nominal exchange rate response following generic inflationary shocks. If the inflation surprise was due to a government spending shock, then the nominal exchange rate in the frictionless model appreciates in normal times as in equation (3.12). As we increase
the friction parameter $\tau$, this appreciation will get smaller and smaller,

$$\frac{\partial \hat{e}_t}{\partial \hat{g}_t} = -\left[\phi - 1 + \frac{\phi}{1 - p}\right] \frac{1}{1 - \gamma} \frac{1}{1 - \tau} \frac{\partial \hat{\pi}_t}{\partial \hat{g}_t}, \quad (3.17)$$

and may even turn into a depreciation for large values of $\tau$. Similarly, at the ZLB the nominal exchange rate depreciates in the frictionless model as in equation (3.13). However, a large enough friction can again switch the sign of the exchange rate response,

$$\frac{\partial \hat{e}_t}{\partial \hat{g}_t} = \left[\frac{1}{1 - p} - \frac{\tau}{1 - p}\right] \frac{1}{1 - \gamma} \frac{\partial \hat{\pi}_t}{\partial \hat{g}_t}. \quad (3.18)$$

The empirical strategy is to estimate the coefficient $\delta$ and, given estimates for $\phi$, $\gamma$, and $p$, infer the value for $\tau$. I allow the estimate for $\tau$ to differ in normal times and at the ZLB, since the latter period featured more financial turmoil in international asset markets. The rest of this section will address some concerns about this empirical strategy.

First, it is unlikely that most generic inflation surprises are due to government spending shocks. However, equations (3.17) and (3.18) hold for any generic inflationary shock in the model, with the exception of $f_t$ which I will discuss below. We can simply replace $g_t$ by some other shock $x_t \neq f_t$, which could be either a demand or supply shock, and still use the estimated exchange rate response to determine the friction parameter $\tau$. I show in Appendix B.5 that the inflation surprises I use appear to be to hitherto unobserved demand shocks. Since there is little a priori reason why conditional deviations from UIP should vary across different types of demand shocks, using these inflation surprises as proxy is unlikely to considerably bias an estimate of $\tau$.

Second, when $f_t$ shocks are a major source of inflation surprises, then this will bias the estimation of $\tau$. In particular, shocks to $f_t$ will add a shock to the estimated exchange rate equation,

$$\Delta \hat{e}_t = \delta \Delta \hat{\pi}_t + \varepsilon_t$$

where $\varepsilon_t$ is the shock to $f_t$, and $\delta$ is $\delta^{NT}$ in normal times (equation (3.17)) or $\delta^{ZLB}$ at the ZLB (equation (3.18)). The estimated coefficient in this regression will be biased depending on the correlation between $\hat{\pi}_t$ and the shock $\varepsilon_t$,

$$E(\hat{\delta}) = \delta\left(1 + \frac{\text{cov}(\hat{\pi}_t, \varepsilon_t)}{\text{var}(\hat{\pi}_t)}\right).$$

In the model, $\text{cov}(\hat{\pi}_t, \varepsilon_t) \geq 0$ if $\sigma \eta > 1$, which implies that $f_t$ shocks will bias the estimates $\hat{\delta}$ away from zero and $\hat{\pi}$ downwards. Thus, if the estimated $\hat{\tau}$ is positive, then this is not a consequence of exogenous shocks to the UIP equation.
Third, for a large open economy, there is also some pass-through to inflation of goods produced in the foreign country $\pi^F_t$, which will mitigate the movements in nominal exchange rate. However, for plausible parameter values this effect is small and can be ignored: With a domestic import share of $\gamma = 0.15$ and a relative size of the foreign country of $n = 3$ (plausible for the US), the foreign import share is just $\psi = 0.05$. This limits the substitution by foreign consumers away or towards foreign goods, and thus the spill-over from domestic inflation, which is typically less than 10% for plausible parameter values. For parameters typical of European countries ($\gamma \approx 0.3$ and $n \geq 9$) the pass-through rate is even smaller. I also split the sample into large and small open economies and find little difference in the estimated exchange rate responses, suggesting that spill-overs do not bias the estimation.

Fourth, if generic inflation shocks are global then the exchange rate effects will be very different from those derived above. When both countries face a simultaneous shock to the excess return of their bonds, the effects on the nominal exchange rate are ambiguous. I consider this possibility in the following section, where I show that inflation surprises in other countries do not forecast current or future inflation in the US, and thus appear to represent local shocks.

### 3.5.1 Inflation Surprises

I construct inflation surprises following Clarida and Waldman (2008): The announced inflation values and the associated professional forecasts for inflation are from Bloomberg Financial Services. Bloomberg surveys numerous professional forecasters on their expectations of the next inflation announcement and I take the median of these expectations as the market expectation. The data covers eight countries from February 2000 until January 2010: the US, Great Britain, the Eurozone, Japan, Sweden, Norway, Switzerland, and Canada. For most countries inflation forecasts are available for the CPI and the core CPI. The definitions of the price indices used are tabulated in Appendix Table B.1 and summary statistics are tabulated in Appendix Table B.2 for headline inflation and appendix Table B.3 for core inflation. I define inflation surprises as the difference between the announced and the expected inflation rate.

Since the empirical test allows inflation surprises, both from demand or supply shocks, I relegate a more detailed analysis to Appendix B.5 and provide only an informal summary here. I show that they contain news about current and future macroeconomic conditions, and are largely exogenous with respect to past macroeconomic conditions. However, contemporaneously and over the next two years, inflation surprises are associated with higher inflation, lower unemployment and (outside the ZLB) higher policy rates. This is consistent with inflation surprises being caused by hitherto unobserved demand shocks, a novel finding to the best of my knowledge.

In addition, neither lagged nor contemporaneous foreign inflation surprises pre-
### 3.5. EMPIRICAL STRATEGY & RESULTS

Table 3.1: Correlation of Inflation Surprise with US Inflation.

<table>
<thead>
<tr>
<th>Timing:</th>
<th>Current Headline</th>
<th>Lead Headline</th>
<th>Current Core</th>
<th>Lead Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Type:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.330</td>
<td>0.535</td>
<td>-0.0849</td>
<td>-0.245</td>
</tr>
<tr>
<td></td>
<td>(0.522)</td>
<td>(0.538)</td>
<td>(0.223)</td>
<td>(0.213)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.997</td>
<td>1.107</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.616)</td>
<td>(0.659)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eurozone</td>
<td>1.744</td>
<td>1.238</td>
<td>-0.312</td>
<td>-0.522</td>
</tr>
<tr>
<td></td>
<td>(2.370)</td>
<td>(2.342)</td>
<td>(0.365)</td>
<td>(0.367)</td>
</tr>
<tr>
<td>Japan</td>
<td>1.804</td>
<td>1.609</td>
<td>-0.0635</td>
<td>-0.209</td>
</tr>
<tr>
<td></td>
<td>(1.365)</td>
<td>(1.391)</td>
<td>(0.310)</td>
<td>(0.279)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.241</td>
<td>0.0560</td>
<td>0.303</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td>(0.467)</td>
<td>(0.476)</td>
<td>(0.233)</td>
<td>(0.242)</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.263</td>
<td>0.433</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.652)</td>
<td>(0.635)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>-1.182</td>
<td>-1.274</td>
<td>-0.571*</td>
<td>-0.590*</td>
</tr>
<tr>
<td></td>
<td>(1.080)</td>
<td>(1.150)</td>
<td>(0.228)</td>
<td>(0.288)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.548***</td>
<td>2.543***</td>
<td>2.133***</td>
<td>2.126***</td>
</tr>
<tr>
<td></td>
<td>(0.139)</td>
<td>(0.140)</td>
<td>(0.0446)</td>
<td>(0.0445)</td>
</tr>
</tbody>
</table>

**Observations**: 109 108 109 108  

**$R^2$**: 0.070 0.071 0.042 0.054  

**F**: 0.980 1.085 1.491 1.436  

**p-value**: 0.450 0.379 0.200 0.218

Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Current = Inflation level in the Month of the Inflation surprise.

Lead = Inflation level in the Month following the Inflation surprise.
dict the inflation levels in the US, the reference country in this analysis. In Table 3.1 I regress current US inflation levels on current and lagged foreign inflation surprises and find p-values that are consistently above 0.2. This suggests that the demand shocks are local in nature, and that have the exchange rate effects described in the preceding section.

Table 3.2: Persistence of Inflation Surprise in Revised Inflation (Monthly Frequency).

<table>
<thead>
<tr>
<th></th>
<th>Headline Inflation</th>
<th>Core Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1) coefficient</td>
<td>0.930***</td>
<td>0.895***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Hansen’s J</td>
<td>14.68</td>
<td>15.82</td>
</tr>
<tr>
<td>p-value</td>
<td>0.91</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Estimated by GMM. HAC standard errors in parentheses.

* p < 0.05, ** p < 0.01, *** p < 0.001

To infer an estimate of the friction, I need to determine the persistence of the inflation shock. As shown in Appendix B.5, the IRF of inflation to an inflation surprise is close to an AR(1) process. In Table 3.2, I restrict the impulse to AR(1) in a GMM estimation, which easily satisfies the over-identification restrictions, with p-values of 0.91 and 0.86 respectively. The estimated persistence for headline inflation corresponds to a value of \( \hat{p} = 0.8 \) at quarterly frequency.

3.5.2 Exchange Rate Response to Inflation Surprises

Next, I examine the exchange rate response to inflation surprises by estimating

\[
\Delta e_{i,t} = \delta_0 + \delta_1 \tilde{\pi}_s^{i,t} + \delta_2 LT_{i,t} + \delta_3 LT_{i,t} \cdot \tilde{\pi}_s^{i,t} + \varepsilon_{i,t},
\]

(3.19)

where \( \Delta e_{i,t} \) is the change in the natural logarithm of the nominal exchange rate of country \( i \), \( \tilde{\pi}_s^{i,t} \) is an inflation surprise,\(^{16}\) and \( LT_{i,t} \) is a dummy variable equal to 1 if the economy is in a liquidity trap. The estimate of \( \delta_1 \) maps into \( \delta^{NT} \), while \( \delta_1 + \delta_3 \) maps into \( \delta^{ZLB} \).

\(^{16}\) Inflation that is not a surprise will already be reflected in the exchange rate.
I classify a country to be a liquidity trap at time $t$ as follows. First, I estimate a Taylor rule for all countries except Japan up to the fourth quarter of 2007 (if available). I then use this rule to forecast the predicted unconstrained interest rate up to 2010Q1 and determine in which quarters the predicted rate is below zero. Second, I check if the nominal interest rate in those quarters is the lowest observed in the sample, so that there is no more room for the central bank to cut rates given its implicit interest rate floor. If a quarter satisfies both conditions then I set the liquidity trap dummy for that quarter equal to 1. The details of the estimation procedure are relegated to Appendix B.7. For Japan the zero bound constraint is binding too frequently in the sample to estimate a Taylor rule. I therefore define Japan to be in a liquidity trap, whenever its policy rate is less than or equal to 0.25.

The exchange rate changes $\Delta e_{i,t}$ are high-frequency as in Clarida and Waldman (2008). For each inflation announcement from January 2005 until January 2010 I calculate the percentage change of the exchange rate against the US Dollar from 5 minutes before the announcement to 5 minutes after the announcement.\textsuperscript{17,18} I tabulate summary statistics for the percentage change in the exchange rate in Appendix Table B.4. All exchange rates are expressed in terms of units of foreign currency so a negative percentage change corresponds to an appreciation (as in the model).

Since equation (3.19) is non-structural it may be helpful to provide some interpretation on its form. First, it is not necessary to include inflation announcements of the foreign country in equation (3.19). Inflation announcements are made at different times across countries so whenever $\tilde{\pi}_{i,t}^{s} \neq 0$, then for all other countries $j \neq i$, $\tilde{\pi}_{j,t}^{s}$, within the 10-minute window that I consider. Thus, equation (3.19) will yield the same result as if I was controlling for foreign inflation surprises within the 10-minute window.

Second, it is not necessary to control if the foreign country is in a liquidity trap or not. Within the small open economy model of section 3.3 the exchange rate response will be the same irrespective of the foreign state. Intuitively, the domestic inflation surprise does not spill over to the large foreign country, so there is no foreign interest rate response even if the foreign country was unconstrained. For plausible parameters of a large open economy such as the US, the spill-overs in a calibrated model are also small. Nevertheless, as one robustness check I limit the estimation to small open economies.

I estimate equation (3.19) as a pooled regression with bootstrapped standard errors corrected for the presence of estimated regressors. The estimates are tabulated in Table 3.3. The first column contains the regression using year-on-year headline inflation surprises. Accordingly, in normal times a one percent inflation surprise is associated with a 0.876% appreciation of the home currency. The standard error of

\textsuperscript{17}The data was supplied by Olsen Data and is recorded at 5-minute frequency.

\textsuperscript{18}I use the British Pound as a reference currency for the US Dollar.
3.5. EMPIRICAL STRATEGY & RESULTS

this estimate is 0.189, so the estimate is statistically significant at the 1% level. With \( \hat{p} = 0.8 \), \( \gamma = 0.15 \) and \( \phi = 1.5 \) this estimate implies a moderate friction in normal times, \( \hat{\tau} = 0.50 \). Higher values for the import share do not affect his estimate very much, e.g. setting \( \gamma = 0.3 \) yields \( \hat{\tau} = 0.54 \).

The estimates for \( \delta_1 \) are very stable across the robustness checks I consider: In column (2) I add country and time fixed effects, in column (3) I restrict the sample to small open economies (excluding the US and the Eurozone), in column (4) I exclude Japan, and in column (5) I use core inflation surprises instead of headline inflation surprises. The estimate for \( \delta_1 \) ranges from -0.84 to -0.93 and is always significant at the 1% level. This result confirms earlier estimates by Clarida and Waldman (2008), who find that a 1% point (quarterly) surprise inflation generates a 0.6% appreciation in the nominal exchange rate in a set of 10 countries using data from July 2001 until December 2005.\(^{19}\) The implied range for \( \hat{\tau} \) is correspondingly tight - it ranges from 0.48 to 0.51 across these estimates.

Turning now to the estimates for the liquidity trap, I tabulate the total exchange rate response (the sum of the first row and the third row) along with the standard error in the two bottom rows of Table 3.3. Accordingly, the exchange rate appreciates by 0.063% for each percentage point of the inflation surprise, with a standard error of 0.366. This is significantly different from the exchange rate response outside the liquidity trap at the 10% level. However, the frictionless model predicted a depreciation in the domestic currency, which suggests that the friction to UIP must have been particularly large at the ZLB. In fact, with \( \hat{p} = 0.8 \) and \( \gamma = 0.15 \) I obtain an estimate of \( \hat{\tau} = 1.26 \), which is significantly larger than in normal times. This estimate is even less sensitive to the choice of \( \gamma \) than the estimate for normal times.

The robustness checks yield similar results, although less precisely estimated than \( \delta_1 \). Reading across columns (1) through (5) in Table 3.3, the estimate ranges from -0.01 (column (5)) to -0.23 (column (4)). In all cases I can reject the hypothesis that the domestic currency depreciates as predicted by the frictionless model. The implied range of \( \hat{\tau} \) for the ZLB estimates is 1.25 to 1.30.

The results are unlikely to be driven by measurement error: First, inflation surprises are followed by significant movements in macroeconomic aggregates, which suggests that they do contain useful information about shocks (see Appendix B.5). Second, the estimates at the ZLB would have been even more negative if there is

\(^{19}\)Nevertheless, the implied appreciation in the real exchange rate contrasts with growing literature, which estimates real exchange depreciations following fiscal shocks (Corsetti, Meier, and Muller (2009); Monacelli and Perotti (2010); Ravn, Schnitt-Grohé, and Uribe (2007)). However, Ilzetzki, Mendoza, and Vegh (2010) find that the real exchange rate appreciates following a fiscal shock for countries with flexible exchange rates. Furthermore, in robustness checks by Corsetti, Meier, and Muller (2009) the real exchange rate appreciates if fiscal shocks are identified using the Ramey and Shapiro (1998) dates. Thus, a real exchange rate appreciation following a fiscal shocks does not appear to be implausible.
3.5. EMPIRICAL STRATEGY & RESULTS

Table 3.3: Exchange Rate Response to Inflation Surprises

<table>
<thead>
<tr>
<th>Inflation Measure</th>
<th>Headline Inflation</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Countries &amp; Time only</td>
<td>Excl. Japan</td>
</tr>
<tr>
<td></td>
<td>Pooled FE (1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Surprise Inflation ($\delta_1$)</td>
<td>-0.867***</td>
<td>-0.831***</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(0.165)</td>
</tr>
<tr>
<td>Liquidity Trap ($\delta_2$)</td>
<td>-0.012</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.087)</td>
</tr>
<tr>
<td>Inflation-Liquidity $\delta_3$</td>
<td>0.804*</td>
<td>0.675*</td>
</tr>
<tr>
<td></td>
<td>(0.417)</td>
<td>(0.366)</td>
</tr>
<tr>
<td>Country &amp; Time FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>485</td>
<td>485</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.107</td>
<td>0.279</td>
</tr>
<tr>
<td>$\delta_1 + \delta_3$</td>
<td>-0.063</td>
<td>-0.156</td>
</tr>
<tr>
<td></td>
<td>(0.366)</td>
<td>(0.330)</td>
</tr>
</tbody>
</table>

Bootstrapped standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

All regressions use CPI non-flash inflation announcements.
3.6 QUANTITATIVE ANALYSIS

bias towards zero, which would imply an even greater distortion to UIP. Therefore, measurement error in the data cannot explain the discrepancy between the estimates and the predictions of the frictionless baseline model.

3.6 Quantitative analysis

Given estimates for $\tau$ in normal times and at the ZLB, I can calibrate the model to assess the quantitative and qualitative importance of departures from UIP. I calibrate the model with a set of standard parameters, which largely follow Christiano, Eichenbaum, and Rebelo (2011) and Bodenstein, Erceg, and Guerrieri (2009). The discount factor $\beta$ is calibrated at 0.99 to match an average real interest rate of 4% per year. The inverse Frisch elasticity $\nu$ is set to 0.5 and the IES $\sigma^{-1}$ is set to 1 to allow for sufficient output variation in a model without capital. The share of government in output $s_g$ is set to 0.2 and the import elasticity $\eta$ to 2 as suggested by Obstfeld and Rogoff (2005). The Calvo probability $\theta$ is calibrated at 0.85 to accord with the small inflation response in this recession.\(^{20}\) The persistence of the shock is set to $p = 0.8$ as estimated in Section 3.5.1.

In Figure 3.1 I display the baseline fiscal multiplier in normal times and at the ZLB, as well as the multipliers in the friction model while varying the import share $\gamma$ from 0 to 0.5. As proven in section 3.3, in the baseline model the fiscal multiplier is less than 1 and decreasing in the import share in normal times, while it is greater than 1 and increasing in the import share at the ZLB. The estimated friction barely affects the multiplier much in normal times - it is still less than 1 and decreasing in the import share. In fact, even much larger frictions, such as those estimated during the ZLB, will not push the fiscal multiplier above 1 during normal times. Small fiscal multipliers appear to be a robust feature during normal times, irrespective of whether UIP holds or not.

The same cannot be said for the ZLB. For small import shares as in the US ($\gamma = 0.15$), the fiscal multiplier in the friction model is “only” 1.5, whereas it assumes a value of 2.5 in the baseline model. Nevertheless, it is above 1 and it would require an extremely large friction - $\tau = 3.5$ - to push the multiplier below 1 for these parameters. However, unlike the baseline model the multiplier in the friction model is now decreasing in $\gamma$. The resulting difference between the closed economy fiscal multiplier and the friction multiplier are quantitatively significant, particularly at import shares relevant for European countries. For example, with $\gamma = 0.3$ the friction multiplier is 30% smaller than the closed economy fiscal multiplier. This suggests that for empirically relevant departures from UIP, exchange rate crowding

\(^{20}\)This calibration is equivalent to a model with firm-specific labor, where the Calvo probability is set to $\theta = 0.7$ (as suggested by Nakamura and Steinsson (2008)) and the elasticity of substitution across goods equals $\varepsilon = 10$. 

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3.6. QUANTITATIVE ANALYSIS

Figure 3.1: Fiscal multipliers for the baseline and friction model in normal times and at the ZLB.
out can be quantitatively important even at the ZLB.

### 3.6.1 A model with capital

In this section I add capital to the model to check the robustness of the results in the previous section. I allow capital to be completely mobile across firms, that have Cobb-Douglas technology with capital share $\alpha$. Aggregate capital depreciates at rate $\delta$ and is accumulated subject to a quadratic adjustment cost $\psi \left( \frac{K_t}{K_{t-1}} - \delta \right)^2 K_{t-1}$, where $K_t$ is capital and $I_t$ is investment. Investment goods are produced using the same weights on domestic and foreign goods as consumption in the baseline model,

\[
\hat{c}_t = (1 - \gamma)\hat{c}_t^H + \gamma\hat{c}_t^F
\]  \hspace{1cm} (3.20)

and with elasticity of substitution $\eta$ between home and foreign goods. Allowing for capital results in two new (log-linearized) first order conditions, an Euler equation for capital,

\[
\dot{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma} \left[ \beta R(1 - \alpha)(\hat{n}_{t+1} - \hat{k}_t) + \beta \hat{q}_{t+1} - \hat{q}_t + \hat{\beta}_{t+1} \right]
\]  \hspace{1cm} (3.21)

and an equation for Tobin’s $q$,

\[
\hat{q}_t = \psi(\hat{k}_t - \hat{k}_{t-1})
\]  \hspace{1cm} (3.22)

This model can no longer be solved analytically for the fiscal multiplier, so I solve this model using the algorithm from Bodenstein, Erceg, and Guerrieri (2009). Coibion, Gorodnichenko, and Wieland (2011) show that this algorithm remains accurate even for large shocks that will push the economy to the ZLB. To make the estimates comparable to the baseline model, I let the ZLB bind for 5 quarters. I set the capital share $\alpha = 0.33$, the depreciation rate at $\delta = 0.02$ and the investment adjustment cost at $\psi = 7$ as in Shapiro (1986). In addition the Frisch elasticity of labor supply $\nu^{-1}$ is reduced to 2/3, and the IES $\sigma^{-1}$ to 0.5, since investment will now induce sufficient output variability.

Importantly, introducing capital does not invalidate the empirical analysis in section 3.5, as the friction enters the UIP relationship in exactly the same fashion. Thus, the same estimated friction can be used to calibrate the model with capital.\footnote{Because the persistence of inflation is endogenous in the capital model I adjust $p$ in equation (3.14) to hit the estimated exchange rate response in Table 3.3.} The fiscal multipliers for the capital model, as well as the capital model with friction are reported in Figure 3.2.

Adding capital does not change the qualitative behavior of fiscal multipliers in normal times: They are small, less than 0.5, both in the standard model and the
The fiscal multipliers are calculated for normal times, binding ZLB, normal times with friction, and binding ZLB with friction.
3.7. CONCLUSION

model with friction. More action occurs at the ZLB: Here, the fiscal multiplier is increasing in the import share in the baseline model with capital, but this is reversed when I add the friction to international asset markets. The differences between the baseline multiplier and the friction multiplier are smaller than in the model without capital, but still quantitatively significant. Furthermore, letting $\gamma = 0.3$ in the friction model again reduces the fiscal multiplier by about 30% relative to the closed economy, which confirms the earlier finding that exchange rate crowding out can be quantitatively important. Finally, the friction fiscal multipliers remain above 1 for reasonable import shares, and are significantly larger than fiscal multipliers in normal times.

In summary, incorporating the empirically estimated friction into the standard models significantly affects the properties of the fiscal multiplier at the ZLB. It will be smaller in the open economy than in the closed economy, because the friction prevents favorable exchange rate adjustments that occur in the baseline model. However, it typically remains above 1, which is large by the standards of the economy literature. This suggests that fiscal policy in the open economy is effective at the ZLB, although not as much as our baseline models may lead us to believe.

3.7 Conclusion

In this paper I have provided both theory and evidence on the open economy fiscal multiplier in a liquidity trap. I show that in open economy New Keynesian models, the fiscal multiplier at the ZLB is greater than 1 and increasing in the import share if there are no frictions to international asset markets. Intuitively, the open economy’s interest rate sensitivity of output is greater (in absolute value), so that it gets a larger boost from a given decline in real interest rates. Indeed, for standard parameterization, the home countries’ net exports rise, so that domestic fiscal expansion in the liquidity trap has a beggar-thy-neighbour effect.

I then show that sufficiently large frictions in international asset markets that manifest themselves as departures from UIP, can overturn both conclusions. In this case, a large friction will prevent the nominal exchange rate depreciation that occurs in the baseline model, so that the terms of trade improve. This can crowd out net exports sufficiently such that the open economy has a smaller multiplier than the closed economy, and that this multiplier is below 1.

To distinguish between these conflicting predictions and determine the likely properties of the open economy fiscal multiplier at the ZLB, I estimate the size of the friction by examining the exchange rate response to generic inflation surprises. While the frictionless model predicts that the nominal exchange rate depreciates by more than 1% for each 1% point of inflation at the ZLB, I estimate essentially a zero response. Thus, large frictions are needed to rationalize this exchange rate response.
A model calibrated with the estimated friction shows quantitatively important deviations from the baseline model at the ZLB. The fiscal multipliers are significantly smaller, even at moderate import shares, and decline as the import share rises. Nevertheless, the fiscal multipliers in the friction model are typically above 1, which is large given the standards of an open economy (e.g., Dornbusch (1976), Ilzetzki, Mendoza, and Vegh (2010)).

As more data on ZLB episodes becomes available, more empirical work will be required to sharpen our estimates of the fiscal multiplier in the current economic environment. In the meantime, as we resort to model-based analysis, the results of this paper suggest that frictions in international asset markets can have important quantitative implications for the open economy fiscal multiplier, particularly at the ZLB. Evidently, more research is needed to understand the sources of these frictions and their connections to the real economy.
Chapter 4
Are Negative Supply Shocks Expansionary at the ZLB?

“As some of us keep trying to point out, the United States is in a liquidity trap: [...] This puts us in a world of topsy-turvy, in which many of the usual rules of economics cease to hold. Thrift leads to lower investment; wage cuts reduce employment; even higher productivity can be a bad thing. And the broken windows fallacy ceases to be a fallacy: something that forces firms to replace capital, even if that something seemingly makes them poorer, can stimulate spending and raise employment.”

Paul Krugman, 3rd September 2011.

4.1 Introduction

The preceding chapter has highlighted that government spending at the ZLB can be very effective. This has lead some economists to argue that demand-side policies, such as fiscal stimulus and forward guidance, are well-suited to address the ZLB problem (e.g., Bernanke (2012), Eggertsson and Krugman (2011), Woodford (2012)). However, these strong expansionary effects arise because such policies increases production costs, which raises inflation expectations through the Phillips curve (1.2), lowers real interest rates and stimulates consumption. Thus, implicit in this argument is that negative supply shocks, i.e. shocks that raise inflation expectations, are expansionary through the inflation expectations channel. As a result, opponents are skeptical whether these models, originally designed and estimated to match normal times, describe the ZLB accurately enough to justify such bold policy choices (e.g., Cochrane (2009), Taylor (2012)).

This chapter contributes to this debate in three ways. First, using a variety of
empirical tests I reject the prediction that negative supply shocks are expansionary at the ZLB. Second, I show that incorporating credit frictions into sticky-price models is important to match this stylized fact. However, this necessity only becomes clear at the ZLB when the central bank’s stabilization policy no longer dampens the financial accelerator. Third, I demonstrate that fiscal policy and forward guidance at the ZLB are significantly less effective in the model with credit friction than in standard sticky-price models, and that this is a robust feature of models where negative supply shocks are contractionary at the ZLB. Thus, contrary to Krugman’s assertion, the behavior of the ZLB economy can be quite similar to that in normal times.

Standard new Keynesian models emphasize the importance of expectations in the propagation of shocks. With a standard Euler equation, consumption today is determined by expected future real interest rates and expected long-run consumption. Temporary, negative supply shocks do not affect long-run consumption, but raise inflation expectations when prices are sticky. In normal times (i.e., outside the ZLB), the central bank raises nominal interest rates enough that expected real interest rates rise (the Taylor principle) and consumption contracts. In contrast, nominal interest rates remain unchanged when the central bank is constrained by the ZLB, so that higher inflation expectations reduce real interest rates and consumption today expands. I call this mechanism the “expectations channel,” since inflation expectations are the key quantity through which supply shocks affect real output at the ZLB.

In my first contribution, I test for the inflation expectations channel by examining two types of evidence. First, I determine the macroeconomic impact of two negative supply shocks at the ZLB: oil supply shocks, and the Japanese earthquake in 2011. My results show that while inflation expectations rise and expected real interest rates fall as predicted by the theory, these negative supply shocks are still contractionary overall. I also provide evidence against a weaker interpretation of the expectations channel. Because expected future nominal rates rise less at the ZLB, supply shocks should be less contractionary than in normal times; however, I document that oil supply shocks appear to be, if anything, more contractionary at the ZLB. This suggests that the expectations channel plays only a limited role in the propagation of such shocks. To the best of my knowledge, this is the first paper to test directly for the expansionary effects of negative supply shocks through the expectations channel at the ZLB.

Second, I argue that inflation risk premia can signal if generic negative supply shocks are also contractionary at the ZLB. In standard models, higher expected inflation raises consumption at the ZLB irrespective of its source, so nominal assets become a hedge — they gain value in deflationary states when consumption is low. Conditional on the ZLB, the inflation risk premium should therefore be negative. However, empirically the one-year inflation risk premium at the ZLB is typically positive, suggesting that investors want to insure against shocks that raise inflation
and lower consumption. This indicates that generic negative supply shocks are not only contractionary at the ZLB, but also a significant contributor to inflation risk over this horizon.

These findings constitute a puzzle for any model with a standard Euler equation, since it predicts that consumption should expand given the lower real interest rates in the data. As my second contribution, I show that incorporating financial frictions in the Euler equation reconciles the theory with the data.\(^1\) Following Gertler and Kiyotaki (2010) and Cúrdia and Woodford (2009), my model features a balance sheet constraint on financial intermediaries that generates an endogenous spread between the borrowing and the deposit rate. Because a negative supply shock reduces profits and share values, the net worth of financial intermediaries falls, tightening their balance sheet constraints. In turn, banks contract loan supply, the borrowing spread rises, and borrowers reduce consumption such that negative supply shocks are contractionary at the ZLB.

While my model is more successful at matching the data, in normal times it behaves similarly to a standard new Keynesian model. Intuitively, the central bank responds more aggressively to amplified inflation and output responses, which endogenously dampens the financial accelerator. This suggests that differences between models are “smoothed-out” in normal times, allowing a variety of models to match these kinds of data. Since this mechanism is absent at the ZLB, many such models will be less successful at matching these data, even if they worked well in normal times. On the flip-side, the ZLB provides a unique testing ground, because one can more easily discriminate among models when propagation mechanisms are uninhibited by central bank actions.

As my third contribution, I show that demand-side policies at the ZLB are less effective in models that match these data. For instance, in a new Keynesian model a government spending shock at the ZLB raises current and expected marginal cost, which increases expected inflation, lowers real interest rates, and stimulates consumption so that the fiscal multiplier exceeds one. Thus, ZLB multipliers are large because higher marginal costs raise consumption through the expectations channel — but this prediction is rejected in the data! In any data-consistent model the rise in inflation expectations from higher marginal costs cannot be expansionary, so that fiscal policy becomes much less effective. Consequently, in the calibrated model with credit frictions, demand-side policies are up to 50% less effective than in a standard new Keynesian model. This suggests that policy makers should be cautious in expecting large positive outcomes from such policies at the ZLB.

This paper is closely related to literature that has explored how standard macroeconomic models make very different, even paradoxical, predictions at the ZLB. My

\(^{1}\)In Section 4.5, I discuss the reasons for this modeling choice as well as other (complementary) mechanisms.
empirical results concern primarily the “Paradox of Toil.” Eggertsson (2010b, 2011, 2012) showed how standard new Keynesian models robustly predict that negative supply shocks (e.g., greater monopoly power, higher labor taxes) are expansionary at the ZLB through the expectations channel. As such, increases in desired labor supply (a positive supply shock) become self-defeating, because the resulting deflation reduces demand and equilibrium employment. In addition, according to the “Paradox of Thrift,” a rise in the desire to save is also self-defeating at the ZLB, because it reduces output so much that aggregate savings fall (Keynes (1936), Krugman (1998), Eggertsson and Woodford (2003), and Christiano (2004)). Finally, according to the “Paradox of Flexibility,” output volatility may rise at the ZLB if prices and wages become more flexible (e.g., Werning (2011), Eggertsson and Krugman (2011)).

Empirical validations of these paradoxes are scarce. In a first examination of the “Paradox of Toil,” Mulligan (2010, 2012) argues that seasonal labor inflows do not appear contractionary and higher minimum wages do not appear expansionary in the data. However, Eggertsson (2010a) disputes that these tests are valid because the shocks are either forecastable or permanent, and therefore cannot raise inflation expectations and work through the expectations channel. Second, Bachmann, Berg, and Sims (2011) show that consumers with above-average inflation expectations, i.e. below average expected real interest rates, have lower willingness to spend. However, in their cross-sectional analysis they cannot test whether aggregate inflation expectations are expansionary at the ZLB as predicted by the expectations channel. My analysis is robust to these critiques, as the supply shocks I use raise aggregate inflation expectations and lower expected real interest, yet still are contractionary.

This stylized fact has important implications for the literature on fiscal multipliers. There is an ongoing debate whether fiscal multipliers are large at the ZLB (e.g., Christiano, Eichenbaum, and Rebelo (2011); Woodford (2011a)) or small (e.g., Cogan, Cwik, Taylor, and Wieland (2010); Drautzburg and Uhlig (2011)). This paper shows that when negative supply shocks are contractionary at the ZLB, then the inflation expectations channel cannot be a source of large multipliers. To the extent that multipliers may be large at the ZLB, my results suggest that these are due to other mechanisms such as consumption-labor complementarities (Nakamura and Steinsson (2011)), low capacity utilization (Christiano, Eichenbaum, and Rebelo (2011)), or high unemployment (Michaillat (2012)).

This paper proceeds as follows. In Section 4.2, I show that temporary, negative supply shocks are expansionary in standard sticky-price models when the ZLB binds. In Section 4.3, I reject this prediction for oil supply shocks and an earthquake. In Section 4.4, I calculate one-year inflation risk premia to determine if generic negative supply shocks are also contractionary at the ZLB. In Section 4.5, I build a model with credit frictions that can match these facts, and in Section 4.6 I derive implications for demand-side policies. I conclude in Section 4.7.
4.2 Predictions from Standard Sticky-Price Models

I derive predictions in the continuous-time adaptation of a standard new Keynesian model from Woodford (2003).\footnote{Braun, Körber, and Waki (2012) demonstrate that the log-linear approximations used here can become inaccurate at the ZLB, and that negative supply shocks may be contractionary in the true non-linear model. However, Christiano and Eichenbaum (2012) argue that the approximations remain valid if output deviates by less than 20% from steady-state as is the case in my calibration. In addition, my findings in Sections 4.2 and 4.3 remain a puzzle in Braun, Körber, and Waki (2012), because even in the non-linear model lower real interest rates should stimulate consumption and the inflation risk premium should be negative.} In Appendix A.2, I illustrate how the more elaborate Smets and Wouters (2007) model also predicts that negative supply shocks are expansionary at the ZLB. Eggertsson (2012) shows further robustness of this prediction in a number of extensions.

Consider a representative agent with separable utility

\[ U_t = C_t^{1-\gamma} - \frac{L_t^{1+\nu}}{1+\nu}, \]

where \( C_t \) is the consumption flow, \( L_t \) is labor supply, \( \gamma \) is the intertemporal elasticity of substitution, and \( \nu \) is the inverse Frisch elasticity. This agent can hold a risk-free nominal bond that pays an instantaneous nominal interest rate \( i(t) \), as well as a complete set of Arrow-Debreu securities. The optimal allocation must satisfy the standard Euler equation,

\[ E_t[d\ln C(t)] = \gamma[i(t) - \pi(t) - \rho - v(t)]dt, \quad (4.1) \]

where \( \pi(t) \) is the instantaneous inflation rate, \( \rho \) the instantaneous discount rate, and \( v(t) \) is an exogenous demand shifter.

Solving the Euler equation forward illustrates that today’s consumption depends on the sum of expected real interest rates and expected consumption in the far future:

\[ \ln C(t) = -\gamma E_t \int_0^\infty [i(t+s) - \pi(t+s) - \rho - v(t+s)]ds + E_t \lim_{T \to \infty} \ln C(T). \quad (4.2) \]

Thus, consumption today is high relative to the long-run consumption level if the expected path of real interest rates is relatively low, and vice-versa.

I assume that the supply side of the model is given by a standard new Keynesian Phillips curve,

\[ d\pi(t) = \rho \pi(t) - \kappa[(\gamma^{-1} + \nu)(\ln C(t) - \ln \bar{C}) + (1 + \nu)u(t)] \quad (4.3) \]

where \( \ln \bar{C} = E_t \lim_{T \to \infty} \ln C(T) \) is the expected long-run consumption level and \( u(t) \) is a temporary, negative supply shifter (e.g., a temporary, negative technology shock).
This paper employs the new Keynesian Phillips curve merely as a convenient way to introduce price stickiness. For the purpose of this analysis the key property of Equation (4.3) is that a temporary, negative supply shock raises expected inflation — a prediction that is verified in the empirical analysis below. This property is shared by other sticky-price models (e.g., Taylor (1979)), and this paper does not take a stand on the relative merits of these settings. However, this prediction contrasts with flexible-price economies where temporary, negative supply shocks trigger expected deflation, because firms immediately raise prices and then let them gradually fall as the supply shock dissipates.

To analyze the effect of a negative supply shock I first differentiate Equation (4.2),

\[
\frac{d \ln C(t)}{du(t)} = -\gamma E_t \int_0^\infty [i(t + s) - \pi(t + s)] ds + \lim_{T \to \infty} E_t C_T du(t).
\]

This reveals that the supply shock can affect consumption by changing either real interest rates or long-run consumption. I restrict my attention to persistent, but temporary supply shocks, so that the second term on the right-hand side (RHS) is zero. Thus, a temporary, negative supply shock can be expansionary if and only if it lowers the expected sum of future real interest rates — the first term on the RHS.

**Summary 2** Given the Euler equation (4.1), a temporary, negative supply shock — an increase in \(u(t)\) — is expansionary if and only if it lowers the sum of expected real interest rates:

\[
\frac{E_t \int_0^\infty [i(t + s) - \pi(t + s)] ds}{du(t)} < 0.
\]

A crucial determinant of the real interest rate response is the interest rate rule of the central bank. I assume that it takes a standard linear form subject to the ZLB,

\[
i(t) = \max\{\bar{i} + \phi_\pi \pi(t) + \phi_y (\ln C(t) - \ln \bar{C}), 0\}. \tag{4.4}
\]

This rule encompasses inflation targeting as a special case \((\phi_\pi \to \infty)\), which is the time-consistent optimal policy rule in this setting.

Suppose that the supply shifter \(u(t)\) follows a Poisson process as in Eggertsson and Woodford (2003). This stochastic process is popular because it is convenient for analyzing the ZLB, but other stochastic processes yield similar results (e.g., Farhi and Werning (2012a)). Initially, the economy is subject to an unanticipated negative supply shock, \(u(0) = \bar{u} > 0\), and \(u(t)\) subsequently evolves according to

\[
du(t) = -\bar{u} dN(t), \quad dN(t) = \begin{cases} 1 \text{ with prob. } \lambda dt & \text{if } N(t) = 0 \\ 0 \text{ with prob. } 1 - \lambda dt & \text{if } N(t) = 0 \\ 1 \text{ with prob. } 1 & \text{if } N(t) = 1 \end{cases}
\]
4.2. PREDICTIONS FROM STANDARD STICKY-PRICE MODELS

where \( N(0) = 0 \). In other words, the system originally starts in the state \( s(0) = \{u(0) = \bar{u}, N(0) = 0\} \), and then stochastically reverts to the absorbing state \( \bar{s} = \{u(t) = 0, N(t) = 1\} \). Thus, the supply shock ceases with probability \( \lambda dt \) over an interval \( dt \). In Appendix A.1, I formally solve the system of differential equations, whereas this section proceeds with a more intuitive discussion of the results.

Suppose first that the ZLB does not bind. Then the economy is split in two regimes — while the supply shock is active \( \ln C(t) \) and \( \pi(t) \) are a constant function of \( u(t) = \bar{u} > 0 \), whereas if the supply shock has ceased the economy returns to steady-state where \( \ln C(t) = \ln \bar{C} \) and \( \pi(t) = 0 \). Therefore, conditional on the first regime, \( E_t d\ln C(t) = -\lambda \bar{C} dt \), which I substitute into the Euler equation to determine the impact of the supply shock

\[
\frac{d\ln C(t)}{d\bar{u}} = -\gamma \frac{(\phi_{\pi} - 1)}{\lambda} \frac{d\pi(t)}{d\bar{u}} - \gamma \frac{\phi_y}{\lambda} \frac{d\ln C(t)}{d\bar{u}}. \tag{4.5}
\]

The terms on the RHS are respectively the change in real interest rates from higher inflation and from movements in consumption. Conditional on a supply shock, Equation (4.5) traces out a downward-sloping line in the \( (\ln C(t), \pi(t)) \)-space with slope \( \frac{d\ln C(t)}{d\pi(t)} = -\frac{\phi_{\pi} - 1}{\lambda + \gamma \phi_y} \), which is plotted in Figure 4.2(a). Intuitively, under the policy rule (4.4) higher expected inflation from a supply shock translates into higher expected real interest rates, which reduces consumption today. Since this curve is derived from the consumer’s FOC, I label it the “Aggregate Demand” (AD) curve as is common in the literature (e.g., Romer (2011)).

Using the supply side (4.3), I substitute \( \frac{d\pi(t)}{d\bar{u}} = \frac{\kappa(\gamma^{-1} + \nu)}{\lambda + \phi_y} \frac{d\ln C(t)}{d\bar{u}} + \frac{\kappa(1 + \nu)}{\lambda + \phi_y} \) and obtain that a negative supply shock (an increase in \( \bar{u} \)) lowers consumption in normal times

\[
\frac{d\ln C(t)}{d\bar{u}} = -\kappa \gamma (\phi_{\pi} - 1)(1 + \nu) \frac{(\lambda + \gamma \phi_y)(\phi_y + \lambda) + (\phi_{\pi} - 1)\kappa(1 + \gamma \nu)}{(\lambda + \gamma \phi_y)(\phi_y + \lambda) + (\phi_{\pi} - 1)\kappa(1 + \gamma \nu)} < 0.
\]

This is illustrated in Figure 4.2(a), which plots the supply side (4.3) as an upward-sloping \( AS \) curve, because higher consumption triggers higher expected inflation by raising marginal costs. A negative supply shock raises expected and actual inflation \textit{ceteris paribus}, so that the \( AS \) curve shifts up to \( AS_2 \). The central bank responds by raising real interest rates, so that consumption contracts. Thus, consistent with Summary 2, negative supply shocks are contractionary in normal times because they raise expected real interest rates.

While in normal times the central bank raises nominal interest rates enough to also raise real rates, this may not occur when the central bank is constrained by the ZLB. For example, if a negative supply shock raises the shadow nominal interest rate from \(-3\%\) to \(-2\%\), then the actual nominal interest rate remains zero. To focus on this case, I follow Eggertsson and Woodford (2003) and introduce a negative demand shock that makes the ZLB bind. In particular, I let \( v(0) = -\bar{v} < 0 \) and
4.2. PREDICTIONS FROM STANDARD STICKY-PRICE MODELS

dv(t) = \tilde{v}dN(t).\) This splits the economy in a ZLB and a non-ZLB regime, and the supply shock only affects the economy in the ZLB state.

At the ZLB, the Euler equation then yields

\[
\frac{d\ln C(t)}{d\bar{u}} = \gamma \frac{1}{\lambda} \frac{d\pi(t)}{d\bar{u}} \Rightarrow \frac{d\ln C(t)}{d\pi(t)} = \gamma \frac{1}{\lambda} > 0.
\]  

(4.6)

Compared to aggregate demand in normal times (Equation (4.5)) the persistence of inflation \(\lambda\) enters analogously. However, there is no change in nominal interest rates so that the terms involving the policy parameters \(\phi_\pi\) and \(\phi_y\) drop out. The resulting \(AD\) curve is now upward-sloping, as shown in Figure 4.2(b), because at the ZLB higher expected inflation lowers expected real interest rates so that consumption expands. The upward-shift in the \(AS\) curve from a negative supply shock therefore raises consumption,

\[
\frac{d\ln C(t)}{d\bar{u}} = \frac{\kappa\gamma(1 + \nu)}{\lambda(\varphi + \lambda) - \kappa(1 + \gamma\nu)} > 0,
\]

because — consistent with Summary 2 — expected real interest rates fall. As shown in Appendix A.1, determinacy at the ZLB requires that the denominator is positive, which implies that the \(AD\) curve is steeper than the \(AS\) curve in Figure 4.2(b).

Summary 3  Consider an economy that satisfies the Euler equation (4.1), the supply side (4.3), and let monetary policy be given by (4.4). Then, a temporary, negative supply shock — an increase in \(u(t)\) — that does not persist after the ZLB binds is expansionary because it lowers expected real interest rates.

The previous stochastic process implies that the supply shock does not affect the economy after the ZLB ceases to bind. However, if a negative supply shock persists beyond the duration of the ZLB, then the central bank may raise real interest rates at those dates, which will impact today’s consumption through the Euler equation. I call these effects “spillovers.” For example, if exit from the ZLB is governed by an exogenous Poisson process with intensity \(\lambda^{ZLB}\), then the change in real interest rates is given by

\[
\frac{d\mathbb{E}_t\int_0^\infty [i(t + s) - \pi(t + s)]ds}{du(t)} = -\frac{d\mathbb{E}_t\int_0^\infty [\pi(t + s)]ds}{du(t)} + \frac{d\mathbb{E}_t\int_0^\infty (1 - e^{-\lambda^{ZLB}s})[i(t + s)]ds}{du(t)}.
\]

\(<0 \text{ (supply shock raises inflation)}\)

\(\text{Exit from ZLB: Spillovers}\)

3In important, recent work Aruoba and Schorfheide (2012), Braun, Körber, and Waki (2012), and Mertens and Ravn (2013) analyze the properties of an economy in the indeterminacy region. I focus on the determinacy case, because in that region a negative supply shock raises expected inflation and lowers expected real interest rates as is empirically the case in Section 4.3.
4.3. NEGATIVE SUPPLY SHOCKS AT THE ZLB

If spillovers are zero (or negative), then negative supply shocks unambiguously lower real interest rates — a sufficient condition for negative supply shocks to be expansionary at the ZLB. However, if spillovers are large and positive, then a negative supply shock might raise expected real interest rates and be contractionary at the ZLB. In fact, large negative supply shocks such as extreme natural disasters and wars will remain contractionary at the ZLB because of such spillover effects. These shocks significantly raise marginal costs of production and inflation expectations such that the economy exits from the ZLB, expected real interest rates rise, and consumption contracts.

Long-term bond yields can reveal whether spillovers are important. According to the expectations hypothesis, long-term bond yields are an average of short-term rates, such that higher expected future policy rates will be reflected in higher long-term bond yields today,

\[
\frac{dy(t, t + s)}{du(t)} = \frac{1}{s} \left[ \int_0^k e^{-\lambda_{ZLB}^t} \frac{di(t + k)}{du(t)} \, dk + \int_0^s (1 - e^{-\lambda_{ZLB}^t}) \frac{di(t + k)}{du(t)} \, dk \right]
\]

where \( y(t, t + s) \) is the annualized yield of a zero-coupon bond with a maturity of \( s \) years. Conversely, if long-term bond yields do not rise, then there are no spillovers and negative supply shocks should be expansionary at the ZLB.

**Summary 4** Spillovers from temporary, negative supply shocks at the ZLB into normal times will be reflected in higher long-term nominal interest rates. Without spillovers, negative supply shocks at the ZLB are expansionary in the baseline model.

This summary motivates the empirical strategy in the following section. I examine two temporary, negative supply shocks and show that they raise inflation expectations but not expected future nominal rates. I then test whether these shocks are in fact expansionary as predicted by standard sticky-price models.

4.3 Negative Supply Shocks at the ZLB

In this section, I examine the effects of two negative supply shocks at the ZLB.

4.3.1 Oil Supply Shocks

The challenge in uncovering oil supply shocks from production and price data is to separate demand from supply shocks. I follow Kilian’s (2009) identification assumption, that oil supply does not respond to demand shocks within a month; this
4.3. NEGATIVE SUPPLY SHOCKS AT THE ZLB

allows oil supply shocks to be identified through a standard Cholesky decomposition. Using this methodology, I extend the original oil supply shock series to December 2011. Similar to the original series, a one-standard-deviation negative oil supply shock raises real oil prices by 0.65% for about 12 months. This is consistent with the measure picking up reductions in supply rather than current or expected negative demand shocks. Importantly, oil supply shocks do not have a permanent effect on real oil prices in the VAR. Therefore, these constitute temporary shocks as required by the theory in Section 4.2.

Since this shock series extends into the current ZLB episode, I include the U.S., the U.K., the Eurozone, Canada, Sweden, and Japan in my estimation. I restrict the baseline estimation to dates when the ZLB binds, which are tabulated in Table 4.1. In practice the central bank interest rate floor is above zero, so dates are determined with the following interest rate cut-offs: 0.5 for Japan before 1998, and 0.25 thereafter; 0.25 for the U.S., Canada, and Sweden; 0.5 for the U.K.; and 1.00 for the Eurozone. Unlike the other countries, Eurozone policy rates have fluctuated below this cut-off, so it is less clear that the ECB was constrained in responding to negative supply shocks. In Appendix A.5, I therefore repeat my analysis excluding the Eurozone and reach essentially unchanged results.

I first test whether negative oil supply shocks raise inflation expectations at the ZLB. The inflation expectations data are taken primarily from Consensus Economics, and supplemented by various national consensus forecasts. Consensus Economics provides annual inflation forecasts at a monthly frequency, and quarterly inflation forecasts at a quarterly frequency. I combine these data sources in a Kalman filter to extract four-quarter-ahead inflation forecasts at a monthly frequency. For example, the March 4Q-ahead inflation forecast captures expected inflation from Q2 this year to Q2 next year. In April the window moves forward to Q3-Q3, in July to Q4-Q4, and in October to Q1-Q1. This is a conservative choice, since it excludes changes in expected inflation for the remainder of the quarter. The details of the data sources and the Kalman filter estimation are relegated to Appendix A.3.

In the baseline specification, I regress changes in 4Q-ahead inflation expectations on lagged values, lagged oil shocks, and lagged controls,

\[
\Delta \pi^{e,4}_t = \alpha + \sum_{j=1}^{12} \beta_j \Delta \pi^{e,4}_{t-j} + \sum_{j=1}^{24} \gamma_j \text{oil}_{t-j} + \sum_{j=1}^{24} \delta_j \text{controls}_{t-j} + \varepsilon_t. \tag{4.7}
\]

\footnote{Kilian and Murphy (2011) find that the short-run supply elasticity is small, thereby validating the analysis in Kilian (2009). I do not follow their methodology, because they impose sign-restrictions on real activity.}

\footnote{In Appendix A.5, I obtain similar results with data up to 2006, so the extension does not drive my findings.}

\footnote{The differencing accounts for the moving window of inflation forecasts.}
4.3. NEGATIVE SUPPLY SHOCKS AT THE ZLB

Since inflation forecasts are published at the beginning of the month, I exclude contemporaneous shocks from the regression because these are unlikely to be contained in the forecasters’ information sets. I include lagged forecast revisions because they have been shown to predict current forecast revisions (Coibion and Gorodnichenko (2011a)). I employ as additional controls changes in industrial production and long-term bond rates to proxy for changes in economic activity. Thus, the regression estimates capture how an oil supply shock affects inflation expectations for a given level of output — that is, the partial derivative \( \frac{\partial \pi(t)}{\partial u(t)} = \kappa (1+\nu) (\lambda+\varphi) \) in the Phillips curve (4.3). All regressions include country fixed effects, and lag lengths are determined by the Akaike Information Criterion (AIC).

The baseline results are tabulated in Figure 4.3(a), along with 95% confidence intervals based on Driscoll and Kraay (1998) standard errors, which are robust to heteroscedasticity, temporal dependence, and cross-country dependence. The standard asymptotic covariance matrix remains valid even though the oil shock in Equation (4.7) is a generated regressor. This is because (a) the oil shocks are the OLS residuals of a first-stage and therefore orthogonal to first-stage sampling uncertainty and (b) even for generated regressors the standard errors are correct under the null of \( \gamma_j = 0 \) \( \forall j \) (see Appendix A.4 for a proof).

In Figure 4.3(a) a one-standard-deviation negative oil supply shock has a positive and statistically-significant short-run effect on 4Q-ahead inflation expectations, which peaks at about 2/5\(^{th}\) of a standard deviation (5 basis points) after three months, and tails off thereafter. In Appendix A.5, I obtain larger responses in inflation expectations when I use a less conservative estimate that includes changes in inflation expectations for the current quarter. In the same appendix, I also demonstrate that oil supply shocks raise inflation swap rates (risk-neutral inflation expectations) at the ZLB, but this regression excludes Japan because of data availability. Thus, for either of these measures oil shocks raise inflation expectations for a given level of output, consistent with the discussion in Section 4.2.

My first measure of monthly economic activity is industrial production (IP). I regress growth in IP on lagged values, and on both contemporaneous and lagged oil shocks:

\[
\Delta y_t = \alpha + \sum_{j=1}^{n} \beta_j \Delta y_{t-j} + \sum_{j=0}^{k} \gamma_j \text{oil}_{t-j} + \varepsilon_{t}^y. \tag{4.8}
\]

While lagged dependent variables are not necessary if the shocks are well-identified, including these terms sharpens the estimates and ensures that the oil shock coefficients do not capture dynamics of output induced by other shocks. In Appendix A.5,

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7In my sample, Consensus Economics forecasts are published between the 4\(^{th}\) and 15\(^{th}\), with a median at the 11\(^{th}\) of the month. Forecasts contained in this survey are likely to be older.

8Coibion (2012) shows that the AIC performs better than BIC lag selection given the short samples employed here. In Appendix A.5, I tabulate results for alternative lag lengths.
I obtain very similar results when these terms are excluded, i.e. the oil supply shocks are largely orthogonal to current economic conditions. This indicates that the identification in Kilian (2009) successfully isolates exogenous supply shocks. As before lag lengths are chosen by AIC.

In Figure 4.3(b) I plot the impulse response function (IRF) of log IP from the estimated Equation (4.8). Following a one-standard-deviation oil supply shock, there is a marginally-significant decline in IP for about five months, with a peak decline of approximately 0.75%. In the following six months the IRF reverts back to zero, and is statistically insignificant. Thus, despite raising inflation expectations at the ZLB, a negative oil supply shock has contractionary effects on IP.

The same conclusion emerges when I estimate Equation (4.8) with the unemployment rate instead of IP. The IRF in Figure 4.3(c) exhibits a statistically-significant increase in the unemployment rate by 0.1 percentage points after a one-standard-deviation negative oil supply shock. This unemployment response begins to dissipate after eleven months, and becomes statistically insignificant twelve months after the shock.

Since the economic contraction from an oil shock may work through investment rather than consumption, it is not obvious that these results constitute a failure of the Euler equation. To address this, I estimate Equation (4.8) using consumption expenditure data for Japan and the U.S., which is available at a monthly frequency. There is strong seasonality in the Japanese data, so I adjust it using the X-12 ARIMA filter from the U.S. Census Bureau. The IRF in Figure 4.3(d) is quite choppy because this series remains volatile, but nonetheless displays a statistically-significant decline in consumption expenditures four months after the oil shock hits. This evidence suggests that the contractionary effects of oil shocks at the ZLB also reflect cutbacks by consumers.

These effects are consistent with the standard Euler equation framework in Section 4.2 only if there is a large increase in long-term nominal bond rates (Summary 4) or if the oil shocks have permanent effects. The latter can be plausibly ruled out based on the temporary effect of supply shocks on real oil prices. In addition, the estimated impulse response functions are insignificant, and very close to zero after four years: -0.2% for IP, 0.04 percentage points for unemployment, and -0.15% for consumption.

To assess whether long-term interest rates have risen, I estimate the impact of oil shocks on bond rates at various maturities. Figure 4.3 shows that bond yields at maturities of three years and greater exhibit a significant *decline* following an oil supply shock. Thus, spillovers of supply shocks into normal times cannot explain the contractionary effects of negative supply shocks. This constitutes a puzzle in the standard set-up of Section 4.2, because without spillovers a temporary, negative supply shock should be expansionary through the expectations channel (Summary 4). For example, given log utility, the decline in ten-year bond yields alone should generate
a 0.5% rise in consumption following an oil supply shock. These results indicate that
other propagation mechanisms trump the expectations channel for negative supply
shocks, and that monetary policy responds to the contractionary effects by loosening
future monetary policy.

Nevertheless, it is possible that the expectations channel mitigates contractionary
effects of oil supply shocks at the ZLB, because a constrained central bank does not
actively raise expected future nominal interest rates. I test this weaker hypothe-
sis by estimating Equation (4.8) using non-ZLB data from 1985-2011, and I plot
the resulting IRFs in Figure 4.4.\textsuperscript{9} Contrary to the previous reasoning, the average
contractionary effects of an oil shock are much stronger at the ZLB than in normal
times. However, the confidence intervals for the ZLB response are large and typically
show substantial overlap with the IRF for normal times, so that one cannot reject
the hypothesis that the responses are the same. Nonetheless, these outcomes provide
evidence against this weaker prediction of the expectations channel.

In Appendix A.5, I show that these results are robust to controlling for inflation
and corporate bond spreads, dropping lagged dependent variables, excluding out-
liers, using HP-filtered IP and unemployment, using non-seasonally-adjusted con-
sumption expenditures, excluding post-2006 data, and alternative lag lengths. In
Appendix A.6, I further corroborate my findings with an event study of the oil
production disruptions from the Libyan civil war.\textsuperscript{10} Despite using a very different
methodology, I also find contractionary effects from this negative supply shock even
though it raises inflation expectations at the ZLB.

4.3.2 The Great East Japan Earthquake

On March 11\textsuperscript{th} 2011, a magnitude-9.0 earthquake off the Japanese eastern coast-
line triggered a tsunami that caused extensive damage to Japanese structures, created
an electricity shortage, and disrupted global supply chains. This was an exogenous,
negative supply shock — to produce the same quantity of output with less capital,
producers had to incur higher costs. Consistent with this logic, the Japanese consen-
sus inflation forecast for 2011 and 2012 rose respectively by 0.3 percentage points
and 0.2 percentage points, as shown in Figure 4.5.\textsuperscript{11}

While professional forecasters expected higher inflation following the earthquake,

\textsuperscript{9}I apply 1985 as a cutoff for three reasons. First, this period broadly features a common monetary
regime. Second, it excludes the major oil shocks around and before 1980, so that my estimates do
not pick up non-linearities from these events. Third, since unemployment is very persistent in the
late 1970s and early 1980s, this cut-off avoids possible non-stationary estimates. Estimates are
similar for other cut-offs around 1985.

\textsuperscript{10}I am grateful to James Hamilton for this suggestion.

\textsuperscript{11}I use February and April forecasts, because the March forecasts were released only shortly after
the earthquake so that some of them were outdated (e.g., the Morgan Stanley forecast).
they revised output forecasts down, as shown in Figure 4.6(a). Japanese output for 2011 was forecasted to be about 1.2% below pre-earthquake predictions. These annual forecasts also mask the severe output losses that occurred during the quarter of the earthquake, when Japanese real output declined at an annualized rate of 7.2%, and real consumption contracted by 4.4%. Japan recovered to its pre-earthquake peak only by the first quarter of 2012. Thus, this negative supply shock had contractionary effects despite raising inflation expectations at the ZLB.

To apply the results from the previous section, this supply shock must be temporary and must not significantly raise expected future nominal interest rates. A priori, the first condition seems reasonable — the capital stock destroyed by the earthquake will be rebuilt as the economy converges to its balanced growth path. In fact, the April survey’s GDP growth forecast for 2012 was revised upward, making up half of the loss from the forecast revision for 2011, as shown in Figure 4.6(a). This suggests that the Japanese economy is catching up to its balanced growth path and that — at most — half of the decline in output could be due to reductions in permanent income. The second condition is also satisfied, as the yield on 10-year government bonds fell from 1.27% on March 10th to 1.19% on March 14th.

As such, the earthquake was a temporary, negative supply shock that reduced expected future real interest rates. However, it was not expansionary, as predicted by the Euler equation framework in Section 4.2, which emphasizes the expectations channel.\textsuperscript{12}

\section*{4.4 Inflation Risk Premia at the ZLB}

The previous section has shown that oil supply shocks and an earthquake are contractionary at the ZLB. This section calculates inflation risk premia, to show that generic negative supply shocks are also contractionary at the ZLB.

\textsuperscript{12}The absence of precautionary savings motives in the baseline model is unlikely to be the source of this discrepancy. In the standard model the first order effect of a negative supply shock at the ZLB is to raise consumption today. Since variations around a higher consumption level are less costly, consumers are likely to reduce their precautionary savings, which further amplifies the first order effects. Thus, endogenous precautionary savings may in fact aggravate the empirical puzzle that the earthquake is not expansionary. Furthermore, the Nikkei VIX index (a measure of uncertainty) was elevated for only one month after the earthquake, so that increases in uncertainty cannot explain the persistence of the contraction.
4.4. INFLATION RISK PREMIA AT THE ZLB

4.4.1 CCAPM Theory

The inflation risk premium is the conditional covariance of marginal utility and inflation,

\[ r_{p\pi}(t) = \text{Cov}_t \left( \frac{dV(t)}{V(t)}, d\pi(t) \right), \]

where \( V(t) \) is the marginal utility of consumption. If marginal utility is high when inflation is high, then the inflation risk premium is positive. In that case, agents dislike nominal assets because they tend to be of low value just as the desire to consume is high. These assets must therefore pay an inflation risk premium to induce agents to hold them. As in Section 4.2, I assume that the marginal utility of consumption is equal to \( V(t) = \zeta C(t)^{-\gamma - 1} \), so that the inflation risk premium is proportional to the covariance of consumption and inflation: \(^{13}\)

\[ r_{p\pi}(t) = -\gamma^{-1} \text{Cov}_t \left( \frac{dC(t)}{C(t)}, d\pi(t) \right). \]

Thus, the inflation risk premium is positive if inflation is associated with low consumption, because then the marginal utility of consumption is high. A positive inflation risk premium then signals that consumers perceive higher-than-expected inflation as bad. Conversely, if the inflation risk premium is negative, then higher inflation is considered good news. \(^{14}\)

The risk premium is determined by the sources of risk in the economy. Consider two independent types of shock: demand shocks, \( dB_v(t) \), raise log consumption and inflation, \( c_v(t) > 0 \) and \( \pi_v(t) > 0 \), and supply shocks, \( dB_u(t) \), lower log consumption and raise inflation, \( c_u(t) < 0 \) and \( \pi_u(t) > 0 \) (subscripts denote partial derivatives). The risk premium is then

\[ r_{p\pi}(t) = -\gamma^{-1} \sigma_v^2 c_v(t) \pi_v(t) - \gamma^{-1} \sigma_u^2 c_u(t) \pi_u(t), \quad (4.9) \]

where \( \sigma_v^2 \) and \( \sigma_u^2 \) are the variances of the shocks. The first term is positive, since supply shocks imply that high inflation is associated with low consumption. The second term is negative, as demand shocks induce the opposite pattern. A priori, the sign of the inflation risk premium is ambiguous, but with data on the risk premium we can determine what kind of shocks are quantitatively more important. In particular, a positive risk premium suggests that investors primarily want to insure against contractionary, negative supply shocks.

However, if the theory in Section 4.2 is correct and negative supply shocks are typically expansionary at the ZLB, then we would expect the inflation risk premium

\(^{13}\)Allowing for shocks to \( \zeta \) does not alter the results in this section.
\(^{14}\)This intuition is similar to the Phillips Curve logic in by Campbell, Sunderam, and Viceira (2013).
4.4. INFLATION RISK PREMIA AT THE ZLB

to be unambiguously negative in this state of the world — when $c^ZLB_u(t) > 0$, both supply shocks and demand shocks should generate a positive covariance of inflation and consumption. Intuitively, in a standard Euler equation framework inflation is always good at the ZLB, because it lowers real interest rates today so that consumption expands through the expectations channel. In contrast, if the standard theory does not hold and negative supply shocks are predominantly contractionary at the ZLB, $c^ZLB_u(t) < 0$, then we may still observe a positive inflation risk premium. In fact, positive inflation risk premia are evidence that contractionary, negative supply shocks (such as oil supply shocks) are quantitatively important at the ZLB.

Summary 5 The standard theory predicts that inflation risk premia should be negative at the ZLB. If instead the inflation risk premium is positive, then this suggests that the standard model is incorrect and that negative supply shocks are typically contractionary at the ZLB.

In Appendix A.7, I show that this intuition from the instantaneous inflation risk carries over to longer-horizon risk premia, and is robust to using Epstein-Zin preferences.

4.4.2 Estimated Inflation Risk Premia

I estimate inflation risk premia from one-year inflation swap contracts. These are financial instruments where one party pays the ex ante known swap rate and the other party pays the realized inflation rate over the contract duration. By comparing the inflation swap rate with ex ante inflation expectations, I derive the inflation risk premium over this horizon.

I use inflation swap rates rather than break-even inflation rates for reasons outlined in Fleckenstein, Longstaff, and Lustig (2012). They show that differences between the Treasury-TIPS spread and swap rates generate persistent, sizable arbitrage opportunities, and that this mispricing originates in the Treasury-TIPS market. This renders the Treasury-TIPS spread an incorrect measure of risk-neutral inflation expectations. In addition, Fleckenstein (2012) shows that these mispricings are typically much less severe in the UK than in the US — UK mispricings are on average zero before the financial crisis and beginning in 2010. These results suggest that inflation swaps are more appropriate measures of risk-neutral inflation expectations, and that market-based inflation expectations for the UK may be more reliable.

I focus on the one-year inflation risk premium to avoid confounding the ZLB, where the inflation risk premium should be negative, with normal times, where the sign is ambiguous. In Appendix A.7, I show that the short end of the risk-premium curve loads relatively more on transient shocks, which are less likely have spillover effects from the ZLB into normal times. In addition, over a one-year horizon exit
4.4. INFLATION RISK PREMIA AT THE ZLB

from the ZLB is unlikely to have occurred, so that any negative supply shocks should be expansionary in the baseline model. Positive one-year inflation risk premia then indicate that negative supply shocks have contractionary effects at the ZLB, which are not due to spillovers or a high probability of exit.

Due to the illiquidity of the Japanese one-year inflation swap market and the absence of matching inflation expectations data, I restrict my attention to the U.S., the U.K., and the Eurozone. For these countries, I match inflation swap rates with consensus inflation forecasts taking into account the indexation lag in swaps. For example, in the U.K. the indexation lag is two months, so that a December 2010 inflation swap contract pays the realized inflation rate from October 2010 to October 2011. I relegate the details of matching inflation swaps with inflation expectations to Appendix A.8.

With matched one-year inflation swap rates $s_\pi(t, t+1)$ and inflation expectations $\pi^e(t, t+1)$, I calculate the inflation risk premium as the simple difference of the two,$^{15}$

$$rp_\pi(t, t + 1) = s_\pi(t, t + 1) - \pi^e(t, t + 1), \quad (4.10)$$

Thus, I assume that the consensus forecasts correctly measures market inflation expectations. This is likely for at least two reasons. First, Ang, Bekaert, and Wei (2007) show that consensus inflation forecasts beat the forecasts of time-series models and term structure models, so that investors are likely to conduct trades based on this superior information. Second, a large fraction of forecasters in the consensus survey are banks that likely participate in inflation swap markets.

I plot the resulting risk premia in Figure 4.7(a) for the U.K., in Figure 4.7(b) for the Eurozone, and in Figure 4.7(c) for the U.S. I do find positive risk premia for the U.K. and the Eurozone at the ZLB, whereas the inflation risk premia for the U.S. are consistently negative. The U.K. one-year inflation risk premia at the ZLB is 25 basis points on average, which is larger than during normal times. This suggests that investors are now more concerned about the contractionary effects of negative supply shocks, perhaps because of greater risk aversion at the ZLB. In the Eurozone inflation risk premia at the ZLB are consistently positive except for two quarters, and on average are equal to 18 basis points. Importantly, these positive inflation risk premia cannot be explained with spillovers or high ZLB-exit-probabilities alone. In the baseline model, inflation risk premia at the ZLB are more negative than in normal times, so a convex combination should be smaller than normal-times-risk-premia. Instead, ZLB-inflation-risk-premia in the U.K. and the Eurozone are similar-sized or larger.

One explanation for persistent negative risk premia in the U.S. is that large mispricings in the TIPS-Treasury market also affect the inflation swap market. Suppose that mispricing is caused by large investors that obtain a convenience yield

$^{15}$This ignores uncertainty over inflation, but this is negligible for reasonable calibrations.
from treasuries as in Krishnamurthy and Vissing-Jorgensen (2012). However, other capital-constraint investors do not value treasury convenience and conduct limited arbitrage via the swap market, pushing down inflation swap rates, and depressing the measured inflation risk premium. This suggests that inflation risk premia are more reliably measured when mispricing is small, such as in the U.K. However, whatever the explanation for the U.S. data, the standard model predicts that inflation risk premia should be negative in all countries, which is rejected for the U.K. and the Eurozone. This suggests that negative supply shocks are not only contractionary at the ZLB, but are also quantitatively important to the marginal investor in these countries.

4.5 A Model with Financial Frictions

The previous two sections have shown that temporary, negative supply shocks are contractionary at the ZLB despite lowering real interest rates, and that these shocks are a quantitatively-important part of inflation risk. These results are inconsistent with any model that features a standard Euler equation (Summary 2 and 5), so that a less restrictive Euler equation is needed to match the data.

Therefore, I allow supply shocks to directly impact the Euler equation (4.1) by endogenously affecting the demand shifter $v(t)$. The consumption response to the supply shock $u(t)$ is then

$$\frac{d \ln C(t)}{du(t)} = -\gamma \frac{d \mathbb{E}_t \int_0^\infty [i(t+s) - \pi(t+s)] ds}{du(t)} + \gamma \frac{d \mathbb{E}_t \int_0^\infty [v(t+s)] ds}{du(t)}.$$

To match the contraction of consumption in the data, the response of the demand shifter $v(t)$ must offset the decline in real interest rates from higher expected inflation. This is illustrated in Figure 4.7, where a negative supply shock endogenously shifts the $AD$ curve to the left, which generates contractionary effects at the ZLB.

There are several mechanisms that can generate this dependency. For example, if nominal wages are sticky and a large fraction of consumers are hand-to-mouth, then a negative supply shock can be contractionary because it lowers real income for these consumers. Another possibility is that negative supply shocks endogenously raise uncertainty in the economy, which will raise precautionary savings for a given real interest rate. A third mechanism is that negative supply shocks transfer wealth abroad, which induces domestic consumers to cut back. I focus on a fourth mechanism where a negative supply shock lowers firm profits and asset values, which reduces net worth at banks. These banks then reduce loan supply, which raises borrowing costs so that borrowers’ consumption contracts.

While other mechanisms may be complementary in explaining my empirical findings, I focus on credit frictions for five reasons. First, credit frictions have arguably
played a prominent role in ZLB episodes such as the Japanese lost decade (Friedman (2000)), the current crisis (Hall (2010b) and Woodford (2010)), and the Great Depression (Bernanke (1983)).

Second, in Appendix A.9 I show that oil supply shocks raise borrowing rates and tighten credit standards in Japan. This is prima facie evidence for credit frictions. If borrowers and lenders were only constrained by their long-run wealth then a temporary supply shock should not impede the capacity of banks to lend and of consumers to borrow.

Third, empirically-reasonable credit frictions are sufficient to rationalize my empirical findings. In Appendix A.9 I verify that the calibrated model is quantitatively consistent with the empirical response of assets spreads to supply shocks. The calibrated credit frictions in turn suffice to generate contractionary effects from negative supply shocks at the ZLB.

Fourth, the financial friction generates positive inflation risk premia at the ZLB, so that a single microfounded mechanism allows the model to match the data in the previous two sections. This contrasts with the hand-to-mouth and sticky-wage model, where assets are still priced by unconstrained Euler equation agents that benefit from higher inflation.

Fifth, wealth-based explanation are not sufficient to explain my findings. In important, recent work Farhi and Werning (2012a,b) investigate the role of incomplete markets, where negative supply shocks can be contractionary by transferring wealth abroad. In Appendix A.10, I show that this wealth transfer generates an immediate, permanent contraction in consumption. Simultaneously, lower real interest rates at the ZLB trigger intertemporal substitution towards the present, which produces smaller contractionary effects in the short-run than in the long-run, contrary to the hump-shaped response in Figure 4.2. Therefore, this mechanism still requires the credit friction to match the larger short-run contraction in the data.

### 4.5.1 Households

Households are modeled as in Cúrdia and Woodford (2009). There is a continuum of households of two types, $b$ and $l$, which split into borrowers and lenders in equilibrium. With probability $\delta dt$ a household draws a new type over an interval $dt$ — either type $l$ with probability $p$ or type $b$ with probability $1 - p$, which are also the frequencies of each type. Whenever a new (but not necessarily different) type is drawn, an insurance contract resets household wealth to the average wealth level. This intermittent consumption insurance guarantees that all households of the same type behave identically in equilibrium.

Each household can deposit at a nominal interest rate $i(t)$, or borrow at a nominal rate $i(t) + \omega(t)$, where $\omega$ is a borrowing spread over the deposit rate. Households cannot engage in any other financial contract, so that financial intermediaries price
and hold all risky assets. Intertemporal optimization implies an Euler equation for each type,

\[ E_t dV^l(t) = [\varrho - i(t) + \pi(t)]V^l(t)dt - \delta(1 - p)[V^b(t) - V^l(t)]dt, \]

\[ E_t dV^b(t) = [\varrho - i(t) + \pi(t) - \omega(t)]V^b(t)dt - \delta p[V^l(t) - V^b_b(t)]dt, \]

where \( V^x = \frac{\zeta^x}{C_x} \) is the marginal utility of type \( x \in \{l, b\} \) assuming log-utility, and \( \zeta^x \) is a constant marginal utility shifter. The marginal utility of borrowers and lenders will differ, because they face different interest rates due to the borrowing spread \( \omega(t) \). Agents also incorporate that they change type with intensity \( \delta \), which yields the capital gain terms in both Euler equations. Setting \( \zeta^b > \zeta^l \) makes consumption by \( b \)-households more valuable, which induces them to borrow if the borrowing spread is not too large.

Households supply perfectly-substitutable labor in a competitive market. The common real wage, \( \frac{W(t)}{P(t)} \), is then equal to the marginal rate of substitution between consumption and leisure for both households, \( \chi^x \frac{L_x(t)^\nu}{V^x(t)} = \frac{W(t)}{P(t)} \), where \( \nu \) is the inverse Frisch elasticity, and \( \chi^x \) is a scalar such that the steady-state labor supply equals one for both household types.

4.5.2 Firms

The supply side of the model features standard new Keynesian price setting. A continuum of monopolistically-competitive firms produces output with a linear production function, \( y(i, t) = e^{a(t)}l(i, t) \), where \( a(t) \) is technology and \( l(i, t) \) is labor input. Technology follows an exogenous Ornstein-Uhlenbeck process with mean-reversion \( \rho_a \) and volatility \( \sigma_a \),

\[ da(t) = -\rho_a a(t)dt + \sigma_a dB_a(t). \]

Given a Calvo intensity \( \alpha \), producers will choose current prices to maximize the share price. For easier comparison with the existing literature, I restrict the supply side to first-order effects, which eliminates price dispersion as a state variable. Thus, aggregate output is linear in labor, \( Y(t) = e^{a(t)}L(t) \), and the continuous-time new Keynesian Phillips Curve is

\[ E_t d\pi(t) = \tilde{\varrho}\pi(t)dt - \kappa \left( \ln \frac{W(t)}{P(t)} - \ln \frac{\bar{W}}{\bar{P}} \right) dt + \kappa(1 + \nu)a(t)dt, \]

where \( \tilde{\varrho} \) is the discount rate of bankers (who own firm shares), and \( \ln \frac{\bar{W}}{\bar{P}} \) is the real wage in the stochastic steady state. Intuitively, when marginal costs are above steady state, inflation is high and falling — that is, \( E_t d\pi(t) < 0 \). The real profits of firms
4.5. A MODEL WITH FINANCIAL FRICTIONS

are the output net of labor costs, \( F(t) = Y(t) - \frac{W(t)L(t)}{P(t)} \), which are distributed to shareholders.

As is standard, the central bank follows an interest rate rule that responds to inflation and the output gap, while being subject to the ZLB constraint,

\[
i(t) = \max\{\bar{i} + \phi_\pi \pi(t) + \phi_y (\ln Y(t) - a(t)), 0\}.
\]

4.5.3 Banking Sector

As in Gertler and Kiyotaki (2010), I assume that each lending household consists of a continuum of agents of two types. A measure \( f \) are workers earning labor income and a measure \( (1-f) \) are bankers managing financial intermediaries. There is complete consumption risk sharing among household members, and at each instant \( \lambda(1-f)dt \) workers switch roles with \( \lambda f dt \) bankers. When bankers switch, the accumulated net worth is paid back to the household, while new-born bankers are endowed with start-up funds. This ensures that bankers will not accumulate enough net worth to void the financial frictions in the model. To the same effect household assets must be spread evenly over banks managed by other households. Finally, when a lending household becomes a borrower its banks are taken over by a household that has become a lender, so that the lender SDF prices the bank.

Only bankers can hold risky assets and intermediate between households. Each banker’s balance sheet is

\[
Q(t)s(t) + b(t) = n(t) + d(t),
\]

where the lower-case variables are bank-specific deposits \( d(t) \), net worth \( n(t) \), real loans to borrowers \( b(t) \), and shares \( s(t) \). \( Q(t) \) is the real price of shares, which pay firm profits \( F(t) \) as dividends. While bankers have specific financial skills, they are subject to an incentive compatibility constraint — they can walk away and divert a fraction \( \theta \) of all assets, at which point the intermediary is wound down and depositors recover the remaining assets. To ensure that bankers do not walk away, the value of the bank must exceed the value of divertible assets. As in Gertler and Kiyotaki (2010), I assume that this incentive constraint is always binding in equilibrium.\(^{16}\)

The optimization problem for the bank is therefore

\[
\max_{\{s(z),b(z),d(z)\}_{z \geq t}} V^I(t)V^B(t) = \int_0^{\infty} \lambda e^{-(\lambda+\theta)t} V^I(t+s)n(t+s)ds
\]

s.t. \( dn(t) = s(t)[dQ(t) + F(t)] + [i(t) - \pi(t) + \omega(t)]b(t) - [i(t) - \pi(t)]d(t) \)

\[
V^B(t) = \theta[Q(t)s(t) + b(t)] \forall t,
\]

\(^{16}\)Subsets of the state space where this assumption is violated occur with negligible probability in the stationary equilibrium of my calibration and are unlikely to affect the quantitative properties of the model.
where $V^B(t)$ is the value of the financial intermediary.

Following Gertler and Kiyotaki (2010) and Maggiori (2011), I guess and verify that the value of the intermediary is a linear function of its net worth, $V^B(t) = \Omega(t)n(t)$, where $\Omega(t)$ is the marginal value of net worth. This constrains the leverage of financial intermediaries to $\phi(t) = \frac{\Omega(t)}{\theta}$. Thus, when the marginal value of net worth is high then banks are allowed to lever up more, because high bank values reduce the incentives to steal.

The optimality conditions for banks determine the excess return of stocks over bonds,

$$
\mathbb{E}_t ER^Q(t) = \mathbb{E}_t \left( \frac{dQ(t) + F(t)}{Q(t)} - r(t) \right) = -\mathbb{E}_t \frac{dQ(t)}{Q(t)} \frac{dV^l(t)}{V^l(t)} - \mathbb{E}_t \frac{dQ(t)}{Q(t)} \frac{d\Omega(t)}{\Omega(t)} + \frac{\mu(t)}{\phi(t)}, \tag{4.11}
$$

where $\mu(t)$ is the shadow value on the banks’ incentive constraint. The RHS shows that excess returns arise for three reasons. First, there is a risk premium if stocks are worth less just as agents want to consume more. Second, as in Maggiori (2011), there is balance sheet risk if stocks lose value when the marginal value of net worth rises. Third, and specific to this model, stocks are cheap if the incentive constraint prevents further stock purchases.\textsuperscript{17}

Because assets are priced by banks the inflation risk premia now incorporates the balance sheet risk. In particular, the instantaneous inflation risk premium is now

$$
 rp(t) = \text{Cov}_t \left( \frac{dV^l(t)}{V^l(t)}, d\pi(t) \right) + \text{Cov}_t \left( \frac{d\Omega(t)}{\Omega(t)}, d\pi(t) \right),
$$

where the first term is the familiar CCAPM risk from Section 4.4, and the new second term captures the exposure of bank balance sheets to inflation.

The FOCs for banks also pin down the borrowing spread over the deposit rate:

$$
\omega(t) = \frac{\mu(t)}{\phi(t)}. \tag{4.12}
$$

Because lending to borrowers is risk-free, the spread depends only on the tightness of the incentive constraint and not on any covariances. If the incentive constraint is very tight due to low net worth, then $\mu(t)$ is high and the spread is large, as depositors do not permit more bank lending. The endogenous demand effects of supply shocks unfold through this channel. As I further discuss below, a negative

\textsuperscript{17} Consistent with equation (4.11), Adrian, Etula, and Muir (2011) show that broker-dealer leverage (equivalent to $\phi(t)$ and $\Omega(t)$) can price a wide range of excess returns in the data.
supply shock lowers profits and asset values, which reduces net worth, tightens the
equity constraint, and raises the borrowing spread.

Verifying the guess for \( V_B(t) \) yields the law of motion for the marginal value of
net worth,

\[
\lambda \frac{1 - \Omega(t)}{\Omega(t)} dt + \mathbb{E}_t \left( \frac{d\Omega(t)}{\Omega(t)} \right) dt + \mathbb{E}_t \frac{d\Omega(t)}{\Omega(t)} A(t) - \delta (1 - p) \left( \frac{V^b(t)}{V^l(t)} - 1 \right) dt + \mu(t) dt = 0.
\]

For example, \( \Omega(t) = 1 \) is the unique bounded solution when there are no frictions,
\( \mu(t) = 0 \) and \( V^b(t)/V^l(t) = 1 \). Intuitively, in a frictionless world the marginal value
of inside funds should equal the marginal value of outside funds (=1). However, if
the banks’ incentive constraint binds, then \( \mu(t) > 0 \) and the marginal value of net
worth will be higher, \( \Omega(t) > 1 \).

The banking sector adds two state variables to the standard model. First, ag-
gregate net worth is the sum of net worth from surviving banks and newly-founded
banks,

\[
dN(t) = [i(t) - \pi(t) - \lambda] N(t) dt + E R(t) Q(t) S(t) dt + \omega(t) B(t) dt + dN^a(t),
\]

where uppercase letters denote aggregate variables. New banks start with net worth
\( dN^a(t) = \psi Q(t) S(t) \), while existing banks accumulate net worth through deposit
interest and excess returns on stocks and lending. The second new state variable is
aggregate real borrowing,

\[
 dB(t) = [i(t) - \pi(t) + \omega(t) - \delta] B(t) dt + (1 - p) [C^b(t) - W^b(t) - \delta S(t) Q(t)] dt
\]

which rises when the consumption of borrowers \( C^b(t) \) exceeds their wage rate \( W^b(t) \),
and declines when inflation reduces the real value of nominal debt. In addition,
the insurance agency nets out debt \( B(t) \) and equalizes asset holdings \( S(t) Q(t) \) for a
fraction \( \delta \) of borrowers and lenders.

### 4.5.4 Calibration

The calibrated parameters are listed in Table 4.2. The household parameters are
taken from Cúrdia and Woodford (2009). The share of lenders is \( p = 0.5 \), the type-
switching intensity is \( \delta = 0.1 \), the discount rate is \( \rho = 0.0125 \), and the steady-state
borrowing spread is \( \bar{\omega} = 0.02 \). The discount rate is lower than in typical calibrations
to make the ZLB bind for small shocks. The marginal utility shifters are \( \zeta^l = 0.72 \)
and \( \zeta^b = 1.34 \), which implies that steady-state debt equals 80% of GDP as in their
calibration.

I calibrate the elasticity of substitution across goods at \( \varepsilon = 40 \), to generate a 2.5%
steady-state profit share as in Basu and Fernald (1997). This value is relatively high,
but its only function is to match the profit share. The slope of the Phillips Curve is separately parameterized at $\kappa = 0.15$ based on estimates from Altig, Christiano, Eichenbaum, and Lindé (2011). Erceg and Lindé (2010) show that DSGE models require very rigid prices and wages at the ZLB to match the small decline in inflation in the recent crisis, which suggests that the estimate from Altig et al. (2011) — which was estimated on data for normal times — is likely an upper bound. Since lower values for $\kappa$ weaken the inflation expectations channel, a calibration along the lines of Erceg and Lindé (2010) will strengthen my results.

The average banking horizon $\lambda^{-1}$ is set to 10 years, and the steady-state leverage ratio to $\bar{\phi} = 2$, similar to Gertler and Kiyotaki (2010). As in their calibration, I use a relatively high Frisch elasticity of $\nu = 2$ to compensate for the lack of labor-market frictions. The fraction of divertible assets, $\theta = 0.7$, is determined by steady-state borrowing spread, and the infusion of net worth to new banks, $\nu = 0.05$, is determined by the steady-state leverage ratio. The interest rate rule parameters for inflation and the output gap are set to standard values, $\phi_\pi = 1.5$ and $\phi_y = 0.5$.

To validate the calibration of the financial accelerator, I verify in Appendix A.9 that the spread in the model and asset spreads in the data exhibit quantitatively similar responses to technology shocks. Thus, the calibrated credit friction is empirically reasonable.

4.5.5 Computation

To make the non-linear solution and mechanics of the model at the ZLB as intuitive as possible, I use an approximation to reduce the state space from three state variables, $\{a(t), N(t), B(t)\}$, to one, $a(t)$. Specifically, I impose that the marginal value of net worth is inversely related to asset prices,

$$\frac{\Omega(t)}{\bar{\Omega}} = \left(\frac{Q(t)}{\bar{Q}}\right)^{-\xi}. \tag{4.13}$$

where $\xi > 0$. For example, with this approximation I can plot consumption and other variables as global functions of the supply shock, which is more transparent than illustrating local results through partial derivatives or impulse-response functions.

Importantly, it is the economics of the problem that makes Equation (4.13) a very good approximation for a judiciously-chosen $\xi$, which I discuss below. In particular, Equation (4.13) captures the following logic in the three-state-variable model. A decline in asset prices $Q(t)$ reduces net worth more than proportionally because

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$^{18}$Altig et al. (2011) estimate the discrete-time Phillips Curve slope $\tilde{\kappa} = 0.014$ at quarterly frequency. Using $\tilde{\kappa} = (1 - \tilde{\alpha})(1 - \beta\tilde{\alpha})/\tilde{\alpha}$ I back out the implied price stickiness $\tilde{\alpha}$ and calculate its continuous-time counterpart $\alpha = (1 - \tilde{\alpha}^4)$. The continuous-time parameter $\kappa$ is then set to $\kappa = \alpha(\alpha + \bar{i} + \bar{\phi}\bar{\omega}) \approx 0.15$. 

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banks are levered up. Thus, without any change in asset-holdings, leverage at banks, $\phi(t)$, will rise. But, in equilibrium, banks are only permitted a higher leverage if the marginal value of net worth is high, as $\phi(t) = \frac{\Omega(t)}{\theta}$. Hence, banks have to reduce their asset holdings, which will raise expected excess returns on assets and the marginal value of net worth $\Omega(t)$. Note that the change in asset prices is still endogenous — the marginal value of net worth will decline only if a negative supply shock lowers asset prices. Thus, the simplification in Equation (4.13) can capture the logic of a more complicated model without sacrificing the economics of the problem.

The parameter $\xi$ controls the strength of this financial accelerator. Higher values of $\xi$ accentuate the financial friction, because a given change in asset prices results in a larger change in the marginal value of net worth. I calibrate $\xi$ at a value such that the one-state-variable model closely mimics the three-state-variable model. First, I determine the linearized solution for the three-state-variable model, and calculate the IRF for output following a technology shock. Second, I set $\xi$ such that the one-state-variable model best matches this IRF. In my baseline calibration, I set $\rho_a = 0.69$ and $\sigma_a = 0.005$, which results in $\xi = 0.6$ (see Appendix A.11.1). This persistence implies a half-life of one year, which is similar to the transient nature of the oil shocks in Section 4.3.1.

For this calibration I also calculate the non-linear solution to the three-state-variable model in Appendix A.11.1, and find that the approximation (4.13) performs very well in normal times and at the ZLB. Since this simplification performs well, I focus on the one-state-variable model and compare it with a standard new Keynesian model. Details of the computational methods are relegated to Appendix A.11.2.

4.5.6 Results

Figure 4.9(a) plots output as a function of the technology shifter $a(t)$ for four cases: the standard new Keynesian model without ZLB, the standard model with ZLB, the one-state-variable model with credit frictions but without ZLB (the “friction model”), and the one-state-variable model with both credit frictions and ZLB. Without ZLB, the standard model has an (almost) linear solution, in which consumption rises in response to technology shocks. Intuitively, the central bank lowers real interest rates in response to the deflation generated by a positive supply shock (Figure 4.9(b)), which raises consumption today. In contrast, when the ZLB binds, the decline in inflation from improved technology raises real interest rates, so that consumption today contracts – the “Paradox of Toil.” Conversely, negative technology shocks are expansionary in the ZLB region because they raise expected inflation and lower real interest rates.

Aggregate consumption in the friction model is more responsive than in the standard model because of the financial friction. For reasonable calibrations, a positive technology shock raises profits by lowering average costs. Intuitively, given a profit
share of 2.5% (Basu and Fernald (1997)), a 2.5% increase in sales raises profits by 2.5% whereas reducing average costs by 2.5% doubles profits. Thus, technology shocks affect profits primarily through average costs, while any sales effects are negligible. Higher profits lead to higher asset prices (Figure 4.10(a)), such that the marginal value of net worth falls (Figure 4.10(b)) and the borrowing spread declines (Figure 4.10(c)). Hence, in normal times both real interest rates and spreads fall following a positive technology shock, so that consumption rises more than in the baseline model where only the former effect is present. Nevertheless, the financial accelerator only has modest effects in normal times, as it makes aggregate consumption approximately 31% more sensitive to technology.

Positive supply shocks remain expansionary when the ZLB binds in the friction model, unlike in the baseline model. In this case, the decline in real borrowing rates from a lower borrowing spread raises consumption by borrowers, which dominates the fall in consumption by lenders from higher real deposit rates. The financial accelerator now has a very large effect on equilibrium outcomes (Figure 4.9(a)): when TFP is 5% above steady state then output in the friction model is almost 3% higher than in the baseline model. Thus, financial accelerator effects that are moderately sized in normal times can have a significant impact at the ZLB. Intuitively, if positive supply shocks lower borrowing spreads in normal times, then output is closer to potential and the central bank lowers real interest rates less. Thus, the central bank dampens the financial accelerator through its interest rate policy. At the ZLB this stabilizing force is absent and the financial accelerator can have large effects.

Since differences between models are “smoothed-out” in normal times, these results suggest that a variety of models can match these data. In contrast, at the ZLB the same models may behave very differently to one another, so that many of them will be empirically less successful. This argues for caution when using models that have only been tested with normal-times data for policy analysis at the ZLB. On the other hand, the ZLB provides a good testing ground to discriminate among models because propagation mechanisms are no longer attenuated by the central bank.

The friction model is also more successful at matching the positive inflation risk premia at the ZLB in the data, as shown in Figure 4.10(d). Intuitively, higher inflation coincides with lower technology, profits, and net worth, and thus a higher marginal value of net worth in banks; this makes banks unwilling to hold assets that load on inflation unless they pay a positive risk premium. In my calibration, this positive balance-sheet risk premium dominates the negative CCAPM inflation risk premium, allowing the model to match the data for the U.K. and the Eurozone. In contrast, the instantaneous inflation risk premium in the standard model becomes negative when the ZLB binds, because it features only the negative CCAPM inflation risk premium.

While the friction model can generate contractionary effects from negative supply shocks at the ZLB, these are attenuated compared to normal times. In contrast, in
the data negative supply shocks appear to be more contractionary at the ZLB (Figure 4.4). But there are plausible extensions that can likely correct this shortcoming. For example, ZLB episodes typically feature large disruption in financial markets as well as a larger-than-usual number of credit-constrained agents. Since these aspects make financial frictions quantitatively more important, the contractionary effects of negative supply shocks through the credit channel are likely to be amplified relative to normal times. Thus, a more detailed model of the ZLB would also match this moment, but at the cost of additional complexity.

In summary, the calibrated credit friction model can qualitatively rationalize contractionary effects of negative supply shocks and positive inflation risk premia at the ZLB. However, the necessity of these credit frictions only becomes apparent when the ZLB binds, because in normal times the central bank endogenously dampens the financial accelerator.

### 4.6 Policy Implications

Since the friction model is more successful at matching the data than the standard new Keynesian model, I explore how the credit friction propagates two prominent policies: forward guidance, and fiscal stimulus through government spending. I capture the former with a disturbance $\varepsilon_i$ in the interest rate rule,

$$i(t) = \max\{\bar{i} + \phi_\pi \pi(t) + \phi_y (\ln Y(t) - a(t)) + \varepsilon_i(t), 0\},$$

which follows an autoregressive process, $d\varepsilon_i(t) = -\rho_i \varepsilon_i(t)dt + \sigma_i dB_i(t)$. This correlated shock is a simple way to capture persistent deviations from standard policy. A negative value for $\varepsilon_i$ at the ZLB implies that interest rates are kept low for an extended period, which has a stimulative effect by reducing long-term bond yields. I set the mean-reversion of the shock as $\rho_i = 0.9$, which is equivalent to a quarterly persistence of 0.8 as is typically assumed.

Similarly, I model government spending as $G(t) = \bar{G}e^{g(t)}$, where $\bar{G}$ is the steady-state value of government spending and $g(t)$ also follows an autoregressive process $dg(t) = -\rho_g g(t)dt + \sigma_g dB_g(t)$. I set the mean-reversion for these processes to $\rho_g = 0.69$, and the share of government spending in output to $\bar{G}/\bar{Y} = 0.2$.\footnote{This is midway between Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011a). I adjust the marginal utility shifters $\zeta$ such that the debt/GDP ratio equals 80%, as in the baseline model.} I calculate the perfect-foresight outcomes for these policies and calibrate the volatilities of $dB_i(t)$ and $dB_g(t)$ as negligible.

Figure 4.11(a) illustrates the effectiveness of forward guidance, by displaying the percentage change in output on impact at various durations of the ZLB for each basis
point reduction in the 10-year bond yield. In both the standard model and the friction model, forward guidance raises output and becomes increasingly more effective the longer the ZLB binds. The logic behind this result is the following: Fix the exit date from the ZLB at \( T_1 < 10 \), and note that the monetary policy shock \( \int_{T_1}^{10} \Delta i(t) dt = -0.1 \) is the same for all \( T_1 \). By the Euler equation (4.1), this alone raises consumption at each point in time from \([0, T_1]\) by \( \Delta \ln C(t \in [0, T_1]) = -\gamma \int_{T_1}^{10} \Delta i(t) dt = 0.1 \gamma \). Thus, the larger \( T_1 \) the more persistent is the rise in consumption. From the Phillips curve (4.3) the more persistent the increase in marginal cost (wages) from higher consumption, the greater the rise inflation, and the greater the decline in real interest rates. This creates a feedback loop whereby lower real interest rates further stimulate consumption, raise inflation, and lower real interest rates. The longer the interval \([0, T_1]\), i.e. the longer the ZLB binds, the stronger is the initial impulse and thus the larger is the equilibrium stimulus.

In the friction model, this policy is much less effective than in the standard model. In the standard model the initial increase in consumption raises labor demand and thus wages. This increase in marginal costs then has stimulative effects by raising expected inflation and lowering real interest rates — but this prediction is rejected in the data! Since the credit friction is designed to match this fact, the rise in marginal costs from increased consumption must be contractionary. Specifically, higher wages reduces profit margins at firms, which depresses asset prices, lowers net worth, and raises the borrowing spread. These effects are quantitatively important as forward guidance in the friction model is 50% less effective than in the standard model when the ZLB binds for four years (Figure 4.11(a)).

The results for fiscal policy are similar (Figure 4.11(b)). Fiscal stimulus is more effective for longer durations of the ZLB, because the spillovers to normal times become smaller. Fiscal policy is also more stimulative in the standard new Keynesian model, where increasing production costs raises inflation expectations, which stimulates consumption through the expectations channel. In contrast, in the model with credit frictions raising production costs is no longer beneficial because it raises credit spreads, which pushes the multiplier below one. In my calibration, this renders fiscal policy 20% less effective than in a standard new Keynesian model when the ZLB binds for four years.

The fiscal multipliers in Figure 4.11(b) are small compared to most of the ZLB literature (Christiano, Eichenbaum, and Rebelo (2011), Woodford (2011a)), because prices in my model are relatively rigid, which weakens the effect of government spending on real interest rates. Erceg and Lindé (2010) also show that calibrations with very sticky prices and wages generate a fiscal multiplier close to one even when the ZLB lasts four years (see their Figure 6). In addition, I abstract from investment, variable capacity utilization, and labor-consumption complementarities, which raise the fiscal multiplier (Christiano, Eichenbaum, and Rebelo (2011)). However, while
4.6. POLICY IMPLICATIONS

these aspects affect the absolute magnitudes of the fiscal multiplier, they are unlikely to alter the attenuation of fiscal multipliers in the friction model relative to the baseline model, which rests on the contractionary effects of raising production costs. In fact, the reduced efficacy of demand-side policies at the ZLB is a robust feature of sticky-price models where negative supply shocks are contractionary at the ZLB. In such models the inflation expectations channel cannot be a source of large demand-side multipliers, because higher inflation is a symptom of higher current and expected marginal costs, which are contractionary in the data. More formally, consider how a temporary government spending shocks impacts the Euler equation (4.2) at the ZLB,

\[ \frac{d \ln C(t)}{dg(t)} = -\gamma \frac{dE_t \int_0^\infty i(t+s)ds}{dg(t)} + \gamma \frac{dE_t \int_0^\infty [\pi(t+s) + v(t+s)]ds}{dg(t)} \]

\[ = \gamma E_t \int_0^\infty \frac{\partial [\pi(t+s) + v(t+s)]}{\partial mc(t+s)} \frac{dmc(t+s)}{dg(t)} ds \]

\[ < 0 \text{ (Supply shocks contractionary)} \]

\[ + \frac{\gamma E_t}{\partial g(t)} \int_0^\infty ds. \]

In any model where negative supply shocks are contractionary at the ZLB, the first term is negative because \( v(t) \) endogenously falls as marginal costs rise. Thus, the inflation expectations channel cannot be a source of large multipliers, because the stimulative effect of inflation expectations is offset by an endogenous response in \( v(t) \) (e.g., a credit spread). This renders fiscal stimulus and forward guidance less effective than in standard new Keynesian models, where they have stimulative effects by raising marginal costs.

The best-case scenario for these policies is when the first term in Equation (4.14) is zero, i.e. when real wages do not respond to demand-side policies. Thus, in Figure 4.10 I also plot the impact of forward guidance and fiscal stimulus when real wages are fixed at their steady-state values. In this scenario, such policies become more effective in the friction model than in a new Keynesian model with rigid wages — with fixed marginal cost, higher output raises profits and asset prices, resulting in lower borrowing spreads. This is reflected in the second term of Equation (4.14). However, even in this setting demand-side policies are much less effective than in the baseline flexible-wage model. Thus, reduced policy effectiveness is a robust feature of models where negative supply shocks are contractionary at the ZLB, because the inflation expectations channel can no longer be a source of large multipliers.

In summary, the friction model reduces policy effectiveness at the ZLB relative
4.7 CONCLUSION

to a standard model. This is a direct consequence of matching the contractionary effects of negative supply shocks in the data, and suggests that policy makers should be cautious in expecting large positive outcomes through the inflation expectations channel.

4.7 Conclusion

This paper shows that negative supply shocks are contractionary at the ZLB. This may sound tautological, but it contradicts the prediction of many macroeconomic models with a standard Euler equation and sticky prices. These models emphasize that negative supply shocks reduce expected real interest rates at the ZLB, which raises consumption today through standard intertemporal substitution. In this paper, I examine two negative supply shocks — oil supply shocks and an earthquake — and show that these shocks are contractionary at the ZLB despite lowering expected future real interest rates. In addition, positive one-year inflation risk premia indicate that generic negative supply shocks are also contractionary at the ZLB and that such shocks are quantitatively important over this horizon. In short, contrary to Krugman’s claim, the ZLB world may not be so “topsy-turvy” after all, and many of the usual rules of economics continue to hold.

The key to reconciling these empirical facts with the theory is to allow supply shocks to have endogenous, negative demand effects. In my model, a negative supply shock reduces net worth of banks, which raises borrowing costs, and thus reduces consumption by borrowers. In addition, the desire of banks to insure against balance-sheet contractions from negative supply shocks generates a positive inflation risk premium. However, the importance of these credit frictions only becomes apparent at the ZLB, because in normal times the central bank “smoothes-out” the financial accelerator. Thus, the ZLB also presents a unique opportunity to discriminate between models, which is likely to be a fruitful area for future research.

These findings have very important policy implications. Demand-side policies at the ZLB are very effective in standard sticky-price models because negative supply shocks are also expansionary. For instance, government spending shocks in such models raise current and expected marginal costs, which raises expected inflation, lowers real interest rates at the ZLB, and stimulates consumption. The fact that raising production costs is contractionary in the data (despite increasing inflation expectations) casts doubt on the quantitative importance of the expectations channel. In any data-consistent model the rise in inflation expectations from higher marginal cost cannot be stimulative, so that demand-side policies are much less effective than in new Keynesian models that build on this mechanism. For example, in the model with credit frictions, demand side policies are up to 50% less stimulative than in a standard new Keynesian model. This suggests that policy makers should not take
4.7. CONCLUSION

Large positive outcomes from the expectations channel for granted. To the extent that such policy multipliers may be large in the data, my results suggest that this is due to other mechanisms. Thus, more research is needed to understand the propagation of fiscal policy and forward guidance, in addition to determining the size of policy multipliers in the data.

Figure 4.1: Impact of negative supply shock in Euler equation framework of Section 4.2

Notes: Left panel shows impact for normal times when the central bank is unconstrained. Right panel shows impact when the ZLB binds. The kink in the AD curve is the exit point from the ZLB.
4.7. CONCLUSION

Figure 4.2: Impulse Response Functions to negative oil supply shocks

(a) 4Q Ahead Expected Inflation
(b) Log Industrial Production
(c) Unemployment
(d) Consumption Expenditures

Notes: IRFs are constructed from autoregressive distributive lag estimates in changes or growth rates and aggregated to levels. 95% confidence intervals are derived by Monte-Carlo draws from a normal distribution with variance equal to the estimated Driscoll-Kraay covariance matrix. Lag lengths are set according to AIC: For the dependent variable they are set to 12 for expected inflation, 48 for industrial production, and 36 for unemployment and consumption. Lag lengths for oil shocks are set to 24 for expected inflation, industrial production, and unemployment, and to 36 for consumption. See Section 4.3.1 for details.
4.7. CONCLUSION

Figure 4.3: Change in nominal bond yield on impact of an oil supply shock

Notes: Point estimates are solid circles. Error bars show the 95% Driscoll-Kraay confidence interval around the point estimates. Estimated based on an autoregressive distributive lag equation, where the dependent variable enters with 36 lags based on the AIC criterion. See Section 4.3.1 for details.

Table 4.1: ZLB Dates

<table>
<thead>
<tr>
<th>Country</th>
<th>Start Date</th>
<th>End Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>October 1995</td>
<td>July 2006</td>
</tr>
<tr>
<td></td>
<td>December 2008</td>
<td>-</td>
</tr>
<tr>
<td>U.S.</td>
<td>December 2008</td>
<td>-</td>
</tr>
<tr>
<td>U.K.</td>
<td>March 2009</td>
<td>-</td>
</tr>
<tr>
<td>Eurozone</td>
<td>January 2009</td>
<td>-</td>
</tr>
<tr>
<td>Canada</td>
<td>May 2009</td>
<td>May 2010</td>
</tr>
<tr>
<td>Sweden</td>
<td>July 2009</td>
<td>June 2010</td>
</tr>
</tbody>
</table>
4.7. CONCLUSION

Figure 4.4: Impulse Response Functions to oil supply shocks 1985-present when ZLB binds (“Baseline”) and when policy rates are unconstrained (“Normal Times”).

Notes: IRFs are constructed from autoregressive distributive lag estimates in changes or growth rates and aggregated to levels. 95% confidence intervals are derived by Monte-Carlo draws from a normal distribution with variance equal to the estimated Driscoll-Kraay covariance matrix. See Section 4.3.1 for details.

Figure 4.5: Consensus Economics forecasts from before Japanese Great Earthquake (February 2011) and after (April 2011)

Notes: Forecasts are for annual GDP and year-on-year inflation. GDP data is annual for 2010 and 2012 and quarterly from 2010Q4 until 2012Q1. CPI data is annual year-on-year inflation. See Section 4.3.2 for details.
Figure 4.6: Inflation Risk Premia for the U.K., Eurozone, and the U.S.

Notes: Calculated as the difference in inflation swap rates and expected inflation rates. For the U.K. and the U.S. monthly risk premia are averaged over a quarter to account for seasonalities. Shaded areas indicate time periods were the ZLB binds as defined in Table 4.1. P-values are calculated over the sub-samples in brackets (U.K. and U.S.) or over pre- and post-ZLB (Eurozone) based on Newey-West standard errors. See Section 4.4 for details.
Figure 4.7: Impact of negative supply shock at ZLB when these shocks have endogenous negative demand effects, as in the friction model of Section 4.5

Notes: The standard shift in the AS is marked by “1” and the endogenous demand effect - the induced shift in the AD curve - is marked by “2.”
4.7. CONCLUSION

Figure 4.8: Model solutions for log aggregate consumption $\ln C(a(t))$ and inflation $\pi(a(t))$ as a function of the technology shifter $a(t)$.

Notes: Shaded areas are intervals of the state space where the ZLB binds. “Standard Model” denotes the standard new Keynesian model, and “Standard Model with ZLB” the same model explicitly incorporating the ZLB constraint. “Friction Model” denotes the model in Section 4.5, where borrowing agents face an endogenous credit spread. See Section 4.5 for details.
Figure 4.9: Model solutions for asset prices, marginal value of net worth, the borrowing spread and the inflation risk premium as a function of the technology shifter $a(t)$.

Notes: Shaded areas are intervals of the state space where the ZLB binds. “Standard Model” denotes the standard new Keynesian model, and “Standard Model with ZLB” the same model explicitly incorporating the ZLB constraint. “Friction Model” denotes the model in Section 4.5, where borrowing agents face an endogenous credit spread. See Section 4.5 for details.
Figure 4.10: Percentage change in output on impact for each basis point change in the 10-year bond rate (left panel) and impact fiscal multiplier (right panel)

(а) Forward Guidance.

(b) Fiscal Multiplier.

Notes: “Standard Model (Flexible Wages)” denotes the standard new Keynesian model with competitive labor market, “Friction Model (Flexible Wages)” denotes the model in Section 4.5 with a competitive labor market, “Standard Model (Rigid Wages)” denotes the standard new Keynesian model with real wages fixed at their steady-state value, and “Friction Model (Rigid Wages)” denotes the model in Section 4.5 with real wages fixed at their steady-state value. See Section 4.6 for details.
### Table 4.2: Parameterization of the Friction Model

<table>
<thead>
<tr>
<th>Param.</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Households</strong></td>
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<td></td>
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<tr>
<td>$\varrho$</td>
<td>Discount Rate</td>
<td>0.012</td>
<td>Own calculations</td>
</tr>
<tr>
<td>$p$</td>
<td>Share of Lenders</td>
<td>0.5</td>
<td>Cúrdia and Woodford (2009)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Type-switching Intensity</td>
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<td>Cúrdia and Woodford (2009)</td>
</tr>
<tr>
<td>$\zeta^l$</td>
<td>Lender Marginal Utility</td>
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<td>Cúrdia and Woodford (2009)</td>
</tr>
<tr>
<td>$\zeta^b$</td>
<td>Borrower Marginal Utility</td>
<td>1.34</td>
<td>Cúrdia and Woodford (2009)</td>
</tr>
<tr>
<td></td>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of Substitution</td>
<td>40</td>
<td>Basu and Fernald (1997)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Slope of Phillips Curve</td>
<td>0.15</td>
<td>Altig et al. (2011)</td>
</tr>
<tr>
<td></td>
<td><strong>Banking Sector</strong></td>
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<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Payout Intensity</td>
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<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Elasticity of Marginal Value of Net Worth w.r.t. Asset Prices</td>
<td>0.6</td>
<td>Own calculations</td>
</tr>
<tr>
<td>$\bar{\phi}$</td>
<td>Steady-State Leverage</td>
<td>2</td>
<td>Gertler and Kiyotaki (2010)$^1$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Fraction of Divertible Assets</td>
<td>0.70</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Infusion of Net Worth</td>
<td>0.05</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td></td>
<td><strong>Central Bank</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Inflation Response</td>
<td>1.5</td>
<td>Standard Parameters</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Output Gap Response</td>
<td>0.5</td>
<td>Standard Parameters</td>
</tr>
<tr>
<td></td>
<td><strong>Shocks</strong></td>
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<tr>
<td>$\rho_a$</td>
<td>Persistence of Technology</td>
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<td>Own calculations</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>SD of Technology Shock</td>
<td>0.005</td>
<td>Own calculations</td>
</tr>
</tbody>
</table>

$^1\bar{\phi} = 2$ matches their steady-state marginal value of net worth.


Bodenstein, Martin, Christopher Erceg, and Luca Guerrieri, “The effects of foreign shocks when interest rates are at zero,” 2010.

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Appendix A

Appendix to Chapter 4

A.1 Formal Solution

In this section I derive analytical expressions for the effects of supply shocks in a standard new Keynesian model outlined in Section 4.2. The Euler equation is given by

$$E_t[d \ln C(t)] = \gamma[i(t) - \pi(t) - \varrho - \upsilon(t)]dt.$$ 

The new Keynesian Phillips curve is given by

$$d \pi(t) = \varrho \pi(t) - \kappa[(\gamma^{-1} + \nu)(\ln C(t) - \ln \bar{C}) + (1 + \nu)u(t)].$$

For convenience I set $\ln \bar{C} = 0$.

I let the Central Bank follow a standard interest rate rule

$$i(t) = \max\{\varrho + \phi_\pi \pi(t) + \phi_y \ln C(t), 0\}.$$ 

This encompasses as a special case the time-consistent optimal policy (as $\phi_\pi \to \infty$).

The disturbances $\upsilon(t)$ and $u(t)$ follow continuous-time poisson processes similar to their discrete-time analogues in Eggertsson and Woodford (2003). Their initial conditions are given by $u(0) = \bar{u}$ and $\upsilon(0) = -\bar{\upsilon}$, and they evolve according to

$$du(t) = -\bar{u}dN(t), \quad dv(t) = \bar{v}dN(t),$$

where $N(0) = 0$ and

$$dN(t) = \begin{cases} 1 & \text{with probability } \lambda dt \text{ if } N(t) = 0 \\ 0 & \text{with probability } 1 - \lambda dt \text{ if } N(t) = 0 \text{ or with probability } 1 \text{ if } N(t) = 1 \end{cases}$$

Thus, the system originally starts in the state $s(0) = \{u(0) = \bar{u}, \upsilon(0) = -\bar{\upsilon}, N(0) = 0\}$, and then stochastically reverts to the state $\bar{s} = \{u(t) = 0, \upsilon(t) = 0, N(t) = 1\}$, which is an absorbing state.
A.1. FORMAL SOLUTION

A.1.1 Case 1: No ZLB

When the ZLB is not a binding constraint in either the original state \( s(0) \) or in the absorbing state \( \bar{s} \), then the system of equations is a set of linear ODEs that can be solved with standard methods. The unique bounded solution is,

\[
\ln C(t) = \frac{-\kappa \gamma (1 + \nu)(\phi_\pi - 1)}{(\lambda + \gamma \phi_y)(\lambda + \varrho) + \kappa (1 + \gamma \nu)(\phi_\pi - 1)} u(t) \\
+ \frac{\gamma (\lambda + \varrho)}{(\lambda + \gamma \phi_y)(\lambda + \varrho) + \kappa (1 + \gamma \nu)(\phi_\pi - 1)} v(t),
\]

\[
\pi(t) = \frac{\kappa (1 + \nu)}{(\lambda + \gamma \phi_y)(\lambda + \varrho) + \kappa (1 + \gamma \nu)(\phi_\pi - 1)} u(t) \\
+ \frac{\kappa (1 + \gamma \nu)}{(\lambda + \gamma \phi_y)(\lambda + \varrho) + \kappa (1 + \gamma \nu)(\phi_\pi - 1)} v(t)
\]

The coefficient on \( u(t) \) in the consumption equation is negative, so that negative supply shocks are contractionary in normal times. Since the coefficient on \( v(t) \) is positive, positive demand shocks are expansionary in normal times. Note that \( \phi_\pi \to \infty \) (optimal policy) and \( \kappa \to \infty \) (flexible prices) both implement the flexible price allocation \( \ln C(t) = \frac{-\gamma (1 + \nu)}{(1 + \gamma \nu)} u(t) \).

A.1.2 Case 2: The ZLB binds

Let \( \bar{v} > 0 \) be sufficiently large s.t. the ZLB binds. Upon exit from the ZLB (which occurs with probability \( \lambda dt \)), the system reverts to Case 1 with all disturbances set to zero. Thus, upon exit at the (stochastic) time \( T \), \( \ln C(t \geq T) = 0 \) and \( \pi(t \geq T) = 0 \). This allows me to rewrite the set of equations as,

\[
\ln C(t) = \gamma \int_t^\infty e^{-\lambda(s-t)}[-\pi(s) - \varrho - \bar{v}] ds,
\]

\[
\pi(t) = \kappa \int_t^\infty e^{-(\lambda + \varrho)(s-t)}[(\gamma^{-1} + \nu) \ln C(s) - \bar{u}] ds.
\]

Note that this system is deterministic, since the only uncertainty was over the exit date from the ZLB. We are thus looking for solutions for \( c : \mathbb{R} \to \mathbb{R} \) and \( \pi : \mathbb{R} \to \mathbb{R} \) conditional on the ZLB binding.\(^1\) These are deterministic functions, although they need not be constant over time.

We can now rewrite the system of differential equations as,

\[
d \ln C(t) = \lambda \ln C(t) dt + \gamma [-\pi(t) - \varrho - \bar{v}] dt,
\]

\[
d \pi(t) = (\varrho + \lambda) \pi(t) dt - \kappa [(\gamma^{-1} + \nu) \ln C(t) + (1 + \nu) \bar{u}] dt.
\]

\(^1\)The following results can also be derived as in Werning (2011).
A.1. FORMAL SOLUTION

Uniqueness of a bounded solution requires that the eigenvalues of the system are both positive. The eigenvalues of this system are

\[ \mu_{1,2} = \frac{2\lambda + \varrho}{2} \pm \sqrt{\left( \frac{2\lambda + \varrho}{2} \right)^2 + \kappa(1 + \gamma \nu) - \lambda(\lambda + \varrho)}, \]

so that uniqueness requires

\[ \Theta \equiv \frac{\kappa(1 + \gamma \nu)}{\lambda(\lambda + \varrho)} < 1. \quad (A.2) \]

The unique solution when \( \Theta < 1 \) can be then be found through standard methods,

\[
\begin{align*}
\ln C(t)^{ZLB} & = \frac{\kappa\gamma(1 + \nu)}{\lambda(\lambda + \varrho) - \kappa(1 + \gamma \nu)} u(t) + \frac{\gamma(\lambda + \varrho)}{\lambda(\lambda + \varrho) - \kappa(1 + \gamma \nu)} v(t) + \frac{\gamma}{\lambda} \varrho, \\
\pi(t)^{ZLB} & = \frac{\kappa(1 + \nu)\lambda}{\lambda(\lambda + \varrho) - \kappa(1 + \gamma \nu)} u(t) + \frac{\kappa(1 + \gamma \nu)}{\lambda(\lambda + \varrho) - \kappa(1 + \gamma \nu)} v(t). 
\end{align*} \quad (A.3)\]

The denominators are positive since \( \Theta < 1 \). Thus, negative supply shocks are now expansionary – the coefficient on \( u(t) \) in the consumption equation is positive. The impact of demand shocks has only changed in magnitude. Note that \( \kappa \to \infty \) again implements the flexible price allocation \( \ln C(t) = -\frac{\gamma(1+\nu)}{(1+\gamma \nu)} u(t) \). However, there are multiple equilibria in this case, since it violates the condition \( \Theta < 1 \). For example, in one equilibrium the ZLB does not bind and \( \pi(t) \) is given by (A.1), whereas in another the ZLB binds and \( \pi(t) \) is given by (A.3).

When \( \Theta > 1 \) we can characterize a set of bounded ZLB equilibria as,

\[
\begin{align*}
\ln C(t)^{ZLB}[A] & = \ln C(t)^{ZLB} + \frac{\gamma}{\lambda - \mu_1} A e^{\mu_1 t}, \\
\pi(t)^{ZLB}[A] & = \pi(t)^{ZLB} + A e^{\mu_1 t}. 
\end{align*} \quad (A.4)\]

where, without loss of generality, \( \mu_1 < 0 \). Equilibria are indexed by \( A \). Note that \( A \) must lie in a subset of the real numbers such that the ZLB constraint binds for \( t \geq t_0 \) where \( t_0 \) is the starting date.

When \( \Theta = 1 \) a set of bounded ZLB equilibria is given by,

\[
\begin{align*}
\ln C(t)^{ZLB}[A] & = \frac{\gamma}{\lambda} A, \\
\pi(t)^{ZLB}[A] & = A. \quad (A.5) 
\end{align*}\]

where \( A \) is such that the ZLB binds.
The model in Section 4.2 is deliberately simple to analytically characterize responses to supply shocks in normal times and at the ZLB. While it abstracts from many complications, I use the Smets-Wouters model to illustrate that negative supply shocks are also expansionary at the ZLB when many such features are added. For instance, this model incorporates capital, investment adjustment costs, variable capacity utilization, fixed costs in production, habits, consumption-labor complementarities, wage stickiness, and indexation in wage- and price-setting. Given the large number of state variables, Smets and Wouters (2007) solve it using a log-linear approximation around the steady-state. I use their estimated parameter values but impose the ZLB on nominal interest rates in the interest rate rule. Since the ZLB makes the model non-linear, I solve it with the algorithm in Bodenstein, Erceg, and Guerrieri (2010b) and Coibion, Gorodnichenko, and Wieland (2012), which can handle the large state space.

In Appendix Figure A.1 I plot the impulse response function for output to a one-standard-deviation negative technology shock in the Smets-Wouters model in normal times and at the ZLB. As in Section 4.2 a negative technology shock is contractionary in normal times but expansionary at the ZLB. Thus, even with these more complicated DSGE models share the same prediction as the standard new Keynesian model.

I conclude by providing some intuition for this result. As in Section 4.2, a negative technology shock raises marginal costs of production, which raises expected inflation and lowers real interest rates at the ZLB. Even with habits, this will raise consumption (albeit more slowly) through the Euler equation. Thus, additional demand must be met with higher labor input and increased capacity utilization. Consumption-labor complementarities will reinforce this expansionary mechanism. With higher labor input, the marginal product of capital also rises, which, in addition to lower real interest rates on bonds, makes investment into capital more attractive. Since both consumption and investment rise the negative supply shock is stimulative. Investment adjustment costs, wage rigidity, and indexation merely slow down this adjustment.

This appendix details the inflation expectations data sources and the Kalman filter used to infer four-quarter ahead inflation expectations from available data.
A.3. INFLATION EXPECTATIONS & KALMAN FILTER

Figure A.1: Impulse response function for output to a one-standard-deviation technology shock in the Smets and Wouters (2007) model.

“Normal times” denotes the IRF when the central bank is constrained. “8 Quarter ZLB” denotes the IRF when the ZLB binds for eight quarters.
A.3. INFLATION EXPECTATIONS & KALMAN FILTER

A.3.1 Data sources

The data sources and their frequency are tabulated in Appendix Table A.1.
### Table A.1: Inflation Expectations Data Sources

<table>
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<tr>
<th>Country</th>
<th>Source</th>
<th>Reference Price</th>
<th>Forecast Frequency</th>
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<td></td>
<td></td>
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<td>Q4-Q4 Forecasts</td>
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<tr>
<td>Japan</td>
<td>Consensus Economics</td>
<td>CPI</td>
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<tr>
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<td>CPI</td>
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<tr>
<td>U.K.¹</td>
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<td>RPIX</td>
<td>Quarterly</td>
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<tr>
<td></td>
<td>HM Treasury</td>
<td>RPIX</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Monthly</td>
</tr>
<tr>
<td></td>
<td>HM Treasury</td>
<td>RPI</td>
<td>-</td>
</tr>
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<td>Quarterly</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Monthly</td>
</tr>
</tbody>
</table>

¹ RPIX forecasts are used in oil shock regressions, and RPI forecast to derive inflation risk premia.
A.3. INFLATION EXPECTATIONS & KALMAN FILTER

A.3.2 Kalman Filter

As in Hamilton (1994), the state space representation of the system is given by,

\[ \xi_t = F \xi_{t-1} + M + v_t \]
\[ y_t = H' \xi_t + w_t \]

where the first equation is the evolution of the state and the second equation is the observation equation. Time units are monthly. The state vector \( \xi_t \) consists of lagged and future one-quarter-ahead inflation forecasts. Let \( \pi_t^{s,s+x} \) denote the time \( t \) inflation forecast from quarter \( s \) to quarter \( s + x \). For example \( \pi_t^{0,4} \) denotes the 4Q-ahead inflation forecast from the current quarter \( s = 0 \). I measure inflation forecast in log points, so that 4Q-ahead inflation forecasts can be calculated as,

\[ \pi_t^{0,4} = \pi_t^{0,1} + \pi_t^{1,2} + \pi_t^{2,3} + \pi_t^{3,4}. \]

The state vector \( \xi_t \) is then equal to,

\[ \xi_t = \left( \pi_t^{-1,0} \quad \pi_t^{0,1} \quad \pi_t^{1,2} \quad \ldots \quad \pi_t^{7,8} \right)'. \]

The transition matrix \( F \) is defined as follows:

\[ F = \begin{cases} 
I_9 & \text{if quarter unchanged} \\
0 \quad I_8 \\
0 \quad \zeta & \text{if new quarter begins}
\end{cases} \]

Thus, if there is no change in the quarter, the state space evolves (in expectations) as \( \pi_t^{s,s+x} = \pi_{t-1}^{s,s+x} \), which is consistent with rational expectations. Whenever a new quarter begins, the inflation forecast from two quarters ago \( \pi_{t-1}^{-1,0} \) drops out of the state space and gets replaced with the inflation forecast from last quarter \( \mathbb{E}_t \pi_{t-1}^{-1,0} = \pi_{t-1}^{0,1} \). The same procedure is repeated for all inflation forecasts in the state space. The 1Q-ahead forecast seven quarters from today, \( \pi_t^{7,8} \) was not in the lagged state space. It is therefore set to be proportional to its counterpart from last quarter, \( \mathbb{E}_t \pi_{t}^{7,8} = \zeta \pi_{t-1}^{7,8} \). I capture the mean of these new expectations in the vector \( M \), which equals

\[ M = \begin{cases} 
0 & \text{if quarter unchanged} \\
0 & \text{if new quarter begins}\\n(1 - \zeta) \mu
\end{cases} \]
A.3. INFLATION EXPECTATIONS & KALMAN FILTER

I allow errors in the state equation to be correlated to capture the notion that shocks affect inflation expectations for several quarters. Specifically, I let the \((i, j)\) element of \(v_t\) be equal to,

\[ v_{ij} = \rho^{|i-j|} \sigma^2_x. \]

In the observation equation, I let the observation vector be

\[ \mathbf{y}_t = \begin{pmatrix} \hat{\pi}_{t,1} & \hat{\pi}_{t,0} & \hat{\pi}_{t,-1,0} & \hat{\pi}_{t,-1,1} & \hdots & \hat{\pi}_{t,7,8} \end{pmatrix}^\prime. \]

where \(\hat{\pi}_{t,s+s+x}\) denotes the observed inflation forecasts from quarter \(s\) to quarter \(s+x\). The first two elements of \(\mathbf{y}_t\) are annual forecasts for next year (first element) and the current year (second element), which are a tent-shaped function of the 1Q-ahead inflation forecasts in the state space. As inflation becomes published, I remove it from the current year forecast to ensure that it can be expressed in terms of the state vector.

The observation matrix \(\mathbf{H}_t^\prime\) will be time varying depending on what forecasts are available. Annual forecasts are available at monthly frequency, but quarterly forecasts are typically only available every 3 months. Nevertheless, this can be easily handled by the Kalman filter. I also allow for white noise errors in the observation equation (correlations are already built into the state space equation). In particular, I let the observation error for annual and quarterly forecasts be \(\sigma^2_a\) and \(\sigma^2_q\) respectively, so that

\[ w_{ii} = \begin{cases} \sigma^2_a & \text{if } i \leq 2 \\ \sigma^2_q & \text{if } i > 2 \end{cases} \]

The parameters estimated in the Kalman filter are \(\mathbf{\Omega} = (\rho, \zeta, \mu, \sigma^2_x, \sigma^2_a, \sigma^2_q)\). I use the standard estimation procedures outlined in Hamilton (1994). With the estimated parameters, I calculate smoothed estimates for the state vector \(\hat{\xi}_t\) and derive the 4Q-ahead inflation forecast

\[ \hat{\pi}_{t,1}^{1,5} = \hat{\pi}_{t,1}^{1,2} + \hat{\pi}_{t,1}^{2,3} + \hat{\pi}_{t,1}^{3,4} + \hat{\pi}_{t,1}^{4,5}. \]

The results are robust to using the unsmoothed (one-sided) inflation expectations. In Appendix Figure A.2 I plot the baseline impulse response function from Section 4.3.1 using inflation expectations both filters. The results are nearly identical.

The basic set-up is slightly modified for the U.S. and the U.K. In the U.S. I use both Consensus Economics and Blue Chip forecasts, which have correlated errors in the observation equation. In the U.K. I also use HM Treasury forecasts to determine RPI inflation. While I use RPIX forecasts in the oil shock estimates, I need the RPI estimates to calculate inflation risk premia. In this case the state space includes two additional variable that capture the difference between the current 4Q-ahead RPI
A.3. INFLATION EXPECTATIONS & KALMAN FILTER

Figure A.2: Impulse Response Functions to Oil Shocks for inflation expectations derived from smoothed estimates ("Baseline") and for inflation expectations derived from the one-sided filter.

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
and RPIX forecasts, $s_{t}^{z,z+4}$ and $s_{t}^{z+4,z+8}$, where $z$ is the first quarter of this year. I then calculate RPI forecasts by $rpi_{t}^{s,s+x} = rpi_{t}^{s,s+x} + w_{1}s_{t}^{z,z+4} + (1 - w_{1})s_{t}^{z+4,z+8}$ where $w_{1}$ is the fraction of quarters in $(s, s + x)$ that lie in the current year.

### A.4 Oil Shocks: Standard Error Correction

This section derives the standard error correction when the generated regressor is the residual of a first stage. I consider a univariate setting, but with more cumbersome notation, this derivation can also be extended to a multivariable setting where the same results obtain (see Wooldridge (2001) p.139-142). The true model is given by

$$y = x\beta + u,$$  \hspace{1cm} (A.6)

where $y$ is the outcome variable (e.g., unemployment), $x$ is the true oil supply shock, and $u$ is a residual. The oil supply shock $x$ is unobserved but can be obtained in a first stage

$$z = w\delta + x.$$  \hspace{1cm} (A.7)

With $z$ (global oil supply) and $w$ (lagged real economic activity) known, $\delta$ is estimated by OLS,

$$\hat{\delta} = (w'w)^{-1}w'z$$  \hspace{1cm} (A.8)

Thus, the estimated residuals are

$$\hat{x} = (I - P_{w})z.$$  \hspace{1cm} (A.9)

where $P_{w} = w(w'w)^{-1}w'$. Note that these residuals are orthogonal to $w$ by definition of OLS.

We can rewrite the second stage as

$$y = \hat{x}\beta + (x - \hat{x})\beta + u,$$  \hspace{1cm} (A.10)

and we are interested in the distribution of $\hat{\beta} = (\hat{x}'\hat{x})^{-1}\hat{x}'y$ – the estimated impact of an oil supply shock on unemployment. Substituting this into the equation, rearranging and multiplying by $\sqrt{N}$ yields,

$$\sqrt{N}(\hat{\beta} - \beta) = (\hat{x}'\hat{x})^{-1}\{\frac{1}{\sqrt{N}}\hat{x}'[(x - \hat{x})\beta + u]\}$$

$$= (\hat{x}'\hat{x})^{-1}\{[\frac{1}{N}\hat{x}'w\beta \sqrt{N}(\hat{\delta} - \delta) + \frac{1}{\sqrt{N}}\hat{x}'u]_{G}\}$$  \hspace{1cm} (A.11)
A.5. OIL SHOCKS: ROBUSTNESS

Focussing on the first term we obtain

\[ G = \beta \frac{1}{N} \hat{x}'w = 0, \tag{A.12} \]

because \( \hat{x} \) (the estimated oil shock) is orthogonal to \( w \) (lagged real activity) by construction. Thus, by Slutsky Theorem the sampling uncertainty in the first stage does not affect the asymptotic variance in the second stage. Hence no correction is necessary.

More generally, so long as one is interested in the hypothesis \( \beta = 0 \) sampling uncertainty also does not affect the asymptotic variance under the null.

A.5 Oil Shocks: Robustness

This appendix investigates the robustness in economic responses to oil shocks. In the following sections I conduct the following checks: letting 4-quarter ahead inflation expectations be measured from the current quarter, measuring inflation expectations using inflation swap rates, excluding the Eurozone, adding controls for inflation and spreads, dropping lagged dependent variables, excluding outliers, using HP-filtered data, using non-seasonally-adjusted consumption, excluding post-2006 data, and using various lag lengths for oil shocks.

A.5.1 4Q-Expectations Measured from Current Quarter

In Section 4.3.1 the change in inflation expectations was calculated ignoring the current quarter. Thus, if an oil shock occurs in April of a given year, I only measured the change in inflation expectations from July onwards. In this subsection, I calculate the 4-quarter ahead inflation expectation starting in the current quarter. For example, for April this will be from Q2 this year until Q2 next year, rather than from Q3 this year to Q3 next year as in the baseline. Thus, this measure will also capture if the oil shock raises inflation expectations from April until June. On the other hand, at the end of a quarter, say in June, it will also capture higher inflation expectations for past two months, thus potentially overstating the total increase in inflation expectations from today. I plot the resulting IRF from Equation (4.7) in Appendix Figure A.3. As expected, it displays a larger increase in inflation expectations than the baseline since the latter is a conservative estimate.

A.5.2 Inflation Expectations Measured from Inflation Swap Rates

This subsection demonstrates that market-based inflation expectations also rise following an oil supply shock by using inflation swap rates. The detailed workings
Figure A.3: Impulse Response Functions to Oil Shocks for 4-quarter ahead inflation expectations measured from the current quarter.

“Baseline” refers to 4-quarter ahead inflation expectations measured from the next quarter. Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
of an inflation swap contract are discussed in Section 4.4.2. Notably, inflation swap rates feature an indexation lag, so that the current swap rates measure inflation expectations based on prices two or three months ago. However, in the Euler equation (4.2) only changes in future prices relative to today matter, so it is important to isolate these changes. To do so, I estimate Equation (4.7) and focus on the coefficients $\gamma_3$, i.e. the impact of an oil shock on inflation swap rates three months from now. Since the three-month-ahead inflation swap takes the current month as a basis, this coefficient will only capture the changes in future prices relative to today – exactly the price changes that matter for an Euler equation consumer.

A downside of using inflation swap data is that Japan does not have a very liquid inflation swap market, so that the only reliable data exists for the U.S., the U.K., and the Eurozone. Since this significantly constrains the sample, I estimate Equation (4.7) without controlling for the bond rate or industrial production. The resulting estimates for $\gamma_3$ at various maturities are plotted in Appendix Figure A.4. I find that an oil supply shock raises inflation expectations at all maturities, although only the one-year rate is significant. The implied forward rates for expected inflation are 0.3% over the first year and zero afterwards. The magnitudes here are somewhat higher than with survey data because the inflation-swap calculation excludes Japan where inflation is less sensitive to shocks. Nevertheless, similarly to the survey data, these results show that oil supply shocks raise inflation expectations for about a year.

A.5.3 Eurozone

I exclude the Eurozone from the sample and repeat the earlier analysis. Since I did not have data on consumption expenditures for the Eurozone, this plot is unchanged and not reported. For the remaining variables the IRFs are essentially identical.

A.5.4 Additional Controls

I include contemporaneous values and twelve lags of changes in inflation and changes in corporate bond spreads in Equation (4.8). The resulting IRFs for oil shocks are plotted in Appendix Figure A.7(c) and are very similar to the baseline results in Section 4.3.1. This suggests that the results are not driven by small-sample covariance of oil shocks with financial shocks or inflation dynamics.

A.5.5 Lagged Dependent Variables

If the oil supply shocks in Kilian (2009) are well-identified, then they would be orthogonal to the current state of the economy, so that excluding lagged dependent variables from Equations (4.7) and (4.8) should yield similar results. In Appendix Figure A.7 I plot the IRFs from this estimation. The results for inflation
Figure A.4: Contemporaneous change in the inflation swap rate three months after an oil supply shock has hit the economy ($\gamma_3$ in Equation (4.7)).

Point estimates are solid circles. Error bars show the 95% Driscoll-Kraay confidence interval around the point estimates. Estimated based on an autoregressive distributive lag equation, where the dependent variable enters with 36 lags based on the AIC criterion. See Section 4.3.1 for details.
A.5. OIL SHOCKS: ROBUSTNESS

Figure A.5: Impulse Response Functions to Oil Shocks excluding the Eurozone from the sample.

(a) 4Q Ahead Expected Inflation

(b) Log Industrial Production

(c) Unemployment

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
A.5. OIL SHOCKS: ROBUSTNESS

Figure A.6: Impulse Response Functions to Oil Shocks controlling for inflation and corporate bond spreads.

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
expectations, unemployment, and industrial production are very similar to the baseline results, which suggests that the oil shock is not picking up dynamics induced by other shocks. If anything, the weakened responses for IP and unemployment indicate that negative oil supply shocks are correlated with improving economic dynamics.

More troubling is that consumption expenditure expand after a negative oil supply shock when I exclude the lagged dependent variables, whereas in the baseline model they contracted (Appendix Figure A.8(d)). However, when I use non-seasonally-adjusted consumption, then the estimated response without lagged dependent variables become more contractionary relative to the baseline (Appendix Figure A.8(e)). With lagged dependent variables included both series agree that negative oil supply shocks reduce consumption expenditures (Figure 4.3(d) and Appendix Figure A.12(a)), which suggests that the noise in consumption expenditure series makes it necessary to control for the lagged state of the economy. Thus, excluding lagged dependent variables does not appear to be informative in this instance.

A.5.6 Outliers

Outliers are determined by jackknifed residuals in the estimated ADLs. Residuals that exceed the 1% critical value are then removed from the sample and the ADLs are re-estimated. The resulting IRFs are plotted in Appendix Figure A.8, together with the baseline results. Typically, the baseline IRF lies within the 95% confidence interval of the outlier-corrected IRF. Quantitatively, the only significant difference is the IRF for industrial production, which now displays less mean-reversion.

A.5.7 HP-filtered data

For industrial production and unemployment I also estimate equation (4.8) using HP-filtered data. The resulting IRFs and 95% confidence intervals are plotted in Appendix Figure A.9. As in the case for outliers the baseline IRF lies within two standard deviations of the HP-filtered IRF. The magnitudes are somewhat smaller, but the contractionary effects of oil shocks remain significant.

I also check if oil shocks affect the economy differently at the ZLB compared to normal times using HP-filtered data. The IRFs in Appendix Figure A.10 display similar results as in the main text, namely that average contractionary effects are stronger at the ZLB, but that uncertainty over these effects is large.

A.5.8 NSA consumption

In Appendix Figure A.11 I plot the IRF for non-seasonally adjusted consumption expenditures in Japan. As with the X-12 adjusted series, the IRF displays a statistically significant contraction in consumption expenditures following an oil shock.
A.5. OIL SHOCKS: ROBUSTNESS

Figure A.7: Impulse Response Functions to Oil Shocks excluding lagged dependent variables in Equations (4.7) and (4.8).

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
A.5. **OIL SHOCKS: ROBUSTNESS**

Figure A.8: Impulse Response Functions to Oil Shocks *excluding outliers* based on jackknifed residual with 1% critical value.

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
A.5. OIL SHOCKS: ROBUSTNESS

Figure A.9: Impulse Response Functions to Oil Shocks for HP-filtered variables.

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix. This result is also robust to excluding outliers. Thus, the measured contraction in consumption is not due to the particular seasonal adjustment mechanism.

A.5.9 Excluding post-2006 data

In Appendix Figure A.12 I plot the impulse response functions when the financial crisis is dropped from the sample. Thus, these IRFs are constructed based only on Japanese data. The results are very similar to the baseline, so the contractionary effects of oil supply shocks are not drive by particularities of recent events. This also shows that extending the Kilian series to include the recent ZLB episode is not a driver of my results.

A.5.10 Lag Lengths

I investigate the sensitivity of my results to changes in lag lengths on the oil shocks. (I already checked for sensitivity to lagged dependent variable lag length above.) In Appendix Table A.2 I tabulate the values of the IRF one month and four months after the shock for 12, 24, and 36 lags of oil shocks. The results are very similar across all these lag lengths and similar to the baseline. In particular, inflation, IP, and unemployment show significant responses one month out, and unemployment
Figure A.10: Impulse Response Functions to oil supply shocks over 1984-now when ZLB binds (“Baseline”) and when policy rates are unconstrained (“Normal Times”) for HP-filtered variables.

IRFs are constructed from autoregressive distributive lag estimates in changes or growth rates and aggregated to levels. 95% confidence intervals are derived by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix. See Section 4.3.1 for details.
A.5. **OIL SHOCKS: ROBUSTNESS**

Figure A.11: Impulse Response Functions of NSA consumption for Japan.

![Impulse Response Functions for Japan](image)

(a) Baseline

(b) Excluding Outliers

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.

and consumption expenditures display significant movements after four months as in the baseline.
Figure A.12: Impulse Response Functions to Oil Shocks \textit{excluding post-2006 data.} 

Two-standard-error confidence intervals are constructed by Monte-Carlo draws from a normal distribution with variance equal to the estimated covariance matrix.
Table A.2: Impact of Oil Shocks on Macroeconomic Variables for Various Lags of Oil Shocks in Equations (4.7) and (4.8).

<table>
<thead>
<tr>
<th>Lags</th>
<th>Inflation Expectations</th>
<th>Industrial Production</th>
<th>Unemployment</th>
<th>Consumption Expenditure</th>
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<td>1 Month</td>
<td>4 Months</td>
<td>1 Month</td>
<td>4 Months</td>
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<tr>
<td>12</td>
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<tr>
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</tbody>
</table>

95% Confidence intervals in brackets based on Driscoll-Kraay s.e. “1 Month” and “4 Months” are the impact and associated 95% CI at 1 month and 4 months after the shock. Baseline specifications are marked in bold font. Dependent variable lags are as in the baseline regressions (12 for inflation, 48 for IP, 36 for unemployment and consumption).
A.6 Oil Shocks: Event Study

This appendix provides corroborating evidence for Section 4.3.1. Rather than constructing oil shocks based on a statistical model as in Kilian (2009), in this section I use an event study methodology around the Libyan civil war. Before the civil war, Libya produced approximately 2% of global oil supply. The beginning of the civil war is typically dated on February 15th, 2011. Foreign intervention officially commenced on March 19th. This conflict caused a significant contraction in Libyan oil production, which by April had declined by almost 90% relative to pre-war levels. Thus, the Libyan civil war constituted a relatively large and exogenous shock to global oil production. Consistent with this interpretation the Kilian (2009) series also displays a 1.4 standard deviation oil shocks over February 2011 and March 2011. This suggests that the Kilian (2009) series does indeed successfully identify exogenous supply shocks. However, while this oil shock is plausibly exogenous, to the extent that foreign governments increase military spending it may be correlated with a positive demand shock. Thus, this event study will be biased against finding contractionary effects from negative supply shocks.

To determine the effects of this oil shock on expected inflation and real economic activity I proceed as in Section 4.3.2. The timing of the oil supply disruptions associated with Libyan civil war are less precise than the timing of the Japanese earthquake – it could plausibly be dated on February 15th, 2011 or the beginning of foreign intervention on March 19th. Thus, I use both these dates in the event study that comprises the U.S., the U.K., and the Eurozone. In Appendix Figure A.13 I compare pre-February 15th forecasts with post-February 15th forecasts, and in Appendix Figure A.14 I compare pre-March 19th forecasts with post-March 19th forecasts. Both figures also display ex-post data.

In all six cases there are significant increases in expected inflation, and even higher ex-post inflation outcomes, consistent with there being a negative supply shock. In addition, in all countries ex-post real output was below the February/March 2011 forecast, and in four out of six cases output forecasts were revised downwards. These negative comovements between expected inflation and output, as well as actual inflation and output, suggest that the Libyan civil war and associated oil supply disruption did indeed constitute have contractionary effects at the ZLB. This is remarkable since this war likely triggered greater military spending, i.e. a positive demand shock. Thus, this event study corroborates the findings from Section 4.3.1 – that oil supply shocks are contractionary at the ZLB despite lowering real interest rates.
Figure A.13: Consensus Economics forecasts from before the Libyan uprising (February 2011) and after (March 2011).

(a) U.S.: GDP

(b) U.S.: Inflation

(c) U.K.: GDP

(d) U.K.: Inflation

(e) Eurozone: GDP

(f) Eurozone: Inflation

Forecasts are for annual GDP and year-on-year inflation. GDP data is annual for 2010 and 2012 and quarterly from 2010Q4 until 2012Q1.
A.6. OIL SHOCKS: EVENT STUDY

Figure A.14: Consensus Economics forecasts from before foreign intervention in the Libyan civil war (March 2011) and after (April 2011).

Forecasts are for annual GDP and year-on-year inflation. GDP data is annual for 2010 and 2012 and quarterly from 2010Q4 until 2012Q1.
A.7. INFLATION RISK PREMIA

A.7 Inflation Risk Premia

This appendix illustrates that the intuition from instantaneous inflation risk premia carries over to longer-horizon risk premia, and is robust to using Epstein-Zin preferences.

A.7.1 s-year Inflation Risk Premia

The one-year risk premium in the model is harder to sign than the instantaneous risk premium. The problems in calculating this quantity are twofold: First, when the economy randomly switches between the ZLB and normal times, then inflation and marginal utility will be non-linear functions of the state variables, which makes it impossible to characterize their distributions analytically. This was not a problem before, because the derivatives $c_u(t)$ and $c_v(t)$, which multiply normal shocks, are fixed over a small time interval. Second, these derivatives must now be solved for within the context of the model, whereas before I didn’t have to specify why $c_u(t)$ is positive or negative at the ZLB.

To make analytical progress on the first issue, I will make the following assumptions: First, inflation is a linear function of the supply and demand shifters, $\pi(t) = \pi_u u(t) + \pi_v v(t)$. Second, the interest rate rule is given by, $i(t) = r + \phi \pi(t)$. Third, I assume that the current state of the world - ZLB or normal times - persists forever. Fourth, the demand and supply shocks follow mean-reverting Ornstein-Uhlenbeck processes, $dx(t) = -\rho_x x(t) + \sigma_x dB_x(t)$, for $x \in \{u, v\}$. Under these assumptions marginal utility and consumption are linear functions of the state variables.

Given these assumptions, I can calculate an explicit solution for the $s$-period inflation risk premium,\(^3\)

$$rp_\pi(t, t+s) = \frac{(1 - e^{-\rho_u s})^2}{2 \rho_u^2 s} \left[ -\frac{\sigma_u^2 \pi_u c_u}{\gamma} \right] - \frac{(1 - e^{-\rho_v s})^2}{2 \rho_v^2 s} \left[ \frac{\sigma_v^2 \pi_v c_v}{\gamma} \right] \quad (A.13)$$

where $c_u = C_u(t)/C(t)$ and $c_v = C_v(t)/C(t)$ are constant parameters. The solution is similar to the instantaneous risk premium in Equation (4.9).\(^4\) Intuitively, when the solution to the model is linear, the $s$-year risk premium strings together a set of constant instantaneous risk premia, while accounting for mean-reversion in the state variables. The weights $\frac{(1 - e^{-\rho_u s})^2}{2 \rho_u^2 s}$ imply that the short-end of the yield

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\(^2\)I ignore sunspot shocks in the ZLB case.
\(^3\)Technically, this calculates a price level risk premium rather than an inflation risk premium. However, I follow the literature that has settled on the latter term.
\(^4\)It can be shown that the partial derivative of $rp_\pi(t, t+s)$ with respect to $s$ evaluated at $s = 0$ equals $rp_\pi(t)$. 

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A.7. INFLATION RISK PREMIA

curve loads relatively more on transient shocks than the long-end as shown in Appendix Figure A.15. For example, at a maturity of 4.75 years, corresponding to a 5-year inflation swap with three month indexation lag, permanent shocks have about 10 times the loading than shocks with a half-life of one year. In contrast, at a maturity of 0.75 years the relative weight is only 1.5. Thus, on the short-end we are more likely to pick up ZLB effects rather than spillovers.

Figure A.15: Weight on inflation risk premia for various maturities and values of $\rho_u$, the mean-reversion of supply shocks, when the state of the economy (ZLB or non-ZLB) persists permanently.

When the ZLB persists forever, there are no spillovers into normal times and supply shocks are contractionary in the baseline model, $c_u(t) > 0$ (Proposition 4). This is problematic, because it will make inflation risk premia unambiguously negative at the ZLB, so that empirical evidence of positive risk premia becomes hard to interpret. To address this issue, I allow supply shocks $dB_u(t)$ to be correlated with demand shocks $dB_v(t)$. If this correlation is negative, then negative supply shocks may be (on average) contractionary at the ZLB because they coincide with negative demand shocks, and this can rationalize a positive risk premia at the ZLB. To simplify the algebra, I assume that the demand and supply shifters are identically distributed, $\sigma_u = \sigma_v = \sigma_x$ and $\rho_u = \rho_v = \rho_x$, and that the correlation between the Brownian Motions is given by $\varphi$. 

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Given these assumptions, the $s$-period inflation risk premium is given by,

$$rp_{\pi}(t, t + s) = \frac{(1 - e^{-\rho_{x}s})^2}{2\rho_{x}^2 s} \left[ -\frac{\sigma_u^2(\pi_u + \varphi\pi_v)}{\gamma} - \frac{\sigma_v^2(\pi_v + \varphi\pi_u)}{\gamma} \right]$$  \hspace{1cm} (A.14)

In normal times, the sign of risk premium in Equation (A.14) is ambiguous when $\varphi = 0$, because negative supply shocks are contractionary and will generate positive risk premia, whereas demand shocks will generate negative risk premia. Thus, if contractionary supply shocks are the most important source of inflation volatility, then the inflation risk premium will be positive.

At the ZLB, negative supply shocks are expansionary in standard models when there are no spillovers, so that the risk premium is unambiguously negative if $\varphi = 0$. However, if the correlation $\varphi$ is negative, then negative supply shocks will be (on average) paired with negative demand shocks, which allows negative co-movement between inflation and consumption, effectively mimicking the behavior of contractionary negative supply shocks. This will be the case when $c_u + \varphi c_v < 0$, making the first term in brackets positive. If, in addition, supply shocks dominate inflation volatility, $\pi_u \gg \pi_v$, then the overall inflation risk premium is positive. Thus, the conclusions are qualitatively similar to the instantaneous case, which is summarized as follows.\footnote{Another possibility to rationalize positive inflation risk premia at the ZLB is to make positive demand shocks deflationary so that $c_v(\pi_v + \varphi\pi_u) < 0$. In fact, if inflation risk premia are positive and $c_u + \varphi c_v < 0$, then it must also be the case that $c_u(\pi_u + \varphi\pi_v) < 0$, because $\varphi < 0$ and $\pi_u \gg \pi_v$. This is the consequence of assuming a statistical correlation between demand and supply shocks: if negative supply shocks are paired with negative demand shocks, then positive demand shocks must be paired with positive supply shocks.}

**Summary 6** *In the standard CCAPM, $s$-year inflation risk premia are positive, in normal times or at the ZLB, if and only if negative supply shocks have contractionary effects on average, and this constitutes a significant fraction of aggregate inflation risk.*

In addition, inflation risk premia also capture supply shocks that occur after the economy has exited from the ZLB, which raises the observed inflation risk premium. Since the probability of exit over a one-year horizon is relatively small, these effects are less likely to show up at short maturities. This intuition is confirmed in Appendix Figure A.16. In this exercise, I let a standard linearized new Keynesian model start at the ZLB and revert permanently to normal times with probability $\lambda dt$ over an interval $dt$. In addition, I abstract from demand shocks and only consider supply shocks. While the solutions for consumption and inflation can be analytically characterized as in Appendix A.1, this is not true for the inflation risk premium. Thus, these results are calculated by numerically integrating over the stochastic ZLB duration. I set $\lambda = 0.5$ implying an expected duration of the ZLB of two years.
A.7. INFLATION RISK PREMIA

In Appendix Figure A.16, the inflation risk premia exhibit hump-shapes at all maturities. These occur as the inflation risk premium is zero at $\rho_u = 0$ because the long-run Phillips curve is vertical and zero at $\rho_u \to \infty$ because such temporary shocks to not affect consumption or inflation. At a nine-month horizon inflation risk premia are predominantly negative as most negative supply shocks over this interval will be expansionary – the only exception being close-to-permanent shocks for which the “Paradox of Toil” does not apply. In contrast, at longer maturities risk premia rise and are predominantly positive as the probability of exit is (cumulatively) higher. Note also that this figure now includes jump risk from the ZLB into normal times.

Figure A.16: Inflation risk premia for various maturities and values of $\rho_u$ (the mean-reversion of supply shocks) when there is stochastic exit from the ZLB.

The calibration is as in Table 4.2, except that $\phi_y = 0$, $\nu = 0$, $\varrho = 0$, and $\sigma_u = 0.1$.

Summary 7 In the standard CCAPM, s-year inflation risk premia at the ZLB are more likely to be negative the smaller s. This is because over a shorter interval exit from the ZLB becomes less likely.

A.7.2 Epstein-Zin Inflation Risk Premia

Piazzesi and Schneider (2006) show that Epstein-Zin utility is a key ingredient
A.7. INFLATION RISK PREMIA

for rationalizing an upward sloping yield curve given the historical correlations of consumption and inflation. Thus, in this subsection I show that my results are robust to this particular preference specification.

I use the simplifying assumption from the beginning of the previous section and consider a consumer with Epstein-Zin utility with IES equal to 1 and risk aversion $\gamma^{-1}$. Then the continuous time limit of the recursive definition of the value function implies,

$$E_t d\ln V(t) = -\frac{\varrho \ln C(t) + \varrho \ln V(t)}{2} + (1 - \varrho)(1 - \gamma^{-1}) \sigma_v^2, \quad (A.15)$$

where $\sigma_v^2$ is the variance of $\ln V(t)$, which is constant in this linear set-up as the current state of the economy persists forever. Solving forward and substituting $\ln C(t) = c_u u(t) + c_v v(t)$ yields,

$$d\ln V(t) - E_t d\ln V(t) = \frac{\varrho}{\varrho + \rho_u} c_u \sigma_u d B_u(t) + \frac{\varrho}{\varrho + \rho_v} c_v \sigma_v d B_v(t). \quad (A.16)$$

The continuous time limit of the SDF is in turn given by,

$$d\ln \Lambda(t) - E_t d\ln \Lambda(t) = -[d\ln C(t) - E_t d\ln C(t)]$$

$$+ (1 - \gamma^{-1})[d\ln V(t) - E_t d\ln V(t)], \quad (A.17)$$

where the second term captures the utility recursion, i.e. deviations from CRRA. Substituting the solution from the previous equation and for consumption yields,

$$d\ln \Lambda(t) - E_t d\ln \Lambda(t) = -c_u \left[1 + \frac{(\gamma^{-1} - 1)\varrho}{\varrho + \rho_u}\right] \sigma_u d B_u(t)$$

$$- c_v \left[1 + \frac{(\gamma^{-1} - 1)\varrho}{\varrho + \rho_v}\right] \sigma_v d B_v(t), \quad (A.18)$$

so that the price of inflation risk is given by,

$$r p_t = -c_u \pi_u \left[1 + \frac{(\gamma^{-1} - 1)\varrho}{\varrho + \rho_u}\right] \sigma_u^2 - c_v \pi_v \left[1 + \frac{(\gamma^{-1} - 1)\varrho}{\varrho + \rho_v}\right] \sigma_v^2. \quad (A.19)$$

This is similar to Equation (4.9) except for the additional term in square brackets involving $\gamma^{-1}$. Standard calibration set $\gamma^{-1} \geq 6$, so that EZ utility amplify risk premia, but do not change their sign. Intuitively, EZ utility also punishes assets that lose value when there is bad news about the future. Due to persistence of shocks in my setting ($\rho_x < \infty$), bad news today is also bad news for the future, so risk premia get amplified (relative to the log utility case) when risk aversion exceeds one.

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A.8 Swap Rate - Inflation Expectation Matching

Appendix Table A.3 illustrates the reference price levels, inflation lags, and inflation expectations sources for inflation swap rates. I match inflation expectations to risk premia as follows: For the Eurozone the ECB releases quarterly 12 month ahead consensus inflation expectations exactly when the previous month’s inflation data is released. For example, in January the forecast horizon is from December to December. The appropriate match for the inflation swap is the October to October expected inflation rate. I therefore add (known) CPI inflation of the previous two months to the forecast, while proportionally downweighting the original 12 month forecast. This procedure is appropriate, if the last two months of the 12 month inflation forecast have a similar inflation rate than the first 10 months. For the U.K. and the U.S. I use the estimates obtained from the Kalman filter procedure in the previous section. For the U.K., I match the December, January, and February swap rates to the Q4-Q4 forecast, which is appropriate given the two month lag, and average over these months to account for seasonal effects. For the U.S. the matched forecast is calculated as the weighted average of the 4-quarter forecasts originating 2 and 3 months back.

<table>
<thead>
<tr>
<th>Country</th>
<th>Reference Price</th>
<th>Inflation Lag</th>
<th>Inflation Expectations Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>CPI</td>
<td>2-3 Months</td>
<td>Kalman filter (Blue-Chip)</td>
</tr>
<tr>
<td>U.K.</td>
<td>RPI</td>
<td>2 Months</td>
<td>Kalman filter (HM Treasury)</td>
</tr>
<tr>
<td>Eurozone</td>
<td>CPI</td>
<td>3 Months</td>
<td>ECB</td>
</tr>
</tbody>
</table>

A.9 Supply Shocks and Credit Frictions

This section shows (a) that oil supply shocks tighten credit availability and raise borrowing rates, and (b) that technology shocks raise asset spreads in a manner that is quantitatively consistent with the model in Section 4.5.

A.9.1 Oil Supply Shocks

First, I determine the effect of oil supply shocks on credit standards for firms and households in Japan. I regress credit standards $y_t$ (reported at quarterly frequency
A.9. SUPPLY SHOCKS AND CREDIT FRICTIONS

in the Japanese loan officer survey) on four of its own lags, on oil shocks, and on recession/recovery dummies,

\[ y_t = \alpha + \sum_{j=1}^{4} \beta_j y_{t-j} + \sum_{j=0}^{4} \gamma_j \text{oil}_{t-j} + I(\text{recession}_t == 1) + I(\text{recovery}_t == 1) + \varepsilon_t, \]
Figure A.17: Impact of oil shocks on credit standards in Japan.

(a) Credit Standards: Small Firms.  
(b) Credit Standards: Medium Firms.  
(c) Credit Standards: Large Firms.  
(d) Credit Standards: Households.

Negative IRFs indicate a tightening of credit standards. 95% confidence intervals are derived by Monte-Carlo draws from a normal distribution with variance equal to the estimated Driscoll-Kraay covariance matrix. Units are calculated as (percentage of respondents selecting “eased considerably” + percentage of respondents selecting “eased somewhat” * 0.5 ) - ( percentage of respondents selecting “tightened considerably” + percentage of respondents selecting “tightened somewhat” * 0.5).
Note that I conduct the regression in levels because the data is already reported as changes in credit conditions.

I plot the resulting impulse response functions in Appendix Figure A.17. The tightening of credit availability persists for about two years for both firms and households, and (consistent with this interpretation) credit tightens more for small firms than for medium firms, which in turn are more affected than large firms. These results indicate that a decline in credit supply is contributing to the contraction following negative supply shocks.

Second, I investigate the effect of oil shocks at the intensive margin. Column one of Appendix Table A.4 replicates the estimate from Figure 4.3 for the 10-year bond rate in Japan. Column two conducts the same regression but with the loan spread as a dependent variable. This spread is calculated as the differences between long-term borrowing rates at Japanese commercial banks and the 10-year bond rate. The results show the decline in bond yields is more than fully offset by the rise in the borrowing spread, i.e. private borrowing becomes more expensive after an oil supply shock. Thus, an Euler equation consumer, who faces these interest rates, would contract her consumption given the rise in borrowing rates. This suggests that both the intensive and extensive margin of loan supply contribute to the contractionary effects of oil supply shocks.

A.9.2 Technology Shocks

Since the model’s forcing process is the technology shock $a(t)$, I verify the quantitative properties of the model utilization-adjusted TFP series for the U.S. from Fernald (2009). This is a closer match with $a(t)$ in the model than using oil supply shocks. The Fernald-series is only available for the U.S., and unfortunately similar series are not available for the other countries in my sample.

I determine the effect of technology shocks on the three-month TED spread, i.e. the spread between the three-month interbank borrowing rate and the three-month Treasury-Bill. While the baseline model in Section 4.5 does not contain an interbank market, it can be easily extended along this dimension following Gertler and Kiyotaki (2010). In such an extension, the TED spread corresponds to the spread $\omega(t)$ in the model – $i(t)$ is the government bond yield and loans to other banks are subject to the incentive constraint and must earn $i(t) + \omega(t)$. Thus, by using the Fernald-series and the TED spread I stay as close to the model as possible.

In Appendix Figure A.19(a) I report the effect of a 1% negative TFP shock on the TED spread, estimated from an ADL equation with 12 lags. The TFP shock raises the TED spread by up to 24 basis points after eight quarters. I compute the corresponding model statistic by linearizing the model around the non-stochastic steady state, setting $\rho_a = 0$ as in the Fernald-series. The IRF is plotted in Appendix Figure A.19(b), and shows that a 1% permanent TFP shock raises the borrowing-lending
### Table A.4: Response of Japanese Bond Rate and Loan Spread to Oil Supply Shock

<table>
<thead>
<tr>
<th></th>
<th>Change in Bond Rate</th>
<th>Change in Loan Spread (Loan Rate - Bond Rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Shock</td>
<td>-0.0416**</td>
<td>0.0588**</td>
</tr>
<tr>
<td></td>
<td>(0.0207)</td>
<td>(0.0233)</td>
</tr>
<tr>
<td>Dep. Var. Lags</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Observations</td>
<td>154</td>
<td>154</td>
</tr>
</tbody>
</table>

Driscoll-Kraay standard errors in parentheses

* p < 0.1, ** p < 0.05, *** p < 0.01
spread by 25 basis points on impact, which subsequently declines to zero. This sug-
gests that the credit frictions in this calibration are empirically reasonable.

A.10 Oil Shocks in Incomplete Markets

This appendix shows how oil shocks propagate in an open economy with incom-
plete markets. The set-up builds on Farhi and Werning (2012a) and Wieland (2012).
There are two countries home (H) and the rest of the world (F). The home economy
is small and must import oil from the rest of the world.

A.10.1 Set-up

Home agents maximize the stream of utility,
\[ \int_0^\infty e^{-\rho t} \left[ \frac{C(t)^{1-\sigma}}{1-\sigma} - \chi \frac{N(t)^{1+\nu}}{1+\nu} \right], \]
where \( C(t) \) is domestic consumption and \( N(t) \) is labor. The inverse of the intertem-
poral elasticity of substitution is \( \sigma \), the inverse of the Frisch elasticity is \( \nu \), and \( \chi \) is
a parameter that determines steady-state labor supply.

Consumption is an aggregate of domestically produced goods \( C_H(t) \) and foreign
produced goods \( C_F(t) \)
\[ C(t) = \left[ (1 - \alpha) \frac{1}{\eta} C_H(t) \frac{\eta-1}{\eta} + \alpha \frac{1}{\eta} C_F(t) \frac{\eta-1}{\eta} \right] \frac{\eta}{\eta-1}, \]
which are themselves aggregates of individual varieties,
\[ C_x(t) = \left( \int_0^1 C_x(t, j) \frac{\psi-1}{\psi} dj \right) \frac{\psi}{\psi-1}, \quad x \in \{H, F\}. \]

The price of home consumption in home currency is given by
\[ P(t) = \left[ (1 - \alpha) P_H(t)^{1-\eta} + \alpha P_F(t)^{1-\eta} \right] \frac{1}{1-\eta}. \]

Home asset holdings of the risk-free bond \( D(t) \) evolve according to
\[ D(t) = i(t) D(t) - P(t) C(t) + W(t) N(t) + \Pi(t), \]
where \( i(t) \) is the nominal interest rate, \( W(t) \) the wage rate, and \( \Pi(t) \) are profits from
firms.

Firms produce output \( Y(t, j) \) of variety \( j \) according to a CES technology,
\[ Y(t, j) = \left[ (1 - \gamma)^{\frac{1}{\psi}} N(t, j)^{\frac{\psi-1}{\psi}} + \gamma \frac{1}{\psi} X(t, j)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \]
A.10. OIL SHOCKS IN INCOMPLETE MARKETS

Figure A.18: TFP Shocks and Credit Spreads

(a) TED Spread.

(b) Model Borrowing Spread.

Left Panel: Impact of 1% permanent, negative TFP Shock on TED Spread (3 month LIBOR - 3 month Treasury Bill). Estimated using an ADL equation with 12 lags of the TED spread and 12 lags of TFP shock. Newey-West standard error bands are drawn from the Variance-Covariance matrix using Monte-Carlo methods. Right Panel: Model impact of a 1% permanent, negative TFP shock on the borrowing-lending spread given the baseline calibration with $\rho_a = 0$. 

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where $X(t,j)$ is oil input, $\gamma$ the steady-state share of oil in production, and $\psi$ its elasticity of substitution with labor input. Firms face standard Calvo pricing frictions where prices can be reset with probability $(\rho \delta dt$ over an interval $dt$.

The foreign economy is large relative to the domestic economy. Thus, the share of home goods in foreign consumption is close to zero $\alpha^{\ast} \rightarrow 0$. Otherwise the foreign economy is set-up as the home economy. I denote foreign quantities with a $\ast$, i.e. $C^{\ast}(t)$ is foreign consumption. In addition the foreign economy is endowed with a stochastic supply of oil $\bar{X}(t)$, that evolves according to

$$d \ln \bar{X}(t) = -\rho_x \ln \bar{X}(t) + \sigma_x dB_x(t).$$

The shock $dB_x(t)$ is the only disturbance in the model. The initial condition is $\ln \bar{X}(0) = 0$. I consider a situation where $dB_x(0) < 0$, i.e. a negative oil supply shock, and $dB_x(t) = 0$ for all $t > 0$. Thus, as in Farhi and Werning (2012a), I compute a perfect foresight equilibrium. This makes the computation of the incomplete market model more convenient as the Pareto weight (defined below) only changes once in value.

I define the following relative prices. The real exchange rate $Q(t)$ is equal to

$$Q(t) = \frac{\mathcal{E}(t)P^{\ast}(t)}{P(t)},$$

where $\mathcal{E}(t)$ is the nominal exchange rate. The terms of trade equal

$$S(t) = \frac{P_F(t)}{P_H(t)},$$

and the relative price of oil equals

$$O(t) = \frac{P_X(t)}{P_H(t)}.$$ 

I assume that the law of one price holds.

The central bank follows a standard interest rate rule subject to the zero bound constraint.

$$i(t) = \max\{\rho + \phi \pi(t), 0\}.$$

A.10.2 Equilibrium

In equilibrium the following first order conditions must be satisfied. Both home and foreign satisfy an Euler equation,

$$\sigma \frac{dC(t)}{C(t)} = (i(t) - \pi(t) - \rho) dt.$$
Labor is supplied allocated according to
\[ \chi C(t)^\sigma N(t)^\nu = \frac{W(t)}{P(t)} \]
where the RHS equals the real wage. Domestic and foreign consumption are related by
\[ C(t) = \Theta C^*(t) Q(t)^{\frac{1}{2}}, \]
where \( \Theta \) is the relative Pareto weight. Relative demands for varieties are given by the standard Dixit-Stiglitz relationships.

The first order condition for the firms reset price \( P^r(t) \) is given by,
\[ \int_0^\infty e^{-\rho s} \int_0^{t+s} \left[ (1 - \varepsilon) + \varepsilon (1 - \tau^L) \frac{W(t+s)/P(t+s)}{MPL(t+s)P^r(t)} \right] Y(t+s,i) ds = 0, \]
where \( MPL(t+s) \) is the marginal product of labor and \( \tau^L \) is a labor input subsidy.

In addition an equilibrium must satisfy the feasibility conditions. Domestic and foreign output must be equal to domestic and foreign consumption respectively
\[ Y(t) = C_H(t) + C^*_H(t), \quad Y^*(t) = C_F(t) + C^*_F(t) = C^*_H(t), \]
where the last two equality follows from the fact that the foreign economy is large. In addition the quantity of oil supplied must equal the quantity of oil demanded,
\[ \bar{X}(t) = X_F(t) + X_H(t) = X_F(t). \]

Finally, net foreign assets (denominated in home currency) evolve according to
\[ dNFA(t) = (P_H(t)C^*_H(t) - P_F(t)C_F(t) - P_X(t)X_H(t)) + i(t)NFA(t), \]
so the home country accumulates assets through exports and draws down assets by importing either consumption goods or oil.

The equilibrium under complete markets features \( \Theta = 1 \), whereas in the equilibrium under incomplete markets \( NFA(0) \) is given.

### A.10.3 Computation

To solve for the equilibrium I log-linearize the equilibrium conditions. Small caps letters denote these log-linearized quantities. I then proceed in two steps. First, I solve for the allocation in the foreign economy. Since this economy is large its allocation is independent of the behavior of the home economy. Next, taking the allocation for the foreign economy as given, I compute the allocation for the home economy.
A.10.4 Foreign Economy

The solution to the foreign economy under complete and incomplete markets is identical. When the ZLB does not bind, then it is given by,

\[ y^*(t) = c^*(t) = \frac{\sigma^{-1}(\phi - 1)\gamma \mu}{\sigma^{-1}\lambda(\phi - 1) + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t), \]

\[ \pi^*(t) = \frac{-\rho_x\gamma \mu}{\sigma^{-1}\lambda(\phi - 1) + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t), \]

\[ n^*(t) = \frac{-\gamma \kappa(\phi - 1)(\sigma \psi)^{-1}(\sigma \psi - 1) + \gamma \rho_x(\rho + \rho_x)}{\sigma^{-1}\lambda(\phi - 1) + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t), \]

where the parameters \( \mu \) and \( \lambda \) are defined as

\[ \mu = \frac{\kappa}{\psi}(1 + \nu \psi), \quad \lambda = \mu + \frac{\kappa}{\psi}(1 - \gamma)(\sigma \psi - 1), \]

and \( \kappa \) is the slope of the new Keynesian Phillips curve. Note that a negative supply shock lowers consumption and output, but may raise GDP (here equal to labor input) depending on parameters.

The price of oil abroad is equal to,

\[ o^*(t) = -\frac{1}{\psi} \frac{\sigma^{-1}(\phi - 1)\gamma \mu}{\sigma^{-1}\lambda(\phi - 1) + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t). \]

It is immediate that negative oil supply shocks lower consumption, raise inflation, and raise real oil prices.

To illustrate the effects of the ZLB, I follow Farhi and Werning (2012a) and consider the case where the central bank interest rate is exogenous with respect to the oil shock (e.g., fixed at zero). In this case, the allocation for the foreign economy is given by,

\[ c^{*,ZLB}(t) = -\frac{\sigma^{-1}\gamma \mu}{-\sigma^{-1}\lambda + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t), \]

\[ \pi^{*,ZLB}(t) = -\frac{-\rho_x\gamma \mu}{-\sigma^{-1}\lambda + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t), \]

\[ n^{*,ZLB}(t) = \frac{\gamma \kappa(\sigma \psi)^{-1}(\sigma \psi - 1) - \gamma \rho_x(\rho + \rho_x)}{-\sigma^{-1}\lambda + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t), \]

Note that \((1 - \gamma)\rho_x(\rho + \rho_x) > \sigma^{-1}\lambda\) is a necessary condition for a unique bounded equilibrium to exist (see also Appendix A.1). The same condition also guarantees that real GDP and real oil prices rise following a negative supply shock,

\[ o^{*,ZLB}(t) = -\frac{1}{\psi} \frac{-\sigma^{-1}\kappa(\nu + \sigma)(1 - \gamma) + \rho_x(\rho + \rho_x)(1 + \gamma \nu \psi)}{-\sigma^{-1}\lambda + (1 - \gamma)\rho_x(\rho + \rho_x)} \bar{x}(t). \]

Thus negative oil supply shocks raise consumption at the ZLB, raise real GDP (labor input), raise inflation, and raise real oil prices.
A.10. OIL SHOCKS IN INCOMPLETE MARKETS

### A.10.5 Home Economy

In a complete market setting, home and foreign are affected symmetrically by the oil shock. Thus the allocation for the home country are the same as abroad, both in normal times,
\[ c_{CM}(t) = c^*(t), \quad \pi_{CM}(t) = \pi^*(t), \]
\[ o_{CM}(t) = o^*(t), \quad n_{CM}(t) = n^*(t), \]
and at the ZLB,
\[ c_{ZLB,CM}(t) = c^*,ZLB(t), \quad \pi_{ZLB,CM}(t) = \pi^*,ZLB(t), \]
\[ o_{ZLB,CM}(t) = o^*,ZLB(t), \quad n_{ZLB,CM}(t) = n^*,ZLB(t). \]

Note in particular, that these solutions are independent of the import share \( \alpha \).

When markets are incomplete I consider the case where \( \alpha \rightarrow 0 \) and \( \nu = 0 \). This significantly simplifies the computation of the new Pareto weight \( \theta = \log \Theta \). As in Farhi and Werning (2012a) the incomplete market allocation is the sum of two components – the complete market allocation and an additional term,
\[ c_{IM}(t) = c_{CM}(t) + \delta_{c}, \quad \pi_{IM}(t) = \pi_{CM}(t) + \delta_{\pi}, \]
\[ o_{IM}(t) = o_{CM}(t) + \delta_{o}, \quad n_{IM}(t) = n_{CM}(t) + \delta_{n}. \]

These additional \( \delta_{IM} \)-terms are independent of time because the home economy is forward looking and thus instantaneously adjusts according to the new wealth level.

With zero initial net financial assets, \( NFA(0) = 0 \), I use the evolution of the net financial asset position to solve for the new Pareto weight \( \theta \). Since the no-ZLB and ZLB region generate different consumption paths, the Pareto weight will also vary accordingly,
\[ \theta = \frac{\rho}{(\rho + \rho_x) [\lambda(\phi - 1) + \sigma(1 - \gamma)\rho_x(\rho + \rho_x)][1 + \sigma(\psi - 1) + (1 - \gamma)(\sigma\eta - 1)]} dB(0), \]

and,
\[ \theta_{ZLB} = \frac{\rho}{(\rho + \rho_x) [-\lambda + \sigma(1 - \gamma)\rho_x(\rho + \rho_x)][1 + \sigma(\psi - 1) + (1 - \gamma)(\sigma\eta - 1)]} dB(0). \]

Note that the Pareto weights may be of different signs if the oil share in production \( \gamma \) is sufficiently small. For instance, suppose that \( \psi < 1 \) so labor and oil are not very substitutable. In normal times a decline in oil supply raises oil prices, which increases import costs for domestic suppliers. In addition, the decline in foreign demand for home goods further deteriorates the home net foreign asset position. Thus, in normal times \( \theta < 0 \) as the foreign country accumulates net assets relative to the domestic
economy. At the ZLB, however, foreign consumption increases after a negative oil supply shock, which raises foreign demand for home goods. The increase in exports is larger than the increase in input costs if $\gamma$ is small, so the home country accumulates net foreign assets and $\theta > 0$. Thus, at the ZLB, a decline in foreign oil production does not necessarily transfer resources from the home country to the foreign country.

Given $\theta$, we can calculate the incomplete market component of the home allocation as follows

$$
\delta_{c}^{IM} = \gamma \theta, \quad \delta_{n}^{IM} = 0, \quad \delta_{o}^{IM} = -(1 - \gamma)\sigma \theta, \quad \delta_{n}^{IM} = -\gamma[(1 - \gamma)(\sigma \eta - 1) + \sigma \psi]\theta. \tag{A.20}
$$

We can now assess whether incomplete markets are sufficient to explain the estimated IRFs in Section 4.3.1. First, temporary negative supply shocks are expansionary in the complete market solution at the ZLB, $c_{ZLB,CM}(t) > 0$ and $n_{ZLB,CM}(t) > 0$. Since these expansionary effects are temporary, we have $c_{ZLB,IM}(t) > \delta_{c}^{ZLB,IM} = c_{ZLB,IM}(\infty)$ and $n_{ZLB,IM}(t) > \delta_{n}^{ZLB,IM} = n_{ZLB,IM}(\infty)$. Therefore, in the short-run a negative oil supply shock at the ZLB should be more expansionary than in the long-run. However, in Figure 4.2 we see the opposite pattern — negative supply shocks are more contractionary in the short-run. Intuitively, in the short-run the economy benefits from higher inflation and lower real interest rates, irrespective of the wealth transfers that matter for the long-run equilibrium. This very general result suggests that incomplete markets by themselves are not sufficient to account for the empirical findings. One still needs a model where the short-run effect of negative supply shocks is contractionary at the ZLB, such as the model in Section 4.5.

Furthermore, there are other aspects of this allocation that suggest that incomplete markets are not sufficient to match the data. First, irrespective of whether the home country becomes a net debtor or creditor, Equation (A.20) shows that consumption and GDP (labor input) move in opposite directions. For example, if wealth declines then consumption falls but real GDP rises to produce the exports necessary to pay down the debt. This contrasts with the empirical results where both consumption and labor input fall following a negative supply shock.

Second, in this allocation $\theta$ and $\theta_{ZLB}$ have opposite signs. Thus, if the incomplete market effects are strong we should see that negative supply shocks have qualitatively different effects in normal times than at the ZLB. However, in the data oil shocks are contractionary in both settings and only differ in their quantitative magnitude.

In summary, the results from this section show that incomplete markets by themselves cannot explain the contractionary effects of oil supply shocks at the ZLB and in normal times. While they may explain part of the contractionary effect, the credit friction is still necessary to match the data. Of course, it is possible that the credit friction and the incomplete market effects reinforce each other, thereby contributing to the contractionary effects of oil supply shocks at the ZLB. I leave the investigation of such interaction effects for further research.
A.1.11. MODEL SOLUTIONS

A.11 Model Solutions

This appendix examines the computational properties of the model. In the first subsection, I examine how closely Equation (4.13) captures the solution in the linearized model with three state variables. In the second subsection, I lay out the computational strategy for the non-linear model with one state variable and compute Euler equation residuals.

A.11.1 Accuracy of State Space Reduction

This section examines how closely equation (4.13) captures the solution to the three state variable model in Section 4.5. First, the model equation of the three-state-variable model and the one-state-variable model are linearized and solved using the Matlab code described in Sims (2002). This code is available for download on Chris Sims’ website. Appendix Figure A.19 displays the resulting impulse response functions of the standard New Keynesian model, the three state variable model, and the one state variable friction model to a 1% TFP shocks. Overall, the models display very similar behavior in the responses of output, consumption, inflation, and the banking variables. While the dynamics of the interest rate spread are somewhat different in the two models, what matters for a borrower’s consumption choice is the integral over the spread IRF. The integrals are very similar across the two models so there is little difference in consumption behavior of borrowers. Thus the one-state-variable model delivers essentially the same results, but greatly simplifies the non-linear computation.

Second, I solve the three-state-variable model using projection methods as outlined in the following section. Since the state space is significantly larger in this model I use polynomials up to degree 5 for each state variable, for a total of $5^3 = 125$ polynomials. While this grid is coarse, it can provide insight if the one-state-variable model also provides a good approximation to the three-state-variable model at the ZLB. In addition, in order to solve this model I have to raise the discount rate slightly to $\varrho = 0.01275$ (compared to $\varrho = 0.0125$ in the baseline) and lower the volatility to $\sigma_a = 0.0001$ (compared to $\sigma_a = 0.005$ in the baseline). In Appendix Figure A.20 I plot IRFs from a 1% positive technology shock without the ZLB. As in the linearized case, the IRFs of the one-state-variable model and the three-state-variable model are very close. Appendix Figure A.21 shows that equation (4.13) also provides a very good approximation at the ZLB. In particular, output expands by similar magnitudes in the one-state-variable model and the three-state-variable model.
A.11. MODEL SOLUTIONS

A.11.2 Accuracy of Non-linear Solution

The model is computed by iterating over the FOCs, while using Chebychev polynomials to approximate equilibrium objects. I use polynomials up to degree 33 in the computation. The initial guess is the (linear) solution to the standard New Keynesian model. Convergence is fast, taking about 20 seconds using the baseline calibration.

The residuals from the two Euler equations (lender and borrower) as well as from the Phillips Curve are plotted in Appendix Figure A.22. The errors in the Phillips Curve are of trivial magnitude ($10^{-14}$) and can be ignored. Similarly, the Euler equation errors ignoring the ZLB are very small ($10^{-13}$). The ZLB introduce larger errors because the approximation will not perfectly capture when the ZLB starts to bind. Thus, the errors are largest at the border of the ZLB and non-ZLB regions. However, the maximum errors are still small: at most 3.5 basis points in the standard model and 1.2 basis points in the friction model.
Figure A.19: IRFs of Standard NK model, Friction Model with Three State Variables, and Friction Model with One State Variable to a 1% TFP shock.

IRFs are calculated from linearized solution to each model.
Figure A.20: IRFs of Friction Model with Three State Variables and Friction Model with One State Variable to a 1% TFP shock assuming that ZLB does not bind.

IRFs are calculated from non-linear model using projection methods.
Figure A.21: IRFs of Friction Model with Three State Variables and Friction Model with One State Variable to a 1% TFP shock when the ZLB binds.

IRFs are calculated from non-linear model using projection methods.
Figure A.22: Residuals of dynamic equations in non-linear solution.

(a) Euler Equation: Lender

(b) Euler Equation: Borrower

(c) Phillips Curve
Appendix B

Appendix to Chapter 3

B.1 Complete Model

B.1.1 Households

Each country is populated by a representative household. The objective function for the household in the home country is given by,

$$\max_{\{C_t, N_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} \beta_s \right) \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi_t \frac{N_t^{1+\nu}}{1+\nu} \right],$$

where $C_t$ is domestic consumption and $N_t$ is domestic labour supply. The discount factor $\beta_t$ follows a stationary stochastic process with steady state value $\beta$. At time $t$, the household knows next period’s value of the stochastic discount factor, $\beta_{t+1}$, as well as its history, but it does not know any other future values with certainty. Discount factor shocks are a standard method to generate a liquidity trap (see Christiano, Eichenbaum, and Rebelo (2011) and Eggertsson and Woodford (2003)). $\sigma$ is the inverse of the elasticity of substitution and $\nu$ is the inverse of the Frisch elasticity of labour supply. Finally, $\chi_t > 0$ governs the steady state labour supply and also follows a stationary stochastic process. For the foreign country, I let $\chi^* = n^{-(\sigma+\nu)} \chi$, to make it produce $n$ times as much output than the home country. The home country becomes a small open economy as I take the limit $n \to \infty$.

The domestic household maximizes its objective (B.1), subject to its budget constraint each period,

$$P_tC_t + E_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_t N_t + T_t \quad \forall t = 0, 1, \ldots$$

(B.2)

where $P_t$ is the price of the domestic consumption good, $W_t$ is the wage rate for domestic labor and $T_t$ are net transfers from the government and firms. $D_{t+1}$ is a vector of payoffs from the portfolio held from time $t$ until $t+1$. I assume that financial
markets are complete, which implies that the stochastic discount factor $Q_{t,t+1}$ is the unique asset pricing kernel for the vector of payoffs $D_{t+1}$.

The optimality conditions for the domestic households are as follows. (For the foreign household there exists an analogous set of first order conditions.) First, the marginal utility of consumption multiplied by the real wage rate must equal the marginal disutility of labor,

$$C_t^{−σ} \frac{W_t}{P_t} = \chi_t N_t^\nu.$$  \hspace{1cm} (B.3)

Second, the household satisfies the Euler equation,

$$\beta_t R_{t+1} E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{−\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] = 1,$$ \hspace{1cm} (B.4)

where $R_{t+1}$ is the gross nominal interest rate on the domestic one-period riskless bond.

The asset pricing equations for the home and foreign safe bonds imply that the excess return of domestic over foreign bonds has a price of zero,

$$E_t \left[ Q_{t,t+1} \left( R_{t+1} - \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} R^\ast_{t+1} \right) \right] = 0.$$ \hspace{1cm} (B.5)

Here $R^\ast_{t+1}$ is the gross nominal interest rate on the foreign bond and $\mathcal{E}_t$ is the nominal exchange rate between the home and the foreign currency. The latter is defined as the quantity of domestic currency for each unit of foreign currency, so that a fall in $\mathcal{E}_t$ corresponds to an appreciation of the domestic currency. Equation B.5 will become an uncovered interest rate parity (UIP) condition in the log-linearized small open economy.

The household in the foreign country also satisfied an Euler equation,

$$\beta_t R^\ast_{t+1} E_t \left[ \left( \frac{C^\ast_{t+1}}{C^\ast_t} \right)^{−\sigma} \left( \frac{P^\ast_t}{P^\ast_{t+1}} \right) \right] = 1,$$ \hspace{1cm} (B.6)

where $C^\ast_t$ is foreign consumption, and $P^\ast_t$ the foreign price level. Foreign consumption $C^\ast_t$ is exogenous with respect to the domestic economy, due to the large size of the foreign country.

**B.1.2 Final Goods Firms**

The production structure in the economy is as follows. In each country there is a continuum of firms that produce intermediate goods with local labor inputs. These intermediate goods are then assembled by final goods firms into the final consumption good.
For both countries, there are two types of perfectly competitive firms. In the home country, one set of firms produces an aggregate $C^H_t$ by combining a continuum of home-produced varieties $C^H_t(j)$, $C^H_t = \left( \int_0^1 C^H_t(j) \varepsilon^{-1} dj \right)^{\frac{1}{\varepsilon}}$, where $\varepsilon$ is the elasticity of substitution between different domestically-produced varieties. The firms’ cost minimization problem implies that home-produced varieties are demanded according to the standard Dixit-Stiglitz demands,

$$C^H_t(j) = \left( \frac{P^H_t(j)}{P^H_t} \right)^{-\varepsilon} C^H_t,$$

where $P^H_t(j)$ is the price of a given variety and $P^H_t = \left( \int_0^1 P^H_t(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the price of the aggregate $C^H_t$. Analogous equations apply to the set of firms in the foreign country that produce $C^F_t$.

The remaining domestic final good firms then combine the aggregates $C^H_t$ and $C^F_t$ into the desired consumption good. In particular, for the home country,

$$C_t = \left[ (1 - \gamma)^{\frac{1}{\eta}}(C^H_t)^{\frac{n-1}{\eta}} + \gamma^{\frac{1}{\eta}}(C^F_t)^{\frac{n-1}{\eta}} \right]^{\frac{\eta}{n-\eta}},$$

where $\gamma$ is the share of imports in aggregate home consumption and $\eta$ the elasticity of substitution between home and foreign goods. I define the degree of openness of an economy to be equal to $\gamma$. Intuitively, when $\gamma = 0$ the home country will never import any goods and it effectively becomes a closed economy. When $\gamma = 1$, then the economy only consumes foreign goods, so it will have to export all its production to finance this consumption. Since the gross amount of trade increases in $\gamma$, I will treat this parameter as measuring the degree of “openness.” Economies with higher values for $\gamma$ are more “open” because they will trade more goods.

Given this particular final consumption good, the optimal choice between the home produced composite and the foreign goods satisfy the standard Dixit-Stiglitz demand equations, so that the price of domestic consumption is given by $P_t = \left[ (1 - \gamma)(P^H_t)^{1-\eta} + \gamma(P^F_t)^{1-\eta} \right]^{\frac{1}{1-\eta}}$, where $P^F_t$ is the domestic price of the foreign consumption aggregate $C^F_t$.

To ensure that larger foreign production does not depress the relative price of foreign goods, I assume that the foreign country consumes primarily its own goods. Thus, I let share of imports of domestic output in foreign consumption be decreasing in $n$, $\psi = \gamma/n$. This ensures that the terms of trade in the steady state equal 1. In the small open economy case, where $n \to \infty$, the share of home goods in foreign consumption approaches zero, $\psi \to 0$.

\footnote{In this case the home economy is equivalent to the model in Christiano, Eichenbaum, and Rebelo (2011).}
B.1. COMPLETE MODEL

B.1.3 Intermediate Goods Firms

In each country there is a continuum of local firms, producing a differentiated product using local labor. In the home country, these firms produce the varieties \( Y_t^H(j) \) according to the production function,

\[
Y_t^H(j) = A_t N_t(j),
\]

where \( N_t(j) \) is domestic labor input into the production of good \( j \) and \( A_t \) is aggregate technology, which I assume to be stationary. Each firm sets its prices on a staggered basis: With probability \( \theta \) its current price remains fixed at last period's price and with probability \( 1 - \theta \) it is optimally reset. The profit maximization problem of firm \( i \) is therefore,

\[
\max_{P_t^H(i)} E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_t^H(i) - (1 - \tau) W_{t+j}(i) \right] Y_{t+j}(i),
\]

where \( \tau = \varepsilon^{-1} \) is an employment subsidy designed to offset the inefficiently low output from monopolistic competition. \( Q_{t,t+j} \) is the stochastic discount factor of domestic residents.

I assume that labor is perfectly substitutable between sectors, which implies that the wage paid in each sector, \( W_t^H(j) \), must be the same in equilibrium. I define aggregate domestic output analogously to consumption \( Y_t^H = \left( \int_0^1 Y_t^H(j) \varepsilon^{-1} dj \right)^{\varepsilon^{-1}} \).

B.1.4 Government

Similar to the Dornbusch model, I assume that the government spends money on domestically produced goods only. In particular, government spending is the same composite good as \( C_t^H \),

\[
G_t = \left( \int_0^1 G_t(j) \varepsilon^{-1} dj \right)^{\varepsilon^{-1}}. \]

Furthermore, I assume that the government chooses among varieties to maximize the aggregator \( G_t \). This assumption greatly simplifies the derivation of the New Keynesian Phillips Curve.

Government purchases are financed by lump-sum taxation and Ricardian equivalence holds in this economy. Subject to the no-Ponzi scheme condition, the government must satisfy the budget constraint,

\[
B_0 = \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} \frac{1}{R_s} \right) G_t - \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} \frac{1}{R_s} \right) T_t, \tag{B.8}
\]

where \( T_t \) are lump-sum taxes and \( B_0 \) are initial government assets. The steady-state share of government spending in output is given by \( g = \bar{G}/\bar{Y} \).
B.1. COMPLETE MODEL

The central bank conducts monetary policy according to a simple Taylor rule, subject to the zero interest rate floor,

\[ R_{t+1} = \max \left\{ \beta^{-1} E_t \left( \frac{P^H_t}{P^H_{t-1}} \right)^{\phi_\pi}, 1 \right\}, \quad (B.9) \]

where \( \phi_\pi > 1 \) to satisfy the Taylor principle. Note that the central bank is responding to the inflation of domestically produced goods (the PPI), not the inflation rate of consumption goods (the CPI).

B.1.5 Prices and Market Clearing

The terms of trade are defined as the ratio between import prices (in domestic currency) and export prices,

\[ S_t = \frac{P^F_t}{P^H_t}. \quad (B.10) \]

I assume that the law of one price holds among individual goods. This defines the nominal exchange rate as ratio of the domestic currency price of good \( j \) to the foreign currency price of good \( j \),

\[ E_t = \frac{P^H_t(j)}{P^H^*(j)} = \frac{P^F_t}{P^F^*}. \quad (B.11) \]

The Backus-Smith condition (Backus and Smith (1993)) in this model is subject to a reduced form friction \( F_t \). Thus, consumption at home relative to abroad is proportional to the ratio of the real exchange rate \( \lambda_t \) and the friction \( F_t \),

\[ \left( \frac{C_t^i}{C_t} \right)^{-\sigma} = \frac{\lambda_t}{F_t}. \quad (B.12) \]

In equilibrium the market clearing conditions for home goods,

\[ Y_t(j) = C_t^H(j) + C_t^{H^*}(i) + G_t(j) \quad \forall \, j, \quad (B.13) \]

as well as for foreign goods have to be satisfied. The definition of the equilibrium in this model is then as follows.

**Definition 8 (Equilibrium)** The equilibrium is a sequence,

\[ \{C_t^H(i), C_t^F, N_t(i), Y_t(i), D_t, G_t(i), T_t, W_t^H, P_t^H(i), P_t^{H^*}, P_t^F, P_t^*, R_{t+1}, R_t^*, S_t, E_t\}_{t=0}^\infty, \]

such that for given \( \{C^*_t, G_t, \beta_t, \chi_t, A_t\}_{t=0}^\infty \) and given initial assets \( \{D_0, B_0\} \) the maximization problems for the domestic and foreign households and firms are satisfied;
the government satisfies its budget constraint (B.8) and allocates spending to maximize \( G_t \); the domestic nominal interest rate follows the Taylor rule (B.9); the foreign real interest rate is determined by the foreign Euler equation (B.6); the Backus-Smith condition (B.12) is satisfied; domestic and foreign prices satisfy the law of one price (B.11); and the market clearing conditions for the home and foreign goods are satisfied.

### B.2 Backus-Smith with Wedge: An Example

Following Bodenstein (2008), consider a two country economy with limited enforcement of international contracts. In particular, they can only be enforced through threat of exclusion from asset markets in the future. Let \( V_t \) denote the home countries value from financial autarky. Then the incentive compatibility constraint must satisfy,

\[
E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}) \geq V_t \quad \forall t
\]

Denote the Lagrange multiplier on this constraint by \( \mu_t \). Then the objective function of the home agent can be rewritten as

\[
\begin{align*}
E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + \{ \mu_t E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}) - V_t \} \right] &= E_0 \sum_{t=0}^{\infty} \beta^t [M_t u(c_t) + \mu_t (u(c_{t+s}) - V_t)]
\end{align*}
\]

where \( M_{t+1} = M_t + \mu_t \) and \( M_0 = 1 \). The first order conditions for the domestic household are,

\[
\lambda_t P_t = [M_t + \mu_t] u'(c_t)
\]

where \( P_t \) is the price of consumption, and

\[
Q_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}
\]

is the home agents SDF. Absent arbitrage opportunities, the home and foreign SDF are equal, which implies (given symmetric initial conditions)

\[
\frac{u'(c_t^*)}{u'(c_t)} = \lambda_t \frac{M_{t+1}}{M_{t+1}^*}
\]

where \( \lambda_t \) is the real exchange rate and \( M_{t+1}^* \) analogously defined for the foreign country. Log-linearizing this equation yields,

\[
\sigma(\hat{c}_t - \hat{c}_t^*) = \hat{\lambda}_t - \hat{f}_t
\]

where \( \hat{f}_t = \hat{m}_{t+1} - \hat{m}_{t+1}^* \), which is the expression (3.1) in the baseline model.
B.3 Solution Method for Baseline Model

The zero lower bound on nominal interest rates makes this model piecewise linear. In particular, let \( x_t = \left( \hat{y}_t \; \hat{c}_t \; \hat{\pi}_t \; \hat{s}_t \; \hat{r}_{t+1} \right)' \) be the vector of endogenous variables and \( \upsilon_t = \left( \hat{g}_t \; \hat{\beta}_t \; \hat{c}_t^* \right)' \) be the vector of exogenous variables. So long as the liquidity trap does not bind, we can write the system of equations as
\[
Ax_t = BE_t x_{t+1} + F \upsilon_t. \tag{B.14}
\]

Given the stochastic process in Example 1, its unique solution when the zero lower bound does not bind is \( x_t = D \upsilon_t \), where \( D = \left( I - A^{-1} B \text{diag}(p) \right)^{-1} A^{-1} F \). Because there are no endogenous state variables in this model, a shock in period \( t \) causes an immediate jump in \( x_t \) to the new equilibrium. If at \( t + 1 \) the shock disappears then the economy jumps back to the steady state. On the other hand, if the shock remains at the same value, then \( x_{t+1} = x_t \). In particular, the fiscal multiplier outside the liquidity trap is the \( D_{11} \) element in the matrix \( D \) scaled by \( g^{-1} \), the inverse of the government’s share in output. Because the system of equations is linear, this multiplier is independent of the size of government spending.

The liquidity trap is somewhat more difficult to handle. When the economy is at the zero lower bound at time \( t \), the system of equations may be written as follows,
\[
A^* x_t = BE_t x_{t+1} + C^* + F \upsilon_t, \tag{B.15}
\]
where \( A^* \) is the same matrix as \( A \) except that the \( A^*_{53} \) entry is equal to zero whereas \( A_{53} = -\phi \). This reflects that in the liquidity trap, the nominal interest rate is bounded at zero and does respond to changes in the inflation rate. Instead, the log-linearized nominal interest rate is equal to \( -\bar{r} \), which is captured in the \( C^*_{51} \) element of the vector \( C^* \). All other elements of this vector are zero.

If the law of motion is given by equation (B.15) for all \( t = 0, 1, ... \), then there is no unique solution to this system of equations because the economy violates the Taylor principle. To avoid this case I require that the economy must ultimately exit the liquidity trap with probability 1. This will imply that in the limit, the law of motion must satisfy equation (B.14) (rather than (B.15)), which we know has a unique solution. Given the stochastic process I have assumed, \( \lim_{j \to \infty} \text{Prob} [ \upsilon_{t+j} = 0 | \upsilon_t \neq 0 ] = 1 \). We therefore know that, in the limit, any shock that causes the zero bound to bind will disappear with probability one. Since we exit the liquidity trap with probability 1, the Taylor principle will hold asymptotically and there will be a unique solution to this problem.
B.4. LARGE OPEN ECONOMY

Now suppose that at time $t$, the shocks $\nu_t$ are such that the zero lower bound binds. Suppose the endogenous variables take on the value $x_t$. We then know that with probability $p$, $\nu_{t+1} = \nu_t$. In this case, the economy at $t+1$ looks just like the economy at $t$. Thus, $x_{t+1} = x_t$ and the liquidity trap will remain binding at $t+1$. With probability $1 - p$, $\nu_{t+1} = 0$. Since the economy exits the liquidity trap, the solution is simply given by (A.1), $x_t = 0$. Quite intuitively, without shocks the economy jumps back to the steady state. Using this logic, I solve equation (B.14) forward, to obtain,

$$x_t = E^* + D^* \nu_t,$$

where $D^* = (I - A^*^{-1}B\text{diag}(p))^{-1}A^* F$ and $E^* = (I - A^*^{-1}B\text{diag}(p))^{-1}A^* C^*$.

The fiscal multiplier in the liquidity trap is now the $D^*_{11}$ element in the matrix $D$ scaled by $g^{-1}$, the inverse of the government’s share in output. Note however, that here fiscal policy is conditional on the liquidity trap. When the economy exits the liquidity trap, then the entire vector $\nu_t$ becomes zero, including the government spending shock. Furthermore, government spending must be small enough such that the economy does not exit the liquidity trap. Theoretically at least, this is defensible because it cleanly isolates fiscal policy in the liquidity trap.

B.4 Large Open Economy

In this section I derive fiscal multipliers for the frictionless model in a large open economy. Let $\psi = \gamma/n$ be the import share of the foreign country, where $n$ is the relative size of the foreign country. Define the function

$$z(\phi) = \frac{\kappa \nu (\phi - p)}{(1 - \beta p)(1 - p) + \kappa (\phi - p)}.$$  \hspace{1cm} (B.16)

Then the fiscal multiplier in the open economy is given by

$$\mu_n = \frac{(1 - p)(1 - \beta p) + \kappa (\phi - p)}{(1 - p)(1 - \beta p) + \kappa (\phi - p) \left\{ 1 + (1 - s_g) \frac{\psi}{\sigma} [1 + \gamma(2 - \gamma - \psi)(\sigma \eta - 1)] \right\} - \gamma \psi \Delta_n},$$  \hspace{1cm} (B.17)

where the new term in the denominator is defined as,

$$\Delta_n = \frac{z(\phi) z(\phi^*) [(1 - \beta p)(1 - p) + \kappa (\phi - p)]^{\frac{1 - s_g}{\sigma} (2 - \gamma - \psi)(\sigma \eta - 1)}^2}{1 + z(\phi^*) \frac{1 - s_g}{\sigma} [1 + \psi(2 - \gamma - \psi)(\sigma \eta - 1)]}.$$  \hspace{1cm} (B.18)

When interest rates are positive in both countries, $z(\phi) > 0$ and $z(\phi^*) > 0$, so that $\Delta_n > 0$. This implies that the fiscal multiplier is larger in the large open economy than in the small open economy. Intuitively, the improvement in the terms of trade.
B.5. THE NATURE OF INFLATION SURPRISES

(due to domestic fiscal expansion) raises demand for foreign products and thus foreign inflation. In response, the foreign central bank raises nominal interest rates such that real rates rise, which mitigates the improvement in the terms of trade. As a result, there is less crowding out of domestic production and the multiplier is higher. This is illustrated in Figure B.2(a).

Figure B.2(a) also shows that the fiscal multiplier is smaller when the home country is unconstrained but the foreign country is at the ZLB. The logic follows from the preceding argument, except that the foreign central bank does not raise nominal interest rates. Consequently, foreign real rates will fall which further amplifies the improvement in the terms of trade and crowds out domestic production.

Figure B.2(b) illustrates the case when both countries are at the ZLB. In this case, $z(\phi) < 0$ and $z(\phi^*) < 0$, so that $\Delta_n > 0$, and the fiscal multiplier is larger than in the baseline model. The logic is similar to above, except that at the ZLB a fiscal expansion generates a deterioration in the terms of trade and thus deflation in the foreign country. If the foreign country is at the ZLB, then its real rates rise which precipitates a further deterioration in the terms of trade and amplifies the fiscal multiplier. Only if the foreign country is unconstrained will domestic real rates fall and cushion the deterioration in the terms of trade, which will result in a smaller fiscal multiplier.

Note that for reasonable parameters the spillovers are small. Intuitively, $\Delta_n$ is multiplied by $\gamma \psi$ which is very small for reasonable import shares and country sizes. Figure B.1 illustrates this for the baseline calibration when $n = 3$ and $\gamma = 0.15$, which is reasonable for the US. (European parameters generate even smaller spillovers given their smaller country size.)

B.5 The Nature of Inflation Surprises

To investigate the nature of inflation surprises, I first test whether they add predictive power to a simple autoregressive model. In particular, denote the forecasted value for inflation by $\hat{\pi}^f_t$, the announced value by $\hat{\pi}^a_t$, and the revised value for inflation by $\hat{\pi}^r_t$. The inflation surprise is then defined as $\hat{\pi}^s_t = \hat{\pi}^a_t - \hat{\pi}^f_t$. In Figure B.2 I plot the residuals from a regression of revised inflation, $\hat{\pi}^r_t$, on 24 of its own lagged values against the residuals from a regression of inflation surprises $\hat{\pi}^s_t$ on 24 lagged values of revised inflation. If inflation surprises added all the missing information to make a perfect forecast for inflation then these residuals should be perfectly correlated, i.e. line up on the 45 degree line. If however, inflation surprises are contaminated by measurement error, then there will be an attenuation bias, which will reduce the correlation. I find that the correlation between the residuals are 0.71 for headline

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2Not all countries in this sample revise initial CPI releases. For these countries, $\hat{\pi}^r_t = \hat{\pi}^a_t$.

3The regressions also include country and time fixed effects.
B.5. THE NATURE OF INFLATION SURPRISES

Figure B.1: Domestic fiscal multipliers

(a) Domestic Fiscal Multiplier: Normal Times

(b) Domestic Fiscal Multiplier: ZLB

In normal times (left panel) and at the ZLB (right panel). Both panels show results from (a) the baseline model (small open economy), (b) when the foreign country is unconstrained and (c) when the foreign country it is at the ZLB. The import share \( \gamma \) is set to 0.15.
inflation and 0.65 for core inflation, which is consistent with the notion that inflation surprises contains news about macroeconomic conditions, and are only somewhat contaminated by measurement error.

Headline inflation surprises over this sub-sample are somewhat predictable based on past levels of (revised) headline inflation: The p-value on the Wald exclusion test of 24 lags of headline inflation is 0.036. While this may reflect some information in revised headline inflation that wasn’t available at the time the forecast is made, I make the conservative choice and purge headline inflation surprises of this correlation. In addition, I control for time and country fixed effects from both headline and core inflation surprises to derive a series of “true,” uncorrelated surprises, which I denote $\tilde{\pi}_t^s$. The correlation between the true inflation surprises, $\tilde{\pi}_t^s$, and the original inflation surprise series, $\hat{\pi}_t^s$, is 0.85 for both headline inflation and core inflation, which suggests that the economic difference between these two series is small. Indeed, all results reported in this section are quantitatively robust to using the original inflation surprise series.\footnote{For headline inflation surprises, the 95% confidence intervals will be larger, but the effects described below are still significant at the 5% level. For core inflation surprises the results reported below strengthen in both magnitude and significance.}

To determine the likely source of inflation surprises, I construct impulse response functions for inflation, unemployment, and central bank nominal interest rates, given a 1% point true inflation surprise. I first regressed true inflation surprises on 24 lags of past inflation, true inflation surprises, unemployment, and central bank rates, and I could not reject that true inflation surprises are unpredictable at the 5% significance level. This is consistent with true inflation surprises being exogenous with respect to past realizations these variables, which allows me to construct IRFs by regressing the outcome variables on 24 lags of true inflation surprises, controlling for country and time fixed effects,

$$y_{i,t} = \gamma_i + \delta_t + \sum_{k=0}^{24} \beta_k \tilde{\pi}_{i,t-k}^s + e_{i,t}, \quad (B.19)$$

and then plotting the coefficients $\{\beta_k\}_{k=0}^{24}$ and associated HAC two-standard-error bands in Figure B.3. The impulse responses for inflation display a significant increase in inflation for at least a year, and appear to follow an AR(1) process.

Unemployment is also below average after the inflation surprise occurred. The peak response is a 0.5% points decline in unemployment after a 1% point headline inflation surprise and a 1.5% point decline after a 1% point core inflation surprise. This suggests that the source of the inflation surprise is in fact an unexpected demand shock. While the effect may seem large, it requires a large real shock to generate a 1% point inflation surprise, given the flatness of the Phillips curve in recent times.\footnote{See e.g. Altig, Christiano, Eichenbaum, and Lindé (2010) and Hall (2011).}
Figure B.2: Predictive Power of Inflation Surprises

Correlation between residuals from a regression of inflation, $\hat{\pi}_t^r$, on 24 of its lagged values and residuals from a regression of inflation surprises, $\hat{\pi}_t^s = \hat{\pi}_t^a - \hat{\pi}_t^f$, on 24 lagged values of inflation, $\hat{\pi}_t^r$. The left panel shows the correlation when headline inflation is used, the right panel when core inflation is used.
B.5. THE NATURE OF INFLATION SURPRISES

Figure B.3: Impulse Response Functions for inflation, unemployment, and central bank nominal interest rates.

(a) Headline Infl. Surprise → Headline Inflation
(b) Core Infl. Surprise → Core Inflation
(c) Headline Infl. Surprise → Unemployment
(d) Core Infl. Surprise → Unemployment
(e) Headline Infl. Surprise → Nom. Policy Rates
(f) Core Infl. Surprise → Nom. Policy Rates

Left side shows impulses for headline inflation surprises, right side shows impulses for core inflation surprises.
The increase of central bank policy rates after inflation surprises further supports the demand shock hypothesis, since unemployment falls even though the central bank tightens monetary policy.\(^6\) I therefore conclude, that the evidence is consistent with inflation surprises being caused by demand shocks, a novel finding to the best of my knowledge.

### B.6 Supplemental Tables for Empirical Section

Table B.1: Price Indices used for Inflation Surprises

<table>
<thead>
<tr>
<th>Economy</th>
<th>Headline</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>CPI Headline NSA</td>
<td>CPI Core NSA</td>
</tr>
<tr>
<td>UK</td>
<td>CPI EU Harmonized NSA</td>
<td>CPI Ex Energy Food Alcohol &amp; Tobacco NSA</td>
</tr>
<tr>
<td>EUZ</td>
<td>Eurozone MUICP All Items NSA</td>
<td>Eurozone MUICP Core NSA</td>
</tr>
<tr>
<td>JAP</td>
<td>CPI Nationwide</td>
<td>CPI Nationwide Ex Fresh Food</td>
</tr>
<tr>
<td>CAN</td>
<td>STCA Canada CPI NSA</td>
<td>STCA Canada CPI Ex the 8 Most Volatile Components and Indirect Taxes NSA</td>
</tr>
<tr>
<td>SWE</td>
<td>Sweden CPI Headline</td>
<td>-</td>
</tr>
<tr>
<td>NOR</td>
<td>Norway CPI</td>
<td>Norway CPI Underlying (ATE)</td>
</tr>
<tr>
<td>CHE</td>
<td>Switzerland CPI</td>
<td>-</td>
</tr>
</tbody>
</table>

### B.7 Taylor Rule Estimation

I estimate the following quarterly regression model for each country in my sample (except for Japan) via OLS:

\[
r_t^* = \alpha + \rho r_{t-1}^* + \phi_\pi \pi_{t+1}^e + \phi_{gy} \Delta y_{t+1}^e + \phi_x x_t^e + \varepsilon_t.
\]

\(^6\)Estimation of the central bank response excludes periods marked as liquidity trap.
### B.7. TAYLOR RULE ESTIMATION

**Table B.2: Inflation Surprises (YoY)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled</td>
<td>-0.003</td>
<td>0.199</td>
<td>-0.9</td>
<td>1</td>
<td>925</td>
</tr>
<tr>
<td>USA</td>
<td>0.021</td>
<td>0.198</td>
<td>-0.4</td>
<td>0.5</td>
<td>84</td>
</tr>
<tr>
<td>UK</td>
<td>0.032</td>
<td>0.181</td>
<td>-0.4</td>
<td>0.6</td>
<td>80</td>
</tr>
<tr>
<td>EUZ</td>
<td>-0.002</td>
<td>0.061</td>
<td>-0.1</td>
<td>0.2</td>
<td>110</td>
</tr>
<tr>
<td>EUZ (flash)</td>
<td>-0.004</td>
<td>0.109</td>
<td>-0.3</td>
<td>0.3</td>
<td>95</td>
</tr>
<tr>
<td>JAP</td>
<td>0.012</td>
<td>0.107</td>
<td>-0.3</td>
<td>0.2</td>
<td>99</td>
</tr>
<tr>
<td>CAN</td>
<td>-0.01</td>
<td>0.242</td>
<td>-0.75</td>
<td>0.6</td>
<td>120</td>
</tr>
<tr>
<td>CHE</td>
<td>-0.04</td>
<td>0.213</td>
<td>-0.55</td>
<td>0.6</td>
<td>120</td>
</tr>
<tr>
<td>SWE</td>
<td>-0.016</td>
<td>0.21</td>
<td>-0.8</td>
<td>0.5</td>
<td>120</td>
</tr>
<tr>
<td>NOR</td>
<td>0</td>
<td>0.325</td>
<td>-0.9</td>
<td>1</td>
<td>97</td>
</tr>
</tbody>
</table>

**Table B.3: Core Inflation Surprises (YoY)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled</td>
<td>-0.014</td>
<td>0.178</td>
<td>-0.8</td>
<td>0.9</td>
<td>476</td>
</tr>
<tr>
<td>USA</td>
<td>-0.06</td>
<td>0.193</td>
<td>-0.8</td>
<td>0.3</td>
<td>84</td>
</tr>
<tr>
<td>UK</td>
<td>-0.005</td>
<td>0.184</td>
<td>-0.3</td>
<td>0.55</td>
<td>53</td>
</tr>
<tr>
<td>EUZ</td>
<td>-0.032</td>
<td>0.123</td>
<td>-0.4</td>
<td>0.2</td>
<td>61</td>
</tr>
<tr>
<td>JAP</td>
<td>0.016</td>
<td>0.12</td>
<td>-0.3</td>
<td>0.9</td>
<td>99</td>
</tr>
<tr>
<td>CAN</td>
<td>0.01</td>
<td>0.191</td>
<td>-0.5</td>
<td>0.775</td>
<td>97</td>
</tr>
<tr>
<td>NOR</td>
<td>-0.023</td>
<td>0.223</td>
<td>-0.55</td>
<td>0.5</td>
<td>82</td>
</tr>
</tbody>
</table>
Here $\pi^e_t$, $\Delta y_t^e$, and $x_t^e$ are real time forecasts on year-on-year inflation, output growth and the output gap respectively. The former two data series are from Consensus Economics, the latter from the OECD. The OECD output gap data is in real time only from 2003 onwards. Since the purpose of these equation is to forecast it is permissible to use the OLS estimates.

While the baseline model did not feature interest rate smoothing or a response to the output gap, the relationship between the fiscal multiplier and the terms of trade response in the liquidity trap is independent of the monetary policy rule. Thus my identification strategy for the fiscal multiplier in the liquidity trap is unaffected by the switch to the above rule, which better matches the data (see Coibion and Gorodnichenko (2011b)).

I estimate this equation using data from the first quarter in 1992, up until the fourth quarter of 2007 (if available). The results are tabulated in Appendix Table B.6. I then generate dynamic forecast up until the first quarter of 2010, which I plot in Appendix Figure B.4. I then set the liquidity trap dummy equal to 1 if

1. based on historical Central Bank behavior, the predicted unconstrained interest rate at $t$, $r^*_t$, is below zero, $r^*_t < 0$.
2. the policy rate at $t$ is the minimum among all observed policy rates in the sample $r_t = \min(\{r_s\}_{s=1}^T)$.

The additional second condition is necessary, because it rules out that interest rates
will be subsequently cut or were previously at a lower level, which violates the premise of a lower bound on interest rates. The episodes that are thus classified as liquidity traps are tabulated in Appendix Table B.5.

Table B.5: Episodes Classified as Liquidity Traps

<table>
<thead>
<tr>
<th>Economy</th>
<th>In the Liquidity Trap</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>Jan 2009 - Jan 2010</td>
</tr>
<tr>
<td>UK</td>
<td>April 2009 - Jan 2010</td>
</tr>
<tr>
<td>EUZ</td>
<td>April 2009 - Jan 2010</td>
</tr>
<tr>
<td>JAP</td>
<td>Dec 2008 - Jan 2010</td>
</tr>
<tr>
<td></td>
<td>Beginning of Sample - June 2006</td>
</tr>
<tr>
<td>CAN</td>
<td>May 2009 - Jan 2010</td>
</tr>
<tr>
<td>CHE</td>
<td>April 2009 - Jan 2010</td>
</tr>
<tr>
<td>SWE</td>
<td>July 2009 - Jan 2010</td>
</tr>
<tr>
<td>NOR</td>
<td>-</td>
</tr>
</tbody>
</table>
### Table B.6: Taylor rule estimates

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>0.931***</td>
<td>0.973***</td>
<td>0.846***</td>
<td>0.914***</td>
<td>0.905***</td>
<td>0.938***</td>
<td>0.958***</td>
</tr>
<tr>
<td>(0.0410)</td>
<td>(0.0404)</td>
<td>(0.0808)</td>
<td>(0.0510)</td>
<td>(0.0397)</td>
<td>(0.0373)</td>
<td>(0.0764)</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.275***</td>
<td>0.150</td>
<td>0.175</td>
<td>-0.0717</td>
<td>-0.0585</td>
<td>0.303***</td>
<td>0.239**</td>
</tr>
<tr>
<td>(0.0813)</td>
<td>(0.110)</td>
<td>(0.187)</td>
<td>(0.141)</td>
<td>(0.0895)</td>
<td>(0.0555)</td>
<td>(0.117)</td>
<td></td>
</tr>
<tr>
<td>EUZ</td>
<td>0.234***</td>
<td>0.302***</td>
<td>0.349***</td>
<td>0.299***</td>
<td>0.425***</td>
<td>0.222***</td>
<td>0.107</td>
</tr>
<tr>
<td>(0.0642)</td>
<td>(0.0701)</td>
<td>(0.0831)</td>
<td>(0.102)</td>
<td>(0.0716)</td>
<td>(0.0504)</td>
<td>(0.190)</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.119**</td>
<td>0.0861</td>
<td>0.0229</td>
<td>0.0502</td>
<td>-0.0729</td>
<td>0.0117</td>
<td>0.156**</td>
</tr>
<tr>
<td>(0.0463)</td>
<td>(0.0526)</td>
<td>(0.0812)</td>
<td>(0.0730)</td>
<td>(0.0429)</td>
<td>(0.0278)</td>
<td>(0.0698)</td>
<td></td>
</tr>
<tr>
<td>CHE</td>
<td>-1.114***</td>
<td>-0.834***</td>
<td>-0.587</td>
<td>-0.369</td>
<td>-0.571***</td>
<td>-0.904***</td>
<td>-0.760</td>
</tr>
<tr>
<td>(0.275)</td>
<td>(0.265)</td>
<td>(0.388)</td>
<td>(0.526)</td>
<td>(0.150)</td>
<td>(0.201)</td>
<td>(0.840)</td>
<td></td>
</tr>
<tr>
<td>SWE</td>
<td>0.968</td>
<td>0.970</td>
<td>0.969</td>
<td>0.819</td>
<td>0.967</td>
<td>0.978</td>
<td>0.932</td>
</tr>
<tr>
<td>NOR</td>
<td>0.968</td>
<td>0.970</td>
<td>0.969</td>
<td>0.819</td>
<td>0.967</td>
<td>0.978</td>
<td>0.932</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Figure B.4: Predicted Policy Rates from Taylor Rule Estimates

(a) USA

(b) UK

(c) EUZ

(d) CAN

(e) CHE

(f) SWE