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Pulsed Radiation by a Phased Semi-Infinite Periodic Planar Array of Dipoles
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I. Introduction
To gain an understanding of the sparsely explored time domain (TD) behavior of periodic arrays of radiating or scattering elements (phased array antennas, frequency selective surfaces and related applications), we have initiated a systematic investigation of relevant canonical TD dipole-excited Green’s functions (GF), which so far include those for infinite and truncated line periodic arrays [1], [2], as well as for infinite planar periodic arrays [3]. Such Green’s functions have been parameterized in terms of TD-Floquet waves (FW) of cylindrical [1] and planar type [3], and truncation-induced TD-FW-modulated diffractions [2].
Such waves on semi-infinite and finite square arrays of dipoles have been investigated in the frequency domain (FD) [4], and shown to be useful in practical array applications [5]. The present contribution extends our TD studies to an infinite periodic sequentially pulsed semi-infinite planar array. The phenomenology associated with truncated TD-FW and truncation-induced diffraction is explained in terms of instantaneous frequencies aided by asymptotic parameterization. Preliminary numerical results demonstrate the efficiency of the TD-FW algorithms.

II. Statement of the Problem
The geometry of the semi-infinite planar array of dipoles oriented along the \( \hat{z} \) direction and excited by transient currents in free space is shown in Fig.1a. The period of the array is \( d_x \) and \( d_z \) in the \( x \) and \( z \) directions, respectively. The \( \mathbf{E} \) field component is simply related to the \( \hat{z} \)-directed magnetic scalar potential \( \mathbf{A} \) which shall be used throughout. A caret \( \sim \) tags time-dependent quantities; bold face symbols define vector quantities; \( \mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z \) denote unit vectors along \( x, y, \) and \( z \), respectively. FD and TD quantities are related by the Fourier transform pair \( A(\omega) = \int_{-\infty}^{\infty} A(t)e^{-j\omega t} dt \), \( A(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega)e^{j\omega t} d\omega \). The phased array FD and TD dipole currents \( J(\omega) \) and \( J(t) \), respectively, are given by

\[
J(\omega) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(\mathbf{r}' - \mathbf{d}_m) \delta(\omega - \omega_{mn}) \left\{ e^{-j\omega \mathbf{d}_m' / c} \right\}
\]

In the \( m, n \)-dependent element current amplitudes multiplying the delta function in (1) the FD portions \( \omega_{mn} \mathbf{z}' / c \) and \( \omega_{mn} \mathbf{z} / c \) account for an assumed (linear) phase difference between adjacent elements in the \( x \) and \( z \) directions, respectively, and \( \eta_{mn} \) and \( \eta_{mn} / c \) denote interelement phase gradients normalized with respect to \( \omega \). The TD portion identifies sequentially pulsed dipole elements, with the element at \( (x', z') = (n\mathbf{d}_m, m\mathbf{d}_n) \) turned on at time \( t_{mn} = (\eta_{mn} \mathbf{d}_m + \eta_{mn} \mathbf{d}_n) / c \).

III. FD and TD Floquet Waves for the Infinite Array
A): Frequency Domain FW. Applying the infinite Poisson summation formula to the doubly-infinite sum over the radiation by each dipole we obtain the total field expressed as \( A^{FW}(r, \omega) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_{mn}^{FW}(r) e^{-j\omega \mathbf{d}_m' / c} \), where the FW \( A_{mn}^{FW}(r) = e^{-j\omega \mathbf{d}_m' / c} \). Here, \( \mathbf{k}_{mn}^{FW} = k_{x} \mathbf{d}_m + k_{y} \mathbf{d}_n + k_{z} \mathbf{d}_x \) denotes the total FW propagation vector, and \( r = x_m + y_n + z \) the distance to the observer. The spectral wavevectors
\( k_{pp} = \omega_{pp} / c + \alpha_p, \quad \alpha_p = 2\pi p / d_s, \quad h_{pp} = \omega_{pp} / c + \omega_q, \quad \omega_q = 2\pi q / d_s \) (2)
characterize FW propagation along \( x \) and \( z \), respectively, and \( h_{pp}(\omega) = (k^2 - k_{pp}^2 - k_{pp}^2)^{-1/2} \), where \( k = \omega / c \), with \( k \) the ambient wavenumber, \( c \) the ambient wave speed, and \( \Im \ h_{pp} \leq 0 \) on the top Riemann sheet. Floquet waves with transverse propagation constants \( h_{pp} < k \) or \( h_{pp} > k \), \( k_{pp} = (k_{pp}^2 + k_{pp}^2)^{-1/2} \), are propagating or evanescent, respectively, in the \( y \)-direction.

B: Time Domain FW. The inversion of the FD-FW is written as \( \tilde{A}_{pq}^{\text{FD}}(r,t) = \int_0^\infty \left\{ F(\omega) \exp(-j2\pi \omega \tau) d\omega \right\} \tilde{A}_{pq} \), in which \( F(\omega) \) accounts for the slowly varying amplitude terms and \( \tilde{A}_{pq} = \tilde{A}_{pq}(1,1) + \tilde{A}_{pq}(1,0) - \omega t \), with \( \tilde{h}_{pp} \), \( \tilde{h}_{pq} \), and \( \tilde{h}_{qq} \) functions of \( \omega \), accounts for all the \( \omega \) phase terms in the exponent. For \( p = q = 0 \), the phase is linearly dependent on \( \omega \) and the inverse Fourier transform is evaluated as \( \tilde{A}_{pq}^{\text{FD}}(r,t) \) [3]. The solutions [3] with \( \tilde{h}_{pq} = \tilde{h}_{pq}(1,1) / (1 - \eta^2) \), \( \tilde{h}_{pq} = \tilde{h}_{pq}(1,2) / (1 - \eta^2) \), \( \tilde{h}_{pq} = \tilde{h}_{pq}(2,2) / (1 - \eta^2) \), are real in the causal domain \( t > t_0 = (\eta_1 x + \eta_2 z) / c \), \( \eta_1 = \sqrt{1 - \eta^2} \) (3).

\( \tilde{A}_{pq}^{\text{FD}}(r,t) = \tilde{A}_{pq}(1,1) \exp(-j2\pi \omega t) d\omega \)

The unit step function \( U(t-t_0) = U(t-t_0) \) arises because real saddle point frequencies \( \tilde{h}_{pq} \) are restricted to \( t > t_0 \), \( t > t_0 \).

IV. FD and TD Floquet Waves for the Semi-Infinite Array

A) Truncation-Induced FD Diffracted Fields. As shown in [4], truncated FD-FW expressions are obtained by deformation of the relevant spectral integration path, followed by uniform asymptotics,

\[
A_{pq}^{\text{FD}}(r,\omega) = \sum_{\text{FD}} A_{pq}^{\text{FD}}(r,\omega) U(\tilde{A}_{pq}^{\text{FD}}(\omega) - \phi) + \sum_{\text{FD}} A_{pq}^{\text{FD}}(r,\omega)
\]
where \( A_{Fw}^{(r,w)} \) are the FWs, and \( A_{Fw}^{(r)} \) are the \( q \)-th cylindrical diffracted fields due to the truncation. The radial wavenumber \( k_{r}(d) = \sqrt{k^2 - k^2_{w}} \) with \( \Im k_{r}(d) < 0 \) implies radial exponential decay when \( |k| < |k_{w}| \) so that only a few \( A_{Fw}^{(r)} \) terms in (5) are necessary away from the truncation. \( \Phi_{P}^{B}(d) \) is the shadow boundary of the truncated FW, which, for propagating FWs, coincides with the FW propagation angle \( \Phi_{P}^{B}(d) = \cos^{-1}(k_{w}/k_{r}(d)) \); \( B(w) = \{ 1 - \exp\left[ i k_{r}(d) \cos \Phi_{P}^{B}(d) \right] \}^{-1} \); \( F(z) \) is the transition function of the Uniform Theory of Diffraction (UTD)\(^{[4]} \), with argument \( \Phi_{P}^{B}(d) = \sqrt{2k_{w}(d)(\cos \Phi - \cos \Phi_{P}^{B}(d))}/2 \) and \( \Phi = \cos^{-1}(z/c) \) is the observation angle measured from the truncation of the array, see Fig.1. Every FW in (5) is the same as that for the infinite array, except that its domain of existence is the region \( \Phi < \Phi_{P}^{B} \).

B: Truncation-Induced TD Diffracted Fields. To parameterize the truncated FW phenomenologies in terms of instantaneous frequencies and wavenumbers, we access the time domain through Fourier inversion of \( A_{Fw}^{(r,w)} \) in (5) which has been obtained by high-frequency asymptotics. This restricts the validity of the truncated TD solution to early observation times near the wavefronts. Each of the \( A_{Fw}^{(r,w)} \) and its corresponding diffracted field \( A_{r,w}^{(r)} \) has a particular arrival time, near which the TD asymptotics will be most accurate. Again we need to distinguish between the dispersive \( q \neq 0 \) and the nondispersive \( q = 0 \) cases. The asymptotic inversion procedure is analogous to that in \([5] \), but differs in detail. We shall only list the results. The "quasi nondispersive" \( q = 0 \) \((\alpha_{q} = 0)\) term is decomposed based on the relations \( k_{w}(d) = \omega/c(1 - \eta_{w}^{2})^{1/2} \) and \( B(w) = 1/2 + \sum \sin\Phi_{P}^{B}(d)/c(1 - \eta_{w}^{2})^{1/2} \). There-
Thus, since local instantaneous frequencies are equal, i.e., 
\[ \omega(t) = \omega_p(t) = \omega_q(t) \]
has a transitional behavior that compensates for the truncation of the py-th TD-FW, moving SB intercepts the stationary observer at \( t \) where the \( \omega \)-th TD diffracted field has a transitional behavior that compensates for the truncation of the \( \omega \)-th TD-FW, and restores total field continuity.

### V. Band Limited Pulse Excitation

When each dipole in (1) radiates a practically useful band-limited (BL) pulse \( G(t-(n_x \pm n_y)z) \), the corresponding band-limited TD-FW \( A_{\omega_p}^{BL} \) for \( \omega_p \neq 0 \) and TD-diffracted field \( A_{\omega_q}^{BL} \) for \( \omega_q \neq 0 \) can be evaluated by including the pulse spectrum \( G(\omega) \) in the impulsive inversion integral. For wideband (short duration) pulses, \( G(\omega) \) can be considered slowly varying with respect to the phases \( \phi(\omega) \) and \( \psi(\omega) \), and can therefore be approximated by its value at the saddle point frequencies \( \omega_{p(\omega)} \) and \( \omega_{q(\omega)} \) for FWs and diffracted fields, respectively. The asymptotic BL-TD fields \( A_{\omega_p}^{BL} \) and \( A_{\omega_q}^{BL} \) are found by multiplying the ordinary asymptotic \( A_{\omega_p}^{FW} \) and \( A_{\omega_q}^{FW} \) by \( G(\omega_{p(\omega)}) \) and \( G(\omega_{q(\omega)}) \), respectively. The FD-FW0 and \( q \neq 0 \) diffracted field are not invertible by \( \omega \)-asymptotics and are calculated by convolving \( G(t) \) with the TD-FW \( A_{\omega_p}^{FW} \) and \( A_{\omega_q}^{FW} \) in (1), respectively. Preliminary numerical experiments have been carried out to test the accuracy of the asymptotic solutions for \( A_{\omega_p}^{FW, BL} \) and to compare the results with a reference solution obtained by an element-by-element summation over the pulsed BL radiation from all dipoles, i.e., \( \hat{G}(t-(n_x \pm n_y)z) \to R_{nm} / [\phi(\omega_{p(\omega)})]^{-1} \), with \( R_{nm} = (n_x \pm n_y)^2 + (z-\lambda_{p(\omega)})^2 + (n_y \pm n_y)^2 \), \( n_x \neq 0 \), \( n_y \neq 0 \), \( n_y \neq 0 \) for all non-negligible element radiations. Figure 1b shows plots for a semi-infinite planar array with interelement phasing \( n_y = n_y = 0 \) (broadside radiation) and interelement spacing \( d_z = 10d_z \), in order to highlight the new phenomena (with respect to the truncation in [2]) due to the \( \phi \)-dispersion relation for diffracted fields (all fields with \( p \neq 0 \) are negligible). BL excitation: normalized Rayleigh pulse \( G(t) = \Re \{ [1 + \cos(\phi(t)) / 1] \} \) (i.e., \( G(0) = 1 \) ); \( \phi(t) = \sin(\omega_{p(\omega)}t) / \omega_{p(\omega)} \), central radiating frequency \( \omega_{p(\omega)} = 2\pi c / d_z \), \( \lambda_{p(\omega)} = 2\pi c / \omega_{p(\omega)} = d_z \) ). Observer location: \( (x, y, z) = (-d_z / 2, d_y, 0) \) so that \( \phi = 93.7^\circ \), and since \( \phi_0 = 90^\circ \) diffracted fields are in transitional behavior. Since \( G(\phi - \phi_0) = 1 \), no FW0 are present. Fields \( A(t) \) are plotted versus normalized time \( t/T \), with \( T = d_z/c \). The almost identical turn-on times \( t/T = 8 \) and \( t/T = 8.02 \) also indicates that diffracted fields are in transitional behavior. The included asymptotic terms \( p = 0 \) and \( |q| \leq 1 \) (solid curve), suffice to give good agreement with the reference solution (dotted curve), demonstrating good convergence of the TD-FW representation.

### REFERENCES

