Essays on Inference from Option Markets

https://escholarship.org/uc/item/35w7m37v

Dossani, Asad

2018

Peer reviewed|Thesis/dissertation
UNIVERSITY OF CALIFORNIA SAN DIEGO

Essays on Inference from Option Markets

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy

in

Economics

by

Asad Rafiq Dossani

Committee in charge:

Professor Allan Timmermann, Chair
Professor Brendan Beare
Professor Joseph Engelberg
Professor James Hamilton
Professor Rossen Valkanov

2018
The Dissertation of Asad Rafiq Dossani is approved and is acceptable in quality and form for publication on microfilm and electronically:

________________________________________

________________________________________

________________________________________

________________________________________

Chair

University of California San Diego

2018
DEDICATION

To my parents, Rafiq and Khairunessa, for giving me love and support.
To my wife, Samina, for giving me love and strength.
To my daughters, Inaya and Imana, for giving me love and faith.
# TABLE OF CONTENTS

Signature Page ........................................................................................................ iii
Dedication ................................................................................................................. iv
Table of Contents .................................................................................................... v
List of Figures .......................................................................................................... viii
List of Tables ........................................................................................................... ix
Acknowledgements ................................................................................................. xi
Vita ........................................................................................................................... xii
Abstract of the Dissertation ..................................................................................... xiii

Chapter 1 Central Bank Tone and Currency Risk Premia ................................. 1
  1.1 Introduction ..................................................................................................... 2
  1.2 Data .............................................................................................................. 6
      1.2.1 Currency Futures and Options ................................................................. 6
      1.2.2 Central Bank Press Conferences .............................................................. 9
  1.3 Methodology ................................................................................................. 10
      1.3.1 Central Bank Tone .................................................................................. 10
      1.3.2 Implied Risk Aversion ............................................................................ 13
      1.3.3 Variance Risk Premium ....................................................................... 20
      1.3.4 Empirical Example .............................................................................. 22
      1.3.5 Net Index Regressions ......................................................................... 23
  1.4 Results .......................................................................................................... 24
      1.4.1 Summary Statistics ............................................................................... 24
      1.4.2 Monotonicity Test ............................................................................... 26
      1.4.3 Regression Results .............................................................................. 26
      1.4.4 Robustness Checks ............................................................................. 28
  1.5 Conclusion ..................................................................................................... 29
  1.6 Acknowledgements ......................................................................................... 31

Chapter 2 Monetary Stimulus and Perception of Risk ...................................... 46
  2.1 Introduction .................................................................................................. 46
  2.2 Data ............................................................................................................. 50
      2.2.1 Effective Monetary Stimulus .................................................................. 50
      2.2.2 Futures .................................................................................................. 51
      2.2.3 Options on Futures .............................................................................. 52
      2.2.4 Inflation Expectations ......................................................................... 54
  2.3 Methodology ................................................................................................. 54
2.3.1 Implied Volatility .................................................. 54
2.3.2 Implied Skewness ................................................. 55
2.3.3 Volatility Regressions ............................................. 56
2.3.4 Skewness Regressions .............................................. 57
2.3.5 Return Regressions ................................................ 58
2.3.6 Cointegration Models ............................................. 59
2.3.7 Inflation Expectations ............................................. 61
2.3.8 Return Predictability ............................................. 62
2.4 Results ................................................................. 63
2.4.1 Plots and Summary Statistics .................................... 63
2.4.2 Volatility and Skewness Regression Results .................... 64
2.4.3 Return Regression Results ........................................ 65
2.4.4 Cointegration Model Results ...................................... 66
2.4.5 Perception of Risk by Asset Class ............................... 67
2.4.6 Inflation Expectations ............................................. 68
2.4.7 Return Predictability ............................................. 69
2.5 Conclusion ............................................................. 69
2.6 Acknowledgements .................................................. 70

Chapter 3 Option Augmented Density Forecasts of Market Return with Monotone Pricing Kernel ........................................ 85
3.1 Introduction .......................................................... 85
3.2 Option augmentation of density forecasts ......................... 89
  3.2.1 Ordinal dominance curve convexification ....................... 89
  3.2.2 Pricing kernel monotonization .................................. 92
  3.2.3 Numerical computation ........................................... 94
3.3 Empirical results: S&P 500 index options ......................... 97
  3.3.1 Data ............................................................... 97
  3.3.2 Estimation ........................................................ 98
  3.3.3 Density forecast evaluation .................................... 101
3.4 Conclusion ............................................................ 107
3.5 Acknowledgements .................................................. 107

Appendix A Central Bank Tone and Currency Risk Premia .................. 108
  A.1 Variance and Volatility Swaps .................................... 108
  A.2 ECB Press Conference ............................................. 109
  A.3 Swiss Franc Outlier ................................................ 110

Appendix B Empirical results for FR, DE, HK, JP, UK ..................... 112
  B.1 Data ............................................................... 112
    B.1.1 Option prices .................................................. 112
    B.1.2 Total returns .................................................. 113
    B.1.3 Risk-free interest rates ...................................... 113
  B.2 Estimation .......................................................... 113
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Net Index by Central Bank</td>
<td>32</td>
</tr>
<tr>
<td>1.2</td>
<td>Empirical Example - Implied Risk Aversion</td>
<td>33</td>
</tr>
<tr>
<td>1.3</td>
<td>Net Index Scatter Plots</td>
<td>34</td>
</tr>
<tr>
<td>2.1</td>
<td>Monetary Stimulus and Inflation Expectations</td>
<td>71</td>
</tr>
<tr>
<td>2.2</td>
<td>Equities</td>
<td>72</td>
</tr>
<tr>
<td>2.3</td>
<td>Bonds</td>
<td>73</td>
</tr>
<tr>
<td>2.4</td>
<td>Commodities</td>
<td>74</td>
</tr>
<tr>
<td>2.5</td>
<td>Currencies A</td>
<td>75</td>
</tr>
<tr>
<td>2.6</td>
<td>Currencies B</td>
<td>76</td>
</tr>
<tr>
<td>3.1</td>
<td>Original and option augmented estimates of Jackwerth (2000)</td>
<td>95</td>
</tr>
<tr>
<td>3.2</td>
<td>Computation of option augmented Jackwerth estimates</td>
<td>96</td>
</tr>
<tr>
<td>3.3</td>
<td>Empirical distributions of probability integral transforms</td>
<td>104</td>
</tr>
<tr>
<td>3.4</td>
<td>Histograms for probability integral transforms</td>
<td>105</td>
</tr>
<tr>
<td>3.5</td>
<td>Marginal calibration plots</td>
<td>106</td>
</tr>
<tr>
<td>B.1</td>
<td>Empirical distributions and histograms: international data</td>
<td>116</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Table 1.1</td>
<td>Adjectives and Nouns</td>
<td>35</td>
</tr>
<tr>
<td>Table 1.2</td>
<td>Central Bank Press Conference Summary Statistics</td>
<td>35</td>
</tr>
<tr>
<td>Table 1.3</td>
<td>Realized GARCH Parameter Estimates</td>
<td>36</td>
</tr>
<tr>
<td>Table 1.4</td>
<td>Monotonicity Test</td>
<td>36</td>
</tr>
<tr>
<td>Table 1.5</td>
<td>Net Index Summary Statistics</td>
<td>37</td>
</tr>
<tr>
<td>Table 1.6</td>
<td>Net Index Summary Statistics: Two and Three Month VRP</td>
<td>37</td>
</tr>
<tr>
<td>Table 1.7</td>
<td>Regression Results: IRA</td>
<td>38</td>
</tr>
<tr>
<td>Table 1.8</td>
<td>Regression Results: 1M VRP</td>
<td>39</td>
</tr>
<tr>
<td>Table 1.9</td>
<td>Regression Results: 2M VRP</td>
<td>40</td>
</tr>
<tr>
<td>Table 1.10</td>
<td>Regression Results: 3M VRP</td>
<td>41</td>
</tr>
<tr>
<td>Table 1.11</td>
<td>Regression Results: Proportional Net Index</td>
<td>42</td>
</tr>
<tr>
<td>Table 1.12</td>
<td>Regression Results: Extended Net Index</td>
<td>43</td>
</tr>
<tr>
<td>Table 1.13</td>
<td>Regression Results: IRA upside &amp; downside</td>
<td>44</td>
</tr>
<tr>
<td>Table 1.14</td>
<td>Regression Results: 1M VRP upside &amp; downside</td>
<td>45</td>
</tr>
<tr>
<td>Table 2.1</td>
<td>Summary Statistics</td>
<td>77</td>
</tr>
<tr>
<td>Table 2.2</td>
<td>Summary Statistics - Futures and Options</td>
<td>78</td>
</tr>
<tr>
<td>Table 2.3</td>
<td>Unit Root and Cointegration Tests</td>
<td>79</td>
</tr>
<tr>
<td>Table 2.4</td>
<td>Volatility and Skewness Regressions</td>
<td>80</td>
</tr>
<tr>
<td>Table 2.5</td>
<td>Return Regressions</td>
<td>81</td>
</tr>
<tr>
<td>Table 2.6</td>
<td>Cointegration Models</td>
<td>82</td>
</tr>
<tr>
<td>Table 2.7</td>
<td>Inflation Expectations</td>
<td>83</td>
</tr>
<tr>
<td>Table 2.8</td>
<td>Return Predictability</td>
<td>84</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Serial dependence of probability integral transforms.</td>
<td>103</td>
</tr>
</tbody>
</table>
Table 3.2. Tests of uniformity of probability integral transforms. .................. 103
Table B.1. Probability integral transform uniformity tests: international data. ....... 115
ACKNOWLEDGEMENTS

Thank you to my committee chair Professor Allan Timmermann for guidance, feedback, and advice throughout the PhD program. Thank you to my dissertation committee members: Professors Brendan Beare, Joseph Engleberg, James Hamilton, and Rossen Valkanov, for valuable feedback and advice. Thank you to participants of the UCSD macroeconomics, econometrics, and finance seminar series for valuable comments. Thank you to my fellow PhD students at UCSD for good company and good conversation over the last six years.

Chapter 1, in full, is currently being prepared for submission for publication of the material. Dossani, Asad. The dissertation author was the sole author of this paper.

Chapter 2, in full, is currently being prepared for submission for publication of the material. Dossani, Asad. The dissertation author was the sole author of this paper.

Chapter 3, in full, is a reprint of the material as it appears in Quantitative Finance 18 (4), 623-625, 2017. Beare, Brendan.; Dossani, Asad. The dissertation author was a primary author of this paper.
VITA

2003 Bachelor of Arts in Economics and South Asian Studies, School of Oriental and African Studies, University of London

2006 Master of Science in Economics, London School of Economics

2011 Master of Science in Financial Economics, Said Business School, University of Oxford

2018 Doctor of Philosophy in Economics, University of California San Diego

PUBLICATIONS

ABSTRACT OF THE DISSERTATION

Essays on Inference from Option Markets

by

Asad Rafiq Dossani

Doctor of Philosophy in Economics

University of California San Diego, 2018

Professor Allan Timmermann, Chair

This dissertation consists of three chapters that analyze the economic information contained in option markets. Option markets are forward looking, and thus contain valuable insight into the beliefs of financial market participants. They can be used to study risk premia and to make forecasts. The Chapter 1, Central Bank Tone and Currency Risk Premia, asks how the tone of central bank press conferences impacts risk premia in the currency market. First, I find that option implied risk aversion increases when central banks are hawkish, and decreases when central banks are dovish. Second, I find that hawkish central bank tone predicts higher future variance risk premia, and vice versa. One explanation for this result is that the tone of a press conference indicates to investors the likelihood of central bank intervention, conditional on the
state of the economy. Chapter 2, Monetary Stimulus and Perception of Risk, investigates the relationship between monetary stimulus and the perception of risk in financial markets, and how this varies across asset classes. First, I document a positive relationship between monetary stimulus and the perception of risk in equity, commodity, and currency markets. I document a negative relationship between monetary stimulus and the perception of risk in bond markets. Second, I establish a cointegrating relationship between monetary stimulus and implied volatility, indicating a positive long run equilibrium relationship in the levels of monetary stimulus and implied volatility. This relationship is present across asset classes. Third, I document the link between monetary stimulus and expected inflation, a possible mechanism by which monetary stimulus affects the perception of risk across financial markets. Chapter 3, Option Augmented Density Forecasts of Market Return with Monotone Pricing Kernel, considers consider an option augmented density forecast of the market return obtained by transforming a baseline density forecast estimated from past excess returns so as to monotonize its ratio with a risk neutral density estimated from current option prices. We find that monotonizing the pricing kernel leads to a modest improvement in the calibration of density forecasts.
Chapter 1

Central Bank Tone and Currency Risk Premia

Abstract

I analyze how the tone of central bank press conferences impacts risk premia in the currency market. I measure tone as the difference between the number of hawkish and dovish phrases made during a press conference. I consider two measures of risk premia. The first measure is implied risk aversion. This is based on the relationship between the option implied, or risk neutral distribution of returns, and the physical, or actual distribution of returns. I find that implied risk aversion increases when central banks are hawkish, and decreases when central banks are dovish. The second measure is the variance risk premium. This is the difference between option implied and realized variance, and reflects the cost of insuring against an unexpected increase in variance. I find that variance risk premia increase when central banks are hawkish, and decrease when central banks are dovish. The magnitudes are economically and statistically significant. A one standard deviation increase in the hawkishness of a press conference increases the one month variance risk premium by 4.7% per year, relative to the average of 28.9% per year.
1.1 Introduction

Central bank press conferences move markets. Press conferences are live, partially unscripted, and provide market participants clues on future policy actions. Press conferences can reveal news not previously available to the markets.\(^1\) This paper asks how the tone of central bank press conferences impacts risk premia in the currency market.

I measure tone as the difference between the number of hawkish and dovish phrases made during a press conference, using the method in Apel & Grimaldi (2012).\(^2\) I examine the impact of tone on two measures of risk premia. The first measure is implied risk aversion, as defined in Jackwerth (2000). It is based on the relationship between the option implied, or risk neutral distribution of returns, and the physical, or actual distribution of returns. The former is estimated using a cross section of option prices, while the latter is estimated using a historical time series of returns. I find that implied risk aversion increases when central banks are hawkish, and decreases when central banks are dovish. This is a contemporaneous result, meaning that central bank tone explains the contemporaneous change in implied risk aversion.

The second measure is the variance risk premium. This is the difference between option implied and realized variance. The variance risk premium reflects the cost of insuring against an increase in variance. Using the method in Carr & Wu (2009), I construct one, two, and three month variance risk premia for each currency. I find that variance risk premia increase when central banks are hawkish, and decrease when central banks are dovish. The magnitudes are economically and statistically significant. A one standard deviation increase in the hawkishness of a press conference increases the one month variance risk premium by 4.7% per year, relative to the average of 28.9% per year. This is a predictive result, meaning that central bank tone predicts future realized variance risk premia.

\(^{1}\)For example, Hansen & McMahon (2016) show that FOMC communication over future monetary policy decisions has a significant impact on both financial and real variables.

\(^{2}\)A hawkish central bank is likely to increase interest rates or tighten monetary policy. A dovish central bank is likely to lower interest rates or loosen monetary policy.
The two risk aversion measures are related in the following way. When implied risk
aversion is higher, it means that options are more expensive, relative to their value if investors
were risk neutral. When options are relatively expensive, the expected return from shorting a
variance swap is higher. This intuition is confirmed by the empirical results. Hawkish central
bank tone contemporaneously increases implied risk aversion, and also predicts higher realized
variance risk premia.

This paper makes two contributions. The first is a joint analysis of the impact of central
bank press conferences on risk premia across a range of countries and currencies. The second is
the use of central bank tone to explain changes in implied risk aversion, and to predict variance
risk premia. To the best of my knowledge, these contributions are novel.

This paper adds to the recent literature on the impact of central bank tone on financial
markets. Schmeling & Wagner (2016) analyze the tone of European Central Bank press con-
ferences, and find that positive tone is associate with higher bond yields, lower implied equity
volatility, lower variance risk premia, and lower credit spreads. They use the method in Loughran
& McDonald (2011) to measure tone, which based on the proportion of negative words contained
in the text.3

Why is central bank tone important for asset prices? It is well documented that central
bank policy decisions impact asset prices.4 However, the impact of their words is relatively
understudied. In part, this is because central bank use of words as a policy tool is relatively recent.
Following the financial crisis of 2007-08, many countries lowered interest rates close to zero.
Since then, they have increasingly relied on forward guidance as a policy tool. Forward guidance
is communication to the public about the future course of monetary policy.5 Individuals, firms,

3Alternative approaches to analyze central bank text are in proposed Lucca & Trebbi (2009) and Acosta (2015).
4For example, Lucca & Moench (2015) document large average excess returns on U.S. equities in anticipation
of monetary policy decisions made at FOMC meetings. Cieslak et al. (2016) show that most of the U.S. equity risk
premium is earned over the weeks corresponding to the FOMC cycle. Boyarchenko et al. (2016) show that Federal
Reserve announcements impact markets independent of changes in conventional monetary policy.
5In the early 2000s, the US Federal Reserve began using forward guidance. For example, in December 2008,
the Fed lowered interest rates close to zero. Additionally, they stated that the Federal Funds rate would remain
exceptionally low for some time due to weak economic conditions. By communicating its intention to keep future
interest rates low, the Fed was hoping to influence then-prevailing decisions. Seven years later, in December 2015,
and investors incorporate this information into their decisions. Consequently, forward guidance has an immediate impact on the economy and financial markets.\footnote{Campbell \textit{et al.} (2012) and Filardo & Hofmann (2014) examine the impact of forward guidance on macroeconomic variables and asset prices. Del Negro \textit{et al.} (2012) and McKay \textit{et al.} (2016) show that the impact of forward guidance is small in practice, relative to theoretical predictions. Swanson (2017) shows that forward guidance has significant impacts on Treasury yields, stock prices, and exchange rates.}

Why do risk premia change in response to the tone of central bank press conferences? One explanation is that the tone of a press conference indicates to investors the likelihood of central bank intervention, conditional on the state of the economy. Suppose that the central bank has a reaction function defined over various future economic states, and this is not known to the investor. Instead, the investor has some expectations. These are then updated in response to new information, such as press conferences.

A hawkish central bank is less likely to provide monetary stimulus in bad states of the world. A dovish central bank is more likely to provide monetary stimulus in bad states of the world. For example, Belke & Klose (2010) compare how the Fed and the ECB reacted to the financial crisis of 2007-08. The Fed was more aggressive in easing monetary policy, relative to the ECB. This is partly attributed to the difference in their respective mandates. The Fed has a dual mandate of stable prices and maximum employment, whereas the ECB’s mandate is price stability. This means the Fed is more dovish than the ECB. As a result, the Fed puts a greater weight on the output gap, and is more likely to provide monetary stimulus in response to a crisis.

Following a press conference, suppose the investor believes that the central bank is more likely to provide monetary stimulus in bad states of the world, i.e. the central bank is dovish. Then the investor has less incentive to pay for insurance in those states, and risk premia decrease. Now suppose instead the investor believes that the central bank is less likely to provide monetary stimulus in bad states of the world, i.e. the central bank is hawkish. Then the investor has more incentive to pay for insurance in those states, and risk premia increase. A well known example of this phenomenon is the Greenspan put. Alan Greenspan, the former chairman of the Fed raised interest rates. At the same time, they communicated their intention to raise interest rates over the coming year.
Federal Reserve, would decrease interest rates when the stock market fell by a certain amount. The central bank essentially provided a put option to investors, protecting them when markets declined.\(^7\)

Why are currencies the best channel to analyze the impact of central banks on risk premia? The first reason is data availability. I have access to option data traded on the Chicago Mercantile Exchange (CME). A prerequisite for this paper’s analysis is liquid option markets, and currencies meet this criteria. In addition, currencies allow the incorporation of data from multiple central banks, each corresponding to its own currency. As the CME is a US based exchange, for any other asset class,\(^8\) the CME option data is relevant only for the Fed, and not other central banks. For example, treasury bond options or S&P 500 options are not significantly impacted by foreign central banks. The Fed data consists of 20 press conferences, while the sample size of all central banks is 157 press conferences.

The second reason is that currencies are disproportionately affected by interest rates, and hence central bank behavior, relative to stocks or commodities. This holds true even if central banks do not directly attempt to influence exchange rates, as they may do with bonds or equities. Like currencies, bonds are also primarily impacted by interest rates and central bank behavior. However, they suffer from the data limitation discussed earlier. Currencies are impacted by the interest rate differential between the home and foreign currency.\(^9\) Currencies typically appreciate in response to a current or an expected future increase in interest rates. And they depreciate in response to a current or an expected future decrease in interest rates. Other macroeconomic data matter for currency markets, and can be interpreted through the lens of how it might impact central banks and interest rates.\(^10\) On the other hand, for stocks and commodities, interest rates

---

\(^7\) A put option’s payoff is decreasing in the price of the underlying asset. Index put options are often used by investors to insure against a fall in the stock market.

\(^8\) Asset classes include stocks, bonds, currencies, and commodities.

\(^9\) An important result in this literature is the forward premium puzzle, documented in Fama (1984) and Hansen & Hodrick (1980). The puzzle is that higher interest rates currencies earn excess returns over lower interest rate currencies.

\(^10\) For example, if inflation or GDP growth is high, the central bank is likely to tighten monetary policy and increase interest rates. Alternatively, if inflation is low or the economy is weakening, the central bank is likely to loosen monetary policy and decrease interest rates.
matter, but they aren’t necessarily the most important factor.\textsuperscript{11}

The remainder of this paper is organized as follows: Section 1.2 describes the data. Section 1.3 presents the methodology. This includes the construction of central bank tone, implied risk aversion, and variance risk premia. An empirical example is presented for exposition, followed by the proposed regression specification. Section 1.4 presents and discusses the results. Section 1.5 concludes.

1.2 Data

This section describes the data. The data consist of currency futures and option contracts, and central bank press conference transcripts.

1.2.1 Currency Futures and Options

Currency futures are traded on the Chicago Mercantile Exchange (CME). I use data on four currency futures contracts: the British pound (GBP), the Canadian dollar (CAD), the euro (EUR), and the Swiss franc (CHF). Prices are sampled at five minute intervals. The data come from DTN IQFeed.\textsuperscript{12} Prices are quoted as the number of US dollars (USD) per unit of foreign currency. Thus, a price increase is an appreciation of the foreign currency, and a depreciation of the US dollar. A price decrease is a depreciation of the foreign currency, and an appreciation of the US dollar. Returns on the US dollar are constructed as a weighted average of the returns on each of the currency contracts. The weights correspond to the proportions in the US dollar index DXY, and are as follows: 14.4\% (GBP), 11.1\% (CAD), 70.1\% (EUR), and 4.4\% (CHF). These are intended to reflect relative trading volumes of each currency to the dollar.

The currency futures market trades 23 hours per day, 5 days per week. Trading begins at 6.00pm EST on Sunday, and stops at 5.00pm EST on Friday. There is a 1 hour trading break from 5.00pm EST to 6.00pm EST each day. Closing prices are at 5.00pm EST from Monday to Friday.

\textsuperscript{11}Stocks are impacted by expectations of future cash flows. Commodities are impacted by their own fundamentals.

\textsuperscript{12}DTN IQFeed is an online provider of live and historical financial market data. Their currency futures data is sourced directly from the CME.
Currency contracts trade in a quarterly cycle, with expiries in March, June, September, and December each year. For all contracts other than the Canadian dollar, they expire the business day preceding the third Tuesday of the contract month, which is usually a Monday. The Canadian dollar contracts expire the business day preceding the third Wednesday of the contract month, which is usually a Tuesday.

The raw data take the form of continuous futures contracts. This is necessary in order to ensure that returns are correctly calculated. Prices are back-adjusted to create a continuous contract. This works by removing price gaps caused by a contract roll. The process starts at the end of the price series, and works its way back. This leaves current prices intact. Prices prior to the last roll date are adjusted. All currency contracts except for the Canadian dollar are rolled two days prior to expiry. The Canadian dollar is rolled three days prior to expiry to ensure that all contracts are rolled on the same day.

Options on the currency futures contracts also trade on the CME. These trade on a monthly cycle. I use option data on each of the four currencies. The data come from OptionWorks via Quandl. Option prices are sampled at the close of each trading day. The data start in June 2009 and end in December 2015. The data take the form of one, two, and three month implied volatility curves.

Implied volatility is a function of the strike price. First, the option price is converted to an implied volatility, $\sigma$, via the Black Scholes model. This is done across the range of traded strike prices, using the Black-Scholes formula. Let $F$ be the futures price, $K$ be the strike price, $T$ be the time to maturity, $\sigma$ be the implied volatility, and $r$ be the interest rate. The call option price $C$ is given by:

13 Quandl is online data provider of financial and economic data. OptionWorks is one of the databases within Quandl that specializes in options on futures.
\[ C = e^{-rT}[FN(d_1) - KN(d_2)] \]
\[ d_1 = \frac{\log(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}} \]
\[ d_2 = d_1 - \sigma\sqrt{T} \]

Next, a polynomial of degree six is fitted to the data. This produces the implied volatility curve, where implied volatility is a function of the strike price. For convenience, the strike price \( K \), is expressed as moneyness \( M \), or the percentage deviation from the current futures price \( F \).

\[
M \equiv \log(K/F)
\]
\[
\sigma(K) = b_0 + b_1M + b_2M^2 + b_3M^3 + b_4M^4 + b_5M^5 + b_6M^6
\]

The raw data consists of the closing futures price, the six model coefficients, the minimum and maximum moneyness, and the time to expiry. The minimum and maximum moneyness are upper and lower bounds. They reflect the fact that option prices only trade in a certain range of the current futures price. Typically, this includes the 5\(^{th}\) and 95\(^{th}\) quantile of the distribution.

Any strike and implied volatility pair can be converted to a call or put option price via the Black Scholes model. Using implied volatility curves does not require that Black Scholes assumptions hold. The implied volatility can be thought of as a normalized option price. It allows for a more intuitive comparison of option prices across strikes, time to maturity, or underlying currencies.

The curves are constructed for constant one month, two month, and three month maturities. Options trade on a monthly cycle. Constant maturity contracts are created by interpolating across traded options with maturities less than and greater than the constant maturity. For example, the one month implied volatility curve is constructed using option prices expiring before and after
one month. This is a standard procedure. For example, this method is used to calculate the VIX, the one month implied volatility of the S&P500 index.

1.2.2 Central Bank Press Conferences

Central bank announcements consist of two parts. The first part is a policy action. Usually, this is an overnight interest rate that is either lowered, raised, or held constant. Policy actions also include asset purchases. The second part is text. Examples of text include press conferences, interest rate decisions, meeting minutes, and economic outlook publications. The text contains useful information pertaining to the state of the economy and the future path of monetary policy. Press conferences tend to have the largest impact on asset prices. This is because they take place in real time and are partially unscripted.¹⁴

I use press conference transcripts for five central banks: The Bank of England (BOE), the Bank of Canada (BOC), the European Central Bank (ECB), the Swiss National Bank (SNB), and the US Federal Reserve (FED). All the press conference data is publicly available on each central bank’s website. The Bank of England, Bank of Canada, and US Federal Reserve hold four press conferences per year. These coincide with alternate interest rate announcements that are made eight times per year. The European Central Bank holds eight press conferences per year, coinciding with each interest rate decision. Through December 2014, it held twelve press conferences per year. Now it holds eight press conferences per year. The Swiss National Bank holds two press conferences per year. These coincide with alternate interest rate announcements that are made four times per year. The total sample consists of 157 press conferences. The data start in June 2009 for all central banks excluding the Fed. The Fed data start in April 2011, at the time it started holding press conferences. All the data end in December 2015.

¹⁴ A press conference typically includes initial prepared remarks, followed by a live question and answer session.
1.3 Methodology

In this section, I present the methodology. First, I measure central bank tone, compute implied risk aversion, and compute variance risk premia. Then, I present an empirical example of a single observation. Finally, I use a regression to measure the impact of central bank tone on implied risk aversion and variance risk premia.

1.3.1 Central Bank Tone

Central bank tone is measured by the relative frequency of hawkish and dovish phrases. I use the method in Apel & Grimaldi (2012). Each phrase is a two word combination, consisting of an adjective followed by a noun. The nouns are common to both hawkish and dovish phrases, while the adjective is used to identify whether the phrase is hawkish or dovish. Table 1.1 lists the adjectives and nouns. A hawkish phrase consists of a hawkish adjective and a noun. A dovish phrase consists of a dovish adjective and a noun. Any combination of adjective and noun is considered a phrase. For example, increased inflation is a hawkish phrase, while slower growth is a dovish phrase.

For each press conference taking place at time $t$, I count the number of hawkish phrases, $\#Hawk_t$, and dovish phrases, $\#Dove_t$. I define the net index, $NI_t$, as the difference between the two.

$$NI_t = \#Hawk_t - \#Dove_t$$

When the net index is greater than zero, tone is hawkish. When the net index is less than zero, tone is dovish. The magnitude of the net index indicates how hawkish or dovish the tone is. Figure 1.1 plots the net index for each central bank.

Apel & Grimaldi (2012) propose this method to analyze the text in Swedish central bank minutes. First, they select a set of phrases that are either hawkish or dovish. Second, they count the number of hawkish and dovish phrases contained in the text. Their measure of tone is based
on the relative proportion of hawkish and dovish phrases contained in the text. They find that the
tone of the Swedish central bank minutes is useful in predicting future interest rate decisions.
Hawkish tone implies that the central bank is likely to tighten monetary policy in the future,
while dovish tone implies that the central bank is likely to ease monetary policy in the future.
The strength of the announcement depends on the numerical value for tone. They conclude that
the minutes contain useful information not captured by observable macroeconomic variables.

In their paper, phrases associated with a stronger economy or higher inflation are hawkish,
while phrases associated with a weaker economy or lower inflation are dovish. This is because
a strong economy or high inflation means the central bank is more likely to tighten monetary
policy. Conversely, if the economy is weak or inflation is low, the central bank is more likely to
ease monetary policy.

The upside to this approach is that tone can be quantified and objectively calculated. It
can later be used in a regression. The downside is that the full content of the announcement is not
captured. Given that text is inherently subjective, there is an inevitable trade-off in objectively
quantifying tone.

Why is this best method to measure central bank tone? Loughran & McDonald (2016)
document that most methods to measure tone or sentiment rely on the frequency of certain words
or phrases in the text.\textsuperscript{15} The relevant words are selected based on what the researcher wishes
to measure. The method I use in this paper, based on Apel & Grimaldi (2012), differs in two
ways from other common methods. First, it uses the frequency of two word combinations rather
a single word. This increases the precision of each phrase, at the expense of fewer phrases
identified. Second, it uses two categories of phrases (hawkish and dovish), instead of one.
Loughran & McDonald (2011) show that the negative words are better at measuring tone, relative
to positive words, and so some studies use only one category of phrases.\textsuperscript{16} This is because
positive words are more frequently negated, relative to negative words. This is more likely to

\textsuperscript{15} Alternatives include machine learning based approaches that are beyond the scope of this paper.
\textsuperscript{16} For example, Schmeling & Wagner (2016) analyze ECB press conferences using only negative words.
be a problem when analyzing single words rather than two word combinations. The use of two
word combinations makes it easier to accurately identify two categories of phrases (hawkish and
dovish).

Apel & Grimaldi (2012) apply their method to Swedish central bank minutes. The central
bank publishes in Swedish, and the analysis is performed in Swedish. Is there a linguistic issue
in applying this method to other central banks? Other than the Swiss National Bank, all the
central banks analyzed in this paper publish in English (including the European Central Bank).
The Swiss National Bank publishes English language versions of their statements; these are what
I use. Even though this list of words is developed in the context of the Swedish central bank, all
the words are commonly used in English. Further, these words are regularly used by economists
and central bankers. There is no reason to believe that the list of words would be any different if
the initial method was developed for an English medium central bank.

One limitation of using the net index to explain to market movements is the difficulty in
capturing market expectations. A dovish press conference may not move the market at all if the
market already expected it. Market movements should depend on how hawkish or dovish a press
conference is relative to expectations, rather than on its own. In subsequent regressions, I get
around this problem by controlling for the market reaction to a press conference. This is done by
including the daily currency return and change in implied volatility over the press conference
period in the regression.\footnote{Another possibility to control for market expectations is to predict the net index using its own lags and other market or macroeconomic data. This then forms the baseline for the market expectation, and hawkishness or dovishness can be measured relative to this. I do not employ this approach in this paper, due to data limitations. The number of press conferences for each central bank is too small to construct accurate forecasts. Macroeconomic data, the basis for central bank policy, is also available only at low frequencies.}

Note that Apel & Grimaldi (2012) calculate the net index differently. They take the
difference between the number of phrases hawkish and dovish phrases, and divide by the total
number of phrases. Their net index is the proportion of hawkish or dovish phrases, rather than
raw value. I avoid this method so that the magnitude of the net index is more meaningful.
Suppose, for example, that the central bank makes nine dovish phrases and one hawkish phrase.
The regular net index is $-8$, while the proportional net index is $-0.8$. In the next period, suppose that the central bank makes two dovish statements and no hawkish statements. The regular net index is $-2$, while the proportional net index is $-1$. Which of the two statements is more dovish? We get different answers depending on which measure we use. The main results are reported using the regular net index. As a robustness check, results are reported using the proportional net index.

A potential downside to not using the proportional net index is that results are affected by the length of the press conference. For example, a longer press conference implies a higher frequency of phrases, but not necessarily a higher proportion of phrases. This is only a problem if there is considerable variation in the length of press conferences. Table 1.2 reports the average length (in words) of each central bank’s press conferences, and the corresponding coefficient of variation (standard deviation divided by average length). First, note that there is considerable variation in average length across central banks. But within central banks, this is generally not the case. Other than the Bank of England, the coefficient of variation is low for each central bank. The standard deviation of the net index for each central bank tends to vary in line with the average length of the press conference. Central banks with longer press conferences have a larger standard deviation of the net index. In the regression analysis, observations for each central bank are pooled together. I control for the difference in press conference length by dividing each central bank’s net index by its own standard deviation. This makes the net index value comparable between central banks, in addition to a more intuitive interpretation.

1.3.2 Implied Risk Aversion

Implied risk aversion is identified from the risk neutral and physical densities of returns. This approach follows the method of Jackwerth (2000). Consider a complete market economy with a representative investor. The investor maximizes expected utility subject to a budget constraint. Let $S$ denote the state, $p(S)$ the physical density, $q(S)$ the risk neutral density, $W(S)$ the investor’s wealth, and $U[W(S)]$ the investor’s utility function. The investor is endowed with
one unit of wealth. Expected utility is given by the following equation:

$$\int p(S) U[W(S)] \, dS$$

In a complete market economy, the investor can purchase state contingent securities. The investor is endowed with one unit of wealth. The budget constraint is given by:

$$\int q(S) W(S) \, dS - 1$$

The investor maximizes expected utility subject to the budget constraint. Let $\lambda$ denote the Lagrange multiplier. The investor solves the following optimization problem:

$$\max \int p(S) U[W(S)] \, dS - \lambda \left( \int q(S) W(S) \, dS - 1 \right)$$

I solve this optimization problem by taking first order conditions. The investor chooses wealth in each state to equate the expected marginal utility of wealth with its relative price. The result is an expression for $U'[W(S)]$.

$$U'[W(S)] = \lambda \frac{q(S)}{p(S)}$$

Next, I differentiate this equation to compute an expression for $U''[W(S)]$.

$$U''[W(S)] = \lambda \left( \frac{p(S)q'(S) - q(S)p'(S)}{[p(S)]^2} \right)$$

Finally, I combine these two equations to get an expression for the Arrow-Pratt coefficient of absolute risk aversion, $ARA(S)$. The absolute risk aversion function is identified from the risk neutral and physical densities, as the Lagrange multiplier is no longer in the expression.

$$ARA(S) = -\frac{U''[W(S)]}{U'[W(S)]} = \frac{p'(S)}{p(S)} - \frac{q'(S)}{q(S)}$$ (1.1)
Closely related to absolute risk aversion function is the pricing kernel, \( m(S) \). This is the ratio of the risk neutral density to the physical density. Intuitively, this is the price of consumption in a given state, relative to the probability that the state occurs.

\[
m(S) = \frac{q(S)}{p(S)}
\]

The absolute risk aversion function is proportional to the derivative of the pricing kernel. The slope of the pricing kernel tells us the sign of the absolute risk aversion function. When the pricing kernel is downward sloping, absolute risk aversion is positive. When the pricing kernel is upward sloping, absolute risk aversion is negative.

\[
ARA(S) = -\frac{m'(S)}{m(S)}
\]

To estimate the risk neutral density, I employ the method of Malz (2014). Recall from section 1.2 that each observation is an implied volatility curve, which is a function of the strike price. I convert implied volatilities into call option prices, via the Black-Scholes formula.\(^{18}\) Let \( F \) be the futures price, \( K \) be the strike price, \( T \) be the time to maturity, \( \sigma \) be the implied volatility, and \( r \) be the interest rate. The call option price \( C \) is given by:

\[
C = e^{-rT}[FN(d_1) - KN(d_2)]
\]

\[
d_1 \equiv \log(F/K) + (\sigma^2/2)T
\]

\[
d_2 \equiv d_1 - \sigma\sqrt{T}
\]

The call option price can be calculated for any strike price, \( K \). Define this as the call valuation function, \( C(K) \). The risk neutral CDF is the derivative of the call price with respect to

\(^{18}\)Using implied volatility curves does not require that Black Scholes assumptions hold. The implied volatility can be thought of as a normalized option price. It allows for a more intuitive comparison of prices across contracts and over time.
the strike price. The risk neutral PDF is the derivative of the CDF with respect to the strike price. The derivatives are approximated by numerical differencing. I calculate the call price across a fine grid of strike prices. This effectively fills in the gaps between the prices of actual traded options. A discrete set of option prices becomes effectively continuous. This is necessary to compute a density function, that is also continuous. The step size of the grid is $\Delta = 0.1\%$. The unit is log return, or the log difference between the strike price and current futures price. The risk neutral CDF $Q(K)$ and PDF $q(K)$ are given by:

$$Q(K) \approx 1 + e^{-rT} \frac{1}{\Delta} \left[ C\left(K + \frac{\Delta}{2}\right) - C\left(K - \frac{\Delta}{2}\right) \right]$$

$$q(K) \approx \frac{1}{\Delta} \left[ Q\left(K + \frac{\Delta}{2}\right) - Q\left(K - \frac{\Delta}{2}\right) \right]$$

This procedure is valid for strikes within the minimum and maximum moneyness. The density is constructed only as far as actual options are traded. Appending tails to the density requires an extrapolation of option prices, rather than interpolation between prices. There are numerous methods to append tails to the distribution. Usually, it requires assuming a particular distribution to fit the tails, such as lognormal or generalized extreme value. I do not append tails to the distribution. Instead, I define the density and compute implied risk aversion only in the range of traded strike prices. This typically includes the 5\(^{th}\) and 95\(^{th}\) quantile of the distribution. Incorporating tails into the distribution does not have a material impact on calculation of implied risk aversion, and the subsequent results. Further, the results are a function only of traded option prices, rather than how the tails are appended.

The physical density is estimated using the Realized GARCH model of Hansen \textit{et al.} (2012), combined with filtered historical simulation. It is constructed using a historical time series. The Realized GARCH model incorporates measurements of volatility based on intraday high frequency data, and it allows for an asymmetric response of volatility to positive and
negative shocks, i.e. leverage effects.¹⁹ Let \( y_t \) be the daily return on the futures contract, \( h_t \) be the latent volatility process, and \( x_t \) be the daily realized volatility constructed from five minute returns. \( \varepsilon_t \) and \( v_t \) are mean zero i.i.d. innovations. The model consists of three equations: the return equation, the GARCH equation, and the measurement equation.

\[
\begin{align*}
  y_t &= \mu + \sqrt{h_t}\varepsilon_t \\
  \log h_t &= \omega + \sum_{i=1}^{q} \alpha_i \log h_{t-i} + \sum_{i=1}^{p} \beta_i \log x_{t-i} \\
  \log x_t &= \zeta + \delta \log h_t + \eta_1 \varepsilon_t + \eta_2 (\varepsilon_t^2 - 1) + v_t
\end{align*}
\]

I set \( p = 1 \) and \( q = 2 \). This is the preferred specification of Hansen et al. (2012). I assume \( \varepsilon_t \) is student t distributed with variance one, and \( v_t \) is normally distributed. For each density, the GARCH model is estimated using daily data.²⁰ For exposition, table 1.3 presents estimates for a single observation. All estimates are computed out of sample, meaning that models are estimated using only data available at the time.

An alternative specification for the GARCH model is to include the net index as an explanatory variable in the GARCH equation. When doing so, the estimated coefficient on the net index is not statistically significant. One interpretation of this finding is that the impact of the net index is primarily captured through option prices and the risk neutral density, rather than the physical density. That said, an alternative explanation is that the sample size of press conference for each currency is too small, leading to limited statistical power.²¹ The statistical power of the net index comes from pooling the central bank observations together. Further, since the GARCH models are estimated out of sample, including the net index would make it difficult to construct a density forecast for the first few press conferences.

Once the GARCH model is estimated, filtered historical simulation (FHS) is used to

---

¹⁹Leverage effects are statistically significant for currencies, as per the results in table 1.3.
²⁰Estimation is by maximum likelihood using the rugarch package in R.
²¹The GARCH model is estimated separately for each currency.
construct a density forecast one month ahead. This process is repeated for each date. The FHS procedure works as follows:

1. Sample $D$ standardized residuals with replacement from the model, where $D$ is the number of trading days in a month.

2. The residuals are fed iteratively through the GARCH model to construct a simulated monthly return.

3. Repeat this process $N$ times to construct $N$ simulated monthly returns. I set $N = 500,000$.

4. Apply a kernel density estimator on the simulated returns to construct the density. The bandwidth choice is based on the plug in method of Wand & Jones (1994).

Using the estimated risk neutral and physical densities, I compute risk aversion as a function of future states. The risk aversion function is proportional to the derivative of the pricing kernel, as per equation 1.2. One point to note is that the function is not smooth. This is because the physical density is estimated using a kernel smoother. The bandwidth is selected to ensure that the density is smooth. However, it does not ensure that the derivative of the density is smooth. Calculating the risk aversion function requires the derivative of the physical density. To get around this problem, I fit the risk aversion function to a polynomial of degree two.

To see how this works, let $r$ be the return on the currency futures contract. When $r$ is positive, the foreign currency appreciates relative to the US dollar. When $r$ is negative, the foreign currency depreciates relative to the US dollar. The polynomial coefficients can be interpreted as the level, slope, and curvature of the risk aversion function. Typically, the three coefficients capture 99% of the variation in the risk aversion function. Thus, a quadratic polynomial is sufficient to capture the variation in the risk aversion function, without significant approximation errors.\textsuperscript{22}

\textsuperscript{22}Gagnon & Power (2012) apply a similar approach to measure changes in risk aversion in the crude oil market.
\[
\text{ARA}(r) = a + br + cr^2
\]

In the majority of cases, risk aversion is positive in states where the foreign currency depreciates \((r < 0)\), and it is negative in states where the US dollar depreciates \((r > 0)\). A positive sign indicates a premium to holding the foreign currency. A negative sign indicates a premium to holding the US dollar. The implication is that there is hedging on both sides of the contract. Investors pay a premium to hedge against declines in the foreign currency as well as declines in the US dollar. This is equivalent to a U-shaped pricing kernel. The hedging demand on both sides isn’t necessarily the same. Net hedging, or the difference between the positive and negative values of risk aversion, need not be zero.

I define implied risk aversion as the probability weighted sum of the distance between the risk aversion function and zero. Implied risk aversion is at the one month horizon. The probabilities come from the physical density estimates. The absolute value of risk aversion tells us the degree of insurance or hedging demand. The sign of this number tells us the direction, i.e. demand for hedging the foreign or domestic currency. The best way to capture the total amount of risk aversion in single number is a weighted sum of the distance between the risk aversion function and zero. In the regression, I use the change in implied risk version over the course of the press conference at time \(t\).

\[
\text{IRA}_t = \sum_r p(r) |\text{ARA}(r)|
\]

\[
\Delta\text{IRA}_t = \log \text{IRA}_t - \log \text{IRA}_{t-1}
\]

To the best of my knowledge, this definition is not present in the literature. Prior studies, such as Jackwerth (2000), assume that the return is a proxy for wealth. In that case, there is no justification for using the absolute value of implied risk aversion when aggregating across
the distribution. This link between returns and wealth is a reasonable assumption when the underlying asset is a market index such as the S&P 500, rather than a currency futures contract. If the absolute value is not used, then positive and negative values of risk aversion cancel each other out. However, if positive and negative values of implied risk aversion actually represent hedging demand on both sides of the contract, it is the magnitude that matters. Then, the absolute value is the correct measure.

1.3.3 Variance Risk Premium

The variance risk premium is the difference between option implied and realized variance. It reflects the cost of insuring against an increase in variance. Suppose that variance is time varying and that investors dislike higher variance. Then, investors are willing to pay a premium to insure against an unexpected increase in variance. Currency variance risk premia have been found to predict future returns. Della Corte et al. (2016) find that low insurance-cost currencies outperform high insurance-cost currencies. Londono & Zhou (2016) find that currency variance risk premia predict future changes in exchange rates. Menkhoff et al. (2012) find a strong link between currency variance risk premia and returns from currency carry trades.

I use the method in Carr & Wu (2009) to quantify the variance risk premium. Their method constructs a variance swap using realized and option implied variance. They show that the variance swap rate is well approximated by a particular portfolio of options. Let $P_t$ be the currency futures price at time $t$, and $R_{t+1}$ be the daily return between time $t$ and $(t + 1)$. The realized variance, $RV_{t,t+\tau}$, is the average daily squared return between time $t$ and $(t + \tau)$.

$$R_{t+1} = \ln(P_{t+1}) - \ln(P_t)$$

$$RV_{t,t+\tau} = \tau^{-1} \sum_{i=1}^{\tau} R_{t+i}^2$$

A variance swap is a forward contract on the realized variance of an underlying asset.
Suppose a swap is initiated at time $t$, and matures at time $(t + \tau)$. $RV_{t,t+\tau}$ is the realized variance over the life of the swap, and $VS_{t,t+\tau}$ is the variance swap rate. I define the variance risk premium, $VRP_{t,t+\tau}$, as the payoff to the holder of a short position in a variance swap. This is given by:

$$VRP_{t,t+\tau} = VS_{t,t+\tau} - RV_{t,t+\tau}$$

The variance swap rate is agreed at initiation of the contract, while the realized variance is determined over the life of the contract. The expected value of the variance swap is zero under the risk neutral measure $\mathbb{Q}$. Thus, the variance swap rate is the risk neutral expectation of realized variance between time $t$ and $(t + \tau)$.

$$VS_{t,t+\tau} = E^{\mathbb{Q}}_t[RV_{t,t+\tau}]$$

The focus of this paper is on realized returns from entering into a variance swap. That said, there is an important distinction between the expected and realized return on a variance swap. The expected return, under the physical measure $\mathbb{P}$ at time $t$ is given by:

$$VRP^e_{t,t+\tau} = VS_{t,t+\tau} - E^{\mathbb{P}}_t[RV_{t,t+\tau}]$$

Note that the literature sometimes defines the variance risk premium as the payoff to the long position in variance swap. The benefit to defining it as I have here is simpler intuition. An increase in the variance risk premium is an increase in the cost of insurance for investors. The literature also sometimes uses volatility (square root of variance) swaps instead of variance swaps. Variance swaps have the advantage that they can be replicated with a static option portfolio, unlike volatility swaps that require a dynamic option portfolio. As a result, in the financial markets, variance swaps are more commonly traded. This is discussed further in appendix A.1.
1.3.4 Empirical Example

I now present an empirical example of the methodological procedure. This is a single observation in the dataset. On September 4, 2014, the ECB held a press conference. It lowered the deposit rate from -0.10% to -0.20%, and it announced the Bank’s intention to begin purchasing non-financial private sector assets. As part of the announcement, the Bank stressed that inflation remained well below its 2% target. A portion of the introductory statement of the ECB press conference is presented in appendix A.2. Relevant phrases for computing the net index are highlighted for exposition. The net index value of this press conference was $-6$, or $-1.3$ standard deviations. The example demonstrates how the net index captures the underlying tone of this particular press conference.

The one month implied risk aversion decreased by 12.0%. The one month, two month, and three month variance risk premia were 0.2%, $-24.1\%$, and $-23.5\%$, respectively. This is relative to the average one month, two month, and three month variance risk premia of 28.9%, 28.0%, and 28.9%, respectively. These numbers reflect the main result of a decrease in implied risk aversion, and lower than average variance risk premia. There is a significant drop in the variance risk premium in the first month following the announcement. The drop is even greater in the second and third month. The second and the third month may reflect persistent announcement affects. They may also be a result of the following month’s press conference by the ECB. That statement, on October 2, 2014, had a net index value of $-7$, or $-1.56$ standard deviations.

Figure 1.2 presents the various steps involved in computing the option implied risk aversion. The top left figure plots the risk neutral density before and after the press conference. The top right figure plots the physical density before and after the announcement. The bottom left figure plots the pricing kernel before and after the press conference. From this picture, the impact of the press conference is a flatter pricing kernel, particularly in the tails of the distribution. The bottom right figure plots the absolute risk aversion function before and after the

---

23 The two and three month average variance risk premia exclude the Swiss Franc outlier. This is discussed in appendix A.3.
press conference. The flatter pricing kernel is reflected in the absolute risk aversion function that is now closer to zero following the press conference. The implied risk aversion is calculated as the probability weighted sum of the distance between the absolute risk aversion function and zero. This decreases as the absolute risk aversion function is closer to zero following the press conference.

### 1.3.5 Net Index Regressions

I run regressions to analyze the impact of the net index on the change in implied risk aversion and variance risk premia. $Y_t$ is the dependent variable, and takes on four possibilities:

$$Y_t \in \{\Delta IRA_t, VRP_{t,t+30}, VRP_{t,t+60}, VRP_{t,t+90}\}$$

The first is the change in the one month implied risk aversion, $\Delta IRA_t$. The next three are the one, two, and three month variance risk premia, given by $VRP_{t,t+30}$, $VRP_{t,t+60}$, $VRP_{t,t+90}$, respectively. Let $t$ be the day of the press conference. $\Delta IRA_t$, is the difference between implied risk aversion at the close of $(t-1)$ and implied risk aversion at the close of $t$. It is contemporaneous with respect to the press conference. The one month variance risk premium, $VRP_{t,t+30}$, is measured from the close of $t$ until the close of $(t+30)$, (i.e. for one month). Note that the calculation begins after the press conference has already occurred. An investor could enter into this short variance swap after observing the press conference. The same logic applies to the two and three month variance risk premia. The regression is given by the following equation:

$$Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t \quad (1.3)$$

$NI_t$ is the net index observed during the press conference. Next, I include four control variables. The first two are the daily return, $Ret_t$, and the daily change in implied volatility, $\Delta IV_t$. Both these variables are measured from the close of the day prior to the press conference to the
close of the day of the press conference. The daily return and change in implied volatility capture
the market impact of the press conference. If the market was surprised by the press conference,
these two variables can account for that. Then, the coefficient on the net index measures the
additional information from the press conference not captured by market movements. The next
two control variables are the prior month variance risk premium, \( VRP_{t-31,t-1} \), and the prior
month change in implied volatility, \( \Delta IV_{t-31,t-1} \). These are intended to capture persistence in the
variance risk premium and implied volatility. Both these variables are measured from the close
of \((t - 31)\) until the close of \((t - 1)\). This is the one month period before the press conference
takes place.

Finally, I include central bank dummy variables. \( CB_t \) consists of central bank dummy
variables for the Bank of England \( (BOE_t) \), Bank of Canada \( (BOC_t) \), European Central Bank
\( (ECB_t) \), and the Swiss National Bank \( (SNB_t) \). The intercept term corresponds to the US Federal
Reserve \( (FED_t) \). This is equivalent to central bank fixed effects.

Observations from all central banks are pooled together. Each central bank’s net index
is divided by its own standard deviation. This makes the net index values across central banks
directly comparable, and justifies pooling of observations. The pooling is critical to ensure
sufficient power in hypothesis tests. The coefficient on the net index can be interpreted as the
impact of a one standard deviation increase in the hawkishness of a press conference.

1.4 Results

This section presents and discusses the results. I begin with summary statistics and a test
of monotonicity in the net index of the dependent variables. Following that is a discussion of the
regression results and their economic significance.

1.4.1 Summary Statistics

Figure 1.3 is a scatter plot of the net index against the four dependent variables (the
change in implied risk aversion, and the one, two, and three month variance risk premia). A
line of best fit is drawn and the correlation coefficient is reported. Each of the four dependent variables has a positive correlation with the net index.

Table 1.2 reports summary statistics for each central bank. The sample period is from June 2009 to December 2015. The statistics include the annual frequency of press conferences, the sample size, the average length (number of words) of each press conference, the corresponding coefficient of variation (standard deviation divided by average length), and the mean and standard deviation of the net index. The mean values of the net index are all negative, indicating that central banks are dovish on average. This is not surprising, given that the sample period was characterized by unprecedented global monetary easing. The standard deviation also varies considerably by central bank. Partly, this is due to the length of the announcement. Central banks with longer statements have a higher net index value, relative to central banks with shorter statements. For example, the ECB tends to have the longest press conferences, and has the highest standard deviation of the net index. In the regression, I divide each central bank’s net index by its own standard deviation. This allows for a more accurate comparison between central banks.

Tables 1.5 and 1.6 present summary statistics for the full sample, and subsamples of the net index. The subsamples consist of the net index above, equal to, and below zero. I report the average change in implied risk aversion, and average values of the one, two, and three month variance risk premia. I also report the Sharpe Ratio corresponding to each value of the variance risk premium. The Sharpe Ratio is the mean return divided by the standard deviation of returns from shorting the variance swap (i.e. earning the variance risk premium). The Sharpe Ratios are approximately twice as large when the net index is positive, relative to when it is negative.

Table 1.6 reports summary statistics including and excluding an outlier observation. On January 15th, 2015, the Swiss National Bank abandoned its peg against the euro. It also lowered interest rates from -0.25% to -0.75%, pushing further into negative territory. The currency rose by 18% against the dollar in a single day. This is approximately 26 times the daily the standard deviation, i.e. a 26 sigma event. This is discussed further in appendix A.3. Regression results are
reported without the outlier observation. When interpreting the summary statistics and regression coefficients as a measure of central tendency, it is better to exclude the outlier. The outlier is relevant only for the two and three month variance risk premia, as it had been over a month since the previous press conference.

### 1.4.2 Monotonicity Test

Tables 1.5 and 1.6 show that both the change in implied risk aversion and the variance risk premia are decreasing in the net index. Patton & Timmermann (2010) propose a test for monotonicity in returns or other financial variables. For this test, observations are sorted by their net index value and split into three equal sized portfolios.\(^{24}\) The null hypothesis is that the dependent variable is not monotonically increasing in the net index. The alternative is that the dependent variable is monotonically increasing in the net index. Table 1.4 reports p-values of this test for four dependent variables: the change in implied risk aversion, the one month variance risk premium, the two month variance risk premium, and the three month variance risk premium. The results indicate monotonicity in the net index for these variables. This finding is consistent with the theory. Namely, that when central banks are hawkish, risk premia are higher. And when central banks are dovish, risk premia are lower. The results are strongest for the variance risk premia, and less strong for implied risk aversion.

### 1.4.3 Regression Results

The regression specification is given by equation 1.3. Table 1.7 reports regression results for the change in implied risk aversion. I use Newey West standard errors with four lags.\(^{25}\) Results are reported with and without central bank dummy variables. The sample period is June 2009 until December 2015. The first and second columns do not include the regressors \(\text{Ret}_t\) and \(\Delta IV_t\). In this case, I am not controlling for the market response to the press conference. The first

\(^{24}\)Observations with a net index value equal to zero are removed. This is to ensure that the net index is strictly decreasing across most observations. The Swiss Franc outlier is excluded from this test.

\(^{25}\)Coefficients are reported with standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively.
and third columns do not include central bank dummy variables. Across all four specifications, the coefficient on the net index is positive and statistically significant. An increase in the net index (i.e. a hawkish press conference) results in an increase in implied risk aversion. The inclusion of $Ret_t$ and $\Delta IV_t$ modestly decrease the coefficient on the net index. This is to be expected, as some of the information coming from the press conference is captured by the market response. The inclusion of central bank dummy variables has no material impact on the magnitude of the coefficient. Based on the third specification, a one standard deviation increase in the net index results in a 0.8% increase in implied risk aversion.

Table 1.8 reports regression results for the one month variance risk premium. The specifications are identical to the previous case. Across all four specifications, the coefficient on the net index is positive and statistically significant. As before, the inclusion of $Ret_t$ and $\Delta IV_t$. modestly decrease the coefficient on the net index. The inclusion of central bank dummy variables has no material impact. Based on the third specification, a one standard deviation increase in the net index results in a 4.7% increase in the one month variance risk premium. This is economically significant, relative to the average one month variance risk premium of 28.9%.

Table 1.9 reports regression results for the two month variance risk premium. The results follow a similar pattern to the one month case. A one standard deviation increase in the net index results in a 6.6% increase in the two month variance risk premium. This is economically significant and higher than the corresponding one month figure. Table 1.10 reports regression results for the three month variance risk premium. The results are quantitatively similar to the corresponding two month case. The standard errors on the net index coefficient are increasing in the horizon of the variance risk premium. Intuitively, the impact of a single press conference is more difficult to identify over longer time periods.26

26The Swiss Franc outlier is excluded from all calculations. This is discussed further in appendix A.3.
1.4.4 Robustness Checks

This section reports regression results for robustness checks. For all regressions, results are reported for the change in implied risk aversion and the one month variance risk premium. The regressors $Ret_t$ and $\Delta IV_t$ are included in the regression. The regressions are run with and without central bank dummy variables.

The original net index proposed by Apel & Grimaldi (2012) divides the difference between hawkish and dovish phrases by the total number of phrases. This is the proportional net index. The magnitude of the net index now represents the proportion of hawkish or dovish phrases, rather than the absolute number. Table 1.11 reports regression results using the proportional net index. The coefficient on implied risk aversion is positive and statistically significant, while the coefficient on the one month variance risk premium is positive but not statistically significant. As discussed in section 1.3.1, using the proportional net index can materially impact the relative magnitude across observations, and the results are sensitive to this.

Apel & Grimaldi (2012) propose the extended net index as a robustness exercise. The extended net index uses additional nouns to identify hawkish and dovish phrases.\textsuperscript{27} Table 1.12 reports regression results using the extended net index. The results follow the same pattern as the original results, but are slightly weaker. For the implied risk aversion regression, the coefficient on the net index is unchanged. For the one month variance risk premium, the coefficient is 4.2%, down from 4.7%.

As an additional robustness check, I separate implied risk aversion and the variance risk premium into upside and downside components. The purpose of this exercise is to determine whether the impact of the net index on implied risk aversion and the variance risk premium is symmetric. Upside (downside) implied risk aversion is the weighted sum of the risk aversion function conditional on it taking a positive (negative) value.

\textsuperscript{27} The additional nouns are listed in table 1.1.
IRA^u = \sum_r p(r)|ARA(r)|\mathbb{1}_{ARA(r)>0}

IRA^d = \sum_r p(r)|ARA(r)|\mathbb{1}_{ARA(r)<0}

Upside (downside) variance is the expected squared return conditional on a positive (negative) return. Upside and downside variance risk premium are thus defined as follows, as per Feunou et al. (2017).

RV^{u}_{t:t+\tau} = \tau^{-1} \sum_{i=1}^{\tau} R^2_{t+i-1:t+i} \mathbb{1}_{R>0}

RV^{d}_{t:t+\tau} = \tau^{-1} \sum_{i=1}^{\tau} R^2_{t+i-1:t+i} \mathbb{1}_{R<0}

VRP^{u}_{t:t+\tau} = V_{S^u_{t:t+\tau}} - RV^{u}_{t:t+\tau}

VRP^{d}_{t:t+\tau} = V_{S^d_{t:t+\tau}} - RV^{d}_{t:t+\tau}

Table 1.13 reports regression results for the upside and downside change in implied risk aversion. The impact of the net index is slightly higher on downside risk aversion, though the difference is not statistically significant. Table 1.14 reports regression results for upside and downside one month variance risk premium. The impact of the net index is slightly higher on upside variance risk premium, but again the difference is not statistically significant. Both these results suggest that the impact of the net index on implied risk aversion and the variance risk premium is symmetric.

1.5 Conclusion

This paper asks how the tone of central bank press conferences impacts risk premia in the currency market. A hawkish press conference results in an increase in implied risk aversion
and higher variance risk premia. A dovish press conference results in an decrease in implied risk aversion and lower variance risk premia. The results are economically and statistically significant, and robust to the inclusion of relevant control variables. The basic intuition of the result is as follows: The tone of a central bank press conference indicates to investors the likelihood of central bank intervention, conditional on the state of the economy. When a central bank is dovish, it is more likely to intervene in bad economic states, and investors pay less for insurance in those states. When a central bank is hawkish, it is less likely to intervene in bad economic states, and investors pay more for insurance in those states. Three broad themes emerge from this paper.

The first theme is that central banks have some ability to influence risk premia in markets. Central banks may have an incentive to choose their words in such a way as to generate a favorable market reaction. This paper treats central bank tone as exogenous. An avenue of a future research is to analyze the predictability of central bank tone itself, using historical financial and macroeconomic data.

The second theme is that investors should incorporate central bank tone into their portfolio decisions. The best case for this is the difference in the Sharpe ratios of shorting a variance swap for different values of the net index. The Sharpe ratio following a hawkish press conference is approximately double that of a dovish press conference. An example of simple trading strategy is to short variance swaps only if a central bank is hawkish, and stay out of the market otherwise.

The third theme is the value of text based information. Most research in empirical finance deals with quantities, e.g. prices, returns, dividends, etc. An any point in time, an investor making a decision has access to qualitative and quantitative information. The qualitative information typically consists of text, e.g. central bank press conferences, news articles, etc. Text based information poses two challenges, relative to quantitative information. The first is that historical data is not as easily available. The second is that there are multiple methods to quantify text, and quantification involves some loss of information. This paper employs a binary classification scheme for text, i.e. treating phrases as hawkish or dovish. It is encouraging that even a simple method produces valuable insights.
1.6 Acknowledgements

Chapter 1, in full, is currently being prepared for submission for publication of the material. Dossani, Asad. The dissertation author was the sole author of this paper.
These figures plot the net index for each central bank. The net index is the difference between the number of hawkish and dovish phrases made during a press conference, i.e. $NI_t = \#Hawk_t - \#Dove_t$. 

Figure 1.1. Net Index by Central Bank
These four plots illustrate the how the change in implied risk aversion is computed. The observation is the European Central Bank press conference on September 4, 2014. The net index value is -6, or -1.34 standard deviations. The top left figure plots the risk neutral density before and after the press conference. The top right figure plots the physical density before and after the announcement. The bottom left figure plots the pricing kernel before and after the press conference. From this picture, the impact of the press conference is a flatter pricing kernel, particularly in the tails of the distribution. The bottom right figure plots the absolute risk aversion function before and after the press conference. The flatter pricing kernel is reflected in the absolute risk aversion function that is now closer to zero following the press conference. Implied risk aversion is calculated as the probability weighted sum of the distance between the absolute risk aversion function and zero.
Figure 1.3. Net Index Scatter Plots

These figures plot the net index on the horizontal axis and the dependent variable on the vertical axis. Observations for all central banks are pooled together. Each central bank’s net index value is divided by its own standard deviation. The units for the net index is standard deviations, and the units for the dependent variable is percentage points. The four dependent variables are the change in implied risk aversion (top left), the one month variance risk premium (top right), the two month variance risk premium (bottom left), and the three month variance risk premium (bottom right). For each figure, a line of best fit is drawn, and the correlation coefficient $\rho$ is reported in the header. The two and three month variance risk premia figures exclude the Swiss Franc outlier.
### Table 1.1. Adjectives and Nouns

<table>
<thead>
<tr>
<th>Hawkish Adjectives</th>
<th>Dovish Adjectives</th>
<th>Nouns</th>
<th>Nouns Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>increased/increasing</td>
<td>decreased/decreasing</td>
<td>inflation</td>
<td>employment</td>
</tr>
<tr>
<td>fast/faster</td>
<td>slow/slower</td>
<td>cyclical position</td>
<td></td>
</tr>
<tr>
<td>strong/stronger</td>
<td>weak/weaker</td>
<td>growth</td>
<td>unemployment</td>
</tr>
<tr>
<td>high/higher</td>
<td>low/lower</td>
<td>price</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>wages</td>
<td>recovery</td>
</tr>
<tr>
<td></td>
<td></td>
<td>oil price</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>development</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>cost</td>
<td></td>
</tr>
</tbody>
</table>

This table presents the adjectives and nouns used to construct the net index. The net index is calculated as the difference between the number of hawkish and dovish phrases made during a press conference. A hawkish phrase consists of a hawkish adjective and a noun. A dovish phrase consists of a dovish adjective and a noun. Any combination of adjective and noun is considered a phrase. For example, higher growth is a hawkish phrase, while lower inflation is a dovish phrase. The final column contains an additional list of nouns used to construct the extended net index.

### Table 1.2. Central Bank Press Conference Summary Statistics

<table>
<thead>
<tr>
<th>Bank</th>
<th>PC/year</th>
<th>N</th>
<th>Avg Len</th>
<th>CV Len</th>
<th>Avg NI</th>
<th>Std Dev NI</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOE</td>
<td>4</td>
<td>26</td>
<td>11,206</td>
<td>0.516</td>
<td>−1.385</td>
<td>3.187</td>
</tr>
<tr>
<td>BOC</td>
<td>4</td>
<td>26</td>
<td>740</td>
<td>0.255</td>
<td>−0.615</td>
<td>1.041</td>
</tr>
<tr>
<td>ECB</td>
<td>8</td>
<td>73</td>
<td>6,749</td>
<td>0.110</td>
<td>−2.311</td>
<td>4.475</td>
</tr>
<tr>
<td>SNB</td>
<td>2</td>
<td>12</td>
<td>1,669</td>
<td>0.244</td>
<td>−0.583</td>
<td>1.256</td>
</tr>
<tr>
<td>FED</td>
<td>4</td>
<td>20</td>
<td>8,347</td>
<td>0.149</td>
<td>−1.000</td>
<td>1.817</td>
</tr>
</tbody>
</table>

This table reports summary statistics for central bank press conferences. The central banks are: Bank of England (BOE), Bank of Canada (BOC), European Central Bank (ECB), Swiss National Bank (SNB), US Federal Reserve (FED). The sample period is June 2009 until December 2015. The sample size is 157. The following statistics are reported for each central bank: the number of press conferences per year, the total sample size, the average length (number of words) of each press conference, the corresponding coefficient of variation (standard deviation divided by average length), the average value of the net index, and the corresponding standard deviation.
This table reports parameter estimates and robust standard errors of the Realized GARCH model for a single observation, the euro futures contract on September 4, 2014. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. Estimation is by maximum likelihood using the rugarch package in R. The Realized GARCH model incorporates measurements of volatility based on intraday high frequency data, and it allows for an asymmetric response of volatility to positive and negative shocks, i.e. leverage effects. Let $y_t$ be the daily return on the futures contract, $h_t$ be the latent volatility process, and $x_t$ be the daily realized volatility constructed from five minute returns. $\epsilon_t$ and $v_t$ are mean zero i.i.d. innovations. The model consists of three equations: the return equation: $y_t = \mu + \sqrt{h_t} \epsilon_t$, the GARCH equation: $\log h_t = \omega + \sum_{i=1}^{q} \alpha_i \log h_{t-i} + \sum_{i=1}^{p} \beta_i \log x_{t-i}$, and the measurement equation: $\log x_t = \zeta + \delta \log h_t + \eta_1 \epsilon_t + \eta_2 (\epsilon_t^2 - 1) + v_t$. I set $p = 1$ and $q = 2$. This is the preferred specification of Hansen et al. (2012). I assume $\epsilon_t$ is student t distributed with variance one and shape $\nu$, and $v_t$ is normally distributed with variance $\lambda$.

### Table 1.4. Monotonicity Test

<table>
<thead>
<tr>
<th>$\Delta IRA_t$</th>
<th>$VRP_{t,t+30}$</th>
<th>$VRP_{t,t+60}$</th>
<th>$VRP_{t,t+90}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>0.068</td>
<td>0.000</td>
<td>0.002</td>
</tr>
</tbody>
</table>

This table reports p-values from the Patton & Timmermann (2010) test of monotonicity in the four dependent variables: the change in implied risk aversion, the one month variance risk premium, the two month variance risk premium, and the three month variance risk premium. The null hypothesis is that the dependent variable is not monotonically increasing in the net index. The alternative is that the dependent variable is monotonically increasing in the net index. For this test, the observations are sorted by the net index value and split into three equal sized portfolios. Observations with a net index value equal to zero are removed. This is to ensure that the net index is strictly decreasing across most observations.
Table 1.5. Net Index Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>$\Delta IRA_t$</th>
<th>$VRP_{t,t+30}$</th>
<th>$SR_{t,t+30}$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>0.092</td>
<td>28.909</td>
<td>0.560</td>
<td>157</td>
</tr>
<tr>
<td>$NI_t &gt; 0$</td>
<td>0.749</td>
<td>35.624</td>
<td>0.856</td>
<td>29</td>
</tr>
<tr>
<td>$NI_t = 0$</td>
<td>0.752</td>
<td>40.660</td>
<td>0.712</td>
<td>42</td>
</tr>
<tr>
<td>$NI_t &lt; 0$</td>
<td>-0.453</td>
<td>20.906</td>
<td>0.414</td>
<td>86</td>
</tr>
</tbody>
</table>

This is the first of two tables that reports summary statistics for the net index. The statistics are reported for the full sample, and three subsamples of the net index (positive, zero, negative). The sample period is June 2009 until December 2015. The summary statistics consist of the change in implied risk aversion, the one month variance risk premium, the one month Sharpe Ratio, and the sample size.

Table 1.6. Net Index Summary Statistics: Two and Three Month VRP

<table>
<thead>
<tr>
<th>Include Outlier</th>
<th>$VRP_{t,t+60}$</th>
<th>$SR_{t,t+60}$</th>
<th>$VRP_{t,t+90}$</th>
<th>$SR_{t,t+90}$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>13.750</td>
<td>0.074</td>
<td>19.349</td>
<td>0.142</td>
<td>157</td>
</tr>
<tr>
<td>$NI_t &gt; 0$</td>
<td>40.130</td>
<td>1.051</td>
<td>43.238</td>
<td>1.047</td>
<td>29</td>
</tr>
<tr>
<td>$NI_t = 0$</td>
<td>40.734</td>
<td>0.756</td>
<td>42.428</td>
<td>0.832</td>
<td>42</td>
</tr>
<tr>
<td>$NI_t &lt; 0$</td>
<td>-8.324</td>
<td>-0.034</td>
<td>0.023</td>
<td>0.000</td>
<td>86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exclude Outlier</th>
<th>$VRP_{t,t+60}$</th>
<th>$SR_{t,t+60}$</th>
<th>$VRP_{t,t+90}$</th>
<th>$SR_{t,t+90}$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>27.965</td>
<td>0.515</td>
<td>28.916</td>
<td>0.442</td>
<td>156</td>
</tr>
<tr>
<td>$NI_t &gt; 0$</td>
<td>40.130</td>
<td>1.051</td>
<td>43.238</td>
<td>1.047</td>
<td>29</td>
</tr>
<tr>
<td>$NI_t = 0$</td>
<td>40.734</td>
<td>0.756</td>
<td>42.428</td>
<td>0.832</td>
<td>42</td>
</tr>
<tr>
<td>$NI_t &lt; 0$</td>
<td>17.489</td>
<td>0.307</td>
<td>17.354</td>
<td>0.230</td>
<td>85</td>
</tr>
</tbody>
</table>

This is the second of two tables that reports summary statistics for the net index. The statistics are reported for the full sample, and three subsamples of the net index (positive, zero, negative). The sample period is June 2009 until December 2015. The summary statistics consist of the two month variance risk premium, the two month Sharpe Ratio, the three month variance risk premium, the three month Sharpe Ratio, and the sample size. The top panel reports results including the outlier, and the bottom panel reports results excluding the outlier. The outlier is discussed further in appendix A.3.
Table 1.7. Regression Results: IRA

<table>
<thead>
<tr>
<th>Dependent Variable ( Y_t )</th>
<th>( \Delta IRA_t )</th>
<th>( \Delta IRA_{t-1} )</th>
<th>( \Delta IRA_{t-2} )</th>
<th>( \Delta IRA_{t-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.026</td>
<td>-1.416**</td>
<td>0.013</td>
<td>-1.284*</td>
</tr>
<tr>
<td></td>
<td>(0.533)</td>
<td>(0.699)</td>
<td>(0.514)</td>
<td>(0.682)</td>
</tr>
<tr>
<td>( NI_t )</td>
<td>0.903***</td>
<td>0.908***</td>
<td>0.845***</td>
<td>0.863***</td>
</tr>
<tr>
<td></td>
<td>(0.327)</td>
<td>(0.314)</td>
<td>(0.324)</td>
<td>(0.311)</td>
</tr>
<tr>
<td>( Ret_t )</td>
<td>0.084</td>
<td>0.055</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.436)</td>
<td>(0.450)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta IV_t )</td>
<td>0.035*</td>
<td>0.028</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.165)</td>
<td>(0.166)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V R P_{t-31,t-1} )</td>
<td>0.023***</td>
<td>0.021***</td>
<td>0.027**</td>
<td>0.024***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>( \Delta IV_{t-31,t-1} )</td>
<td>-0.004</td>
<td>-0.005</td>
<td>0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.145</td>
<td>0.177</td>
<td>0.161</td>
<td>0.188</td>
</tr>
<tr>
<td>( N )</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

\[ Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 V R P_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t \]

This table reports results from the above regression for the change in implied risk aversion. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. \( NI_t \) is the net index, \( Ret_t \) is the daily return, \( \Delta IV_t \) is the daily change in implied volatility, \( V R P_{t-31,t-1} \) is the prior month variance risk premium, and \( \Delta IV_{t-31,t-1} \) is the prior month change in implied volatility. \( CB_t \) consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and second regressions, the restriction \( \gamma_1 = \gamma_2 = 0 \) is imposed. \( \gamma_1 \) and \( \gamma_2 \) are unrestricted in the third and fourth regressions. In the first and third regressions, the restriction \( \delta = 0 \) is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, \( \delta \) is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant.
### Table 1.8. Regression Results: 1M VRP

<table>
<thead>
<tr>
<th>Dependent Variable ($Y_t$)</th>
<th>$VRP_{t,t+30}$</th>
<th>$VRP_{t,t+30}$</th>
<th>$VRP_{t,t+30}$</th>
<th>$VRP_{t,t+30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>19.705***</td>
<td>10.036</td>
<td>19.776***</td>
<td>10.649</td>
</tr>
<tr>
<td>$NI_t$</td>
<td>5.757**</td>
<td>5.614**</td>
<td>4.703**</td>
<td>4.527**</td>
</tr>
<tr>
<td></td>
<td>(2.401)</td>
<td>(2.246)</td>
<td>(2.261)</td>
<td>(2.109)</td>
</tr>
<tr>
<td>$Ret_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.463</td>
<td>5.882</td>
<td>(3.549)</td>
<td>(3.668)</td>
</tr>
<tr>
<td>$∆IV_t$</td>
<td>0.399**</td>
<td>0.376**</td>
<td>(0.165)</td>
<td>(0.166)</td>
</tr>
<tr>
<td>$VRP_{t-31,t-1}$</td>
<td>0.515***</td>
<td>0.484***</td>
<td>0.552***</td>
<td>0.523***</td>
</tr>
<tr>
<td></td>
<td>(0.107)</td>
<td>(0.107)</td>
<td>(0.115)</td>
<td>(0.119)</td>
</tr>
<tr>
<td>$∆IV_{t-31,t-1}$</td>
<td>0.356***</td>
<td>0.325***</td>
<td>0.428***</td>
<td>0.397***</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.105)</td>
<td>(0.103)</td>
<td>(0.109)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.163</td>
<td>0.178</td>
<td>0.177</td>
<td>0.190</td>
</tr>
<tr>
<td>$N$</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 ∆IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 ∆IV_{t-31,t-1} + \delta CB_t + \varepsilon_t$

This table reports results from the above regression for the one month variance risk premium. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. $NI_t$ is the net index, $Ret_t$ is the daily return, $∆IV_t$ is the daily change in implied volatility, $VRP_{t-31,t-1}$ is the prior month variance risk premium, and $∆IV_{t-31,t-1}$ is the prior month change in implied volatility. $CB_t$ consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and second regressions, the restriction $\gamma_1 = \gamma_2 = 0$ is imposed. $\gamma_1$ and $\gamma_2$ are unrestricted in the third and fourth regressions. In the first and third regressions, the restriction $\delta = 0$ is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, $\delta$ is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant.
Table 1.9. Regression Results: 2M VRP

<table>
<thead>
<tr>
<th>Dependent Variable ((Y_t))</th>
<th>(VRP_{t,t+60})</th>
<th>(VRP_{t,t+60})</th>
<th>(VRP_{t,t+60})</th>
<th>(VRP_{t,t+60})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>17.220***</td>
<td>12.105</td>
<td>17.302***</td>
<td>12.012</td>
</tr>
<tr>
<td></td>
<td>(3.908)</td>
<td>(8.554)</td>
<td>(3.927)</td>
<td>(8.558)</td>
</tr>
<tr>
<td>(NI_t)</td>
<td>7.077**</td>
<td>7.064***</td>
<td>6.559**</td>
<td>6.245**</td>
</tr>
<tr>
<td></td>
<td>(2.773)</td>
<td>(2.670)</td>
<td>(2.961)</td>
<td>(2.733)</td>
</tr>
<tr>
<td>(Ret_t)</td>
<td></td>
<td></td>
<td>3.278</td>
<td>5.522</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5.935)</td>
<td>(4.963)</td>
</tr>
<tr>
<td>(\Delta IV_t)</td>
<td>0.224</td>
<td>0.376**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.398)</td>
<td>(0.340)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(VRP_{t-31,t-1})</td>
<td>0.611***</td>
<td>0.627***</td>
<td>0.646***</td>
<td>0.523***</td>
</tr>
<tr>
<td></td>
<td>(0.101)</td>
<td>(0.103)</td>
<td>(0.115)</td>
<td>(0.107)</td>
</tr>
<tr>
<td>(\Delta IV_{t-31,t-1})</td>
<td>0.353***</td>
<td>0.360***</td>
<td>0.383**</td>
<td>0.407***</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.108)</td>
<td>(0.151)</td>
<td>(0.126)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.217</td>
<td>0.242</td>
<td>0.219</td>
<td>0.248</td>
</tr>
<tr>
<td>(N)</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
</tr>
</tbody>
</table>

\[ Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t \]

This table reports results from the above regression for the two month variance risk premium. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. \(NI_t\) is the net index, \(Ret_t\) is the daily return, \(\Delta IV_t\) is the daily change in implied volatility, \(VRP_{t-31,t-1}\) is the prior month variance risk premium, and \(\Delta IV_{t-31,t-1}\) is the prior month change in implied volatility. \(CB_t\) consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and second regressions, the restriction \(\gamma_1 = \gamma_2 = 0\) is imposed. \(\gamma_1\) and \(\gamma_2\) are unrestricted in the third and fourth regressions. In the first and third regressions, the restriction \(\delta = 0\) is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, \(\delta\) is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant. Results are reported without the Swiss Franc outlier, which is discussed further in appendix A.3.
Table 1.10. Regression Results: 3M VRP

<table>
<thead>
<tr>
<th>Dependent Variable ($Y_t$)</th>
<th>$VRP_{t,t+90}$</th>
<th>$VRP_{t,t+90}$</th>
<th>$VRP_{t,t+90}$</th>
<th>$VRP_{t,t+90}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>16.875***</td>
<td>10.235</td>
<td>16.910***</td>
<td>10.082</td>
</tr>
<tr>
<td></td>
<td>(4.164)</td>
<td>(9.069)</td>
<td>(4.158)</td>
<td>(9.023)</td>
</tr>
<tr>
<td>$NI_t$</td>
<td>7.234**</td>
<td>7.240**</td>
<td>6.984*</td>
<td>6.513*</td>
</tr>
<tr>
<td></td>
<td>(3.194)</td>
<td>(3.216)</td>
<td>(3.933)</td>
<td>(3.633)</td>
</tr>
<tr>
<td>$Ret_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.525</td>
<td>5.029</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.815)</td>
<td>(5.636)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta IV_t$</td>
<td>0.080</td>
<td>0.191</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.535)</td>
<td>(0.421)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$VRP_{t-31,t-1}$</td>
<td>0.668***</td>
<td>0.708***</td>
<td>0.723***</td>
<td>0.523***</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.115)</td>
<td>(0.115)</td>
<td>(0.120)</td>
</tr>
<tr>
<td>$\Delta IV_{t-31,t-1}$</td>
<td>0.440***</td>
<td>0.465***</td>
<td>0.505***</td>
<td>0.407***</td>
</tr>
<tr>
<td></td>
<td>(0.155)</td>
<td>(0.130)</td>
<td>(0.227)</td>
<td>(0.173)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.172</td>
<td>0.216</td>
<td>0.172</td>
<td>0.219</td>
</tr>
<tr>
<td>$N$</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t$

This table reports results from the above regression for the three month variance risk premium. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. $NI_t$ is the net index, $Ret_t$ is the daily return, $\Delta IV_t$ is the daily change in implied volatility, $VRP_{t-31,t-1}$ is the prior month variance risk premium, and $\Delta IV_{t-31,t-1}$ is the prior month change in implied volatility. $CB_t$ consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and second regressions, the restriction $\gamma_1 = \gamma_2 = 0$ is imposed. $\gamma_1$ and $\gamma_2$ are unrestricted in the third and fourth regressions. In the first and third regressions, the restriction $\delta = 0$ is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, $\delta$ is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant. Results are reported without the Swiss Franc outlier, which is discussed further in appendix A.3.
Table 1.11. Regression Results: Proportional Net Index

<table>
<thead>
<tr>
<th>Dependent Variable ($Y_t$)</th>
<th>$\Delta IRA_t$</th>
<th>$\Delta IRA_{t+30}$</th>
<th>$VRP_{t+30}$</th>
<th>$VRP_{t+30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.178</td>
<td>-1.470**</td>
<td>17.817***</td>
<td>8.542</td>
</tr>
<tr>
<td></td>
<td>(0.501)</td>
<td>(0.676)</td>
<td>(4.810)</td>
<td>(12.917)</td>
</tr>
<tr>
<td>$NI^p_t$</td>
<td>0.622*</td>
<td>0.622*</td>
<td>2.820</td>
<td>2.750</td>
</tr>
<tr>
<td></td>
<td>(0.355)</td>
<td>(0.332)</td>
<td>(4.670)</td>
<td>(4.685)</td>
</tr>
<tr>
<td>$Ret_t$</td>
<td>0.111</td>
<td>0.086</td>
<td>5.940*</td>
<td>6.372*</td>
</tr>
<tr>
<td></td>
<td>(0.436)</td>
<td>(0.448)</td>
<td>(3.588)</td>
<td>(3.695)</td>
</tr>
<tr>
<td>$\Delta IV_t$</td>
<td>0.037*</td>
<td>0.030*</td>
<td>0.424**</td>
<td>0.401**</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.169)</td>
<td>(0.170)</td>
</tr>
<tr>
<td>$VRP_{t-31,t-1}$</td>
<td>0.030***</td>
<td>0.027***</td>
<td>0.575***</td>
<td>0.546***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.118)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>$\Delta IV_{t-31,t-1}$</td>
<td>0.005</td>
<td>0.003</td>
<td>0.453***</td>
<td>0.422***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.106)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.143</td>
<td>0.168</td>
<td>0.171</td>
<td>0.185</td>
</tr>
<tr>
<td>$N$</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta NI^p_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t$

This table reports results from the above regression for two dependent variables: the change in implied risk aversion and the one month variance risk premium. It reproduces the results in the third and fourth columns of tables 1.7 and 1.8, using the proportional net index instead of the regular net index. The proportional net index is based on the proportion of hawkish or dovish phrases in a press conference, rather than the number of phrases. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. $NI^p_t$ is the proportional net index, $Ret_t$ is the daily return, $\Delta IV_t$ is the daily change in implied volatility, $VRP_{t-31,t-1}$ is the prior month variance risk premium, and $\Delta IV_{t-31,t-1}$ is the prior month change in implied volatility. CB$t$ consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and third regressions, the restriction $\delta = 0$ is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, $\delta$ is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant.
Table 1.12. Regression Results: Extended Net Index

<table>
<thead>
<tr>
<th>Dependent Variable ($Y_t$)</th>
<th>$\Delta IRA_t$</th>
<th>$\Delta IRA_t$</th>
<th>$VRP_{t,t+30}$</th>
<th>$VRP_{t,t+30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.073</td>
<td>-1.407**</td>
<td>19.257***</td>
<td>9.828</td>
</tr>
<tr>
<td></td>
<td>(0.498)</td>
<td>(0.684)</td>
<td>(4.659)</td>
<td>(12.908)</td>
</tr>
<tr>
<td>$NI^e_t$</td>
<td>0.768**</td>
<td>0.808***</td>
<td>4.205*</td>
<td>3.954*</td>
</tr>
<tr>
<td></td>
<td>(0.327)</td>
<td>(0.302)</td>
<td>(2.226)</td>
<td>(2.211)</td>
</tr>
<tr>
<td>$Ret_t$</td>
<td>0.103</td>
<td>0.069</td>
<td>5.585</td>
<td>6.013</td>
</tr>
<tr>
<td></td>
<td>(0.436)</td>
<td>(0.449)</td>
<td>(3.629)</td>
<td>(3.763)</td>
</tr>
<tr>
<td>$\Delta IV_t$</td>
<td>0.036*</td>
<td>0.029</td>
<td>0.406**</td>
<td>0.384**</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.018)</td>
<td>(0.168)</td>
<td>(0.168)</td>
</tr>
<tr>
<td>$VRP_{t-31,t-1}$</td>
<td>0.028***</td>
<td>0.025***</td>
<td>0.559***</td>
<td>0.530***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.115)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>$\Delta IV_{t-31,t-1}$</td>
<td>0.003</td>
<td>0.000</td>
<td>0.432***</td>
<td>0.403***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.103)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.155</td>
<td>0.183</td>
<td>0.176</td>
<td>0.189</td>
</tr>
<tr>
<td>$N$</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta NI^e_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t$

This table reports results from the above regression for two dependent variables: the change in implied risk aversion and the one month variance risk premium. It reproduces the results in the third and fourth columns of tables 1.7 and 1.8, using the extended net index instead of the regular net index. The extended net index includes additional nouns, as presented in table 1.1, to identify hawkish and dovish phrases. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. $NI^e_t$ is the extended net index, $Ret_t$ is the daily return, $\Delta IV_t$ is the daily change in implied volatility, $VRP_{t-31,t-1}$ is the prior month variance risk premium, and $\Delta IV_{t-31,t-1}$ is the prior month change in implied volatility. $CB_t$ consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and third regressions, the restriction $\delta = 0$ is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, $\delta$ is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant.
Table 1.13. Regression Results: IRA upside & downside

<table>
<thead>
<tr>
<th>Dependent Variable ($Y_t$)</th>
<th>$\Delta IRA_u$</th>
<th>$\Delta IRA_u^*$</th>
<th>$\Delta IRA_d$</th>
<th>$\Delta IRA_d^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>−0.091</td>
<td>−1.173</td>
<td>0.169</td>
<td>−1.440</td>
</tr>
<tr>
<td></td>
<td>(0.437)</td>
<td>(0.601)</td>
<td>(0.632)</td>
<td>(0.786)</td>
</tr>
<tr>
<td>$NI_t$</td>
<td>0.815***</td>
<td>0.826***</td>
<td>0.879***</td>
<td>0.905***</td>
</tr>
<tr>
<td></td>
<td>(0.289)</td>
<td>(0.281)</td>
<td>(0.373)</td>
<td>(0.355)</td>
</tr>
<tr>
<td>$Ret_t$</td>
<td>0.216</td>
<td>0.199</td>
<td>−0.090</td>
<td>−0.134</td>
</tr>
<tr>
<td></td>
<td>(0.395)</td>
<td>(0.409)</td>
<td>(0.497)</td>
<td>(0.510)</td>
</tr>
<tr>
<td>$\Delta IV_t$</td>
<td>0.024</td>
<td>0.019</td>
<td>0.047**</td>
<td>0.038**</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$VRP_{t-31,t-1}$</td>
<td>0.021***</td>
<td>0.019**</td>
<td>0.034***</td>
<td>0.031***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\Delta IV_{t-31,t-1}$</td>
<td>−0.001</td>
<td>−0.003</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.164</td>
<td>0.186</td>
<td>0.150</td>
<td>0.181</td>
</tr>
<tr>
<td>$N$</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-31,t-1} + \gamma_4 \Delta IV_{t-31,t-1} + \delta CB_t + \epsilon_t$

This table reports results from the above regression for two dependent variables: the change in upside implied risk aversion and the change in downside implied risk aversion. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. $NI_t$ is the net index, $Ret_t$ is the daily return, $\Delta IV_t$ is the daily change in implied volatility, $VRP_{t-31,t-1}$ is the prior month variance risk premium, and $\Delta IV_{t-31,t-1}$ is the prior month change in implied volatility. $CB_t$ consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and third regressions, the restriction $\delta = 0$ is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, $\delta$ is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant.
**Table 1.14. Regression Results: 1M VRP upside & downside**

<table>
<thead>
<tr>
<th>Dependent Variable ((Y_t))</th>
<th>(VRP^{\mu}_{t,t+30})</th>
<th>(VRP^{\mu\mu}_{t,t+30})</th>
<th>(VRP^{\mu d}_{t,t+30})</th>
<th>(VRP^{dd}_{t,t+30})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>13.531***</td>
<td>3.232</td>
<td>10.231***</td>
<td>9.162**</td>
</tr>
<tr>
<td></td>
<td>(2.133)</td>
<td>(9.219)</td>
<td>(2.954)</td>
<td>(4.345)</td>
</tr>
<tr>
<td>(NI_t)</td>
<td>3.394**</td>
<td>3.133**</td>
<td>2.814*</td>
<td>2.856*</td>
</tr>
<tr>
<td></td>
<td>(1.577)</td>
<td>(1.474)</td>
<td>(1.575)</td>
<td>(1.531)</td>
</tr>
<tr>
<td>(Ret_t)</td>
<td>4.225*</td>
<td>4.325*</td>
<td>-1.430</td>
<td>-1.223</td>
</tr>
<tr>
<td></td>
<td>(2.466)</td>
<td>(2.596)</td>
<td>(2.310)</td>
<td>(2.245)</td>
</tr>
<tr>
<td>(\Delta IV_t)</td>
<td>0.464</td>
<td>0.398</td>
<td>-0.042</td>
<td>-0.039</td>
</tr>
<tr>
<td></td>
<td>(0.326)</td>
<td>(0.333)</td>
<td>(0.211)</td>
<td>(0.198)</td>
</tr>
<tr>
<td>(VRP_{t-1,T,t-1})</td>
<td>0.124</td>
<td>0.115</td>
<td>0.438***</td>
<td>0.405***</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.098)</td>
<td>(0.110)</td>
<td>(0.120)</td>
</tr>
<tr>
<td>(\Delta IV_{t-1,T,t-1})</td>
<td>-0.026</td>
<td>-0.045</td>
<td>0.489***</td>
<td>0.445**</td>
</tr>
<tr>
<td></td>
<td>(0.143)</td>
<td>(0.148)</td>
<td>(0.171)</td>
<td>(0.180)</td>
</tr>
<tr>
<td>CB Dummies</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.050</td>
<td>0.077</td>
<td>0.110</td>
<td>0.130</td>
</tr>
<tr>
<td>(N)</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
</tbody>
</table>

\[Y_t = \alpha + \beta NI_t + \gamma_1 Ret_t + \gamma_2 \Delta IV_t + \gamma_3 VRP_{t-1,T,t-1} + \gamma_4 \Delta IV_{t-1,T,t-1} + \delta CB_t + \epsilon_t\]

This table reports results from the above regression for two dependent variables: the one month upside variance risk premium and the one month downside variance risk premium. Each central bank’s net index is divided by its own standard deviation. Thus, the coefficient on the net index can be interpreted as the affect of a one standard deviation increase in hawkishness. Results are reported with and without central bank dummy variables. Coefficients are reported with Newey West standard errors in parentheses. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is June 2009 until December 2015. \(NI_t\) is the net index, \(Ret_t\) is the daily return, \(\Delta IV_t\) is the daily change in implied volatility, \(VRP_{t-1,T,t-1}\) is the prior month variance risk premium, and \(\Delta IV_{t-1,T,t-1}\) is the prior month change in implied volatility. \(CB_t\) consists of central bank dummy variables for the BOE, BOC, ECB, and SNB. In the first and third regressions, the restriction \(\delta = 0\) is imposed, meaning that each central bank has the same constant term. In the second and fourth regressions, \(\delta\) is unrestricted, so the constant term corresponds to the FED. The dummy variable coefficients are not reported as they are insignificant.
Chapter 2

Monetary Stimulus and Perception of Risk

Abstract

This paper investigates the relationship between monetary stimulus and the perception of risk in financial markets, and how this varies across asset classes. First, I document a negative relationship between monetary stimulus and the perception of risk in equity, currency, and commodity markets. For example, a one standard deviation increase in monetary stimulus implies a 0.636% decrease in S&P 500 implied volatility. In contrast, I document a positive relationship between monetary stimulus and the perception of risk in bond markets. Second, I identify a cointegrating relationship between monetary stimulus and implied volatility across asset classes, indicating a positive long run relationship in the levels of monetary stimulus and implied volatility. Third, I document a positive relationship between monetary stimulus and inflation expectations, a possible explanation for why monetary stimulus impacts equity and bond markets in opposite ways.

2.1 Introduction

How does monetary stimulus drive the perception of risk financial markets? The financial crisis of 2008 and subsequent recession led to unprecedented levels of monetary stimulus. The risk taking channel of monetary policy suggests that monetary stimulus should reduce market participants’ perception of risk, and thereby encourage risk taking. This is one of the mechanisms
by which monetary policy is effective during downturns. This paper studies the relationship between monetary stimulus and the perception of risk in financial markets, and how this varies across asset classes. I document a negative relationship between monetary stimulus and the perception of risk in equity markets, and a positive relationship between monetary stimulus and the perception of risk in bond markets. For example, a one standard deviation increase in monetary stimulus is associated with a 0.636% decrease in S&P 500 implied volatility. In addition, I document a negative relationship between monetary stimulus and the perception of risk in commodity and currency markets, but to a smaller degree relative to equity markets.


Between December 2008 and December 2015, the Federal Funds rate reached the zero lower bound (ZLB). During this time, a variety unconventional monetary policies were implemented, including asset purchases and forward guidance. To effectively measure monetary policy at the ZLB, Wu & Xia (2016) proposed the shadow short rate (SSR), estimated using a Shadow Gaussian affine term structure model GATSM. The short term interest rate is the maximum of the SSR and zero. The SSR is allowed to take on negative values, and thus varies considerably during the ZLB period. One of the drawbacks of using the SSR is its lack of robustness, meaning that it displays considerable variation depending on the specification of the term structure model used. It is also not directly comparable between ZLB and non ZLB periods, as negative short rates are not interest rates at which agents can actually transact at. As an alternative, Krippner (2014) proposes a measure of monetary policy called Effective Monetary Stimulus (EMS). This measure integrates the difference between expected actual interest rates relative to the neutral interest rate, as estimated from the Shadow GATSM. The measure is directly comparable between ZLB and non ZLB periods, and is robust relative to the
specification of the Shadow GATSM used. In this paper, I use EMS as the independent variable to summarize the stance of monetary policy.

Perceptions of risk in financial markets are measured from option prices and realized returns. From option prices, I compute implied volatility and implied skewness. All else equal, an increase in volatility is an increase in the perception of risk, while an increase in skewness is a decrease in the perception of risk. I use the term ‘perception of risk’ rather than simply ‘risk’ because of the forward looking nature of option prices. Both implied volatility and implied skewness reflect expectations over future risk, rather than the realization of risk. Equity, commodity, and currency markets exhibit a negative relationship between monetary stimulus and implied volatility. In bond markets, monetary stimulus has no significant impact on implied variance. Equity markets exhibit a positive relationship between monetary stimulus and skewness, while bond markets exhibit a negative relationship between monetary stimulus and skewness. In commodity and currency markets, monetary stimulus has no significant impact on skewness.

In addition, I establish a cointegrating relationship between monetary stimulus and implied volatility across asset classes. This implies the existence of a long run equilibrium relationship between the levels of implied volatility and monetary stimulus. In the long run, there is a positive relationship between monetary stimulus and implied volatility, and this holds across asset classes. Over the sample period, as economic conditions have improved, the levels of monetary stimulus and implied volatility have declined over time.

The broad summary of these results are as follows: Monetary stimulus is associated with a decline in the perception of risk in equity, currency, and commodity markets. And it is associated with an increase in the perception of risk in bond markets. Risky assets become relatively more attractive than bonds.

Separate from the perception of risk, I examine how monetary stimulus impacts average returns and the distribution of returns. In equity markets, monetary stimulus is associated with positive average returns, and a relatively lower likelihood of large negative returns. In bond
markets, monetary stimulus is associated with negative average returns, and a relatively higher likelihood large negative returns. Both these findings are consistent with idea that risk falls for equities, but rises for bonds. Commodities display similar results to equity markets, but are considerably weaker. In currency markets, monetary stimulus is associated with positive average returns, and a relatively higher likelihood of large positive returns. This implies a positive relationship between monetary stimulus and the likelihood of US dollar deprecation, consistent with standard economic intuition.

What can explain these results? One possibility is the role of inflation expectations. One of the primary risks associated with expansionary monetary policy is increased inflation. During the ZLB period, many market participants believed that unconventional monetary policy would lead to higher inflation, though this did not materialize in a significant way. For example, Gambacorta et al. (2014) show that unconventional monetary policy leads to an increase in inflation, though the impact is weaker relative to conventional monetary policy. If inflation expectations are higher, this increases the attractiveness of risky assets relative to safe assets. Bonds are vulnerable to higher inflation, as they payout a fixed nominal return. I document a positive relationship between monetary stimulus and inflation expectations. This result makes intuitive sense, as expansionary monetary policy should increase future inflation. Inflation expectations are derived from yields on constant maturity Treasury securities and inflation indexed constant maturity Treasury securities. If inflation expectations rise in response to monetary stimulus, this explains why equities and other risky assets become more attractive relative to bonds.

In an additional exercise, I examine whether monetary stimulus predicts future returns. Why should predictability exist? Suppose there exists a risk return relationship, meaning that all else equal, an increase in risk today means that future returns should be higher, and vice versa. Then if monetary stimulus has a positive relationship with risk, it should predict higher future returns, and vice versa. The results indicate weak evidence for return predictability. The results are strongest in bond markets, where an increase in monetary stimulus predicts higher
future returns. In equity markets, an increase in monetary stimulus predicts lower future returns. Both these results are consistent with the risk return relationship. There is no evidence of return predictability in commodity or currency markets.

This paper makes the following contributions: First, I document a positive relationship between monetary stimulus and the perception of risk in equity, commodity, and currency markets. I document a negative relationship between monetary stimulus and the perception of risk in bond markets. Second, I establish a cointegrating relationship between monetary stimulus and implied volatility, indicating a positive long run equilibrium relationship in the levels of monetary stimulus and implied volatility. This relationship is present across asset classes. Third, I document the link between monetary stimulus and expected inflation, a possible mechanism by which monetary stimulus affects the perception of risk across financial markets. The remainder of the paper is organized as follows: Section 2.2 describes the data, Section 2.3 proposes the methodology, Section 2.4 presents the results, and Section 2.5 concludes.

2.2 Data

2.2.1 Effective Monetary Stimulus

Krippner (2014) proposes effective monetary stimulus (EMS) as a measure of the stance of monetary policy. The EMS is derived from a two factor shadow term structure model. Specifically, the K-ANSM(2), with a fixed 12.5 basis point lower bound, and yield curve data with maturities from 0.25 to 30 years. The EMS is the integral of the difference between the expected path of the shadow short rate (SSR) truncated at zero, and the long run neutral rate. Wu & Xia (2016) propose the shadow short rate (SSR) as a measure of the stance of monetary policy at the zero lower bound (ZLB). The SSR is the shortest maturity rate estimated from the shadow yield curve. It is effectively equal to the policy rate (i.e. Fed funds rate) during non ZLB periods. However, during ZLB periods, the SSR can take on negative values. Krippner (2014) argues that the EMS is a better measure of the stance of monetary policy relative to the SSR
for the following reasons: First, it is directly comparable between ZLB and non ZLB periods. This is not the case for the SSR, as negative shadow rates do not represent actual interest rates faced by economic agents. Second, the EMS is more robust than the SSR, with respect to model specification, data used, and estimation method.

EMS and the corresponding SSR are computed at the daily frequency, starting in November 1985. The data come from Leo Krippner’s website.\footnote{https://www.rbnz.govt.nz/research-and-publications/research-programme/additional-research/measures-of-the-stance-of-united-states-monetary-policy} Figure 2.1 plots the EMS and SSR. The plots are from June 2009 onwards, the time period corresponding to the option price data. The units of EMS are SSR are percentage points. Positive values of EMS imply monetary stimulus, while negative values imply monetary contraction. The EMS is positive throughout the sample. In the original series, in some years prior to 2009, the EMS takes on negative values. Higher values of the SSR imply tight monetary policy, while lower values imply loose monetary policy. The SSR is negative from the start of the sample until July 2016, and positive since then. From 2009 until 2014, the EMS is at record high levels, corresponding to the implementation of unconventional monetary policy. From 2014 until the present, EMS is decreasing while the SSR is increasing. This corresponds to the normalization of monetary policy and eventual move away from the ZLB.

### 2.2.2 Futures

I use futures and options on futures on thirteen underlying assets across four asset classes. The four asset classes are equities, bonds, commodities, and currencies. The equities asset class consists of the S&P 500 and the Nasdaq 100. The bonds asset class consists of the 10 year Treasury bond and the 30 year Treasury bond. The commodities asset class consists of crude oil, gold, and silver. The currencies asset class consists of the Australian Dollar, the British Pound, the Canadian Dollar, the Euro, and the Swiss Franc. All currencies are traded against the US
The futures data are sampled daily, and come from DTN IQFeed. The raw data take the form of continuous futures contracts. This is necessary in order to ensure that returns are correctly calculated. Prices are back-adjusted to create a continuous contract. This works by removing price gaps caused by a contract roll. The process starts at the end of the price series, and works its way back. This leaves current prices intact. Prices prior to the last roll date are adjusted.

### 2.2.3 Options on Futures

I use options on futures on the same thirteen underlying assets across four asset classes. The option data come from OptionWorks via Quandl. Option prices are sampled at the close of each trading day, and have a one month time to expiry. The data start in June 2009 and end in January 2018. The data take the form of one month implied volatility curves.

Implied volatility is a function of the strike price. First, the option price is converted to an implied volatility, $\sigma$, via the Black Scholes model. This is done across the range of traded strike prices, using the Black-Scholes formula. Let $F$ be the futures price, $K$ be the strike price, $T$ be the time to maturity, $\sigma$ be the implied volatility, and $r$ be the interest rate. The call option price $C$ is given by:

\[
\begin{align*}
C &= e^{-rT}\left[FN(d_1) - KN(d_2)\right] \\
\frac{d_1}{\sigma\sqrt{T}} &= \frac{\log(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}} \\
\frac{d_2}{\sigma\sqrt{T}} &= d_1 - \sigma\sqrt{T}
\end{align*}
\]

---

2Prices are quoted as the number of US dollars (USD) per unit of foreign currency. Thus, a price increase is an appreciation of the foreign currency, and a depreciation of the US dollar. A price decrease is a depreciation of the foreign currency, and an appreciation of the US dollar.

3DTN IQFeed is an online provider of live and historical financial market data. Their data is sourced directly from the relevant exchanges.

4Quandl is online data provider of financial and economic data. OptionWorks is one of the databases within Quandl that specializes in options on futures.
Next, a polynomial of degree six is fitted to the data. This produces the implied volatility curve, where implied volatility is a function of the strike price. For convenience, the strike price \( K \), is expressed as moneyness \( M \), or the percentage deviation from the current futures price \( F \).

\[
M \equiv \log\left(\frac{K}{F}\right)
\]

\[
\sigma(K) = b_0 + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + b_5 M^5 + b_6 M^6
\]

The raw data consists of the closing futures price, the six model coefficients, the minimum and maximum moneyness, and the time to expiry. The minimum and maximum moneyness are upper and lower bounds. They reflect the fact that option prices only trade in a certain range of the current futures price. Typically, this includes the 5\(^{th}\) and 95\(^{th}\) quantile of the distribution. The computation of implied volatility is robust to small changes in these quantiles. Most of the information about implied volatility comes from the center of the distribution.

Any strike and implied volatility pair can be converted to a call or put option price via the Black Scholes model. Using implied volatility curves does not require that Black Scholes assumptions hold. The implied volatility can be thought of as a normalized option price. It allows for a more intuitive comparison of option prices across strikes, time to maturity, or underlying assets.

The curves are constructed for a constant one month maturity. Options trade on a monthly cycle. Constant maturity contracts are created by interpolating across traded options with maturities less than and greater than the constant maturity. For example, the one month implied volatility curve is constructed using option prices expiring before and after one month. This is a standard procedure. For example, this method is used to calculate the VIX, the one month implied volatility of the S&P500 index.
2.2.4 Inflation Expectations

Inflation expectations are derived from yields on constant maturity Treasury securities and inflation indexed constant maturity Treasury securities. The returns from inflation-indexed bonds rise with inflation, and thus are effectively hedged against changes in inflation. They pay a fixed real return, rather than a fixed nominal return. This is a market based measure of expected future inflation. I use two series to capture inflation expectations: The 5-year breakeven inflation rate, and the 10-year breakeven inflation rate. These are the implied values of expected inflation over the next 5 and 10 years, respectively.

Note that this measure of inflation expectations includes a potential risk premium. Thus, these are risk neutral expectations of future inflation. If investors are willing to pay a premium to protect against inflation risk, then this would over estimate true expectations. However, changes in risk neutral expectations are likely highly correlated with changes in true expectations.

2.3 Methodology

2.3.1 Implied Volatility

Let $K$ be the strike price, and $C(K)$ and $P(K)$ be the corresponding call and put option prices, respectively. Let $M(K)$ be the minimum of the call and put price. As per Andersen & Bondarenko (2007), the model free implied variance $MFIV$ is given by the following equation:

\[
M(K) \equiv \min[P(K), C(K)]
\]

\[
MFIV = 2 \int_0^\infty \frac{M(K)}{K^2} dK
\]

\[
MFIV \approx 2 \sum_i \frac{M(K_i) K_{i+1} - K_{i-1}}{2K_i^2}
\]

5 Federal Reserve Bank of St. Louis, 5-Year Breakeven Inflation Rate [T5YIE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/T5YIE, March 27, 2018.

6 Federal Reserve Bank of St. Louis, 10-Year Breakeven Inflation Rate [T10YIE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/T10YIE, March 27, 2018.
The integrals are approximated with numerical differencing, across a equally spaced fine grid of strike prices, indexed by $K_i$. This is straightforward to compute, as the raw data take the form of implied volatility curves, allowing the computation of option prices for any strike within the minimum and maximum moneyness. In subsequent regressions, the volatility, $Vol_t$, is defined as the log of the square root of the model free implied variance. Volatility is computed daily, and the $t$ subscript indicates the day.

$$Vol_t \equiv \log \sqrt{MFIV_t}$$ \hfill (2.1)

Why take logs? It turns out that volatility over time is positively skewed. Occasionally, volatility takes on large values, typically during times of market distress. When using the volatility in regressions, the results are affected by these outliers, and the residuals exhibit positive skewness. The log of volatility does the best job of reducing outliers and reducing the skewness of the OLS residuals. Using the log of volatility has the added advantage that the regression coefficients can be interpreted as the percentage change in volatility resulting from a unit change in the independent variable.

### 2.3.2 Implied Skewness

Next, I split the integral into two parts, as per Feunou et al. (2015). The first part is the integral from 0 to $F$, the current futures price. Define this as the downside model free implied variance $MFIV^d$. The second part is the integral from $F$ to $\infty$. Define this as the upside model free implied variance $MFIV^u$.

$$MFIV = 2 \int_0^F \frac{M(K)}{K^2} \, dK + 2 \int_F^\infty \frac{M(K)}{K^2} \, dK$$

$$MFIV^d \equiv 2 \int_0^F \frac{M(K)}{K^2} \, dK$$

$$MFIV^u \equiv 2 \int_F^\infty \frac{M(K)}{K^2} \, dK$$
The skewness, $\text{Skew}_t$, is defined as one hundred times the difference between upside and downside model free implied variance, divided by the total model free implied variance.

$$\text{Skew}_t \equiv 100 \times \frac{MFIV^u_t - MFIV^d_t}{MFIV_t}$$ (2.2)

This measure of skewness was first proposed by Feunou et al. (2011). It is scale-invariant and dimensionless. It is bounded between -100 and +100. They show that this measure is coherent, and satisfies the properties proposed by Groeneveld & Meeden (1984).

### 2.3.3 Volatility Regressions

I consider two models to capture the relationship between monetary stimulus and volatility. The first is a regression model, and the second is a cointegration model. First, note that both monetary stimulus and volatility contain unit roots. The first column of table 2.3 presents p-values from the Dickey Fuller GLS unit root test for monetary stimulus, expected inflation, and the variance of each asset. The null hypothesis is that there is a unit root. The monetary stimulus series has p-value of 0.798, implying the presence of a unit root. For the variance of all of the individual assets, the null hypothesis of a unit root is not rejected at 5% level. For all but two of the individual assets (Japanese Yen and Swiss Franc), the null hypothesis of a unit root is not rejected at 10% level. The second column of table 2.3 reports the autoregressive coefficient, i.e. the estimated value of $\beta$ from the following regression, where $Y_t$ is the dependent variable

$$Y_t = \alpha + \beta Y_{t-1} + \epsilon_t.$$

For all the volatility series, the autoregressive coefficients are typically around 0.97 or 0.98. In a large sample, economic intuition suggests that implied volatility is bounded. However, for a finite sample, implied volatility behaves as if it is a unit root. That is, the implied volatility cannot be statistically distinguished from a unit root. The same logic holds for both monetary stimulus and inflation expectations. Let $\Delta EMS_t$ and $\Delta Vol_t$ be the first difference of $EMS_t$ and
I regress the change in monetary stimulus, $\Delta EMS_t$, on the change in volatility, $\Delta Vol_t$, and a constant. The existence of a constant implies a time trend in the level of volatility, which may or may not exist. Robustness checks indicate that the constant term is insignificant and has no material impact on the coefficients of interest. The regressions are estimated using OLS with Newey West standard errors. In all subsequent regressions, I divide $\Delta EMS_t$ by its own standard deviation. Thus, the coefficient $\beta$ can be interpreted as the impact of a one standard deviation increase in monetary stimulus.

$$\Delta Vol_t = \alpha + \beta \Delta EMS_t + \epsilon_t$$

(2.3)

### 2.3.4 Skewness Regressions

I consider two approaches to measure the impact of monetary stimulus. First, I regress skewness, $Skew_t$, on a constant, the change in monetary stimulus, $\Delta EMS_t$, and lagged skewness, $Skew_{t-1}$. Note that skewness is a persistent variable, but is not a unit root.

$$Skew_t = \alpha + \beta \Delta EMS_t + \gamma Skew_{t-1} + \epsilon_t$$

(2.4)

The second approach is to separately measure the impact of monetary stimulus on upside and downside volatility. While this doesn’t tell us the impact on skewness directly, it does provide us a sense of the magnitude of relative changes in upside and downside volatility, both of which are used to compute skewness. First, I define upside volatility, $Vol^u_t$ as the log of the square root model free upside variance, $MFIV^u_t$, and downside volatility, $Vol^d_t$, as the log of the...
square root of model free downside variance, $MFIV_i^d$.

\[
\begin{align*}
Vol_i^u & \equiv \log \sqrt{MFIV_i^u} \\
Vol_i^d & \equiv \log \sqrt{MFIV_i^d}
\end{align*}
\]

Next, I estimate a regression model separately for upside and downside volatility. The model the same specification as the model for volatility in the previous section.

\[
\begin{align*}
\Delta Vol_i^u &= \alpha + \beta \Delta EMS_t + \epsilon_t \\
\Delta Vol_i^d &= \alpha + \beta \Delta EMS_t + \epsilon_t
\end{align*}
\]

### 2.3.5 Return Regressions

I consider two approaches to examine the impact of monetary stimulus on returns. Note that the daily return, $Ret_t$, is defined as the log difference in the futures price.

\[
Ret_t \equiv \log F_t - \log F_{t-1}
\]

In the first approach, I regress the daily return $Ret_t$ on a constant and the change in monetary stimulus, $\Delta EMS_t$.

\[
Ret_t = \alpha + \beta \Delta EMS_t + \epsilon_t \quad (2.5)
\]

The second approach uses a quantile regression rather than an OLS regression. In addition to impacting the mean return, the impact of monetary stimulus may vary across the distribution of returns. Let $\theta$ be the quantile of interest. I regress the daily return quantile $q_\theta(Ret_t)$ on a constant and the change in monetary stimulus, $\Delta EMS_t$. I estimate three quantile regressions:
\[ q_\theta(\text{Ret}_t) = \alpha + \beta \Delta EMS_t + \varepsilon_t \quad (2.6) \]

\[ \theta \in \{0.25, 0.50, 0.75\} \]

### 2.3.6 Cointegration Models

The cointegration model is proposed on the basis of a cointegrating relationship between monetary stimulus and volatility. The hypothesis is that there is a long run equilibrium relationship between the levels of volatility and monetary stimulus, and that the short run impact depends on deviations from the long run equilibrium. Further, the short run and long run impact of monetary stimulus on volatility need not be the same. Let \( Y_t \) be a vector consisting of volatility and monetary stimulus.

\[
Y_t \equiv \begin{bmatrix} \text{Vol}_t \\ \text{EMS}_t \end{bmatrix}
\]

Next, consider a general vector autoregression of order two, and its corresponding error correction form.

\[
Y_t = \mu + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + u_t \quad (2.7)
\]

\[
\Delta Y_t = \mu + \Pi_1 Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + u_t \quad (2.8)
\]

If the two elements in \( Y_t \) are cointegrated, then the matrix \( \Pi_1 \) is of reduced rank. The Johansen test examines whether the matrix \( \Pi_1 \) is of reduced rank. The third column in table 2.3 reports the maximum eigenvalue test statistic from the Johansen cointegration test, assuming a constant term in the cointegration relationship. This is a bivariate test of one cointegrating vector between volatility and monetary stimulus. The null hypothesis is no cointegration. One, two, and
three stars indicate rejection of the null hypothesis at the 10%, 5%, and 1% level, respectively. The null hypothesis is rejected at the 5% level for all assets other than Crude Oil, and is rejected at the 1% level for eight out of thirteen assets.

To examine the short and long run impacts of monetary stimulus on volatility, I estimate a structural vector error correction model. We can decompose $\Pi_1$ into two elements: $\Pi_1 = \alpha \beta'$, where $\alpha$ is the loading matrix or error correction terms, and $\beta$ is the cointegrating vector. Note that $\alpha$ measures the speed of reversion to long run equilibrium, while $\beta$ captures the long run equilibrium relationship. The model works as follows: Let $u_t = B \varepsilon_t$, where $\varepsilon_t \sim N(0, I_k)$, and $\varepsilon_t$ represents structural errors to $Y_t$. Rewrite the ECM as follows:

$$\Delta Y_t = \mu + \Pi_1 Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + B \varepsilon_t$$

(2.9)

Consider the Beveridge Nelson moving average representation of $Y_t$.

$$Y_t = \Xi \sum_{i=1}^{t} u_i + \sum_{j=0}^{\infty} \Xi_j^* u_{t-j} + y_0^*$$

The first part is integrated of order one and captures the common trends in the system. The second part is integrated of order zero, and it is assumed that $\Xi_j^*$ converges to 0 as $j \to \infty$. Then, the common trends are given by $\Xi B \sum_{t=1}^{\infty} \varepsilon_t$. Thus, the long run effects of structural shocks are captured by the matrix $\Xi B$, and the short run effects of structural shocks are captured by the matrix $B$.

For this model to be identified, a restriction on the matrix $B$ is required. I assume that monetary stimulus is contemporaneously exogenous with respect to volatility. This amounts to imposing the following restriction: $B_{21} = 0$. This interpretation is consistent with previous regressions, where monetary stimulus is the independent variable. With this framework, I estimate the short run and long run impact on volatility of a one standard deviation shock to monetary stimulus. I also estimate the error correction term, i.e. the speed at which volatility adjusts to prior deviations from the long run equilibrium relationship. Estimation is via maximum
2.3.7 Inflation Expectations

A primary result of this paper is a negative relationship between monetary stimulus and the perception of risk in equity markets, and a positive relationship between monetary stimulus and the perception of risk in bond markets. One possible explanation for this phenomenon is the role of inflation expectations. An increase in inflation makes equities more attractive relative to bonds. The returns from holding equities are more likely to keep up with inflation, as corporate earnings and dividends will also rise with inflation. On the other hand, the real return from holding bonds is lower when inflation is higher, since coupon and principal payments are fixed in nominal terms.

Suppose there exists a positive relationship between monetary stimulus and inflation expectations. If this is the case, then an increase in monetary stimulus makes it more attractive to hold equities relative to bonds, confirming the primary result of this paper. Thus, it makes sense to test the existence of a positive relationship between monetary stimulus and inflation expectations.

Like volatility, inflation expectations is a unit root process. Inflation expectations is also cointegrated with monetary stimulus. Table 2.3 reports unit root and cointegration tests for 5 and 10 year inflation expectations, using the same specifications as the tests for volatility of each asset. The null hypothesis of no cointegration is rejected at the 5% level for 5 and 10 year inflation expectations.

As with volatility, I estimate two models: a regression model and a cointegration model. Define $\Delta IE_t^k$ as the change in $k$ year inflation expectations, where $k$ is equal or 5 or 10.

\[
\Delta IE_t^k \equiv IE_t^k - IE_{t-1}^k
\]

First, I regress the change in inflation expectations, $\Delta IE_t^k$, on the change in monetary...
stimulus, $\Delta EMS_t$, and a constant. This is a short run model that does not assume a long run relationship between the variables.

$$\Delta IE_t^k = \alpha + \beta \Delta EMS_t + \varepsilon_t$$  

(2.10)

Second, I estimate a structural vector error correction model. It has the same specification as the model for volatility. $Y_t$ is a vector containing $k$ year inflation expectations as the first element, and monetary stimulus as the second element.

$$Y_t \equiv \begin{bmatrix} IE_t^k \\ EMS_t \end{bmatrix}$$

I then estimate the following error correction model. Note that the specification is identical to the model estimated in section 2.3.6.

$$\Delta Y_t = \mu + \Pi_1 Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + B \varepsilon_t$$  

(2.11)

I assume that monetary stimulus is contemporaneously exogenous with respect to inflation expectations. Thus, the restriction $B_{21} = 0$ is imposed. With this framework, I estimate the short run and long run impact on inflation expectations of a one standard deviation shock to monetary stimulus. I also estimate the error correction term, i.e. the speed at which inflation expectations adjusts to prior deviations from the long run equilibrium relationship.

### 2.3.8 Return Predictability

As an additional exercise, I test whether the variable $\Delta EMS_t$ predicts future returns. The logic is as follows: If monetary stimulus impacts risk, it may also impact future returns due to the risk return relationship. For example, the intertemporal capital asset pricing model (ICAPM), by Merton (1973), suggests a riskier investment should earn a higher return.

I consider two methods to test return predictability. In the first method, I regress the return
at time \((t + 1)\), \(R_{t+1}\), on the change in monetary stimulus at time \(t\), \(\Delta EMS_t\), and a constant. The estimated value of \(\beta\) measures the extent to which monetary stimulus predicts future returns.

\[
R_{t+1} = \alpha + \beta \Delta EMS_t + \epsilon_t
\]  

(2.12)

In the second method, I split the sample into groups. The first group consists of returns following an increase in monetary stimulus, i.e. \(R_{t+1}\) conditional on \(\Delta EMS_t > 0\). This is approximately 42% of the sample. The second group consists of returns following a decrease in monetary stimulus, i.e. \(R_{t+1}\) conditional on \(\Delta EMS_t < 0\). This is approximately 50% of the sample. Note that days when \(\Delta EMS_t = 0\) are discarded. This is approximately 8% of the sample. I compute the average annual return for each sample, and the difference between the two measures the economic magnitude of return predictability.

\[
\begin{align*}
R_{t+1}^u &= R_{t+1} I_{\Delta EMS_t > 0} \\
R_{t+1}^d &= R_{t+1} I_{\Delta EMS_t < 0}
\end{align*}
\]

2.4 Results

2.4.1 Plots and Summary Statistics

Figure 2.1 plots the Effective Monetary Stimulus, Shadow Short Rate, and 5 and 10 year inflation expectations. The sample period is from June 2009 until January 2018. The correlation between monetary stimulus and the shadow short rate is -0.82. The correlation between monetary stimulus and the 5 and 10 year expectation inflation are 0.47 and 0.72, respectively. Table 2.1 presents a full set of summary statistics of these variables.

Figure 2.2 plots the average monthly volatility and average monthly skewness for the S&P 500 and the Nasdaq 100. Figure 2.3 plots the average monthly volatility and average
monthly skewness for the 10 year Treasury and the 30 year Treasury. Figure 2.4 plots the average monthly volatility and average monthly skewness for Crude Oil, Gold, and Silver. Figures 2.5 and 2.6 plot the average monthly volatility and average monthly skewness for the Australian Dollar, the British Pound, and the Canadian Dollar, Euro, Japanese Yen, and Swiss Franc. The data are computed from option prices as described earlier in sections 2.3.1 and 2.3.2. The left hand side plots are annualized volatility. The right hand side plots are skewness, which is bounded between -100 and +100. All of the series experience considerable variation, with the appearance of some common trends between them. Though the series are computed at a daily frequency, the figures plot the monthly average. This makes it easier to visually identify trends.

Table 2.2 reports summary statistics for all futures and options data. For each asset, I report the average annual return, the average annual volatility, and the average skewness. Standard deviations are reported in parenthesis below each value. Returns are computed from futures contracts, while volatility and skewness are computed from corresponding option contracts. Equities have the high annual returns, high volatility, and large negative skewness. Bonds have low annual returns, low volatility, and skewness close to zero. Commodities have low annual returns, the high volatility, and negative skewness. Currencies have low annual returns, moderate volatility, and negative skewness.

2.4.2 Volatility and Skewness Regression Results

Table 2.4 reports volatility and skewness regression results for all asset classes, as per equations 2.3 and 2.4. For equities, an increase in monetary stimulus is associated with a decrease in volatility and an increase in skewness. Both are consistent with a reduction in the perception of risk. For example, a one standard deviation increase in monetary stimulus reduces S&P 500 volatility by 0.636% and increases skewness by 0.126. The results for the Nasdaq 100 are similar, though lower in magnitude, particularly for skewness.

For bonds, an increase in monetary stimulus is associated with no significant change in volatility, and a decrease in skewness. This is consistent with an increase in the perception of
risk. For example, a one standard deviation increase in monetary stimulus reduces skewness by 0.313 and 0.269 for the 10 and 30 year Treasury, respectively.

For commodities, an increase in monetary stimulus is associated with a decrease in volatility for crude oil and gold, and no significant change for silver. There is no significant change in skewness for any of the contracts. This is consistent with a reduction in the perception of risk, though to a smaller extent relative to equities.

For currencies, an increase in monetary stimulus is associated with a decrease in volatility across all contracts. For example, a one standard deviation increase in monetary stimulus is associated with a 0.264% decline in Euro volatility. The magnitudes are similar across currencies. An increase in monetary stimulus is also associated with a modest increase in skewness for the British Pound and Euro, and no significant impact on other currencies. These results are consistent with a reduction in the perception of risk.

### 2.4.3 Return Regression Results

Table 2.5 reports return and quantile regression results for all asset classes, as per equations 2.5 and 2.6. For equities, an increase in monetary stimulus is associated with positive average returns. For example, a one standard deviation increase in monetary stimulus is associated with a return of 0.144% for the S&P 500. The quantile regression results indicate that the impact on the return varies across the distribution. The coefficients are highest for the $25^{th}$ quantile, relative to the $50^{th}$ and $75^{th}$ quantiles. This indicates that the positive average effect is driven primarily by a lower likelihood of a negative return.

For bonds, an increase in monetary stimulus is associated with negative returns. The quantile regressions indicate that the impact on the return varies across the distribution. The coefficients are highest in magnitude for the $25^{th}$ quantile, relative to the $50^{th}$ and $75^{th}$ quantiles. This indicates that the negative average effect is driven primarily by a higher likelihood of a negative return. The magnitudes for the 30 year Treasury are higher than for the 10 year Treasury. This is because longer dated bonds are more sensitive to interest rate changes.
For commodities, an increase in monetary stimulus is associated with a positive average returns on crude oil and silver, and no significant return on gold. The quantile regressions for crude oil show a similar pattern to equities. That is, the effects are largest for the 25th quantile, indicating that the average effect is driven primarily by a lower likelihood of a negative return. The coefficients for gold and silver are insignificant.

For currencies, an increase in monetary stimulus is associated with an increase in average returns. The quantile regressions indicate the opposite pattern to the case of equities. With currencies, the largest impacts are on the 75th quantile, rather than the 25th. This indicates that the effect on returns is driven primarily by a higher likelihood of a positive return. This pattern holds for all currencies other than the Japanese Yen, which has no significant results.

2.4.4 Cointegration Model Results

Table 2.6 reports cointegration model results for all asset classes, as per equation 2.9. The cointegration models estimate both the short run and long run impact of a one standard deviation increase in monetary stimulus on volatility. For equities, an increase in monetary stimulus is associated with a short run decline in volatility, and a long run increase in volatility. For example, a one standard deviation increase in monetary stimulus implies a 1.447% decrease in volatility in the short run, and a 0.479% increase in volatility in the long run. The magnitude of the short run impact is more than double the magnitude coming from the regression model estimated earlier. The error correction terms are negative and significant, and equal to -0.034 for both contracts. This implies that 3.4% of the deviation from the long run equilibrium is corrected each day.

For bonds, an increase in monetary stimulus is associated with both a short run and a long run increase in volatility. This is unlike the regression model, which indicated no significant impact of monetary stimulus on volatility. The impact is much larger for the 10 year treasury relative to the 30 year treasury. The error correction terms are negative and significant, and smaller in magnitude relative to equities.

Within commodities, the short run and long run relationship between monetary stimulus
and crude oil volatility is negative. For gold and silver, there is an insignificant short run relationship, and a positive long run relationship. The error correction terms are negative and significant, and similar in magnitude relative to equities. The crude oil model is likely to be misspecified, as it is the only asset for which the cointegration test did not reject the null hypothesis of no cointegration.

Currencies follow a similar pattern to equities. There is negative short run relationship and a positive long run relationship between monetary stimulus and volatility. The long run effects are present for all currencies, while the short run effects are significant for all currencies excluding the British Pound and Japanese Yen. For all assets, the error correction terms are negative and significant, and smaller in magnitude relative to equities.

Why is the long run relationship between monetary stimulus and volatility positive? The positive long run relationship exists across asset classes, even when the short run relationship is negative. The sample period, June 2009 through January 2018, coincides with an overall improvement in economic and financial conditions. This improvement in conditions has led to the normalization of monetary policy and slow withdrawal of monetary stimulus. At the same time, improvement in conditions also led to lower levels of risk, hence the decline in both monetary stimulus and volatility.

2.4.5 Perception of Risk by Asset Class

For equities, an increase in monetary stimulus is associated with a decline in volatility, an increase in skewness, positive average returns, and a relatively lower likelihood of a negative return. These results are consistent with a decline in the perception of risk. In magnitudes, the declines in risk for equities are stronger than for any other asset class.

For bonds, an increase in monetary stimulus is associated with no change or modest increase in volatility, a decrease in skewness, negative average returns, and a relatively higher likelihood of a negative return. These results are consistent with an increase in the perception of risk. Bonds are the only asset class for which the perception of risk is unambiguously higher.
For commodities, an increase in monetary stimulus is associated with a decline in volatility, no change in skewness, positive average returns, and a relatively lower likelihood of a negative return. These results are consistent with a decline in the perception of risk. Quantitatively, the results are weaker relative to equities. The results are strongest for Crude Oil, and weaker for Gold and Silver.

For currencies, an increase in monetary stimulus is associated with a decline in volatility, no change in skewness, positive average returns, and a relatively higher likelihood of a positive return. The results are broadly consistent with a decline in the perception of risk. The higher likelihood of a positive return is equivalent to a higher likelihood of a fall in the value of the US dollar, relative to the foreign currency. This makes intuitive sense, as monetary stimulus means an increase in the supply of US dollars, thus a greater likelihood of US dollar depreciation. Note that this pattern holds for all currencies other than the Japanese Yen. This is likely because the Japanese yen is considered a safe haven currency, much like the US dollar.

### 2.4.6 Inflation Expectations

A possible explanation for the results is the role inflation expectations. An increase in inflation expectations would increase the attractiveness of risky assets relative to safe assets. It would explain why monetary stimulus is associated with a decrease in risk for equities, commodities, and currencies, but an increase in risk for bonds. For currencies, it would also explain the greater likelihood of a decline in the value of the US dollar.

Table 2.7 presents regression and cointegration model results for inflation expectations, as per equations 2.10 and 2.11. The regression indicates a positive relationship between monetary stimulus and inflation expectations, both 5 and 10 year. In the cointegrating model, there is a positive short run and long run relationship between monetary stimulus and inflation expectations. The impacts are also larger as compared with the regression model. Finally, the error correction terms are negative and significant. These results confirm the hypothesis that monetary stimulus impacts the perception of risk, partly through its impact on inflation expectations.
2.4.7 Return Predictability

Table 2.8 presents the results for whether changes in monetary stimulus predicts future returns. The hypothesis is that changes in monetary stimulus should have the same directional impact on both risk and returns. This means if monetary stimulus contemporaneously increases risk, it should predict higher future returns. If monetary stimulus contemporaneously decreases risk, it should predict lower future returns. The first column indicates the level and statistical significant of the regression coefficient, as per equation 2.12. The second and third column presents the average annual return when $\Delta EMS_t > 0$ and when $\Delta EMS_t < 0$, respectively.

The overall results are mixed. For the S&P 500, the regressions coefficient is negative and significant at the 10% level. The difference between annual returns is economically significant. For example, the average annual return on days following an increase in monetary stimulus is 12.17%, relative to 20.18% on days following a decrease in monetary stimulus. There is no predictability for the Nasdaq 100.

The bond market displays the greatest amount of return predictability. The regression coefficients are positive and significant for both the 10 and 30 year Treasury. The economic significance is also large. For example, the average annual return on the 10 year Treasury on days following an increase in monetary stimulus is 5.36%, relative to 0.86% on days following a decrease in monetary stimulus.

The results for both the S&P 500 and bonds are in line with expectations, which is that changes in monetary stimulus should have the same directional impact on both risk and returns. For currencies and commodities, none of the contracts other than the Australian dollar exhibit any return predictability.

2.5 Conclusion

The primary takeaway from this paper is that monetary stimulus has heterogeneous impacts on the perception of risk across asset classes. In the short run, monetary stimulus
increases the attractiveness of risky assets such as equities, commodities, and currencies, relative to safe assets such as bonds. One possible explanation for these results is expected inflation. Monetary stimulus has a positive relationship with expected inflation, and this is why risky assets become more attractive relative to safe assets that pay a fixed nominal return. Furthermore, there is a positive long run relationship between the levels of monetary stimulus and volatility, as demonstrated by the cointegration results. This occurs due an overall improvement in economic conditions that leads to the normalization of monetary policy and a reduction in overall risk.

These results are important, because they demonstrate the importance of monetary policy in influencing market participants’ perception of risk. This is relevant for both policy makers as well as market participants. Much attention has been made in recent years to how unconventional monetary policy is impacting the financial markets. Even if it is the case that monetary policy is not targeting asset prices directly, the response of asset prices is economically significant. And the response of asset prices helps to understand the extent to which the risk taking channel of monetary policy is working. This is an important consideration, as the Federal Reserve continues on the path of normalization of monetary policy, and a withdrawal of monetary stimulus.

2.6 Acknowledgements

Chapter 2, in full, is currently being prepared for submission for publication of the material. Dossani, Asad. The dissertation author was the sole author of this paper.
Figure 2.1. Monetary Stimulus and Inflation Expectations

These figures plot Effective Monetary Stimulus, Shadow Short Rate, and average monthly 5 and 10 year inflation expectations. Effective Monetary Stimulus and the Shadow Short Rate are derived from a two factor shadow term structure model. The 5 and 10 year inflation expectations are derived from yields on constant maturity Treasury securities and inflation indexed constant maturity Treasury securities. The units for all series are percentage points. The series are sampled daily, and the sample period is from June 2009 until January 2018.
These figures plot the average monthly annualized volatility and average monthly skewness for two contracts in the equities asset class: the S&P 500 and the Nasdaq 100. The data are computed from one month option prices. The left hand side plots are annualized volatility, reported in percentage points. The right hand side plots skewness, which is bound between -100 and +100. The sample period is from June 2009 until January 2018.
Figure 2.3. Bonds

These figures plot the average monthly annualized volatility and average monthly skewness for two contracts in the bonds asset class: the 10 year Treasury and the 30 year Treasury. The data are computed from one month option prices. The left hand side plots are annualized volatility, reported in percentage points. The right hand side plots skewness, which is bound between -100 and +100. The sample period is from June 2009 until January 2018.
These figures plot the average monthly annualized volatility and average monthly skewness for three contracts in the commodities asset class: Crude Oil, Gold, and Silver. The data are computed from one month option prices. The left hand side plots are annualized volatility, reported in percentage points. The right hand side plots skewness, which is bound between -100 and +100. The sample period is from June 2009 until January 2018.
These figures plot the average monthly annualized volatility and average monthly skewness for three contracts in the currencies asset class: the Australian Dollar, the British Pound, and the Canadian Dollar. The data are computed from one month option prices. The left hand side plots are annualized volatility, reported in percentage points. The right hand side plots skewness, which is bound between -100 and +100. The sample period is from June 2009 until January 2018.
These figures plot the average monthly annualized volatility and average monthly skewness for three contracts in the currencies asset class: the Euro, the Japanese Yen, and the Swiss Franc. The data are computed from one month option prices. The left hand side plots are annualized volatility, reported in percentage points. The right hand side plots skewness, which is bound between -100 and +100. The sample period is from June 2009 until January 2018.
<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Monetary Stimulus</td>
<td>13.99</td>
<td>6.49</td>
<td></td>
</tr>
<tr>
<td>Shadow Short Rate</td>
<td>−1.64</td>
<td>1.97</td>
<td>−0.82</td>
</tr>
<tr>
<td>5 year Inflation Expectations</td>
<td>1.71</td>
<td>0.29</td>
<td>0.47</td>
</tr>
<tr>
<td>10 year Inflation Expectations</td>
<td>2.00</td>
<td>0.31</td>
<td>0.72</td>
</tr>
</tbody>
</table>

This table reports summary statistics for the following four time series: Effective Monetary Stimulus, Shadow Short Rate, 5 year Inflation Expectations, and 10 year Inflation Expectations. The first column reports the mean of the series, the second column reports the standard deviation of the series, and the third column reports the correlation between Effective Monetary Stimulus and the series. The units of the mean and standard deviation are percentage points. The data are sampled daily, and are from June 2009 until January 2018.
Table 2.2. Summary Statistics - Futures and Options

<table>
<thead>
<tr>
<th></th>
<th>Returns</th>
<th>Volatility</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>15.32</td>
<td>16.73</td>
<td>-31.23</td>
</tr>
<tr>
<td></td>
<td>(16.83)</td>
<td>(5.94)</td>
<td>(6.60)</td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>18.93</td>
<td>18.58</td>
<td>-33.09</td>
</tr>
<tr>
<td></td>
<td>(16.95)</td>
<td>(5.54)</td>
<td>(4.99)</td>
</tr>
<tr>
<td>10Y Treasury</td>
<td>3.74</td>
<td>5.80</td>
<td>-5.82</td>
</tr>
<tr>
<td></td>
<td>(5.92)</td>
<td>(1.44)</td>
<td>(9.07)</td>
</tr>
<tr>
<td>30Y Treasury</td>
<td>5.29</td>
<td>10.73</td>
<td>-2.86</td>
</tr>
<tr>
<td></td>
<td>(11.26)</td>
<td>(2.40)</td>
<td>(6.61)</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>-7.80</td>
<td>33.49</td>
<td>-13.37</td>
</tr>
<tr>
<td></td>
<td>(24.63)</td>
<td>(10.95)</td>
<td>(7.33)</td>
</tr>
<tr>
<td>Gold</td>
<td>3.02</td>
<td>17.17</td>
<td>-4.60</td>
</tr>
<tr>
<td></td>
<td>(16.02)</td>
<td>(4.45)</td>
<td>(10.04)</td>
</tr>
<tr>
<td>Silver</td>
<td>0.20</td>
<td>30.50</td>
<td>-5.55</td>
</tr>
<tr>
<td></td>
<td>(29.05)</td>
<td>(8.74)</td>
<td>(9.22)</td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>3.52</td>
<td>11.83</td>
<td>-14.49</td>
</tr>
<tr>
<td></td>
<td>(12.94)</td>
<td>(3.33)</td>
<td>(6.48)</td>
</tr>
<tr>
<td>British Pound</td>
<td>-2.05</td>
<td>9.43</td>
<td>-9.70</td>
</tr>
<tr>
<td></td>
<td>(8.99)</td>
<td>(2.92)</td>
<td>(6.22)</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>-1.72</td>
<td>9.30</td>
<td>-9.66</td>
</tr>
<tr>
<td></td>
<td>(8.62)</td>
<td>(2.66)</td>
<td>(5.80)</td>
</tr>
<tr>
<td>Euro</td>
<td>-1.64</td>
<td>10.21</td>
<td>-8.47</td>
</tr>
<tr>
<td></td>
<td>(8.95)</td>
<td>(2.71)</td>
<td>(6.78)</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>-1.57</td>
<td>10.78</td>
<td>4.03</td>
</tr>
<tr>
<td></td>
<td>(9.47)</td>
<td>(2.49)</td>
<td>(7.62)</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>0.55</td>
<td>10.33</td>
<td>-2.29</td>
</tr>
<tr>
<td></td>
<td>(10.91)</td>
<td>(2.76)</td>
<td>(7.35)</td>
</tr>
</tbody>
</table>

This table reports summary statistics for all futures and options data. For each asset, I report the average annual return, the average annual standard deviation, and the average skewness. The units for annual return and annual standard deviation is percentage points, while Skewness is bound between -100 and +100. Each estimate is reported with its corresponding standard deviation in parenthesis below. Returns are computed from futures contracts, while volatility and skewness are computed from corresponding option contracts. The data are sampled daily, and are from June 2009 until January 2018.
Table 2.3. Unit Root and Cointegration Tests

<table>
<thead>
<tr>
<th></th>
<th>Unit Root</th>
<th>Autoregressive Coef</th>
<th>Cointegration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Monetary Stimulus</td>
<td>0.798</td>
<td>1.001</td>
<td></td>
</tr>
<tr>
<td>5 year Inflation Expectations</td>
<td>0.056</td>
<td>0.991</td>
<td>16.50**</td>
</tr>
<tr>
<td>10 year Inflation Expectations</td>
<td>0.317</td>
<td>0.995</td>
<td>18.03**</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.243</td>
<td>0.971</td>
<td>36.98***</td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>0.181</td>
<td>0.969</td>
<td>38.74***</td>
</tr>
<tr>
<td>10Y Treasury</td>
<td>0.545</td>
<td>0.976</td>
<td>32.08***</td>
</tr>
<tr>
<td>30Y Treasury</td>
<td>0.401</td>
<td>0.979</td>
<td>24.53***</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>0.304</td>
<td>0.989</td>
<td>12.59</td>
</tr>
<tr>
<td>Gold</td>
<td>0.136</td>
<td>0.973</td>
<td>38.95***</td>
</tr>
<tr>
<td>Silver</td>
<td>0.239</td>
<td>0.982</td>
<td>29.35***</td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>0.364</td>
<td>0.986</td>
<td>17.97**</td>
</tr>
<tr>
<td>British Pound</td>
<td>0.249</td>
<td>0.986</td>
<td>18.30**</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>0.424</td>
<td>0.986</td>
<td>17.73**</td>
</tr>
<tr>
<td>Euro</td>
<td>0.169</td>
<td>0.983</td>
<td>19.42**</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>0.091</td>
<td>0.976</td>
<td>26.09***</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>0.079</td>
<td>0.981</td>
<td>25.22***</td>
</tr>
</tbody>
</table>

This table presents results of unit root and cointegration tests for each time series. The first three series are Expected Monetary Stimulus, and 5 and 10 year inflation expectations. The remainder are the log of volatility of each individual asset. The first column reports p-values from the Dickey Fuller GLS unit root test. The null hypothesis is that there is a unit root. The second column reports the autoregressive coefficient, i.e. the estimated value of $\beta$ from the following regression: $Y_t = \alpha + \beta Y_{t-1} + \epsilon_t$. The third column reports the maximum eigenvalue test statistic from the Johansen cointegration test, assuming a constant term in the cointegration relationship. This is a bivariate test of one cointegrating vector between log of volatility and Effective Monetary Stimulus. The null hypothesis is no cointegration. One, two, and three stars indicate rejection of the null hypothesis at the 10%, 5%, and 1% level, respectively. The data are sampled daily, and are from June 2009 until January 2018.
Table 2.4. Volatility and Skewness Regressions

<table>
<thead>
<tr>
<th></th>
<th>$Y_t$</th>
<th>$\Delta Vol_t$</th>
<th>$Skew_t$</th>
<th>$\Delta Vol_t^u$</th>
<th>$\Delta Vol_t^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>$-0.636^{***}$</td>
<td>0.126**</td>
<td>$-0.601^{***}$</td>
<td>$-0.648^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.165)</td>
<td>(0.051)</td>
<td>(0.154)</td>
<td>(0.171)</td>
<td></td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>$-0.590^{***}$</td>
<td>0.072*</td>
<td>$-0.584^{***}$</td>
<td>$-0.596^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td>(0.037)</td>
<td>(0.148)</td>
<td>(0.154)</td>
<td></td>
</tr>
<tr>
<td>10Y Treasury</td>
<td>0.056</td>
<td>$-0.313^{***}$</td>
<td>0.054</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.100)</td>
<td>(0.119)</td>
<td>(0.123)</td>
<td></td>
</tr>
<tr>
<td>30Y Treasury</td>
<td>$-0.073$</td>
<td>$-0.269^{***}$</td>
<td>$-0.102$</td>
<td>$-0.049$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.106)</td>
<td>(0.075)</td>
<td>(0.109)</td>
<td>(0.107)</td>
<td></td>
</tr>
<tr>
<td>Crude Oil</td>
<td>$-0.404^{***}$</td>
<td>$-0.002$</td>
<td>$-0.405^{***}$</td>
<td>$-0.399^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.118)</td>
<td>(0.048)</td>
<td>(0.116)</td>
<td>(0.122)</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>$-0.264^{**}$</td>
<td>$-0.018$</td>
<td>$-0.277^{**}$</td>
<td>$-0.254^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.122)</td>
<td>(0.064)</td>
<td>(0.125)</td>
<td>(0.127)</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>$-0.157$</td>
<td>0.035</td>
<td>$-0.151$</td>
<td>$-0.169^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.045)</td>
<td>(0.122)</td>
<td>(0.132)</td>
<td></td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>$-0.265^{***}$</td>
<td>0.039</td>
<td>$-0.250^{***}$</td>
<td>$-0.275^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.060)</td>
<td>(0.087)</td>
<td>(0.112)</td>
<td></td>
</tr>
<tr>
<td>British Pound</td>
<td>$-0.209^{***}$</td>
<td>0.074*</td>
<td>$-0.191^{**}$</td>
<td>$-0.228^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.045)</td>
<td>(0.077)</td>
<td>(0.088)</td>
<td></td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>$-0.371^{***}$</td>
<td>0.022</td>
<td>$-0.366^{***}$</td>
<td>$-0.373^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.100)</td>
<td>(0.044)</td>
<td>(0.097)</td>
<td>(0.105)</td>
<td></td>
</tr>
<tr>
<td>Euro</td>
<td>$-0.264^{**}$</td>
<td>0.118*</td>
<td>$-0.209^{**}$</td>
<td>$-0.301^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.070)</td>
<td>(0.101)</td>
<td>(0.147)</td>
<td></td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>$-0.205^{**}$</td>
<td>$-0.077$</td>
<td>$-0.222^{**}$</td>
<td>$-0.192^{*}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.048)</td>
<td>(0.105)</td>
<td>(0.105)</td>
<td></td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>$-0.231^{**}$</td>
<td>0.031</td>
<td>$-0.236^{**}$</td>
<td>$-0.230^{*}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.045)</td>
<td>(0.116)</td>
<td>(0.121)</td>
<td></td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta \Delta E M S_t + \varepsilon_t$

$Y_t = \alpha + \beta \Delta E M S_t + \gamma Y_{t-1} + \varepsilon_t$

This table presents regression results for volatility and skewness. The first column presents results as per the first equation above, where $Y_t = \Delta Vol_t$, i.e. the change in log of volatility. The second column presents results as per the second equation above, where $Y_t = Skew_t$, i.e. skewness. The third and fourth columns present results as per the first equation above, for $Y_t = \Delta Vol_t^u$ and $Y_t = \Delta Vol_t^d$, i.e. upside and downside volatility, respectively. For each regression, I report the estimated value of $\beta$, and its corresponding Newey West standard error in parenthesis below. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is daily, from June 2009 until January 2018, and the sample size is $N = 2234$. 

80
Table 2.5. Return Regressions

<table>
<thead>
<tr>
<th>$Y_t$</th>
<th>$Ret_t$</th>
<th>$q_{0.25}(Ret_t)$</th>
<th>$q_{0.50}(Ret_t)$</th>
<th>$q_{0.75}(Ret_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>0.144***</td>
<td>0.131***</td>
<td>0.078***</td>
<td>0.065***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.025)</td>
<td>(0.016)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>0.134***</td>
<td>0.128***</td>
<td>0.070***</td>
<td>0.084***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.027)</td>
<td>(0.020)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>10Y Treasury</td>
<td>−0.031***</td>
<td>−0.056***</td>
<td>−0.039***</td>
<td>−0.028***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.010)</td>
<td>(0.009)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>30Y Treasury</td>
<td>−0.139***</td>
<td>−0.161***</td>
<td>−0.149***</td>
<td>−0.121***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.016)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>0.123***</td>
<td>0.127***</td>
<td>0.112***</td>
<td>0.066*</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.045)</td>
<td>(0.034)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>Gold</td>
<td>0.018</td>
<td>0.009</td>
<td>0.020</td>
<td>−0.030</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.025)</td>
<td>(0.020)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Silver</td>
<td>0.096*</td>
<td>0.055</td>
<td>0.048</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.049)</td>
<td>(0.032)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>0.065**</td>
<td>0.030</td>
<td>0.041**</td>
<td>0.059***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.024)</td>
<td>(0.018)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>British Pound</td>
<td>0.045***</td>
<td>0.031**</td>
<td>0.045***</td>
<td>0.066***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.016)</td>
<td>(0.012)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>0.059***</td>
<td>0.061***</td>
<td>0.041***</td>
<td>0.055***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.012)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Euro</td>
<td>0.059***</td>
<td>0.023</td>
<td>0.051***</td>
<td>0.081***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.017)</td>
<td>(0.013)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>−0.005</td>
<td>−0.022</td>
<td>−0.009</td>
<td>−0.008</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.012)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>0.048**</td>
<td>0.022</td>
<td>0.028**</td>
<td>0.054***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.017)</td>
<td>(0.013)</td>
<td>(0.015)</td>
</tr>
</tbody>
</table>

$Y_t = \alpha + \beta \Delta E M S_t + \epsilon_t$

This table presents regression results for returns and its quantiles. The first column presents results from an OLS regression, as per the equation above, where $Y_t = Ret_t$, i.e. the return. The second, third, and fourth columns present results from a quantile regression, as per the equation above, where $Y_t = q_{0.25}(Ret_t)$, $Y_t = q_{0.50}(Ret_t)$, and $Y_t = q_{0.75}(Ret_t)$. These are the 25th, 50th, and 75th quantiles of returns, respectively. For each regression, I report the estimated value of $\beta$, and its corresponding standard error in parenthesis below. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is daily, from June 2009 until January 2018, and the sample size is $N = 2180$.  

81
### Table 2.6. Cointegration Models

<table>
<thead>
<tr>
<th></th>
<th>Short Run</th>
<th>Long Run</th>
<th>Error Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>$-1.447^{***}$</td>
<td>$0.479^{***}$</td>
<td>$-0.034^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.212)</td>
<td>(0.044)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>$-1.139^{***}$</td>
<td>$0.293^{***}$</td>
<td>$-0.034^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.173)</td>
<td>(0.022)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>10Y Treasury</td>
<td>$0.332^{**}$</td>
<td>$0.325^{***}$</td>
<td>$-0.028^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.030)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>30Y Treasury</td>
<td>$0.113$</td>
<td>$0.106^{***}$</td>
<td>$-0.021^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
<td>(0.010)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>$-0.705^{***}$</td>
<td>$-0.198^{***}$</td>
<td>$-0.011^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.120)</td>
<td>(0.019)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Gold</td>
<td>$-0.248$</td>
<td>$0.482^{***}$</td>
<td>$-0.035^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.154)</td>
<td>(0.038)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Silver</td>
<td>$-0.158$</td>
<td>$0.579^{***}$</td>
<td>$-0.027^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.141)</td>
<td>(0.055)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>$-0.312^{**}$</td>
<td>$0.239^{***}$</td>
<td>$-0.015^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.021)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>British Pound</td>
<td>$-0.119$</td>
<td>$0.239^{***}$</td>
<td>$-0.014^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.110)</td>
<td>(0.015)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>$-0.428^{***}$</td>
<td>$0.005^{***}$</td>
<td>$-0.014^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.083)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Euro</td>
<td>$-0.304^{**}$</td>
<td>$0.154^{***}$</td>
<td>$-0.017^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.129)</td>
<td>(0.014)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>$-0.069$</td>
<td>$0.059^{***}$</td>
<td>$-0.023^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>$-0.300^{**}$</td>
<td>$0.251^{***}$</td>
<td>$-0.021^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.119)</td>
<td>(0.024)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

\[ \Delta Y_t = \mu + \Pi_1 Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + B \varepsilon_t \]

This table reports results from cointegration models. I estimate a structural vector error correction model, as per the the equation above. \( Y_t \) includes both volatility and monetary stimulus. The first and second column report the short run and long run impact on volatility of a one standard deviation shock to monetary stimulus, respectively. The third column reports the error correction term. Each estimate is reported with its corresponding standard error in parentheses below. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is daily, from June 2009 until January 2018, and the sample size is \( N = 2234 \).
<table>
<thead>
<tr>
<th></th>
<th>Regression</th>
<th>Short Run</th>
<th>Long Run</th>
<th>Error Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 year Inf Exp</td>
<td>0.003***</td>
<td>0.006***</td>
<td>0.003***</td>
<td>−0.010***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>10 year Inf Exp</td>
<td>0.004***</td>
<td>0.007***</td>
<td>0.005***</td>
<td>−0.009***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

\[
\Delta IE_t^k = \alpha + \beta \Delta EMS_t + \varepsilon_t
\]

\[
\Delta Y_t = \mu + \Pi_1 Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + B \varepsilon_t
\]

This table reports results for inflation expectations. The first column reports regression results, as per the first equation above. \( \Delta IE_t^k \) is the change in \( k \) year inflation expectations, and \( k \) is equal to either 5 or 10. I report the estimated value of \( \beta \). Next, I estimate a structural vector error correction model, as per the second equation above. \( Y_t \) includes both \( k \) year inflation expectations and monetary stimulus. The second and third columns report the short run and long run impact on inflation expectations of a one standard deviation shock to monetary stimulus, respectively. The fourth column reports the error correction term. Each estimate is reported with its corresponding standard error in parentheses below. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The sample period is daily, from June 2009 until January 2018, and the sample size is \( N = 2161 \).
<table>
<thead>
<tr>
<th></th>
<th>Ret_{t+1}</th>
<th>ΔEMS_{t} &gt; 0</th>
<th>ΔEMS_{t} &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>−0.036*</td>
<td>12.17</td>
<td>20.18</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(15.16)</td>
<td>(18.11)</td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>−0.015</td>
<td>19.01</td>
<td>19.30</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(15.42)</td>
<td>(17.89)</td>
</tr>
<tr>
<td>10Y Treasury</td>
<td>0.021**</td>
<td>5.36</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(6.14)</td>
<td>(5.77)</td>
</tr>
<tr>
<td>30Y Treasury</td>
<td>0.027*</td>
<td>6.30</td>
<td>2.95</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(11.62)</td>
<td>(10.98)</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>−0.004</td>
<td>0.50</td>
<td>−11.49</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(22.78)</td>
<td>(25.85)</td>
</tr>
<tr>
<td>Gold</td>
<td>−0.006</td>
<td>0.39</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(15.05)</td>
<td>(16.99)</td>
</tr>
<tr>
<td>Silver</td>
<td>0.001</td>
<td>0.71</td>
<td>3.35</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(28.67)</td>
<td>(29.65)</td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>−0.033*</td>
<td>−1.22</td>
<td>8.54</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(12.48)</td>
<td>(13.42)</td>
</tr>
<tr>
<td>British Pound</td>
<td>0.001</td>
<td>−1.87</td>
<td>−2.66</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(8.93)</td>
<td>(8.30)</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>−0.007</td>
<td>−2.33</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(8.40)</td>
<td>(8.82)</td>
</tr>
<tr>
<td>Euro</td>
<td>−0.013</td>
<td>−2.49</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(9.38)</td>
<td>(8.73)</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>0.006</td>
<td>−1.87</td>
<td>−4.80</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(8.91)</td>
<td>(9.81)</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>−0.000</td>
<td>0.34</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(9.17)</td>
<td>(12.60)</td>
</tr>
</tbody>
</table>

\[ Ret_{t+1} = \alpha + \beta \Delta EMS_t + \varepsilon_t \]

This table reports return predictability results. The first column reports the estimated value of β from the above regression equation, with standard errors in parentheses below. One, two, and three stars indicate p-values below 0.10, 0.05, and 0.01, respectively. The second column reports the average annual return and corresponding standard deviation in parentheses, on days following an increase in monetary stimulus, i.e. ΔEMS_t > 0. The third column reports the average annual return and corresponding standard deviation in parentheses, on days following an decrease in monetary stimulus, i.e. ΔEMS_t < 0. The units of the annual return is percentage points. The sample period is daily, from June 2009 until January 2018, and the sample size is N = 2180.
Chapter 3

Option Augmented Density Forecasts of Market Return with Monotone Pricing Kernel

Abstract

Basic financial theory indicates that the ratio of the conditional density of the future value of a market index and the corresponding risk neutral density should be monotone, but a sizeable empirical literature finds otherwise. We therefore consider an option augmented density forecast of the market return obtained by transforming a baseline density forecast estimated from past excess returns so as to monotonize its ratio with a risk neutral density estimated from current option prices. To evaluate our procedure, we compare baseline and option augmented monthly density forecasts for the S&P 500 index over the period 1997–2013. We find that monotonizing the pricing kernel leads to a modest improvement in the calibration of density forecasts. Supplementary results supportive of this finding are given for market indices in France, Germany, Hong Kong, Japan and the United Kingdom.

3.1 Introduction

A sizable literature in empirical finance documents a curious regularity: pricing kernel estimates for market portfolios frequently fail to be monotone. By pricing kernel, we mean
the ratio $q/p$, where $p$ is the **physical density**—that is, the density of the market return at some future date conditional on current information—and $q$ is the corresponding **risk neutral density**. The physical density is typically estimated from historical market returns, while the risk neutral density is estimated from the prices of options written on the market portfolio. A pricing kernel that fails to be nonincreasing implies, perversely, the existence of contingent claims written on the market portfolio whose returns stochastically dominate the market return (Beare (2011)), and runs counter to the ubiquitous financial intuition that payoffs in bad states of the world ought to be more expensive than payoffs in good states of the world. The nonmonotone shape of empirical pricing kernel estimates was first observed in a trio of papers by Aıt-Sahalia & Lo (2000), Jackwerth (2000) and Rosenberg & Engle (2002), and has become known as the pricing kernel puzzle. More recent empirical studies of pricing kernel monotonicity, largely confirming the existence of puzzling nonmonotone behavior, include Bakshi et al. (2010), Golubev et al. (2014), Chaudhuri & Schroder (2015), Härdle et al. (2014) and Beare & Schmidt (2016).

A variety of explanations for the pricing kernel puzzle have been proposed in the literature; see, for instance, Hens & Reichlin (2012). In our view the most persuasive explanation was put forward by Chabi-Yo et al. (2007). Those authors emphasize the role of proper conditioning on relevant state variables in the construction of empirical pricing kernels. Specifically, since the risk neutral density is estimated using option prices that reflect all currently available information, it is imperative that we also estimate the physical density conditional on current information. For instance, option prices are heavily dependent upon the current level of market volatility. If we estimate the unconditional physical density rather than the physical density conditional on current volatility, then there will be a mismatch between the scales of the estimated physical and risk neutral distributions, potentially leading to spurious u-shaped or n-shaped pricing kernels in times of high or low volatility respectively. Recent papers focusing on the role of omitted volatility factors in explaining pricing kernel nonmonotonicity include Chabi-Yo (2012), Christoffersen et al. (2013) and Song & Xiu (2016). More generally, spurious nonmonotonicity of the empirical pricing kernel may result when the estimated physical density is not taken conditional on any...
state variable that plays a role in the pricing of options. This is deeply problematic because many state variables that influence option prices may not be available for physical density estimation, or may be difficult to quantify.

In this paper we propose a method for transforming an estimate of the physical density so that its ratio with an estimate of the risk neutral density is guaranteed to be monotone. We refer to the transformed density as an option augmented density forecast, as it augments a density forecast constructed from historical market return data with information contained in current option prices. We study the relative empirical performance of transformed and untransformed physical density estimates for a variety of market indices. The point of this exercise is to shed light on the nature of failures of empirical pricing kernel monotonicity. Suppose, on the one hand, that the true (unobservable) pricing kernel is monotone, and that violations of empirical pricing kernel monotonicity can be attributed to misestimation of the physical density—this might be due to the omission of relevant state variables, model misspecification, or parameter uncertainty, for instance. In this case, our transformation has the effect of imposing a valid shape constraint on the physical density, which we might reasonably expect to improve its empirical performance as a density forecast of future returns. On the other hand, suppose that the true pricing kernel is not monotone, due to perverse pricing of options. In this case, our transformation imposes an invalid shape constraint upon the estimated physical density, which we might expect to worsen its empirical performance as a density forecast. The relative empirical performance of the transformed and untransformed physical density estimates may therefore tell us something about the source of empirical pricing kernel nonmonotonicity, with superior performance of the transformed estimates pointing toward misestimation of the physical density as the culprit, and superior performance of the untransformed estimates pointing toward option mispricing. From a practical perspective, the former case is of interest because our option augmentation procedure may be used to improve density forecasts of market returns based on historical data, with obvious implications for risk management, whereas the latter case is of interest because option trading strategies along the lines of those considered by Constantinides et al. (2008) and Constantinides...
et al. (2011) may be effective.

We provide empirical results for a range of market indices, but focus attention on the S&P 500 index, which has received particular attention in the literature on the pricing kernel puzzle. Using monthly S&P 500 index option data from the years 1997–2013, we find that option augmentation of physical density forecasts obtained from a state-of-the-art time series model for market returns, the Realized GARCH model of Hansen et al. (2012), appears to modestly improve the calibration of density forecasts. In Appendix B, we provide supplementary empirical results for market index options in France, Germany, Hong Kong, Japan and the United Kingdom. We again find that imposing pricing kernel monotonicity leads to modest improvements in measures of density forecast calibration.

Our contribution is related to recent papers by Linn et al. (2017), Cuesdeanu & Jackwerth (2016) and Sala & Mira (2017). Like us, these authors emphasize that apparent pricing kernel nonmonotonicity may be a spurious byproduct of failing to condition on current information in the estimation of the physical density. Linn et al. (2017) and Cuesdeanu & Jackwerth (2016) propose novel estimators of the pricing kernel that avoid direct estimation of the physical density, but instead rely critically on the pricing kernel being either constant over time, or varying over time in a manner that can be explained by systematic covariation with market volatility. The former authors obtain results that are supportive of pricing kernel monotonicity, while the latter obtain more mixed results.

Sala & Mira (2017) propose an option augmented estimator of the physical density that is very different to ours, but shares the aim of improving physical density forecasts by incorporating information present in option prices. Like us, they estimate risk neutral and physical densities separately using option prices and historical return data respectively. They then use a Bayesian technique to shrink the estimated physical density toward a risk premium adjusted version of the estimated risk neutral density, yielding their option augmented estimate of the physical density. This does not ensure monotonicity of the implied pricing kernel, so the empirical performance of our option augmentation procedure is more directly related to the presence or absence of pricing
kernel monotonicity. The performance of the Sala & Mira (2017) procedure instead depends on the extent to which the true physical density resembles a risk premium adjusted version of the risk neutral density.

The remainder of the paper is structured as follows. In Section 3.2 we introduce our transformation of the physical density that monotonizes its ratio with the risk neutral density, and prove a simple mathematical result that motivates the application of this transformation to estimated densities in the presence of pricing kernel monotonicity. We also discuss the computation of our transformation. In Section 3.3 we present our empirical results using S&P 500 index option data. Section 3.4 concludes. Supplementary empirical results for France, Germany, Hong Kong, Japan and the United Kingdom are given in Appendix B.

3.2 Option augmentation of density forecasts

In Section 3.2.1 we introduce an operator that transforms a distribution in such a way as to monotonize its density ratio with respect to another distribution over a given interval. In Section 3.2.2 we explain how this operator can be used to construct density forecasts of market returns that imply pricing kernel monotonicity over a given region, and why we might expect these to perform well when pricing kernel monotonicity is satisfied. In Section 3.2.3 we explain how to numerically compute our transformation.

3.2.1 Ordinal dominance curve convexification

Let $\mathcal{D}$ be the collection of all cumulative distribution functions (cdfs) $F : [0, \infty] \rightarrow [0, 1]$ that are continuous, strictly increasing, and satisfy $F(0) = 0$ and $F(\infty) = 1$. We equip $\mathcal{D}$ with the Kolmogorov-Smirnov metric $d_{KS}$, defined by

$$d_{KS}(F, G) = \sup_{x \in [0, \infty]} |F(x) - G(x)|, \quad F, G \in \mathcal{D}.$$ 

To each cdf $F \in \mathcal{D}$ there corresponds a unique inverse $F^{-1} : [0, 1] \rightarrow [0, \infty]$ which we
call the quantile function for $F$. Given two cdfs $F, G \in \mathcal{D}$, the composition $G \circ F^{-1}$ is called the ordinal dominance curve (odc) for $G$ and $F$. Every odc is a nondecreasing map from the closed unit interval to itself.

The shape of the odc tells us something about the ratio of the densities of $F$ and $G$. Suppose that $F$ and $G$ admit continuous probability density functions (pdfs) $f$ and $g$, strictly positive on some open interval $(a, b) \subseteq (0, \infty)$. Then the density ratio $f/g$ is nonincreasing over $(a, b)$ if and only if the odc $G \circ F^{-1}$ is convex over $(F(a), F(b))$. This can be seen by differentiating the odc:

$$\frac{d}{du} G(F^{-1}(u)) = \frac{g(F^{-1}(u))}{f(F^{-1}(u))}.$$ 

Since $F^{-1}$ is continuous and strictly increasing, the right-hand side of this equality is nondecreasing in $u$ if and only if the density ratio $f/g$ is nonincreasing, which corresponds to the slope of the odc $G \circ F^{-1}$ being nondecreasing. The relationship between the convexity of the odc and monotonicity of the density ratio was used by Carolan & Tebbs (2005) and Beare & Moon (2015) to develop statistical tests of density ratio monotonicity.

Let $\mathcal{E}$ denote the collection of all functions from $[0, 1]$ to $[0, 1]$. All odcs belong to this space. Given an open interval $(v, w) \subseteq (0, 1)$, we define the operator $GCM^{(v, w)} : \mathcal{E} \to \mathcal{E}$ as follows:

$$GCM^{(v, w)}(\phi)(u) = \sup \{ \psi(u) : \psi \in \mathcal{E}, \psi \text{ is convex over } (v, w), \text{ and } \psi \leq \phi \}.$$ 

We call $GCM^{(v, w)}$ the greatest convex minorant over $(v, w)$. It can be shown by arguing as in Eggermont & LaRiccia (2001) (pp. 225—226), that $GCM^{(v, w)}(\phi)$ is convex over $(v, w)$ and everywhere no greater than $\phi$, for any $\phi \in \mathcal{E}$.

Given a cdf $F \in \mathcal{D}$ and an open interval $(v, w) \subseteq (0, 1)$, we define the operator $A^{(v, w)}_F : \mathcal{D} \to \mathcal{D}$ in the following way:

$$A^{(v, w)}_F G = GCM^{(v, w)}(G \circ F^{-1}) \circ F, \quad G \in \mathcal{D}.$$
We let $A_F = A_F^{(0,1)}$ for brevity. The operator $A_F^{(v,w)}$ will play a key role in the construction of our option augmented density forecasts. Some properties of $A_F^{(v,w)}$ that will be important for us are gathered in the following result.

**Proposition 3.2.1.** For every cdf $F \in \mathcal{D}$ and every open interval $(v, w) \subseteq (0, 1)$, the operator $A_F^{(v,w)} : \mathcal{D} \to \mathcal{D}$ has the following properties.

(a) For every $G \in \mathcal{D}$, the odc of $A_F^{(v,w)} G$ and $F$ is convex over $(v, w)$.

(b) For every $G \in \mathcal{D}$ for which the odc of $G$ and $F$ is convex over $(v, w)$, we have $A_F^{(v,w)} G = G$.

(c) $A_F^{(v,w)}$ is weakly contractive with respect to the Kolmogorov-Smirnov metric. That is, for every pair of cdfs $G, H \in \mathcal{D}$, we have

$$d_{KS}(A_F^{(v,w)} G, A_F^{(v,w)} H) \leq d_{KS}(G, H).$$

**Proof.** The first two properties are obvious, so we will prove only the third. The key is Marshall’s lemma (Marshall (1970); see also Lemma 2.2 of Durot & Tocquet (2003)) which states that the maximal distance between the greatest convex minorants of any two functions can be no greater than the maximal distance between the functions. Since $F$ is a bijection between $[0, \infty]$ and $[0, 1]$, the Kolmogorov-Smirnov distance between $A_F^{(v,w)} G$ and $A_F^{(v,w)} H$ may be written as

$$d_{KS}(A_F^{(v,w)} G, A_F^{(v,w)} H)$$

$$= \sup_{x \in [0, \infty]} \left| GCM^{(v,w)}(G \circ F^{-1})(F(x)) - GCM^{(v,w)}(H \circ F^{-1})(F(x)) \right|$$

$$= \sup_{u \in [0, 1]} \left| GCM^{(v,w)}(G \circ F^{-1})(u) - GCM^{(v,w)}(H \circ F^{-1})(u) \right|.$$

By Marshall’s lemma, the latter quantity is bounded by the maximal distance between the odcs $G \circ F^{-1}$ and $H \circ F^{-1}$. Again using the fact that $F$ is a bijection between $[0, \infty]$ and $[0, 1]$, we
obtain

\[
d_{KS} \left( \mathcal{F}^{v,w}_F G, \mathcal{F}^{v,w}_F H \right) \leq \sup_{u \in [0,1]} \left| G(F^{-1}(u)) - H(F^{-1}(u)) \right|
\]

\[
= \sup_{x \in [0,\infty]} |G(x) - H(x)|
\]

\[
= d_{KS}(G,H),
\]

as claimed. \qed

3.2.2 Pricing kernel monotonization

Let \( P_t \) be the cdf describing the value of a market index at time \( t + 1 \), conditional on all relevant information available at time \( t \). We call \( P_t \) the physical cdf. Let \( Q_t \) denote the corresponding risk neutral cdf, which could be inferred, for instance, from the prices at time \( t \) of vanilla options at a dense range of strikes expiring at time \( t + 1 \). We assume that \( P_t \) and \( Q_t \) belong to \( \mathcal{D} \), and admit pdfs \( p_t \) and \( q_t \) that are strictly positive on \((0,\infty)\). The density ratio \( q_t / p_t \) is called the pricing kernel.

While \( Q_t \) may be inferred from option prices, it can be difficult or impossible in empirical work for us to accurately characterize \( P_t \), as we may not be in possession of all relevant information upon which \( P_t \) is based. Instead it is common to base an estimator \( \hat{P}_t \) of \( P_t \) on a time series model—frequently some variant of GARCH—fitted to historical market return data. In this section we take no stance on how the estimator \( \hat{P}_t \) is constructed, allowing it to be any cdf in \( \mathcal{D} \) admitting a pdf \( \hat{p}_t \) strictly positive on \((0,\infty)\).

The so-called pricing kernel puzzle is the observation that the empirical pricing kernel \( q_t / \hat{p}_t \) frequently fails to be nonincreasing when \( \hat{P}_t \) is constructed from historical return data. Instead, it sometimes exhibits increasing behavior around the center of the return distribution, as in Jackwerth (2000), or at high return levels, as in Bakshi et al. (2010). Regions over which the pricing kernel increases are inconsistent with standard asset pricing models and imply perverse behavior in options markets Beare (2011). It was pointed out by Chabi-Yo et al. (2007) that
nonmonotonicity of the empirical pricing kernel $q_t / \hat{p}_t$ may be due to the fact that the estimator \( \hat{p}_t \) poorly captures the shape of \( p_t \) when the latter is based on all relevant information available at time \( t \) regarding the value of the market index at time \( t + 1 \), and the former is based only on historical market returns. The same point was made more recently by Linn et al. (2017), Cuesdeanu & Jackwerth (2016) and Sala & Mira (2017).

Suppose the true but unobserved pricing kernel $q_t / p_t$ is nonincreasing. In this case it may be possible to use this information to improve the estimated physical distribution $\hat{P}_t$. We propose to do precisely that, by replacing $\hat{P}_t$ with $A_{Q_t} \hat{P}_t$. More generally, if we only believe $q_t / p_t$ to be nonincreasing over some interval $(Q_t^{-1}(u), Q_t^{-1}(v))$, where $0 \leq u < v \leq 1$, then we propose to replace $\hat{P}_t$ with $A_{Q_t}^{(u,v)} \hat{P}_t$. We call $A_{Q_t}^{(u,v)} \hat{P}_t$ an option augmented distributional forecast, and the corresponding pdf an option augmented density forecast.

From Proposition 3.2.1(a) we know that the odc of $A_{Q_t}^{(u,v)} \hat{P}_t$ and $Q_t$ is guaranteed to be convex over $(v, w)$, which corresponds to the implied pricing kernel being nonincreasing over $(Q_t^{-1}(u), Q_t^{-1}(v))$. From Proposition 3.2.1(b) we know that if the empirical pricing kernel $q_t / \hat{p}_t$ is nonincreasing over $(Q_t^{-1}(u), Q_t^{-1}(v))$ then $A_{Q_t}^{(u,v)} \hat{P}_t = \hat{P}_t$, so that our option augmentation procedure has no effect. Proposition 3.2.1(c) tells us something deeper about the option augmented distributional forecast $A_{F_t}^{(u,v)} \hat{P}_t$, which we will state as a separate result.

**Proposition 3.2.2.** If $P_t, Q_t \in \mathcal{D}$ are such that the odc of $P_t$ and $Q_t$ is convex over some open interval $(u, v) \subseteq (0, 1)$, then for every $\hat{P}_t \in \mathcal{D}$, we have

$$d_{KS}(A_{Q_t}^{(u,v)} \hat{P}_t, P_t) \leq d_{KS}(\hat{P}_t, P_t).$$

**Proof.** Since the odc of $P_t$ and $Q_t$ is convex over $(u, v)$, Proposition 3.2.1(b) implies that $A_{Q_t}^{(u,v)} P_t = P_t$. The desired result now follows from Proposition 3.2.1(c) by setting $G = \hat{P}_t$ and $H = P_t$. \[\square\]

Proposition 3.2.2 tells us that if the pricing kernel $q_t / p_t$ is nonincreasing over
or, equivalently, if the odc of $P_t$ and $Q_t$ is convex over $(u, v)$—then the option augmented distributional forecast $\mathcal{A}^{(u,v)}_{Q_t} \hat{P}_t$ is guaranteed to be weakly closer in the Kolmogorov-Smirnov sense to $P_t$ than is the unaugmented distributional forecast $\hat{P}_t$. This is true regardless of how $\hat{P}_t$ is constructed. Guaranteed improvements to estimation brought about by shape constraints have been shown to occur in other contexts; see, for instance, Chernozhukov et al. (2010), Proposition 4.

While Proposition 3.2.2 establishes an encouraging property of our option augmentation procedure, there is no reason to consider the Kolmogorov-Smirnov distance to be the most natural with which to assess the approximation of $P_t$. It is difficult to imagine extending Proposition 3.2.2 to allow for more general distances without obtaining a corresponding generalization of Marshall’s lemma, which seems unlikely. We can nevertheless investigate the performance of our procedure empirically using a range of potentially more relevant criteria. This task will be undertaken in Section 3.3.

### 3.2.3 Numerical computation

Our option augmentation procedure can be implemented by computing the convex hull of a collection of points in the plane. To illustrate, we shall revisit the pricing kernel estimate obtained by Jackwerth (2000) in arguably the seminal paper on the pricing kernel puzzle. In the top two panels of Figure 3.1 we reproduce the risk neutral and physical density estimates given by Jackwerth (2000), Fig. 2, and display their ratio, the implied pricing kernel. These estimates correspond to one month returns on the S&P 500 index on April 15, 1992. The same plots appear in Beare (2011), Figs. 4.1–4.2 and Beare & Schmidt (2016), Fig. 1. Note that the estimated pricing kernel is nonmonotone, and exhibits an increasing region around the center of the return distribution, as well as a smaller increasing region at very low return levels. We will apply our option augmentation procedure to the Jackwerth (2000) density estimates, resulting in a monotone pricing kernel estimate.

The input to our procedure consists of estimated risk neutral and physical cdf values,
which we record over a fine grid of \( n \) payoff values spread across the support of the payoff distribution. In our example with the Jackwerth (2000) estimates, the grid consists of \( n = 98 \) evenly spaced points running from 0.68 to 1.26. We take the \( n \) pairs of risk neutral and physical cdf values recorded at these grid points, and display them as a scatter plot in Figure 3.2. Filled and unfilled circles represent pairs of cdf values, with risk neutral probability measured by the horizontal axis and physical probability measured by the vertical axis. We also include an artificial data point at the upper-left corner \((0, 1)\).

Option augmented cdf values are obtained by computing the convex hull of the \( n + 1 \) points in our scatter plot. The pairs of cdf values that form vertices of the boundary of the convex hull are displayed with filled circles, while the pairs in the interior of the convex hull are displayed with unfilled circles. Popular scientific computing packages typically include fast algorithms for computing the convex hull of a set of points in the plane; we used the convhull
command in MATLAB. For a given payoff value $x$ in our grid of $n$ values, the option augmented cdf value $A_QP(x)$ can be read off Figure 3.2 by finding the vertical distance between $Q(x)$ on the horizontal axis, and the lower boundary of the convex hull. This is a simple calculation, once the convex hull has been computed. If we wanted to monotonize the pricing kernel over only a subset of payoff values, this could be done by restricting the convex hull computation to include only those $(Q(x), P(x))$ pairs with $Q(x)$ belonging to a specified subinterval $(u, v) \subseteq (0, 1)$.

In the bottom two panels of Figure 3.1 we display the outcome of applying our option augmentation procedure to the Jackwerth (2000) estimates. Option augmentation has shifted the estimate of the physical pdf so that it now appears to be closer to the risk neutral pdf. The risk neutral pdf is of course unchanged, and the pricing kernel is monotone by construction. The flat region around the middle of the pricing kernel corresponds to the lengthy linear segment of the lower part of the boundary of the convex hull shown in Figure 3.2, running from around 0.1 to around 0.9 on the horizontal axis. The shorter flat region in the pricing kernel at lower return levels is difficult to see in Figure 3.2 because those return levels correspond to risk neutral probabilities very close to zero, where there are many nearby points in the scatter plot.
3.3 Empirical results: S&P 500 index options

In Section 3.3.1 we describe our data set, and in Section 3.3.2 we explain how we estimated the physical and risk neutral densities. In Section 3.3.3 we present our empirical results: the relative performance of the baseline time series physical density forecasts and option augmented physical density forecasts.

3.3.1 Data

The dataset used for our analysis is essentially the same as that of Beare & Schmidt (2016). It may be helpful to refer to that paper for additional details.

Our data include the prices of European call and put options written on the S&P 500 from January 1997 to December 2013. We collect prices on one day during each month, when the time-to-maturity is between 18 and 22 days. Each price is the bid-ask average of the closing prices reported by the Option Price Reporting Authority.1 Alongside each option price we observe the corresponding index level, time-to-maturity and strike price. We exclude options with strike prices deviating from the index level by more than 15%. The time intervals of 18–22 days between the dates at which we record option prices and the corresponding expiry dates are approximately, but not perfectly, non-overlapping. Some overlap is unavoidable because there is one expiry date each month, occurring on the Thursday before the third Friday of the month, and these expiry dates are approximately but not exactly separated by one month. We exclude six months from our analysis due to missing, incomplete or inconsistent data, leaving us with option data for a total of 199 months.

On each day at which we record option prices, we use the one-month Treasury bill rate to determine the risk-free discount rate corresponding to the time-to-maturity of the options. Following Bondarenko (2000), pp. 29–30, we then impute the corresponding forward price of the S&P 500 index by averaging the sum of the call-put differential (inflated by the prevailing

risk-free rate) and strike price over those strikes within 15 points of the current S&P 500 index value. This approach to determining the forward price avoids synchronization issues associated with mixing data from the futures and options markets. We then discard all in-the-money (ITM) options from our data set, and convert out-of-the-money (OTM) call option prices to artificial ITM put option prices by applying put-call parity based on the imputed forward price.

The time series model we use to estimate the physical density is the Realized GARCH model of Hansen et al. (2012). This model takes as inputs a daily time series of S&P 500 returns, and also a daily time series of realized kernels, which are measurements of intraday volatility based on high frequency return data Barndorff-Nielsen et al. (2008). We obtained these data from the Oxford-Man Institute Realized Library2 (Gerd et al. (2009)). Both time series run from January 1996 to December 2013. Realized kernel measurements are similar to the realized variance, but provide a more accurate estimate of quadratic variation due to their robustness to market microstructure noise and to endogenous data spacing and market friction; see Barndorff-Nielsen et al. (2008) for details. See Figure 3 of Beare & Schmidt (2016) for a visual comparison of the volatility measurements produced by our Realized GARCH model with risk neutral volatilities.

3.3.2 Estimation

For each of the 199 dates at which we record option prices, we estimate the risk neutral density corresponding to the value of the S&P 500 index at the date, 18–22 trading days hence, at which all the options expire. While many estimators for the risk neutral density are available, we choose to use the positive convolution approximation (PCA) method of Bondarenko (2003); see also Bondarenko (2000) for an extended version of the same paper. PCA is a nonparametric method that produces smooth well behaved densities while minimizing the possibility of overfitting, and has been shown in simulations to have good small sample properties compared to other parametric and nonparametric alternatives.

2http://realized.oxford-man.ox.ac.uk.
The method of PCA provides an estimate of the risk neutral density that belongs to a particular class of admissible pdfs. That class is determined by the specification of a kernel function \( k : [0, \infty)^2 \rightarrow [0, \infty) \) such that \( k(x, \cdot) \) is a pdf supported on \([0, \infty)\) for each \( x \in [0, \infty) \).

Given the choice of kernel function, we define the class of pdfs

\[ \mathcal{W} = \left\{ \int_0^\infty k(x, \cdot)u(x)dx : u \text{ is a pdf supported on } [0, \infty) \right\}. \]

Elements of \( \mathcal{W} \) are uncountable mixtures of the pdfs \( k(x, \cdot), x \in [0, \infty) \). We further restrict the class of admissible pdfs by requiring that our risk neutral density estimator precisely implies the observed forward price \( f \) of the S&P 500 index, paid at the time our options expire. We retain only those members of \( \mathcal{W} \) that imply the exact forward price by setting

\[ \mathcal{W}_0 = \left\{ w \in \mathcal{W} : \int_0^\infty xw(x)dx = f \right\}. \]

To select our risk neutral density estimator \( \hat{q} \) from the class \( \mathcal{W}_0 \) we use the put option prices \( p_1, \ldots, p_m \) observed at strikes \( s_1 < \cdots < s_m \), with prices at ITM strikes imputed from OTM call prices as discussed in Section 3.3.1. Our risk neutral density estimator \( \hat{q} \) is the element of \( \mathcal{W}_0 \) that minimizes the quantity

\[ \sum_{i=1}^m \left( p_i - \frac{1}{1 + r} \int_0^{s_i} \int_0^y \hat{q}(x)dx dy \right)^2, \]

where \( r \) is the prevailing risk-free discount rate. The double integral here derives from the well-known formula of relating the risk neutral density to the second derivative of the associated put option pricing function. It is clear from this relation that our estimator \( \hat{q} \) is the member of \( \mathcal{W}_0 \) that minimizes the sum of squared put option pricing errors.

The preceding discussion of PCA differs in certain details from that of Bondarenko (2000) and Bondarenko (2003), who required the kernel \( k \) to be of the form \( k(x, y) = \phi(y - x) \) for some pdf \( \phi \). In this case the mixture distribution \( \int_0^\infty k(x, \cdot)u(x)dx \) is simply the convolution of
\( \phi \) and \( u \). Carefully annotated Matlab code to implement PCA written by Bondarenko and dated July 2005 extends his earlier work to allow for mixtures that do not correspond to convolution. We use this code to implement our risk neutral density estimation. In it, Bondarenko specifies \( k(x, \cdot) \) to be the pdf of a log-normal random variable whose logarithm has mean \( \ln x \) and standard deviation proportional to the Black-Scholes implied volatility at moneyness \( x/f \). The code is available upon request to readers seeking additional details.

For each of the 199 risk neutral densities estimated as described in the preceding subsection, we require an estimate of the corresponding physical density. To this end we employ the log-linear Realized GARCH \((p, q)\) model of Hansen et al. (2012). This model extends the classic GARCH model of Bollerslev (1986) in two main directions: it incorporates measurements of volatility based on high frequency intraday return data, and it allows leverage effects—that is, asymmetric responses of volatility to positive and negative innovations. The Realized GARCH model is based on three equations referred to respectively as the return equation, the GARCH equation, and the measurement equation:

\[
\begin{align*}
\log y_t &= \mu + \sqrt{h_t} \varepsilon_t \\
\log h_t &= \omega + \sum_{i=1}^{p} \beta_i \log h_{t-i} + \sum_{j=1}^{q} \gamma_j \log x_{t-j} \\
\log x_t &= \xi + \delta \log h_t + \eta_1 \varepsilon_t + \eta_2 (\varepsilon_t^2 - 1) + \nu_t.
\end{align*}
\]

Here, the observed data are \( y_t \), the ratio of the values of the S&P 500 index on days \( t \) and \( t - 1 \), and \( x_t \), the daily realized kernel constructed from high frequency intraday return data as in Barndorff-Nielsen et al. (2008). The terms \( \varepsilon_t \) and \( \nu_t \) are iid innovations assumed to satisfy \( E(\varepsilon_t) = E(\nu_t) = 0 \) and \( E(\varepsilon_t^2) = 1 \), and \( h_t \) is a latent volatility process. Note that \( t \) indexes consecutive trading days, and does not correspond to the 199 dates at which we observe option prices. At each of these 199 dates, we estimate the Realized GARCH model using all current and prior daily observations of \( x_t \) and \( y_t \), going back to the beginning of 1996. We set \( p = 1 \) and
\( q = 2 \), which is the preferred specification of Hansen et al. (2012) for SPY returns, and also the specification used by Beare & Schmidt (2016). Estimation is by maximum likelihood assuming that \( \nu_t \) is Gaussian and that \( \epsilon_t \) is Student-\( t \) distributed, rescaled to have variance one. To achieve this we used version 1.3-6 of the rugarch package of Alexios Ghalanos.\(^3\)

Each time we fit the Realized GARCH model using return data leading up to one of the 199 dates at which we observe option prices, we use the fitted model to simulate the physical density corresponding to the estimated risk neutral density for that date. This is achieved using a resampling procedure sometimes referred to as filtered historical simulation Barone-Adesi et al. (1998). Depending on the time-to-maturity of options, we seek to simulate the distribution of the index value \( h \) trading days ahead, where \( h \) is between 18 and 22. Let \( t = 0 \) index the date at which option prices are observed, and \( t = -T + 1, \ldots, 0 \) index the time span of the series used to fit the Realized GARCH model. We begin by resampling pseudo-innovations \( \epsilon_1^*, \ldots, \epsilon_h^* \) and \( \nu_1^*, \ldots, \nu_{h-1}^* \) from the fitted innovations \( \hat{\epsilon}_{-T+1}, \ldots, \hat{\epsilon}_0 \) and \( \hat{\nu}_{-T+1}, \ldots, \hat{\nu}_0 \). We then feed these pseudo-innovations iteratively into the estimated return, GARCH and measurement equations to obtain simulated returns \( \log y_1^*, \ldots, \log y_h^* \) that depend only on the data observed up to time \( t = 0 \). The simulated market index value for time \( t = h \) is then given by \( S_h^* = S_0 \prod_{i=1}^{h} y_i^* \), where \( S_0 \) is the observed market index value at time \( t = 0 \). We repeat this entire procedure 100,000 times to obtain 100,000 realizations of \( S_h^* \). Our estimated physical density is then obtained by applying a kernel density estimator to these simulated values, with Gaussian kernel and bandwidth selected as in Sheather & Jones (1991). We construct 199 estimated physical densities in this fashion, to pair with our 199 estimated risk neutral densities.

### 3.3.3 Density forecast evaluation

Denote by \( \hat{P}_t \) the 199 physical cdfs, and by \( Q_t \) the corresponding 199 risk neutral cdfs, constructed as described in Section 3.3.2. The index \( t \) now corresponds to the date at which the options used to construct the risk neutral cdf \( Q_t \) expire, and ranges over the period 1997–2013 at

\(^3\)https://cran.r-project.org/web/packages/rugarch/index.html.
intervals of approximately 20 trading days. Denote by $S_t$ the value of the S&P 500 index at time $t$. Following Diebold et al. (1998), we will evaluate the performance of the distributional forecasts $\hat{P}_t$ by assessing whether the associated sequence of probability integral transforms (PITs) $\hat{P}_t(S_t)$ forms an independent and identically distributed (iid) sequence of random variables distributed uniformly on the unit interval, i.e. distributed $\mathcal{U}(0, 1)$. See also Gneiting et al. (2007) and Elliott & Timmermann (2016), ch. 18, for further discussion of the evaluation of distributional forecasts.

We will compare the performance of the distributional forecasts $\hat{P}_t$ to that of two modified versions of $\hat{P}_t$ obtained by using the transformation described in Section 3.2 to monotonize the pricing kernel implied by $\hat{P}_t$ and $Q_t$. In the notation of Section 3.2, $\mathcal{A}_Q\hat{P}_t$ is the cdf obtained by monotonizing the pricing kernel everywhere, and $\mathcal{A}_{Q, (0.25, 0.75)}\hat{P}_t$ is the cdf obtained by monotonizing the pricing kernel between the 0.25 and 0.75 quantiles of the risk neutral distribution. The second of these distributional forecasts might be expected to outperform the first if the prices of far out-of-the-money options are unreliable.

We first investigate whether our three sequences of PITs—$\hat{P}_t(S_t)$, $\mathcal{A}_Q\hat{P}_t(S_t)$ and $\mathcal{A}_{Q, (0.25, 0.75)}\hat{P}_t(S_t)$—are serially independent. The data are clear on this point: there is no evidence of serial dependence in any of the three sequences. In Table 3.1 we report the first order autocorrelations and associated p-values of the raw PITs and of their centered squares and cubes. The estimated autocorrelations are numerically and statistically close to zero. Higher order autocorrelations, not reported, were also found to be numerically and statistically close to zero. The lack of serial correlation in the centered squares of the PITs is particularly salient. It is well known that financial return data are conditionally heteroskedastic; as noted by Diebold et al. (1998), failure to accurately model this feature of the data will manifest as serial correlation in the centered squares of the PITs. Our results therefore indicate that the Realized GARCH model does a good job of capturing the pattern of heteroskedasticity in the data, and that the imposition of pricing kernel monotonicity leaves this property intact.

It remains for us to assess the distributional uniformity of our three sequences of PITs. In Table 3.2 we report the outcome of Kolmogorov-Smirnov, Cramér-von Mises and Anderson-
Table 3.1. Serial dependence of probability integral transforms.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{P}_t(S_t)$</th>
<th>$\mathcal{A}_{Q_t} \hat{P}_t(S_t)$</th>
<th>$\mathcal{A}_{Q_t}^{(0.25,0.75)} \hat{P}_t(S_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIT</td>
<td>autocorrelation</td>
<td>-0.040</td>
<td>-0.039</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.572)</td>
<td>(0.591)</td>
<td>(0.544)</td>
</tr>
<tr>
<td>(PIT − 0.5)$^2$</td>
<td>autocorrelation</td>
<td>-0.013</td>
<td>-0.048</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.858)</td>
<td>(0.502)</td>
<td>(0.808)</td>
</tr>
<tr>
<td>(PIT − 0.5)$^3$</td>
<td>autocorrelation</td>
<td>-0.054</td>
<td>-0.045</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.453)</td>
<td>(0.532)</td>
<td>(0.449)</td>
</tr>
</tbody>
</table>

Table 3.2. Tests of uniformity of probability integral transforms.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{P}_t(S_t)$</th>
<th>$\mathcal{A}_{Q_t} \hat{P}_t(S_t)$</th>
<th>$\mathcal{A}_{Q_t}^{(0.25,0.75)} \hat{P}_t(S_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolmogorov-Smirnov</td>
<td>statistic</td>
<td>0.992</td>
<td>1.063</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.308)</td>
<td>(0.232)</td>
<td>(0.398)</td>
</tr>
<tr>
<td>Cramér-von Mises</td>
<td>statistic</td>
<td>0.251</td>
<td>0.192</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.187)</td>
<td>(0.283)</td>
<td>(0.365)</td>
</tr>
<tr>
<td>Anderson-Darling</td>
<td>statistic</td>
<td>1.346</td>
<td>1.338</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.118)</td>
<td>(0.121)</td>
<td>(0.196)</td>
</tr>
</tbody>
</table>

Darling tests of the null hypothesis that our PITs are drawn from the $U(0,1)$ distribution. Alongside the test statistics we report exact p-values computed by simulation. We find that none of the three test statistics yields a rejection of uniformity at the 10% level for any of the three PIT sequences. Moreover, the test statistics are fairly similar for the different PIT sequences. They do not give much of an indication of whether pricing kernel monotonization improves or worsens the calibration of density forecasts.

To investigate the uniformity of PITs further, we inspected plots of the empirical distributions of the three sequences of PITs, and corresponding histograms. The empirical distributions are displayed in Figure 3.3 accompanied by uniform 95% confidence bands. Under uniformity, the empirical distributions should be close to the $45^\circ$ line. We see in panel (a) that the empirical distribution of the PITs produced by the Realized GARCH model without pricing kernel monotonization falls below the $45^\circ$ line at moderate quantiles, and rises above the $45^\circ$ line at higher quantiles. When we apply full pricing kernel monotonization, producing the empirical
distribution in panel (b), we eliminate the deviations from the 45° line at moderate quantiles at the cost of mildly exacerbating them at higher quantiles. When we apply pricing kernel monotonization at moderate quantiles only, producing the empirical distribution in panel (c), we obtain a picture quite similar to that shown in panel (a), but with departures from the 45° line at moderate quantiles mitigated somewhat.

Histograms for our three sequences of PITs are provided in Figure 3.4. One aspect of all three histograms that immediately stands out is that far too few PITs fall into the tenth and final bin. This indicates that all three distributional forecasts are assigning too much probability to very high returns. Turning our attention to the remaining bins, the histogram in panel (b), corresponding to full pricing kernel monotonization, appears the closest to uniformity, while the histogram in panel (a), corresponding to no monotonization, appears the least uniform. The histogram in panel (c), corresponding to partial monotonization, provides an intermediate picture fairly similar to that in panel (a).

Gneiting et al. (2007) recommend the use of marginal calibration plots to provide further information about the behavior of a sequence of density forecasts. We provide these in Figure 3.5. The marginal calibration plot for \( \hat{P}_t \) graphs the quantity

\[
\frac{1}{T} \sum_{t=1}^{T} \hat{P}_t (S_{t-1} (1 + x)) - \frac{1}{T} \sum_{t=1}^{T} 1 \{ S_t / S_{t-1} \leq 1 + x \}
\]
as a function of different return levels \( x \). This will be close to zero if the forecasts \( \hat{P}_t \) assign probabilities to the events \( S_t/S_{t-1} \leq 1 + x \) whose average is close to the empirical frequency with which \( S_t/S_{t-1} \leq 1 + x \). Marginal calibration plots for \( \mathcal{A}_Q \hat{P}_t \) and \( \mathcal{A}_Q^{(25\%, 75\%)} \hat{P}_t \) are defined similarly. We see in Figure 3.5 that, at return levels up to around 1\%, the marginal calibration plots corresponding to no monotonization or partial monotonization stray far from zero, whereas the marginal calibration plot corresponding to full monotonization stays fairly close to zero. At return levels above around 2\% marginal calibration is worsened by monotonization, but the deterioration is perhaps not as stark as the improvement at lower return levels. In our view, taken together with the empirical distribution plots in Figure 3.3 and the histograms in Figure 3.4, the marginal calibration plots in Figure 3.5 provide fairly compelling informal evidence that the density forecasts with full pricing kernel monotonization are the best calibrated of our density forecasts.

As discussed by Elliott & Timmermann (2016), ch. 18, proper scoring rules can provide a simple and effective means for ranking alternative density forecasts. A popular proper scoring rule is the continuous ranked probability score. This is computed for \( \hat{P}_t \) as

\[
\text{CRPS} = \int_{-\infty}^{\infty} \text{BS}(x) \, dx = \int_{-\infty}^{\infty} \frac{1}{T} \sum_{t=1}^{T} \left( \hat{P}_t(S_{t-1}(1+x)) - \mathbb{1}(S_t/S_{t-1} \leq 1 + x) \right)^2 \, dx,
\]
and similarly for $\mathcal{A}_{Q_{t}} \hat{P}_{t}$ and $\mathcal{A}^{(.25,.75)}_{Q_{t}} \hat{P}_{t}$. The quantity BS($x$) is the Brier score for predicting the binary event that returns are equal to or less than $x$; that is, $S_{t}/S_{t-1} \leq 1 + x$. We computed the continuous ranked probability scores for our three series of density forecasts to be 0.0260 without pricing kernel monotonization, 0.0256 with full monotonization, and 0.0258 with partial monotonization. Lower scores indicate better performance, so the full pricing kernel monotonization yields the best continuous ranked probability score. The difference between the best and worst scores is not, however, statistically significant, with the associated p-value being 0.23. On the other hand, the practically relevant Brier score BS(0) corresponding to the prediction of positive/negative returns is 0.2475 without pricing kernel monotonization, 0.2388 with full pricing kernel monotonization, and 0.2433 with partial pricing kernel monotonization. The difference between the best and worst Brier scores is borderline statistically significant, with the associated p-value being 0.052.

Additional empirical results for index options in France, Germany, Hong Kong, Japan and the United Kingdom are provided in Appendix B. Our results with these data are somewhat more favorable toward our option augmentation procedure than are our results with the US data:
there is stronger evidence that imposing pricing kernel monotonicity improves the probabilistic calibration of density forecasts.

3.4 Conclusion

The picture to emerge from our empirical assessment of the effect of imposing pricing kernel monotonicity upon density forecasts of S&P 500 index returns is somewhat murky. This is perhaps to be expected. Using the same S&P 500 index option data, Beare & Schmidt (2016) find that pricing kernel monotonicity can be statistically rejected at the 5% level in slightly less than half of the months in the sample. We would not expect pricing kernel monotonization to have much effect upon density forecasts in months where the pricing kernel is approximately monotone. Nevertheless, the empirical distributions, histograms and marginal calibration plots appearing in Figures 3.3–3.5 provide reasonably compelling evidence that pricing kernel monotonization can lead to an improvement in the probabilistic calibration of density forecasts. Further evidence of this is provided by supplementary results for index option data from France, Germany, Hong Kong, Japan and the United Kingdom given in Appendix B. Our results suggest that failures of pricing kernel monotonicity may be informative about future returns, and usefully incorporated into return density forecasts.

3.5 Acknowledgements

Chapter 3, in full, is a reprint of the material as it appears in Quantitative Finance 18 (4), 623-625, 2017. Beare, Brendan.; Dossani, Asad. The dissertation author was a primary author of this paper.
Appendix A

Central Bank Tone and Currency Risk Premia

A.1 Variance and Volatility Swaps

There is an important distinction to be made between variance and volatility swaps. Volatility is the square root of variance, and is often more frequently quoted than variance in the context of options. It is theoretically possible to construct either a variance or volatility swap. In practice, variance swaps are traded more often than volatility swaps. Most variance swaps, including currency variance swaps, are traded in over the counter markets. One exception is S&P500 variance futures traded on the Chicago Board Options Exchange (CBOE).

The reason is variance swaps are preferred to volatility swaps is ease of replication. Demeterfi et al. (1999) show that a variance swap can be replicated with a static portfolio of options and dynamic hedging of the underlying asset. However, replicating a volatility swap requires a dynamic portfolio of options. In fact, from a pricing and replication perspective, a volatility swap is best thought of as a derivative of a variance swap.

As discussed, most of the literature on the variance risk premium has focused on its predictive power over future currency returns. If the variance risk premium is used only as a predictor, it does not matter how easy or difficult it is to replicate in the market. There is no reason to favor variance swaps over volatility swaps. Since this paper is focused on actual returns from variance swaps, there is a strong case for using variance swaps instead of volatility swaps.
A trading strategy based on central bank announcements would be much easier to implement using variance swaps rather than volatility swaps.

A.2 ECB Press Conference

This is a portion of the introductory statement of the ECB press conference on September 4, 2014. It contained six dovish phrases and no hawkish phrases. One of the dovish phrases appears in this portion, and is highlighted in bold.

Ladies and gentlemen, the Vice-President and I are very pleased to welcome you to our press conference. We will now report on the outcome of today’s meeting of the Governing Council, which was also attended by the Commission Vice-President, Mr Katainen.

Based on our regular economic and monetary analyses, the Governing Council decided today to lower the interest rate on the main refinancing operations of the Eurosystem by 10 basis points to 0.05% and the rate on the marginal lending facility by 10 basis points to 0.30%. The rate on the deposit facility was lowered by 10 basis points to -0.20%. In addition, the Governing Council decided to start purchasing non-financial private sector assets. The Eurosystem will purchase a broad portfolio of simple and transparent asset-backed securities (ABSs) with underlying assets consisting of claims against the euro area non-financial private sector under an ABS purchase programme (ABSPP). This reflects the role of the ABS market in facilitating new credit flows to the economy and follows the intensification of preparatory work on this matter, as decided by the Governing Council in June. In parallel, the Eurosystem will also purchase a broad portfolio of euro-denominated covered bonds issued by MFIs domiciled in the euro area under a new covered bond purchase programme (CBPP3). Interventions under these programmes will start in October 2014. The detailed modalities of these programmes will be announced after the Governing Council meeting of 2 October 2014. The newly decided measures, together with the targeted longer-term refinancing operations which will be conducted in two weeks, will have a sizeable impact on our balance sheet.

These decisions will add to the range of monetary policy measures taken over recent months. In particular, they will support our forward guidance on the key...
ECB interest rates and reflect the fact that there are significant and increasing differences in the monetary policy cycle between major advanced economies. They will further enhance the functioning of the monetary policy transmission mechanism and support the provision of credit to the broad economy. In our analysis, we took into account the overall subdued outlook for inflation, the weakening in the euro area growth momentum over the recent past and the continued subdued monetary and credit dynamics. Today's decisions, together with the other measures in place, have been taken with a view to underpinning the firm anchoring of medium to long-term inflation expectations, in line with our aim of maintaining inflation rates below, but close to, 2%. As our measures work their way through to the economy they will contribute to a return of inflation rates to levels closer to 2%. Should it become necessary to further address risks of too prolonged a period of low inflation, the Governing Council is unanimous in its commitment to using additional unconventional instruments within its mandate.

A.3 Swiss Franc Outlier

On January 15, 2015, the Swiss National Bank abandoned its peg against the euro. They also lowered interest rates from -0.25% to -0.75%, pushing further into negative territory. While they did not hold a press conference on that day, they did release a statement explaining why they abandoned the peg. That statement is reproduced below:

The Swiss National Bank (SNB) is discontinuing the minimum exchange rate of CHF 1.20 per euro. At the same time, it is lowering the interest rate on sight deposit account balances that exceed a given exemption threshold by 0.5 percentage points, to 0.75%. It is moving the target range for the three-month Libor further into negative territory, to between 1.25% and 0.25%, from the current range of between 0.75% and 0.25%.

The minimum exchange rate was introduced during a period of exceptional overvaluation of the Swiss franc and an extremely high level of uncertainty on the financial markets. This exceptional and temporary measure protected the Swiss economy from serious harm. While the Swiss franc is still high, the overvaluation has decreased as a whole since the introduction of the minimum exchange rate. The economy was able to take advantage of this phase to adjust to the new
situation.

Recently, divergences between the monetary policies of the major currency areas have increased significantly – a trend that is likely to become even more pronounced. The euro has depreciated considerably against the US dollar and this, in turn, has caused the Swiss franc to weaken against the US dollar. In these circumstances, the SNB concluded that enforcing and maintaining the minimum exchange rate for the Swiss franc against the euro is no longer justified.

The SNB is lowering interest rates significantly to ensure that the discontinuation of the minimum exchange rate does not lead to an inappropriate tightening of monetary conditions. The SNB will continue to take account of the exchange rate situation in formulating its monetary policy in future. If necessary, it will therefore remain active in the foreign exchange market to influence monetary conditions.

The currency rose by 18% against the dollar in a single day. This is approximately 26 times the daily the standard deviation, i.e. a 26 sigma event. There were other ramifications such as low liquidity in the Swiss Franc for a considerable period of time. Various foreign exchange brokers lost heavily, as client margin requirements did not cover such an extreme move. The outlier has a significant impact on the results. When interpreting the summary statistics and regression coefficients as a measure of central tendency, it is better to exclude the outlier. That said, outliers like this serve to illustrate the powerful impact of central banks on currency markets.
Appendix B

Empirical results for FR, DE, HK, JP, UK

Here we report empirical results using market index options for five additional countries: France, Germany, Hong Kong, Japan and the United Kingdom. We will be more brief than we were in Section 3.3 with the US data.

B.1 Data

Our additional data cover five major market indices: the CAC 40 (France), the DAX (Germany), the Hang Seng Index (Hong Kong), the Nikkei 225 (Japan) and the FTSE 100 (United Kingdom). For each index our data include option prices, total returns and risk-free interest rates.

B.1.1 Option prices

The option price data come from Bloomberg. They consist of 30-day implied volatilities across a range of strike prices. 30-day implied volatilities are constructed using options expiring before and after 30 days, similar to how the VIX is constructed using S&P 500 options. The strike prices correspond to moneyness, expressed as a percentage of the current spot price. Data are available for the following levels of moneyness: 80%, 90%, 95%, 97.5%, 100%, 102.5%, 105%, 110%, and 120%. In early parts of the sample, the 80% and 120% strikes are not available. The data are sampled at the close of the first trading day of each month, so that the data are almost non-overlapping. The data run from January 2006 to December 2016 for all indices other
than the FTSE 100, which runs from October 2007 to December 2016. There are 132 months of data for all indices other than the FTSE 100, which has 111 months of data.

B.1.2 Total returns

The total return data come from Bloomberg. The total return index measures the return from investing in the index assuming dividends are reinvested. Data are sampled at the close of each trading day, running from January 1996 through December 2016.

B.1.3 Risk-free interest rates

The interest rate data come from Global Financial Data.\footnote{http://www.globalfinancialdata.com.} We use the one month T-bill rate whenever it is available; otherwise we use the three month T-bill rate. In most cases the one month rate is available.

B.2 Estimation

To estimate risk neutral densities we again used the PCA method of Bondarenko (2003), as described in Section 3.3.2. To estimate physical densities we fit the GJR-GARCH(1,1,1) model of Glosten et al. (1993) to excess total returns. This is essentially a GARCH(1,1) model augmented with an additional term that allows volatility to respond asymmetrically to positive and negative returns: the so-called leverage effect. It requires only daily return data, not high frequency intraday data.

B.3 Results

As in Section 3.3.3, we used our estimated physical and risk neutral densities to construct sequences of PITs \( \hat{P}_t(S_t) \) and \( \hat{Q}_t(S_t) \) corresponding to no monotonization and full monotonization of the empirical pricing kernel. Under correct probabilistic calibration, these should
be iid sequences of $U(0, 1)$ random variables. Tests of serial correlation of the kind reported in Table 3.1 indicated no evidence of serial correlation; to conserve space we do not report these.

In Table B.1 we report the outcome of Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling tests of the null hypothesis that our PITs are drawn from the $U(0, 1)$ distribution. Comparing the test statistics with and without pricing kernel monotonization, it is noteworthy that monotonization always results in a smaller—sometimes much smaller—test statistic, indicating a smaller measured departure from uniformity. Uniformity can occasionally be rejected at the 5% or 10% level when monotonicity is not imposed, but never when it is imposed. In Figure B.1 we display empirical distribution functions and histograms for our PITs with and without pricing kernel monotonization, which are again generally supportive of the hypothesis that pricing kernel monotonization improves PIT uniformity. The results for the United Kingdom are somewhat ambiguous, with the empirical distributions suggesting improved uniformity, but the histogram for the monotonized PITs exhibiting a large spike at the fourth quintile.
<table>
<thead>
<tr>
<th>Country</th>
<th>Kolmogorov-Smirnov</th>
<th>Cramér-von Mises</th>
<th>Anderson-Darling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>statistic</td>
<td>(p-value)</td>
<td>statistic</td>
</tr>
<tr>
<td>France</td>
<td>1.306 (0.050)</td>
<td>0.696 (0.768)</td>
<td>1.298 (0.138)</td>
</tr>
<tr>
<td>Germany</td>
<td>1.045 (0.178)</td>
<td>0.609 (0.768)</td>
<td>1.304 (0.135)</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>0.783 (0.624)</td>
<td>0.609 (0.768)</td>
<td>1.126 (0.243)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.870 (0.359)</td>
<td>0.609 (0.768)</td>
<td>1.431 (0.086)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.424 (0.042)</td>
<td>0.759 (0.670)</td>
<td>1.696 (0.032)</td>
</tr>
</tbody>
</table>
Figure B.1. Empirical distributions and histograms: international data.
Bibliography


Bauer, Michael D, & Rudebusch, Glenn D. 2013. The signaling channel for Federal Reserve
bond purchases.


Belke, Ansgar Hubertus, & Klose, Jens. 2010. (How) do the ECB and the Fed react to financial market uncertainty? The Taylor rule in times of crisis.


Chabi-Yo, Fousseni, Garcia, Rene, & Renault, Eric. 2007. State dependence can explain the risk


Cieslak, Anna, Morse, Adair, & Vissing-Jorgensen, Annette. 2016. Stock returns over the FOMC cycle.


Del Negro, Marco, Giannoni, Marc P, & Patterson, Christina. 2012. The forward guidance puzzle.


Filardo, Andrew J, & Hofmann, Boris. 2014. Forward guidance at the zero lower bound.


Gambacorta, Leonardo, Hofmann, Boris, & Peersman, Gert. 2014. The effectiveness of unconventional monetary policy at the zero lower bound: A cross-country analysis. Journal of
Money, Credit and Banking, 46(4), 615–642.


Ghalanos, Alexios. 2015. Introduction to the rugarch package.(Version 1.3-1).


Krippner, Leo. 2014. Measuring the stance of monetary policy in conventional and unconventional environments.

Krippner, Leo. 2015. A comment on Wu and Xia (2015), and the case for two-factor shadow
short rates.


Schmeling, Maik, & Wagner, Christian. 2016. Does central bank tone move asset prices?


