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The escalating food demand by a growing and increasingly affluent global population is placing unprecedented pressure on the limited land and water resources of the planet, underpinning concerns over global food security and its sensitivity to shocks arising from environmental fluctuations, trade policies, and market volatility. Here, we use country-specific demographic records along with food production and trade data for the past 25 years to evaluate the stability and reactivity of the relationship between population dynamics and food availability. We develop a framework for the assessment of the resilience and the reactivity of the coupled population–food system and suggest that over the past two decades both its sensitivity to external perturbations and susceptibility to instability have increased.

The common tenet of current demographic theories is that global population growth is not limited by food availability (1). As such, demographic projections often do not account for resource limitations but are based on changes in demographic parameters (infant mortality, women’s fertility rates, childbearing age, or life expectancy) under different scenarios of economic development, urbanization, and sociocultural change. The idea that food availability might control population growth dates back to Malthus (2), who anticipated a constraint on demographic growth by resource limitation. Malthus’ theory, however, failed to account for technological advances that would have enhanced food production worldwide (3). The related Kuznets-like debate (4) mirrors the now data-intensive one on income and resource redistribution (5), nicely subsumed by the construction of a unified language for sustainability and policy analyses (6). Indeed, long-run economic growth is often slowed by chronic food insecurity, and the case has been made for interventions aimed at speeding food security rather than waiting for the effects of economic growth (7).

In the last two centuries—particularly in the decades after World War II—mechanized agriculture, irrigation, nitrogen fertilizers, new cultivars, and other innovations brought about by the industrial and green revolutions have led to unprecedented increases in crop yields (3, 7). In modern times, famine and destitution are induced by limitations in the access to food, rather than by lack of food per se (8). In this regard, entitlement relations, social stability (9), and resource governance seem to control food security more than food availability (8). In the early 1980s, it was noted that Malthusian analyses were inconclusive at the global scale (8). Has the situation changed since this view was put forth? The world’s population has increased by 60%, while crop yields are reaching a plateau (10). In recent years a few crises have occurred, notably in 2007–2008 and 2010–2011, which have affected food security worldwide. In both 2007 and 2010, extreme environmental conditions (e.g., droughts, extensive wildfires) occurred in conjunction with an increasing demand for agricultural products, which led to a dramatic increase in food prices, a symptom of food scarcity (11). To protect their populations, some countries decided to reduce or ban exports (12), and some trade-dependent nations started to panic. The aftermath of those crises saw episodes of social unrest and a sharp increase of food-insecure populations worldwide (13). The effects of local shocks in the production system were adverted globally as a result of the globalization of the food market (12, 14, 15). The occurrence of these episodic food crises and the associated spikes in food prices can be interpreted as warning signs of limitations in global food availability (16, 17). The emergence of global food scarcity is confirmed by a number of recent studies suggesting that the limited resources of the planet [primarily, land (18) and water (19)] would soon become insufficient to meet the escalating demand for food, fibers, and biofuels by the increasingly numerous and affluent human population (10, 20–23). Such conclusions indicate that we may be at the verge of a global-scale Malthusian catastrophe (24, 25). One then wonders how sensitive global food security is to perturbations arising from drought occurrences, changes in energy and trade policies, or food price spikes.

In this manuscript, we use data analyses to evaluate the extent to which resource availability is constraining population growth and investigate using a theoretical model the reactivity (instantaneous response) and the resilience (asymptotic response) of the coupled dynamics of population growth and food supply with respect to external perturbations. To that end, we develop a population dynamics model that accounts for the effect borne by food availability (through domestic production and trade) on the demographic growth of each country. We investigate the stability of the coupled demographic growth dynamics and identify the countries that are most vulnerable to environmental and market shocks. By using global production and trade data in conjunction with complex network analysis, we point at the nations endowed with the highest (and lowest) susceptibility to perturbations in production and trade—the hotspots of resilience and reactivity.

**Theoretical Framework**

**Analysis of Demographic and Food Data.** Our analysis assumes that population growth is in the long term constrained by food availability, and thus the carrying capacity of a nation—defined as the maximum population size that can be sustained with the

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Significance

The past few decades have seen an intensification of international food trade and the increase in the number of countries that depend on food imports. As an effect of the associated globalization of food, local shocks in food production, combined with the adoption of new national or regional energy and trade policies, have recently led to global food crises. Here we develop a framework to investigate the coupled global food-population dynamics, and evaluate the effect of international trade on global food security. We find that, as the dependency on trade increases, the global food system is losing resilience and is becoming increasingly unstable and susceptible to conditions of crisis.

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food resources that nation has access to—can be calculated as the ratio between the total amount of food available for human consumption (e.g., in calories per day) and the average diet (e.g., in calories per capita per day) for a given country (14).

The estimation of the carrying capacity parameter is performed year by year between 1986 and 2010, using production data available for each country along with per capita consumption data and information on trade flows from the food trade network (15, 23). In particular, in our analysis, we evaluate \( i \) the local carrying capacities based solely on local production data \( K_i(t) = R_{ij}(t)/cal(t) \), with \( R_{ij}(t) \) and \( cal(t) \) being the food being the local food production and the average per capita calorie consumption (23) in country \( i \) and year \( t \); and (ii) the global carrying capacities \( K_{ij}(t) = R_{ij}(t)/cal(t) \), taking into account also exports and imports of food from and into each country; thus the food calories available in country \( i \) are \( R_{ij}(t) = H_i + \sum_{j=1}^{N} (T_{j} - T_{ij}) \), with \( T_{j} \) being the trade from \( j \) to \( i \), based on trade data (Materials and Methods).

At the same time, we use country-specific population data for the same 24-y period (26) and denote with \( x_i(t) \) the population of country \( i \) on year \( t \).

Caution should be taken, of course, in a number of assumptions. The Malthusian framework based on the global supply of calories seems suitable for addressing the relationship between population dynamics and resource supply. With a perfect food-population balance, we expect that \( x \approx K_t \). However, because of food deficits, food stocks, and economic factors controlling production and trade, and delays induced by natural population dynamics, the situation is more complex. Nevertheless, there are some general patterns that are here captured in a classification of the world’s countries based on the relative magnitude of the three variables \( K_i, K_T \), and \( x \) (Materials and Methods and SI Text): Countries with \( x \approx K_t > K_T \) (group A) are net exporters and their exports substantially reduce the resources available to their populations; countries with \( x \approx K_t > K_i \) (group B) exhibit an interdependency between demographic growth and food imports. These countries cannot sustain their population relying only on their local resources. In countries with \( x \approx K_t \approx K_T \) (group C) food trade does not have a substantial impact on food availability. Finally, in group D are those countries that are affected by an overall food deficit.

Stability and Reactivity of the Food Trade System. To model the food–demographic system, we generalize the network model proposed in ref. 14. In this model, each nation that has a population smaller than its carrying capacity exports all of the food that is not consumed domestically (we are assuming that no food is stocked).

To account for the effect of trade on food security and population dynamics, we use food trade data (15, 27) and set up a generalized logistic model with delay (28) in which the carrying capacities at time \( t \) depend on the populations of other nations at time \( t - \tau \). The coupled population dynamics equations for \( N \) nations thus read:

\[
\frac{dx_i(t)}{dt} = \alpha_i x_i(t) \left( 1 - \frac{x_i(t)}{K_i(x_i(t), t)} \right) = f_i(x, t, \tau),
\]

for \( i = 1, \ldots, N \) and where \( K_i(x, t-\tau) = K_{i0}(t-\tau) + \sum_{j=1}^{N} (K_j(x, t-\tau) - x_j(t-\tau))y_{ij}(t) - x_i(t-\tau) \) is the trade-dependent carrying capacity of country \( i \) at year \( t \), with \( y_{ij}(t) \) being the fraction of country \( j \)'s imports going to \( i \) (Materials and Methods). Thus, in this model the carrying capacity depends on the net food trade matrix, \( \phi \), obtained from the Food and Agricultural Organization of the United Nations (FAO) trade data \( T \) (Materials and Methods). The time lag, \( \tau \), is introduced to account for the fact that changes in food production and trade likely affect population dynamics with a delay associated with the existence of food stocks, unsustainable use of resources, and inertia in the demographic response to changes in food availability. The above model of global population dynamics has a number of weaknesses and strengths: on the one hand, as explained below, it does not account for food stocks and fish; on the other, it accounts for the observed patterns of trade while providing a novel framework for the analysis of the resilience and sensitivity to shocks of the coupled population–resource system.

Because of the ongoing escalating changes in population, diets, and trade, the food–population system is not in an equilibrium/stationary state; rather, it is undergoing trajectories of change and therefore, its resilience cannot be evaluated as an attribute of a steady state of the system but as the response of those trajectories to perturbations (price shocks, droughts, and changes in energy or trade policies) (11). To measure the system’s asymptotic sensitivity to perturbations, we then evaluate the real part \( \text{Re}(\lambda_i) \) (with \( i = 1, \ldots, N \)) of the eigenvalues of the Jacobian matrix, \( \mathbf{J} \), of the system \( \mathbf{x}(x, t, \tau) \) in the case of short delays \( \tau \leq 1 \) year; Materials and Methods). If \( \text{Re}(\lambda_i) > 0 \), then the trajectory \( x_i(t) \) will exponentially diverge, indicating that the population dynamics of nation \( i \) are sensitive to perturbations in the long time limit.

The real part eigenvalues of \( \mathbf{J} \) characterize the stability of the asymptotic (i.e., \( t \to \infty \)) properties of the dynamics, but it does not provide any insights into their short-term response to perturbations. Even though the underlying dynamics are asymptotically stable, the short-term response to a shock may entail a transient that long-lived effects of the perturbation before it decays and the dynamics converge to their stable configuration. Indeed, in the short term, perturbations can grow substantially before they decay; these transient perturbations may undermine global food security and lead to episodic food crises. A system that experiences such initial amplifications of the perturbations is called reactive (29). To determine the possible existence of transient growth, we analyze the reactivity, defined as the maximum initial amplification rate immediately after a perturbation (29). To that end, we also investigate the transient stability of the system. It can be shown (29, 30) that the reactivity \( \lambda_{\text{react}} \) can be computed as the maximum real part of the eigenvalues of the matrix \( \mathbf{H} = (\mathbf{J} + \mathbf{J}^*)/2 \). If \( \text{Max}(\text{Re}(\lambda_i)) < 0 \) and \( \text{Max}(\text{Re}(\lambda_{\text{react}})) > 0 \), then the equilibrium point is stable but the system is said to be reactive. We note that, if \( \lambda_0(t) \to 0 \), then \( \lambda_0(t) \to 0 \) for \( t \ll \tau \) and thus reactivity can also be used to develop an early warning signal for systems approaching instability (17, 30).

Finally, we detect the directions of larger instabilities by studying the localization of the eigenvectors \( \mathbf{v} \) and \( \mathbf{w} \) corresponding to the larger real part eigenvalues, \( \lambda_i \) and \( \lambda_{\text{react}} \) of \( \mathbf{J} \) and \( \mathbf{H} \), respectively; this is done by analyzing the components of \( \mathbf{v} \) and \( \mathbf{w} \): each component corresponds to a country, and its magnitude is indicative of the impact of a potential perturbation on that node. If one or few more eigenvector components are much larger than the others, then the system is said to be localized (Materials and Methods), and the ability of perturbations to propagate through the system is reduced.

Results and Discussion

This study evaluates food security by relating population growth to the availability of an amount of food calories sufficient to meet country-specific demands. This analysis does not address the nutritional aspects of food security (e.g., adequate calorie consumption or protein requirements) (31), nor the environmental, economic, or institutional factors controlling food production and trade; rather, it relies on data analyses and a theoretical model based on generalized logistic population growth constrained by food availability. Both data-driven analysis and theoretical modeling account for some aspects of the observed global patterns of food production and trade without modeling the underlying environmental and societal drivers. As a result, this framework is not suitable for the development of future projections, but provides some criteria to interpret and identify recent and ongoing changes in global food security.
Countries belonging to group A include exporting countries in which population growth is driven by local (i.e., domestic) resources without accounting for the fact that part of these resources are currently exported (Fig. 1A). In the case of group B (Fig. 1B), the dynamics appear to be driven by food available both through domestic production and trade. These trade-dependent countries (Fig. 2) would not have been able to sustain their observed changes in population without importing food. Most of these countries are thus increasingly relying on resources they do not control. These results suggest that for nations belonging to group B an interrelationship exists between population growth and access to food through domestic production and trade. Such a dependency, however, is a necessary but not sufficient condition for the existence of food controls on population growth. In fact, the existence of interdependency does not imply causality, and it could be argued that the increase in resource availability is a consequence rather than a driver of population growth. Though there is a clear biological limit to population size imposed by the maximum amount of food the planet can provide (32), it has been argued that no conclusive evidence exists for constraints on demographic growth placed by food availability (1). Proponents of this argument (i.e., no food limitation on demographic growth) typically invoke the development of new technology and trade as means that have allowed food availability to keep up with demographic growth. However, recent environmental science literature on food security tends to show that mankind is about to drain the planet’s ability to feed us (10, 20), which suggests the existence of food limitations on demographic growth.

The fact that groups A and B are both relying on the same pool of resources suggests that their population growth is not sustainable (14) in the long run, unless food production increases through agricultural intensification (e.g., closure of yield and harvest gaps) or development of new technologies (10, 33, 34). As previously highlighted by ref. 14, the conflicting needs of these two groups are expected to lead in the long run to a strong competition for resources. If production declines or demand increases, exporting countries (group A) will likely reduce their exports as it happened during the recent food crises of 2007–2008 and 2010–2011, when, to respond to escalating food prices, a number of exporting countries reduced or banned their exports (12, 35). This situation generated panic in trade-dependent countries that saw their food security undermined by the unreliability of the global food market.

In group C, trade does not substantially alter food availability or population growth (or vice versa). Countries from the fourth group (group D; Fig. 2) are in a state of chronic food stress (Fig. 1D). Their populations have grown above the levels that would have been compatible with the rates of food supply (from both domestic production and trade) and the average country-specific diets—cal—reported by the FAO. We interpret this discrepancy as the result of a number of factors, including the access that these populations might have to food through unreported small-scale farming and fishing, that were not accounted for by the FAO data sets used in this study (Materials and Methods).

Regardless, group D exhibits clear signs of food limitation as evidenced by the poorer diets of these countries (we found an average gross diet of ~2,359 kcal·cap−1·d−1, including waste) (36) with respect to the other three groups (group A, ~3,018 kcal·cap−1·d−1; group B, ~2,767 kcal·cap−1·d−1; group C, ~2,655 kcal·cap−1·d−1; Fig. S1). Moreover, unlike groups A–C, where a surplus of resources still exists, food-stressed countries (group D) are characterized by a comparably small food deficit, indicating again the possible presence of unaccounted resources (Fig. S4).

In general, trade appears to enhance the variability of resources available to countries from group A, indicating that, as these countries increase their exports, their access to food (through domestic production and international trade) becomes on average more uncertain. However, in countries that cannot rely only on their local resources (groups B and D), trade slightly decreases the uncertainty of food availability (Fig. S8). These results indicate that countries from these four groups differ both in terms of their access to food (i.e., self-sufficient or not; with surplus or deficit) and the effect of globalization (through trade) on fluctuations in food availability.

We then investigate the resilience and reactivity of the entire system and evaluate how countries in the above four groups differ in their sensitivity to shocks. The Lyapunov analysis shows that the resilience of food security has decreased in the course of

Fig. 1. Typical examples of population, food production, and trade dynamics for four countries representative of the four different groups presented in the text. Black dots indicate demographic records, and the continuous line represents our evaluation of \(K = K_f\) (in red) and \(K = K_t\) (in green).
the study period (1986–2010) as suggested by the increase in the number of unstable countries. The number of the most sensitive countries (defined as the nodes with \( \lambda > -0.05 \)) has increased throughout the study period (Fig. 4A). These results indicate that in the last few years there has been an increase in food insecurity, as suggested by the overall decreasing stability of the coupled dynamics of population and food availability. Reactivity follows the same trend as asymptotic stability. As for the case of the Lyapunov exponents, the number of most reactive countries (\( \lambda_{HI} > -0.005 \)) increases with time. Countries that are most vulnerable to external perturbations are here identified as those with consistently positive values of \( \lambda_{HI} \); for these countries, even a small initial perturbation may grow and become a serious source of future instability. Though the sensitivity to perturbations exhibited a generalized increase in time (Fig. S2), countries from groups B and C appear to be more prone to instability than self-sufficient countries from group A (Fig. 3C and D). This result suggests that food insecurity is increasing worldwide and trade-dependent countries are particularly vulnerable to food crises.

Finally, we show that the leading eigenvectors of the reactivity matrix (corresponding to the eigenvalue \( \lambda_{HI} \)) are localized, meaning that not all countries are affected in the same way by shocks, but few nations are more vulnerable than others. In particular, our analysis may be suitable to identify the directions of major fragility and susceptibility to crisis (Fig. 4B). Combined, both the reactivity and asymptotic stability analyses show that the resilience of the coupled food–population dynamics is declining and the system is becoming increasingly susceptible to instabilities.

We stress that the modeling framework developed in this study accounts for the structure of the food trade network by using the same trade links as in the FAO food trade data (Materials and Methods) from which the trade matrix, \( \Phi \), is obtained. The amount of food exported by each country, however, is not obtained from the trade data but calculated as the difference between country-specific values of supply and demand. Therefore, the model does not account for the existence of food stocks, because the entire amount of food exceeding the demand is made available to other countries through international trade. Because of this assumption, the carrying capacities used in this modeling framework may in general differ from those calculated in the empirical analysis underlying the classification of the world’s countries shown in Fig. 2. In other words, we model a solidarity scenario in which all food is used either domestically or traded, without accounting for the food stocks existing particularly in countries from groups A–C (Fig. 3A). The role of food stocks is difficult to evaluate because, to date, country-specific assessments of grain reserves are still missing. Estimates available for the world as a whole indicate that grain stocks have decreased from \( \sim 30\% \) of the annual rate of food consumption in the 1990s to \( \sim 20\% \) (37, 38). Thus, the ability of stocks (and their international transfers) to contribute to the resilience of the global food system has substantially decreased in the past 10–15 y. Despite the important role of food stocks (38) and other food sources not accounted for in this study (e.g., seafood), the analysis of such scenario provides a theoretical baseline for the evaluation of the resilience of the global food system.

Both the asymptotic stability and the reactivity analyses indicate that the global food system is losing resilience and is becoming increasingly susceptible to crises. The cases of food shortage and peaking prices that occurred in the past few years (12) are consistent with such findings: the escalating demand for agricultural products resulting from demographic growth, shifts to more resource-demanding diets, and new bioenergy policies have increased the pressure on agricultural production (23). At the same time, crop yields have been stagnating (10) while the environmental costs of cropland expansion are too high to justify advocating for an extension of agriculture (18, 20). As a result, humanity is rapidly exhausting the safety margins underlying the resilience of the global food system (39). In the past 25 y, the growing demand for agricultural products has been partly met by increasing the reliance on international trade (15, 40). Because trade allows populations to consume food produced elsewhere beyond self-sufficiency, export bans issued in response to episodic

Fig. 2. Map displaying the geographic distribution of countries belonging to the four groups: (A) Exporting countries whose food trade has an impact on the carrying capacity, \( K_r \). (B) Trade-dependent countries where population relies on food available through both domestic production and trade (import). (C) Countries where the impact of trade on food availability is negligible. (D) Countries exhibiting clear signs of food limitation as evidenced by the poorer diets.
escalations in food prices have affected the reliability of the global food trade. Trade dependency thus appears to limit food security in importing countries (12). The framework developed in this paper allowed us to suggest how, combined, food globalization through international trade and fluctuations in supply–demand around the world are undermining the resilience of the global food system.

Materials and Methods

Food Trade Network and Production Data. Data from the FAO (27) allowed for reconstruction of the detailed directed trade (described by a matrix $T$ with $T_{ij}$ being the trade from $i$ to $j$) (15, 41) for each year, from 1986 through 2010, between 260 unique nodes, of 149 primary commodities and 100 secondary commodities. The FAO data used in this trade analysis (27) also includes food aid. Production values of only the 149 primary commodities were used to avoid double accounting issues that occur because secondary commodity production incorporates some portion of the already accounted for primary products they are derived from (15). To determine the total amount of each commodity available for human consumption in each individual country the average proportion used for feed and seed and other uses was calculated from the FAO Food Balance Sheets (15) and removed both from the production and trade data. All trade and production of commodities were converted to calorie equivalent (kcal/ton) based on ref. 27 (nutritive factors) (15). Fragmented population, trade and production values for the countries comprising the former Yugoslavia were combined as a single network node, allowing for the longer population, production, and trade records required for further analyses. The states emerging from the former Czechoslovakia, and the countries of Ethiopia and Eritrea, and West Germany and East Germany were similarly combined. Conversely, the countries resulting from the disintegration of the Soviet Union were investigated separately, but the analysis of their food–population dynamics was started on 1993 rather than 1986. Last, only nodes with populations greater than 1 million people were used resulting in a network with $n = 143$ distinct country nodes. Corresponding population data were also obtained and all data adjusted to match changes in country boundaries following ref. 42. Country-specific estimates of per capita daily calorie consumption were determined (for every year) from FAO Food Balance Sheets (15) using the methods described in ref. 23.

Country Classifications. Groups of countries are classified based on the distance among the three 25-y-long time series of $x_i$, $K_i$, and $\phi_i$, expressed as

$$\Delta x_i^T = \sum_j (K_{ij}(y) - x_i(t))/x_i(t)$$

and $\Delta K_i^T = \sum_j (K_{ij}(y) - x_i(t))/x_i(t)$. We then set an arbitrary threshold $\theta = 0.5$ (the results of this study are not sensitive to small changes of the threshold) and define four country groups as follows (Figs. S4–S11): group A: $\Delta x_i^T/\Delta x_i^T \geq \theta$, $\Delta K_i^T/\Delta x_i^T \geq \theta$ and $\Delta K_i^T/\Delta x_i^T < \theta$; group B: $\Delta x_i^T/\Delta x_i^T > \theta$ and $\Delta K_i^T/\Delta x_i^T > \theta$; group C: $\Delta x_i^T/\Delta x_i^T = 1$; group D: $\Delta x_i^T < \theta$ and $\Delta K_i^T < -\theta$.

Food–Demographic Delayed Logistic Model. The input of the delayed coupled food–demographic model is the net trade matrix $\phi$ calculated as the difference between exports and imports: $\phi_{ij} = T_{ij} - T_{ji}/\text{cal}$. $\phi_{ij}$ is the per capita food calorie consumption in country $i$. The net trade matrix is time dependent because both $T$ and cal change year by year. From $\phi_i$, we then build the fraction of net food import for each country: $\phi_i^\text{cal} = \sum_j \phi_{ij}/\text{cal}$. We note that the carrying capacity $K_i(x, t) = K_\text{loc} + \sum_j (K_{ij}(y) - x_i(t))x_i^\text{cal} - (K_i(x, t) - x_i(t))$ can be calculated in a closed form as $K_i(x, t) = (2l - l^2x_i^\text{cal})^{-1}(K_\text{loc} + (1-l^2x_i^\text{cal})x_i(t))$ for every year $t$, where $l$ is the identity matrix. Notice that, whereas in the empirical analysis, the trade-dependent carrying capacities were calculated directly from the data, in the model-based analyses, $K_i$ is determined as the solution of these nested equations because of the assumptions made by this model on food redistribution through trade (i.e., all of the food not used domestically is exported to the connected nodes). Assuming that food availability does not influence directly the demographic growth parameter $\alpha_i$ in Eq. 1, we calculate $\alpha$ directly from demographic data as the country-specific

\[
\alpha_i = \frac{\text{cal} \cdot \phi_i^\text{cal}}{\phi_i^\text{cal} + K_i(x, t)}
\]
average growth rate, $\alpha_i = \sum_{j=1}^{N} (x_i(y+1) - x_i(y)) / x_i(y)$ for $i = 1, \ldots, N$ and where $y_j = 1986$ and $y_j = 2010$. As noted, Eq. 1 accounts for the delayed effect of the carrying capacity demographic on dynamics. We assume that the delay $\tau$ for any country is small with respect to the timescale of its population's exponential growth or decay, i.e., $\tau \ll 1 / \alpha_i$, where $\alpha_i = \sum_{j=1}^{N} y_j / N$. In particular in our case $\tau \approx 10^6$ and thus we set $\tau = 1$ for all countries, $i = 1, \ldots, N$. Within this “small delay” assumption, the set of coupled nonlinear delay Eq. 1 maintains some of the properties of the corresponding nondelayed equations (i.e., in the case $\tau = 0$). Thus, for small delays, the equations are still mathematically tractable and the properties of linearized dynamics can still be investigated through a “classical” Jacobian analysis (43). To estimate the robustness of the coupled population growth-food supply system to perturbations such as regional changes in food production, we first study the eigenvalues of the Jacobian associated to the set of differential equations given by Eq. 1. If $x_i(t)$ is a stable trajectory of the state variables $x_i(t)$, a small displacement $\delta x_i(t)$ from $x_i(t)$ should tend to zero. Because the displaced state $x_i(t) = x_i(t) + \delta x_i(t)$ is also a solution of 1, we have that $\delta x_i(t) = f(x_i(t), t)$.

Subtracting 1 from this equation and linearizing $f(x_i(t), t)$ around $x_i(t)$ at time $t$ we obtain

$$\delta x_i = f_1^i(\delta x_i + f_2^i(\delta x_i - x_i(t))) = f_1^i(\delta x_i) + f_2^i(\delta x_i - x_i(t)),$$

where $\delta x_i = x_i(t) - x_i(t)$, and

$$f_1^i = \frac{\partial f_i}{\partial x_i} \bigg|_{x_i(t)} \quad \text{and} \quad f_2^i = \frac{\partial f_i}{\partial x_i} \bigg|_{x_i(t)} = \frac{x_i(t)}{K_i} \bigg|_{x_i(t)},$$

where $D = \bigg[ (1 - s e^{-\alpha_i t}) \bigg] / \bigg[ (1 - s e^{-\alpha_i t}) \bigg]$. Assuming $f_1^i = f_2^i = \frac{x_i(t)}{K_i} \bigg|_{x_i(t)}$ and expressing the solution as $\delta x_i = \mathrm{e}^{\lambda \tau}$, using Eq. 2, we find that the eigenvalues $\lambda_i$ of the system satisfy the equation

$$f_1^i \lambda_i^2 + f_2^i \lambda_i - \frac{x_i(t)}{K_i} \bigg|_{x_i(t)} = 0.$$ 

Eq. 6 describes the linearized dynamics for short $\tau$; in fact, based on Eq. 2, the linearized dynamics can be expressed as $\delta x_i(t) = \lambda_i \delta x_i(t)$, with $J = (1 - e^{-\alpha_i t})^{-1} f_1^i + f_2^i$. Thus, the problem is reduced to finding the eigenvalues and eigenvectors of the matrix $J$. We find that increasing the time lag $\tau$ has a weak positive effect on the stability of the system (Figs. S12 and S13). However, for larger values of $\tau$, completely novel solutions can appear that are fundamentally different from those obtained in the limit of small lags (43). The properties of the coupled food-population dynamics with longer delays, however, are beyond the scope of this work. Likewise, the feasibility of the system is investigated in the assumption of short delay through the eigenvalues of the matrix $H = (1 - e^{-\alpha_i t})^{-1}/(2 - e^{-\alpha_i t})$, where $T$ denotes the transposition operation. Localization is inferred by computing the inverse participation ratio (IPR) (44) of the leading eigenvectors corresponding to the larger eigenvalues $\lambda_i$. However, for larger values of $\lambda_i$, completely novel solutions can appear that are fundamentally different from those obtained in the limit of small lags (43). The properties of the coupled food-population dynamics with longer delays, however, are beyond the scope of this work. Likewise, the feasibility of the system is investigated in the assumption of short delay through the eigenvalues of the matrix $H = (1 - e^{-\alpha_i t})^{-1}/(2 - e^{-\alpha_i t})$, where $T$ denotes the transposition operation. Localization is inferred by computing the inverse participation ratio (IPR) (44) of the leading eigenvectors corresponding to the larger eigenvalues $\lambda_i$ and $\lambda_j$, i.e., $\text{IPR}[i] = 1 / \sum_{n=1}^{\infty} | \langle x \rangle_n |^2$ and $\text{IPR}[j] = 1 / \sum_{n=1}^{\infty} | \langle y \rangle_n |^2$. The system is considered localized if $\text{IPR} < N$; indeed, we find that this is the case for both $\lambda_i$ and $\lambda_j$ in all years.

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Correction

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The authors note that Fig. 3 and its corresponding legend appeared incorrectly. The corrected figure and its corrected legend appear below.

The authors also note that on page 6905, left column, first paragraph, lines 15–19 “countries from groups B and C appear to be more prone to instability than self-sufficient countries from group A (Fig. 3 C and D).” This result suggests that food insecurity is increasing worldwide and trade-dependent countries are particularly vulnerable to food crises” should instead appear as “countries from groups A and B appear to be more prone to instability (Fig. 3 C and D).” These results suggest that long-term food insecurity is increasing worldwide and also affecting self-sufficient (group A) countries. Trade-dependent countries are particularly vulnerable to food crises (Fig. 3D).” These errors do not affect the conclusions of the article.

Fig. 3. (A) Total food excess (in calories) calculated as the sum of the difference between supply (through production and trade) and consumption for all countries in each group, g (i.e., \( R_T(y) - C(y) = \sum_{i \in g} (K_T(y) - x_i) \cdot cal_i \)), where cal_i is the average daily per-capita diet of country i and x_i is its population. (B) Ratio of the coefficients of variation (i.e., SD: mean) of food production (\( K_L \)) and trade-dependent supply (\( K_T \)) for each country group as a function of time. (C) Fraction of most sensitive (i.e., unstable) countries within each group for different years. (D) Distribution of the most reactive countries [given by \( \text{Max}(\text{Re}(\lambda_h)) \)] for the four groups, considering all years from 1986 to 2010. Food trade-dependent countries (group B) are on average more reactive than group A, C, and D nations. (Lower) Map of reactive countries (see also Fig. S3 for most unstable countries) for two different years (1987 and 2008). All calculation have been done with a delay \( \tau = 1 \text{ y} \).

The authors note that Figs. S2 and S3 appeared incorrectly, and that the legends for Figs. S2, S4, S7, S10, and S11 appeared incorrectly. The SI has been corrected online.

**Fig. S2.** Probability density function (PDF) of the reactivity $\lambda_H$ of the nations belonging to the four groups for four different years.

**Fig. S3.** Map of unstable countries (i.e., $\lambda > 0$, in red) on three different years: 1987 (Left), 1997 (Center), and 2009 (Right).
Fig. S4. Group A nations ($x \approx K_i > K_T$): net exporters and their exports substantially reduce the resources available to their populations.
Fig. 57. Group B nations (κ ≈ K_r > K_L) exhibit an interdependency between demographic growth and food imports. These countries cannot sustain their population relying only on their local resources.
Fig. S10. Group C nations ($r \geq K_t \approx K$): countries where food trade does not have a substantial impact on food availability.
Fig. S11. Group D: countries that are affected by an overall food deficit.