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Publication Date
1963-10-18
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Contract No. W-7405-eng-48

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October 18, 1963
Final-State Interactions in the Decay $\eta \rightarrow 3\pi$†

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In this Letter we present the results of an analysis of the Dalitz-Fabri plot of 97 eta decays, $\eta \rightarrow \pi^+ \pi^- \pi^0$. The etas were produced in the reaction $\pi^+ p \rightarrow \pi^+ p \eta$, by using $\pi^+$ of 1170 MeV/c (76 events) and 1050 MeV/c (21 events) incident on the Alvarez 72-in. hydrogen chamber. Our sample differs from previously published samples in two important respects. First, our background is negligible. Second, the contaminating decay mode $\eta \rightarrow \pi^+ \pi^- \gamma$, which is $26 \pm 8\%$ as probable as the $\pi^+ \pi^- \pi^0$ mode, has been cleanly separated out and removed.

We do not present here the complete Dalitz-Fabri plot, but only its projection on the $T_0$ axis, where $T_0$ is the kinetic energy of the $\pi^0$. We first compare our spectrum with that given in the compilation of Berley et al. The comparison is shown in Fig. 1. Agreement is only fair. In particular, our data show a more rapid decrease in intensity for $T_0$ greater than about 30 MeV than does the compilation. Our belief is that the disagreement is due to the unsubtracted background and the unseparated $\pi^+ \pi^- \gamma$ decays contained in the compilation.
We now compare our spectrum with two theories. The first theory we call the linear-matrix-element theory.\(^6\),\(^7\) We fit our spectrum to the formula

\[
dN/dT_0 = C |1 + a y \exp(i\beta)|^2 \phi(y) = C(1 + 2a y \cos\beta + a^2 y^2)\phi(y),
\]

where \(y = 2(T_0/T_0^{\text{max}}) - 1\), so that \(-1 \leq y \leq +1\); and where \(\phi(y)\) is the Lorentz-invariant phase space. The constant \(C\) is chosen to normalize the area to 97 counts. We find a minimum \(\chi^2 = 6.1\) for \(\cos\beta = -1\) and \(a = 0.71 \pm 0.09\). The expected \(\chi^2\) is 4.0 and the \(\chi^2\) probability for a fit as bad or worse is about 20\%. The best fit to this theory is the "linear-matrix-element" curve shown in Fig. 2.

From these parameters one can predict\(^6\) the branching ratio

\[
R \equiv \Gamma_\eta(000)/\Gamma_\eta(\pm 0) = (3/2)P/[1 + (a^2/4)],
\]

where \(P = 1.1\) corrects for \(m_0 \neq m_{\pi^+}\). Inserting our best-fit value \(a = 0.71\), we obtain the prediction \(R = 1.50 \pm 0.04\). This can be compared with the directly measured value,\(^8\)

\[
R = 0.83 \pm 0.32.
\]

The \(\chi^2\) probability for agreement between the predicted and measured values of \(R\) is 3.8\%. Thus the agreement is poor.

We next compare our spectrum with the theory of Brown and Singer.\(^9\),\(^10\) In order to explain the unexpectedly large competition of the isospin-violating decay \(\eta \to \pi^+\pi^-\pi^0\) with the electromagnetic decay \(\eta \to \pi^+\pi^-\gamma\), they postulate that \(\eta \to 3\pi\) proceeds via \(\eta \to \sigma + \pi^0\), followed by \(\sigma \to \pi^+\pi^-\) or \(\sigma \to \pi^0\pi^0\). Here \(\sigma\) represents an \(I=0\) dipion resonance with \(0^{++}\) quantum numbers. Angular-momentum conservation forbids \(\eta \to \sigma + \gamma\), so that the \(3\pi\) mode is enhanced but the \(\pi^+\pi^-\gamma\) mode is not. Following Brown and Singer,\(^10\) we fit our spectrum to the expression

\[
dN/dT_0 = C \phi(T_0)[(T_0 - A)^2 + B^2]^{-1},
\]

where \(\phi(T_0)\) is phase space, \(C\) normalizes the area to 97 counts,
\[ A = \frac{1}{2m_\eta} (m_\eta - m_0)^2 - m_\sigma^2 \] and \( B = m_\sigma \Gamma_\sigma / 2m_\eta \). We find \( \chi^2_{\text{min}} = 2.7 \), where 4.0 is expected. The best-fit parameters are

\[ m_\sigma = 381 \pm 5 \text{ MeV}, \]

and

\[ \Gamma_\sigma = 48 \pm 8 \text{ MeV}. \]

The best-fit curve is shown in Fig. 2, labeled "Brown and Singer." From the parameters \( m_\sigma \) and \( \Gamma_\sigma \), Brown and Singer can predict the branching ratio \( R \). We shall not write down their formula. Using our results (2) and their formula, we obtain the prediction

\[ R = 1.02 \pm 0.07. \]

The \( \chi^2 \) probability for agreement with the measured value (1) is 57%.

In summary, our data are in poor agreement with the linear-matrix-element theories, and in excellent agreement with the \( I = 0, J = 0 \) dipion-resonance hypothesis of Brown and Singer. However, we can not rule out the possibility that other hypotheses involving final-state interactions might also fit the data. In particular we emphasize that it is only after we assume the existence of the \( \sigma \) resonance that we can determine the parameters of Eq. (2). Thus it is not possible in our experiment to determine whether the resonance actually does or does not exist.

It is a pleasure to acknowledge the advice and support of Luis W. Alvarez. One of us (E. C. F.) wishes to express his gratitude for the hospitality shown to him by the Alvarez bubble chamber group, and to acknowledge financial support during part of the experiment from the Lawrence Radiation Laboratory and from Yale University.
FOOTNOTES AND REFERENCES

† Work done under the auspices of the U. S. Atomic Energy Commission.

1. D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters 10, 114 (1963), have compiled 511 charged eta decays from eight different experiments. (For references see their Table I.) They estimate that not more than \( \sim 100 \) of the events are background. The \( \pi^+ \pi^- \gamma \) decays were not separated out, and should constitute \( \sim 100 \) of the nonbackground events. Thus perhaps as many as 200 of the 500 events are spurious. The 69 events labeled "Berkeley-c in Table I were our preliminary results, obtained before we had eliminated a small background and separated out the \( \pi^+ \pi^- \gamma \) decays.

2. This is demonstrated in Fig. 1 of Ref. 3, which contains 76 of our present 97 events. First, the figure shows that essentially all events of the type \( \pi^+ p \rightarrow \pi_1^+ p \pi^+ \pi^- \pi^0 \) are due to \( \eta \) production, with \( \eta \rightarrow \pi^+ \pi^- \pi^0 \). Second, the figure (and our calculation) shows that about 16% of the events are ambiguous with respect to the \( \pi^+ \), when \( \pi_1^+ \pi^- \pi^0 \) and \( \pi_2^+ \pi^- \pi^0 \) both have the \( \eta \) mass. We always choose that combination yielding a mass closest to \( m_\eta = 548.0 \). Therefore we choose the wrong \( \pi^+ \) in 8% of the cases. We have examined our Dalitz plot with the ambiguous events deleted and also with the ambiguous positive pions interchanged, and find no distinguishable change in the shape. Third, we discard events with "m \( (e^+ e^-) \)" < 100 MeV to eliminate \( \pi^+ p \rightarrow \pi^+ p e^+ e^- \cdots \) from our sample. This cutoff also eliminates any event \( \pi^+ p \rightarrow \pi^+ p \pi^- \pi^- \pi^0 \), where the \( \pi^- \) and one \( \pi^+ \) have the same direction in the laboratory. We have examined the cutoff events and find that about ten of them correspond to \( \eta \) production and decay into \( \pi^+ \pi^- \pi^0 \). Adding these events to the spectrum for \( T_0 \) produces no detectable effect on the shape of the spectrum. Finally, we have examined the events discarded
because they satisfy $\pi^+ p \to \pi^+ p \pi^+ \pi^-$, with a Coulomb scatter on one track (1-C fit. See Ref. 3). Six of these events are probably $\eta \to \pi^+ \pi^- \pi^0$. None satisfy the cutoff criterion on the error in $m^2$ (neutral) described in Ref. 4.


4. The technique is described in Ref. 3. See especially Fig. 3b. If we relax our cutoff on the error in $m^2$ (neutral) = $m^2$ ($\pi^0$ or $\gamma$), our 97 $\pi^+ \pi^- \pi^0$ events become 146 $\pi^+ \pi^- \pi^0$ events with a small (but not easily measured) contamination from $\pi^+ \pi^- \gamma$. The $T_0$ spectrum of these 146 $\pi^+ \pi^- \pi^0$ events is not distinguishable from that of our reduced sample of 97, which has nearly zero contamination. Thus the error cutoff does not distort the $T_0$ spectrum.


10. L. M. Brown and P. Singer, Three-Pion-Decay Modes of Eta and K Mesons and a Possible New Resonance (submitted to Physical Review) compare their theory with the data on K and eta decay compiled in Ref. 1. We are grateful to these authors for several enlightening discussions of their preprint.
11. The off-diagonal error term is \( \delta\Gamma_\sigma \delta m_\sigma = 12 \text{ (MeV)}^2 \).
12. Our values for \( m_\sigma \) and \( \Gamma_\sigma \), Eq. (2), may be compared with the values \( m = 395 \pm 10 \text{ MeV} \) and \( \Gamma = 50 \pm 20 \text{ MeV} \) for a \( \pi^+ \pi^- \) resonance observed by N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters 9, 139 (1963), and assigned \( I = 0 \) or 1 by them. The agreement is striking, but could be accidental.

The existence of this resonance has not yet been directly confirmed, either in other experiments [see for instance C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg, D. Miller, J. Schultz, J. Steinberger, T. H. Tan, H. Brugger, P. Kramer, and R. Plano, Phys. Rev. Letters 9, 322 (1962)] or in the present experiment. It is not possible to prove conclusively the existence of the \( \sigma \) resonance in the present experiment, mainly because the width \( \Gamma_\sigma = 48 \text{ MeV} \) is not small compared with \( T_0^{\text{max}} = 84 \text{ MeV} \).

Assuming the existence of the resonance, we determine the parameters of Eq. (2). Thus we do not regard our results as sufficient to confirm the observation of Samios et al.

13. If \( \eta \rightarrow 3\pi \) went exclusively via \( \eta \rightarrow \sigma + \pi_0^0 \) (d for direct), and if the width \( \Gamma_\sigma \) were zero, then \( \pi_0^0 \) would not interfere with either of the neutral pions from \( \sigma \rightarrow 2\pi^0 \). The direct pion, \( \pi_0^0 \), would be distinguishable by its energy in the \( \eta \) frame. Then \( R = (1/2)\rho \approx 0.55 \) follows from the hypothesis \( I_\sigma = 0 \).

In the limit \( \Gamma_\sigma \rightarrow \infty \), any-one-of the three neutral pions could be regarded
as direct, so that the $|\text{amplitude}|^2$ for $\eta \to 3\pi^0$ would be enhanced over the case $\Gamma_\sigma = 0$ by a factor $|(1+1+1)/\sqrt{3}|^2 = 3$, because of the three possible assignments for $\pi_d^0$. In that limit, one has $R = (3/2)P = 1.7$. The $3\pi^0$ are then in the totally symmetric $I = 1$ state.
FIGURE LEGENDS

Fig. 1. Spectrum of kinetic energy of $\pi^0$ from $\eta \rightarrow \pi^+\pi^-\pi^0$. Solid circles represent the present experiment; open triangles represent the compilation of Ref. 1, renormalized so as to give the same area. The smooth curve is a best fit of our data to the theory of Brown and Singer, and is included in this figure only to aid comparison of the present experiment with the previous compilation. (The same smooth curve appears in Fig. 2.)

Fig. 2. Spectrum of the $\pi^0$ kinetic energy from $\eta \rightarrow \pi^+\pi^-\pi^0$. The experimental points are from the present experiment. The three smooth curves correspond to phase space, the linear-matrix-element theories, and the $I = 0, J = 0$ dipion-resonance theory of Brown and Singer.
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