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Author
Leskovar, Branko.

Publication Date
1971-05-01
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AEC Contract No. W-7405-eng-48

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GENERALIZED ANALYSIS OF PHASE-SENSITIVE DETECTION CIRCUIT OPERATING CHARACTERISTICS AT THE SIGNAL DETECTION IN THE PRESENCE OF NOISE

Branko Leskovar
Lawrence Radiation Laboratory
University of California
Berkeley, California 94720
June 1971

Previous analysis of phase-sensitive detection circuits\(^1\),\(^2\) have been based on the assumption that the additive noise is nonexistent in the detector reference channel. However, in contemporary experimental research instrumentation and phase-lock systems, a significant amount of noise can be present in the reference channel.\(^3\) Such case will be discussed, assuming that the input signal and the reference wave are in the presence of independent, stationary, and additive Gaussian noise.

The generalized detection circuit under consideration is shown in Fig. 1. The circuit input consists of a signal \(s(t)\) superimposed on a broad-band noise \(n_s(t)\). After filtering, the sum \(s_1(t) = s(t) + n_s(t)\) is applied to the balanced phase-sensitive detector. The detector reference input consists of a reference wave \(r(t)\) superimposed on a broad-band noise \(n_r(t)\). After filtering, the sum \(r_1(t) = r(t) + n_r(t)\) is applied to the detector and differential circuit. The generalized and normalized form of the detector operating characteristics is

\[
S_0/\eta_d \left( N_r^2 + N_s^2 \right)^{1/2} = (\pi/2)^{1/2} \Delta F_1(x_s, u_r, \psi, n),
\]

where \(S_0\) represents the output signal; \(\eta_d\) is the detection efficiency; \(N_r\) and \(N_s\) are rms values of the narrow-band noise in the reference channel and signal channel, respectively; \(x_s = S/N_s\) is the input signal-to-noise ratio; \(u_r = R/N_r\) is the reference wave-to-noise ratio; \(n = N_s/N_r\) is the signal channel noise-to-reference channel noise ratio; \(F_1(x_s, u_r, \psi, n)\) denotes a confluent hypergeometric function of the form \(\Delta F_1(-1/2;1;x_s, u_r, \psi, n)\); \(\Delta\) signifies the difference of the hypergeometric functions; \(\psi\) is the phase angle between the input signal and reference wave. Detector characteristics nonlinearity \(N_A^*(x_s)\) relating to the input signal-to-noise ratio is described by a generalized equation

\[
N_A^*(x_s) = 1 - (n-n^{-1}) \Delta F_1(x_s, u_r, n)/[x_s x_r^* F_1(u_r, n)].
\]

The terms \(F_1(x_s, u_r, n)\) and \(F_1(u_r, n)\) denote confluent hypergeometric functions of the form \(\Delta F_1(-1/2;1;x_s, u_r, \psi, n)\) and \(F_1(1/2;2;u_r, n)\), respectively. \(N_A^*(x_s)\) as a function of \(x_s\) is calculated and plotted in Figs. 2-3. From the curves it can be seen that \(N_A^*(x_s)\) increases with increasing \(x_s\), for given \(u_r\) and \(n\). Also, \(N_A^*(x_s)\) decreases when the ratio of \(u_r/x_s\) increases, particularly when \(u_r > 1\). Furthermore, when the ratio \(n\) is large, the minimization of \(N_A^*(x_s)\) requires a very high value of the ratio \(u_r/x_s\). For a fixed ratio \(n\) and phase angle \(\psi\), the proper choice of \(u_r\) and \(x_s\) minimizes \(N_A^*(x_s)\) to an acceptable amount. This is important in a wide dynamic range, wide-band detection application using solid state circuit components where the ratio \(u_r\) is often close in value to the ratio \(x_s\), resulting in a large nonlinearity. Curves in Fig. 3 show that for \(n = 1\) and \(x_s = 1\), the increase of the \(u_r/x_s\) ratio by one order of magnitude will decrease the nonlinearity by almost three orders of magnitude. Furthermore, \(N_A^*(x_s)\) is reduced even more for larger amounts of \(x_s\).
For \( \mu_r = 10 \), increasing \( \mu_r \) by one order of magnitude decreases the nonlinearity by more than four orders of magnitude.

Detector characteristics nonlinearities \( N_B^*(\psi) \) and \( N_C^*(\psi) \) relating to the phase angle \( \psi \) for operating points \( \psi_B = (2m+1)\pi/2 \) and \( \psi_C = 2m\pi \), where \( m = 0,1,2, \ldots \), are described by equations

\[
N_B^*(\psi) = 1 - \frac{n+n^{-1}}{1 + F_1(x_s, \mu_r, \psi, n)} \quad \text{and} \quad N_C^*(\psi) = 1 - \frac{n+n^{-1}}{1 + F_1(x_s, \mu_r, \psi, n)}
\]

Hypergeometric functions \( F_1(x_s, \mu_r, \psi, n) \) are of the same form as in the expression for \( N_A^*(\psi) \); \( \Delta \) signifies the difference of functions of the form \( F_1(x_s, \mu_r, \psi, n) \). The nonlinearity \( N_B^*(\psi) \) depends strongly upon \( \psi, x_s, \mu_r \), and \( n \), as shown in Fig. 4. To obtain the most information about \( N_B^*(\psi) \) behavior, the nonlinearity is calculated and plotted as a function of \( x_s \). The curves show that for \( n = 10^{-3} \) and \( \mu_r = 1 \), the nonlinearity \( N_B^*(x_s) \) is practically independent of \( x_s \) over a wide dynamic range. On the contrary, \( N_B^*(x_s) \) has a strong dependence on \( \psi, n, \) and \( \mu_r \). From the curves it follows that the proper choice of \( \mu_r \) for a given \( n \) and \( x_s \) minimizes the nonlinearity.

Any other value of \( \mu_r \) can be considered as nonoptimum.

The curves for the nonlinearity \( N_C^*(x_s) \) in Fig. 5 show that particular values of \( \mu_r \) minimize the nonlinearity. For this purpose it is important to avoid the regions where \( N_C^*(x_s) \) curves have maximums. Analysis and the curves show that the presence of the reference channel noise has a dominant role in the detector optimum operating conditions.

From a comparison of the nonlinearity of \( N_B^*(x_s) \) and \( N_C^*(x_s) \) curves it can be seen that nonoptimum values of \( \mu_r, x_s, \) and \( n \) increase \( N_B^*(x_s) \) one-half order of magnitude in the worst case. However, \( N_C^*(x_s) \) nonoptimum values of \( \mu_r, x_s, \) and \( n \) increase \( N_C^*(x_s) \) two orders of magnitude. Consequently, the phase angle \( \psi_B = (2m+1)\pi/2 \) should be chosen as the operating point wherever possible.

**ACKNOWLEDGMENT**

The author would like to thank Ruth L. Hinkins for computer programming.
FOOTNOTE AND REFERENCES

This work was performed as part of the program of the Physical-Chemical Biodynamics Instrumentation Research and Development Group of the Lawrence Radiation Laboratory, Berkeley and was supported by the U. S. Atomic Energy Commission, Contract No. W-7405-eng-48.


FIGURE CAPTIONS

Fig. 1 Generalized phase-sensitive detection circuit under consideration.

Fig. 2. Nonlinearity $N_A^*(x_s)$ versus signal-to-noise ratio $x_s$, for the signal channel noise-to-reference channel noise ratio $n = 10^{-3}$ and $10^3$.

Fig. 3 Nonlinearity $N_A^*(s)$ versus signal-to-noise ratio $x_s$, for the signal channel noise-to-reference channel noise ratio $n = 1$.

Fig. 4 Nonlinearity $N_B^*(x_s)$ versus signal-to-noise ratio $x_s$, for the signal channel noise-to-reference channel noise ratio $n = 10^{-3}$ and $10^3$.

Fig. 5 Nonlinearity $N_C^*(x_s)$ versus signal-to-noise ratio $x_s$, for the signal channel noise-to-reference channel noise ratio $n = 10^{-3}$ and $10^3$. 
INPUT SIGNAL AND NOISE
\[ s(t) + n_s(t) \]
\[ x_s = S/N \]

TIME IN Variant NARROW BAND FILTER
\[ x_s = S/N \]

INPUT SIGNAL AND NOISE
\[ s_i(t) = s(t) + n_i(t) \]
\[ s(t) = S \cos \omega t \]
\[ n_i(t) = n_{sP}(t) \cos \omega t - n_{sQ}(t) \sin \omega t \]

PHASE SENSITIVE DETECTOR
\[ r_i(t) = r(t) + n_r(t) \]
\[ r(t) = R \cos(\omega t + \theta) \]
\[ n_r(t) = n_{rP}(t) \cos \omega t - n_{rQ}(t) \sin \omega t \]
\[ \mu_r = R/N_r \]

REFERENCE WAVE AND NOISE
\[ r(t) + n_r(t) \]
\[ \mu_r = R/N_r \]

DIFFERENTIAL CIRCUIT
\[ s_o = \gamma_d S_R \]
\[ \gamma_d (N_s^2 + N_n^2)^{1/2} \]

NORMALIZED OUTPUT
\[ s_o \]

TIME IN Variant NARROW BAND FILTER

Fig. 1
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