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MISSING MASS AS AN ALTERNATIVE TO RAPIDITY-GAP IN THE EXPERIMENTAL STUDY OF DIFFRACTION

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ABSTRACT

The measurement of diffractive cross sections by selecting events with large rapidity gaps in the final state—across which the pomeron is expected to dominate—is impeded by neutrals that escape detection. To surmount this problem in a few special but important cases, we derive formulas that relate the gaps of interest to easily calculable missing masses. The reliability of these formulas is tested by applying them to a sample of events with known kinematics.

Diffractive dissociation may be defined as a reaction where (a) the produced particles are grouped into two clusters separated by a large gap in rapidity, each cluster carrying the same quantum numbers as one of the incident particles, and (b) the reaction amplitude is "pomeron-dominated"—exhibiting the factorizability and dependence on intercluster rapidity-gap characteristic of pomeron exchange. Such a definition is flawed by the circumstance that pomeron dominance at best is achieved only in the asymptotic limit as the rapidity gap, \( \Delta \rightarrow \infty \). For finite gaps, contributions from lower-lying singularities make the experimental definition of diffraction inherently approximate and dependent on the choice of a minimum accepted gap \( \Delta_0 \); different choices of \( \Delta_0 \) are appropriate to different levels of measurement accuracy and different levels of precision in the analysis. We do not here enter the delicate question of choosing the minimum gap but discuss practical measurement problems assuming that a definite value of \( \Delta_0 \) has been agreed on.

The selection of large rapidity-gap \((\Delta > \Delta_0)\) events by a direct measurement of particle momenta is usually frustrated by the presence of possible undetected neutral particles. What we propose in this paper is that for the measurement of two important types of diffraction, the failure to observe all produced particles may be no greater an impediment than the intrinsic ambiguity in the definition of diffraction due to the finiteness of rapidity gaps. For the reaction types in question we argue that a direct measurement of rapidity gaps may be replaced by a statistically adequate gap determination based on missing mass variables.

The first case may be described as dissociation of one of the two incident particles, the other surviving; the large gap considered is adjacent to the survivor (Fig. 1a and 1b). The second case involves two large gaps, each adjacent to one of the initial particles (Fig. 1c). These latter events may be viewed either as dissociation of particle a (particle b remaining intact), or as dissociation of particle b (particle a remaining intact). These events are described as double-pomeron exchange (DPE).

Missing Mass Equivalence to Rapidity-Gap

To measure dissociation of particle b, particle a remaining intact (Fig. 1a), we wish to select events so that the rapidity-gap
between particle $a$ and the next one is $\Delta > \Delta_0$. But if the latter particle is neutral, $\Delta$ cannot be directly measured. To overcome this difficulty, we now derive a relation between the gap $\Delta$ and the missing mass $M_B^2 = (p_a + p_b - p_1^c)^2$ which can be obtained from measurement of the single momentum $p_a^c$. An identical treatment with $\tilde{a} \rightarrow b$ applies to the dissociation of particle $a$ (Fig. 1b). We start from the general formula for the squared mass of two particles of mass $m_1$ and $m_2$:

$$s_{12} = m_1^2 + m_2^2 + 2m_1m_2 \cosh(y_1 - y_2) - 2p_{12}^2 s_{12} \simeq m_1^2 e^{|y_1 - y_2|}\quad (1).$$

where $\mu_1$ is the transverse mass $\left(\frac{m_1^2 + |p_{12}^t|^2}{2}\right)^{1/2}$, $p_{12}^t$ the transverse momentum and $y_1$ the rapidity of particle 1. With this approximate formula, valid for large $|y_1 - y_2|$, if we call $y_B$ the rapidity of the center of mass of cluster B, the center-of-mass energy squared is given by:

$$s = \mu_1 \mu_B^A e^{y_B - y_A} \simeq \mu_1 \mu_B^A e^{y_B - y_1}\quad (2),$$

assuming the transverse momentum of cluster B to be negligible with respect to its mass $M_B$. We calculate $y_B$ by using the definition of center of mass, measuring all momenta in the rest frame of particle A:

$$M_B \sinh(y_N - y_B) \simeq M_B e^{y_B - y_1} + \sum_{i=1}^{N-1} p_{1i} = \sum_{i=1}^{N-1} \mu_1 \sinh(y_N - y_1)\quad (3).$$

If particle 1 is well separated from $b$ in rapidity so that $e^{y_N - y_1} >> e^{y_N - y_2}$, we can approximate the sum by the leading term $p_{11}$ to get

$$M_B e^{y_B - y_1} \simeq \mu_1 e^{y_B - y_1}\quad (4).$$

Inserting this into Eq. (2) gives us

$$s = \mu_1 \mu_B^A e^{y_B - y_1}\quad (5).$$

Assuming on the average that $\langle p_{11}^t \rangle \simeq 350 \text{ MeV}/c$ for all the particles and that particle 1 is a pion, we get

$$\Delta \simeq \ln \left(\frac{s}{M_B^2}\right) + \ln \left(\frac{\langle \mu_1 \rangle}{\langle \mu_2 \rangle}\right)\quad (6),$$

with $\langle \mu_1 \rangle = \left(\frac{m_1^2 + \langle p_{11}^t \rangle}{2}\right)^{1/2}$. If particle 1 is a pion, the second term vanishes. If particle 1 is a proton

$$\ln \left(\frac{\langle \mu_2 \rangle}{\langle \mu_1 \rangle}\right) \simeq -1.$$

We may use formula (6) to measure either the total cross section $\sigma_{ab \rightarrow AB}$ for the dissociation of particle $b$ by selecting all events in the range of $M^2$ such that according to formula (6), $\Delta > \Delta_0^2$, or the differential cross section $^3 \frac{d\sigma}{dt} \mid_{d(M_B^2/s)}$ by binning the diffractive events in $M_B^2$ and $t = (p_a - p_1^c)^2$.

In the case of double-pomeron events (Fig. 1c), the two gaps $\Delta_1$ and $\Delta_2$ separating particles $a'$ and $b'$ respectively, from adjacent particles, are given by

$$\Delta_1 \simeq \ln \left(\frac{s}{M_B^2}\right) + \ln \left(\frac{\langle \mu_1 \rangle}{\langle \mu_2 \rangle}\right)\quad (7a),$$

$$\Delta_2 \simeq \ln \left(\frac{s}{M_A^2}\right) + \ln \left(\frac{\langle \mu_1 \rangle}{\langle \mu_2 \rangle}\right)\quad (7b),$$

where $M_B$ and $M_A$ are the masses recoiling against $a$ and $b$, respectively. A measurement of the DPE cross section is then possible.
by assuming pomeron dominance for events in the range of $M_A^2$, $M_B^2$ such that according to these formulae, both $\Delta_1$ and $\Delta_2$ are $> \Delta_0$.

By binning the events in $t_1 = (p_A - p_a)^2$, $t_2 = (p_B - p_b)^2$, $M_A^2$ and $M_B^2$, the differential cross section $d\sigma_{\text{DPE}}/dt_1 dt_2 d(M_A^2/s) d(M_B^2/s)$ can also be measured.

Comparison with Experiment: Single Diffraction and Double Pomeron Exchange Selections

We have tested Eqs. (6), (7a) and (7b) by considering a sample of events of the reaction $\pi^- p \rightarrow \pi^+ \pi^- p$ at 205 GeV/c. The final particle momenta were determined by a 4-constraint fit and labeled $\alpha$, $\beta$, $\gamma$ and $\delta$ in the order of increasing rapidity. Further, the events have been selected with particle $\alpha$ and $\delta$ being respectively a proton and a $\pi^-$, so that they could be at the same time $\pi^-$-dissociation, proton-dissociation and double-pomeron exchange candidates.

In what follows, we will test these formulae for $\Delta_0 = 2$.

Figure (2a) shows the measured rapidity gap $R_{\text{op}}$ between the proton and the nearest pion ($\pi^-$) vs $\ln(s/M_{\pi^+}^2)$ for all of the selected events. The line of unit slope and intercept (approximately) minus one represents our expectation based on formula (6). The measurement of $\pi^-$-dissociation by the rapidity selection ($R_{\text{op}} > 2$) gives 95 events while the missing mass formula ($\ln(s/M_{\pi^+}^2) > 3$) gives 94 events. These two number compare quite well.

Proton dissociation events displayed in Fig. 2b, where $R_{\gamma\delta}$ is represented versus $\ln(s/M_{\text{pr}^+\pi^+}^2)$, can be chosen by making either the cut $R_{\gamma\delta} > 2$ or $\ln(s/M_{\text{pr}^+\pi^+}^2) > 2$. The first method gives 46 events while the second one gives 48 events. Here again the two numbers compare quite well.

A study of DPE events has been made by plotting $R_{\text{op}}$ vs $R_{\gamma\delta}$ (Fig. 2e). In such a plot 12 events are selected with both these gaps larger than $\Delta_0 = 2$. The corresponding plot of $\ln(s/M_{\pi^+}^2)$ vs $\ln(s/M_{\text{pr}^+\pi^+}^2)$ gives also 12 events (among which eight were chosen by the first method) and consequently give the same upper limit as in Ref. 6.

To show the advantage of our formulas in the presence of neutrals, we have performed a fake experiment by pretending in $\pi^-$-dissociation (or proton-dissociation) that the nearest particle $n_B (n_\gamma)$ to the proton (fast $\pi^-$) is neutral. The selection based on the "measurable" rapidity gaps $R_{\text{op}}$ (and $R_{\text{pr}}$) $> 2$ (Figs. 2c and 2d), gives 107 events (79 events) to be compared with the 94 (48) events selected by the missing mass criteria of formula (6). Clearly, a direct rapidity selection may substantially overestimate the number of so-called diffractive events. The entire analysis was repeated for $\Delta_0 = 3$. Table I summarizes the results of this experiment.

In conclusion, we have shown that relations (6), (7a) and (7b) can replace a direct rapidity-gap measurement with a missing-mass measurement in the study of beam and target diffraction as well as double-pomeron exchange.

We wish to thank G. F. Chew for having suggested this study and we are grateful to our NAL-BERKELEY collaborators for having allowed us the use of the necessary experimental data and F. Winkelmann for useful comments.
FOOTNOTES AND REFERENCES

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† On leave of absence from the University of Paris VI, Paris, France.


2. It follows from Eq. (6) that a selection of diffractive events on the basis of a minimum gap $\Delta_0$ admits events of increasing $M_B^2$ with increasing $s$.

3. The dependence on $M_B^2/s$, which is equivalent to a dependence on $\Delta$, must eventually be verified to be that characteristic of pomeron exchange.

4. The dependence on $M_B^2/s$ and $M_A^2/s$ must be shown to be that expected from double-pomeron exchange.

5. For a detailed study of this reaction and the experiment, see (a) D. Bogert, et al., Phys. Rev. Letters 21, 1271 (1973); (b) C. S. Abrams, et al., LBL-2112 (1973) and (c) L. Stutte, et al., LBL-2460 (1974) for an estimate of the background (< 10%).


7. Some events in the sample belonged to none of the three categories since they did not have a large gap ($\Delta > \Delta_0$) either next to $\alpha$ or next to $\beta$. The large gap was between $\beta$ and $\gamma$ as in quasi-two body processes.

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**TABLE I**

<table>
<thead>
<tr>
<th>Processes</th>
<th>Selection</th>
<th>Missing mass formula for rapidity gap</th>
<th>Measured rapidity gap $(R_{np}$ and $R_{np}$)</th>
<th>&quot;Measured&quot; rapidity gap for faked $\pi^0$ $(R_{np}$ and $R_{np})$</th>
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</table>
FIGURE CAPTIONS

1. Rapidity diagram for: (a) particle b dissociation, (b) particle a dissociation, (c) double-pomeron exchange (DPE).

2. (a) Measured rapidity gap $R_{\phi\beta}$ between the proton and the nearest pion vs $\ln(s/M_{3\pi}^2)$.
   (b) Measured rapidity gap $R_{\gamma\delta}$ between the fast $\pi^-$ and the nearest $\pi$ vs $\ln\left(\frac{s/M_{2\pi}^2}{p^-_{\pi^+}\pi^-_{\text{slow}}}\right)$.
   The lines with the unit slope are our expectations from formula (6). The broken lines represent the cuts corresponding to $\Delta_0 = 2$.
   (c) "Measured" rapidity gap $R_{\alpha\gamma}$ vs $\ln\left(\frac{s/M_{2\pi}^2}{p^-_{\pi^+}\pi^-_{\text{slow}}}\right)$, ($\beta$ is the fake $\pi^0$).
   (d) "Measured" rapidity gap $R_{\beta\delta}$ vs $\ln\left(\frac{s/M_{2\pi}^2}{p^-_{\pi^+}\pi^-_{\text{slow}}}\right)$, ($\gamma$ is the fake $\pi^0$).
   (e) $R_{\phi\beta}$ vs $R_{\gamma\delta}$.
   (f) $\ln\left(\frac{s/M_{3\pi}^2}{p^-_{\pi^+}\pi^-_{\text{slow}}}\right)$ vs $\ln\left(\frac{s/M_{2\pi}^2}{p^-_{\pi^+}\pi^-_{\text{slow}}}\right)$. 
Fig. 1
Fig. 2
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