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Do experiments and astrophysical considerations suggest an inverted neutrino mass hierarchy?

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The recent results from the Los Alamos neutrino oscillation experiment, together with assumptions of neutrino oscillation solutions for the solar and atmospheric neutrino deficit problems, may place powerful constraints on any putative scheme for neutrino masses and mixings. Assuming the validity of these experiments and assumptions, we argue that a nearly unique spectrum of neutrino masses emerges as a fit, if two additional astrophysical arguments are adopted: (1) the sum of the light neutrino masses is \( \sim 5 \text{ eV} \), as large scale structure simulations with mixed cold plus hot dark matter seem to suggest; and (2) \( r \)-process nucleosynthesis originates in neutrino-heated ejecta from type-II supernovae. In this fit, the masses of the neutrinos must satisfy \( m_{\nu_1} \approx m_{\nu_2} \approx 2.7 \text{ eV} \) (where \( \nu_e \) is split from a sterile species \( \nu_s \) by \( \sim 10^{-6} \text{ eV} \)) and \( m_{\nu_3} \approx m_{\nu_s} \approx 1.1 \text{ eV} \) (where these species are split by \( \sim 10^{-2} \text{ eV} \)). We discuss alternative neutrino mass spectra that are allowed if we decline to adopt certain experiments or astrophysical models.

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In this paper we examine constraints on the possible spectrum of neutrino masses in light of recent experiments and astrophysical models. Surprisingly, and perhaps disturbingly, we find that if we insist on having a significant amount of light neutrino dark matter, and assume that \( r \)-process nucleosynthesis originates in neutrino-heated type-II supernova ejecta, then standard interpretations of the experiments force us into adopting an inverted neutrino mass hierarchy.

Three experiments, or sets of experiments, have been widely interpreted as suggesting evidence for neutrino oscillations and, therefore, the existence of neutrino mass. We discuss each of these in turn.

(1) The recent report of evidence for \( \bar{\nu}_\mu = \bar{\nu}_e \) oscillations found by the Liquid Scintillation Neutrino Detector (LSND) experiment at Los Alamos is very exciting [1]. This result by itself, if correct, seems to be consistent with a fair range in \( \delta m_{\text{2}\mu}^2 \approx |\Delta m_{\text{2}\mu}^2| \) (the notation \( \delta m_{\text{2}\mu}^2 \) and similar notation hereafter are adopted for convenience, and should be interpreted as the mass-squared differences between the appropriate neutrino mass eigenstates). If indeed the events from the LSND experiment are interpreted as vacuum neutrino oscillations, then the mass-squared difference and vacuum mixing angle must fall in a range \( 5 \times 10^{-2} \lesssim \delta m_{\text{2}\mu}^2 \lesssim 20 \text{ eV}^2 \) and \( 10^{-3} \lesssim \sin^2 \theta_{\text{2}\mu} \lesssim 1 \) [1]. However, much of this range has been eliminated by the upper limits obtained from previous experiments, so that only intermitten ranges of \( \delta m_{\text{2}\mu}^2 \) and \( \sin^2 \theta_{\text{2}\mu} \) survive. These allowed ranges are centered on \( \delta m_{\text{2}\mu}^2 \approx 6 \text{ eV}^2 (\sin^2 \theta_{\text{2}\mu} \approx 3 \times 10^{-3}) \), \( \delta m_{\text{2}\mu}^2 \approx 10 \text{ eV}^2 \) (\( \sin^2 \theta_{\text{2}\mu} \approx 2 \times 10^{-3} \)), and \( 0.2 \lesssim \delta m_{\text{2}\mu}^2 \lesssim 2 \text{ eV}^2 \) with \( 10^{-3} \lesssim \sin^2 \theta_{\text{2}\mu} \lesssim 4 \times 10^{-2} \). In what follows, we shall adopt \( \delta m_{\text{2}\mu}^2 \approx 6 \text{ eV}^2 \) and \( \sin^2 \theta_{\text{2}\mu} \approx 3 \times 10^{-3} \) as representative of the LSND results. It is reported that these adopted values are a “best fit” to the LSND \( \nu_\mu \rightleftharpoons \nu_e \) vacuum oscillation data which are unpublished [1]. Our qualitative conclusions will be unchanged by adopting any \( \delta m_{\text{2}\mu}^2 \gtrsim 2 \text{ eV}^2 \), and our numerical results are generalizable in obvious fashion for these cases. We will discuss how our conclusions might change if we were to adopt \( \delta m_{\text{2}\mu}^2 \lesssim 2 \text{ eV}^2 \) for the LSND result.

(2) The apparent deficit in the expected flux of solar neutrinos has been confirmed by a number of experiments [2]. Whether a solution to this problem demands new neutrino physics, or an alteration of the solar model, has been extensively debated [3]. However, with the new calibration of the GALLEX experiment, and the identification of the principal neutrino deficit as originating near the \( \text{Be}(e^-,\nu_e)^7\text{Li} \) electron capture line, a consensus is building that neutrino oscillations are at the root of the solution [4]. Either a matter-enhanced Mikheyev-Smirnov-Wolfenstein (MSW) neutrino flavor transformation [5], or large mixing angle vacuum neutrino oscillations [6], involving the electron neutrino could explain the solar neutrino deficit. Perhaps the best choices of neutrino mixing parameters for solving this problem are those for the so-called “small-angle solution,” with \( \delta m_{\alpha\alpha}^2 \approx 10^{-6} - 10^{-5} \text{ eV}^2 \) and \( \sin^2 2\theta_{\alpha\alpha} \approx 5 \times 10^{-3} \). Here the subscript \( \alpha \) refers to a neutrino species \( \nu_\alpha \), for which the small-angle solution could be either \( \nu_\mu \), \( \nu_\tau \), or a sterile species \( \nu_s \). The “large mixing angle solution” and vacuum oscillation solution encompass other ranges of vacuum neutrino properties: \( \delta m_{\alpha\alpha}^2 \approx 10^{-6} - 10^{-5} \text{ eV}^2 \) and \( \sin^2 2\theta_{\alpha\alpha} \gtrsim 0.4 \), or \( \delta m_{\alpha\alpha}^2 \approx 10^{-10} \text{ eV}^2 \) and \( \sin^2 2\theta_{\alpha\alpha} \approx 0.75 \), respectively. In neither of these latter...
two solutions could $\nu_\alpha$ be a sterile species [6,7].

(3) High-energy cosmic rays incident on the upper atmosphere produce large numbers of $\pi^+$ and $\pi^-$. The decay of these particles produces $\nu_e$, $\nu_\mu$, and $\nu_\tau$, which should occur in the ratio $(\nu_\mu + \nu_\tau)/(\nu_e + \nu_\mu) \approx 2$. Instead, Kamiokande, and perhaps other experiments, observe this ratio to be close to unity [8]. This apparent deficit itself, together with the zenith angle dependence of the ratio of muon-type to electron-type neutrinos, has been argued [8,9] to be evidence for $\nu_\mu = \nu_\tau$ vacuum oscillations with $\delta m_{\mu\tau}^2 \sim 10^{-3} - 10^{-1}$ eV$^2$. Here $\nu_\mu$ probably could be only $\nu_\mu$ or $\nu_\tau$, as the vacuum mixing angles required to explain the data are very large: $\sin^2 2\theta_{\mu\tau} \gtrsim 0.4$. A sterile candidate for $\nu_\beta$ having such a large mixing with $\nu_\mu$ would be disallowed because it would increase the primordial $^4$He abundance by increasing the number of degrees of freedom extant at the epoch of primordial nucleosynthesis [7].

If all of the above experiments, and their interpretations in terms of neutrino oscillations, are correct then it is clear that $\nu_\alpha$ cannot be the same as $\nu_\beta$. This is because the three distinctive mass splittings implied by these experiments do not overlap. Furthermore, adoption of the putative LSND result forces us to conclude that there is no mutually consistent identification for $\nu_\alpha$ and $\nu_\beta$ with the introduction of a sterile neutrino species. The LSND limit $\delta m_{\mu\nu}^2 > 0.2$ eV$^2$ implies that $\nu_\beta \neq \nu_e$, so $\nu_\beta = \nu_\tau$; it also implies that $\nu_\alpha \neq \nu_\mu$, and $\nu_\alpha = \nu_\tau$ is then inconsistent with (3), so $\nu_\alpha = \nu_e$. This conclusion remains true, even if we adopt a rigorous treatment for the mixing of three or more neutrinos.

Two additional astrophysical arguments may provide yet a different set of insights into the possible masses and mixings of light neutrinos. These astrophysical considerations are (a) the possible requirement of a significant contribution to the closure density of the universe from light neutrino dark matter, and (b) the best proposed site for the synthesis of the $r$-process elements is the neutrino-heated ejecta from type-II supernovae ("hot bubble") [12]. This putative $r$-process site has the advantage that it yields the observed solar system abundance distribution of $r$-process nuclei, and each supernova makes an amount of $r$-process material which is in accord with models of galactic chemical evolution. No other proposed $r$-process site can accomplish these feats without the introduction of ad hoc parameters. Additionally, the conditions which determine nucleosynthesis in the post-core-bounce "hot bubble" environment are expected to be independent of the mesy details of the supernova explosion mechanism. The conditions conducive to the $r$ process will arise in the late stages of all successful type-II supernovae which leave hot neutron star remnants.

However, it has been shown that matter-enhanced neutrino flavor transformation ($\nu_e = \nu_{\mu(r)}$ or $\bar{\nu}_e = \bar{\nu}_{\mu(r)}$) can affect supernova dynamics and nucleosynthesis [13]. In fact, $r$-process nucleosynthesis from neutrino-heated supernova ejecta cannot occur unless the material in the hot bubble has an excess of neutrons over protons. In turn, the neutron-to-proton ratio in this environment is determined by the spectra of the $\nu_e$ and $\bar{\nu}_e$. These facts have been used to place broad limits on the mixing parameters of a light $\nu_e$ with $\nu_\mu$, and/or $\nu_\tau$ possessing cosmologically significant masses [14].

Detailed numerical calculations which include the neutrino-neutrino scattering contributions to the neutrino effective masses confirm that $r$-process nucleosynthesis is sensitive to neutrino flavor mixing [15,16]. In fact, the studies in [14-16] show that our adopted "best-fit" LSND parameters, $\delta m_{\mu\nu}^2 \approx 6$ eV$^2$ and $\sin^2 2\theta_{\mu\nu} \approx 3 \times 10^{-3}$, are not consistent with the $r$-process originating in neutrino-heated supernova ejecta if the vacuum neutrino masses satisfy $m_{\nu_\mu} > m_{\nu_\tau}$. Given the above LSND parameters, this neutrino mass hierarchy guarantees matter-enhanced $\nu_e = \nu_\mu$ transformation in the hot bubble, with resulting hardening of the $\nu_e$ spectrum, and consequent reduction of the neutron fraction below acceptable levels. References [14-16] show that hot bubble $r$-process nucleosynthesis could only be compatible with
the reported LSND results if $\delta m^2_{\mu} < 2 \text{ eV}^2$.

However, it has been shown that matter-enhanced $\nu_e \rightarrow \nu_{\mu}$ transformation in the hot bubble will not result in a proton excess. In this case, there are no obvious conflicts with $r$-process nucleosynthesis [17]. A necessary condition for matter-enhanced antineutrino flavor transformation is that there be an inverted hierarchy of neutrino masses, $m_{\nu_e} > m_{\nu_\mu}$. If this inverted mass scheme obtains, then $r$-process nucleosynthesis would be compatible with all LSND parameters.

It is not completely clear, however, whether such matter-enhanced antineutrino transformation with the LSND parameters would yield a $\nu_e$ spectrum compatible with that inferred from the SN 1987A data taken by the IMB and Kamiokande detectors [18]. These detectors were known to be sensitive to primarily $\nu_e$ through the reaction $\nu_e + p \rightarrow n + e^+$. Antineutrino transformation would certainly increase the average energy of $\nu_e$ over the standard case with no flavor transformation. However, the inferred range of temperature for the $\nu_e$ energy distribution in [18] is marginally compatible with $\nu_e \rightarrow \nu_{\mu}$, within the statistical uncertainties of the SN 1987A data. Furthermore, the underlying neutrino emission model on which the analysis of [18] and similar studies are based, is now known to be incorrect (e.g., they assume that the radius of the neutron star is fixed, the neutrino spectra are blackbody, these spectra cool with time, and the luminosity of each neutrino species is given by the first three assumptions—all of these points are wrong). Therefore, any constraints on antineutrino transformation derived from SN 1987A would be, at best, model dependent. Recent work by Mayle and Wilson [19] which incorporates matter-enhanced antineutrino transformation with our adopted LSND “best-fit” parameters yields a $\nu_e$ spectrum which would give an acceptable signal in the IMB or Kamiokande detectors for SN 1987A. However, future water Čerenkov neutrino detectors (e.g., super Kamiokande) would certainly see a clear signal for matter-enhanced antineutrino oscillations for a galactic supernova.

How seriously should we take these $r$-process considerations? It should be borne in mind that the calculations in [14–16] have some limitations and caveats. First, we do not know with complete certainty where the $r$-process originates. Second, studies of neutrino flavor transformation in the post-core-bounce supernova environment have only examined two-neutrino mixing [13–16]. If, for example, the $\nu_e$ and $\nu_{\mu}$, which have identical energy spectra in the supernova, also had nearly degenerate masses and mixed with the $\nu_e$ in a similar fashion, then for a given neutrino energy and $\delta m^2_{\mu}(\nu)$, we would expect the resonance regions for $\nu_e \rightarrow \nu_{\mu}$ and $\nu_e \rightarrow \nu_{\tau}$ to be overlapping. Is it then possible to get destructive quantum interference of the neutrino flavor conversion amplitudes at resonance? These amplitudes are known to be energy dependent [15], so that destructive interference in some narrow energy region of the neutrino spectrum would be countered with constructive interference in another. In this way it is clear that one could not engineer destructive interference-induced reduction in the degree of neutrino flavor transformation over the whole neutrino energy spectrum. Nevertheless, this issue bears further examination.

If we adopt as correct experiments (1)–(3), along with their interpretations in terms of neutrino oscillations, and adopt the astrophysical arguments (a) and (b), then there is a fairly unique set of neutrino masses and mixings which emerges as a fit. In this fit, the masses of the neutrinos must satisfy $m_{\nu_e} \approx m_{\nu_\mu} \approx 2.7 \text{ eV}$ (where $\nu_e$ is split from a sterile species, $\nu_\mu$, by $\sim 10^{-5} \text{ eV}$) and $m_{\nu_\mu} \approx m_{\nu_\tau} \approx 1.1 \text{ eV}$ (where these species are split by $\sim 10^{-2} \text{ eV}$). Here, and in what follows, when we say one neutrino species is split from the other by a certain amount, we mean that the corresponding neutrino vacuum mass eigenvalues differ by this amount. In this particular fit, we would adopt $\sin^2 2\theta_{ee} \sim 10^{-3}$, $\sin^2 2\theta_{e\mu} \sim 1$, and $\sin^2 2\theta_{e\tau} \sim 10^{-2}$. In this fit, we have employed our adopted “best-fit” LSND result and we assume that $\sum_{i} m_{\nu_i} \approx 5 \text{ eV}$. This mass spectrum could be altered in obvious fashion if we adopt slightly different values for the LSND results and $\sum_{i} m_{\nu_i}$. Note that the production of $\nu_e$ in the early Universe with $\sin^2 2\theta_{ee} \sim 10^{-2}$ is negligible, as would be required from considerations of big bang nucleosynthesis and the observed $^4\text{He}$ abundance [7]. This justifies the exclusion of $\nu_e$ in the sum $\sum_{i} m_{\nu_i}$.

The absence of an observation of neutrino-less double $\beta$ decay has been argued to place a limit on the Majorana mass of the $\nu_e$ of $\sim 1 \text{ eV}$ (actually, a weighted sum of the masses of all light neutrino species is constrained to be less than $\sim 1 \text{ eV}$, but the $\nu_e$ usually makes the biggest contribution to the sum) [20]. With this limit, it is clear that the neutrino masses in the above scheme would have to be Dirac, unless there were a fortuitous cancellation in the weighted sum over neutrino masses. From the tritium end-point experiments, the current upper limit on the mass (Dirac or Majorana) of the $\nu_e$ is $7.2 \text{ eV}$ [21].

Note that it is the LSND result which forces us to contemplate an inverted neutrino mass hierarchy and/or the introduction of a sterile neutrino species. If we were to accept the LSND result, but give up atmospheric neutrino oscillations and the hot bubble $r$-process, then we could have $m_{\nu_e} \approx m_{\nu_\mu} \approx 1.1 \text{ eV}$ and $m_{\nu_\mu} \approx 2.7 \text{ eV}$, with $\nu_e$ and $\nu_\mu$ split by $\sim 10^{-5} \text{ eV}$, a value sufficient to give an MSW solution for the solar neutrino problem (assuming $\sin^2 2\theta_{ee} \sim 10^{-2}$). This scheme has the obvious advantage that it does not require the introduction of a sterile neutrino species, but we note that it does have a curious neutrino mass hierarchy (essentially, this is an inverted neutrino mass hierarchy, since the second family $\nu_\mu$ is heavier than the third family $\nu_\tau$). This scheme could be consistent with either Majorana or Dirac neutrino masses. If we were to modify this scheme by requiring atmospheric neutrino oscillations, but dropping the MSW mechanism in the sun, then we could have $m_{\nu_e} \approx m_{\nu_\mu} \approx 2.5 \text{ eV}$ (with these species split by $\sim 10^{-2} \text{ eV}$ and $\sin^2 2\theta_{e\mu} \sim 1$), and $m_{\nu_\mu} \approx 0$ with $\sin^2 2\theta_{e\mu} \sim 10^{-2}$, with no mass inversion and no sterile species. We could restore the MSW mechanism in the sun for this latter scheme by simply adding a sterile neutrino species split from the $\nu_e$ by $\sim 10^{-3} \text{ eV}$ with $\sin^2 2\theta_{ee} \sim 10^{-2}$. This scheme then accounts for all con-
It is tempting to visit a scheme where $m_{\nu_e} \approx m_{\nu_x} \approx 2.5$ eV, with these species split by $\sim 10^{-6}$ eV, to give the MSW mechanism in the sun (assuming $\sin^2 2\theta_{12} \sim 10^{-7}$), and where $m_{\nu_x} \approx m_{\nu_\tau} \approx 0$, with these species split by $\sim 10^{-1}$ to $\sim 10^{-2}$ eV, to give atmospheric neutrino oscillations. Note, however, that the large mixing between $\nu_\mu$ and $\nu_\tau$ required for atmospheric neutrino oscillations, $\sin^2 2\theta_{13} \sim 1$, is probably precluded by big bang nucleosynthesis considerations [7]. If we drop the sterile neutrino species from this scheme, then we can explain all of the above constraints (1, 2, a, b), except for the atmospheric neutrino deficit. Since the explanation of the atmospheric neutrino deficit in terms of neutrino oscillations is far from settled, this may be an attractive scenario if the LSND result holds up.

Finally, we note that there is a qualitatively different fit of neutrino masses and mixings consistent with all our constraints (1, 2, 3, a, b), if we adopt $\delta m^2_{12} \ll 1$ eV$^2$ for the LSND result. In this case, we can have $m_{\nu_e} \approx m_{\nu_x} \approx 1.6$ eV. With this scheme, $\nu_e \equiv \nu_\tau$ matter-enhanced flavor transformation with $\delta m^2_{12} \sim 10^{-6}$ eV$^2$ gives the solution to the solar neutrino problem, $\nu_\mu \equiv \nu_e$ vacuum oscillations give the LSND result, and $\nu_\mu \equiv \nu_\tau$ vacuum oscillations explain the atmospheric neutrino deficit if $\delta m^2_{13} \sim 10^{-2}-10^{-1}$ eV$^2$ [22]. With this nearly degenerate scheme of neutrino masses, no inverted neutrino mass hierarchy is required. Furthermore, there is no need to introduce a sterile neutrino species. Conflicts with $r$-process nucleosynthesis are avoided in this scheme, because the neutrino mass-squared differences are too small to give matter-enhanced neutrino or antineutrino flavor transformation in the hot bubble $r$-process nucleosynthesis region.

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