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TRANSPORT OF INTENSE ION BEAMS*  
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Summary

The possibility of using intense bursts of heavy ions to initiate an inertially confined fusion reaction has stimulated interest in the transport of intense unneutralized heavy ion beams by quadrupole or solenoid systems. We have examined this problem in some detail, using numerical integration of the coupled envelope equations for the quadrupole case. The general relations which emerge are used to develop examples of high energy transport systems and as a basis for discussing the limitations imposed by a transport system on achievable intensities for initial acceleration.

Solution of the Envelope Equations

The envelope equations of Kapchinskij and Vladimirskij (KV) are 1).

\[
\frac{d^2 a}{ds^2} = - K_x(s) a_x + \frac{c_2}{\beta^2} \frac{N r_0}{A} a_x a_y
\]

\[
\frac{d^2 a}{ds^2} = - K_y(s) a_y + \frac{c_2}{\beta^2} \frac{N r_0}{A} a_x a_y ,
\]

where \( a_x \) = beam half-width, height; \( \pi c \) = normalized emittance; \( q \) and \( A \) are the ion charge state and atomic weight, \( N \) is the number of particles per unit length, \( r_0 \) is the classical proton radius and \( K_x, y \) = \( \frac{B(s)}{B_0} \) for quadrupoles

\[
K_x, y = \frac{1}{4} \left[ \frac{B(s)}{B_0} \right]^2 \text{ for solenoids (in a frame rotating at the Larmor frequency).}
\]

If \( K(s) \) is a step-wise function of \( s \), of constant magnitude or zero, Eqs. (1) can be put into dimensionless form by setting

\[ s = K^{-1/2} q \quad \text{and} \quad a = K^{-1/4} e^{1/2} (\beta_i) -1/2 u: \]

\[
\frac{d^2 u_x}{d\theta^2} = - S_x(\theta) u_x + \frac{1}{3} u_x + \frac{Q}{u_x a_y}
\]

\[
\frac{d^2 u_y}{d\theta^2} = - S_y(\theta) u_y + \frac{1}{3} u_y + \frac{Q}{u_y a_x}
\]

where \( Q = \frac{4q^2}{A} \frac{N r_0}{\beta_i e^2 K_0} \) and \( S(\theta) \) is a step function of unit amplitude. If \( S(\theta) \) is periodic, the necessary aperture and the current for a matched beam can be expressed as functions of \( Q \). For quadrupoles,

\[
a = C_1 \left( \frac{A}{q} \right)^{1/3} B_0^{-1/3} (\beta_i)^{-1/3} c_2^{1/3} u_m^{1/3} \]

\[
I = C_2 \left( \frac{A}{q} \right)^{1/3} B_0^{2/3} (\beta_i)^{5/3} c_2^{1/3} u_m^{1/3} - \frac{Q}{u_m}
\]

where \( C_1 = \left( \frac{m_i c}{e} \right)^{1/3} = 1.46 \) [MKSA units]

\[
C_2 = \left[ \frac{4n}{\mu_0} \right]^{5/6} \left( \frac{m_i c^2}{e} \right)^{1/6} = 3.66 \times 10^6 \]

\( B_0 \) and \( a \) are defined by \( K = \frac{B_0}{B_0 a} \), and \( u_m \) is the maximum value of \( u_x, y \) for the periodic solution of Eqs. (2). \( B_0 \) is in Teslas, \( a \) and \( c \) in meters and \( I \) in electrical amperes.

For a continuous solenoid, let \( u_x = u_y = u \), a constant for a matched beam. Equations (2) yield the relation \( Q = 2(u^2 - 1/u^2) \), from which one obtains:

\[
a = C_3 \left( \frac{A}{q} \right)^{1/2} B_0^{-1/2} e^{1/2} u
\]

\[
I = C_4 \left( \frac{A}{q} \right) (\beta_i) c (u^2 - \frac{1}{u^2}) = \frac{C_4}{C_3} \left( \frac{A}{q} \right) (\beta_i) B_0^{2/3} a^2 \left( 1 - \frac{1}{u^2} \right)
\]

where \( C_3 = \left( \frac{2m_i c}{e} \right)^{1/2} = 2.5 \)

\( C_4 = \frac{1}{4} \left( \frac{4n}{\mu_0} \right)^{5/6} = 2.5 \times 10^6 \)

\( Q \), or the corresponding \( u_m \) or \( u \), can be regarded as a free parameter measuring the influence of the space-charge force on particle motion. In the quadrupole case the relation between \( Q \) and \( u_m \) depends on the lattice structure. It is convenient to use the phase advance per period, \( \mu = \int ds = \int \frac{d\theta}{\beta} = \int \frac{d\theta}{u_x} \), as the space charge parameter, since it has a more immediate physical significance than \( Q \) or \( u_m \). In Fig. 1 is plotted \( Q/u_m \) and \( u_m^{1/3} \) for a FODO lattice with equal drift and magnet lengths and a phase advance per period of 120° at zero intensity. It is evident from the figure and from the form of Eqs. (4) for the solenoid case that, on the basis of these simple considerations, there is no limit to the current which can be transported, provided that the aperture can be made large enough and the variation in individual particle motion with intensity is tolerable.

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Transport at High Energy

As an application of these equations, we consider a situation in which a beam is extracted from an accelerator, passed through a buncher and allowed to drift some distance to shorten the pulse and increase the current to meet the targeting requirements. It is assumed that the rate of increase of current with distance is sufficiently slow that the transverse motion will adjust itself adiabatically to the matched conditions if it is matched at the entrance to the channel, where the current is low. We further assume that the elements at the end which focus the beam onto the pellet are adjusted to accept the phase-space configuration of the peak of the current pulse, which requires that there be a substantial overlap of the phase space ellipses for peak and lower intensities. The quantity, \( n \), also shown in Figure 1, is the fraction of the zero-intensity phase-space area lying inside the higher intensity ellipse, assuming an emittance independent of intensity. It can be seen that requiring \( n \) to be larger than, say, 50% sets a definite limit on peak current for a given quadrupole field. Table I gives four examples: energy, peak current and emittance are target requirements. \( B_q \) was chosen arbitrarily for \( U^{38} \) and as high as seemed realistic for \( U^{127} \) because of the constraint on \( n \).

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>Examples of High Energy Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion Type</td>
<td>( U^{38} )</td>
</tr>
<tr>
<td>Energy (GeV)</td>
<td>100</td>
</tr>
<tr>
<td>( I_{peak} ) (kA)</td>
<td>3.0</td>
</tr>
<tr>
<td>( c \times 10^{-5} ) m-radians</td>
<td>4.0</td>
</tr>
<tr>
<td>( B_q ) (T)</td>
<td>1</td>
</tr>
<tr>
<td>( a ) (cm)</td>
<td>4.2</td>
</tr>
<tr>
<td>( u ) (deg)</td>
<td>107</td>
</tr>
<tr>
<td>( n )</td>
<td>.95</td>
</tr>
</tbody>
</table>

Solenoid focusing does not look favorable for the cases considered. It is not difficult to show that

\[ n = 1 - \frac{4}{\pi} \tan^{-1} \frac{u}{u+1}, \]

whence \( u < 2.5 \) for \( n > .50 \).

Transport at Low Energy

Equations (3) and (4) indicate that particle current must be much reduced at lower energy. Hence, to provide a final high current, the accelerator system is required to build up the current by orders of magnitude by some combination of stacking in transverse space and longitudinal compression.

Additional considerations will affect the application of Eqns. (3) and (4). We assume that there is no need to transport a reduced current as well as the highest current through the same system, or equivalently, that the lower current portion may have a lower emittance. This would then permit the zero-intensity phase shift \( \mu \) to approach the pass-band limit of 180° and \( \mu \) to be made as small as allowed by the aperture or other considerations. However, a strong field \( B_q \) may result in quadrupole lengths and drift lengths that are too short, relative to the aperture, to permit fields that are reasonably linear and defined in length (as was assumed in the analysis). For the strong quadrupole case, then, we introduce the additional requirement that the ratio of aperture radius to quadrupole length not exceed a limiting value \( R \) and this results in the following limit on particle current in the FODO lattice with equal drift and quadrupole lengths:

\[ I = \frac{C_6}{q} \left( \frac{B_q}{U_m} \right)^3 \left( \frac{R}{2} \right)^{180°} \]

where \( C_6 = \frac{1}{16} \left[ \frac{m c^2}{u} \right]^{1/2} \left( \frac{1}{r_p} \right)^{1/2} \) and \( \theta \) is the cell length in the scaled variable, \( \theta \).

The quantity \( \frac{\mu}{u_m} \) will depend on the phase advances, but has a maximum value close to unity.

Two other limitations should be kept in mind. First, the electrostatic potential in the beam can become comparable to the kinetic energy and, second, if \( B_q \) or \( B_s \) approaches \( 2[B_s]_a \), ions entering a lens at radius \( a \), will be turned back at low intensity for a quadrupole and at any intensity for a solenoid. Both potential/kinetic energy and \( B_s/2[B_s]_a \) must be much less than unity for the paraxial ray approximation used in this paper to be valid.

For a numerical example, we consider a beam of \( U^{38} \) at a kinetic energy of 1.0 MeV and the same normalized emittance as the 100-GeV example of Table I. With a strong \( B_q \) and no restriction on \( R \), the first column of Table II shows that a current of 4.65 amperes can be transported by the FODO system. Restricting \( R \) to about 0.5 reduces the current to 1.0 ampere (column 2); with reduced \( B_q \), a better compromise is found at 2.42 amperes in column 3 but with somewhat larger aperture roughly proportional to the ratio \( I/B_q \) in consistency with Eqns. (3) and the approximate constancy of \( Q/U_m \).
motion becomes highly non-linear except for the moderate change in current, for a large number of modes (see also Ref. 4). The lowest threshold occurs at $u = 1.6$, which would imply that Eqns. (4) have a very limited range of validity. On the other hand, the KV distribution has special mathematical properties and we prefer to believe that Eqns. (4) are probably qualitatively correct for more realistic distributions which are probably stable.

c) Since the Hamiltonian is not a constant of the motion for a quadrupole transport line, it is not possible to construct stationary (i.e., periodic) solutions by the technique outlined in the previous paragraph. As a partial step away from the KV distribution we examined a "self-inconsistent" problem by tracing individual particle trajectories in a field with a linear part generated by the periodic solution of the envelope equation, plus cubic terms appropriate to a parabolic density profile of the same outer dimensions. For intensities such that $\mu$ is less than $10^{-2}$, we find a large growth in amplitude of some particles and the development of an island structure in their phase space, indicative of resonant behavior in the periodic non-linear field. We are thereby led to suspect that a quadrupole transport system may be subject to unstable behavior at high intensity.

References
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