Subitizing, Finger Gnosis, and the Representation of Number

Marcie Penner-Wilger (mpwilger@connect.carleton.ca)
Lisa Fast (lisa@hume.ca)
Jo-Anne LeFevre (jo-anne_lefevre@carleton.ca)
Centre for Applied Cognitive Research, Institute of Cognitive Science, Carleton University, Ottawa, ON K1S 5B6 Canada

Brenda L. Smith-Chant (bresmith@trentu.ca)
Department of Psychology, Trent University, Peterborough, ON K9J 7B8 Canada

Sheri-Lynn Skwarchuk (s.skwarchuk@uwinnipeg.ca)
Faculty of Education, University of Winnipeg, MB R3B 2E9 Canada

Deepthi Kamawar (dkamawar@connect.carleton.ca)
Institute of Cognitive Science and Department of Psychology, Carleton University, Ottawa, ON K1S 5B6 Canada

Jeffrey Bisanz (jeff.bisanz@ualberta.ca)
Department of Psychology, University of Alberta, Edmonton, AB, T6G 2E9 Canada

Abstract
What precursor abilities form the building blocks of numerical representations? Two abilities were investigated: the ability to mentally represent small numerosities, indexed by subitizing speed (Butterworth, 1999), and the ability to mentally represent one’s fingers, indexed by finger gnosia (Butterworth, 1999; Penner-Wilger & Anderson, 2008). We examined the longitudinal relation between these abilities in Grade 1 and tasks assessing numerical representation in Grade 2—symbolic number comparison and number-line estimation— for 100 Canadian children. Finger gnosia (but not subitizing speed) in Grade 1 was related to children’s symbolic distance effect in number comparison and to the linearity of children’s estimates in Grade 2. Thus, children with better finger gnosia scores had lower symbolic distance effects and more accurate estimates, reflecting a more precise mapping between numerals and their associated magnitude.

Keywords: number representation; numeracy; math development; subitizing; finger gnosia, number comparison; estimation; numerical distance effect.

Precursors to the Representation of Number
What precursor abilities form the building blocks of numerical representations? Subitizing, the ability to quickly enumerate small sets without counting, and finger gnosia, the ability to mentally represent one’s fingers, are related to children’s number system knowledge and calculation skill in Grade 1 (Penner-Wilger et al., 2007). We hypothesized that this relation occurs because subitizing and finger gnosia facilitate the development of number representations (Butterworth, 1999). In the current paper, we test this hypothesis by examining the relation between the precursors, subitizing and finger gnosia, in Grade 1 and tests designed to assess the strength of numerical representations: magnitude comparison and number line estimation in Grade 2.

Subitizing is a developmentally and evolutionarily primary numerical ability that is seen both in infants as well as other species (Dehaene, 1992). Benoit, Lehalle, & Jouen (2004) asserted that subitizing is a necessary component for the mapping of number words to numerosities, as subitizing “allows the child to grasp the whole and the elements at the same time” (p. 21). In Butterworth’s (1999, 2005) theory of numeracy development, subitizing is an index of our (exact) numerosity representations and forms the core numerical ability upon which all others are built.

How are non-symbolic representations of number related to more abstract symbolic representations of number? The prevailing view is that symbolic representations of number (number words, numerals, etc.) acquire meaning by being mapped onto non-symbolic representations (Brannon, 2005; Butterworth, 1999, Dehaene, 1997; Diester & Nieder, 2007; Verguts & Fias, 2004). In contrast, Ansari (2008; Holloway & Ansari, 2008) suggests that symbolic and non-symbolic representations may be distinct. Regardless of the form of the relation between symbolic and non-symbolic representations of number, humans are able to recognize common representational content in different vehicles (e.g., dots, number words, numerals, etc.). Thus, even if non-symbolic and symbolic representations are not built upon one another, both forms must be linked to the semantic representation of number.

Finger gnosia is hypothesized to support the mapping of symbolic and non-symbolic representations of number, thus increasing the range and precision of numerical representations. Fayol and Seron (2005) propose that the fingers are well suited as a tool to link non-symbolic and symbolic representations of number, as, unlike linguistic representations “finger representations exhibit an iconic relation to numerosities, since they preserve the one-to-one matching relation between the represented set and the fingers used to represent it” (p. 16). Butterworth (1999) likewise hypothesizes that children’s use of fingers to represent numerosities in the course of numerical development helps to ‘bridge the gap’ from numerosity representations to more abstract number words. Thus, the ability to mentally represent one’s fingers is thought to aid
in the mapping of non-symbolic representations (indexed by subitizing) onto symbolic representations of number, building up a full number system.

Three views exist on the relation between finger gnosion and the representation of number. On the localizationist view, finger gnosion is related to numerical abilitiés because the two abilities are supported by neighboring brain regions, and these regions tend to have correlated developmental trajectories. On this view, there is no direct causal link between the representation of finger and number (Dehaene et al., 2003). In contrast, on the functional view, finger gnosion and numerical abilities are related because the fingers are used to represent quantities and perform counting and arithmetic procedures. As a result, the representation of numbers and of fingers becomes entwined (Butterworth, 1999). On the redeployment view, finger gnosion is related to math ability because part of the functional complex for number representation overlaps with the functional complex for finger representation. On this view, finger and number share a common neural resource that supports both representations (Penner-Wilger & Anderson, 2008). In summary, despite the distinct mechanisms proposed for the link between finger and number representation, each view hypothesizes a relation between finger gnosion and numerical representations.

Numerical comparison and numerical estimation have been proposed as indices of the strength of number representations (Butterworth & Reigosa, 2007; Holloway & Ansari, 2008). Butterworth and Reigosa (2007), assert that mathematical difficulties stem in part from slower and less efficient processing of numerical information, specifically the estimation and comparison of numerosities. In the current paper we investigate the relations among subitizing, finger gnosion, number comparison and estimation to determine whether the precursors are related to tasks designed to assess numerical representations.

**Number Comparison**

Number comparison involves recognition and judgment of the relative magnitude of numerosities, and is used as an index of the semantic representation of number (McCloskey, 1992). Multiple forms of number comparison tasks exist with both symbolic and non-symbolic stimuli. In number comparison tasks, the outcome result is the distance effect: participants are faster to judge pairs with large differences or splits (e.g., 2 vs. 7) than pairs with small splits (e.g., 2 vs. 3). The distance effect is hypothesized to reflect the mapping between external and mental representations of number, with larger distance effects reflecting noisier mappings (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Holloway & Ansari, 2008). Both adults and children show a distance effect, but the effect is attenuated in adults (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Duncan & McFarland, 1980; Noël, Rousselle & Mussolin, 2005). Larger distance effects are thought to reflect the mapping between external and internal representations of numerical magnitude (Holloway & Ansari, 2008).

Holloway and Ansari (2008) examined the concurrent relation between the distance effect and math achievement in typically developing 6- to 8-year-old children. Both non-symbolic and symbolic number comparison tasks were used. They found that the symbolic distance effect (in response time), but not the non-symbolic, was related to both math fluency and calculation skill. Further analyses showed that this relation held for both math measures for the 6-year-olds, for math fluency for the 7-year-olds, and was not significant for the 8-year-olds.

**Number-line Estimation**

Estimation “is a process of translating between alternative quantitative representations, at least one of which is inexact.” (Siegler & Booth, 2005, p. 198). Number-line estimation, more specifically, is hypothesized to provide direct information about representations of numerical magnitude (Siegler & Booth, 2005). Siegler and colleagues assert that the linearity of children’s estimates is an index of the quality of their numerical representations, with more linear estimates reflecting better representations (Siegler & Booth, 2004).

In earlier grades and for larger-scale number lines, the relation between the targets and children’s estimates is best fit by a logarithmic function. The shift from log to linear representations happens between Kindergarten and Grade 2 for 0-100 number lines and between Grade 2 and Grade 6 for 0-1000 number lines (Siegler & Booth, 2004; Siegler & Opfer, 2003). The linearity of children’s estimates correlates with their concurrent math achievement for Kindergarten through grade four (Booth & Siegler, 2006; Siegler & Booth, 2004).

**Predictions: Subitizing, Finger Gnosis, and Numerical Representations**

**Subitizing.** On Butterworth’s view (1999, 2005), subitizing will predict both number comparison and estimation performance, because subitizing forms the core numerical ability upon which all others are built. On Ansari’s view (Ansari, 2008; Holloway & Ansari, 2008), subitizing will not predict performance on either task, because both tasks used in the current paper are symbolic. Ansari proposes that symbolic representations of number are not built upon the non-symbolic representation, indexed by subitizing, and therefore that the relation between subitizing and symbolic number representation is not a precursor relation.

**Finger Gnosis.** On Butterworth’s view (1999), finger gnosion will predict both number comparison and estimation performance, because finger gnosion facilitates the mapping of non-symbolic representations onto symbolic representations of number. On Dehaene’s view (Dehaene et al., 2003), finger gnosion may also predict both number comparison and estimation performance, but only due to the shared developmental trajectory of the brain regions involved in the representation of finger and number. On Penner-Wilger and Anderson’s view (2008), finger gnosion
will predict both number comparison and estimation performance, as all three tasks make use of a common underlying neural resource, which originally evolved as part of the functional complex supporting the representation of fingers and has since been redeployed as part of the functional complex supporting the representation of number, serving both uses.

**The Current Research.** The primary goal of this study was to examine the longitudinal relations between subitizing, finger gnosis, and tasks designed to assess numerical representations; a secondary goal was to examine the concurrent relation between numerical representation tasks and math outcome measures. To this end, we assessed children’s subitizing and finger gnosis in Grade 1 and their performance on number comparison, number-line estimation, and standardized math outcome measures (including KeyMath Numeration subtest, Woodcock-Johnson Calculation subtest, and addition fluency) in Grade 2. We hypothesized that finger gnosis would predict number comparison and estimation performance. On Butterworth’s view (1999) subitizing would predict comparison and estimation performance. Consistent with Butterworth’s view, we previously found that subitizing predicted concurrent performance on number system knowledge and calculation skill in Grade 1 (Penner-Wilger et al., 2007, 2009). Based on the view of Holloway and Ansari (2008), however, subitizing would not predict performance on the symbolic comparison and estimation tasks used in the current experiment. The participants in the current study are the same as in Penner-Wilger et al. (2007), where we examined number system knowledge and calculation skill concurrently in Grade 1. Here we extend that work to determine whether subitizing and finger gnosis are longitudinal predictors of tasks assessing number representations in Grade 2.

**Method**

**Participants**

Grade 1 children (N = 148) were selected for the current paper from a larger group who were recruited for the Count Me In longitudinal project, which involved children from seven schools in three Canadian cities. The children were tested in May or June each year. Of the Grade 1 children, 112 participated the following year, in Grade 2. Data were missing for 12 of these 112 children on both of the tasks of interest: number comparison and number line estimation. Thus, the present analyses are based on the 100 children (51 boys, 49 girls, mean age, in years:months, 6:10 in Grade 1, range 5:7 to 7:4) who had completed Grade 1 data and at least one Grade 2 outcome measure.

**Materials and Procedure**

Children completed the subitizing and finger gnosis tasks, along with the processing speed and vocabulary tasks, in Grade 1. The outcome tasks (numeration, calculation, and addition fluency) were completed in Grade 2.

**Subitizing.** Children were shown arrays of one to six dots and were asked to state ‘how many’ dots as quickly and accurately as possible. Children initiated the trial by pressing the space bar. The trial ended when the child stated the numerosity of the set and the experimenter pressed a key and then typed in the response. There were three trials of each array size, each in a different random pattern. Two practice sets (of 1 and 7 dots) were included as the first two trials. The median subitizing latency was computed from the latencies of the nine trials showing one, two, and three dots.

**Finger Gnosis.** The Finger Gnosis measure is based on Noël (2005). Ten trials were conducted on each hand, beginning with the dominant hand. In each trial, two fingers were lightly touched below the first knuckle. The child’s view of the touches was obstructed with a cloth cover raised from the child’s wrist. After the cloth cover was lowered, the child pointed to the two fingers that had been touched. A point was awarded for each correct identification of a touched finger in a trial, with a maximum of 20 points per hand. The total score across both hands was used as the dependent measure, with a maximum score of 40.

**Processing Speed.** To assess processing speed, we implemented a computer-based simple choice reaction time task. Two types of stimuli (an X or an O) were displayed for 1 second, preceded by a 500 ms fixation point. Children were instructed to press the key corresponding to the target letter shown on the screen. The display then cleared and the next trial began automatically 1 second later. There were 24 trials. The median response time for pressing the correct key in response to the stimuli was used as the dependent measure.

**Vocabulary.** Receptive language was measured using the Peabody Picture Vocabulary Test–Third Edition, form B (Dunn & Dunn, 1997). It was included primarily as a measure of verbal, non-mathematical knowledge. Dunn and Dunn cite the split-half reliability coefficient for Form B for seven year olds as .95.

**Number Comparison.** The number comparison task was designed based on the numerical condition from Landerl, Bevan, and Butterworth (2004). The child was shown two numerals on the screen (from 1 to 9) and was asked, “Which number is more than the other number?” Children indicated their response by pressing a yellow key on the side that was more (z on the left or . on the right). For each trial, there was a 500-ms delay prior to the stimulus presentation. Stimuli were displayed until the child responded or until a 3-s maximum was reached.

Stimuli varied on two dimensions, physical size (large vs. small font) and numerical size. For congruent trials, the number that was larger numerically was also larger physically. For incongruent trials, the number that was larger numerically was smaller physically. Numerical distance was defined with small splits as a distance of 1 (e.g., 2 3) and large splits as a distance of 5 (e.g., . 7). Half of the trials were congruent and half were incongruent. There were 40 trials preceded by two practice trials. The
stimuli pairs were taken from Landerl et al. (2004) with 24 trials of the six small-split combinations (1-2, 2-3, 3-4, 6-7, 7-8, 8-9) and 16 trials of the four large-split combinations (1-6, 2-7, 3-8, 4-9). Two pseudo-random orders were created.

Three dependent variables were computed: overall response time for correct trials, overall accuracy, and distance effect calculated as in Holloway and Ansari (2008) to capture the increase in RT from large to small splits controlling for individual differences in RT ((small split RT – large split RT)/large split RT).

**Number-Line Estimation.** Number line estimation was measured using a computerized test of numerical estimation skill (Siegler & Opfer, 2003). Children were shown a target number between 1 and 1000 at the top of the screen and used the mouse to position a vertical line at the appropriate spot on a number line starting at 0 and ending at 1000. The computer recorded the location and the solution latency.

Order of the 25 trials was randomized separately for each child. The stimuli were chosen based on Laski and Siegler (2007) and were balanced with four targets between 0 and 100, four between 900 and 1000, two targets from each other decade and distances matched from the endpoints. The targets were: 6, 994, 18, 982, 59, 991, 97, 903, 124, 876, 165, 835, 211, 789, 239, 761, 344, 656, 383, 617, 420, 580, 458, 542, and 500.

Regression was used to calculate the relation between the actual values of the presented numbers and the locations that the child chose for those numbers on the number line. Larger $R^2$ values are indicative of a close correspondence between the number line locations and the presented numbers.

**KeyMath Numeration.** Children completed the Numeration subtest of the multi-domain math achievement test, the KeyMath Test-Revised (Connolly, 2000). This test covers concepts such as quantity, order, and place value (on later items). Most of the items in the range for these children require knowledge of the symbolic number system. The reported alternate form reliability coefficient for the grade-scaled Numeration subtest is .75 (Connolly, 2000). Connolly provides a split-half reliability coefficient of .81 for spring Grade 2.

**Woodcock-Johnson Calculation.** Children completed the calculation subtest of the Woodcock Johnson Psycho-Educational Battery - Revised (Woodcock & Johnson, 1989). This calculation measure involves all four operations (addition, subtraction, multiplication, and division), although most of the questions that were attempted by the children in the present study involved addition or subtraction. This test has a median reliability of .85 and a one-year test-retest correlation of .89 for Grades 2 through 4 (Woodcock & Johnson, 1989). The WJ-R manual cites the split-half reliability for six year olds as .928, $SEM(W)=5.7$ ($N=309$).

**Addition Fluency.** The children solved 16 single-digit sums displayed on the computer screen. In Grade 2, the sums were greater than ten. This task has a stop condition of five sequential errors and trials timed out if the child did not respond within 20 seconds. The child initiated each trial by pressing the ‘GO’ button. When the child spoke their answer, the experimenter pressed a key to stop the timer and typed in their response. Each child's median addition latency was computed from their correct trials.

**Results**

Descriptive statistics for each measure are shown in Table 1. All results are significant at $p < .05$ unless otherwise noted.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing speed$^1$</td>
<td>652</td>
<td>108</td>
</tr>
<tr>
<td>Vocabulary$^2$</td>
<td>109</td>
<td>11</td>
</tr>
<tr>
<td>Subitizing$^3$</td>
<td>1261</td>
<td>198</td>
</tr>
<tr>
<td>Finger gnosis$^2$</td>
<td>30.7</td>
<td>3.8</td>
</tr>
<tr>
<td>Distance effect</td>
<td>.12</td>
<td>.14</td>
</tr>
<tr>
<td>Estimation linearity</td>
<td>.68</td>
<td>.26</td>
</tr>
<tr>
<td>Numeration$^3$</td>
<td>12.3</td>
<td>3.2</td>
</tr>
<tr>
<td>Calculation$^3$</td>
<td>100</td>
<td>14</td>
</tr>
<tr>
<td>Add fluency$^4$</td>
<td>3773</td>
<td>1208</td>
</tr>
</tbody>
</table>

1 Milliseconds; 2 Number correct; 3 Standardized score; 4 Grade-Scaled Score

**Distance Effect in Number Comparison**

A significant distance effect was found in both accuracy and RT. Participants were more accurate comparing numbers with splits of five ($M = 90\%$, $SD = 13\%$) than numbers with splits of one ($M = 84\%$, $SD = 13\%$), $F(1, 85) = 53.10, MSE = 28.76$. Participants were faster comparing numbers with splits of five ($M = 1150\ ms$, $SD = 262\ ms$) than numbers with splits of one ($M = 1278\ ms$, $SD = 269\ ms$), $F(1, 85) = 49.35, MSE = 14335$. The distance effect in accuracy was not correlated with subitizing or finger gnosis, and is not discussed further in this paper.

**Do Subitizing and Finger Gnosis Jointly and Independently Predict Number Comparison?**

To determine whether subitizing and finger gnosis predict magnitude comparison performance, both jointly and independently, multiple regression was performed. The symbolic distance effect in Grade 2 was predicted from subitizing and finger gnosis in Grade 1. In both this and the following regression, gender, processing speed, and receptive vocabulary were included as control variables. As shown in Table 2, only finger gnosis significantly predicted the symbolic distance effect, accounting uniquely for 10% of the variability in the distance effect.

**Do Subitizing and Finger Gnosis Jointly and Independently Predict Number Line Estimation?**

To determine whether subitizing and finger gnosis predict performance of number line estimation, both jointly and independently, multiple regression was performed.
Estimation linearity in Grade 2 was predicted from subitizing and finger gnosis in Grade 1. As shown in Table 2, after accounting for the control variables only finger gnosis significantly predicted estimation linearity, accounting uniquely for 7% of the variability in linearity.

Table 2: Standardized regression coefficients and model $R^2$ values for each regression analysis.

<table>
<thead>
<tr>
<th>Predictors</th>
<th>Distance Effect</th>
<th>Estimation Linearity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing speed</td>
<td>-.09</td>
<td>-.13</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>.02</td>
<td>.35**</td>
</tr>
<tr>
<td>Gender</td>
<td>.00</td>
<td>.33**</td>
</tr>
<tr>
<td>Subitizing</td>
<td>.08</td>
<td>-.14</td>
</tr>
<tr>
<td>Finger gnosis</td>
<td>-.35**</td>
<td>.27**</td>
</tr>
<tr>
<td>Model $R^2$ (total)</td>
<td>.12</td>
<td>.36**</td>
</tr>
</tbody>
</table>

Significance levels: * $p < .05$, ** $p < .01$.

Are Number Comparison and Estimation Related to Math Skills?

To determine whether number comparison and estimation were related to concurrent math outcomes in Grade 2 partial correlations were performed controlling for gender, vocabulary, and processing speed. Correlations are shown in Table 3.

Table 3: Correlations among Grade 2 measures.

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Distance effect</td>
<td>.12</td>
<td>.39**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Estimate linear.</td>
<td>-.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Numeration</td>
<td>-.12</td>
<td>.39**</td>
<td>.41**</td>
<td></td>
</tr>
<tr>
<td>4. Calculation</td>
<td>-.18</td>
<td>.38**</td>
<td>.41**</td>
<td>.62**</td>
</tr>
<tr>
<td>5. Add fluency</td>
<td>.19</td>
<td>-.29*</td>
<td>-.31*</td>
<td>-.62**</td>
</tr>
</tbody>
</table>

Significance levels: * $p < .05$, ** $p < .01$, df = 67.

Number comparison. In contrast to Holloway and Ansari (2008), the symbolic distance effect was not correlated with performance on the KeyMath Numeration subtest, Woodcock-Johnson Calculation subtest, or Addition fluency. This finding is surprising given that the same age group was investigated in both studies, the studies had a similar number of participants ($N = 87$ vs. 100 in the current paper), and one of the outcome measures, the Woodcock-Johnson calculation subtest, was the same. There was, however, a task difference: We included trials in which physical size and magnitude conflicted whereas Holloway and Ansari did not. Thus, it is possible that the relation between the symbolic distance effect and math outcomes is not robust across task variability.

Number-line estimation. Consistent with Siegler and Booth (2004), we found that the linearity of children’s estimates were correlated with performance on the KeyMath Numeration subtest, Woodcock-Johnson Calculation subtest, and addition fluency.

Discussion

The primary goal of this paper was to examine the longitudinal relations between subitizing, finger gnosis, and tasks designed to assess numerical representations. We found that finger gnosis in Grade 1 was related to children’s symbolic distance effect in number comparison and to the linearity of children’s estimates in Grade 2. Children with better finger gnosis scores had smaller symbolic distance effects, reflecting a more precise mapping between numerals and their associated magnitude. The relation between the symbolic distance effect and finger gnosis is consistent with the view that finger gnosis facilitates the mapping between non-symbolic (magnitude) representations and symbolic representations. Children with better finger gnosis scores also made more precise estimates, again reflecting a more precise mapping between numerals and their associated magnitude.

Subitizing in Grade 1 was not related to either measure of symbolic number representation in Grade 2. This finding is not consistent with the predictions of Butterworth (1999), whereby subitizing was hypothesized to relate to the symbolic distance effect and to estimation. Holloway and Ansari (2008) hypothesize that symbolic and non-symbolic representations are distinct, contrary to prevailing views that symbolic representations are built on non-symbolic representations. The pattern of results may, therefore, be quite different if non-symbolic versions of comparison and estimation tasks were used. Further work will explore the relation between the precursors and non-symbolic representations.

A secondary goal was to examine the concurrent relation between numerical representation tasks and math outcome measures. The linearity of children’s estimates was related to all investigated math outcomes including: KeyMath numeration subtest, Woodcock-Johnson Calculation subtest, and addition fluency. In contrast, the symbolic distance effect was not related to any of the investigated math outcomes.

In conclusion, finger gnosis was related to all indices of the symbolic representation of number. This finding may reflect a developmental phenomenon whereby the mental representations of fingers and of number become linked functionally, through the practiced use of fingers to represent numerosities (Butterworth, 1999). Alternatively, the relation between finger and number representations may be one of identity, wherein the relation reflects a shared underlying representational form (Penner-Wilger & Anderson, 2008).

Acknowledgments

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