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Cliodynamics: The Journal of Quantitative History and Cultural Evolution

## A Dynamic Analysis of American Socio-Political History

A Review of Ages of Discord: A Structural Demographic Analysis of American History by Peter Turchin (Beresta Books, 2016) Peter J. Richerson University of California, Davis

These days it is a commonplace of news and opinion pieces that politics in the United States has become more polarized over the past half century. A few of these commentators note that the present scene resembles the years just before the Civil War. No one I have read or heard has a sensible causal account of this pattern, with the exception of Turchin's Age of Discord (AD). AD applies his and colleagues' Structural-Demographic Theory (SDT) to understanding the political history of the United States 1780-present. The theory has been applied heretofore to agrarian states such as China, France and England. The US was a somewhat peculiar agrarian state up until about the Civil War, after which it transitioned to an industrial state, so accounting for its political dynamics is a fresh challenge for SDT. For readers who are new to Turchin's oeuvre two bits of background are useful to understand AD. First, it develops theoretical models using coupled differential equations, an approach all but universal in the natural sciences but heretofore uncommon in the social sciences. Second, SDT has been applied to a number of other cases, some of which have considerably deeper histories than the United States. On the one hand, support for SDT generally is rather stronger than the short history of the U.S. would suggest. On the other hand we tend to think of the Industrial Revolution as having led to societies that are fundamentally different from their agrarian predecessors. Many observers, especially Americans, think that the U.S. is exceptional among industrialized nations. AD makes a case that the U.S., and probably other industrialized nations, have only fairly subtly modified the basic political-economic dynamics of stratified societies that go back several millennia.

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# Background: Theory Building in the Case of Complex Dynamic Systems

The strategy taken in AD is to recognize that human history consists of complex dynamic systems with many interacting parts. The universe is full of such systems. Take the solar system. It consists of the sun. 8 planets, many moons. dwarf planets and other bodies. Each of these bodies exerts gravitational attraction on all the others. The solar system is relatively simple and the mathematical tools of calculus developed by Newton and Leibnitz can be deployed to make rather precise predictions about the motions of planets. For example, the orbit of the earth around the sun is slightly perturbed by gravitational interactions with other bodies in the solar system. The eccentricity of its orbit (the degree to which it is circular versus elliptical) is perturbed by gravitational interactions with Jupiter and Saturn. Although "many body" problems like the solar system can be intractable, the slight mass of even large planets relative to the sun makes tractably simple approximations feasible. The accurate prediction of the earth's orbital variation led to Mulutin Milanković's theory of how orbital perturbations drove the quasi-periodic variation of Pleistocene climates (Havs. Imbrie, et al. 1976).

As the natural sciences developed, complex time-varying problems were usually modeled using coupled differential or difference equations. For example, ecologists study the interactions between predators and prey by coupling two differential equations representing the population dynamics of each species. Predation is an interaction effect. The nutrition predators obtain from prey tends to increase predator populations and diminish prey populations. Predator-prey models usually exhibit complex dynamics. In the simplest cases these take the form of regular cycles but the dynamics are often more complex, leading to chaotic variation, for example. Particularly in the low-diversity Arctic, predatorprey dynamics are often fit fairly well with comparatively simple models (Stenseth, Falck, et al. 1998).

The social sciences and history, by contrast, have mostly developed theory without using dynamic models. The theory of cultural evolution is an exception because some of its founders were population geneticists who were very familiar with dynamic systems modeling (Cavalli-Sforza and Feldman 1981). A small school of evolutionary economists have attempted to convince their colleagues of the utility of dynamic models, so far with little success (Bowles 2003; Cordes, Richerson, et al. 2014; Nelson and Winter 1982).

Theory building in fields studying complex dynamic systems requires an intimate connection between models and data. Coupled differential equation models are easy to construct and often have quite different behavior depending upon parameter combinations for the same model, not to mention structurally

different models. To do scientifically interesting work as opposed to pure math requires the theorist to explore models in a parameter space that reflects the real world (Efferson and Richerson 2007). The fundamental challenge is the formidable complexity and diversity of many of the systems we are interested in. such as ecosystems or human societies. The recognition of complexity and diversity should interest historians and humanistic social scientists who frequently accuse scientists of being reductionists. An important practical challenge is that information about these complex systems is generally very limited. We never have anything like a complete description of the system. The complexity and limited data problems interact. As the ecological data analysts Burnham and Anderson (2002: 20) say "we believe that 'truth' (full reality) in the biological sciences has essentially infinite dimension, and hence full reality cannot be revealed with only finite samples of data and a 'model' of those data.... We can only hope to identify a model that gives a good approximation of the data available." Recently much work, such as Burnham and Anderson's, has gone into devising techniques to find models that are good approximations (Gerbault, Allaby, et al. 2014; McElreath 2015). These methods take advantage of the falling cost of computation to use models to simulate the data. systematically varying the parameters of the model and asking which parameter combinations best fit the data. In other words, the strategy is to analyze the data with models that one believes are good candidates to describe the actual causal processes underlying the behavior of the system under study. Interestingly, the best fitting models are often rather simple. A small amount of noisy data about a complex system can have only a limited amount of real information about the system. If the analyst is lucky, some dominant processes will have a sufficiently large effect to be detectable with the data to hand. Complex systems teach scientists humility! Conventional statistical analysis using general linear models like OLS regression suffer from a number of deficiencies according to those employing these methods. For example, using variance explained as a goodness of fit parameter tends to result in "overfitting" of data, fitting the noise in the data, leading to poor prediction of out-of-sample data.

Ironically, these advances in quantitative data analysis have given a new rationale for non-quantitative approaches to complex dynamical systems, another thing that should interest humanists. Verbal models are imprecise but they can keep qualitative track of a larger number of variables than a formal quantitative model (Leijonhufvud 1997). According to Leijonhufvud, a macroeconomist, *theories* are the general structure of our overall knowledge about a field of complex dynamic systems like economies, generally expressed verbally, perhaps with illustrative simple models. Theories are what advanced textbooks attempt to describe. *Models* are tractable formalisms that we might want to use to

understand a key element, or few interacting elements, of our system. A model of the whole theory would be of intractably high dimension. Thus, a theory like Structural Demographic Theory (SDT) aspires to have a large toolkit of models, and the art of the sciences of dynamic systems is diagnosing the problem at hand and applying the right tool to further our understanding of it. Selecting the right model usually requires a certain amount of cut and try, competing one plausible model against others, looking for the most effective one. In an evolving system, this year's best model may be poor next year or in a superficially similar system over the hill. The dominant processes that we can hope to detect are liable to vary in detail from time to time and place to place. If full reality is tantamount to infinite dimensionality there is no guarantee that we can discover a satisfactory understanding of our system of interest, and if we can it will be because a relatively simple process dominates its dynamics in the here and now.

Peter Turchin is a theoretical ecologist by background so his toolkit very much includes the use of coupled differential equations and the general strategy of theory building described here. In AD, he makes the case that a particular mathematically-informed theory, SDT, can help explain the changing fortunes of human societies throughout history.

### Structural Demographic Theory

Structural Demographic Theory is a theory along the lines articulated by Leijonhufvud. It is derived from the verbal theories of Ibn Khaldun (1958 [1377]) and Jack Goldstone's (1991) quantitative models of mass mobilization potential and political stress. The basic starting point for SDT is that human societies, like many biological and physical systems, are complex sets of interacting dynamic parts. A general lesson from the study of complex dynamic systems is that feedbacks between parts easily generate oscillations and other non-linear behavior even in quite simple systems. Human intuition is generally poor at guessing the dynamics of such systems but if a few processes dominate the dynamics, one can often get a good quantitative approximation to the system's behavior with a fairly simple model. STD aims to develop such models.

SDT was further elaborated by Turchin in *Historical Dynamics: Why States Rise* and *Fall* (Turchin 2003). It imagines a society divided into laboring and elite classes. The laboring class is prone to a Malthusian dynamic in which population growth during good times leads to a labor surplus and falling wages. Elites during good times also grow, and as wages for labor begin to fall, elites grow rich on the profits from cheap labor, a Marxist element in the theory. But as the incomes of the elite increase relative to laborers' wages, laborers are attracted to try to enter the elite class. Upward mobility, and often higher rates of internal increase, cause the elite class to swell, resulting in elite overproduction and escalating

competition within the elite class or between established and aspiring elites. Inequality within the elite class grows. As more resources are required to support the growing elite class, ideological and regional ties are used to recruit allies in both the elite and, (when possible) the laboring classes in a competitive struggle to retain access to the resources needed to retain elite status. The immiseration of the laboring class, growing conflict within the elite class, and often the fiscal bankruptcy of the state, eventually destabilize the society, making it prone to revolution, civil war, or invasion. In an instability crisis, the size of the elite and perhaps also the laboring class is reduced. Wages rise and the reduced, and now unified, elite operate a political system that temporarily benefits everyone.

*Historical Dynamics* shows how most verbal theories of the rise and fall of states have failed to get at the mechanisms involved. For example, students of the collapse of societies frequently articulate a model that corresponds to the logistic equation when cast into mathematics. Societies are said to overshoot their carrying capacity and collapse. But the logistic model smoothly reduces growth rates as they approach carrying capacity, preventing overshoots. The SDT theory pictures a system in which elites are mutualists when comparatively rare but predatory in the elite overproduction phase of the cycle. Models of this type have the non-linear feedback necessary to drive cycles and collapses.

The empirical part of *Historical Dynamics* takes advantage of the availability of the relatively few long-term historical data sets for ancient and early modern agrarian states, including China, France, and Russia. In a later book Turchin and Nefedov (2009) analyze several cycles from England, France, Rome, and Russia. This background is important for understanding *Ages of Discord*. The general case for SDT and its tendency to lead to boom and bust dynamics is made in this earlier work. The main question for *Ages of Discord* is whether SDT also operates in the context of a rapidly modernizing, democratic society that is in some was quite different from ancient and early modern agrarian states and empires.

### **Ages of Discord**

The first chapter of *AD* briefly reviews the empirical evidence addressed in Turchin (2003) and Turchin and Nefedov (2009). The second chapter of *AD* shows how to translate these basic ideas into mathematical models. The models have to be rather simple to be tractable, but the approach is also flexible and can be adapted to the peculiarities of a given country's history. For example, in most agrarian societies the state treasury is an important component of the cycle, but in the 19<sup>th</sup> century, the US state was fiscally so unimportant as to not figure in the run-up to the crisis of the Civil War. The 19<sup>th</sup> Century US also had unusually high rates of immigration coupled with mass migration to the frontier, one increasing and one decreasing the population pressure on the laboring class. The 20<sup>th</sup>

century dynamics are complicated by rapid economic growth per capita, automation, and the demographic transition all of which affect the tendency toward the growth and immiseration of the laboring class. This approach reflects the distinction between theories and models. Ecologists are used to the idea that species and ecosystems are diverse and evolving entities and that models of them have to adapt accordingly. *AD* lays out a toolkit of models that can be adapted to the case at hand, the United States 1780–2015.

The second part of the book develops the empirical measures and proxies necessary to put SDT to the test. Some of the time series necessary are reasonably straightforward, such as birth and death rates. Some are only slightly problematic such as immigration rates. Others, such as the real wage, are rather difficult, but economic historians and other experts have gone to a lot of trouble to craft reasonable measures. Estimates for things like average well-being can be estimated by multiple measures including economic (wages) and biological (height) proxies. When these agree, one is obviously on firmer ground than if they disagree. Other important variables are more difficult to quantify. Elites are hard to define in an open society with gradations in wealth, the lack of legal definitions of class, and a lack of good data on incomes before the income tax. Turchin nevertheless makes a good case that a crude index like the size of the largest fortune divided by the average wage tracks with other estimates of income inequality. Intra-elite competition is proxied by the number of lawyers per capita and elite fragmentation by the level of polarization in the House of Representatives. Some of the proxies are quite clever. For example, the proportion of new counties named for local as opposed to national notables increased markedly in the run-up to the Civil War and stayed high through the succeeding age of discord. The frequency of riots and homicides are useful indices of instability. All of the basic variables in SDT can be quantified to a reasonable degree of precision.

The third and fourth parts of *AD* analyze the political history of the US using SDT and the measurements of aggregate well-being, aggregate elite overproduction and competition, and sociopolitical instability. The broad picture is that the US has gone through one full secular cycle from rising concord to discord and back to rising concord (1780–1930) and is in the midst of a second (1930–present). After the Revolution, political instability and its drivers decline until about 1830 when a reversal occurs leading to increased instability leading to an age of discord (1860–1920). Then another reversal occurs leading to an age of concord (1940–1960), followed by another reversal. We presently live in a time of increasing discord, especially in the U.S., as we all know.

The third part of *AD* is an account of the first full secular cycle of US history 1780 to 1930. A major objective of this section is to explain the Antebellum trend

reversal that ended the Era of Good Feelings and led to the Civil War. Importantly, Turchin notes that we cannot expect SDT to predict specific events like the Civil War. SDT is something like the theory of seafloor spreading. From that theory geologists can accurately compute estimates of the amount of stress shifting plates put on faults. Events like earthquakes occur rather unpredictably when local structures locking the fault fail, allowing it to relieve stress in a more or less catastrophic earthquake. Political earthquakes are more likely to occur during periods of high instability but they are inherently difficult to predict with precision.

The Era of Good Feelings around 1830 coincided with external conflicts with Indians on the frontier which tended to unify ethnically and religiously diverse European frontier settlers and competition with the British Empire, including the War of 1812, tended to unify those on the Eastern Seaboard. Elite competition was low as Northern bankers and merchants had an economic symbiosis with the planter class of the South. These trends began to reverse during the presidency of Andrew Jackson. High rates of population growth began to drive down wages. Population density increases were complicated by rising rates immigration but also by the westward expansion of the frontier. The latter two trends approximately balanced each other. Rural population growth ceased on the Eastern Seaboard and an intense period of urbanization began. Stature declined and wages began to fall. Urban elites took advantage of falling wages to increase their consumption. Elite numbers, particularly in the North, grew. Since Independence, the US Government had been dominated by the Southern planter class but the growth of Northern elites increased the competition for elected and appointed government posts. The rise of manufacturing in the North reduced the earlier symbiosis between Southern and Northern elites. Northern elites began to want to use tariffs to protect their nascent industries from competition whereas Southerners favored cheap imports. Northern elites fractured as those from the new manufacturing industries split politically from the merchants and bankers with wealth derived for their old symbiosis with Southern planters. High rates of immigration and the resulting low wages also led to nativist movements in the Anglo-Protestant working class. These tensions led to a proliferation of political parties.

Turchin translates these ideas into a quantitative SDT model of population dynamics, wages, and elite dynamics. These models are then used to create an estimate of mass mobilization potential (roughly the number of poor young urban men who could be recruited by feuding elites to their causes) and an estimate of elite mobilization potential (roughly the number of under-employed elites frustrated in their search for advancement). Both rose dramatically in the Antebellum period. The multiple of these two estimates he calls the political stress index. Observed political instability also rises in the run-up to the Civil War with about a ten year lag relative to the political stress index.

The Civil War itself impoverished Southern elites and enriched Northern ones. After the war, levels of political violence receded some, but assassinations, riots, and violent strikes continued to be common in the late 19<sup>th</sup> and early 20<sup>th</sup> centuries. Political instability reached another peak around 1920. A secondary cycle with a period of about 50 years is frequently observed in SDT analyses.

The Progressive Era ushered in another trend reversal, sparked by a consolidation of elites that reduced intra-elite competition. According to Turchin, U.S. elites came to fear revolutionary violence in the period 1900–1920 and began to advocate reforms that increased the incomes of the working class. During the Progressive Era and New Deal period, a series of reforms, including strong restrictions on immigration, improved the welfare of the working class. The Great Depression trimmed the fortunes of the many in the elite class. All measures of well-being increased from minima around 1910 for the next half-century. After 1920, the elite share of income and wealth declined toward a broad minimum 1950–1980. Levels of political violence declined accordingly.

The fourth part of *AD* considers the beginning of the second secular cycle in US history 1930-present. Another trend reversal occurred around 1960, ushering in the beginning of a new secular cycle. Thus the full first secular cycle in US history had a period of nearly a century and a half. The more recent data are fuller and allow for better estimates and more proxies for key SDT variables. Turchin models how labor oversupply has resulted in stagnation and declines of real wages for the lower half of the income distribution. A growing gap between the supply and demand for labor opened in 1970 due to a combination of increased immigration, greater participation in the paid workforce by women, and by a growing trade imbalance that shifted many manufacturing jobs overseas. The consensus among elites that peace with labor unions and good wages for workers were important objectives broke down. Union busting on the part of business became routine and business lobbying increased. Turchin uses the decline in the real minimum wage as a proxy for this cultural shift in elite attitudes that critics often call "neoliberalism." A three-factor model of wage changes using GDP/capita, labor supply and demand, and the neoliberalism proxy account very well for the flattening of real wages after 1970. Elite overproduction follows the stagnation in the real wage as more people seek elite qualifications. After 1990, overproduction causes elite income relative to wages to decline. Several proxies suggest that intra-elite competition has been rising. For example, the cost of winning election to the House of Representatives has doubled since 1985 while the number of candidates rose from 1233 in 2000 to 1711 in 2012.

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Once again Turchin computes a political stress indicator. It has been rising at an increasing rate since 1970. 2000–2012 it rose from 7 to 37! He suggests that the US is approaching a level of destabilization at which the country will be vulnerable to unpredictable violent upheaval. The unexpected Trump victory in 2016 (after *AD* went to press) is perhaps a foreshock of what might come. Trump's appeal was to former manufacturing workers most badly affected by wage stagnation and to regional elites who perceive themselves as losing to their West Coast and Northeastern competitors. Turchin is not particularly pessimistic, since if we understand the cycles of integration-disintegration that SDT predicts, we are in a position to put in place policies to stop them. A pessimist might worry that there is enough time. Even if SDT is the correct theory, it will take some time to convince a large enough segment of intellectual elite of that and bring SDT into the political discourse.

Turchin's style of analysis in AD is an interesting blend of verbal and quantitative modeling and data analysis. SDT is a theory in Leijonhufyud's terms. It is a verbal sketch of a rather general process. It manifests itself in somewhat different ways in different times and places. Applying it to particular cases depends upon the availability of data and oft-times rough proxy measures are the best that can be had. An immense amount of information is embedded in historians' conventional verbal descriptions of the phenomena they study. But unaided human intuition is poor at understanding complex non-linear systems. Building dynamic models and fitting them to data are prosthetics for the mind in this regard. Still, Turchin rests his case in AD on informal graphical arguments rather than computationally intensive fitting procedures (for which most readers will be grateful!). This approach is justified by the aims of the book. He wants to convince social scientists, and intellectuals more generally, of the value of the general theoretical approach. There are not yet theories to compete with SDT, although we can imagine what some competitors might look like. For example, one might make a model based on Piketty's (2014) idea that the growth of the wealth of the elite class is basically a stable growth path but subject to random shocks rather than a more deterministic cyclical process. When they exist, a more formal gold standard data analysis based on competing models will be warranted.

One impediment to progress on the application of SDT or similar theories to industrial societies is the short history of such societies. The U.S. has undergone only one cycle under industrial conditions. To some extent one cycle is an n of 1 for understanding cycles. For the deeper history of agrarian societies the case for cycles is much stronger because we observe multiple cycles. Quite a number of societies have industrial transformations reaching as far, or a little further, back than the U.S. If most of them have a pattern that is similar to the U.S. that would be some evidence. But many of them suffered the same 20<sup>th</sup> century shocks of the

World Wars and the Great Depression making distinguishing a cycle from a shock hypothesis hard.

### Conclusion

Whether SDT is ultimately the correct explanation for ages of discord in the U.S. and elsewhere is *almost* beside the most important point of *AD*. That point is that human societies are dynamic systems interacting with each other and set within dynamic environmental systems. The natural way to theorize about such systems is to make dynamic models of them using coupled differential or difference equations. That is how it is done in the sciences ranging from physics to ecology and evolution. In the social sciences, the mathematically most sophisticated discipline, economics, traditionally models equilibria, not dynamics, and treat dynamic problems with a comparative statics approach. Hard-argued attempts to convince economists to adopt a dynamic approach to theory (Bowles 2003; Nelson and Winter 1982) has only spawned a small, low-prestige subfield of evolutionary economics (and prestige ranking is especially virulent in economics). There is a thriving field of cultural evolution theory based a modeling strategy borrowed from evolutionary biology (Cavalli-Sforza and Feldman 1981). Quantitatively ambitious social scientists need to learn to use coupled differential equation models and the modern approaches to data analysis that give such models a central place. Often a practical way to up your game is to look for collaborators in the math-heavy sciences that use coupled dynamic models every day. Theorists are often hungry for new empirical phenomena to model. They may also teach courses that can bring a smart graduate student up to speed. You don't need SDT to tell you that there is need for haste in understanding the boom and bust cycles of complex societies!

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