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Kramer, GJ
Chen, L
Fisher, RK
et al.

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Fractional Resonances between Waves and Energetic Particles in Tokamak Plasmas


Princeton Plasma Physics Laboratories, P.O. Box 451, Princeton, New Jersey 08543, USA
University of California-Irvine, Irvine, California 92697, USA
Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou, 310027 China

General Atomics, P.O. Box 85608, San Diego, California 92186, USA

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From numerical simulation and analytical modeling it is shown that fast ions can resonate with plasma waves at fractional values of the particle drift-orbit transit frequency when the plasma wave amplitude is sufficiently large. The fractional resonances, which are caused by a nonlinear interaction between the particle orbit and the wave, give rise to an increased density of resonances in phase space which reduces the threshold for stochastic transport. The effects of the fractional resonances on spatial and energy transport are illustrated for an energetic particle geodesic acoustic mode but they apply equally well to other types of MHD activity.

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Resonant interactions between waves and particles are ubiquitous in the physical world. Resonances between waves and drift-orbit transit frequencies of fast particles in magnetized plasmas play an important role in fast-ion transport and mode excitation. The well known linear resonance condition in tokamaks between a wave with frequency \( \omega \) and the particle poloidal drift-orbit transit frequency, \( \omega_p \), is given by \( \omega = \omega_p + n \omega_\phi \) with \( n \) the toroidal mode number of the wave, \( \omega_\phi \) the toroidal transit frequency, and \( p \) the bounce harmonic number which is an integer in this expression [1–3]. In this Letter, however, we show that resonances between waves and particles can also occur when \( p \) is fractional. This increases the number of available wave-particle resonances significantly, and we explore the effects of fractional resonances on fast-ion transport. Although fractional resonances in nonlinear plasma media have been noticed previously [4,5], this Letter is the first to show that fractional resonances can cause particle losses in a magnetic confinement device. The fractional wave-particle resonances were first found in simulations of particles interacting with an energetic particle geodesic acoustic mode (E-GAM) [6], and they can be explained with a theoretical model presented in this Letter. The fractional resonances are not restricted to E-GAMs but they also appear when other MHD activity is present such as Alfvén eigenmodes at realistic mode amplitudes.

Wave-particle interactions can be studied in detail for the E-GAM because it is a global toroidally symmetric \((n = 0)\) electrostatic mode with mode frequencies in the range of \(10\) to \(30\) kHz. It can be excited to large amplitudes; density fluctuations of \( \delta n/n = 10\% \) have been found experimentally [7] and can expel large numbers of fast ions. In accordance with Ref. [6] we have modeled the E-GAM as a time-varying electrostatic potential [Fig. 1(a)] which is a flux function. From this potential the radial electric field in the plasma was obtained [Fig. 1(d)]. The magnitude of the potential in the plasma center, \(5\) kV, was obtained by comparing the measured density fluctuations with the ones obtained from the displacement induced by the mode [7].

The full orbit following code SPIRAL [8], which calculates single particle orbits in toroidal geometry by solving the Lorentz equations, was used to calculate particle trajectories in the plasma in the presence of the E-GAM. Periodic excursions of the particle orbit and energy were found under the influence of a \(15\) kHz E-GAM with a central electrical potential of \(5\) kV as shown in Fig. 1 for a \(50\) keV deuteron launched on a passing orbit. Initially, the particle gains energy from the mode until its orbit becomes so large that the phase between the mode changes and the particle loses energy to the mode until it returns to its initial position and the whole process repeats itself.

In order to determine the resonances between the poloidal transit frequencies of the particles and a \(15\) kHz E-GAM we have launched a number of deuterons with energies between \(5\) and \(150\) keV with an initial pitch \(v_y/v\) of \(0.5\) at a major radius of \(2.02\) m at the midplane. The selected energy range gives poloidal transit frequencies between \(4\) and \(40\) kHz. For each particle the maximum and minimum energy excursion was determined and plotted against the initial transit frequency for a number of values of the central electrical potential as shown in Fig. 2.

At the lowest mode amplitude \(V_{\text{central}} = 0.005\) kV which is \(0.1\%\) of typical E-GAM amplitudes only the linear \(p = 1, 2, \) and \(3\) resonances are visible. When the mode amplitude is increased to \(V_{\text{central}} = 0.5\) kV several fractional resonances appear between the \(p = 1\) and \(2\) resonances while at \(30\) kHz the \(1/2\) resonance appears. The increased mode amplitude also broadens the resonances. This can be explained by the large excursions that the particle makes at those mode amplitudes. The time-averaged bounce

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period over one energy excursion cycle is now resonant with the mode frequency. At a central potential of 1.25 kV the threshold for resonance overlap between the linear and fractional resonances is passed as can be inferred from the spikes appearing in the energy traces near the linear resonances. Finally, at $V_{\text{central}} = 5$ kV, the experimental value of the central potential, the orbits in the frequency interval up to the $p = 1$ resonance have become fully stochastic due to resonance overlap while at higher frequencies additional new subharmonic resonances have appeared.

The fractional resonances found in the simulations can be understood by writing down the Hamiltonian, $H = e + e\delta \Phi$, of a charged particle with charge $e$, and kinetic energy, $e$, in a time-varying electrical potential: $\delta \Phi = \Phi_0 \sin(\omega t - k_r r)$ with $\omega$ the mode frequency and $k_r$ its wave number. Resonances correspond to secular behavior and are obtained when $\langle dH/dt \rangle \neq 0$ with $\langle \cdots \rangle = \frac{1}{T} \int_0^T \cdots dt$ and $T$ the fast time scale. The unperturbed particle position is given by $r = r_0 + \rho_0$ with $r_0$ the drift center and $\rho_0$ the drift-orbit radius, and its radial drift velocity is $\dot{\rho}_0 = v_d r = v_d \sin(\theta)$. The angle coordinate, $\theta$, and particle transit frequency, $\dot{\theta} = \omega_\theta$, are both functions of particle energy, pitch, and drift center. The mode induces a perturbation to the energy, $e = e_0 + \delta e$, and drift center, $r_c = r_0 + \delta r_c$, whereby the effects in energy dominate over the change in drift-orbit center. The change of energy in time, given by $\delta \dot{e} = v_d e \delta E$ ($\delta E$ is the electric field associated with the potential $\delta \Phi$), can be cast

$$\delta \dot{e} = k_r v_d e \Phi_0 \sin(\theta_0 \pm (\omega t - k_r r)),$$

where the $\pm$ sign accounts for the fact that the mode forms beat waves with the particle transit frequency at the sum and difference frequency. The change of energy over time

$$\delta E = \frac{k_r v_d e \Phi_0}{2} \left[ \cos(\theta_0 \pm (\omega t - k_r r)) - \cos(\pm k_r r) \right]$$

can be obtained from a simple integration over time and by using the leading term of the particle transit frequency: $\theta_0 = \omega_\theta$. The next step is to expand the perturbed particle transit frequency as

$$\dot{\theta} = \omega_\theta + (d_e \omega_\theta) \delta e + (d_r \omega_\theta) \delta r$$
(where \(d_y \equiv dy/dx\)). Integrating this expression with time gives \(\theta_0 = \omega_0 t\) for the first term and \(\delta \theta = d_y \omega_0 \int \delta \text{d}t\) for the second term while the third term is omitted because \(\delta r\) is small. In a similar manner the change in the drift-orbit radius is calculated as \(r = \frac{\omega_0}{\omega_0} - \frac{v_0}{\omega_0} \cdot \frac{2}{(\omega_0 - \omega)^2} \int \delta \text{d}t\), where \(\omega_0 = v_0 \sin(\theta_0)\) and \(\delta \theta = \int [d_y, v_0] \delta e \sin(\theta_0) + \omega_0 \cos(\theta_0) \delta \theta\). Integrating over time gives \(\rho_0 = -v_0 \cos(\theta_0) / \omega_0\), while \(\delta \rho\) is evaluated as

\[
\delta \rho = \sum_{i=2}^{7} z_i \cos(\alpha_i),
\]

where \(z_i\) and \(\alpha_i\) are given in Table I. Wave-particle resonances are now obtained from \((dH/dt) = \omega e\Phi_0 \text{Re} \left(e^{i(\omega t - k_r r_\perp + \rho + \delta \rho)}\right) \neq 0\). The terms with \(\rho_0\) and \(\delta \rho\) contain cosine functions so we can use the Jacobi-Anger identity to express \((dH/dt)\) as the product of Bessel functions (with \(\psi = \omega t - k_r (r_\perp + \rho_0 + \delta \rho)\)).

The fractional resonances increase the resonance density in phase space as shown in Fig. 3. This can lead to below the fundamental cyclotron resonance while in the fractional resonances the subharmonic resonances appear above the fundamental resonance [Fig. 2(a)].

It is interesting to note the analogy between the fractional resonances and resonant heating below the cyclotron frequency as described in Refs. [9, 10]. In the cyclotron case the heating, \(\delta W_c = v_1 e \delta E\), is caused by the product of the perpendicular velocity, \(v_\perp\), and perpendicular electric field, \(\delta E\), while the cyclotron frequency, \(\omega_c\), is constant. In the fractional resonances case the heating is given by \(\delta \tilde{e} = v_\perp e \delta E\), where \(v_\perp\) plays the same role as \(v_1\) in the cyclotron case, and the electric field, \(\delta E\), is equivalent to \(\delta E\). The particle transit frequency, \(\omega_T\), plays a similar role as \(\omega_c\), but \(\omega_T\) depends on the drift velocity. Another difference between the cyclotron and fractional resonances case is that the subharmonic resonances appear at frequencies

![FIG. 3 (color online). Energy-pitch phase space with linear [solid (green online) line] and a small number of fractional resonances [dashed (red online) line] shown. The locations of the three ensembles that were used to investigate the effects of the fundamental resonance, the half resonance, and nonresonance on particle transport are indicated with the black squares.](image)
increased transport in two ways: (i) stochastic transport can occur when the fractional and linear resonances overlap, while (ii) resonant transport can occur when the fractional resonances are well separated.

Stochastic transport occurs at sufficiently high mode amplitudes as can be seen in Fig. 2(d) where a stochastic region is formed below 18 kHz in the region where the linear resonances were located. The appearance of the fractional resonances between the linear resonances decreases the threshold for stochastic orbits. Another important observation that can be made from Fig. 2(d) is that when the region has become stochastic the particles gain more energy from the mode than they lose to the mode which leads to a damping of the mode. In carefully designed experiments with cocurrent and countercurrent beam injection this damping may be observed experimentally.

Resonant transport was studied in simulations with three populations of 10 000 particles each with their guiding centers at \( R, Z = (2.02, 0.0) \) m, an energy of 50 keV, and pitches of 0.40, 0.73, and 0.88, respectively, so that the particles are resonant with the fundamental resonance, resonant with the half resonance, and nonresonant, respectively, as indicated in Fig. 3 by the black squares. After the particles were distributed uniformly over the equilibrium drift orbit, the E-GAM was switched on for 0.27 ms and the spread in guiding centers relative to the unperturbed drift orbit and the spread in energy were calculated as a function of mode amplitude (Fig. 4).

From Fig. 4 it can be seen that the particles that were launched at the fundamental resonance are affected by very low wave amplitudes, which spread them in space and energy. The particle population at the half resonance is only affected when the mode amplitude reaches a threshold of 1 kV which is 20% of the amplitude seen in experiments. The nonresonant population is hardly affected by the mode. These results show clearly that the inclusion of the fractional resonances enhances the fast-ion transport and a larger part of phase space is affected by the mode than only the regions around the linear resonances.

Preliminary evidence of fast-ion transport at fractional harmonics of the mode frequency is observed during strong E-GAM activity in the DIII-D tokamak where the E-GAM was strongly excited during the current ramp-up phase with equal amounts (2.2 MW) of cocurrent and countercurrent neutral beam injection (NBI) similar to the experiments reported in Ref. [7] whereby the E-GAM is strongly driven by the countercurrent NBI (particle energy: 81 keV) while the cocurrent beam (particle energy: 76 keV) is injected in a region of phase space where the \( p = 1/2 \) resonance is residing (Fig. 3). In Fig. 5 the spectrum of E-GAM oscillations as measured with a Mirnov coil is compared with the spectrum of losses to a fast-ion detector (FILD) [11]. A peak in the loss spectrum occurs at the half harmonic that does not appear in the instabilities spectrum which is consistent.

FIG. 4. Standard deviation of the (a) radial and (b) energy excursion of an ensemble of ions as a function of E-GAM mode amplitude for the fundamental resonance, the half resonance, and nonresonant particles.

FIG. 5 (color online). Magnetic [dashed (black) line] and FILD [solid (red) line] spectrum. Losses at half the E-GAM frequency are only observed with FILD.

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with the theoretical prediction that $\rho$ should contain fractional harmonic oscillations at large mode amplitude.

From numerical simulation and analytical modeling it was found that fast ions can resonate with plasma waves at fractional values of the particle drift-orbit transit frequency when the plasma wave amplitude is sufficiently large. The fractional resonances, which are caused by a nonlinear interaction between the particle orbit and the wave, give rise to an increased density of resonances in phase space which reduces the threshold for stochastic fast-ion transport.

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