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Authors
Hoang, T.F.
Cork, B.
Crawford, H.J.

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T.F. Hoang, B. Cork, and H.J. Crawford

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Effective Radii from High Energy Nuclear Reactions

T. F. Hoang

1749 Oxford Street
Berkeley, California 94709

Bruce Cork

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

H. J. Crawford

Space Sciences Laboratory
Berkeley, California 94720

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An effective nuclear radius is introduced to account for opacity according to the optical model. For hadron-nucleus (h-A) reactions, it assumes \( R = r(A^{1/3} - a/A^{1/3}) \) where \( r \) is the radius parameter and \( a \approx (\lambda/r)^2 \), \( \lambda \) being the absorption mfp. A similar relation exists for heavy-ion (HI) reactions with \( A^{1/3} = A_1^{1/3} + A_2^{1/3} \), i.e. add two radii without overlapping. Properties of \( r \) and \( \lambda \) are investigated using high-energy h-A and HI data; the experimental ratios of \( <r_p>/<r_\pi> \) and \( <\lambda_\pi>/<\lambda_p> \) show properties of quark-counting, with \( <\lambda_p> = 1/m_\pi \).
1. Introduction

It is well known that geometrical models such as the Glauber theory [1] and the Chou-Yang model [2] are adequate to describe high energy elastic scatterings of hadrons and nuclei. In this paper we investigate nuclear reaction cross-sections using the optical model of Fernbach, Serber and Taylor [3]. Our aim is to study the characteristic parameters of the model and to get an insight into some properties of nuclear matter at high energies.

We shall be concerned with both hadron-nucleus (h-A) and heavy-ion (HI) reactions, Sec. 2. We find that the hadron-nucleus cross-section Eq. (4-a) can be expressed in a simple form by means of an effective radius $R$, Eq. (8), which differs from the usual scaling law $R = rA^{1/3}$ by a term $-\lambda^2/4rA^{1/3}$ due to absorption specified by the mean-free-path (mfp) $\lambda$. This term is negligible, so that $R = rA^{1/3}$ for large $A$ and small $\lambda$, i.e. black nuclei as in $p$-A reactions, Sec. 4. A validity test for $R$ will be made, Sec. 3, using the photoproduction data of DESY-MIT [4].

The properties of $r$ and $\lambda$ are investigated using the nuclear data of $p$, $\pi^\pm$, $K^\pm$ and $\bar{p}$ reactions from Serpukhov [5], Sec. 4. We shall see that the ratios of $r$ and $\lambda$ for $p$ and $\pi$-reactions unravel interesting properties of quark contents, Sec. 6.

The HI cross-sections, Eq. (4-b), is similar to that of p-A, Eq. (4-a). In this case, $R$ is given by adding two radii without overlapping, Sec. 3, in contrast to the Bradt-Peters formula. An analysis of the LBL data [6] indicates that $r$ is comparable to that of p-A, but the mfp is different, Sec. 5. For like-nuclei, it is possible to relate the cross-section to that of p-A by a simple expression, Eq. (12). This relationship will be used to further test our approach, Sec. 7.

Remarks are made on the applications of Eqs. (4-a) and (4-b).
2. Derivation of Nuclear Cross-Sections

We begin with the hadron-nucleus reaction, denoted by $\sigma_h(A)$, $A$ being the mass number of the target nucleus of radius $R$. According to the optical model [3], it is given by the absorption cross section as follows

$$\sigma_{\text{abs}} = \pi R^2 \int_0^R \left(1 - e^{-2x/\lambda} \right) \frac{db^2}{R^2}$$

(1-a)

where $b$ is the impact parameter, $2x = 2\sqrt{R^2 - b^2}$ the traversal and $\lambda$ the mfp of absorption, see Fig. 1a. We recall the result

$$\sigma_h(A) = \pi R^2 \left[ 1 - 2\left( 1 - (1 + 2R/\lambda)e^{-2R/\lambda} \right) (\lambda/2R)^2 \right]$$

(2-a)

no distinction being made between the proton and the neutron of the nucleus, for simplicity.

For the heavy-ion (HI) reaction, we proceed in the same way. First, consider the case of like-nuclei and let $R_1 = R_2 = R$ be the radius and $a$ the distance between the centers, Fig. 1b; then

$$\sigma = 2\pi R^2 \int_0^a \frac{da^2}{2R} \int_0^{\sqrt{a^2}} \left(1 - e^{-2x/\lambda} \right) \frac{db^2}{R^2}$$

(1-b)

An integration yields

$$\sigma = 2\pi \left[ 2R^2 - 2\lambda^2 + \frac{\lambda^4}{R^2} \{3 - (3 + 6R/\lambda + 4R^2/\lambda^2)e^{-2x/\lambda}\} \right]$$(2-b)

For unlike-nuclei, $R_1 \neq R_2$, we assume the following substitution

$$2R = R_1 + R_2$$

(3)

We may use the large $A$ approximation: $2R/\lambda \gg 1$ to simplify the above result and assume the scaling law

$$R = rA^{1/3}$$
We find from (2-a) the hadron-nucleus cross-section

\[ \sigma_h(A) = \pi r^2 \left( \frac{A^{1/3} - a}{A^{1/3}} \right)^2 \]  

(4-a)

where

\[ a \equiv (\lambda/2r)^2 \]  

(5-a)

Likewise, we obtain for the HI cross-section from (2-b)

\[ \sigma(A_1, A_2) = \pi r^2 \left( \frac{A_1^{1/3} + A_2^{1/3} - c}{A_1^{1/3} + A_2^{1/3}} \right)^2 \]  

(4-b)

with

\[ c \equiv 2(\lambda/r)^2 \]  

(5-b)

Note that (4-b) resembles the well-known Bradt-Peters formula [8] with the "overlapping parameter"

\[ b = c/(A_1^{1/3} + A_2^{1/3}) \]  

(6)

decreasing with nuclear size as has been observed experimentally [9] and that it has been proposed previously [10] but remained unnoticed.

In the same approximation, these cross-sections may be written as

\[ \sigma_h(R) = \pi R^2 e^{-2(\lambda/2R)^2} \]  

(7-a)

\[ \sigma(R_1, R_2) = \pi (R_1 + R_2)^2 \cdot e^{-4\lambda^2/(R_1 + R_2)^2} \]  

(7-b)

Note that these relations fall less rapidly than Eqs. (4) and fit the light nucleus better.
3. The Effective Radius

The physical meaning of the hadron-nucleus cross-section [4a] is more apparent if we introduce an effective radius

\[ R = r(A^{1/3} - a/A^{1/3}) \]  

(8)

then \( \sigma_h(A) = \pi R^2 \). Note that \( R = rA^{1/3} \) at large A limit or \( a = 0 \) i.e. \( \lambda = 0 \).

Likewise, the HI cross-sections are obtained by replacing

\[ A^{1/3} \rightarrow A_1^{1/3} + A_2^{1/3} \ , \ a \rightarrow c \]

i.e. adding two radii without overlapping.

We note that the A dependence of \( R \) turns out to be similar to that of the rms radius of the Fermi-type distribution of nuclear matter

\[ \rho(r) = \frac{\rho_0}{1 + e^{(r-R)/t}} \]

where \( R = r_0A^{1/3} \) is the half-way radius, \( r_0 \) being the rms radius parameter, and \( t \) the surface thickness:

\[ \sqrt{\langle r^2 \rangle} = r_0A^{1/3} - \frac{\pi t^2}{3r_0A^{1/3}} \]  

(9)

to the order \( \mathcal{O}(1/A^{5/3}) \) [11].

For a test of (8), we use the nuclear radii measured by DESY-MIT [4] using photoproduction of \( p^0 \) at 7.5 GeV/c

\[ \gamma + A \rightarrow p^0 + A \]

\[ \rightarrow \pi^+ + \pi^- \]

Their measurements are plotted against \( A^{1/3} \) in Fig. 2, errors \( \pm 2\% \) for \( A > 27 \) are omitted for simplicity; the line shows the fit with (8). The parameters are
\[ r = 1.14 \pm 0.01 \text{ fm} \]
\[ a = 0.22 \pm 0.36 \text{ } \]

Our estimate of \( r \) agrees with their value \( 1.12 \pm 0.02 \text{ fm} \). Whereas the surface thickness \( t \) estimated according to (8) and (9) is 0.28 fm, somehow smaller than 0.56 fm used in their analysis [4].

From (5-a) we find for the mfp of \( \rho^0 \)

\[ \lambda_{\rho} = 1.07 \pm 0.16 \text{ fm} \]

As \( \rho^0 \rightarrow \pi^+ + \pi^- \), we deduce for that of \( \pi \):

\[ \lambda_{\pi} = 2.14 \pm 0.32 \text{ fm} \]

in excellent agreement with \( 2.18 \pm 0.24 \text{ fm} \) estimated from \( \pi \)-A reaction cross-sections at 3.9 GeV of a previous analysis [12], and also with the Serpukhov data to be discussed in the next section.

4. Hadron-Nucleus Reactions

We now proceed to investigate the properties of hadron-nucleus reactions using the Serpukhov data of \( p, \pi^\pm, K^\pm, \) and \( \bar{p} \) at \( P_{\text{lab}} = 6.6 \text{ to } 60 \text{ GeV/c} \) [5]. These data, except for \( \bar{p} \) to be discussed later, are analyzed with Eq. (4-a). We then compute the corresponding mfp by (5-a). The parameters and fitting errors are summarized in Table I.

In Fig. 3 is shown a typical fit for \( p\)-A cross-section at 20 GeV/c. In general the fits are good, except that for light nuclei, there is a systematic deviation as is seen from the figure.

Consider the radius parameter \( r \). We note that it is well defined, with rather small errors, that it is practically independent of energy, and that it is different for \( p\)-A and \( \pi \) or \( K\)-A reactions.
As regards the mfp $\lambda$, the errors are rather large. That the measured cross-sections, according to [5], are energy independent implies that the same holds for estimates $r$ and $\lambda$. We now investigate the dependence of $\lambda$ on $A$, and find from our analysis of the $p$-$A$ data

$$\frac{d\lambda}{dR} = 0.053 \pm 0.024$$

consistent with zero, indicating an independence of $A$.

Therefore, for a given projectile, we may take the averages of $r$ and of $\lambda$. We get for $p$-$A$ reactions

$$\langle r_p \rangle = 1.30 \pm 0.01, \langle \lambda_p \rangle = 1.44 \pm 0.23$$

in fm, and likewise for $\pi$-$A$ reactions

$$\langle r_\pi \rangle = 1.23 \pm 0.03, \langle \lambda_\pi \rangle = 2.14 \pm 0.21.$$ 

We find $\langle r_p \rangle > \langle r_\pi \rangle$ and $\langle \lambda_p \rangle < \langle \lambda_\pi \rangle$; likewise $\langle r_p \rangle > \langle r_K \rangle$ and $\langle \lambda_p \rangle < \langle \lambda_K \rangle$.

Note that $\langle \lambda_p \rangle = \hbar/m \omega_c$. Further discussions will be resumed in Sec. 6.

Turn now to the $\bar{p}$-reactions. We find that here the least-squares fits with Eq. (4-a) yield a wrong sign to the parameter $a$. This in turn leads to an imaginary $\lambda$ according to Eq. (5); whereas $a$ is in fact small compared to $r$. We have investigated this point further by fitting the data with the exact absorption cross-section (2-a). We found the same $r$ and $\lambda$ consistent with zero; the values thus obtained are listed in Table I. Thus we tentatively set $a = 0$ and assume

$$\sigma_\bar{p} = \sigma_o A^{2/3} \quad (10)$$

with $\sigma_o = \pi r_p^2$. This $A$-dependence is different from those for $p$, $\pi$, and
K-nucleus reactions and agrees with the BNL results [13]. The estimates of \( r_p \) are listed in Table I. Note that

\[
\langle r_p \rangle = 1.33 \pm 0.03 \text{ fm}
\]
is comparable to \( \langle r_p \rangle \) and that the nucleus is black in this case. This fit for \( P_{\text{lab}} = 13.3 \text{ GeV/c} \) is shown in Fig. 3; the log plot is linear in contrast with that of p-A, which is slightly curved downward for small \( A \).

5. Heavy-Ion Reactions

As for heavy-ion (HI) reactions, we shall use the LBL experiment [6] of \( ^{56}\text{Fe} \) at 1.88 GeV/N on targets from H to U. Their data for \( \Delta Z > 1 \) are reproduced in Fig. 4. The solid line is our fit with Eq. (4-b), the parameters are

\[
\begin{align*}
 r &= 1.31 \pm 0.01 \text{ fm} \\
 c &= 4.45 \pm 0.15
\end{align*}
\]

Note the slight curvature compared to the straight line of their fit using the Bradt-Peters formula, see Sec. 2. As the mid-range of \( A_1^{1/3} + A_2^{1/3} \) of their data is 7.4, the overlapping parameter \( b \) is \(-0.70 \pm 0.02\) according to (6), compared to \( 0.83 \pm 0.12 \) of their fit, their \( r \) being \( 1.35 \pm 0.02 \text{ fm} \). Their parameters were used for anomalon investigation [14].

Our \( r \) is comparable to that of p-A in Table I; whereas the mfp deduced from (5-b) is \( 1.96 \pm 0.34 \text{ fm} \), about \( \sim 20\% \) longer than that of p-A as obtained in the previous section. This difference may be due to the fact that the mechanism of \( Z \)-changing HI reactions is more complicated than for p-A reactions, see Sec. 7 for further discussion on this point.
Finally, it should be noted that Eq. (4-b) should not be applied to the H-target by setting zero or a small value for the corresponding A-value in the formula, as is used in the literature. Indeed, we see that in doing so, Eq. (4-b) is reduced to the same form as (4-a) for p-A reactions. But, the parameter c which is \( a_p \) remains unaffected. Consequently, the cross-section thus computed is underestimated.

To illustrate this point which is sometimes overlooked, consider the mfp of \(^{56}\text{Fe}\) in lucite \( C_5H_8O_2 \). Assuming a density \( d = 1.18 \ \text{g/cm}^3 \) and using Eq. (4-b) for all elements, we find a mfp 7.85 cm compared to the experimental value 8.14 \( \pm \) 0.04 cm [14]. Whereas the correct estimate using (4-a) for H yields 8.11 cm in excellent agreement.

6. Properties of \( r \) and \( \lambda \)

We have investigated various nuclear reactions in terms of Eqs. (4-a) and (4-b) with two parameters \( r \) and \( \lambda \). We find that both \( r \) and \( \lambda \) depend on the nature of the projectile. It follows that the nuclear size is not an intrinsic property, but depends on the nature of the probe.

For p-A reactions, we get \( \langle r_p \rangle = 1.30 \pm 0.01 \ \text{fm} \). Recall that the rms radius of the Fermi distribution is \( \sqrt{3/5} \) times that of the uniform density distribution. Thus our estimate agrees with \( \sim 1 \ \text{fm} \) found by Glauber and Mitthial [1b].

Referring to Table I, we note that the averages of mfp for \( \pi^+ \) and \( \pi^- \) reactions are practically the same; being 2.19 \( \pm \) 0.35 and 2.09 \( \pm \) 0.26 fm, respectively. This indicates that the difference between the density distributions of p and n inside a nucleus is rather small. Note that these mfp agree with 2.05 fm estimated by Chou and Yang [2b].
However, we expect that the parameters $r$ and $\lambda$ are different for $K^+$ and $K^-$, since their absorption cross-sections are different. It would be interesting to investigate this point.

Consider now the averages of $r$ and $\lambda$ for $p$ and $\pi$-$A$ reactions discussed in Sec. 4. We may estimate the average cross-section per nucleon, assuming the average nucleon density to be $\rho_0 = 0.17 \text{ N/fm}^3$. We find

$$\bar{\sigma}_{pN} = 40.8 \pm 6.5 \text{ mb} \quad , \quad \bar{\sigma}_{\pi N} = 26.8 \pm 3.4 \text{ mb}$$

in agreement with well known free nucleon cross-sections.

As for the ratio of average $r$, we get

$$\frac{<r_p>}{<r_\pi>} = 1.07 \pm 0.03$$

comparable to that of rms radius of $p$ and $\pi$ estimated by Chou [16a] from elastic $pp$ and $\pi p$ scatterings, i.e.

$$\frac{<r_p^2>^{1/2}}{<r_\pi^2>^{1/2}} = 1.12 \pm 0.03 \quad ;$$

whereas the Yale-FNL experiment [16b] yields $r_p/r_\pi = 1.08$.

We note that according to the geometrical picture, this ratio can be expressed in terms of numbers of constituent quarks as follows

$$\left(\frac{3}{2}\right)^{1/3} = 1.15 \quad .$$

In this regard, we note that this property holds also for the mfp. Indeed, if $\lambda_q$ is the quark mfp, then

$$\frac{1}{\lambda_p} = \frac{3}{\lambda_q} \quad , \quad \frac{1}{\lambda_\pi} = \frac{2}{\lambda_q} \quad .$$

We therefore expect
Experimentally, we find

\[ \frac{\langle \lambda_\pi \rangle}{\langle \lambda_p \rangle} = 1.49 \pm 0.28 \]

in agreement with quark-counting. In passing, we note that from the parameters of the Fermi distribution of the BNL data of \( K^\pm \) reactions on C and Cu [17] we estimate \( \lambda_K \approx 2.14 \text{ fm} \) in agreement with those in Table I.

7. Conclusion

In summary, the heavy-ion (HI) cross-section derived from the optical model, Eq. (4-b), is of the Bradt-Peters type and is similar to that of the hadron-nucleus (hA), Eq. (4-a). This property leads us to introduce an effective radius for h-A reactions:

\[ R = r(A^{1/3} - a/A^{1/3}) \]  

the parameters \( r \) and \( a = (\lambda/2r)^2 \) being characteristic of the incident hadron. As \( \lambda_p \) is less than \( \lambda_\pi \) and \( \lambda_K \), the nucleus appears to be more transparent to meson than proton induced reactions; whereas all h-A cross-sections follow the same \( R^2 \) dependence.

As for the HI reactions, we note that in the case of like-nuclei, the cross-section \( \sigma(A,A) \) can be expressed in terms of \( \sigma_p(A) \) of p-nucleus of the same A. Indeed, we find according to Eqs. (4-a) and (4-b)

\[ \frac{\sigma(A,A)}{\sigma_p(A)} = \frac{r}{r_p} \left( 4 - \frac{c - 4a_p}{A^{2/3} - a_p} \right) . \]  

(12)
It is interesting to note that this ratio is not sensitive to the large $A$ approximation, i.e. $2R/\lambda \gg 1$ as is required by Eqs. (4) and Eqs. (5). Indeed, consider the case of $p-\alpha$ and $\alpha-\alpha$. With our parameters for $p-A$ and HI reactions, we get from (12): $\sigma(a,a)/\sigma_p(a) = 2.56$ in agreement with the experimental ratios $2.48 \pm 0.04$ of another LBL experiment at 2.1 GeV/N [7] and $2.73 \pm 0.46$ of the recent ISR experiment of colliding beams of 31.5 GeV protons and 63 GeV $\alpha$'s [18]. Recall that these $\sigma(a,a)$ measurements refer to total rather than $Z$-changing reaction cross-sections.

Further investigations of HI cross-sections with BEVALAC data are in progress by one of the authors (HC) and will be reported elsewhere.

Acknowledgement

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References

Table I—Parameters of hadron-nucleus reactions
Serpukhov data Ref. [5]

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<th>mfp $\lambda$(fm)</th>
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<td>$1.28 \pm 0.01$</td>
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Figure Captions

1. Reaction cross-sections according to the optical model: (a) hadron-nucleus and (b) two like-nuclei.

2. Nuclear radii from photoproduction of $p^0$, measured by the DESY-MIT Collaboration, Ref. [4]. The curve represents the fit with the effective radius $\mathcal{R} = r(A^{1/3} - a/A^{1/3})$, $r = 1.14 \pm 0.01$ fm and $a = 0.22 \pm 0.36$, see Sec. 3.

3. Cross-sections of $p$–A and $\bar{p}$–A from Serpukhov, Ref. [5]. The lines are fits using Eq. (4-a), the parameters are listed in Table I.

4. Heavy-ion cross sections for $^{56}$Fe at 1.88 GeV/N, LBL $\Delta Z > 1$ data, Ref. [6]. The curve is the fit with Eq. (4-b). Note its non-linearity in contrast to the Bradt-Peters formula, see text.
Figure 1
Figure 2

DESY-MIT

$\gamma - A \ 7.5\text{GeV/c}$

$A^{1/3}$

$R(\text{fm})$

XBL 847-8528

Figure 2
Figure 3

Serpukhov
\( \bar{p} - A \) 13.3 GeV/c

\( p - A \) 20 GeV/c

\( \sigma_{\bar{p}A} (\text{fm}^2) \)

\( \sigma_{pA} (\text{fm}^2) \)

A
Figure 4
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