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Microscopic and Macroscopic Calculations of One-body Damping in Fission*

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ABSTRACT

A microscopic method for calculating the damping of collective motion into intrinsic states is described. For a particular example, the excitation of neutron levels in the fission of $^{236}$U, the results of this method are compared with a simple semi-classical approach based on the kinetic theory of gases.

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As a part of a continuing study [1-3] of the damping of large-scale collective motion into intrinsic excitation, we have solved the time-dependent Schrödinger equation for a sequence of shapes corresponding to the fission of $^{236}\text{U}$. These calculations, which are described in detail in ref. 3, were motivated by the fact that classical hydrodynamical calculations of the fission process [4] seem to be incompatible with the strong damping observed in heavy-ion collisions [5]. We wanted to compare the actual flow of energy into single-particle states with that calculated using a classical viscosity. We find, as have others [6,7], that the energy absorbed is extremely large for a classically determined sequence of shapes that is consistent with the experimentally observed asymptotic kinetic energy release.

The microscopic calculations that have been performed so far are not self-consistent, and two major defects remain even when the viscosity has been varied in the hydrodynamical calculations until the total energy absorbed is the same as that found in a microscopic calculation for the same sequence of shapes: 1) The calculations no longer agree with experiment, and 2) the energy absorption profiles differ along the trajectory. These calculations [6] served to dramatize the fact that the concept of hydrodynamic viscosity poorly represents the damping mechanism that creates the internal excitation in the single particle calculations. Hydrodynamic viscosity is basically a two-body phenomenon involving momentum transfer across viscosity shears due to collisions. In contrast to this the independent particle systems (with or without residual
interactions) seem to become excited mainly from the motion of the collective potential well in which they are contained.

In order to obtain results from the hydrodynamical calculations that would be more nearly comparable to the microscopic calculations it was necessary to find a classical damping mechanism analogous to that seen in the microscopic calculations. Such a mechanism is obtained [8,9] from classical kinetic theory by carrying the system energy expression, for volume conserving systems, to second order in the appropriate expansion parameter (the ratio of the wall velocity to the average velocity of the particles). We have used this method to calculate the damping to be expected from such a mechanism and have compared it with the microscopic calculations performed earlier.

The Microscopic Calculation

In order to study the internal excitation energy associated with the collective motion for a fissioning nucleus we solved the time-dependent Schrödinger equation for the neutron levels in $^{236}$U. The sequence of shapes (provided by Nix [4]) corresponds to a viscous classical hydrodynamical calculation for the saddle to scission motion. A viscosity of 0.02 terapoise was used, which is the value that seems to be indicated by the experimental results. We also considered another sequence of shapes derived from these by introducing a mass asymmetry that increases linearly from the saddle to a value of 1.4 at scission.

In all cases the time-dependent single-particle potential well was generated using the procedure of ref. 10 to insure that the normal diffuseness is approximately constant over the entire surface.
A sequence of these potentials is shown in fig. 1. No residual interactions or spin-orbit terms were included.

The time-dependent Schrödinger equation was solved in a fixed deformed harmonic oscillator basis with a deformation corresponding to a shape intermediate along the trajectory. Accuracy checks showed this method to be quite adequate both for the original trajectory and the mass asymmetric one. The time-dependent equations were solved by a predictor-corrector method [11], the first few points having been determined by a Runge-Kutta procedure.

Figure 2 shows the results that were obtained. At the bottom of this figure a dashed line labeled $E_{\text{coll}}^0$ indicates how the hydrodynamical collective kinetic energy increases during the motion in the calculation that was used to generate the shape sequence [4]. The solid line labeled $E_t^a$ is the sum of this kinetic energy and the dissipated energy. (Both curves are scaled down by a factor of $144/236$ so they can more easily be compared with the microscopic calculation which was done for the neutrons alone.)

For comparison with the collective kinetic energy found in the classical hydrodynamic calculation we calculated the microscopic quantity $E_{\text{coll}}$ [12], where

$$E_{\text{coll}}(t) = \int \frac{1}{2} \rho \vec{v}^2 \, dr.$$  \hspace{1cm} (1)

The local flow velocity is calculated from the expression $\vec{v} = \vec{j}/\rho$. Where $\rho$ and $\vec{j}$ are, respectively, the quantum mechanical density and current defined by the expressions,
\[ \rho(\mathbf{r},t) = m \sum_i \psi_i^* \psi_i, \]  

and

\[ \mathbf{j}(\mathbf{r},t) = \hbar \sum_i \text{Im}(\psi_i^* \cdot \nabla \psi_i), \]  

where, the wave functions \( \psi_i(\mathbf{r},t) \) are solutions to the time-dependent Schrödinger equation in the moving potential well. This definition has considerable intuitive appeal since it clearly applies to simple translations and would seem to give a reasonable result even for fairly turbulent flow.

The comparison of the dissipation energy in the two cases is not so straightforward. Because of the axial symmetry of the system the magnetic quantum number \( m \) of a given level must remain the same in the course of the motion. The same is true for the parity \( \pi_z \) in the case of reflection symmetric shapes. Consequently, even if the motion of the potential is extremely slow (adiabatic), the system may end up in an excited state since there is no way for a particle to change to an empty level that moves down through the Fermi surface if its quantum numbers \( m \) and \( \pi_z \) are different from the levels being crossed.

In our calculations a substantial part of the apparent excitation energy is of this type. This part of the energy \( E_s \) (where the subscript \( s \) indicates that it has its origin in symmetry effects) is simply the difference between the ground state of the system \( E_0 \) (filling the lowest levels) and the "adiabatic" energy \( E_a \) (where the quantum numbers appropriate to the system are conserved).
\[ E_s = E_a - E_0 \]  
\[ E_0 = \sum_{i=1}^{N} E_i(\beta), \text{ lowest N levels,} \]  
\[ E_a = \sum_{i=1}^{N} E_i(\beta), \text{ lowest N levels having the appropriate quantum numbers.} \]

In these expressions \( \beta \) is a one-dimensional deformation parameter measuring the distance along the dynamical path.

The total excitation energy \( E_t \) is defined as the difference between the total energy \( E^* \) of the system described by the time-dependent Schrödinger equation and the ground state at that same value of \( \beta \), where

\[ E_t = E^* - E_0, \]

\[ E^* = \sum_{i=1}^{N} E_i[\beta(t)], \]

and the \( E_i[\beta(t)] \) are the time-dependent energy levels. Of course, \( E_t \) contains \( E_s \) (the apparent excitation arising purely from symmetry), as well as \( E_{\text{coll}} \). If the motion is slow this is a serious defect that makes the calculation meaningless. If residual interactions were taken into account, the system would always remain in its ground state for adiabatic changes in the potential well. The defect does not seem to be so serious in the case of rapid motion since the levels in a rapidly changing potential are more likely to retain their nodal structure (keeping \( m \) and \( n_z \) approximately the same) than to rearrange in order to follow a new level coming in from above.
In fig. 2 both the total excitation energy $E_t$ and that part arising from symmetry $E_s$ are plotted against time for the two cases we have considered. The reason for comparing these two calculations was to determine whether the microscopic dynamics would give preference to asymmetric scission shapes as has often been speculated [13-15]. Indeed, the damping into intrinsic states is considerably less for the trajectory leading to an asymmetric mass division.

The Macroscopic Calculation

For comparison with the microscopically determined energy flow into intrinsic states we also calculated the dissipation to be expected on the basis of the classical one-body damping expression mentioned earlier [8,9]. The rate of energy absorption is given, in this theory, by the surface integral

$$\dot{E} = \rho_m \bar{v} \int \hat{n}^2 d\sigma,$$

where $\rho_m$ is the mass density, $\bar{v}$ the average velocity of the particles ($3/4 v_F$ in the Fermi-gas model), and $\hat{n}$ is the normal velocity of the nuclear surface. Note that unlike ordinary viscous damping, this expression contains no adjustable parameters.

In fig. 3 we have compared the energy absorption calculated from eq. (9) both with the viscous energy absorption present in the calculations used to generate the shapes, and with the microscopic calculations described earlier. For this purpose we have used the label $E_d^{\text{max}}$ for the points derived from the expression

$$E_d^{\text{max}} = E^* - E_0 - E_{\text{coll}}.$$

(10)
Similarly, we have used the label $E_{d}^{\text{min}}$ to label the points derived from the expression

$$E_{d}^{\text{min}} = E^{*} - E - E_{\text{coll}}$$

since this is the smallest excitation energy consistent with our calculations [obtained by taking the "adiabatic" energy $E_{a}$ as ground state instead of $E_{0}$ as is done in eq. (10)].

Discussion

The results of this work serve to draw attention to the importance of single particle damping for large-scale collective motion. They show that the energy dissipation profile is quite different than that associated with a hydrodynamic (or "two-body") viscosity. They show (as has been frequently speculated) that asymmetric fission is preferred over symmetric fission for the nucleus $^{236}\text{U}$ on the basis of microscopic dynamical considerations.

They also show that the results for one-body dissipation treated quantum mechanically and classically are very close one to each other and it seems that this type of dissipation is more nearly comparable to that which takes place in nuclei since an independent particle description certainly applies. Classical hydrodynamical calculations performed recently with this new type of damping give rise to a distinctly different sequence of shapes than those used here [16].

Another interesting result is that the microscopic collective kinetic energy is greater than that for irrotational flow indicating that some turbulence is generated by the collective motion. Consequently, we are inclined to question the applicability of the traditional
hydrodynamical saddle to scission trajectories. Both the dissipation and internal flow may well prefer a different sequence of shapes. An important step in the continuation of this work would be to perform the microscopic calculations in potential wells following the new shape sequence obtained with the use of one-body damping [16] to see if the damping along the path to scission is more nearly like that of the classical prediction.

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References


12. According to a suggestion of S. E. Koonin.

Figure Captions

Fig. 1. The 10, 30, 50, 70 and 90 percent countours are shown for three shapes in the saddle to scission sequence used in the calculations described here. The algebraic method of ref. 10 was used to create the diffuseness.

Fig. 2. The results of our microscopic calculations [3] for the intrinsic excitation of the neutrons in $^{236}$U for a hydrodynamically [4] determined sequence of shapes from saddle to scission. The dashed line is the corresponding hydrodynamic collective kinetic energy (irrotational flow assumed) and the solid line is obtained by adding the internal energy arising from the viscous damping of the motion. The circles represent the results of our calculation for $E_{\text{coll}}$, the collective kinetic energy; $E_t$, the total excitation; and $E_s$, which is that part of the total excitation connected with the symmetry of the potential. The triangles represent these same quantities in the case when reflection asymmetry is introduced.

Fig. 3. The largest and smallest amounts of internal excitation consistent with our microscopic calculations are plotted as $E_{d}^{\text{max}}$ and $E_{d}^{\text{min}}$ for the two cases considered (symmetric and asymmetric). For comparison the one-body damping from eq. (9) is shown as a solid line and the viscous damping (from the calculation used to generate the shapes) is shown as a dashed line.
Fig. 1
Fig. 2
Fig. 3

- Symmetric
- Asymmetric

One-body damping

Viscous damping

$E_d^{\text{max}}$

$E_d^{\text{min}}$

$E_d$
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