HEAT TRANSFER IN FRACTURED GEOTHERMAL RESERVOIRS WITH BOILING

Karsten Pruess

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HEAT TRANSFER IN FRACTURED GEOTHERMAL RESERVOIRS WITH BOILING

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Abstract

Analytical and numerical techniques are used to study non-isothermal flow of water and steam in idealized fractured porous media. We find that heat conduction in the rock matrix can substantially alter the flowing enthalpy in the fractures. Effects of matrix permeability and fracture spacing are demonstrated for production from and injection into vapor-dominated and liquid-dominated geothermal reservoirs.

Introduction

Most high-temperature geothermal reservoirs are highly fractured systems (Brook et al., 1978). The fractures provide conduits through which fluid and heat can flow at sufficiently large rates to attract commercial interest. The rock matrix has a low flow capacity, but it stores most of the heat and fluid reserves. The fractures represent a very small fraction of the void volume, and probably contain less than 1% of total fluid and heat reserves in realistic cases. Sustained production from a fractured reservoir is only possible if the depletion of the fractures can be replenished by leakage from the matrix. The rate at which heat and fluid can be transferred from the matrix to the fractures is therefore of crucial importance for an assessment of reservoir longevity and energy recovery. Yet most work in the area of geothermal reservoir evaluation and analysis has employed a "porous medium"-approximation, which amounts to assuming instantaneous (thermal and hydrologic) equilibration between fractures and matrix. Effects...
of matrix/fracture interaction for single-phase reservoirs have been investi-
gated in a number of papers (Bodvarsson, 1969; Kasameyer and Schroeder, 1976; Bodvarsson and Tsang, 1982). Few researchers have studied the behavior of boiling fractured reservoirs (Moench et al., 1978, 1980; Pinder et al., 1979). Moench's work addresses the question of pressure transients during drawdown and build-up tests in fractured reservoirs. The present paper focuses on a complementary aspect, namely, enthalpy transients. We use simple analytical expressions to analyze fluid and heat transfer between a porous rock matrix and fractures. It is shown that heat conduction in a matrix with low permeability can substantially increase flowing enthalpy in the fractures. This affects fluid mobility, and has important consequences for energy recovery and reservoir longevity. We present results of numerical simulations which illustrate these effects and show their dependence upon matrix permea-
bility and fracture spacing.

Conductive Enhancement of Flowing Enthalpy

Production from the fractures causes pressures to decline, and initiates fluid flow from the matrix into the fractures. Assuming Darcy's law to hold in the matrix, and neglecting gravity effects, the mass flux being discharged into the fracture system can be written:

\[ \dot{n} = \sum_{\beta=\text{liquid, vapor}} \phi_{\beta} = -k_m \sum_{\beta} \frac{k_{\beta} \rho_{\beta}}{\mu_{\beta}} (\nabla p)_n \]  

Here \((\nabla p)_n\) is the normal component of pressure gradient at the matrix/fracture interface. In a boiling reservoir, pressure gradients are accompanied by temperature gradients. Idealizing the reservoir fluid as pure water substance, the temperature gradient is given by the Clapeyron equation:
\( \nabla T = \frac{(v_v - v_\ell)(T + 273.15)}{h_v - h_\ell} \nabla p \) \hspace{1cm} (2)

Therefore, there is a one-to-one correspondence between mass flux and the conductive heat flux which is given by

\[ q = -kVT \] \hspace{1cm} (3)

The total heat flux discharged into the fracture system is

\[ G = \sum_\beta h_\beta F_\beta - K(VT)_n \] \hspace{1cm} (4)

In the matrix, heat is stored in rocks and fluids. In the fractures, heat resides solely in the fluid filling the void space. The heat flux given by (4) has to be carried, therefore, by the mass flux given by (1). Upon entering the fracture system, the fluid heat content is enhanced by the absorption of the conductive heat flux. From \( G = h_p \) we obtain the effective flowing enthalpy of the fluid entering the fractures:

\[
h = \frac{\frac{k_\ell}{\mu_\ell} \rho_\ell \left[ \frac{k_{lim}}{h + \frac{k_{lim}}{k_m k_\ell} (h_v - h_\ell)} \right] + \frac{k_v}{\mu_v} \rho_v h_v}{\frac{k_\ell}{\mu_\ell} \rho_\ell + \frac{k_v}{\mu_v} \rho_v} \] \hspace{1cm} (5)

Here we have defined a limiting effective permeability, dependent upon heat conductivity and temperature, as:

\[
k_{lim}(K,T) = k \frac{\mu_\ell v_\ell (v_v - v_\ell)(T + 273.15)}{(h_v - h_\ell)^2} \] \hspace{1cm} (6)
The enhancement of flowing enthalpy occurs because the conductive heat flux vaporizes part (or all) of the liquid which is discharged into the fractures. The effect depends upon the ratio of heat conductivity $K$ to effective permeability for the liquid phase, $k_m k_L$. The smaller the permeability of the matrix, the smaller is the mass flux which has to absorb the conductive heat flux, resulting in a stronger enhancement of flowing enthalpy. From (5) it can be seen that the fraction of liquid flux vaporized is

$$\nu = \frac{k_{\text{lim}}}{k_m k_L}$$

(7)

Thus, for $k_m k_L = k_{\text{lim}}$ all liquid is vaporized, giving rise to discharge of saturated steam from the matrix ($h = h_v$). Larger permeability ($k_m k_L > k_{\text{lim}}$) results in discharge of two-phase fluid, while for $k_m k_L < k_{\text{lim}}$ superheated steam is produced even if a mobile liquid phase is present in the matrix. Pruess and Narasimhan (1982a) have shown that gravity effects diminish the limiting effective permeability, hence the conductive enhancement of flowing enthalpy. This is easily understood by noting that gravity-driven flow does not require non-zero pressure gradients, and therefore need not be accompanied by conductive heat transfer. $k_{\text{lim}}$ is plotted as a function of temperature in Figure 1 for the no-gravity case. Figure 2 shows the effective flowing enthalpy of fluid discharged into the fractures as a function of matrix permeability calculated from (5). Curves are given for different values of vapor saturation; in these calculations the relative permeabilities were assumed to be given by Corey's equation with $S_{LR} = 0.3$, $S_{SR} = 0.05$ (Faust and Mercer, 1979).
\[ k_L = [s^*]^4 \]  
\[ k_V = (1-s^*)^2 (1-[s^*]^2) \]

where \[ s^* = \frac{S_L - S_{LR}}{1 - S_{LR} - S_{SR}} \]

It is seen that conductive enhancement of flowing enthalpy becomes significant for \( k < 10^{-15} \) m\(^2\), and becomes very large for smaller permeability.

**Numerical Simulations of Fractured Reservoir Behavior**

The above considerations are borne out by numerical simulations. We employ an idealized model of a fractured reservoir, with three perpendicular sets of infinite, plane, parallel fractures of equal aperture \( \delta \) and spacing \( D \) (see Figure 3). Modeling of transient fluid and heat flow is accomplished with the "multiple interacting continua" method (MINC). This method is conceptually similar to, and is a generalization of, the well-known double-porosity approach (Barenblatt et al., 1960; Warren and Root, 1963).

The classical double-porosity work employed a quasi-steady approximation for "interporosity" flow between rock matrix and fractures, which in later work has been improved by using time-dependent analytical solutions (Duguid and Lee, 1977; Evans, 1981). These approximations are applicable only to flow of fluids with small and constant compressibility. The MINC-method on the other hand treats interporosity flow entirely by numerical methods. This makes possible a fully transient treatment of interporosity flow which is applicable to problems with coupled fluid and heat flow, and to multiphase fluids with large and varying compressibilities, such as steam-water mixtures. Also, the MINC-description is applicable to reservoirs with irregular and statistical fracture distributions, although the calcula-
tions reported below were made for highly idealized fracture patterns. A detailed account of the foundations of the MINC-method has been given in (Pruess and Narasimhan, 1982b). In the present paper we shall give a brief summary of the methodology, followed by illustrative applications to geothermal reservoir problems.

In order to numerically model flow processes in geothermal reservoirs (or, for that matter, in any subsurface flow systems), it is necessary to partition the system under study into a number of volume elements $V_n$ ($n = 1, 2, \ldots, N$). Then the appropriate conservation equations for mass, energy, and momentum can be written down for each volume element (see appendix). These equations hold true irrespective of size, shape, heterogeneities, etc. of the volume elements $V_n$. This geometric flexibility can be most fully exploited within an integral finite difference formulation which is locally one-dimensional, avoiding any reference to a global coordinate system (Edwards, 1972). However, the conservation equations in integral finite difference form are useful only if the allowable partitions $V_n$ ($n = 1, \ldots, N$) are suitably restricted on the basis of geometric and thermodynamic considerations. Indeed, for practical applications we need to be able to relate fluid and heat flow between volume elements to the accumulation of fluid and heat within volume elements. Fluid and heat flow are driven by gradients of pressure and temperature, respectively, and these can be expressed in terms of average values of thermodynamic variables if (and only if) there is approximate thermodynamic equilibrium within each element. In porous media, this requirement will be satisfied for any suitably "small" subregion, as thermodynamic conditions generally vary continuously and smoothly with position. The situation can be quite different in fractured
media, where changes in thermodynamic conditions as a consequence of boiling or cold water injection may propagate rapidly in the fracture network, while migrating only slowly into the rock matrix. Thus, thermodynamic conditions may vary rapidly as a function of position in the vicinity of the fractures.

Based on the different response times, the MINC-method makes the approximation that thermodynamic conditions in the matrix depend locally only upon the distance from the nearest fracture. For the idealized fracture distribution shown in Figure 3, the requirement of approximate thermodynamic equilibrium then gives rise to a computational mesh as shown in Figure 4. The matrix is partitioned into a series of nested volume elements, defined according to the distance from the nearest fracture. Modeling of fluid and heat flow in such a system of interacting continua is straightforward within an integral finite difference formulation (Narasimhan and Witherspoon, 1976). The matrix-fracture interaction is described in purely geometrical terms, and the relevant geometric quantities, i.e. element volumes, interface areas, and nodal distances, can be easily obtained in closed form. The pertinent expressions are given by Pruess and Narasimhan (1982b).

In reservoir regions where thermodynamic conditions vary slowly as a function of position, it is not necessary to have separate volume elements within each of the elementary units depicted in Figure 4. Instead, corresponding nested volumes in neighboring units, which are identified by an index number in Figure 4, can be lumped together into one computational volume element. Element volumes and interface areas scale proportional to the number of elementary units which are lumped together, whereas nodal distances remain unchanged. The scaling procedure can be further generalized.
by applying the same scaling laws to grid blocks of arbitrary shape and size. Thus we arrive at a two-step procedure for defining a computational mesh for a fractured reservoir. The first step is to construct a mesh just as would be done for a porous-medium type system, with small grid blocks near wells, etc. ("primary mesh"). The second step is to sub-partition each grid block into several continua, the respective volumes, interface areas, and nodal distances of which are obtained by appropriate scaling from the quantities pertaining to the basic fractured unit ("secondary mesh").

Properly speaking, in regions where small grid blocks are desirable for spatial resolution, e.g. near wells, one should attempt to model individual fractures. A description based on average fracture spacings is appropriate at greater distances from wells, where less detail is available and desirable. However, we believe it useful to be able to extend a fracture description based on average parameters ( spacings, widths, orientations) to small volume elements, because this is applicable in cases where no detailed information about individual fractures near a well is available.

The calculations presented below were made with LBL's geothermal simulators SHAFT79 and MULKOM. These simulators solve coupled mass- and energy-balance equations for two-phase flow of water and steam, employing Newton-Raphson iteration and direct solution technique. The thermophysical properties of water substance are accurately represented by the steam table equations as given by the International Formulation Committee (1967). The governing equations employed in the simulators are given in the appendix; more details on the mathematical and numerical treatment can be found in (Pruess and Schroeder, 1980; Pruess and Narasimhan, 1982b). The parameters used in our model studies are summarized in Table 1.
(i) Radial Flow. The radial flow problem uses parameters applicable to a typical well at The Geysers, and assumes a mobile liquid phase \( (k_f = 0.143 \text{ at } S_g = 0.70) \). The "primary" mesh is one-dimensional, consisting of a sequence of concentric cylinders. For the fractured reservoir simulation, each of the primary grid blocks is sub-partitioned into a number of interacting continua, with geometrical parameters based on a regular fracture distribution as shown in Figure 3, and employing the scaling procedure as outlined above. This results in a "secondary" mesh as schematically depicted in Figure 5. Global fluid flow occurs exclusively in the fracture system. The matrix acts as a "one-way street" for fluid and heat flow, replenishing the fracture system as pressures and temperatures decline due to production accompanied by boiling.

Figure 6 gives computed enthalpy transients for fractured media with different matrix permeability, and for a uniform porous medium with the same average permeability, defined as

\[
k_2 \approx 2k_f \frac{\delta}{D}
\]

The factor 2 in (9) takes into account that, due to the presence of three orthogonal fracture sets, there are two fracture sets available for flow in any given direction. During the first 800 seconds the enthalpy response is governed by wellbore storage, and subsequently an increase in flowing enthalpy is noted which is stronger for smaller matrix permeability. The calculations show, in agreement with the analytical results presented above, that for \( k_m k_f < k_{lim} \), the produced enthalpy rises to above \( 2.8 \text{ MJ/kg} \) (superheated steam) within minutes, whereas for \( k_m k_f > k_{lim} \), a two-phase steam/water mixture is produced. For a more detailed discussion of these results, see (Pruess and Narasimhan, 1982a).
(ii) **Areal Depletion Problem** We have studied the depletion of an areal 7 km x 3 km reservoir, with parameters similar to those used by Bodvarsson et al., (1980) in their assessment of the geothermal reservoir at Baca, New Mexico. Results for produced enthalpies and flowing downhole pressures are given in Figures 7 and 8. Two basic depletion patterns are observed, depending upon whether matrix permeability is (relatively) "high" or "low". For "low" \( k_m = 10^{-17} \) m\(^2\), the boiling process is localized in the vicinity of the fractures, with vapor saturations in the matrix decreasing as a function of distance from the fractures. The opposite pattern is observed for "high" \( k_m = 9 \times 10^{-17} \) m\(^2\), \( 10^{-15} \) m\(^2\), where the depletion process causes a boiling front to rapidly move into the matrix, giving rise to largest vapor saturations in the interior of the matrix, away from the fractures. This is analogous to what has been noted for porous medium-type reservoirs (Pruess et al., 1982).

It is apparent from Figure 7 that produced enthalpy depends much more strongly upon matrix permeability than upon fracture spacing. Enthalpy increases with decreasing matrix permeability, in agreement with the discussion given above. From Figure 8 it can be noted that pressure decline is more rapid in case of higher enthalpy, due to the fact that the mobility of two-phase fluid generally decreases with increasing enthalpy. The crucial importance of matrix permeability is particularly evident in the case of \( k_m = 10^{-17} \) m\(^2\), where fracture spacings of 5 m and 50 m, respectively, result in virtually identical enthalpy and pressure response, even though the matrix/fracture contact areas differ by a factor of 10.

It is seen that fractured reservoirs can behave quite differently than a porous medium with the same average permeability and porosity. The
differences depend mainly on the permeability contrast between matrix and fractures, and they do not, in general, diminish for small fracture spacing.

(iii) Five Spot For a more realistic assessment of the depletion of a naturally fractured boiling reservoir, we investigated a five-spot production/injection strategy for the reservoir discussed in the previous example. The primary mesh as given in Figure 9 takes advantage of flow symmetry. The production/injection rate was fixed at 30 kg/s, which corresponds to the more productive wells in the Baca reservoir.

Our results show that without injection, pressures will decline rapidly in all cases. The times after which production-well pressure declines below 0.5 MPa are: 1.49 yrs for a porous medium, 2.70 yrs for a fractured reservoir with D = 150 m, $k_m = 9 \times 10^{-17}$ m$^2$, and 0.44 yrs for $D = 50$ m, $k_m = 1 \times 10^{-17}$ m$^2$. The fractured reservoir with large $k_m$ ($9 \times 10^{-17}$ m$^2$) shows a greater longevity than a porous reservoir with the same average permeability and porosity. The reason for this is that the large matrix permeability provides good fluid supply to the fractures, while conductive heat supply is limited. Therefore, vapor saturation in the fractures remains relatively low, giving good mobility and a more rapid expansion of the drained volume.

The results obtained with 100% injection demonstrate the great importance of injection for pressure maintenance in fractured reservoirs with low permeability. Simulation of 90 years for the porous medium case, and 42 and 103 years, respectively, for fractured reservoirs with $D = 50$ m and $D = 250$ m ($k_m = 10^{-17}$ m$^2$), showed no catastrophic thermal depletion or pressure decline in either case. These times are significantly in excess of the 30.5 years needed to inject one pore volume of fluid. Figure 10 shows temperature and pressure profiles along the line connecting production and injection.
wells for the three cases studied after 36.5 years of simulated time. The temperatures of the porous-medium case and the fractured reservoir with 
\(D = 50\) m agree remarkably well, indicating an excellent thermal sweep for the latter (see also Bodvarsson and Tsang, 1982). The temperature differences \(\Delta T = T_m - T_f\) between matrix and fractures are very small: after 36.5 years, we have \(\Delta T = 0.2^\circ C\) near the production well, \(0.001^\circ C\) near the injection well, and less than \(5^\circ C\) in between. In the \(D = 50\) m case, produced enthalpy remains essentially constant at \(h = 1.345\) MJ/kg. It is interesting to note that this value is equal to the enthalpy of single-phase water at original reservoir temperature \(T = 300^\circ C\). Thus, there is an approximately quasi-steady heat flow between the hydrodynamic front at \(T \approx 300^\circ C\) and the production well, with most of the produced heat being supplied by the thermally depleting zone around the injector.

At the larger fracture spacing of \(D = 250\) m, the contact area between matrix and fractures is reduced, and portions of the matrix are at larger distance from the fractures. This slows thermal and hydrologic communication between matrix and fractures, causing the reservoir to respond quite differently to injection. More of the injected water remains in the fracture system, so that produced enthalpy and boiling rates are reduced. After 36.5 years, thermal sweep is much less complete, with temperature differences between matrix and fractures amounting to \(16^\circ C\), \(118^\circ C\), and \(60^\circ C\), respectively, near producer, near injector, and in between. Temperatures increase monotonically away from the injection well at \(D = 250\) m, whereas for \(D = 50\) m there is a region of lower temperature around the production well caused by heat loss in boiling. Two-phase conditions with intense boiling occur within 200 m of the producer after 36.5 years for \(D = 50\) m fracture spacing. In contrast, the fracture system becomes completely water filled after 30 years in the case with \(D = 250\) m.
For the particular production and injection rates employed in this study, reservoir depletion is slow enough that even at a large fracture spacing of $D = 250 \text{ m}$, most of the heat reserves in the matrix can be produced. We are presently investigating energy recovery in the presence of a prominent short-circuiting fault or fracture between production and injection wells, under which conditions less favorable thermal sweeps are expected.

Conclusions

The results of our analytical and numerical studies can be summarized as follows:

1. Fractured boiling reservoirs can behave quite differently from porous medium-type reservoirs with the same average permeability and porosity, even in cases where fracture spacing is small. Assumptions of thermodynamic equilibrium between rock matrix and fractures, which are frequently made in geothermal reservoir analysis, can lead to completely erroneous results.

2. If matrix permeability is low, heat transfer to the fractures by means of conduction becomes very important, giving rise to an enhancement of flowing enthalpy.

3. For production without reinjection, reservoir response depends much more strongly upon matrix permeability than upon fracture spacing. Produced enthalpy will generally increase with decreasing matrix permeability.

4. Fractured geothermal reservoirs may produce superheated steam even if there is plenty of liquid water flowing in the matrix.
5. Full reinjection can be an effective means for pressure maintenance in fractured reservoirs. When no major short-circuiting faults or fractures are present, it appears that excellent thermal sweeps can be achieved even in cases with large fracture spacing of several hundred meters.

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Appendix: Mass - and energy - balances

The simulators SHAFT79 and MULKOM solve discretized versions of the following mass-and energy-balance equations:

\[
\frac{d}{dt} \int_{V_n} \phi \rho dv = \int_{\Gamma_n} F \cdot n d\Gamma + \int_{V_n} q dv \tag{A.1}
\]

\[
\frac{d}{dt} \int_{V_n} U dv = \int_{\Gamma_n} G \cdot n d\Gamma + \int_{V_n} q dv \tag{A.2}
\]

Mass flux is approximated by Darcy's law, which expresses a momentum balance with negligible inertial force

\[
F = \sum_{\beta = \text{liquid, vapor}} F_{\beta} = -k \sum_{\beta} \frac{k_{\beta}}{\mu_{\beta}} \rho_{\beta} \left[ \nabla \rho - \rho_{\beta} g \right] \tag{A.3}
\]

Energy flux contains conductive and convective terms

\[
G = -k \nabla T + \sum_{\beta} h_{\beta} F_{\beta} \tag{A.4}
\]

and the volumetric internal energy of the rock/fluid mixture is

\[
U = \phi \rho_u + (1-\phi) \rho_R C_R T \tag{A.5}
\]

The main assumptions made in the above formulation are as follows:

1. The systems described by SHAFT79 or MULKOM are approximated as mixtures of rock and single-component fluid in liquid and vapor form. (2) Liquid, vapor, and rock are in local thermodynamic equilibrium, i.e. at the same temperature and pressure, at all times. (3) Capillary pressure effects are neglected.

It is to be noted that the equations given above hold for porous and fractured media alike. Experimental work has established that fracture flow obeys Darcy's law, with fracture permeability related to fracture aperture as (Witherspoon et al., 1980).

\[
k_f = \frac{\varphi^2}{12} \tag{A.6}
\]
References


2. Numerical Solution Techniques for Liquid-and Vapor-Dominated

International Formulation Committee (1967), "A Formulation of the Thermodynamic
Properties of Ordinary Water Substance," IFC Secretariat, Duesseldorf, Germany.

to Geothermal Reservoirs with Fracture and Pore Permeability," Proceedings,
Second Workshop on Geothermal Reservoir Engineering, Stanford University,
Stanford, California.

Moench, A.F. (1978), "The Effect of Thermal Conduction upon Pressure Drawdown
and Buildup in Fissured, Vapor-Dominated Reservoirs," Proceedings, Fourth
Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford,
California.

Moench, A.F., and Denlinger, R. (1980), "Fissure-Block Model for Transient
Pressure Analysis in Geothermal Steam Reservoirs," Proceedings, Sixth
Workshop on Geothermal Reservoir Engineering, Stanford University,
Stanford, California.

vol. 12, no. 1, pp 57-64.

Pinder, G.F., Ramey, H.J. Jr., Shapiro, A., and Abriola, L. (1979), "Block
Response to Reinjection in a Fractured Geothermal Reservoir," Proceedings,
Fifth Workshop on Geothermal Reservoir Engineering, Stanford University,
Stanford, California.


Nomenclature

\( C_R \): Specific heat of rock (J/kg°C)

\( D \): Fracture spacing (m)

\( F \): Mass flux (kg/m²•s)

\( g \): Gravitational acceleration (9.81 m/s²)

\( G \): Heat flux (W/m²)

\( h \): Specific enthalpy (J/kg)

\( k \): Absolute permeability (m²)

\( K \): Heat conductivity (W/m°C)

\( k_f \): Fracture permeability, (m²)

\( k_{lim} \): Limiting effective permeability (m²)

\( k_\beta \): Relative permeability for phase \( \beta \), dimensionless

\( k_2 \): Average permeability of fracture system (m²)

\( p \): Pressure (Pa)

\( q \): Rate of mass sink or source (kg/m³•s)

\( q \): Conductive heat flux (W/m²)

\( r \): Radial coordinate (m)

\( S \): Saturation (void fraction), dimensionless

\( S_{lr} \): Irreducible liquid saturation, dimensionless

\( S_{sr} \): Irreducible vapor saturation, dimensionless

\( T \): Temperature (°C)

\( u \): Internal energy of fluid (J/kg)

\( U \): Volumetric internal energy of rock/fluid mixture (J/m³)

\( v \): Specific volume (m³/kg)

\( V_n \): Volume element (m³)

\( z \): Vertical coordinate (m)
\( \delta \): Fracture aperture (m)

\( \Gamma_n \): Surface element (m²)

\( \nu \): Fraction of liquid flux vaporized, dimensionless

\( \phi \): Porosity, dimensionless

\( \mu_\beta \): Viscosity of phase \( \beta \) (Pa·s)

\( \rho_\beta \): Density of phase \( \beta \) (kg/m³)

**Subscripts**

- \( f \): Fracture
- \( l \): Liquid
- \( m \): Matrix
- \( n \): Normal component
- \( R \): Rock
- \( v \): Vapor
- \( \beta \): Phase (\( \beta = \) liquid, vapor)
TABLE I: Parameters Used in Simulations

<table>
<thead>
<tr>
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<th>Radial Flow Problem</th>
<th>Depletion Problem</th>
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<td>Formation</td>
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<tr>
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<td>Corey-curves</td>
<td>$S_{fr} = .30, S_{ar} = .05$</td>
<td>$S_{fr} = .30, S_{ar} = .05$</td>
</tr>
<tr>
<td>Initial Conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature</td>
<td>$T = 243 \text{ }^{\circ}\text{C}$</td>
<td>$300 \text{ }^{\circ}\text{C}$</td>
</tr>
<tr>
<td>liquid saturation</td>
<td>$S_L = 70%$</td>
<td>$99%$</td>
</tr>
<tr>
<td>Production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>wellbore radius</td>
<td>$r_w = .112 \text{ m}$</td>
<td></td>
</tr>
<tr>
<td>skin</td>
<td>$s = -.5.18$</td>
<td></td>
</tr>
<tr>
<td>effective wellbore radius</td>
<td>$r_w^* = r_w e^{-s} = 20.0 \text{ m}$</td>
<td></td>
</tr>
<tr>
<td>wellbore storage volume</td>
<td>$V_w = 27.24 \text{ m}^3$</td>
<td></td>
</tr>
<tr>
<td>production rate</td>
<td>$q = 20 \text{ kg/s}$</td>
<td>$82.5 \text{ kg/s}(b), 30 \text{ kg/s}(c)$</td>
</tr>
<tr>
<td>Injection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rate</td>
<td>---</td>
<td>$30 \text{ kg/s}(c)$</td>
</tr>
<tr>
<td>enthalpy</td>
<td>---</td>
<td>$5 \times 10^5 \text{ J/kg}$</td>
</tr>
</tbody>
</table>

(a) fractures modeled as extended regions of high permeability, with a width of $\approx .2 \text{ m}$
(b) rectangular reservoir
(c) five-spot
Figure 1: Limiting effective permeability.
Figure 2: Effective flowing enthalpy.
Figure 3: Idealized model of fractured reservoir.
Figure 4: Basic computational mesh for fractured porous media, shown here for simplicity for a two-dimensional case. The fractures enclose matrix block of low permeability, which are subdivided into a sequence of nested volume elements.
Figure 5: Schematic diagram of "secondary" mesh for computing radial flow in a fractured reservoir.
Figure 6: Enthalpy transients for radial flow problem.
Figure 7: Produced enthalpy for areal depletion problem.
Figure 8: Pressure decline for areal depletion problem.
Figure 9: "Primary" mesh for five-spot well pattern.
Figure 10: Temperature and pressure profiles for five-spot after 36.5 years.
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