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More is up… and right: Random number generation along two axes

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Abstract
Research on the mental representation of numbers has focused on a horizontally aligned mental number line, but more and more findings have begun to implicate a vertical orientation as well. We investigate the relationship between these two orientations when people generate random numbers. In the horizontal condition, people generated larger numbers when they looked right as opposed to left. In the vertical condition, people generated larger numbers when they looked up as opposed to down. We present two main results based on analyses that compare the two spatial orientations. First, we show that the vertical effect was stronger than the horizontal one. Second, we show a weak correlation between the vertical and the horizontal effect, potentially suggesting a shared underlying mechanism.

Keywords: mental number line; SNARC; numerical representation; mathematical cognition; metaphor

Introduction
People use numbers for practically everything: counting coins, ordering dinner, making an appointment, filling out tax forms, and more. But how do they represent numbers in the head? Research on numerical representation has focused on the link between numbers and spatial cognition. Neuroimaging studies consistently find that the intraparietal sulcus is implicated in numerical as well as spatial tasks (Hubbard, Piazza, Pinel, & Dehaene, 2005). And across several neuropsychological disorders, deficits in spatial cognition are correlated with deficits in numerical cognition (Zorzi, Priftis & Umiltà, 2002; Rotzer, Loenneker, Kucian, Martin, Klaver & von Aster, 2009).

One proposal as to how we represent numbers is the idea of a horizontally oriented mental number line, where smaller numbers are associated with left percutual space and larger numbers with right perceptual space (at least in Western cultures). Evidence for such a representation comes from the Spatial-Numerical Association of Response Code effect (SNARC), which revealed that people respond faster to relatively larger numbers with their right hand, and faster to relatively smaller numbers with their left hand (Dehaene, Bossini & Giraux, 1993). This effect has been replicated in over 100 experiments (Wood, Nuerk, Willmes & Fischer, 2008), and similar effects have been found with pointing (Fischer, 2003), body movements (Hartmann, Grabherr, & Last, 2011), handwriting (Perrone, de Hevia, Bricolo, & Girelli, 2010) and many other methodologies.

While there is much converging evidence for horizontally oriented numerical representations, more and more findings are emerging that also support the presence of a vertical mental number line. For example, when people are moved upwards by a lifting chair while they generate a “random” sequence of numbers, generated numbers are “higher” than when the chair is moving downwards (Hartmann, Grabherr & Last, 2011). Similarly, an upwards directed eye movement predicts that the next number in a randomly generated sequence will be “higher” than the preceding number (Loetscher, Bockisch, Nicholls, & Brugger, 2010). Relatively larger numbers also facilitate upwards directed saccades (Schwarz & Keus, 2004) and upwards directed spatial attention (Pecher & Boot, 2011).

Evidence for a vertical representation of number also comes from language processing: When people read sentences that contain the word “more”, people are faster to respond with an upwards oriented response button as opposed to a downwards oriented response button (Sell & Kaschak, 2012). The opposite is true for sentences that contain the word “less”. Finally, the classic SNARC paradigm, too, works with vertically oriented response buttons (Ito & Hatta, 2004; Müller & Schwarz, 2007; Shaki & Fischer, 2012), where larger numbers facilitate responses to a high button and smaller numbers to a low button.

The potential existence of two orientations along which numbers are represented naturally leads to the question: What is the relation between the horizontal and vertical mental number line? From the get-go, research on the horizontal number line emphasized the cultural nature of spatial numerical associations, where the orientation of the horizontal axis is thought to stem from a culture’s writing direction (Dehaene et al., 1993; Göbel, Shaki, & Fischer, 2011; for a related perspective focusing on cultural aspects, see Núñez, 2011). As reading and writing are very entrenched behaviors, one could imagine the horizontal mapping to be stronger than the vertical one.

The vertical SNARC effect, on the other hand, has been suggested to come from embodied interactions with the world. Cognitive linguists working on Conceptual Metaphor Theory (e.g., Lakoff, 1987) argue that we build up a mental connection between verticality and quantity because we repeatedly experience a correlation between these two domains in our environment (e.g., when we pour water into a glass, as quantity increases, verticality increases as well). Given that the vertical mapping is also connected to entrenched patterns of language use (“this is a high number”, “rents are rising”), one could imagine vertical SNARC effects to be stronger than horizontal ones.

Holmes and Lourenco (2011, 2012) explicitly compared the two orientations by pitting them against each other:
When participants had to respond to a top/left and to a bottom/right button, people were quicker to respond to the left button with smaller numbers and to the right button with larger numbers. As this mapping goes against the vertical mental number line but produces a regular horizontal SNARC effect nonetheless, Holmes and Lourenço conclude that the horizontal orientation “trumps” the vertical.

We follow up on the work by Holmes and Lourenço (2011, 2012) by providing another comparison between horizontal and vertical mappings with a different task, namely, a random number generation task. We pursue two main questions: First, we compare the relative strength of the horizontal and the vertical effect. Second, we look to see whether the horizontal and the vertical effect are related to each other across individuals. Thus, rather than pitting the two orientations against each other, we take an individual differences perspective, comparing an individual participant’s propensity to align numbers on the vertical axis to her propensity to align numbers horizontally. This approach is inspired by work suggesting considerable individual differences in how numbers are mapped onto space (e.g., Fischer & Campens, 2008; Fischer, 2008; Beecham, Reeve, & Wilson, 2009). Moreover, studies on individual difference have been used for a range of different phenomena to investigate the question whether different tasks potentially share the same underlying mechanism (e.g., see Stanovich & West, 2000). Thus, if the vertical and the horizontal effect are related across individuals, they can be seen as tapping into the same system.

Experiment

The task was a random number generation task used by Loetscher et al. (2008) and Hartmann et al. (2011) designed to study spatial numerical associations. Participants called out numbers during rhythmic head movements. In one block, head movements were along the horizontal axis, in another, along the vertical. In line with the horizontal SNARC effect and the previous findings of Loetscher et al. (2008), we expected numbers to be larger when people look towards the right. In line with the vertical SNARC effect, we expected numbers to be larger when people were looking upwards.

Procedure

Participants were asked to call out numbers between 1-30 to a beat of 0.5 Hz, played by an electronic metronome (following the procedure of Loetscher et al., 2008). There were three blocks: A horizontal block, vertical block, and straight-ahead block. The order of horizontal vs. vertical block was counter-balanced across participants. The straight block was always last. We asked participants to generate 40 numbers in the straight block and 80 numbers in the vertical and horizontal one. Half the participants started left in the horizontal block and down in the vertical block, and the other half started with the right and up positions.

We built up the procedure in pieces: We first instructed participants to perform the rhythmic head movements to the beat, “as large as possible while still being comfortable”. Then, we introduced the random number generation component, participants were told to be “as random as possible” and to avoid counting sequences. We reminded participants that randomness in this context means that each number has equal likelihood, and that each number is independent from the preceding one (see Towse & Cheshire, 2007). They were also asked not to call out the number while performing the movement but when the head was stationary in the corner positions of each axis. To avoid bias, the experimenter never mentioned numbers or spatial language to describe numbers (“high number”, “large number”). Following Loetscher et al. (2008), we asked participants to close their eyes while performing the task.

Participants

Sixty-five UC Merced undergraduates (all native speakers of English) participated in the experiment for extra credit in a social sciences course. A total of 6 participants (9% of the total data) were excluded from the analyses because they were unable to finish the task (frequent self-interruptions, incapability of following the beat even after sustained practice).

Analysis

Loetscher et al. (2008) binned numbers into large (>15) and small (<15) numbers, but we took a more direct approach, analyzing all generated numbers as a continuous measure. We performed two separate analyses, one on absolute numbers (whether the average was larger for one position over the other), and another on relative numbers (whether the average difference to the preceding number in the sequence was smaller or larger).

We analyzed the data with mixed models using R (R Core Team, 2012) and the package lme4 (Bates, Maechler & Bolker, 2012). Our analysis controlled for the by-participant variability in the response (e.g., some participants might generate overall larger numbers than others), as well as for differential responses to the head turning manipulation.

Because Holmes and Lourenço (2012) had found that it matters whether people are exposed to a vertical or a horizontal block first (ibid. 1049, footnote 4), it was necessary to control for the effect of Block Order. It was necessary to control for the effect of Starting Orientation (left/down vs. right/up) because people tend to have a counting or “runs” strategy (see Towse & Cheshire, 2007). Such a strategy could create spurious spatial mappings if Starting Orientation were not controlled for. For example, a participant who tended to count upwards and start at the down position might generate numbers that are, on average, higher in the up position than those in the down position.

1 In other words, the model included both random intercepts and slopes (cf., Barr, Levy, Scheepers & Tily, 2013). We also tested the interaction of Head Position with the control variables Block
Finally, we also controlled for potential long-term changes in each block to see whether the horizontal and vertical effects would become stronger or weaker as the experiment progressed.

Results

Compressive scaling Before examining the effect of spatial position, we looked at whether participants’ randomly generated numbers would exhibit a small number bias (see Loetscher & Brugger, 2007). A regression of frequency on number reveals that on average, per each increase of number by 1, frequency decreased by 8.96 (SE = 1.78) (F(1,28)=25.35, p=0.000025, R²=0.46). There was no interaction between the small number bias and response orientation (horizontal, vertical, straight) (F(2,84)=2.539, p=0.085). Thus, within each condition (horizontal, vertical, straight), the small number bias was of similar magnitude.

Order effects For absolute numbers, the control variables (Block Order, Starting Orientation, Trial Order) did not interact with Head Position in the horizontal block (χ²(3)=1.36, p=0.71) or in the vertical block (χ²(3)=3.47, p=0.32). For relative numbers, the control variables also failed to produce any interaction in the horizontal block (χ²(3)=3.56, p=0.31) and in the vertical block (χ²(3)=4.78, p=0.19). This suggests that the effects reported below are relatively independent from these other factors.

Absolute numbers Numbers generated were on average 0.26 (SE = 0.24) larger when people looked to the right versus to the left, but the effect was not significant (χ²(1)=1.2, p=0.27). There was, however, a significant effect of vertical position (χ²(1)=7.91, p=0.0049), with numbers being 0.67 (SE = 0.24) larger up as opposed to down. To test whether the difference between the two axes is significant, we coded “up” and “right” together and “left” and “down”, combining them into a single factor “Position”. There was no interaction between “Position” and “Axis Orientation”. Thus, for absolute numbers, there is no conclusive evidence for the vertical effect being stronger.

Relative numbers There was a significant effect of horizontal position (χ²(1)=4.31, p=0.038), with numbers being +0.52 (SE = 0.25) larger than the preceding number when people looked right versus left, and a significant effect of vertical position (χ²(1)=8.13, p=0.004), with numbers being +1.34 (SE = 0.46) larger than the preceding number in the up position versus the down position (see Fig. 1). An analysis that combined both axis orientations also yielded a significant interaction of Axis Orientation and Position (χ²(1)=10.706, p=0.001), with the vertical effect predicted to be stronger by +0.82 (SE = 0.47).

Effect sizes Standardized effect measures showed stronger effects for the vertical than for the horizontal condition for absolute and relative numbers. This is also reflected in the larger coefficients for the vertical condition in the analyses reported above, as well as the significant interaction between Axis and Position for relative numbers.

Table 1: Effect sizes for absolute and relative numbers by condition.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Cohen’s d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal, absolute</td>
<td>0.17</td>
</tr>
<tr>
<td>Vertical, absolute</td>
<td>0.43</td>
</tr>
<tr>
<td>Horizontal, relative</td>
<td>0.44</td>
</tr>
<tr>
<td>Vertical, relative</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Individual differences We analyzed individual differences by looking at difference scores for right minus left (henceforth “horizontal bias”) and for up minus down (henceforth “vertical bias”). For the horizontal condition, 61.5% (40 participants) showed a horizontal bias (positive difference score), in line with the SNARC effect for both absolute and relative numbers. Similarly, 61.5% of all participants showed a vertical bias for both absolute and relative numbers.

While only ~15% (10 people) had no horizontal bias and no vertical bias (hence, showing opposite effects of what was predicted by both mappings), about 38% (25 people) had both a horizontal bias and a vertical bias simultaneously. However, the majority of participants (30 people, 46%) had either one bias or the other. Table 2 summarizes this result:

Table 2: Summary of individual differences by condition.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both horizontal</td>
<td>25</td>
</tr>
<tr>
<td>Both vertical</td>
<td>25</td>
</tr>
<tr>
<td>Horizontal</td>
<td>15</td>
</tr>
<tr>
<td>Vertical</td>
<td>15</td>
</tr>
<tr>
<td>No horizontal bias</td>
<td>10</td>
</tr>
<tr>
<td>No vertical bias</td>
<td>10</td>
</tr>
</tbody>
</table>

2 As there are no standardized effect size measures for mixed models, we chose Cohen’s d as shorthand. To calculate this measure, we used a by-participants analysis (averaging over trials).
Discussion

Vertical versus horizontal mappings For both absolute and relative numbers, we found stronger effects for the vertical than for the horizontal axis (as indicated by Cohen’s d and model coefficients), and for relative numbers, the vertical axis produced significantly stronger results than the horizontal axis.

Why did we find the vertical mapping to be stronger than the horizontal one? And, does this necessarily stand against the results of Holmes and Lourencó (2011, 2012) discussed above? We are cautious to conclude that these differences in effect size reflect a straightforward difference in the “strength” or “entrenchment” of the underlying mappings. There are several alternative reasons for why one mapping could lead to stronger or more consistent effects than the other. For example, people often perform smaller vertical head movements than horizontal ones (Glenn & Vilis, 1992; Pelz, Hayhoe, & Loeber, 2001), which could have made vertical head movements more salient. People’s vertical saccades are also known to be slower and less accurate than their horizontal saccades (Collewijn, Erkelens, & Steinman, 1988). And research on the vertical-horizontal illusion shows that people generally overestimate vertical extent more than horizontal extent (Finger & Spelt, 1947; Chapanis & Mankin, 1967; Prinzmetal & Gettleman, 1993). Even though the difference in saccades and the vertical-horizontal illusion might be deemed irrelevant given that our task required participants to close their eyes, overall, these results point to fundamental asymmetries between horizontal and vertical space. Thus, it is not impossible that we found vertical effects simply because vertical space is more salient than horizontal space.

This alternative explanation opens up many interesting avenues for future research. As there are considerable interindividual differences in the amount and degree to which individuals move their head (Fuller, 1992; Stahl, 1999), one could correlate each participant’s “head movement propensity” with the size of the vertical or horizontal effect. Ideally, one would like to correlate the strength of the head movement with the results of the number generation task on a trial-by-trial basis. The vertical and horizontal biases are predicted to be stronger for relatively larger movements. Moreover, there are also individual differences in the strength of the vertical-horizontal illusion (e.g., Coren & Porac, 1987). The susceptibility to this illusion could also be correlated with the horizontal or vertical bias. Here, people who have a stronger vertical-horizontal illusion should show stronger vertical effects. If, however, vertical space is more “salient” across the board, it does not fully account for the difference between Holmes and Lourencó (2011, 2012) and the present study, because presumably, the same vertical-horizontal asymmetries should be at play.

Here, it is noteworthy that many studies that found vertical effects either invoke random number generation (present study, Loetscher et al., 2010 and Hartmann et al., 2011) or approximate quantity information such as the words “more” or “less” (Sell & Kaschak, 2012; Pecher &

Table 2: Individuals with horizontal or vertical bias.

<table>
<thead>
<tr>
<th>Vertical</th>
<th>Horizontal</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>25</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>15</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

The preceding discussion of individual differences examined propensity to show vertical or horizontal effects categorically (sign of the difference score), yet it is also useful to carefully consider the relative strength of the vertical or horizontal bias per person.

The solid lines that are shown in Fig. 2 reveal an apparent correlation between the vertical and the horizontal bias. For both absolute numbers and relative numbers, this correlation became significant (absolute: t(63) = 2.62, p = 0.011; relative: t(63) = 2.35, p = 0.022). However, visual inspection and influence diagnostics revealed that there were a few individuals with substantial leverage on the data. If data points with large Cook’s distance (over 4/(N-k-1)) were excluded, both correlations cease to be significant (absolute: t(57) = 0.9, p = 0.37; relative: t(57) = 0.57, p = 0.57).

**Figure 2:** Correlations of horizontal bias (x-axis) and vertical bias (y-axis) for (a) absolute numbers and (b) relative numbers. Dashed lines represent correlation without influential points; solid lines with all data.
Boot, 2011). Moreover, Holmes and Lourenco (2012) found vertical effects only after priming magnitude. This invites the hypothesis that the vertical mapping might be stronger in tasks that invoke a more approximate number system. The small number bias observed in our participants’ responses would support this view, as a compressive scaling of the mental number line is associated with the idea of the approximate number system.

However, there are also linguistic reasons to expect that vertical effects are stronger with approximate magnitude representations: We frequently use the words “high” and “low” and “rising” and “falling” to talk about numbers, but we do not use horizontal spatial language the same way. The linguistic vertical metaphors are degree words that underspecify the exact quantity. The underspecification of verbal metaphors might make the vertical mapping particularly amenable for approximate magnitude representations as opposed to exact quantity representations. Finally, if, as cognitive linguists have claimed (Lakoff, 1987), the vertical mapping really comes from embodied interactions with the world, a connection between approximate magnitude and verticality might ultimately have physical origins. The environmental correlation between verticality and quantity often involves uncountable quantities rather than exact numbers, for instance, when pouring liquid into a container, or when creating a pile of pebbles. The horizontal mapping, on the other hand, might be more connected to exact numerical representations because of its connection to writing and symbolic representations of numbers, which are ideal for representing exact sequences (e.g., calendars, numbers on keyboards, rulers).

**Absence of order effects** The absence of any order effects in the current study is somewhat surprising. That Block Order did not affect the results suggests that whatever mapping is most preferred by a participant is not primed by being exposed to a vertical or a horizontal block first. One could imagine that the vertical or horizontal effects are entirely task dependent, resulting only after a bit of exposure to the up/down or left/right going movements. The absence of an interaction between Trial Order and Head Position suggests that this was not the case. Thus, it appears that participants responded in line with horizontal or vertical SNARC effects from the very beginning of each block. In other words, the spatial numerical associations appear to have been relatively stable.

**Individual differences** Finally, analyzing the data of separate participants revealed considerable differences between individuals, similar to other studies that have found considerable differences in the way people respond to numerical cognition tasks (Fischer & Campens, 2008; Fischer, 2008; Beecham, Reeve, & Wilson, 2009). The most prevalent pattern was that participants either had a horizontal bias or a vertical bias, with a considerable number (~38%) having both and only a handful (~15%) having neither. There was a weak correlation between a participant’s vertical bias and a participant’s horizontal bias. This may initially seem to suggest overlapping mechanisms for the vertical and the horizontal mappings. However, closer inspection revealed that this correlation was largely due to a few individuals. Based on the results obtained in the current study, it is clear that more research is needed to determine whether the connection between the vertical and horizontal mapping holds across different tasks.

**Conclusions**

We found that randomly generated numbers were “higher” when people looked upwards and when people looked to the right. We found the vertical effect to be stronger than the horizontal one. There were also considerable inter-individual differences: Some people were more easily affected by the vertical manipulation, others, more easily by the horizontal manipulation. Across individuals, there was a weak correlation between the vertical and the horizontal effect. Future research needs to find out under which conditions vertical effects are stronger than horizontal ones, and whether the weak relationship between these two effects holds across different experimental paradigms.

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