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Abstract

We make two remarkable observations about the pion condensate state based on a self-consistent theory. The condensate becomes energetically more important at high temperature, eventually creating a density isomer. Secondly in the condensed state the pressure is anisotropic. Consequences for high energy nuclear collisions and for neutron stars are discussed.

In a related work, pion condensation in zero temperature matter was investigated in a relativistic field theory solved in the mean field approximation. Unlike earlier work on the subject, the theory was constrained to possess the known saturation properties of nuclear matter. Here we investigate matter in the same theory at finite temperature. Of course finite temperatures are interesting because pion condensation is believed to be an important mechanism involved in the cooling of neutron stars. Moreover, nuclear collisions at high energy, if they produce dense matter, certainly produce it at finite temperature.

The meson fields considered are the chargeless scalar and vector mesons \( \sigma \) and \( \omega \) and the pseudoscalar isovector pion field. The first two are Yukawa coupled to the 8-component nucleon field, and the pions are vector coupled to the axial vector isospin current of the nucleons. The field equations are solved for the mean values of the fields, evaluated for a finite temperature medium.

The properties of the normal state can be used to determine all parameters of the theory except for the effective pion-nucleon coupling, \( g \). This is a most important parameter of the theory, and one which we shall take great care in determining. It would be unacceptable to simply use the vacuum value. This would drastically overestimate the importance of the condensate state.

Through the derivative coupling in our Lagrangian, the pions interact with nucleons dominantly in the \( p \)-wave state as it should be. We can represent...
this by the diagram \[ \pi \rightarrow \pi \] corresponding to a pion absorption on a particle in the Fermi sea exciting a particle-hole state, and the subsequent re-emission of the pion. In addition to this, other processes are believed to be important which are not explicit in our theory. One of these is the absorption of a pion to create a \( \Delta \) isobar; \[ \pi \rightarrow \pi \] where the double line represents the isobar. This process encourages condensation and is very important because it is not inhibited by the Pauli-exclusion principle. Acting in the opposite direction are the repeated scatterings of the particle-hole excitations by the nucleon-nucleon interaction. In the particle-hole state, having quantum numbers of the pion, this can be discussed in terms of the Landau parameter \( g' \). The above two diagrams should be replaced by the renormalized \( p \)-wave interaction defined by \[ \pi \rightarrow \pi \] where the particle state can be either nucleon or isobar. While these additional processes would be extremely difficult to incorporate in a theory of the fully developed condensate state, such as ours, they are relatively easy to incorporate in the pion self-energy at the pion condensation threshold. The reason is that at threshold the infinitesimal pion field does not affect the nucleon states so that the above contributions can be calculated on the unperturbed basis. Such a calculation of the pion condensation threshold has been carried out by many authors, initially by Migdal\(^2\) and Sawyer and Scalapino.\(^3\) The finite temperature propagator was first calculated by Ruck, Gyulassy and Greiner,\(^4\) and more recently by Hecking, Weise, and Akhoury.\(^5\) We make use of this most recent calculation of the condensate critical density as a function of temperature.

Our strategy in brief is to renormalize the pion-nucleon coupling \( g_\pi \) in our theory to such a value as to reproduce at each temperature, the critical density as calculated by Hecking et al.\(^5\) In this way we constrain our theory to the bulk properties of the normal state, and the best estimates, to date, of the pion condensation threshold density.

We make two remarkable observations concerning the pion condensed state, which are illustrated in the figures. In figure 1 is shown the binding energy per nucleon as a function of density at various temperatures between 0 and 100 MeV. At low temperature, the condensate makes very little contribution to the energy, when the theory is constrained by the bulk properties of nuclear matter, as was found in our earlier work.\(^1\) But at higher temperatures the condensate makes an increasingly important contribution. At a temperature a little higher than 50 MeV the condensate produces an isomer, which by \( T = 100 \text{ MeV} \)
is lower than the normal state at that temperature. This dramatic softening in
the equation of state will strongly influence the hydrodynamical flow of hot
dense matter, if a transition to the condensed state occurs. This is likely in
the region above the critical density, since the free energy in the pion con­
densed state is lower than in the normal state at the same energy and baryon
densities. Moreover because the temperature is much higher in the condensed
state, a subsequent transition to a quark matter phase may be facilitated, if
the density is high enough.

There is another observation with interesting consequences. As is well
known, the pion condensed state corresponds to a specific alignment of spin and
isospin, an isospin lattice, having an orientation in space with a wave number
\( k \approx 1.5 \text{ fm}^{-1} \). What has not been shown previously is that in this direction the
pressure is greater than in the transverse directions. This is true at any
temperature but the anisotropy grows both with temperature and density, as shown
in figure 2. There are several consequences. First, if a pion condensed state
is formed in a nuclear collision, the anisotropy in pressure will cause the
nuclear material to be ejected in bulk preferentially in the condensate direc­
tion. The second consequence has to do with neutron stars. They are bound by
gravity and as a consequence their density increases toward their center. If
conditions allow for the development of a pion condensed state in the dense
interior, the anisotropic pressure will tend to deform the core of the star.
This tendency is opposed by gravity, so that a stable deformation will be
reached. A crude estimate, based on the \( T = 0 \) conditions of symmetric matter
and assuming for simplicity Newtonian mechanics and a uniform mass density,
yields a ratio of axis of about 1.5/1. The two most questionable assumptions in
this calculation, the symmetric matter condensate and uniform density are both
conservative with respect to the estimate of the deformation. Of course there
are other factors such as high angular frequencies and strong magnetic fields
that will effect the shape of a real star. But the above estimate indicates
that the condensate in addition to effecting the internal structure, will play
a vital role in the large scale dynamics of the star.

In summary we have found density isomers in a self-consistent theory
of the pion condensed state. We have also noted that the pressure in the con­
densate state is not isotropic as has been assumed previously. This general
property of the condensed state can be inferred by examining the stress-energy	ensor.
Fig. 1. Binding energy for asymmetric nuclear matter. Solid lines are normal and dashed pion condensate state. The critical density follows the trajectory in the p-T plane calculated in Ref. 5 for \( g' = 6 \).

Fig. 2. Pressure of symmetric nuclear matter. Solid lines are the normal and dashed the condensate state. For the condensate state the pressure, parallel to the condensate direction is greater than in the perpendicular direction.

References

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