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THE VELOCITY DEPENDENCE OF BUBBLE-TRACK DENSITY

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The Velocity Dependence of Bubble-Track Density

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The foregoing analysis does not require knowledge of the connection between the bubble density \( g \) and the mean rate at which the moving particle loses energy. In this note the indirect nature of this relationship is studied.

In track-recording instruments, such as emulsion or bubble chambers, the particle traversing the sensitive volume makes its passage known by a series of energy transfers to electrons of the medium. Some, but generally only a part, of this energy is in a form which can be utilized by the detector.

Let \( o(\beta, w)\, dw \) be the cross section for transfer of an amount of energy between \( w \) and \( w + dw \) to an electron of the detector by an energetic particle of velocity \( \beta \). Then the mean energy loss in unit path is

\[
\kappa(\beta) = n \int_0^\infty w o(\beta, w)\, dw ,
\]  

(1)
where \( n \) is the electron density. The particle detector responds to these energy transfers, but transitions in certain energy intervals are efficient in producing a track, in others, inefficient. For track formation this causes the integrand to be weighted by a certain efficiency function \( \varepsilon(w) \) characteristic of the detector (and perhaps its condition of operation). Thus

\[
g(\beta) = n \int_0^\infty \varepsilon(w)\sigma(\beta,w)dw.
\]

(2)

In a bubble chamber high local concentrations of heat (thermal spikes) are believed to initiate the growth of bubbles. On the other hand, in emulsion a number of electrons must be raised to conduction bands in the silver halide crystal, and subsequently trapped, before it is rendered developable. Very large values of \( w \) do not contribute to the track in either instrument because such energy transfers produce knock-on electrons which leave the track locus. Therefore, for this reason alone, \( g \) cannot be proportional to the rate of energy loss \( k \) for either track-recording instrument.

It is usual, in discussing the problem of energy loss of a charged particle as it penetrates matter, to break the integral of Eq. (1) into two parts:

\[
k = n \int_0^w w\sigma(\beta,w)dw + n \int_{w_1}^\infty w\sigma(\beta,w)dw
\]

\[
= k_1 + k_2.
\]

(3)
The first integral, which is the energy-loss rate in soft collisions, depends on the properties of the stopping material. The limit, \( w_1 \), is chosen high enough so that in the second integral the collision cross section of the moving particle with an electron is unaffected by the electron binding. This cross section is well approximated by

\[
s(\varepsilon, w)dw = \frac{2\pi^2 r_o^2 m c^2}{\beta^2} \left[ 1 - \frac{w(1 - \beta^2)}{2mc^2} \right] \frac{dw}{w^2}
\]  

(4)

for \( w > w_1 \). Here \( m \) is the electron mass and \( r_o \), its classical radius. The moving particle carries \( z \) units of charge.

For small values of \( w \) the transition probabilities from the initial state to excited states of relative energy \( w_\alpha \), extending into the continuum, are governed by the oscillator strengths \( f_\alpha \). The net effect is expressed by the value of the mean excitation potential, \( I \). It is given by \( \ln I = \sum_\alpha f_\alpha \ln w_\alpha \). The actual values of the integrals are

\[
k_1 = n \int_0^{w_1} w_\sigma(w)dw \approx \frac{2\pi^2 r_o^2 m c^2}{\beta^2} \left[ \ln \frac{2mc^2 \beta^2 \gamma^2 w_1}{\gamma^2 w_1} - \beta^2 - \delta \right]
\]

(5)

and

\[
k_2 = n \int_{w_1}^{\infty} w_\sigma(w)dw \approx \frac{2\pi^2 r_o^2 m c^2}{\beta^2} \left[ \ln \frac{2mc^2 \beta^2 \gamma^2 w_1}{w_1} - \beta^2 \right]
\]

(6)

when \( w_1 \ll mc^2 \).
The quantity \( \delta \) has been calculated by Sternheimer.\(^3\) It evaluates the effect of polarization of the medium on the ionization loss.

In complete analogy with Eq. (3) we write

\[
g = n \int_0^{w_1} E(w) \sigma(\beta, w) dw + n \int_{w_1}^{\infty} E(w) \sigma(\beta, w) dw
\]

\[
= g_1 + g_2.
\]

One of the simplest models for the formation of a bubble track assumes that delta rays are the sites of bubbles. Energy transfers that are too small -- less than \( w' \), for example -- are not supposed to contribute to the bubble density, and delta rays of high energy, greater than \( w'' \), leave the particle trajectory, and do not form part of the track.

In using this model the number of delta rays in the interval \( w' \) to \( w'' \), rather than the energy lost to them, determines the bubble density. Suppose \( w' > w_1 \), then \( g_1 = 0 \) and \( E(w) = 1 \) in the interval \( w' \) to \( w'' \). Then the density of useful delta rays is

\[
g \approx \frac{2m_0^2 r^2 \beta^2}{\beta^2} \left[ \frac{1}{w'} - \frac{1}{w''} - \frac{(1 - \beta^2)}{2mc^2} \ln \frac{w''}{w'} \right].
\]

\[
\approx g/\beta^2.
\]
(It is supposed that $\beta$ is high enough so that $w''$ is less than the maximum possible energy transfer to an electron.) This expression for $g$ does not pass through a minimum as the velocity increases. The model, however, doubtless is too simplified, especially for liquids containing atoms of high atomic number. The formula (8) fails and $g_1$ is not zero when $w_1$ cannot be taken less than $w'$. The function $\bar{g}(w)$ then does not vanish in the interval of $w$ where $\sigma(\lambda, w)$ is affected by the electron binding—namely, where $w$ is not very much greater than $I$. A relativistic rise beyond the minimum somewhat paralleling that in the restricted energy-loss function $h$ then can occur, and the relation, $g = 08^{-2}$, will be inexact.

The relative roles of single and multiple energy losses in the formation of a bubble also can be discussed in a general way.

Suppose that in a volume of linear dimension $\lambda$ an amount of energy $\epsilon$ must be deposited for a bubble to form. Only very near the end of its range does the energy-loss rate of an electron pass through a maximum. (For it to initiate a bubble, the delta-ray energy must be transformed to heat locally by such processes as collisions of the second kind.) Where an electron stops is a probable site for bubble formation if the electron energy corresponding to a residual range $\lambda$ exceeds $\epsilon$.

For example, in propane the delta-ray range, $R$, is approximately

$$ R = 6 \times 10^{-6} \omega^{1.5} \text{ cm,} $$

where $\omega$ is in kev. The magnitude of $\lambda$ is perhaps $2 \times 10^{-6}$ cm. 

(9)
The average energy deposited in $2 \times 10^{-6}$ cm of delta-ray track at the terminus is therefore about 450 ev. When $\epsilon$ is less than this figure, a bubble may be expected to form where a delta ray stops. However, by underexpanding a bubble chamber, the threshold energy for bubble formation can be made larger than 450 ev.

Electrons scatter strongly at low velocities. The electron path may be bent back on itself, and the energy dissipated in a small volume thus increased. For electrons, the energy-loss straggling also is very high. Nevertheless, as $\epsilon$ increases it becomes more and more improbable for a lone delta ray to initiate a bubble.

If energy $\epsilon$ is not deposited by a single delta ray, penetration of the same volume by more than one electron can cause $\epsilon$ to be exceeded. The theory of Landau$^6$ provides one with the distribution of energy losses in a small segment of the particle path that results from plural elementary energy transfers. This energy of molecular ionization and excitation, however, may not all remain close to the point of transfer, because some delta rays have appreciable ranges, and the energy sometimes may escape by radiation. These various circumstances so complicate the analysis that the detailed discussion for the present remains qualitative. The function $E(w)$ characterizing the instrument is best found empirically.

If a bubble chamber is operated cold or underexpanded, the relativistic rise in bubble density should tend to be suppressed if bubbles form on single delta rays, and accentuated if they are produced by the cooperative effect of a number of small transfers of energy. This behavior could be the basis of an experiment to study the relative importance of single and multiple processes.

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References


