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SU(3) Decay Amplitudes of Pentaquarks into Decuplet Baryons

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Abstract

Evidence for the existence of exotic, pentaquark baryons was first found in the invariant mass spectrum of a baryon-meson pair, with the baryon in the octet of flavor SU(3). Exotic pentaquarks may also decay into members of the spin $\frac{3}{2}$ baryon decuplet. We calculate relations between decay rates of the exotic pentaquark into a baryon decuplet and an octet of pseudoscalar mesons based on SU(3) flavor symmetry, including the leading flavor symmetry breaking term. We consider all possible representations for an exotic pentaquark, namely, that it belongs to either a $\mathbf{10}$, $\mathbf{27}$ or $\mathbf{35}$ representation of flavor SU(3). In addition, we use the approximate SU(6) spin-flavor symmetry of baryons to derive relations between the reduced matrix elements of our calculation and those of an existing analogous calculation for decays into the baryon octet. By comparing several decay rates of exotics into both octet and decuplet baryons, our results could be used to elucidate the SU(3) nature of pentaquarks.
I. INTRODUCTION

Recent evidence for pentaquark states (see, e.g. [1]) has generated excitement and a resurgence of interest in hadron spectroscopy. For the recent review of the experimental status of pentaquark searches see [2] and references therein. The very existence of these exotic states provides a challenge for theorists. It is fair to say that our understanding of the mechanism by which strong forces bind quarks into baryons is crude. There is no theoretical consensus on the nature of the pentaquark states or even as to whether they are predicted by QCD [3], let alone on the production mechanisms [4] of exotics in different experimental setups.

Flavor symmetry may help us gain some insights into the nature of the forces that bind quarks. Pentaquarks form one or more multiplet of states that belong to an irreducible representation of \( SU(3) \)-flavor symmetry. For a state with the quantum numbers of four quarks and one antiquark the possible exotic representations are \( \mathbf{10} \), \( \mathbf{27} \) or \( \mathbf{35} \). Which of these representations is found in nature is determined by strong interaction dynamics. Therefore, a determination of which \( SU(3) \) multiplet observed pentaquarks fall into would advance, if only minimally, our understanding of the forces that bind them. With only a few members of a multiplet observed, distinguishing between representations is challenging. \( SU(3) \) relations indicate different patterns of decays for different representations.

The implicit assumption for an \( SU(3) \)-symmetry analysis is the approximate validity of the symmetry. Symmetry breaking effects originate from difference in light quark masses (and the fact that the largest, \( m_s \) is not negligibly small relative to the hadronic scale). Electromagnetic interactions also break the symmetry, but the effect is usually much smaller. Our analysis incorporates symmetry breaking effects to first order in \( m_s \). A wealth of evidence accumulated over 40 years shows that \( SU(3) \) flavor symmetry relations hold to about 30% accuracy. Therefore, our relations for pentaquark decays are expected to be valid to a precision of approximately \((0.3)^2 \sim 10\%\). Of course, the analysis is a prerequisite for a test of the validity of \( SU(3) \) symmetry in this new hadronic realm.

Baryons exhibit yet another approximate symmetry [5]: \( SU(6)_c \) (the ”c” stands for ”contracted”). The origin of the symmetry is not completely understood. With some technical assumptions it has been shown that \( SU(6)_c \) appears as a symmetry of baryons in the large \( N_c \) expansion of QCD [6]. Implications of the symmetry, including \( SU(6)_c \) breaking effects, have been investigated in detail. The \( 1/N_c \) expansion enables one to investigate systematically these symmetry breaking effects. The comparison with experiment shows that this expansion works fairly well [7]. Hence it is reasonable to expect that \( SU(6)_c \) holds as an approximate symmetry for properties of pentaquarks.

In this paper we calculate decay amplitudes of pentaquarks into decuplet baryons and pseudoscalar mesons in terms of several reduced matrix elements using \( SU(3) \)-flavor symmetry including first order symmetry breaking. Together with the amplitudes for the decays into octet baryons in Ref. [8] they give the complete picture of decay amplitudes into ground state baryons. Previous work had considered the symmetry relations that follow in the \( SU(3) \) symmetry limit [9], ignoring symmetry breaking effects. Furthermore we use \( SU(6)_c \) symmetry to give all of the reduced matrix elements for all of the exotic multiplets decaying into both octet and decuplet baryons in terms of four independent parameters. To this end we assume that the exotic pentaquark multiplets are all members of the 700 irreducible representation of \( SU(6) \). We have not investigated relations that arise from assuming the exotics fall into other irreducible representations of \( SU(6) \). We have only included flavor symmetry
breaking of $SU(6)$, but neglected symmetry breaking suppressed by $1/N_c$ that includes, for example, spin-symmetry breaking. That is, we work consistently to leading order in $1/N_c$ and to first order in $m_s$.

The paper is organized as follows. The results of the calculation are presented in Secs. V, VI, and VII and the brief Sec. II explains the contents of those tables. The method used to perform the calculation leading to those tables is described in section III. The $SU(6)$ calculation relating the reduced matrix elements is presented in section IV. The reader who needs to use the results may safely skip Secs. III and IV, and can first refer to Sec. II for instructions on how to use the tables, and then consult them in Secs. V–VII.

II. HOW TO READ OUR RESULTS

We here briefly explain how to use the tables in Secs. V, VI, and VII, how to further sharpen those results using the $SU(6)$ symmetry relations in Sec. IV and give some examples.

Consider the pentaquark decay $\Xi^{-10} \rightarrow \Xi^{*-}\pi^-$, with exotic final state flavor $ssdd\bar{u}$. From the table in Sec. V we read that the amplitude for this process is $A(\Xi^{-10} \rightarrow \Xi^{*-}\pi^-) = -5\sqrt{3}\gamma_{10}$. Similarly, $A(\Xi_0^{10} \rightarrow \Xi^{*-}\pi^+) = 5\gamma_{10}$. It follows immediately that

$$\Gamma(\Xi^{-10} \rightarrow \Xi^{*-}\pi^-) = 3\Gamma(\Xi_0^{10} \rightarrow \Xi^{*-}\pi^+). \quad (1)$$

This relation is a consequence of isospin symmetry. Relations that do not follow solely from isospin must account for differences in kinematics. These arise from two sources, phase space and momentum dependence of the reduced matrix elements in the amplitudes. In addition, since these differences are a consequence of $SU(3)$ breaking, one must also account for $SU(3)$ breaking in the amplitude, and our results include symmetry breaking to linear order in the symmetry breaking parameter, $m_s$.

While accounting for phase space differences is trivial, for the kinematic dependence of the amplitudes we need further understanding of the dynamics. Soft pion theorems require that pseudoscalar mesons are derivatively coupled. Hence the amplitude is at least linear in the momentum, and we make the assumption that this linear coupling is present. In particular, this means that our results are a simultaneous expansion in the $SU(3)$ symmetry breaking parameter and in the momentum of the pseudoscalar meson. We see then that rates are proportional to the third power of momentum. The analysis in section IV makes this explicitly clear since it introduces the coupling of the pseudoscalar mesons with an axial current as an interpolating field. For example, from the same table we have also $A(\Xi^{-10} \rightarrow \Sigma^{*-}K^-) = 5\sqrt{3}\gamma_{10}^*\bar{p}^i$ and hence

$$\frac{1}{|\vec{p}|^3}\Gamma(\Xi^{-10} \rightarrow \Sigma^{*-}K^-) = \frac{1}{|\vec{p}|^3}\Gamma(\Xi^{-10} \rightarrow \Xi^{*-}\pi^-) = \frac{3}{|\vec{p}|^3}\Gamma(\Xi_0^{10} \rightarrow \Xi^{*-}\pi^+), \quad (2)$$

where it is understood that $\vec{p}$ stands for the momentum of the final state particle in the relevant reaction. Some decays, particularly when the final state includes a $K$-meson, are not kinematically allowed and should not be included in the relations. In this example $m_K + m_{\Sigma^*} = 1880$ MeV, and a candidate for $\Xi^{-10}$ has been reported[10] with mass 1862 MeV, so the first relation in Eq. (2) above becomes meaningless.

We include in the tables all possible decay modes consistent with flavor conservation, regardless of whether the process is kinematically allowed. The amplitudes for kinematically forbidden modes may be useful in computing off-shell amplitudes.
Referring now to section IV we see, from Eq. (41), that $SU(6)_c$ implies $\gamma_{\Xi_{10}} = 0$. So, to this order the decay rates above vanish. This means, for example, that compared to the decay rate for $\Xi_{10}^{--}$ into members of the baryon octet, which are allowed in the symmetry limit, the decays into members of the decuplet are suppressed by at least two powers of $m_s$:

$$\frac{1}{|\vec{p}|^3} \Gamma(\Xi_{10}^{--} \to \Xi^{*--} \pi^-) \lesssim (0.3)^2 \sim 0.1. \quad (3)$$

This result is peculiar to the $\Xi_{10}$ multiplet. In fact the roles of allowed and suppressed decays are inverted in Eq. (3) if the $\Xi^{--}$ belongs to the $\Xi_{35}$ multiplet, and the ratio is order of unity if in the $\Xi_{27}$. We emphasize that we only retain terms of leading order in $1/N_c$. In particular, terms that break $SU(6)_c$ through spin operators are neglected.

In the examples of Eq. (2), the amplitudes were all given in terms of a single reduced matrix element. This is not generally the case. Consider, for example, decays of a $\Xi'_{27}^{--}$ that belongs to the $\Xi_{27}$: 

$$\frac{1}{|\vec{p}|^3} \Gamma(\Xi'_{27}^{--} \to \Sigma^{*-} K^-) = | -\sqrt{2/3} \alpha_{27}^* + 4 \sqrt{2/3} \delta_{27}^* - 2 \sqrt{2/3} \epsilon_{27}^* - 24 \sqrt{6} \zeta_{27}^* |^2 \quad (4)$$

$$\frac{1}{|\vec{p}|^3} \Gamma(\Xi'_{27}^{--} \to \Xi^{*-} \pi^-) = | -\sqrt{2/3} \alpha_{27}^* + 4 \sqrt{2/3} \delta_{27}^* - 2 \sqrt{2/3} \epsilon_{27}^* + 24 \sqrt{6} \zeta_{27}^* |^2. \quad (5)$$

We see that for the $\Xi_{27}$ the amplitudes differ at order $m_s$.

There is an implicit spin-wavefunction factor in our amplitudes. We have omitted this from our expressions, since it is straightforward to re-introduce it. For example, the amplitude for $\Xi_{10}^{--} \to \Xi^{*-} \pi^-$ discussed at the opening of this section involves a spin-$\frac{1}{2}$ to a spin-$\frac{1}{2}$ plus spin-0 decay. The amplitude has a factor of

$$\frac{1}{\sqrt{2}} \chi(\Xi^{*-})^\dagger [\hat{\vec{p}} \cdot \vec{\sigma}] \chi(\Xi_{10}^{--}) = \frac{1}{\sqrt{2}} \left( \chi(\Xi^{*-})^\dagger \chi(\Xi_{10}^{--})^\dagger - \chi(\Xi^{*-})^\dagger \chi(\Xi_{10}^{--})^\dagger \right), \quad (6)$$

where $\chi$ is a bi-spinor, $\vec{\sigma}$ are Pauli matrices, $\hat{\vec{p}}$ is a unit vector in the direction of the pion momentum, and the last equality follows from assuming $\vec{p}$ is along the $z$-axis. In fact, with $\vec{p}$ along the $z$-axis, the same factor occurs for transitions between spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons, with the understanding that only the middle two components of the four component spin-$\frac{3}{2}$ spinor contribute. The case of spin-$\frac{3}{2} \to$ spin-$\frac{3}{2}$ is slightly different. The factor in these amplitudes is

$$\frac{1}{\sqrt{20}} \left( 3 \chi_+^\dagger \chi'_- + \chi_+^\dagger \chi'_+ - \chi_-^\dagger \chi'_- - 3 \chi_-^\dagger \chi'_+ \right), \quad (7)$$

where, again, we have specialized to the case of $\vec{p}$ along the $z$-axis, and $\chi$ and $\chi'$ are four component spinors.

### III. $SU(3)$ CALCULATION

In this section the calculation of $SU(3)$ relations is described. The computational algorithm is the same as that implemented in [8], where we refer the interested reader for details.
A. \( (\overline{10}, \frac{1}{2}) \) Decay

The tensor decomposition
\[
8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35 \tag{8}
\]
shows that the decay of a \( \overline{10} \) pentaquark into baryon decuplet and octet of pseudoscalar mesons is forbidden in the limit of \( SU(3) \) flavor symmetry. The first contribution to the decay amplitude is due to the flavor breaking piece in the strong interactions Hamiltonian. So, the matrix element to be calculated is
\[
\langle 8 \otimes 10 | O^i_j (T^8_8)_{ji} | \overline{10} \rangle. \tag{9}
\]
Here \( O^i_j \) is a decay operator due to the chiral symmetry breaking term in QCD Hamiltonian that transforms like the \( T^8_8 \) component of an octet tensor under flavor \( SU(3) \). Vector \( \langle 8 \otimes 10 \rangle \) is the direct product of the pseudoscalar meson octet and baryon decuplet states. Vector \( | \overline{10} \rangle \) stands for a \( \overline{10} \) pentaquark. Taking the product of the tensor decomposition Eq. (8) with its conjugate,
\[
8 \otimes \overline{10} = 8 \oplus \overline{10} \oplus 27 \oplus 35 \tag{10}
\]
we see that there are only two irreducible matrix elements parameterizing Eq. (9), corresponding to the product of two \( 8 \)'s and of two \( 27 \)'s in the meson × baryon and \( T^8_8 \times \) pentaquark tensor decompositions. The explicit form of the meson \( (M^j_r) \), baryon \( (B^j_r) \), and \( \overline{10} \) pentaquark \( (P^j_i) \) tensors is given by Eqs. (5) and (6) in [8]. (Note that in the present paper we use symbols \( P \) for pentaquark states and \( D \) for decuplet states.) The independent components of the completely symmetric baryon decuplet tensor are
\[
\begin{align*}
D^{111} &= \Delta^{++}, & D^{112} &= \frac{\Delta^+}{\sqrt{3}}, & D^{122} &= \frac{\Delta^0}{\sqrt{3}}, & D^{222} &= \Delta^-, \\
D^{113} &= \frac{\Sigma^{*+}}{\sqrt{3}}, & D^{123} &= \frac{\Sigma^{*0}}{\sqrt{6}}, & D^{223} &= \frac{\Sigma^{*-}}{\sqrt{3}}, \\
D^{133} &= \frac{\Xi^{*0}}{\sqrt{3}}, & D^{113} &= \frac{\Xi^{*-}}{\sqrt{3}}, & D^{333} &= \Omega^{-}.
\end{align*}
\tag{11}
\]
The explicit form of the irreducible \( 8 \) and \( 27 \) representations in the decomposition of the direct product of meson and baryon tensors is
\[
\begin{align*}
8^i_j &= \epsilon^{irs} (M^i_t)_r^j (D^t)_i q s j, \\
27^i_{jk} &= \epsilon^{rs(i} (M^{(k)}_r)_t^i (D^t)_s j l - \frac{1}{5} \delta^{i(i} S^{k)}_{s l}.
\end{align*}
\tag{12}
\]
Here parentheses stand for symmetrization. The \( 27 \) tensor is symmetric both in upper \( (ik) \) and lower \( (jl) \) indices. Hermitian conjugates are used because both tensors are wavefunctions of \( bra \) vectors and therefore transform as complex conjugate under \( SU(3) \) flavor group. Analogously, the corresponding \( 8 \) and \( 27 \) in the decomposition of the product of \( T^8_8 \) and pentaquark tensor are
\[
\begin{align*}
8^i_j &= \frac{1}{\sqrt{2}} \epsilon^{irs} (T^8_8)^i_r q P^q s j, \\
27^i_{jk} &= \epsilon^{rs(i} (T^8_8)^k_r P^r s j l - \frac{1}{5} \delta^{i(i} S^{k)}_{r s l}.
\end{align*}
\tag{13}
\]
The extra factor of $1/\sqrt{2}$ is a convenient, although largely irrelevant, normalization. Finally, we write the matrix element in Eq. (9) in terms of two parameters (reduced matrix elements):

$$\langle 8 \otimes 10|O_j(T_8)^2|10 \rangle = \beta_{27}^* \cdot (8^\dagger)^i_j (8^\dagger)^l_i + \gamma_{27}^* \cdot (27^\dagger)^i_j (27^\dagger)^l_i.$$  

At this point it is a simple, but tedious and extensive, task to expand this expression in terms of the tensor components. This is easily done with the aid of a symbolic manipulator program. The result is shown as a table in Sec. V.

**B. (27, $\frac{1}{2}$) Decay**

The tensor decomposition in Eq. (8) contains a 27, so the strong decay amplitude of a 27 pentaquark already proceeds through the flavor symmetric part of the Hamiltonian. The matrix elements

$$\langle 8 \otimes 10|O|27 \rangle = \alpha_{27}^\ast \cdot e^{ijk} (M^\dagger)^i_j (D^\dagger)^{kl} W_{iq}^{lr},$$  

where $|27 \rangle$ stands for a pentaquark state, are all proportional to the single constant $\alpha_{27}^\ast$ and listed in Sec. VII. The components of the tensor $W_{iq}^{lr}$ are given in [3].

Turning to the contributions to the decay amplitude at first order in $SU(3)$ symmetry breaking, we analyze the generic matrix element

$$\langle 8 \otimes 10|O_j(T_8)^2|27 \rangle.$$  

The tensor decompositions

$$8 \otimes 27 = 8 \oplus 10 \oplus 16 \oplus 27 \oplus 35 \oplus 35 \oplus 64$$  

and Eq. (8) give that this matrix element is parameterized by five reduced matrix elements:

$$\langle 8 \otimes 10|O_j(T_8)^2|27 \rangle = \beta_{27}^\ast \cdot (8^\dagger)^i_j (8^\dagger)^l_i + \gamma_{27}^\ast \cdot (10^\dagger)^{ijk} (10)_{ijk} + \delta_{27}^\ast \cdot (27^\dagger)^i_j (27 \ast)_{ijkl} + \epsilon_{27}^\ast \cdot (27^\dagger)^i_j (27b)_{ijkl} + \zeta_{27}^\ast \cdot (35^\dagger)^m_{ijkl} (35)_{ijkl}.$$  

The relevant tensors in this decomposition coming from meson×baryon states are given by Eq. (12) and

$$(10^\dagger)^{ijk} = (D^\dagger)^{r(ik)} (M^\dagger)^k_l$$

$$(35^\dagger)^m_{ijkl} = (D^\dagger)^{ijkl} (M^\dagger)^m_{ijkl} - \frac{1}{5} \delta^m_{ijkl} (10^\dagger)^{ijkl}.$$  

Their 27×$T_8$ counterparts are composed of the $T_8$ Gell-Mann matrix and the tensor $W_{ik}^{kl}$:

$$(8)^i_j = (T_8)^i_s W^{si}_{rj}$$

$$(10)^{ijk} = \epsilon^{rs(t)} W^{sr}_{rq} (T_8)^q_s$$

$$(27a)^{ik}_{jl} = (T_8)^r_j W^{ik}_{br} - \frac{1}{5} \delta^{(i}_{l} (8)^k_{j})$$

$$(27b)^{ik}_{jl} = (T_8)^{ij} W^{k}_{jl} - \frac{1}{5} \delta^{(i}_{l} (8)^k_{j})$$

$$(35)^{ijkl} = (T_8)^i_q W^{ijkl}_{mr} - \frac{1}{6} \delta^{(i}_{m} (10)^{ijkl}).$$  

All tensors correspond to irreducible representations of $SU(3)$ and therefore are symmetric and traceless. The amplitudes for specific decay modes that follow from Eq. (18) are listed in Sec. VII.
C. \((35, \frac{3}{2})\) Decay

The tensor decomposition Eq. (8) contains a 35, so, as was the case with the 27, the decay of the 35 pentaquark into a baryon decuplet state and a pseudoscalar meson proceeds through the flavor symmetric part of the Hamiltonian. Contrast this with the decay into a baryon octet and a pseudoscalar meson, which is forbidden in the \(SU(3)\) symmetry limit \[8\].

The values of matrix elements

\[
\langle 8 \otimes 10 | O | 35 \rangle = \alpha_{35}^* \cdot (M^\dagger)_i^m (D^\dagger)_j^k F_{ijkl}^{ijkl},
\]

where \(|35\rangle\) stands for a pentaquark state, are all proportional to the single parameter \(\alpha_{35}^*\) and listed in the table in Sec. VII. The components of the tensor \(F_{ijkl}^{ijkl}\) can be found in \[8\]. It follows from tensor decomposition

\[
35 \otimes 8 = 10 \oplus 27 \oplus 28 \oplus 35 \oplus 35 \oplus 64 \oplus 81
\]

that the flavor symmetry breaking part of the decay amplitude is parameterized by four reduced matrix elements:

\[
\langle 8 \otimes 10 | O^j_i (T_8)^i_j | 35 \rangle = \beta_{35}^* \cdot (10^\dagger)^{ijk} (10)^{ijk} + \gamma_{35}^* \cdot (27)^j_i (27)^i_j
\]

\[
+ \delta_{27}^* \cdot (35^\dagger)^{ijkl} (35_a)^{ijkl} + \epsilon_{27}^* \cdot (35^\dagger)^{ijkl} (35_b)^{ijkl}.
\]

The relevant tensors in the decomposition of \(35 \times (T_8)\) are:

\[
(10)^{ijk} = (T_8)^r_s F^{rijk}_r
\]

\[
(27)^{ijkl} = \epsilon_{rs(t(j)k)}^q (T_8)^s_q
\]

\[
(35a)^{ijkl} = (T_8)^r_m F^{ijkl}_r - \frac{1}{6} \delta_m^{ijkl}
\]

\[
(35b)^{ijkl} = (T_8)^{ijk} (35^\dagger)^{ijkl} - \frac{1}{6} \delta_m^{ijkl}.
\]

The amplitudes for the specific decay modes in Eq. \[23\] are listed in Sec. VII.

IV. \(SU(6)\) CALCULATION

In the previous section we saw how the decay amplitudes of pentaquarks to a baryon decuplet are given in terms of a few reduced matrix elements. The decays of pentaquarks into a baryon octet are similarly given in terms of a different set of reduced matrix elements. \(SU(3)\)-flavor symmetry gives no relation among the decay parameters for the decuplet and octet final states. However, \(SU(6)\) spin-flavor symmetry does relate the matrix elements of decays into octet and decuplet final states.

Spin-flavor \(SU(6)\) is an approximate symmetry of baryons. It was motivated by the success of the non-relativistic quark model, in which the \(SU(3)\) of flavor and the separate \(SU(2)\) symmetry of spin (an internal symmetry in the non-relativistic limit) are immediately promoted to a full \(SU(6)\), and is supported empirically. More recently it has been shown that, given some mild technical assumptions, contracted \(SU(6)\) is a symmetry of the baryon sector of QCD in the large \(N_c\) limit \[6, 7\], and that the pattern of symmetry breaking implied
by the large-$N_c$ works well\cite{11}. While $SU(6)$ works relatively well for strong decays, its predictions are poor for non-leptonic weak decays\cite{12}.

The baryon octet $(8, \frac{1}{2})$ and decuplet $(10, \frac{3}{2})$ states comprise the 56 multiplet of $SU(6)$. Hence pentaquark decays into the two multiplets are related in the $SU(6)$ limit. Moreover, the $(\bar{1}, 1, 1, 1)_{700}$ representation of $SU(6)$ contains, among others, the $(\bar{10}, \frac{1}{2})$, $(27, \frac{1}{2})$, and $(35, \frac{3}{2})$ representations. This suggests that pentaquark multiplets exist not just in one of the representations discussed in Sec.\cite{11} but rather in all, with the appropriate spin\cite{13}. In the following we assume that pentaquarks are members of this 700 representation, so we will be able to relate the reduced matrix elements of the $(\bar{10}, \frac{1}{2})$, $(27, \frac{1}{2})$, and $(35, \frac{3}{2})$ representations. The reader should bear in mind, however, that individually each of these $SU(3)$ representations may fit into other $SU(6)$ representation and have, possibly, different spin, in which case there are no relations among decays of the $\bar{10}$, $27$, and $35$ representations.

The $SU(6)$ calculation proceeds much as the $SU(3)$ calculation. Following\cite{12} one builds $SU(6)$ tensors as a direct product of irreducible $SU(2)$-spin and $SU(3)$-flavor tensors. The $SU(2)$-spin tensors for spin $\frac{1}{2}$ and $\frac{3}{2}$ representations are

$$
\chi^1 = \left(\frac{1}{2}, \frac{1}{2}\right), \quad \chi^2 = \left(\frac{1}{2}, \frac{3}{2}\right), \quad \chi^{112} = \frac{1}{\sqrt{3}} \left(\frac{1}{2}, \frac{1}{2}\right), \quad \chi^{122} = \frac{1}{\sqrt{3}} \left(\frac{3}{2}, \frac{3}{2}\right), \quad \chi^{222} = \left(\frac{2}{2}, \frac{3}{2}\right).
$$

(25)

The 56-multiplet of baryonic states is represented by the tensor\cite{12}

$$(\mathbf{D}^\dagger)_{aibjck} = \chi^\dagger_{ij} (\mathbf{D}^\dagger)_{abc} + \frac{1}{3\sqrt{2}} \left[ \epsilon_{ij} \chi^\dagger_k \epsilon_{a\xi b\eta d} (\mathbf{B}^\dagger)_{\xi}^c + \epsilon_{jk} \chi^\dagger_i \epsilon_{c\xi d\eta} (\mathbf{B}^\dagger)_{\xi}^a + \epsilon_{ki} \chi^\dagger_j \epsilon_{r\xi s\eta} (\mathbf{B}^\dagger)_{\xi}^d \right].
$$

(26)

Here $a, b, c = 1, 2, 3$ are $SU(3)$ indices and $i, j = 1, 2$ are $SU(2)$ indices; $\epsilon_{ij}$ and $\epsilon_{abc}$ are invariant completely antisymmetric $SU(2)$ and $SU(3)$ tensors. The factor $\frac{1}{3\sqrt{2}}$ ensures the correct normalization. The tensor in Eq.\cite{26} is completely symmetric under permutation of the combined $SU(6)$ indices $(ai, bj, ck)$ and $SU(2)$ and $SU(3)$ tensors. Again we need complex conjugate tensors because $\chi$ represents a bra vector that transforms as a complex conjugate representation of $SU(6)$. Hermitian conjugation for $SU(2)$ tensors is performed by lowering the indices with $\epsilon_{ij}$:

$$
\chi^\dagger_i = \epsilon_{qr} \chi^r, \quad \chi_{ij}^\dagger = \epsilon_{ir} \epsilon_{js} \epsilon_{kt} \chi^{rs}.
$$

(27)

The tensor $(\bar{1}, 1, 1, 1)_{700}$ of $SU(6)$ is completely symmetric in its four upper (quark) indices $(ai, bj, ck, dl)$ and has one lower (antiquark) index $(ft)$. It vanishes if any upper index is contracted with the lower one. These properties unambiguously (up to an overall normalization factor) define the embedding of $(\bar{10}, \frac{1}{2})$ of $SU(3) \times SU(2)$ into 700 of $SU(6)$ by

$$
P(\bar{10})_{ft} = P_{fg} \chi_{ft} \left[ \epsilon^{gab} \epsilon^{hcd} \epsilon^{ij} \epsilon^{kl} + \epsilon^{gac} \epsilon^{hbd} \epsilon^{ik} \epsilon^{jl} + \epsilon^{gad} \epsilon^{hbc} \epsilon^{il} \epsilon^{jk} \right]
$$

$$
- \frac{1}{6} \delta^a_b \delta^c_d P_{gh} \chi_{rf} \left[ \epsilon^{gab} \epsilon^{hcd} \epsilon^{rj} \epsilon^{kl} + \epsilon^{gac} \epsilon^{hbd} \epsilon^{rk} \epsilon^{jl} + \epsilon^{gad} \epsilon^{hbc} \epsilon^{rl} \epsilon^{jk} \right].
$$

(28)

The embedding of $(27, \frac{1}{2})$ and $(35, \frac{3}{2})$ into 700 is even simpler and is given by

$$
P(27)_f = W_{fg} \epsilon^{gad} \epsilon^{bkl} (\chi^i \delta^j_l + \chi^j \delta^i_l) + \text{permutations}
$$

$$
P(35)_f = \epsilon^{gbd} \chi(ij) \delta^i_l).
$$

(29)
In order to have properly normalized tensors (so that each state occurs with unit weight in \((P^\dagger)_{aibjckdl}^f P_{aibjckdl}^{\dagger f}\)), we write the 700 decomposition in terms of the tensors in Eqs. (28) and (29) as

\[
P_{aibjckdl}^{\dagger f} = \frac{1}{6\sqrt{2}} P(\mathbf{10})_{aibjckdl}^{\dagger f} + \frac{1}{12\sqrt{2}} P(27)_{aibjckdl}^{\dagger f} + \frac{1}{2\sqrt{5}} P(35)_{aibjckdl}^{\dagger f}.
\]  

(30)

A consistent way of including pseudoscalar mesons into the SU(6) calculation is by the use of the axial current as an interpolating field which transforms as \((8, 1)\) under SU(3) \times SU(2) \[12\]. This is a part of the decomposition of \((\mathbf{1}, 1)_{35}\) of SU(6). So, the SU(6) calculation is essentially chiral perturbation theory calculation, where pseudoscalar mesons are components of axial current of broken chiral symmetry. Therefore when using SU(6) relations for the decay amplitudes one should multiply the amplitude by the momentum of meson in the final state. In the limit of unbroken SU(6) the momentum is the same for all mesons. Following Ref. \[12\] the SU(6) tensor for the pseudoscalar octet is written in terms of the standard octet tensor \(M^a\) as

\[
(M^\dagger)^a_{bj} = (M^\dagger)^a_{bj}((\vec{s} \cdot \hat{n})^t)^i_j, \quad \text{where} \quad (\vec{s} \cdot \hat{n})^i_j = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}
\]

(31)

and \(\hat{n}\) is an arbitrary unit vector denoting the direction of the momentum of the meson. Of course, our symmetry relations (34) and (41) are independent of this direction.

In the limit of unbroken SU(6) the decay modes

\[
(\mathbf{10}, \frac{1}{2}) \rightarrow (\mathbf{8}, \frac{1}{2}) + \text{mesons}
\]

\[
(27, \frac{1}{2}) \rightarrow (\mathbf{8}, \frac{1}{2}) + \text{mesons} \quad \text{and} \quad (27, \frac{1}{2}) \rightarrow (\mathbf{10}, \frac{3}{2}) + \text{mesons}
\]

\[
(35, \frac{3}{2}) \rightarrow (\mathbf{10}, \frac{3}{2}) + \text{mesons}
\]

(32)

are incorporated into the single matrix element

\[
\langle 35 \otimes 56 | O | 700 \rangle,
\]

(33)

where \(O\) is the SU(6) invariant part of the effective decay Hamiltonian and therefore are all proportional to a single parameter \[13\]. This gives the following relations between the SU(3) reduced matrix elements \(\alpha_{\mathbf{10}}\) and \(\alpha_{27}\) from \[3\] and \(\alpha_{27}^*\) and \(\alpha_{35}^*\) from Secs. \[\text{VI}\] and \[\text{VII}\]

\[
\alpha_{27} = \frac{\sqrt{2}}{3} \alpha_{\mathbf{10}}, \quad \alpha_{27}^* = \sqrt{\frac{2}{3}} \alpha_{\mathbf{10}} \quad \text{and} \quad \alpha_{35}^* = \sqrt{2} \alpha_{\mathbf{10}}.
\]

(34)

The transitions

\[
(\mathbf{10}, \frac{1}{2}) \rightarrow (\mathbf{10}, \frac{1}{2}) + \text{mesons} \quad \text{and} \quad (35, \frac{3}{2}) \rightarrow (\mathbf{8}, \frac{1}{2}) + \text{mesons}
\]

(35)

are forbidden in the limit of exact SU(6) symmetry. The corresponding amplitudes are due to flavor symmetry breaking terms in the effective decay Hamiltonian. The simplest SU(6) generalization of the flavor symmetry breaking operator \(T_8\) of SU(3)\(_F\) is obtained by combining it with \(\delta_j^i\) of SU(2)\(_\text{spin}\):

\[
(T_8)^{ai}_{bj} = (T_8)^a_ib^i_j\delta_j^i.
\]

(36)
This operator belongs to the \((8,0)\) representation of \(SU(3) \times SU(2)\) and therefore transforms as a 35 under \(SU(6)\) as well. \(SU(3) \times SU(2)\) decomposition of \((1,1)_{35}\) of \(SU(6)\) contains also \((8,1)\) and \((1,1)\). The corresponding operators are \(1/N_c\) suppressed and are not included in our analysis.

The tensor decomposition of products of \(SU(6)\) representations

\[
(\overline{1},1)_{35} \otimes (1,1,1)_{56} = (\overline{1},2,1,1)_{1134} \oplus (\overline{1},1,1,1)_{700} \oplus (2,1)_{70} \oplus (1,1,1)_{56}
\]

\[
(\overline{1},1)_{35} \otimes (\overline{1},1,1,1)_{700} = (\overline{1},2,1,1,1)_{56} \oplus (\overline{2},2,1,1,1)_{5670} \oplus (\overline{1},1,1,1,1)_{4536}
\]

\[
\oplus (\overline{2},1,1,1,1)_{3080} \oplus (\overline{1},2,1,1)_{1134} \oplus (\overline{1},1,1,1,1)_{700} \oplus (\overline{1},1,1,1,1)_{700} \oplus (1,1,1)_{56}
\]

shows that there are four reduced matrix elements contributing to the decay amplitude:

\[
\langle 35 \otimes 56|O_{ab}^{ij}(T_8)_{ai}^{bj}|700\rangle = 56 \cdot 56 \oplus 700 \cdot 700 \oplus 700 \cdot 700 \oplus 1134 \cdot 1134. \tag{38}
\]

The corresponding irreducible tensors in the decomposition of \(35 \otimes 56\), the product of the final baryon and meson states, are (here \(A,B,C,\ldots = 1,\ldots 6\) are \(SU(6)\) indices):

\[
\begin{align*}
(56)_{ABC} &= (D^\dagger)_{(ABS}(M^\dagger)_C^S), \\
(700)_{ABCD}^{F} &= (D^\dagger)_{(ABC}(M^\dagger)_D^F) - \frac{1}{9}(56)_{(ABC}\delta_D^F), \\
(1134)_{ABCD}^{F} &= (D^\dagger)_{ABC}(M^\dagger)_D^F - (D^\dagger)_{DBC}(M^\dagger)_A^F \\
&\quad + \frac{1}{40} \left( (D^\dagger)_{ABS}(M^\dagger)_C^S\delta_D^F + (D^\dagger)_{ACS}(M^\dagger)_B^S\delta_D^F - (D^\dagger)_{BDS}(M^\dagger)_C^S\delta_A^F - (D^\dagger)_{CBS}(M^\dagger)_B^S\delta_A^F \right) \\
&\quad + \frac{1}{8} \left( (D^\dagger)_{CBS}(M^\dagger)_A^S\delta_B^F + (D^\dagger)_{DBS}(M^\dagger)_C^S\delta_A^F - (D^\dagger)_{ABS}(M^\dagger)_D^S\delta_F^F - (D^\dagger)_{ACS}(M^\dagger)_D^S\delta_B^F \right) \\
&\quad + \frac{3}{20} \left( (D^\dagger)_{CBS}(M^\dagger)_A^S\delta_D^F - (D^\dagger)_{DBS}(M^\dagger)_B^S\delta_A^F \right). \tag{39}
\end{align*}
\]

We need hermitian conjugate tensors because they represent bra vectors. The \(56\) is completely symmetric under permutation of \(A,B,C\). The \(700\) is completely symmetric under permutation of \(A,B,C,D\) and vanishes if its upper index is contracted with a lower one. The 1134 has mixed symmetry: the indices \(A,B,C\) are symmetrized, while \(A,D\) are antisymmetrized. This fixes the first two terms of 1134. "The tail" is determined by tracelessness of 1134, i.e., the tensor must vanish if its upper index is contracted with a lower one.

The relevant irreducible tensors in the decomposition of \(35 \otimes 700\), the product of the pentaquark and the symmetry breaking Hamiltonian, are:

\[
\begin{align*}
(56)_{ABC} &= (T_8)_S^R P_R^{ABC}, \\
(700a)_{F}^{ABCD} &= (T_8)_S^F P_S^{ABCD} - \frac{1}{9}(56)_{ABC}\delta_F^D, \\
(700b)_{F}^{ABCD} &= (T_8)_S^{(D} P_S^{ABC)S} - \frac{1}{9}(56)_{ABC}\delta_F^D, \\
(1134)_{F}^{ABCD} &= (T_8)_S^{D} P_S^{ABCS} - (T_8)_S^A P_S^{DBCS} - \frac{1}{5} \left[ (56)_{ABC}\delta_F^D - (56)_{DBC}\delta_F^A \right]. \tag{40}
\end{align*}
\]

These tensors satisfy the same symmetry requirements as those in Eq. (39).

At first order in \(SU(3)\) breaking the amplitudes of a \(\overline{10}\) pentaquark decay into baryon octet plus pseudoscalar mesons are given in terms of four reduced matrix elements \(\beta,\beta'\),
\[ \gamma_{\Pi}, \delta_{\Pi}, \text{and} \epsilon_{\Pi}, \text{the same number as for the} \ SU(6) \text{decay amplitudes calculated in this section. Matching between the} \ SU(3) \text{and} \ SU(6) \text{calculations allows one to express all the reduced matrix elements in the present paper and} [8] \text{in terms of these four parameters only:} \]

\[
\begin{align*}
\beta^*_{10} &= \frac{4}{15} \left( -10\gamma_{\Pi} + 5\delta_{\Pi} + 2\sqrt{3}\epsilon_{\Pi} \right), \\
\gamma^*_{10} &= 0, \\
\beta_{35} &= -\frac{1}{\sqrt{3}} \left( -10\gamma_{\Pi} + 5\delta_{\Pi} + 2\sqrt{3}\epsilon_{\Pi} \right), \\
\gamma_{35} &= 0, \\
\beta^*_{35} &= -\frac{5}{\sqrt{6}} \gamma_{\Pi} + \sqrt{\frac{3}{2}} \delta_{\Pi} + \frac{17}{15\sqrt{2}} \epsilon_{\Pi}, \\
\gamma^*_{35} &= \frac{1}{4\sqrt{2}} \epsilon_{\Pi}, \\
\delta^*_{35} &= -\frac{1}{2\sqrt{6}} \beta_{\Pi} + \frac{5}{3\sqrt{6}} \gamma_{\Pi} - \frac{1}{3\sqrt{6}} \delta_{\Pi} - \frac{11}{30\sqrt{2}} \epsilon_{\Pi}, \\
\epsilon^*_{35} &= \frac{1}{2\sqrt{6}} \beta_{\Pi} + \frac{5}{6\sqrt{6}} \gamma_{\Pi} - \frac{1}{6\sqrt{6}} \delta_{\Pi} - \frac{11}{60\sqrt{2}} \epsilon_{\Pi}, \\
\beta_{27} &= -\frac{1}{9\sqrt{5}} \gamma_{\Pi} + \frac{45\sqrt{5}}{128} \delta_{\Pi} + \frac{225\sqrt{15}}{\epsilon_{\Pi}}, \\
\delta_{27} &= -\frac{5}{6\sqrt{3}} \gamma_{\Pi} + \frac{1}{6\sqrt{3}} \delta_{\Pi} + \frac{7}{45} \epsilon_{\Pi}, \\
\gamma_{27} &= -\frac{52}{9\sqrt{5}} \gamma_{\Pi} + \frac{142}{45\sqrt{5}} \delta_{\Pi} + \frac{808}{225\sqrt{15}} \epsilon_{\Pi}, \\
\epsilon_{27} &= \frac{1}{9} \epsilon_{\Pi}, \\
\zeta_{27} &= -\frac{1}{3\sqrt{3}} \beta_{\Pi} + \frac{5}{18\sqrt{3}} \gamma_{\Pi} - \frac{1}{18\sqrt{3}} \delta_{\Pi} - \frac{4}{45} \epsilon_{\Pi}, \\
\psi_{27} &= \frac{1}{3\sqrt{3}} \beta_{\Pi} + \frac{5}{9\sqrt{3}} \gamma_{\Pi} - \frac{1}{9\sqrt{3}} \delta_{\Pi} - \frac{8}{45} \epsilon_{\Pi}, \\
\beta^*_{27} &= \frac{16\sqrt{2}}{9} \gamma_{\Pi} - \frac{52\sqrt{2}}{45} \delta_{\Pi} - \frac{248\sqrt{2}}{225\sqrt{3}} \epsilon_{\Pi}, \\
\delta^*_{27} &= \frac{1}{3\sqrt{2}} \beta_{\Pi} - \frac{5}{18\sqrt{2}} \gamma_{\Pi} + \frac{1}{18\sqrt{2}} \delta_{\Pi} + \frac{1}{60\sqrt{6}} \epsilon_{\Pi}, \\
\gamma^*_{27} &= \frac{5}{27\sqrt{2}} \gamma_{\Pi} - \frac{1}{27\sqrt{2}} \delta_{\Pi} - \frac{135\sqrt{6}}{\epsilon_{\Pi}}, \\
\epsilon^*_{27} &= -\frac{1}{3\sqrt{2}} \beta_{\Pi} - \frac{5}{9\sqrt{2}} \gamma_{\Pi} + \frac{1}{9\sqrt{2}} \delta_{\Pi} + \frac{1}{30\sqrt{6}} \epsilon_{\Pi}, \\
\zeta^*_{27} &= \frac{1}{36\sqrt{6}} \epsilon_{\Pi}. \quad (41)
\end{align*}
\]

From the first two lines in Eq. (41) we see a remarkably simple pattern. In the leading order all decay amplitudes in (32) are proportional to the single parameter \( \alpha_{\Pi} \). The decay modes in (35), forbidden in the symmetry limit, are also proportional to a single parameter, \( (10\gamma_{\Pi} - 5\delta_{\Pi} - 2\sqrt{3}\epsilon_{\Pi}) \) at first order in \( m_s \).
V. DECAY AMPLITUDES OF 10 INTO DECTPLET BARYONS

Decay amplitudes of 10 pentaquarks to the baryon decuplet are forbidden in the SU(3) symmetry limit. Some decay modes are also kinematically forbidden according to the reported values for pentaquark masses $\Theta^+_{10} \sim 1540$ MeV [1] and $\Xi^- \sim 1862$ MeV [10]. In the first order of SU(3) breaking all isomultiplets in the 10 representation are equidistant, so the multiplet masses, in MeV, are roughly

$$\Theta^+_{10} \sim 1540, \quad N_{10} \sim 1647, \quad \Sigma_{10} \sim 1755, \quad \Xi_{10} \sim 1862.$$ \hspace{1cm} (42)

It follows that the decay of a $\Theta^+$ into baryon decuplet states is kinematically forbidden. The only decay modes of $\Theta^+_{10}(0, +1)$ allowed by flavor conservation are $\Delta^+(1/2, 0)K^0(-1/2, +1)$ and $\Delta^0(-1/2, 0)K^+/(1, 2, 0)$. These decays are forbidden: 1540 MeV vs $1240+495=1735$ MeV. In fact, the only kinematically allowed modes of members of the 10 are $N_{10} \to \Delta \pi$, $\Sigma_{10} \to K \Delta$, $\Xi_{10} \to \Xi^* \pi$, $\Xi_{10} \to \Xi^\ast \pi$ (1735 MeV).

The table below shows the values of the amplitudes in terms of two reduced SU(3) matrix elements. We include in the table all modes regardless of whether the process is kinematically allowed. The amplitudes for kinematically forbidden modes may be useful in computing off-shell amplitudes.

<table>
<thead>
<tr>
<th>$\Theta^+$</th>
<th>$\beta^+_{10}$</th>
<th>$\gamma^+_{10}$</th>
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<tr>
<td>$\Delta^+ K^0$</td>
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</tr>
<tr>
<td>$\Delta^0 K^+$</td>
<td>$\sqrt{2}$</td>
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</tr>
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</table>

<table>
<thead>
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<th>$N_{10}$</th>
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<th>$\gamma^\ast_{10}$</th>
</tr>
</thead>
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<tr>
<td>$\Delta^0 \pi^0$</td>
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<td>$\sqrt{3}$</td>
</tr>
<tr>
<td>$\Delta^+ \pi^0$</td>
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<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>$\Delta^0 \pi^+$</td>
<td>$\sqrt{2}$</td>
<td>$\sqrt{3}$</td>
</tr>
<tr>
<td>$\Delta^+ \pi^+$</td>
<td>$-1$</td>
<td>$\sqrt{2}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_{10}$</th>
<th>$\beta^\ast_{10}$</th>
<th>$\gamma^\ast_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^0 \pi^0$</td>
<td>$-1$</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>$\Delta^+ \pi^+$</td>
<td>$\sqrt{2}$</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

| $\Sigma^\ast K^0$ | $\sqrt{2}$ | $\sqrt{3}$ |
| $\Sigma^+ K^+$ | $-1$ | $\sqrt{2}$ |

| $\Xi^\ast K^0$ | $-1$ | $\sqrt{2}$ |
| $\Xi^+ K^+$ | $\sqrt{2}$ | $\sqrt{3}$ |

| $\Xi^{\ast -} \pi^+$ | $1$ | $\sqrt{2}$ |
| $\Xi^{\ast -} K^0$ | $\sqrt{2}$ | $\sqrt{3}$ |
| $\Xi^{\ast 0} K^0$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast 0} \pi^+$ | $\sqrt{2}$ | $\sqrt{3}$ |
| $\Xi^{\ast 0} K^+$ | $-1$ | $\sqrt{2}$ |

| $\Xi^{\ast -} \pi^-$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast -} K^0$ | $\sqrt{2}$ | $\sqrt{3}$ |
| $\Xi^{\ast 0} K^0$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast 0} \pi^-$ | $\sqrt{2}$ | $\sqrt{3}$ |
| $\Xi^{\ast 0} K^+$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast -} K^0$ | $\sqrt{2}$ | $\sqrt{3}$ |

| $\Xi^{\ast +} \pi^0$ | $1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} K^0$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} \pi^-$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} K^+$ | $\sqrt{2}$ | $\sqrt{3}$ |

| $\Xi^{\ast +} K^0$ | $1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} \pi^0$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} \pi^-$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} K^+$ | $\sqrt{2}$ | $\sqrt{3}$ |

| $\Xi^{\ast +} K^0$ | $1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} \pi^0$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} \pi^-$ | $-1$ | $\sqrt{2}$ |
| $\Xi^{\ast +} K^+$ | $\sqrt{2}$ | $\sqrt{3}$ |
VI. DECAY AMPLITUDES OF 27 PENTAQUARK MULTIPLET

Below we list the decays of the twentyseven states that form this irreducible representation of $SU(3)$ in terms of one parameter, $\alpha_{27}^*$, at zeroth order in $SU(3)$ symmetry breaking, plus five more parameters, $\beta_{27}$, $\gamma_{27}$, $\delta_{27}$, $\epsilon_{27}^*$, and $\zeta_{27}^*$ of order $m_s$. It is worth mentioning that there are several new “exotic decays”. For example, the decays $\Sigma_{27}^{++} \to \Delta^{++} K_{S,L}$ and $\Sigma_{27}^{-} \to \Delta^{-} K^{-}$ have final state flavor $uusd$ (or $uuuds$) and $ddsu$, respectively.

\[ \begin{array}{ccccccc} 
\Theta_{27}^{++} & \alpha_{27}^* & \beta_{27}^* & \gamma_{27}^* & \delta_{27}^* & \epsilon_{27}^* & \zeta_{27}^* \\
\Delta^{++} K^0 & -1 & 0 & 0 & -8 & 4 & 0 \\
\Delta^+ K^+ & \sqrt{1/3} & 0 & 0 & 8/\sqrt{3} & -4/\sqrt{3} & 0 \\
\end{array} \]

\[ \begin{array}{ccccccc} 
\Theta_{27}^+ & \alpha_{27}^* & \beta_{27}^* & \gamma_{27}^* & \delta_{27}^* & \epsilon_{27}^* & \zeta_{27}^* \\
\Delta^+ K^0 & -\sqrt{2/3} & 0 & 0 & -8\sqrt{2/3} & 4\sqrt{2/3} & 0 \\
\Delta^0 K^+ & \sqrt{2/3} & 0 & 0 & 8\sqrt{2/3} & -4\sqrt{2/3} & 0 \\
\end{array} \]

\[ \begin{array}{ccccccc} 
\Theta_{27}^0 & \alpha_{27}^* & \beta_{27}^* & \gamma_{27}^* & \delta_{27}^* & \epsilon_{27}^* & \zeta_{27}^* \\
\Delta^0 K^0 & -\sqrt{1/3} & 0 & 0 & -8\sqrt{1/3} & 4\sqrt{1/3} & 0 \\
\Delta^- K^+ & 1 & 0 & 0 & 8 & -4 & 0 \\
\end{array} \]

\[ \begin{array}{ccccccc} 
N_{27}^{'+} & \alpha_{27}^* & \beta_{27}^* & \gamma_{27}^* & \delta_{27}^* & \epsilon_{27}^* & \zeta_{27}^* \\
\Delta^{++}\pi^- & -\sqrt{2/15} & 3\sqrt{3/10} & 0 & -28/5\sqrt{2/15} & 2/5\sqrt{2/15} & 0 \\
\Delta^+\pi^0 & 2/3\sqrt{5} & -3/\sqrt{5} & 0 & 56/15\sqrt{5} & -4/15\sqrt{5} & 0 \\
\Delta^0\pi^+ & \sqrt{2/3}\sqrt{5} & -3/\sqrt{10} & 0 & 28/15\sqrt{2/5} & -2/15\sqrt{2/5} & 0 \\
\Sigma^{++} K^0 & 4/3\sqrt{2/5} & 3/\sqrt{10} & 0 & 112/15\sqrt{2/5} & -8/15\sqrt{2/5} & 0 \\
\Sigma^{*0} K^+ & -4/3\sqrt{5} & -3/2\sqrt{5} & 0 & -112/15\sqrt{5} & 8/15\sqrt{5} & 0 \\
\end{array} \]

\[ \begin{array}{ccccccc} 
N_{27}^0 & \alpha_{27}^* & \beta_{27}^* & \gamma_{27}^* & \delta_{27}^* & \epsilon_{27}^* & \zeta_{27}^* \\
\Delta^+\pi^0 & -1/3\sqrt{2/5} & 3/\sqrt{10} & 0 & -28/15\sqrt{2/5} & 2/15\sqrt{2/5} & 0 \\
\Delta^0\pi^0 & 2/3\sqrt{5} & -3/\sqrt{5} & 0 & 56/15\sqrt{5} & -4/15\sqrt{5} & 0 \\
\Delta^-\pi^+ & \sqrt{2/15} & -3\sqrt{3/10} & 0 & 28/5\sqrt{2/15} & -2/5\sqrt{2/15} & 0 \\
\Sigma^{*0} K^0 & 4/3\sqrt{5} & 3/2\sqrt{5} & 0 & 112/15\sqrt{5} & -8/15\sqrt{5} & 0 \\
\Sigma^{*-} K^+ & -4/3\sqrt{2/5} & -3/\sqrt{10} & 0 & -112/15\sqrt{2/5} & 8/15\sqrt{2/5} & 0 \\
\end{array} \]

\[ \begin{array}{ccccccc} 
\Delta^{++}_{27} & \alpha_{27}^* & \beta_{27}^* & \gamma_{27}^* & \delta_{27}^* & \epsilon_{27}^* & \zeta_{27}^* \\
\Delta^{++}\pi^0 & 1/2 & 0 & 27/2 & 1 & -2 & -9 \\
\Delta^{++}\eta & -\sqrt{3/2} & 0 & 9\sqrt{3/2} & -\sqrt{3} & 2\sqrt{3} & -15\sqrt{3} \\
\Delta^+\pi^+ & \sqrt{1/6} & 0 & 9\sqrt{3/2} & \sqrt{2/3} & -2\sqrt{2/3} & -3\sqrt{3} \\
\Sigma^{*-} K^+ & -\sqrt{1/6} & 0 & 9\sqrt{3/2} & \sqrt{2/3} & 2\sqrt{2/3} & 15\sqrt{6} \\
\end{array} \]
<table>
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<tr>
<th>( \Delta_{27}^\pm )</th>
<th>( \alpha_{27}^\pm )</th>
<th>( \beta_{27}^\pm )</th>
<th>( \gamma_{27}^\pm )</th>
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<td>(\sqrt{2}/3)</td>
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<td>( \Sigma^+K^+ )</td>
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<td>2/3</td>
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<td>( \Delta^0\pi^0 )</td>
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<td>(-9/2)</td>
<td>(-1/3)</td>
<td>2/3</td>
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<tr>
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<td>0</td>
<td>(9\sqrt{3}/2)</td>
<td>(\sqrt{2}/3)</td>
<td>(-2\sqrt{2}/3)</td>
<td>(-3\sqrt{6})</td>
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<tr>
<td>( \Sigma^{0}K^0 )</td>
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<td>0</td>
<td>(9)</td>
<td>(-2/3)</td>
<td>4/3</td>
<td>30</td>
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<td>0</td>
<td>(9/\sqrt{2})</td>
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<th>( \beta_{27}^+ )</th>
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<td>0</td>
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<td>(-4)</td>
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<td>0</td>
<td>(4/\sqrt{3})</td>
<td>(4/\sqrt{3})</td>
<td>36(\sqrt{3})</td>
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<td>0</td>
<td>0</td>
<td>(-2)</td>
<td>(-2)</td>
<td>18</td>
</tr>
<tr>
<td>( \Delta^+K^\pm )</td>
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<td>0</td>
<td>(2\sqrt{3})</td>
<td>(2\sqrt{3})</td>
<td>(-18\sqrt{3})</td>
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<tr>
<td>( \Sigma^{++}\pi^0 )</td>
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<td>(2\sqrt{2}/3)</td>
<td>(2\sqrt{2}/3)</td>
<td>18(\sqrt{6})</td>
</tr>
<tr>
<td>( \Sigma^{0}\pi^+ )</td>
<td>(\sqrt{1/6})</td>
<td>0</td>
<td>0</td>
<td>(-2\sqrt{2}/3)</td>
<td>(-2\sqrt{2}/3)</td>
<td>(-18\sqrt{6})</td>
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<td>0</td>
<td>(-2\sqrt{2}/3)</td>
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<tr>
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<td>(-\sqrt{3}/2)</td>
<td>0</td>
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<td>(-2\sqrt{2}/3)</td>
<td>(-18\sqrt{2})</td>
</tr>
<tr>
<td>( \Sigma^{0}\pi^+ )</td>
<td>(\sqrt{1/6})</td>
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<td>0</td>
<td>(-2\sqrt{2}/3)</td>
<td>(-2\sqrt{2}/3)</td>
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<td>(-2\sqrt{2}/3)</td>
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<td>(-18\sqrt{6})</td>
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</table>

14
\[
\begin{array}{cccccccc}
\Sigma_{27}^+ & \alpha_{27}^+ & \beta_{27}^+ & \gamma_{27}^+ & \delta_{27}^+ & \epsilon_{27}^+ & \zeta_{27}^+ \\
\Delta^- K^- & -1 & 0 & 0 & 4 & 4 & -36 \\
\Sigma^* - \pi^- & \sqrt{1/3} & 0 & 0 & -4/\sqrt{3} & -4/\sqrt{3} & -36\sqrt{3} \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\Sigma_{27}^0 & \alpha_{27}^0 & \beta_{27}^0 & \gamma_{27}^0 & \delta_{27}^0 & \epsilon_{27}^0 & \zeta_{27}^0 \\
\Delta^+ K^- & 1/2\sqrt{3} & -3/\sqrt{5} & -18/\sqrt{5} & 2/5\sqrt{5} & 2/5\sqrt{5} & 6/5 \\
\Delta^0 K^0 & 1/2\sqrt{15} & -3/5 & -6\sqrt{3}/5 & 2/5\sqrt{15} & 2/5\sqrt{15} & 2\sqrt{15} \\
\Sigma^* - \pi^0 & -\sqrt{3}/10 & \sqrt{3}/10 & -6\sqrt{6}/5 & -2/5\sqrt{6}/5 & -2/5\sqrt{6}/5 & 2\sqrt{30} \\
\Sigma^* + \eta & \sqrt{2}/5 & 3/\sqrt{10} & 0 & 4/5\sqrt{2}/5 & 4/5\sqrt{2}/5 & 12\sqrt{10} \\
\Sigma^* + \pi^+ & -\sqrt{3}/10 & \sqrt{3}/10 & -6\sqrt{6}/5 & -2/5\sqrt{6}/5 & -2/5\sqrt{6}/5 & 2\sqrt{30} \\
\Xi^0 K^+ & 2/\sqrt{15} & 3/\sqrt{10} & -6\sqrt{6}/5 & 4/5\sqrt{2}/15 & 4/5\sqrt{2}/15 & -4\sqrt{30} \\
\Xi^* - K^+ & 2/\sqrt{15} & 3/\sqrt{10} & -6\sqrt{6}/5 & 4/5\sqrt{2}/15 & 4/5\sqrt{2}/15 & -4\sqrt{30} \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\Sigma_{27}^0 & \alpha_{27}^0 & \beta_{27}^0 & \gamma_{27}^0 & \delta_{27}^0 & \epsilon_{27}^0 & \zeta_{27}^0 \\
\Delta^+ K^- & \sqrt{1}/30 & -6/\sqrt{5} & -6\sqrt{6}/5 & 2/5\sqrt{2}/15 & 2/5\sqrt{2}/15 & 2\sqrt{30} \\
\Delta^0 K^0 & \sqrt{1}/30 & -6/\sqrt{5} & -6\sqrt{6}/5 & 2/5\sqrt{2}/15 & 2/5\sqrt{2}/15 & 2\sqrt{30} \\
\Sigma^* - \pi^0 & -\sqrt{3}/10 & \sqrt{3}/10 & -6\sqrt{6}/5 & -2/5\sqrt{6}/5 & -2/5\sqrt{6}/5 & 2\sqrt{30} \\
\Sigma^* + \eta & \sqrt{2}/5 & 3/\sqrt{10} & 0 & 4/5\sqrt{2}/5 & 4/5\sqrt{2}/5 & 12\sqrt{10} \\
\Sigma^* - \pi^+ & -\sqrt{3}/10 & \sqrt{3}/10 & -6\sqrt{6}/5 & -2/5\sqrt{6}/5 & -2/5\sqrt{6}/5 & 2\sqrt{30} \\
\Xi^0 K^0 & 2/\sqrt{15} & 3/\sqrt{10} & -6\sqrt{6}/5 & 4/5\sqrt{2}/15 & 4/5\sqrt{2}/15 & -4\sqrt{30} \\
\Xi^* - K^+ & 2/\sqrt{15} & 3/\sqrt{10} & -6\sqrt{6}/5 & 4/5\sqrt{2}/15 & 4/5\sqrt{2}/15 & -4\sqrt{30} \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\Lambda_{27} & \alpha_{27}^- & \beta_{27}^- & \gamma_{27}^- & \delta_{27}^- & \epsilon_{27}^- & \zeta_{27}^- \\
\Sigma^* + \pi^- & -2/3\sqrt{2}/5 & 9/2\sqrt{10} & 0 & -32/15\sqrt{2}/5 & -32/15\sqrt{2}/5 & 0 \\
\Sigma^* 0 & 2/3\sqrt{2}/5 & -9/2\sqrt{2}/10 & 0 & 32/15\sqrt{2}/5 & 32/15\sqrt{2}/5 & 0 \\
\Sigma^* + \pi^+ & 2/3\sqrt{2}/5 & -9/2\sqrt{10} & 0 & 32/15\sqrt{2}/5 & 32/15\sqrt{2}/5 & 0 \\
\Xi^0 K^0 & \sqrt{2}/5 & 9/2\sqrt{10} & 0 & 16/5\sqrt{2}/5 & 16/5\sqrt{2}/5 & 0 \\
\Xi^* - K^+ & -\sqrt{2}/5 & -9/2\sqrt{10} & 0 & -16/5\sqrt{2}/5 & -16/5\sqrt{2}/5 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\Xi_{27}^+ & \alpha_{27}^+ & \beta_{27}^+ & \gamma_{27}^+ & \delta_{27}^+ & \epsilon_{27}^+ & \zeta_{27}^+ \\
\Sigma^* + K^0 & \sqrt{2}/3 & 0 & 0 & -4/\sqrt{2}/3 & 2\sqrt{2}/3 & 2\sqrt{2}/3 \\
\Xi^0 \pi^+ & -\sqrt{2}/3 & 0 & 0 & 4/\sqrt{2}/3 & -2\sqrt{2}/3 & 2\sqrt{2}/3 \\
\Xi^0 K^0 & -2/3 & 0 & 0 & 8/3 & -4/3 & -48 \\
\Xi^* \pi^0 & -2/3 & 0 & 0 & 8/3 & -4/3 & 48 \\
\Xi^* - \pi^+ & \sqrt{2}/3 & 0 & 0 & -4/\sqrt{2}/3 & 2\sqrt{2}/3 & -2\sqrt{2}/3 \\
\end{array}
\]
VII. DECAY AMPLITUDES OF 35 PENTAQUARK MULTIPLET

Below we list the decays of the thirtyfive states that form this irreducible representation of $SU(3)$ in terms of one parameter, $\alpha^*_{35}$, at zeroth order in $SU(3)$ symmetry breaking, plus
four more parameters, $\beta_{35}^*$, $\gamma_{35}^*$, $\delta_{35}^*$, and $\epsilon_{35}^*$ of order $m_s$. As was the case with the 27, there are several new “exotic decays”. A truly exotic example is $\Phi_{35}^- \to \Omega^- K^-$ with final state quark content $ss\bar{s}u$.

\[
\begin{array}{cccccc}
\Theta_{35}^{++} & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^{++} K^+ & 1 & 0 & 0 & -8 & 16 \\
\Theta_{35}^+ & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^+ K^0 & 1/2 & 0 & 0 & -4 & 8 \\
\Delta^0 K^+ & \sqrt{3}/2 & 0 & 0 & -4\sqrt{3} & 8\sqrt{3} \\
\Theta_{35}^0 & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^0 K^0 & \sqrt{3}/2 & 0 & 0 & -4\sqrt{3} & 8\sqrt{3} \\
\Delta^- K^+ & 1/2 & 0 & 0 & -4 & 8 \\
\Theta_{35}^- & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^- K^0 & 1 & 0 & 0 & -8 & 16 \\
\Delta_{35}^{++} & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^{++} \pi^+ & 1 & 0 & 0 & 4 & 16 \\
\Delta_{35}^+ & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^+ \pi^0 & \sqrt{2}/5 & 0 & 0 & 4\sqrt{2}/5 & 16\sqrt{2}/5 \\
\Delta^+ \pi^- & -\sqrt{3}/5 & 0 & 0 & -4\sqrt{3}/5 & -16\sqrt{3}/5 \\
\Delta_{35}^0 & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^+ \pi^- & \sqrt{1}/10 & 0 & 0 & 2\sqrt{2}/5 & 8\sqrt{2}/5 \\
\Delta^+ \pi^0 & \sqrt{3}/10 & 0 & 0 & 4\sqrt{3}/5 & 16\sqrt{3}/5 \\
\Delta^0 \pi^+ & -\sqrt{3}/10 & 0 & 0 & -2\sqrt{6}/5 & -8\sqrt{6}/5 \\
\Delta_{35}^0 & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^0 \pi^- & \sqrt{3}/10 & 0 & 0 & 2\sqrt{6}/5 & 8\sqrt{6}/5 \\
\Delta^0 \pi^0 & \sqrt{3}/5 & 0 & 0 & 4\sqrt{3}/5 & 16\sqrt{3}/5 \\
\Delta^- \pi^- & -\sqrt{1}/10 & 0 & 0 & -2\sqrt{2}/5 & -8\sqrt{2}/5 \\
\Delta_{35}^- & \alpha_{35}^* & \beta_{35}^* & \gamma_{35}^* & \delta_{35}^* & \epsilon_{35}^* \\
\Delta^0 \pi^- & \sqrt{3}/5 & 0 & 0 & 4\sqrt{3}/5 & 16\sqrt{3}/5 \\
\Delta^- \pi^0 & \sqrt{2}/5 & 0 & 0 & 4\sqrt{2}/5 & 16\sqrt{2}/5 \\
\end{array}
\]
<table>
<thead>
<tr>
<th>$\Delta_{35}^+$</th>
<th>$\alpha_{35}^*$</th>
<th>$\beta_{35}^*$</th>
<th>$\gamma_{35}^*$</th>
<th>$\delta_{35}^*$</th>
<th>$\epsilon_{35}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}\pi^0$</td>
<td>$1/4\sqrt{3}/5$</td>
<td>$3\sqrt{15}/4$</td>
<td>$-\sqrt{15}/2$</td>
<td>$-3/2\sqrt{3}/5$</td>
<td>$3/2\sqrt{3}/5$</td>
</tr>
<tr>
<td>$\Delta^{++}\eta$</td>
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<td>$3\sqrt{5}/4$</td>
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<td>$-3\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
</tr>
<tr>
<td>$\Delta^+\pi^+$</td>
<td>$1/2\sqrt{10}$</td>
<td>$3\sqrt{5}/2$</td>
<td>$-\sqrt{15}/2$</td>
<td>$-3\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
</tr>
<tr>
<td>$\Sigma^+K^+$</td>
<td>$-1/2\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
<td>$\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
<td>$-3\sqrt{5}/2$</td>
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<td>$4$</td>
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<th>$\gamma_{35}^*$</th>
<th>$\delta_{35}^*$</th>
<th>$\epsilon_{35}^*$</th>
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<td>$\Delta^{++}\pi^-$</td>
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<td>$-\sqrt{15}/2$</td>
<td>$-3\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
</tr>
<tr>
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<td>$-1/2\sqrt{3}/5$</td>
<td>$-1/2\sqrt{3}/5$</td>
<td>$1/2\sqrt{3}/5$</td>
</tr>
<tr>
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<td>$\sqrt{5}/4$</td>
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<td>$3\sqrt{5}/2$</td>
</tr>
<tr>
<td>$\Delta^{0}\pi^0$</td>
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<td>$1/2\sqrt{3}/5$</td>
</tr>
<tr>
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<td>$-3\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
</tr>
<tr>
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<td>$-3\sqrt{5}/2$</td>
<td>$3\sqrt{5}/2$</td>
</tr>
<tr>
<td>$\Sigma^{*0}K^0$</td>
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<td>$\sqrt{5}/3$</td>
<td>$\sqrt{15}$</td>
<td>$-\sqrt{15}$</td>
<td>$-\sqrt{15}$</td>
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<tr>
<td>$\Sigma^{*-}K^+$</td>
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<td>$1/2\sqrt{15}/2$</td>
<td>$\sqrt{5}/6$</td>
<td>$\sqrt{15}/2$</td>
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<th>$\beta_{35}^*$</th>
<th>$\gamma_{35}^*$</th>
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<th>$\epsilon_{35}^*$</th>
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<td>$-3\sqrt{3}/4$</td>
<td>$-3\sqrt{3}/4$</td>
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<tr>
<td>$\Sigma^{*+}\pi^0$</td>
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<td>$\sqrt{6}$</td>
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<tr>
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<td>$2\sqrt{2}$</td>
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\Xi^{*} K^{-} & \sqrt{3}/2 & 0 & 2\sqrt{3} & 2\sqrt{3} & -10\sqrt{3} \\
\Omega^{-} \pi^{-} & 1/2 & 0 & -6 & 2 & -10 \\
\end{array} \]

\[ \begin{array}{cccccc}
\Omega_{35}^{-} & \alpha_{35}^{*} & \beta_{35}^{*} & \gamma_{35}^{*} & \delta_{35}^{*} & \epsilon_{35}^{*} \\
\Xi^{*} K^{-} & 1/2 & 3 & 0 & 0 & -12 \\
\Xi^{*} K^{0} & 1/2 & 3 & 0 & 0 & -12 \\
\Omega^{-} \eta & \sqrt{1/2} & -3\sqrt{2} & 0 & 0 & -12\sqrt{2} \\
\end{array} \]

\[ \begin{array}{cccccc}
\Phi_{35}^{-} & \alpha_{35}^{*} & \beta_{35}^{*} & \gamma_{35}^{*} & \delta_{35}^{*} & \epsilon_{35}^{*} \\
\Omega^{-} K^{0} & 1 & 0 & 0 & 4 & -32 \\
\phi_{35}^{-} & \alpha_{35}^{*} & \beta_{35}^{*} & \gamma_{35}^{*} & \delta_{35}^{*} & \epsilon_{35}^{*} \\
\Omega^{-} K^{-} & 1 & 0 & 0 & 4 & -32 \\
\end{array} \]

VIII. ACKNOWLEDGMENTS

This work was supported in part by the Department of Energy under Grant DE-FG03-97ER40546.

[14] We use the common notation for irreducible representation of SU(N): \((n_1, n_2, \ldots, n_k)\), where \(n_i\) is the number of boxes in the first, second etc. column of the Young tableaux starting from the left. The \(\overline{\mathbf{n}}\) is equal to \(N - n\) which corresponds to the fundamental representation that is complex conjugate to \(n\). The subscript shows the dimension of the representation. Where it doesn’t result in ambiguity we use only dimension to identify a representation.
[15] The resulting amplitudes are products of SU(2) spin amplitudes \(\mathbf{6}\) and \(\mathbf{7}\) and SU(3) flavor amplitudes.
[16] The numbers in parentheses stand for isospin and strangeness.