Title
Temporal Factorisation and the Dynamics of Cognitive Agent States

Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 28(28)

ISSN
1069-7977

Author
Treur, Jan

Publication Date
2006

Peer reviewed

Temporal Factorisation and the Dynamics of Cognitive Agent States

Jan Treur[1] (treur@cs.vu.nl)
Vrije Universiteit Amsterdam, Department of Artificial Intelligence
De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands

Abstract

Dynamics play a major role in a variety of disciplines. This paper contributes to the identification of common principles underlying approaches to dynamics used within cognitive and noncognitive disciplines. As a central, unifying principle, the temporal factorisation principle is introduced, formalised and illustrated. This principle expresses, that every temporal relationship of the form 'past pattern implies future pattern' can be factorised into a relationship of the form 'past pattern implies present state' and a relationship of the form 'present state implies future pattern'. It provides a conceptual framework which unifies various approaches to dynamics, including the cases where cognitive agent states are modelled.

Introduction

The physicalist perspective on cognition views cognition as one of the phenomena of nature. A natural consequence of such a position is that it is a challenge to relate principles behind cognition to principles in nature, or even to search for common principles behind the physical world and cognition. In particular, for cognitive agent models the issue of grounding or embedding them in the physical world is challenging. Having common principles behind nature and cognitive agent models gives a new perspective on this issue, and can contribute to the development of unified modelling approaches that are applicable in different (cognitive and noncognitive) scientific areas. In this paper such a unifying principle is identified and shown to play a crucial role in different disciplines, such as Physics and Cognitive Science: temporal factorisation. Roughly spoken, the temporal factorisation principle claims that if a certain (past) pattern of events leads to a certain (future) pattern of events, then there exists a state property *p* such that the past pattern leads to a (present) state where this property *p* holds, and any state where the state property *p* holds leads to the future pattern. This postulated state property *p* is called a mediating state property for the 'past pattern implies future pattern' relationship. It enables one to factorise this temporal relationship into two others, which in general are simpler: a 'past pattern implies present state' and a 'present state implies future pattern' relationship.

Examples of mediating state properties from Physics include velocity, momentum (obtained by temporal factorisation of past-future relationships between patterns in the position (and mass) of an object over time), and force (obtained by temporal factorisation of past-future relationships between patterns in velocity or momentum over time). In approaches in Cognitive Science, mental states can be postulated by the temporal factorisation principle applied to the dependence of future behaviour patterns on patterns of past sensory inputs.

In this paper it is shown how certain approaches described in the literature on Philosophy of Mind can be generalised beyond the cognitive domain and incorporated in a conceptual framework based on temporal factorisation. It is shown, for example, how a more general notion of representational content for mediating state properties can be specified, based on a generalisation of Kim (1996, pp. 200-202)'s relational specification approach for representational content of mental state properties. Moreover, the temporal-interactivist approach to the dynamics of mental states as put forward in Bickhard (1993) and formalised in Jonker and Treur (2003) is shown to be a special case of temporal factorisation.

First, the temporal factorisation principle is introduced. It is shown how its formulation does not commit to any determinism assumption, and how it relates to views on the dynamics of the world by Descartes and Laplace. Examples of temporal factorisation from the physical domain are discussed. Next, it is shown how the relational specification approach to representational content of mental state properties can be used to formulate the temporal factorisation principle in more detail, and to define a form of representational content for mediating state properties resulting from temporal factorisation. Using an appropriate formal language it is shown how the temporal factorisation principle can be formalised. Furthermore, it is discussed how, within Cognitive Science, mental state properties can be viewed as mediating state properties, resulting from temporal factorisation. Finally, it is shown how the Dynamical Systems Theory (DST; e.g., Port and Gelder, 1995) relates to the principle of temporal factorisation.

Relating Past, Present and Future

Descartes (1633) introduced a perspective on the world that sometimes is called the *clockwork universe*. This perspective claims that with sufficiently precise understanding of the world's dynamics at some starting time, the future can be predicted by applying a set of 'laws of nature'. Descartes emphasizes that after such a starting time nothing (not even God) except these laws of nature determines the world's dynamics. This view on the world's dynamics is often compared to a clockwork. The view assumes that the laws of nature provide systematic relationships between world states over time, in the sense that (properties of) past world states imply (properties of) future world states: *past states* → *future states*. The clockwork universe view has been developed further by Newton, Leibniz, Laplace and others. Laplace

[1] Part of this work was performed at a position at Utrecht University, Department of Philosophy, The Netherlands

(1825) sketches how an intellect could be able to determine future world states from a present world state, that by itself is the effect of past world states.[2]

The Temporal Factorisation Principle

The view expressed by Laplace (1825) assumes that the dynamics of the world can be described in the form of (a) relationships between past world states and the present world state, and (b) relationships between the present world state and future world states: *past states* $\rightarrow$ *present state* & *present state* $\rightarrow$ *future states*. To analyse in more detail the temporal relationships pointed at by Descartes and Laplace, the *temporal factorisation principle* can be used. This principle, as introduced in this paper, is formulated in terms of temporal relationships between past patterns, present states, and future patterns. Here a *past pattern a* refers to a property of a number of states or events (possibly at different time points) in the past, and a *future pattern b* refers to a property of a number of states or events (possibly at different time points) in the future.[3] To put it in a nuttshell, the temporal factorisation principle states that any systematic temporal '*past pattern implies future pattern*' relationship $a \rightarrow b$ between a past pattern a and a future pattern b can be factorised in the form of two temporal relationships $a \rightarrow p$ and $p \rightarrow b$ for some state property p of the present world state. More specifically, the principle claims that for any '*past pattern implies future pattern*' relationship $a \rightarrow b$ *there exists* a world state property p (expressed in the ontology for state properties) such that temporal relationships '*past pattern implies present state property*' $a \rightarrow p$ and '*present state property implies future pattern*' $p \rightarrow b$ hold.[4] In short:

$$a \rightarrow b \Rightarrow \exists p \;\; a \rightarrow p \;\; \& \;\; p \rightarrow b$$

The postulated state property p is called a *mediating state property* for the given '*past pattern implies future pattern*' relationship.

The principle claims that the description of the present world state contains sufficient information so that we can forget about the temporal pattern a in the past if we want to understand why the temporal pattern b occurs in the future. The mediating state property in the present state may be viewed to represent the past pattern and the future pattern in the present state. It will be shown how the relational specification approach to representational content of mental state properties, as proposed by Kim (1996), can be extended beyond the cognitive area to the situation here.

Note that the temporal factorisation principle itself does not claim that any '*past pattern implies future pattern*', '*past pattern implies present state*' or '*present state implies future pattern*' relationships can be found. Due to the conditional, it only claims that <u>if</u> a '*past pattern implies future pattern*' relationship is available, then also '*past pattern implies present state*' and '*present state implies future pattern*' relationships can be found. To make this more precise, if Descartes' view is interpreted in the sense that dynamics can be described by '*past pattern implies future pattern*' relationships (D), and Laplace's view is interpreted in the sense that dynamics can be described by '*past pattern implies present state*' and '*present state implies future pattern*' relationships (L), then the temporal factorisation principle (TFP) logically connects the two: Descartes' view interpreted as D and the temporal factorisation principle TFP together imply Laplace's view interpreted as L, i.e.,

$$\text{D} \;\&\; \text{TFP} \Rightarrow \text{L}.$$

So, the temporal factorisation principle can be used to explain the shift in history, from Descartes's view to Laplace's view. While Descartes' and Laplace's views each can be considered to assume a deterministic world, the temporal factorisation principle is not based on such an assumption, due to the conditional. Temporal factorisation addresses those cases and those aspects of the world where '*past pattern implies future pattern*' relationships can be found, but does not in any way claim that such relationships can always be found for all aspects of the world. Thus, the principle supports all forms of partial determinism, or, in other words, any perspective between a fully deterministic world and a fully non-deterministic world. For a more extensive discussion about (non)determinism, see, for example, Earman (1986), Dennett (2003, pp. 25-96).

Some Examples of Temporal Factorisation

To illustrate the temporal factorisation principle, as a first example the notion 'momentum' of a moving object in classical mechanics is taken. For cognitive examples, see below. For a moving object in free space its future positions depend on its past positions (and not only on its position in the present). Within Physics the notion 'momentum' has been postulated to mediate this dependency. Different histories of an object can lead to the same momentum in the present state. The future of the object only (given the object's current position) depends on this momentum in the present state, not on the specific history. This was the criterion by which the concept momentum was introduced in Physics in history. Therefore the state property momentum can be understood as a mediating state property for past and future patterns in (change of) position of an object; the temporal factorisation principle postulates the existence of this state property. The state property momentum abstracts from the various histories that could have happened and would have resulted in the same future pattern. In the other time

[2] 'We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes.' (Laplace, 1825)

[3] An example of a past pattern, referring to different points in time, is: at some state in the past state property c occurred and since then to the present it persisted. An example of a future pattern, referring to different time points in the future is: if in some future state property c occurs, then in some later state property d will occur. A special, simple case of a past (or future) pattern is the occurrence of a state property in some single past (or future) state.

[4] So, notice that the notation $\rightarrow$ is used here to indicate logical implication (between temporal properties).

direction, a momentum indicates from what pattern it originated, no matter what future will arise, so it abstracts from futures. Similarly the concept 'force' from classical mechanics within Physics can be considered a postulated mediating state property obtained by temporal factorisation for past to future patterns in momentum: it mediates between a (past) state with given momentum to a (future) state with changed momentum.[5]

Temporal Relational Specification for Mediating State Properties

A mediating state property *p* for a '*past pattern a implies future pattern b*' relationship, as postulated by the temporal factorisation principle can be considered to carry information both about the past pattern *a* and about the future pattern *b*; it in a way represents both the past pattern and the future pattern in the present state. One of the more recent approaches to representational content for internal (mental) agent states described within the Philosophy of Mind literature, is the *relational specification approach*; cf. Kim (1996, pp. 200-202). This approach turns out to be suitable for the more general case, beyond the cognitive area, for mediating state properties as well. A temporal relational specification can be viewed as the specification of temporal relationships of a (mental) state to other patterns distant in space and time. Kim explains from a philosophical perspective how a mental state property can be considered an intrinsic internal state property, whereas its relational specification expresses how it relates to other items in the world; he did not formalise his view.

The concept of relational specification can be extended to obtain representational content of a mediating state property as postulated by the temporal factorisation principle as follows. Addressing the future direction first, if *p* is a mediating state property related to some future pattern *b* (and some past pattern *a*), then the actual occurrence of *p* at some time point *t* leads to the actual occurrence of *b* in the future after *t*. Indeed, a relational specification can be identified expressing the relationship between this mediating state property *p* and the subsequent future is (i.e., pattern *b*). A similar analysis can be made for the past relationships. Given past pattern *a* that is assumed to lead to a mediating state property *p*, a relational specification can be identified to express this temporal relationship. Thus, state property *p* can be considered to represent in the present state the fact that the past pattern *a* occurred. Combining the past and future perspective, the fact that *p* is a mediating state property between future pattern *b* and past pattern *a*, can be relationally specified in a temporal manner by a scheme of the following type:

if before t, past pattern a occurs, then at t, state property p holds
if at t, state property p holds, then after t, future pattern b will occur

These two temporal relationships are a '*past pattern implies present state property*' and a '*present state property implies future pattern*' relationship, respectively. Together they can be considered to provide a relational specification of the representational content of the mediating state property *p*, which takes into account both the past and the future.

Formalising Temporal Factorisation

In this section, it is shown how the temporal factorisation principle can be expressed in a formal language. First, this language is briefly introduced. Next, it is shown how past and future patterns can be expressed in this language, and it is shown how temporal relationships between past patterns, future patterns and present states are expressed. Finally, it is shown how the temporal factorisation principle as a whole can be expressed in the language. To specify and formalise temporal relationships that play a role in temporal factorisation, an expressive formal language is needed that allows to refer to patterns over time. Furthermore, it should be possible to express the existential quantifier for state properties, which occurs in the temporal factorisation principle. The Temporal Trace Language (TTL) is such a language (Jonker and Treur, 2002; Bosse et al., 2006).

The language TTL is based on traces (or trajectories), time points, and state properties as primitive notions. A state can be parameterised by a trace in which it occurs and a time point at which it occurs. The language is built up as follows. A *state ontology* is a specification (in sorted predicate logic) of a vocabulary (i.e., names for sorts, constants, functions and predicates). A *state* for ontology Ont is an assignment of truth-values {true, false} to the set At(Ont) of ground atoms expressed in terms of Ont. The *set of all possible states* for state ontology Ont is denoted by STATES(Ont). The set of *state properties* STATPROP(Ont) for state ontology Ont is the set of all propositions over ground atoms from At(Ont).[6] A fixed *time frame* T is assumed, which is linearly ordered. Depending on the application, the time frame T may be dense (e.g., the real numbers), or discrete (e.g., the natural numbers). A *trace* or *trajectory* γ over a state ontology Ont and time frame T is a mapping $\gamma : T \rightarrow STATES(Ont)$. The set of (names for) traces over state ontology Ont is denoted by TRACES(Ont).

The set of *dynamic properties* DYNPROP(Ont) over state ontology Ont is the set of temporal statements that can be formulated with respect to traces based on the state ontology Ont in the following manner. Given a trace γ over state ontology Ont, a state of the world at time point t is syntactically denoted by state(γ, t). These states can be related to state properties p expressed in Ont, via the formally defined (in TTL syntax) satisfaction relation |= (used as a binary infix predicate), i.e.: state(γ, t) |= p , which denotes that state property p holds in trace γ at time t (this has a similarity with the Holds-predicate in situation calculus). Based on these statements, dynamic properties can be formulated in a formal manner in a sorted predicate logic with sorts TIME for time points, TRACES for traces and STATPROP for state formulae, using quantifiers, among others, over time, traces and state formulae, and the usual logical connectives such as $\neg$, $\wedge$, $\vee$,

[5] For a more detailed historical case study, see Treur (2005)

[6] When no confusion is expected, the argument Ont will be left out: STATPROP.

⇒, ∀, ∃. Within TTL (real and integer) numbers can be used for time, but also within state properties.[7]

To formalise the temporal factorisation principle, formalisations are needed for the temporal relationships between past patterns, present states and future patterns with respect to a given time point *t*. As a first step it is shown how past patterns and future patterns can be specified. The basic idea is that, for example, a past pattern refers to a specific set of past traces (up to some time point *t*). The way in which this reference takes place is by expressing a pattern in the form of a (temporal) property that the traces in the set have in common, or, in other words, that characterises this set of traces. To express this property characterising a pattern, the language TTL is used.

A *past statement* for γ and t is a temporal statement φ(γ, t) where γ and t are free variables, such that each time variable different from t is restricted to the time interval before t. In other words, for every time quantifier for a variable s a restriction of the form s ≤ t, or s < t is required within the statement. A *past pattern* is any past statement. A trace γ satisfies a past pattern φ(γ, t) for t if φ(γ, t) is true. The set of past statements over state ontology Ont with respect to time point t is denoted by PFOR(Ont, γ, t). Similarly as the past statements, FFOR(Ont, γ, t) denotes the set of *future statements* over state ontology Ont with respect to trace γ and time point t: γ and t are free variables and every time quantifier for a variable s is restricted by s ≥ t or s > t. A *future pattern* is any future statement.

Given the specification of past and future patterns defined above, the temporal relationships can be defined as follows. A *past pattern implies future pattern* relationship for state ontology Ont at any given time point t is specified as an implication ∀γ [φ(γ, t) ⇒ ψ(γ, t)] where γ ranges over the sort TRACE, φ(γ, t) ∈ PFOR(Ont, γ, t), ψ(γ, t) ∈ FFOR(Ont, γ, t). A *past pattern implies present state* relationship for a state ontology Ont and time point t is specified as a logical implication ∀γ [φ(γ, t) ⇒ state(γ, t) |= p] for a given state property p ∈ STATPROP(Ont) and φ(γ, t) ∈ PFOR(Ont, γ, t), whereas γ ranges over the sort TRACE. A *present state implies future pattern* relationship for a state ontology Ont and time point t is specified as a logical implication ∀γ [state(γ, t) |= p ⇒ ψ(γ, t)] for a given state property p ∈ STATPROP(Ont) and ψ(γ, t) ∈ FFOR(Ont, γ, t), whereas γ ranges over the sort TRACE.

Using the notions defined above, the temporal factorisation principle over state ontology Ont expresses that for any past and future formulae φ(γ, t) ∈ PFOR(Ont, γ, t), ψ(γ, t) ∈ FFOR(Ont, γ, t) with respect to t, for which for any trace γ and time point t the implication φ(γ, t) ⇒ ψ(γ, t) holds, there exists a state property p ∈ STATPROP(Ont) such that for all traces γ and time points t the implications φ(γ, t) ⇒ state(γ, t) |= p & state(γ, t) |= p ⇒ ψ(γ, t) hold, or in concise format[8]:

$$\forall\gamma, t\ [\ \varphi(\gamma, t)\ \Rightarrow\ \psi(\gamma, t)\ ]\ \Rightarrow\ \exists p\ [\ \forall\gamma, t\ [\ \varphi(\gamma, t)\ \Rightarrow\ state(\gamma, t) \models p\ ]\ \&\ \forall\gamma, t\ [\ state(\gamma, t) \models p\ \Rightarrow\ \psi(\gamma, t)\ ]\ ]$$

where γ ranges over the sort TRACE, t over sort TIME and p over sort STATPROP.

Temporal Factorisation and Mental States

One of the challenges in the cognitive agent domain is to describe how a (human or animal) agent's behaviour depends on its past experiences (e.g., sensing of stimuli). As relationships between future patterns of an agent's behaviour and past patterns of its stimuli may be quite complex, the notion of mental state has been postulated as a mediating state between past stimuli and future behaviour. The mental state of an agent depends on its past, and the agent's future behaviour depends on its mental state. In this sense the temporal factorisation principle applies. The postulated mental state properties play an important role in the explanation and prediction of behaviour. An example concerning an agent's belief state illustrates the case in some more detail below.

To illustrate how temporal factorisation relates to the notion of mental state property, consider an agent's reaction on its observation of the presence of food at a position *P*:

if *at any time t' ≤ t the agent observed food at position P*
then *if at some t'' ≥ t the agent observes the opportunity to go to P,*
then at some time point t''' ≥ t'' the agent will go to P.

The above specification describes a temporal relationship between past (observation) events and future behaviours, without taking into account internal, mental states; as such it is a description from a behaviourist perspective; e.g., Kim (1996). Here, it is assumed that at the moment that the opportunity to go to *P* is observed, the food at *P* may not be observed anymore. The mental state property

belief that food is present at position P

can be seen as a temporal factorisation of this temporal past to future relationship. Its temporal relational specification can be obtained in a simplified form in the following manner. The past pattern is described by φ(γ, t), which is the past statement

$$\exists t1 \leq t\quad state(\gamma, t1) \models observed(food\_present\_at(P))$$

which states that there exists a past time point at which the agent observed food at *P*. Moreover, the future pattern is described by ψ(γ, t), which is the future statement

$$\forall t2 \geq t\ [\ state(\gamma, t2) \models observed(opportunity\_to\_go\_to(P))\ \Rightarrow\ \exists t3 \geq t2\ \ state(\gamma, t3) \models performed(go\_to(P))\ ]$$

expressing that as soon as an opportunity is observed, the agent goes to *P*. Temporal factorisation of ∀γ, t [φ(γ, t) ⇒ ψ(γ, t)] for this case is obtained by the following temporal relational specifications for the belief state:

[7] For a detailed description of TTL's syntax and semantics, see (Bosse et al., 2006)

[8] Notice that for this formalisation the following three features of TTL are crucial: (1) it is based on an expressive first-order language for temporal relationships, (2) traces are first class citizens in the language, which means that variables and quantification over traces are possible, and (3) state properties are first class citizens, which means that variables and quantification over state properties are possible.

$$\forall\gamma, t\ [\ \varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models \text{belief(food\_present\_at(P))}\ ]\ \&$$
$$\forall\gamma, t\ [\ \text{state}(\gamma, t) \models \text{belief(food\_present\_at(P))} \Rightarrow \psi(\gamma, t)\ ]$$

This states that if there is a past observation of food at *P*, then a belief state concerning this fact is there, and if such a belief state is there, then the agent will go to *P* as soon as the opportunity is observed. This also shows how the functional or causal role of a mental state property (e.g., Kim, 1996, pp. 75-80) can be described by the temporal factorisation principle.

In recent literature in the area of Cognitive Science and Philosophy of Mind, cognitive functioning is studied from an interactivist perspective (e.g., (Bickhard, 1993; Jonker and Treur, 2003). Bickhard (1993) emphasises the relation between the (mental) state of a system (or agent) and it's past and future in the interaction with its environment.[9] His view suggests that mental states are related to interaction histories on the one hand, and to future interactions, on the other hand. In (Jonker and Treur, 2003) a formalisation is proposed which takes into account the temporal aspects of this interactivist perspective. The general idea is as follows. Suppose for an agent a mental state property *p* is given, which relates to a pattern of past interaction (up to a given time point *t*), on the one hand, and to a pattern of future interaction on the other hand. Let $\varphi(\gamma, t)$ be a specification of this pattern of past interaction and $\psi(\gamma, t)$ a specification of the pattern of future interaction. The temporal-interactivist approach considers that the mental state property *p* holding in the present can mediate in this process as follows:

$$[\ \varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p\ ]\ \&\ [\ \text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)\ ]$$

These specifications express a temporal factorisation of the past to future relationship $\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$; this shows how the temporal factorisation principle subsumes the temporal-interactivist approach to mental states. An illustration of the temporal-interactivist approach by an extension of the belief example above is as follows. Take $\varphi(\gamma, t)$ the past statement

$$\exists t1 \leq t\ [\ \text{state}(\gamma, t1) \models \text{observed(food\_present\_at(P))}\ \&$$
$$\forall t2\ \ t1 \leq t2 \leq t \Rightarrow \neg\ \text{state}(\gamma, t2) \models \text{observed(not(food\_present\_at(P)))}$$

This is a more complex pattern than the one above; in this case it involves all time points in an interval between t_1 and t, which makes more clear how a complete history can be taken into account instead of only one past state. Moreover, as before $\psi(\gamma, t)$ is the future statement

$$\forall t2 \geq t\ \ [\ \text{state}(\gamma, t2) \models \text{observed(opportunity\_for(go\_to(P)))} \Rightarrow$$
$$\exists t3 \geq t2\ \ \text{state}(\gamma, t3) \models \text{performed(go\_to(P))}\ ]$$

Then the belief state property p is a mediating state property for this past to future relationship $\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$.

Temporal Factorisation and DST

Dynamics in domains such as Physics, Chemistry, and Biology, has been addressed in history by the development of the Dynamical Systems Theory (DST). In recent times, it has been proposed to apply the DST approach to cognition as well (e.g., Port and van Gelder, 1995). One of the assumptions underlying DST is the assumption on state-determined systems (cf. van Gelder and Port, 1995; Ashby, 1952/1960). In this section the state-determined system assumption is discussed in relation to the temporal factorisation principle. Van Gelder and Port (1995), following Ashby (1960), explain what a dynamical system is in the following manner. A *system* is a set of changing aspects (or state properties) of the world. A *state* at a given point in time is the way these aspects or state properties are at that time; so a state is characterised by the state properties that hold. The set of all possible states is the *state space*. A *behaviour* of the system is the change of these state properties over time, or, in other words, a succession or sequence of states within the state space. Such a sequence in the state space can be indexed, for example, by natural numbers (*discrete* case) or real numbers (*continuous* case), and can also be called a *trace* or *trajectory*. Following Ashby, such a system is *state-determined* if:

> A system is state-determined only when its current state always determines a unique future behaviour. Three features of such systems are worth noting. First, in such systems, the future behaviour cannot depend in any way on whatever states the system might have been in *before* the current state. In other words, past history is irrelevant (or at least, past history only makes a difference insofar as it has left an effect on the current state). Second, the fact that the current state determines future behaviour implies the existence of some *rule of evolution* describing the behaviour of the system as a function of its current state. (…) Third, the fact that future behaviours are uniquely determined means that state space sequences can never fork. (Gelder and Port, 1995, p. 6).

According to some, a dynamical system is just a state-determined system (Giunti, 1995). For others, in particular those involved in DST, a dynamical system is a state-determined system for which the state properties are described by assignments of numerical values to a given set of variables (van Gelder and Port, 1995). According to Ashby (1960), a main question for a scientist is how to obtain an appropriate state ontology such that based on this ontology for a certain state it can be found out how it is going to change to a different state, according to a certain rule of evolution.[10] The hypothesis is that such a state ontology

[9] 'When interaction is completed, the system will end in some one of its internal states - some of its possible final states. (…) The final state that the system ends up in, then, serves to implicitly categorise together that class of environments that would yield that final state if interacted with. (...) The overall system, with its possible final states, therefore, functions as a *differentiator* of environments, with the final states implicitly defining the differentiation categories. (…) Representational content is constituted as indications of potential further interactions.' (Bickhard, 1993)

[10] 'Because of its importance, science searches persistently for the state-determined. As a working guide, the scientist has for some centuries followed the hypothesis that, given a set of variables, he can always find a larger set that (1) includes the given variables, and (2) is state-determined. Much research work consists of trying to identify such a larger set, for when it is too small, important variables will be left out of account, and the behaviour of the set will be capricious. The assumption that such a larger set exists is implicit in almost all science, but, being fundamental, it is seldom mentioned explicitly.' (Ashby, 1960, p. 28).

always can be found. At first sight, this seems to be close to the consequent $\exists p \quad a \rightarrow p \ \& \ p \rightarrow b$ of the temporal factorisation principle, especially in the claim that certain state properties exist. However, in Ashby's formulation much emphasis is put on the relationship $p \rightarrow b$, almost remaining silent about how p is brought about based on past events. Therefore it might be more fair to state that his position is expressed most sincerely by only part of the consequent: $\exists p \ p \rightarrow b$. In contrast to Ashby's bias on the '*present to future*' relationship, in the formulation of the consequent of the temporal factorisation principle an equal balance between past and future has been achieved. A second difference between Ashby's state-determined system assumption and the temporal factorisation principle is the conditional $a \rightarrow b$ used in the latter. This conditional may also be expected to be a silent assumption in Ashby's view. The temporal factorisation principle makes this assumption explicit in the form of a conditional. This conditional makes a crucial difference in the sense that temporal factorisation does not assume a deterministic system, whereas Ashby's notion of state-determined system is deterministic, and therefore his notion is more limited.

For a more detailed illustration, an analysis of the state property (instantanious) velocity or change rate in a continuous process, which plays a central role in DST, involves the notion of limit. For example, temporal relational specification of velocity $p(t)$ at t in the form of a '*past pattern implies present state*' relationship $a \rightarrow p$ is given by

$$lim_{t' \uparrow t} \ ((x(t') - x(t)) / (t' - t)) = w \ \Rightarrow \ p(t) = w$$

which relates past state properties at $t' < t$ to the mediating state property $p(t)$ at t; similarly

$$p(t) = w \ \Rightarrow \ lim_{t'' \downarrow t} \ ((x(t'') - x(t)) / (t'' - t)) = w$$

for the future perspective). Application of the temporal factorisation principle explains the mathematical theorem that if a past to future relationship in the form of a smoothness condition is fulfilled, then at t a derivative exists for the function x of t (i.e., the function is differentiable at t).

Discussion

The more popular, physicalist views on cognition in Philosophy of Mind, consider cognition as a phenomenon of nature. A challenge then is to relate principles behind cognition to principles in nature, or to search for common principles: how can physical architectures, functioning on the basis of principles valid in the physical world, show cognition; which principles make that possible? A central principle was identified and discussed, which deals with dynamics both in the physical world and in cognitive processes. From a historical perspective, this temporal factorisation principle seems rather fundamental in scientific development (e.g., the development of areas within Mathematics and Physics such as calculus, and classical mechanics). It postulates the existence of mediating state properties that can be used to decompose any temporal 'past pattern implies future pattern' relationship into two simpler temporal relationships: a 'past pattern implies present state' relationship and a 'present state implies future pattern' relationship. In this paper, in addition, a formalisation of this temporal factorisation principle was put forward.

The temporal factorisation principle has been shown to be a basic assumption underlying standard approaches to dynamics in disciplines such as Physics and Cognitive Science; such approaches include Dynamical Systems Theory (DST, cf. Port and Gelder, 1995), and functionalist and interactivist approaches to cognition (cf. Kim, 1996, Bickhard, 1993, Jonker and Treur, 2003). By providing this unification, the temporal factorisation principle provides a new perspective on the issue of grounding or embedding of cognitive agent models in the physical world. For example, it provides a new perspective on the temporal-interactivist approach to the dynamics of mental states as described in Jonker and Treur (2003), as being an instance of a more general principle of nature.

References

Ashby, W.R. (1952/1960). *Design for a Brain*. Chapman & Hall, London, 1952. Revised, second edition, 1960.

Bickhard, M.H. (1993). Representational Content in Humans and Machines. *Journal of Experimental and Theoretical Artificial Intelligence*, vol. 5, pp. 285-333.

Bosse, T., Jonker, C.M., Meij, L. van der, Sharpanskykh, A., & Treur, J. (2006). A Temporal Trace Language for the Formal Analysis of Dynamic Properties. Technical Report, Vrije Universiteit Amsterdam. URL http://www.few.vu.nl/~treur/TTL.pdf

Dennett, D.C. (2003), *Freedom Evolves*. Penguin Books.

Descartes, R. (1633). *The World or Treatise on Light*. Withdrawn from publication, 1633. In: Descartes, R., *The World and Other Writings*. (S. Gaukroger ed.) Cambridge Texts in the History of Philosophy. Cambridge University Press.

Earman, J., (1986). *A Primer on Determinism*. Reidel Publishing Company, Kluwer, Dordrecht.

Gelder, T.J. van, and Port, R.F., (1995). It's About Time: An Overview of the Dynamical Approach to Cognition. In: Port and van Gelder (1995), pp. 1-43.

Giunti, M. (1995). Dynamical Models of Cognition. In: Port and van Gelder (1995), pp. 549-572.

Jonker, C.M., and Treur, J., (2002). Compositional Verification of Multi-Agent Systems: a Formal Analysis of Pro-activeness and Reactiveness. *Int. J. of Cooperative Information Systems*, vol. 11, pp. 51-92.

Jonker, C.M., and Treur, J., (2003). A Temporal-Interactivist Perspective on the Dynamics of Mental States. *Cognitive Systems Research Journal,* vol. 4, 2003, pp. 137-155.

Kim, J. (1996). *Philosophy of Mind*. Westview press.

Laplace, P.S. (1825). *Philosophical Essays on Probabilities*. Springer-Verlag, New York, 1995. Translated by A.I. Dale from the 5th edition of 1825.

Port, R.F., Gelder, T. van (eds.) (1995). *Mind as Motion: Explorations in the Dynamics of Cognition*. MIT Press, Cambridge, Mass..

Treur, J., (2005). States of Change: Explaining Dynamics by Anticipatory State Properties. *Philosophical Psychology*, vol. 18, 2005, pp. 441-471.