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Construction and Characterization of a Frequency-Controlled, Picometer-Resolution, Displacement Encoder-Actuator

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We have constructed an actuator/encoder whose generated displacement is controlled through the resonance frequency of a microwave cavity. A compact, 10-µm-range, digitally-controlled actuator executing frequency-coded displacement with picometer resolution is described. We consider this approach particularly suitable for metrologic-precision scanning probe microscopy.

The easy mechanical tunability of microwave cavities has been exploited in all imaginable applications from broadband, mechanically-tuned microwave oscillators and filters to gravitational wave detection thanks to the ever growing variety of resonator structures and frequency locking schemes. High sensitivity transducers based on microwave cavities have been reported as well.

In this paper we describe a displacement actuator system - a microwave cavity and controller - designed specifically for Scanning Probe Microscopy (SPM) applications.

Positioning in scanning probe microscopy represents a particular challenge due to the enormous dynamic range (> 10⁷) of the piezo-electric actuators used. While scan ranges of 3-100 µm are common, sub-angstrom (< 10 pm) resolution is achieved reproducibly at low temperature and somewhat randomly in less controlled conditions. A search for a reasonable way to build a metrologic instrument capable of making measurements with precision comparable to that of crystallographic data, is the motivation of this work. The minimum requirement to achieve that, is an active-feedback-controlled distance encoder with ≈ 10 µm working range, a > 1 kHz bandwidth response combined with picometer-scale accuracy, linearity, and long-term stability. We demonstrate a device and control system that meets all these requirements in a smaller, simpler configuration than other methods under development. In addition, it is self-calibrating; relying on the speed of light to set the distance scale.

The resonant frequencies, \( f_n \), of the TEM\(_{00n} \) modes of a coaxial resonator are related to its length, \( L \), through \( f_n = nc/2L \). The frequency’s independence of all other dimensions makes these modes the most suitable for our purpose, in spite of the fact that they are relatively low quality factor (Q) modes. To keep it compact, we choose to build a 15 mm length cavity working at its lowest frequency (n=1) mode, at 10 GHz. The practical realization required building a variable-length cavity, with a coupling that is minimally perturbed by the length change, as well as a sub-millihertz-resolution microwave control system with \( 10^{-12} \) stability. Choosing the cavity frequency and allowing closed-loop control to match the resonator length, forms our frequency-controlled, displacement encoder-actuator.

Figure 1 shows a cross section of the constructed mechanical assembly. The adjustable length is realized by cutting the cavity along a plane perpendicular to the axis and equidistant from the ends. There is no current flowing in the walls there and therefore the Q and the resonant frequency of the cavity are unaffected by a small gap. The left half-cavity is tied to the end of the external tube housing the device - indicated as the “Fixed” Half in the figure - setting the coarse length of the cavity. The holder of the “Moving” Half - attached to the right end of the housing tube - is variable length. A stacked-piezo tube (Piezomechanic HPSt 150/14-10/12), kept under compressive load throughout its stroke, allows elastic elongation of the concentric aluminum tube over a 10 µm range. The shape of the variable-length holder is designed to minimize strain on the cavity wall. By applying a layer of damping material (not shown) around the flexure tube, the stability of the feedback loop could be extended from about 2 kHz to 8 kHz. Both half-cavity pieces were fabricated out of aluminum 7075 and subsequently gold plated, resulting in a loaded Q of 1280.

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**FIG. 1.** (Color online) Cross section of the displacement encoder assembly, which incorporates the coupling structure, the resonant cavity, a mechanical flexure, and an optical interferometer along the axis to independently measure the relative displacement of the cavity halves. The moving half-cavity is machined into the end of an aluminum tube stretched by a concentric stacked-piezo tube.
FIG. 2. The cavity resonance frequency is measured with a microwave version of a Pound-Drever-Hall lock, which generates a signal proportional to the difference between the source and cavity frequencies. This is used as an error signal in a feedback loop to lock the cavity to the source, transferring the stability of the rubidium frequency reference, and the precise, digital tunability of the microwave source, to the cavity length. Double-lined arrows indicate the path of high frequency ($\geq 8$ GHz) signals, while single-lined arrows indicate low frequency signals. The inset indicates the spectrum of the generated signal: a carrier and two phase-modulation-like sidebands. Amp: Amplifier, Is: Isolator, Mxr: Mixer, Flt: Filter, Circ: Circulator.

While the 15 mm cavity length is set by the choice of the 10 GHz working frequency, the 3.5 mm and 11 mm inner and outer diameter were chosen as a compromise between quality factor, coupling dynamic range, and other conveniences. An oversized coaxial waveguide (whose inner diameter matches that of the cavity’s inner conductor), ending with a half-toroidal curved wall, brings the microwaves close to the fixed half-cavity end wall, and a taper element allows connecting it to a standard SMA connector. Coupling is done with an iris (small hole) on the cavity/waveguide wall, which is partially blocked by a floating copper pill held in place by a dielectric screw. Moving the pill in front of the iris allows precise control of the coupling, over a range of $0.8 < \beta < 1.2$, where $\beta = (1 + |S|)/(1 - |S|)$, and $S$ is the scattering parameter on resonance; the top (bottom) sign is chosen for over- (under-) coupled cases. We have found that a return loss greater than 50 dB optimizes the signal-to-noise ratio in our system, necessitating a smoothly-working adjustable coupling.

To evaluate the motion generated by the piezo tube actuator, an optical interferometer was added along the axis of the cavity. A single-mode-fiber-coupled 1310 nm laser diode was used as a light source. Light was brought through a fiber-coupled circulator to a lens assembly consisting of a gradient index lens and a 3 mm radius plano-concave lens affixed to the moving half-cavity. The lens ensemble has a 3 mm working distance, assuring that light rays stepping to air from glass are perpendicular to the concave surface - which has no anti-reflection coating - providing a proper reference surface to a flat mirror placed at the beam waist. The light is focused onto the aluminum-coated end of a polished quartz rod held by the fixed half-cavity. The lens collects the back-reflected light, as well as the $\sim 4\%$ reflected off the concave glass-air interface and couples them back into the single-mode fiber. The fiber-coupled circulator directs them into a photodiode; the observed interference signal reflects the distance change between the concave lens surface and the end of the quartz rod, allowing to monitor the relative motion of the half cavities.

The cavity resonant frequency is compared to that generated by a home-built microwave source, shown in the block diagram in figure 2. We have implemented a microwave version of what is commonly known in optics as a Pound-Drever-Hall (PDH) lock. Driving the piezo-actuator to match the cavity resonance frequency to that of the microwave signal, the cavity frequency inherits the stability and digital tunability of the source, stable to $\sim 10$ mHz, and adjustable in the range of $9810 \pm 15$ MHz, with better than millihertz precision.

The frequency modulated 9.8 GHz signal is generated in multiple steps. All signal sources are phase-locked directly or indirectly to a rubidium frequency reference (SRS PRS10), ensuring $2 \times 10^{-12}$ stability over 100 s. The fine frequency control and modulation is implemented at 1.8 GHz, and then converted up to 9.8 GHz using an 8 GHz low phase-noise source (Wenzel multiplied crystal oscillator (MXO)). A 1810 $\pm$ 15 MHz VCO (ZCommunications ZRO1820A1LF) is controlled via a fractional-N phase-lock-loop (PLL) that internally includes a direct digital synthesizer (DDS) chip with a 48-bit frequency tuning word (Analog Devices AD9956). The reference for the PLL is a 100 MHz crystal oscillator (Wenzel SC-cut XO) with low close-in phase noise. This configuration provides digital frequency control with
The described system is used as an actuator in closed-loop active feedback mode, controlling the motion through setting the frequency value. The exponential settling time constant - limited by the mechanical motion - for a 5 kHz (8 nm) step in the setpoint is about 20 µs. While adding payload, i.e. the mass of a sample holder, will reduce the bandwidth, we estimate that an X-Y scanner using this actuator would be able to collect up to 5 full-scale, high-resolution images per second.

The precision of the system is determined by the quality of the microwave sources used. The frequency analysis of the error (PDH) signal is shown both for closed- and open-loop operation in figure 3. The open-loop noise spectral density curve shows a minimum of 60 fm Hz$^{-1/2}$ around 500 Hz as the low frequency noise is dominated by the cavity frequency noise; we have made no particular effort to isolate vibrational and acoustical noise from the cavity. As the mechanical noise diminishes at higher frequencies, the frequency difference noise starts to get dominated by the phase noise of the microwave signal sources. Turning on the feedback loop actively compensates the vibration and acoustic noise, but also converts phase noise to length fluctuations within the loop bandwidth.

Above the loop bandwidth (1 kHz in the figure), the closed loop signal practically matches the open loop signal. From a noise and stability point-of-view, the optimal bandwidth is where the diminishing (in frequency) mechanical noise intersects the increasing frequency noise of the source. In practice however, the bandwidth is chosen as a compromise between RMS noise and settling time. Limiting the feedback bandwidth to 1 kHz, and taking the distance noise to be 60 fm Hz$^{-1/2}$ everywhere in this range allows us to claim a less than 2 pm RMS noise in a 1 kHz bandwidth.

Obviously, the fiber interferometer built into the middle of the cavity was never meant to match the precision claimed, but it was useful to characterize the overall system behavior, such as trouble shooting, verifying scan range, piezo-hysteresis, etc. Figure 3 illustrates the limitations of the interferometer measurement, showing subsequent fringes obtained from a full range scan. We fit parabolas to locate successive minima or maxima (blue and green curves) and extracted the frequency difference corresponding to the λ/2 periodicity with ±0.5% precision, i.e. about ±3 nm, reproducibly. We averaged the fringe spacing from an equal number of forward and backward scans (10 each) to compensate for drifts.

The ±0.5% irreproducibility is due to small changes in the alignment and coarse length/frequency adjustment upon rebuilding the assembly. This precision would not allow us to detect any deviations from the ideal relation $f = c/2L$ if the gap between the cavity halves is less than 10 µm. Our differential measurement would be indistinguishable from $\Delta L = -c\Delta f/2f^2$. To verify the useful range of our device, we have simulated the cavity frequency as a function of the gap width in Ansys HFSS. The quadratic correction results in a 1% and 5% deviation in the linearized frequency-distance relationship at 45 and 120 µm respectively. The 5% deviation with a 120 µm gap was sufficiently significant to verify with our interferometer within the experimental error (figure 4). The back extrapolation of the values confirms that less than 0.1% deviation from the ideal slope is expected in...
ment tools, capable of picometer precision. The de-
cocrowave cavities make excellent displacement measure-
ing to explore the limitations of our mechanical design. PDH-like system advantage is a fast response time allow-
limited by the phase stability of the source. The realized

temperature variation of the cavity wall resistivity
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stringent environmental control is needed to eliminate

capitalize on the stability offered by frequency standards,
in perpendicular directions with minimal cross-talk. To
stage of an SPM would require two encoders to operate

tactice parameters obtained by bulk crystallography.

Incorporating such an encoder into the positioning
stage of an SPM would require two encoders to operate
in perpendicular directions with minimal cross-talk. To
 capitalize on the stability offered by frequency standards,
stringent environmental control is needed to eliminate
temperature variation of the cavity wall resistivity\[7\], the
dielectric constant of the gas within\[8\], just as well as the
rest of the scanner to limit thermal drift of all mechanical
parts involved. The effective “thermal coefficient” of the
encoder is expected to be \(1.7 \times 10^{-7} \text{ K}^{-1}\) (for 15 mm
cavity length), almost an order of magnitude smaller than
that of Invar.

We note that the precision of any other Automatic Fre-
cquency Control (AFC) method\[2\] would also be ultimately
limited by the phase stability of the source. The realized
PDH-like system advantage is a fast response time allowing
to explore the limitations of our mechanical design.

We have demonstrated that compact, coaxial mi-
crowave cavities make excellent displacement measure-
ment tools, capable of picometer precision. The de-
scribed device has a range of 10 µm, limited by the chosen
piezo-actuator, and a noise floor of 60 fm Hz\(^{-1/2}\), limited
by the quality of the signal generators. The actuator sys-
tem is capable of self-calibration to an accuracy of better
than 0.1% of the displacement range, although for ranges
larger than 10s of micrometer, a non-linear correction
will be needed to account for the larger gap. Incorpor-
ating displacement encoders using these principles into
SPM would go a considerable way towards realization of
a metrological SPM.

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\[10\] This required using an analog PID (SRS SIM960) because of
a 70 µs delay associated with the digital electronics. For other
measurements reported, the digital PID was used, limiting the
bandwidth to 1 kHz. We checked that the settling time-constant
of the VCO was faster than 20 µs.

\[7\] Drift of about 2.5 nm K\(^{-1}\) can be expected based on the
thermal coefficient of resistivity for common, high-conductivity
metals.

\[8\] It was observed that thermal and humidity fluctuations caused
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