UNIVERSITY OF CALIFORNIA, SAN DIEGO

Neutrino Flavor and Spin Transformations in Astrophysical Environments

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by

James Y. Tian

Committee in charge:

George M. Fuller, Chair
Professor Dusan Keres
Professor Aneesh V. Manohar
Professor Kurt Marti
Professor Mark H. Thiemens

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The dissertation of James Y. Tian is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

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DEDICATION

To my family, friends, and all those who have supported me.
EPIGRAPH

I think I can safely say that nobody understands quantum mechanics.

—Richard Feynman
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VITA

2010  B. S. in Astrophysics *summ cum laude*, University of California, Los Angeles

2012  M. S. in Physics, University of California, San Diego

2014  C. Phil. in Physics, University of California, San Diego

PUBLICATIONS


We discuss and explore the neutrino spin and flavor evolution of neutronization burst neutrinos inside an Oxygen-Neon-Magnesium (ONeMg) type IIa supernova. In addition, we look at the flavor evolution of neutrinos originating from the disk-like (temporary) remnant of a binary neutron star (BNS) merger environment. Computer simulations were run to simulate coherent, forward (non direction changing) scattering of Majorana type neutrinos in these environments. We find in our simulations that, a large neutrino mass ($\approx 10eV$), a flat electron fraction profile, and a high neutrino luminosity are needed to produce a significant spin coherence effect. However, if such a spin transformation does occur (e.g. by other non-linear feedback mechanisms that we did not
consider here), these spin transformations can have a significant, qualitative, effect on the subsequent flavor evolution of the neutrinos. In the case of the BNS merger environments, we found bipolar spectral swaps which are reminiscent of neutrino flavor evolution in the neutrino driven wind environments in supernovae. These flavor transformations can potentially increase the electron fraction and thereby weaken the $r$-process in these environments.
Chapter 1

Introduction

1.1 Motivation

1.1.1 The Nature of Neutrinos

An outstanding question about the nature of neutrinos is whether they are Dirac in nature (they are not their own antiparticles) or Majorana in nature (they are their own antiparticles). A potential way to ascertain which of these two possible types of particles neutrinos actually are would be an observation of a neutrinoless double beta decay. If it’s possible to have a neutrinoless double beta decay, then neutrinos must be Majorana in nature. If neutrinos are Dirac in nature, then the neutrinoless double beta decay would be impossible. The spin coherence that we seek to explore in this thesis could potentially be a different, astronomical, probe of this true nature of neutrinos. Spin transformations which change the spin of neutrinos would turn a neutrino or antineutrino into a sterile neutrino if neutrinos were Dirac in nature. If neutrinos are Majorana in nature, then a spin transformation would turn neutrinos into antineutrinos and vice versa.

Suppose we are able to detect neutrinos from the neutronization burst epoch (an epoch where primarily electron neutrinos are emitted, and few antineutrinos or other
flavored neutrinos are) of a galactic (or other nearby) supernova. If we find that a significant portion of antineutrinos are present in this neutronization burst, that would be an indication that neutrinos are Majorana in nature, and that spin coherence transformed some existing neutrinos into antineutrinos. If neutrinos were Dirac in nature, then a strong spin coherence effect would only imply that here on Earth we detect fewer neutrinos (since they were turned sterile) than we would expect. Therefore, knowing how plausible it is that there occurs a strong spin coherence effect inside a supernova neutronization burst would allow us to know whether we can use supernovae as an astronomical probe into this Dirac or Majorana nature of neutrinos. We explore a potential spin coherence effect inside an ONeMg supernova in chapter 2.

1.1.2 \(r\)-Process in Supernovae and BNS Mergers

One of the primary reasons to study neutrinos in explosive astrophysical environments such as supernovae and BNS mergers is to ascertain the effect these neutrinos may have on the potential \(r\)-process occurring in these environments. The \(r\)-process, or rapid neutron capture process (rapid here meaning faster than the rate of beta decay of heavy neutron rich isotopes), is the method by which some heavy nuclei (heavier than iron) such as germanium or xenon are made. The other primary process by which heavy nuclei are made is the \(s\)-process, or slow neutron capture process, thought to occur in asymptotic giant branch (AGB) stars. Currently it is still not certain where exactly the \(r\)-process elemental abundances come from. The two top candidates right now are supernovae and BNS mergers. Both of these environments radiate a huge amount of neutrinos and so it would be prudent to investigate the behavior of these neutrinos in order to better understand the possibility for these environments to be the ultimate source of the \(r\)-process materials.

The primary method by which neutrinos may affect the \(r\)-process in these envi-
environments is through their effect on the electron fraction \((Y_e)\). The electron fraction is defined as:

\[
Y_e \equiv \frac{n_{e^-} - n_{e^+}}{n_b}
\]  

(1.1)

Where \(n_e\) is the number density of electrons, \(n_{e^+}\) is the number density of positrons, and \(n_b\) is the number density of baryons. By charge neutrality and the fact that \(n_p + n_n = n_b\) (number of protons plus number of neutrons equals total number of baryons), the electron fraction inside these explosive environments is also a measure of how neutron rich they are with a lower \(Y_e\) meaning a more neutron rich environment.

The primary method by which neutrinos affect \(Y_e\) is through the neutrino capture and antineutrino capture processes

\[
\nu_e + n \rightarrow p + e^-
\]

\[
\bar{\nu}_e + p \rightarrow n + e^+
\]  

(1.2)

which convert neutrons to protons and vice versa. Notice here that only the electron flavor neutrinos participate in these interactions, and also that neutrinos tend to destroy neutrons while antineutrinos tend to create neutrons. Therefore, as long as weak interactions are not decoupled from the environment, we need to know about the evolution of the flavor and spin-state of neutrinos and antineutrinos as they move out and interact with the supernovae or merger environments.

In the spin coherence case, it is expected (and indeed is borne out by simulations which we will show in chapter 2) that significant spin transformations can only occur near a resonance condition which requires \(Y_e \lesssim 1/3\) (see section 2.5.1). In this case, the electron fraction may not be ideal (a bit too high) for creating a strong \(r\)-process in the first place. Thus, emphasis on the potential impact of neutrinos on the \(r\)-process is made in the BNS merger case in chapter 3 (see specifically section 3.5.2) where the flavor
transformations do not require the electron fraction to take on some specific value. In this case, the flavor transformations could have significant impact on the rates of the reactions given in equation 1.2. A detailed analysis of neutrino flavor transformation in these environments is therefore warranted.

1.2 Background

1.2.1 Flavor Transformations

Here we look briefly at the behavior of neutrinos, particularly the evolution of their flavor states, in vacuum. Neutrinos are typically produced via some weak interaction, (e.g. neutron decay $n \rightarrow p^+ + e^- + \bar{\nu}_e$ which produces an electron antineutrino) in some flavor state, $|\nu_e\rangle$, $|\nu_\mu\rangle$, or $|\nu_\tau\rangle$. The neutrino flavor states, electron, muon, and tau neutrinos are defined by the weak interactions which pairs them with corresponding charged leptons, the electron, the muon, and the tau particles. For instance, to be specific, one could define an electron neutrino as the neutrino state which may share a charged current interaction vertex with an electron but not the muon or tau.

From the solar neutrino problem and the resolution thereof, it was discovered that the flavor eigenstates are not in fact mass eigenstates, which we will denote as $|\nu_1\rangle_V$, $|\nu_2\rangle_V$, $|\nu_3\rangle_V$ (we have appended a subscript “V” here to specify that these are mass eigenstates of the vacuum Hamiltonian in order to distinguish these states from the instantaneous mass eigenstates in matter which we will discuss in the next section). If the mass eigenstates did coincide with the flavor states, e.g. if the neutrinos were all massless, or all had a fixed mass for fixed flavor, then no neutrino flavor transformations would occur in vacuum. These sets of states are related to each other by a unitary transformation:
\[ |\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^* |\nu_i\rangle_V, \quad \alpha = e, \mu, \tau, \quad i = 1, 2, 3 \quad (1.3) \]

Where \( U_{\alpha i}^* \) is the Pontecorvo-Maki-Nakagawa-Sakata matrix.

For simplicity and for clarity, we will look only at a model two flavor case. The two flavor model we present here will give us an intuition and insight into all the qualitative features of neutrino flavor evolution without the added complexities of dealing with three flavors. It is not difficult to generalize our arguments to the full three flavor case, it is simply much more mathematically complex. In the two flavor case, the unitary matrix can be made to be real and orthogonal and the transformation between flavor states and mass states will be given by:

\[ |\nu_e\rangle = \cos \theta_V |\nu_1\rangle_V + \sin \theta_V |\nu_2\rangle_V \quad (1.4) \]
\[ |\nu_x\rangle = -\sin \theta_V |\nu_1\rangle_V + \cos \theta_V |\nu_2\rangle_V \quad (1.5) \]

For the purposes of this thesis, we will always be analyzing coherent neutrino flavor evolution. As such, the neutrino state (generally denoted \(|\Psi\rangle\)) evolves according to a Schrodinger like equation:

\[ H|\Psi\rangle = i \frac{\partial}{\partial t} |\Psi\rangle. \quad (1.6) \]

Once we know what the Hamiltonian is, we can solve this equation to obtain the neutrino flavor state as a function of time. In vacuum (no interactions), a particle has energy (working in units of \( \hbar = c = 1 \)) \( E = \sqrt{p^2 + m^2} \). Since a neutrino is extremely light, it will almost always be highly relativistic, and for detectors on Earth, the neutrinos will always be highly relativistic. Therefore we may Taylor expand this expression for energy
to second order:

\[ E \approx p + \frac{m^2}{2p}. \]  

(1.7)

Since the masses of the neutrinos are very small, the momenta of different neutrino mass states will be very nearly identical. It will be a constant, and so the relevant vacuum Hamiltonian is expressed in the vacuum mass basis (which is defined to be the eigenstates of the vacuum Hamiltonian) is simply:

\[
  H_{V,m} = \begin{pmatrix}
    \frac{m_1^2}{2p} & 0 \\
    0 & \frac{m_2^2}{2p}
  \end{pmatrix}
\]  

(1.8)

Where \( m_1, m_2 \) are the masses corresponding to the two mass eigenstates. It is standard when analyzing flavor transformations to remove the trace from the Hamiltonian. We can subtract a constant times the identity matrix to make our Hamiltonian traceless. This will only remove an overall phase factor from our calculations and will not affect the flavor evolution. The traceless form of the Hamiltonian is found easily (with \( \Delta m^2 \equiv m_2^2 - m_1^2 \)):

\[
  \bar{H}_{V,m} = \frac{\Delta m^2}{4p} \begin{pmatrix}
    -1 & 0 \\
    0 & 1
  \end{pmatrix}.
\]  

(1.9)

We can see here, from a mathematical standpoint, that since the eigenstates of flavor are not coincident with the mass eigenstates, a flavor eigenstate will evolve in time even in vacuum and therefore a neutrino’s flavor will not be conserved.

1.2.2 Mikheyev Smirnov Wolfenstein effect

Many of the results of this thesis build upon or resemble a well known phenomenon in neutrino flavor evolution called the Mikheyev-Smirnov-Wolfenstein (MSW)
effect. It is for this reason that we wish to give a brief overview of the MSW effect here. In matter, there will be an additional term in the Hamiltonian governing the interaction of neutrinos with matter. This term is given by the neutral and charged current weak interaction’s of the neutrino on electrons and nuclei. In the flavor basis, the Hamiltonian can be expressed as:

$$H_{M,f} = \sqrt{2} G_F \begin{bmatrix} n_e - n_n/2 & 0 \\ 0 & -n_n/2 \end{bmatrix}$$  \hspace{1cm} (1.10)$$

Here $G_F$ is the Fermi coupling constant, and $n_e$ is the local number density of electrons while $n_n$ is the local number density of neutrons. Again, we change this Hamiltonian into a traceless form (the contributions from $n_n$ will cancel):

$$\tilde{H}_{M,f} = \frac{\sqrt{2}}{2} G_F n_e \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$ \hspace{1cm} (1.11)$$

In the previous section, we gave the vacuum Hamiltonian in the mass basis. In order to solve problems, we need to pick one basis and stick with it, so we wish now to express the vacuum Hamiltonian in the flavor basis so that we can find the new eigenstates of the Hamiltonian (let us assume for now, that the Hamiltonian is time-independent). Of course, the flavor basis vacuum Hamiltonian and mass basis vacuum Hamiltonian are related by a similarity transformation:
\[ \bar{H}_{V,f} = U_{V} \bar{H}_{V,m} U_{V}^{\dagger} \]

\[ \begin{bmatrix} \cos \theta_{V} & \sin \theta_{V} \\ -\sin \theta_{V} & \cos \theta_{V} \end{bmatrix} \begin{bmatrix} \Delta m^2 \\ \frac{4p}{4p} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_{V} & \sin \theta_{V} \\ -\sin \theta_{V} & \cos \theta_{V} \end{bmatrix}^{T} \]

\[ \frac{\Delta m^2}{4p} \begin{bmatrix} -\cos 2\theta_{V} & \sin 2\theta_{V} \\ \sin 2\theta_{V} & \cos 2\theta_{V} \end{bmatrix} \]

So the total Hamiltonian, in the flavor basis is:

\[ \bar{H}_{T,f} = \frac{\Delta m^2}{4p} \begin{bmatrix} -\cos 2\theta_{V} & \sin 2\theta_{V} \\ \sin 2\theta_{V} & \cos 2\theta_{V} \end{bmatrix} + \sqrt{2} G_{F} n_{e} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \]  

(1.13)

One can see (and we also know from basic linear algebra) that the traceless property of our Hamiltonian is preserved under similarity transformations. For notational simplicity, let’s define \( \Delta_{V} \equiv \frac{\Delta m^2}{2p} \) and \( \Delta' \equiv \sqrt{2} G_{F} n_{e} \) so that our Hamiltonian is now:

\[ \bar{H}_{T,f} = \frac{1}{2} \begin{bmatrix} -\Delta_{V} \cos 2\theta_{V} + \Delta' & \Delta_{V} \sin 2\theta_{V} \\ \Delta_{V} \sin 2\theta_{V} & \Delta_{V} \cos 2\theta_{V} - \Delta' \end{bmatrix} \]  

(1.14)

Finding the 2 Eigenvectors for this Hamiltonian is therefore just a matter of plug-and-chug:

\[ |\nu_{\pm}\rangle_{M} = \begin{pmatrix} -\Delta' + \Delta_{V} \cos 2\theta_{V} + \sqrt{\Delta_{V}^2 + \Delta'^2 - 2\Delta_{V} \Delta' \cos 2\theta_{V}} \\ \Delta_{V} \sin 2\theta_{V} \\ 1 \end{pmatrix}. \]  

(1.15)

The Eigenvalues of the matrix in equation 1.14 are:

\[ \lambda_{\pm} = \pm \frac{1}{2} \sqrt{\Delta_{V}^2 + \Delta'^2 - 2\Delta_{V} \Delta' \cos 2\theta_{V}} \]  

(1.16)
Note that because we have removed a constant times the identity from our Hamiltonian, the actual energy eigenvalues will be some constant plus (minus) these eigenvalues, which is why at first there appears to be a negative eigenvalue. Following standard notation, we define:

$$\Delta_M \equiv \sqrt{\Delta_V^2 + \Delta'^2 - 2\Delta_V\Delta' \cos 2\theta_V}$$

(1.17)

So that the mass eigenvalues are:

$$M_{1,2} = \text{const} \pm \frac{\Delta_M}{2}.$$  \hspace{1cm} (1.18)

Now we focus our attention on our Eigenvectors. We note first that the total Hamiltonian is of the form:

$$\tilde{H}_{T,m} = \frac{1}{2} \begin{pmatrix} -a & b \\ b & a \end{pmatrix}$$

(1.19)

Where we have made the definitions $$a \equiv \Delta_V \cos 2\theta_V - \Delta'$$ and $$b \equiv \Delta_V \sin 2\theta_V$$. With these definitions, our eigenvectors have a nicer form:

$$|\nu\rangle_M = \begin{pmatrix} -a \pm \sqrt{a^2+b^2} \\ b \\ 1 \end{pmatrix}.$$  \hspace{1cm} (1.20)

This suggestive form suggests to us to draw a right triangle with $$\phi$$ defined as one angle of the triangle with the adjacent side of length $$a$$ and the opposite side of length $$b$$. With these definitions, our Eigenvector becomes:

$$|\nu\rangle_M = \begin{pmatrix} \frac{-\cos \phi \pm 1}{\sin \phi} \\ \sin \phi \\ 1 \end{pmatrix}.$$  \hspace{1cm} (1.21)
Normalizing these eigenvectors gives us:

\[ |\nu_\pm\rangle_M = \frac{1}{\sqrt{2 \pm 2\cos\phi}} \begin{pmatrix} -\cos\phi \mp 1 \\ \sin\phi \end{pmatrix} \]  

(1.22)

So that the diagonalization matrix is:

\[ S = \begin{pmatrix} -1 - \cos\phi & 1 - \cos\phi \\ \sqrt{2 + 2\cos\phi} \sin\phi & \sqrt{2 - 2\cos\phi} \sin\phi \\ \sqrt{2 + 2\cos\phi} \cos\phi & \sqrt{2 - 2\cos\phi} \cos\phi \end{pmatrix} \]  

(1.23)

Notice that our Eigenvectors are orthonormal, and so our diagonalization matrix must be orthogonal. In fact, with clever use of trigonometric identities, one can find explicitly:

\[ S = \begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \]  

(1.24)

Thus, to change bases from our old flavor states \( |\nu_\alpha\rangle_V \) to this new eigenstates, we have \( |\nu_i\rangle_M \) (where now the \( i = 1, 2 \) corresponds to the \( \pm \) in our previous notation):

\[ |\nu_j\rangle_M = \sum_{\alpha=e,x} S^\dagger_{ji} |\nu_\alpha\rangle \quad j = 1, 2 \]  

(1.25)

If we want our transformation matrix to mirror the transformation matrix in vacuum, we should define:

\[ \theta_M = \frac{\phi}{2} \]  

(1.26)

So that our transformation matrix is:
\[ U_M = \begin{bmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{bmatrix} \] (1.27)

Let us look at what \( \phi \) is in terms of known variables. By drawing the triangle we can see that:

\[
\tan \phi = \frac{\Delta V \sin 2\theta_V}{\Delta V \cos 2\theta_V - \Delta'}
= \frac{\tan 2\theta_V}{1 - \frac{\Delta'}{\Delta V} \sec 2\theta_V}
\] (1.28)

(1.29)

We can now make the definitions \( L_e \equiv \frac{2\pi}{\Delta} \) and \( L_V \equiv \frac{2\pi}{\Delta_V} \) and we can immediately see, we recover equation 9.47 in [Bahcall]:

\[
\tan 2\theta_M = \frac{\tan 2\theta_V}{1 - \frac{L_V}{L_e} \sec 2\theta_V}
\] (1.30)

We can see from this form that the presence of matter changes the effective mixing angle. Let us take one particular limit to see if we are at least on the right track. If \( n_e = 0 \), then \( \Delta' = 0 \) and we can see that in that case \( \phi = 2\theta_V \) and we recover our vacuum mixing (this limit works for our Eigenvalues as well). Also note that due to our definition of \( \phi \), it is a strictly increasing function of \( \Delta' \) which itself is just directly proportional to \( n_e \). We can therefore see that as \( n_e \) increases, the effective mixing angle increases (in a non-trivial manner of course). Let’s see what happens when \( \Delta_V / \Delta' = \cos 2\theta_V \). In this case, the diagonal elements of our Hamiltonian, equation 13, both become 0. As a consequence, \( a = 0 \), and we have \( \phi = \frac{\pi}{2} \) and therefore \( \theta_M = \frac{\pi}{4} \), so we can get maximal mixing even when \( \theta_V \approx 0 \) (but not if it’s actually is 0, because in that case we have a 0/0 triangle). As \( n_e \to \infty \), we can see that \( \phi \to \pi \) so that \( \theta_M \to \frac{\pi}{2} \), meaning that now the electron neutrino state \( |\nu_e\rangle \) is now entirely aligned with \( |\nu_2\rangle_M \). Thus, if an electron neutrino is produced in an environment in which \( \Delta' >> \Delta_V \), it is very nearly identical
with the second mass eigenstate. If the electron density then decreases slowly, so that the
adiabatic approximation holds, this electron neutrino will stay in the mass state 2. By the
time it reaches the vacuum, the mass eigenstate 2 in matter will have transformed into the
vacuum mass eigenstate \( |\nu_2\rangle_M \rightarrow |\nu_2\rangle_V \), but now since \( \theta_V \ll 1 \), this mass eigenstate 2
is mostly coincident with the second neutrino flavor (muon or tau neutrino), and not the
electron neutrino! Thus our electron neutrino has turned from an electron neutrino into a
different neutrino even though the vacuum mixing angle is very small.

1.2.3 Collective Neutrino Oscillations

Beyond the vacuum flavor oscillations discussed in section 1.2.1 and the resonant
flavor transformation via the MSW effect discussed in section 1.2.2, it has been found
that under certain circumstances neutrinos may undergo collective neutrino oscillations
wherein neutrinos of all different energies collectively transform in flavor in some way.
In all of our simulations presented in this thesis, we also found collective behavior and so
we will give a brief overview here.

As we have seen earlier, the neutrino flavor transformations in vacuum and in
matter are all dependent on the neutrino energy. Collective oscillations can only arise
if there is nonlinear feedback from all of the neutrinos in our ensemble on each other.
Neutrinos can (forward) scatter off of other neutrinos and we can obtain an effective
neutrino-neutrino Hamiltonian (for the \( i \)th neutrino):

\[
H_{\nu\nu,i} = \sqrt{2} G_F \sum_a (1 - \cos \theta_{ia}) n_{\nu,a} |\Psi_{\nu,a}\rangle \langle \Psi_{\nu,a}| - \sqrt{2} G_F \sum_a (1 - \cos \theta_{ia}) n_{\bar{\nu},a} |\Psi_{\bar{\nu},a}\rangle \langle \Psi_{\bar{\nu},a}| \tag{1.31}
\]

Where \( n_{\nu,a} \) and \( n_{\bar{\nu},a} \) are the local number densities of neutrinos and antineutrinos in the “\( a \)”
state respectively (we sum over all possible flavor states). We can see that each neutrino
is affected by the ensemble of all other neutrinos that it meets along its trajectory. This interaction from neutrino to neutrino is highly nonlinear, and it really is this interaction which can give rise to collective behavior.

The easiest way to understand and visualize these collective neutrino oscillations is through the neutrino flavor isospin (NFIS) formalism. This formalism makes a mapping between the 2 dimensional unitary transformations of the neutrino flavor states with rotations in 3 dimensional Euclidean space (so that 2 dimensional complex vectors representing the flavor state are mapped into 3 dimensional real vectors). This formalism would require too much exposition to reproduce here, but it is developed and described in detail in [3–5].

Collective flavor oscillations usually begin to develop where the neutrinos are still very dense (for a supernova or BNS merger environment, this would be close to the central remnant) and the neutrino-neutrino Hamiltonian locks all of the neutrino flavors together into a synchronized oscillation mode where all of the neutrinos of all energies change in flavors as one (at a frequency which is a kind of weighted average of the neutrinos’ vacuum oscillation frequencies). Electron neutrinos and muon neutrinos swap flavors back and forth in synchronized motion. As the number density of neutrinos drop, typically a stepwise bipolar spectral swap develops where (in the normal mass hierarchy) low energy (below a certain energy threshold) electron neutrinos and muon neutrinos swap in flavor, but high energy neutrinos are untouched (in the inverted mass hierarchy, it is the high energy neutrinos which swap flavors). These two collective behavior, synchronized flavor oscillations and bipolar flavor swaps, appear in many of the results of this thesis.
1.3 BULB Code

All simulations carried out in this thesis were performed using the BULB code, originally written by the authors of [3–13], modified to fulfill our specific needs. The original BULB code, written in FORTRAN 90, was made to simulate coherent flavor evolution of neutrinos emitted isotropically from a central spherical surface inside a supernova called the neutrinosphere. Originally, BULB did not have provisions to simulate spin transformations, or the geometry of a Binary Neutron Star (BNS) merger remnant disk.

The most extensive modifications were made in order to incorporate spin transformations into BULB. The underlying object of interest, the neutrino’s state vector ($|\Psi\rangle$), had to be expanded from 2 or 3 dimensional (depending on the number of flavors we wanted to simulate) to be 4 or 6 dimensional. In addition, the dimensions of the Hamiltonian had to be similarly expanded.

Modifications made in order to simulate the BNS merger environment were not as extensive, mostly due to the fact that we chose to simulate only the polar trajectory neutrino in order to not have to change the underlying architecture of BULB (see Sec. 3.3.2 for details). In this section, we will go over in detail first the original BULB code and then the modifications that were made in order to perform the simulations that we wanted to make.

A word of warning about using the BULB code in FORTRAN: BULB was written using implicit variables in order to not have to define explicitly all of the dummy variables used in the code. As such, one must be extremely careful in not making typos when modifying BULB.
1.3.1 **Original BULB**

**Overview**

BULB was written specifically to simulate coherent neutrino flavor evolution. As such, the main object of simulation in the code is a state vector $|\Psi\rangle$ and $|\bar{\Psi}\rangle$, which are the (multidimensional) arrays named “psi” and “psibar” defined in the main program “bulb-new.F90”. The main output of BULB is then a readout of all the values of the “psi” array at one kilometer intervals from some inner starting simulation radius to the final simulation radius (for us, the final radius will always be 5000km). The code is modular in design, and uses the six following modules: “angles.F90”, “helectron.F90”, “keepclocks.F90”, “neutrinodist.F90”, “parallel.F90”, and “params.F90”.

BULB was written to be able to calculate both single angle and multi angle simulations of neutrino flavor evolution. For the purposes of multi angle simulations, different angle bins (of which there were 400 in previous simulations) would be calculated by different processor cores in a supercomputer. One could assign multiple angle bins to each core; however, a single angle bin was never parallelized to be calculated by multiple cores. Therefore, all parallelization efforts put into BULB were made in order to calculate multi angle neutrino bulb simulations. As we only deal with single angle calculations in this thesis, we can simply neglect everything in BULB which deals with parallelization. This means we can neglect everything in the module “parallel.F90” and also all lines of code which are enclosed in “#ifdef mpi, ..., #endif” or “#ifdef mpe, ..., #endif” blocks.

Because BULB evolves the state vectors, $|\Psi\rangle$ and $|\bar{\Psi}\rangle$, and not a density matrix, it must simultaneously evolve four or six state vectors, for two flavor and three flavor simulations respectively, in order to accommodate each of the initial flavors and also both neutrinos and antineutrinos. In addition, even in single angle mode, BULB needs to track the state vector for each energy bin, of which there are typically 400. We track
neutrinos of energies in the range $[0.2 \text{ MeV}, 80 \text{ MeV}]$. All of this information is kept track of in a two six dimensional complex arrays, originally defined in the line of code “complex(kind=real8),dimension(nflavor,nflavor,negy,ncospproc,nphi,2):: psi” (and a corresponding one for psibar). The first dimension of size “nflavor” (number of flavors) denotes which of the initial state kets we are looking at, while the second dimension of size “nflavor” denotes the components of that state ket. The third dimension of size “negy” (number of energy bins) denotes the energy bin. The fourth dimension of this array, of size “ncospproc” (number of angle bins per processor), is used only in multi angle simulations. For us, this will always be 1 and we don’t have to worry about it. The fifth dimension, of size “nphi”, was originally written in the code to provide a possibility of making a more complex multi angle simulation where there was no assumed azimuthal (in emission angles) symmetry of the evolution of $|\Psi\rangle$. This feature was never implemented, and again will always be 1. Lastly, the sixth dimension of the array denotes whether we are at the beginning (1) or end (2) of the particular radial step that we are taking in our evolution (one should not get confused that this last dimension denotes whether we are looking at neutrinos or antineutrinos, antineutrino information is contained in “psibar”).

For our flavor-only evolution simulations (for the BNS merger environment), nflavor is equal to two or three, and negy is equal to 400. Therefore, the array “psi” (and “psibar”) will contain 3200 complex numbers for two flavor simulations and 7200 complex numbers for three flavor simulations. The main output file “wvsav.dat”, written in unformatted binary FORTRAN output, will print out “psi” and “psibar” at roughly one kilometer intervals (so usually BULB will print out “psi” to “wvsav.dat” on the order of 5000 times). For a single angle flavor-only simulation, then, “wvsav.dat” contains on order 16 million or 36 million complex numbers (for two and three flavor simulations respectively) and is typically sized around 200mb or 500mb.

Finally, the vacuum and matter Hamiltonians are written in the output file “he.sav”
while the neutrino-neutrino Hamiltonian is written in the output file “hvv.sav”. There is also a summary output file, “bulb.out”, which shows the energy averaged probabilities contained in $|\Psi\rangle$ and $|\bar{\Psi}\rangle$ at one kilometer intervals and the final survival probabilities for each energy bin. This file also prints out all the main parameters used in the simulation (luminosity, average energies, mixing angles, etc.) as well as the length of time the calculations took.

**Modules**

As mentioned earlier, the module “parallel.F90” is only used for multi angle simulations and so we will not discuss it here. In addition, the module “angles.F90” is used to set up the angle binning in a multi angle simulation and will also not be relevant to our discussion. The module “keepclocks.F90” is used simply to record how long BULB takes to complete each of the steps of the calculation (as well as a total time). Thus, the important modules for our calculations are “helectron.F90”, “neutrinodist.F90”, and “params.F90”.

The purpose of the “helectron.F90” module is to initialize the density profile of the supernova we are examining, to figure out the vacuum Hamiltonian, $H_V$, using the provided mixing angles and mass-squared differences, and to find the matter Hamiltonian, $H_M$, at each simulation radius. This module contains the subroutine “hvac_init” which, as its name suggests, is used to initialize the vacuum Hamiltonian. For two flavor simulations, it simply calculates $\cos^2\theta$ and $\sin^2\theta$ for the flavor basis Hamiltonian shown in equation 1.12 (Programmer’s note: the variables containing this information, “cos2v” and “sin2v”, are defined in the module “params.F90”, not in this module). For three flavor simulations, it explicitly calculates the total three dimensional vacuum Hamiltonian using the full Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and stores it in a real $3 \times 3$ array “hvac”. It should be noted here that the CP violating phase ($\delta_{CP}$) is always
set to 0 and is not included in the coding. (Programmer’s note: as the array “hvac” is a type real array, not a complex array, it appears that the original authors of BULB did not intend to implement a non-zero CP violating phase.)

Having initialized the vacuum Hamiltonian, the module “helectron.F90” goes on to initialize the matter density profile through the subroutine “helec_init”. This subroutine allows the user to choose between three different possible profiles, a \( \rho \propto 1/r^3 \) density profile, a \( \rho \propto e^{-r/r_0} + 1/r^3 \) density profile, and a density profile read in from a text file. If one chooses to read in a density profile from a text file, the code will use the modules “spline.f” and “splint.f” in order to set up a spline (in our case, a piecewise third order polynomial fit) of the density profile so that we can obtain the density at each specific radius that we need. The density profile text file will usually only give the density at large (kilometer to hundreds of kilometer sized) radial intervals whereas we need to know the density in radial intervals of the calculation step size which is typically of order less than one meter. Finally, having initialized the vacuum Hamiltonian and the density profile, a vacuum plus matter Hamiltonian is returned by the function “helec”. It should be noted that “helec”returns an array which includes both the vacuum and matter terms of the Hamiltonian, already summed together.

The “neutrinodist.F90” module initializes the neutrino distribution. For BULB, the neutrinos are all assumed to be in a Fermi-Dirac distribution with a degeneracy parameter of 3. The average neutrino energies are also declared (as parameters) inside this module. If one wants to change the average energies around, one must modify this module and re-make the program (Programmer’s note: it is not clear why the original authors chose to make the average energies parameters instead of variables). This module also normalizes the Fermi-Dirac distributions used to the luminosities for each flavor (details for how this is done for a disk-like geometry is provided in chapter 3, for a spherical geometry the process is analogous).
Lastly, “params.F90” simply contains most of the fixed parameters that we use in our simulations. Most of these fixed parameters are simply physical constants like the speed of light, the Planck’s constant, \( \pi \), etc. But the number of flavors that we are simulating is also contained within this module. (Programmer’s note: not all constants and parameters are contained within this module, for example the solar mass and the mass of a proton are defined in the module “helectron.F90”.) Parameters which we might want to change from simulation to simulation are generally read into the program through the text file “bulb.in.new”.

Main Program

Having taken care of all the preliminary calculations in the various modules, the main program of BULB will only have three major tasks. The first major task is to find the neutrino-neutrino Hamiltonian, the second task is to actually evolve the wave function (i.e. take a step), and the third task is to write out all of the results. The neutrino-neutrino Hamiltonian for a single angle simulation can be written as [6]:

\[
H_{\nu\nu} = \frac{\sqrt{2} G_F}{2\pi R_{\nu}^2} \sum_{\alpha} \int \left[ \rho_{\nu\alpha}(E') f_{\nu\alpha}(E') \frac{L_{\nu\alpha}}{\langle E_{\nu\alpha} \rangle} \right. \\
\left. - \rho_{\bar{\nu}\alpha}(E') f_{\bar{\nu}\alpha}(E') \frac{L_{\bar{\nu}\alpha}}{\langle E_{\bar{\nu}\alpha} \rangle} dE' \right].
\]  

(1.32)

Here \( \rho_{\nu\alpha}, \rho_{\bar{\nu}\alpha} \) are the density matrices for neutrinos and antineutrinos respectively constructed out of the state kets \( |\Psi\rangle, |\bar{\Psi}\rangle \), \( f_{\nu\alpha} \) is the normalized energy distribution of the neutrinos, \( R_{\nu} \) is the radius of the neutrinosphere, and \( L_{\nu\alpha}, \langle E_{\nu\alpha} \rangle \) are the luminosity and average energies of the neutrinos respectively. \( D(r/R_{\nu}) \) is a geometric factor:

\[
D(r/R_{\nu}) \equiv \frac{1}{2} \left[ 1 - \sqrt{1 - \left( \frac{R_{\nu}}{r} \right)^2} \right]^2.
\]  

(1.33)
Most of the complicated coding involving finding the neutrino-neutrino Hamiltonian has to do with multi angle simulations. There, the Hamiltonian is dependent on all of the neutrinos in all of the different angle bins so that care must be taken in order to evolve all of the angle bins in lock step and to share all of the neutrino flavor state information between all the different processors. A single angle simulation is much simpler, all that needs to be done is to get the geometric factors and integrations (summations for discrete computing) correct and to create a density matrix out of the state kets.

Actually evolving the state kets is more complicated, even for a single angle simulation. The basic differential equation that we need to solve after obtaining the full Hamiltonian is:

$$i \frac{\partial}{\partial r} |\Psi\rangle = H |\Psi\rangle.$$

We solve this differential equation using the Magnus method [11, 14, 15] in order to be able to use relatively large step sizes. What BULB does in order to make sure it is using appropriately sized steps is to start off with a given step size and then to make same calculation using two steps which are half the size of the original. If the final state ket differs between the two cases by more than the error tolerance, BULB will cut the step size down by half and redo the calculations. BULB will then keep reiterating this process until a convergence is found. (Programmer’s note: current error checking makes sure no component in any state ket differs by more than one part in a million between the two cases.) Periodically, BULB will also increase the step size to see if it can get away with using a larger step size to make the calculations more efficient. This way, BULB’s adaptive step sizes make sure that the calculations are proceeding as efficiently and as fast as possible without introducing too much numerical errors into the calculations.

The subroutine which actually evolves the state ket from step to step is called “onestep”. In order to use the Magnus method, BULB needs to be able to diagonalize the Hamiltonian. Diagonalization and exponentiation of the Hamiltonian is done in the
subroutine “exphcal”. Diagonalization itself in BULB for three flavor simulations relies on the linear algebra package “zheevh3” (called upon by exphcal), while two flavor diagonalization is written directly into exphcal.

After evolving the state kets, BULB will write the results to the files “he.sav”, “hvv.sav”, “wvdat.sav”, and “bulb.out” which were described above. Of these files, only “bulb.out” is formatted and can be opened with a standard text editor. The other files, in order to save space, are unformatted and require some program to unpack the binary data into a format that is usable. There are a few subroutines written specifically to write information to the disk, those are “wv_store”, which stores the full state ket information (including phases), “wrtegy”, which writes out the probabilities obtained from the state kets for each energy bin (for multi angle simulations it averages these probabilities over the angle bins), and “wrdens” which writes out an energy (and angle for multi angle simulations) averaged probabilities obtained from state kets. All other write statements are included at various points in the main code of BULB.

1.3.2 BULB Modifications

In order to perform the calculations that we wanted to make in this thesis, for a supernova flavor transformation calculation which includes spin transformations, and for a BNS merger flavor transformation calculation, we had to modify BULB to our purposes. In order to accommodate spin coherence, significant modifications had to be made, while for the BNS merger environment, only minor modifications had to be made in the coding of BULB in order to accommodate the new geometry used. Most of the changes in initial conditions required to simulate a BNS merger environment (e.g. in the initial luminosities, average energies, and matter density profiles, etc.) were able to be accommodated by BULB without modifying any coding. For further details for the difference in geometry between a BNS merger environment and a spherical bulb
simulation, see section 3.3.1.

The inclusion of possible spin transformations in the way neutrinos evolve requires us to change the fundamental object of evolution in BULB. For this thesis, we will consider Majorana type neutrinos. If neutrinos are Majorana in nature, then a spin transformation is really a transformation of a neutrino into an antineutrino and vice versa. If neutrinos are Dirac in nature, spin transformations would change active neutrinos and antineutrinos into sterile states. Therefore, instead of the 2 or 3 dimensional $|\Psi\rangle$ and $|\bar{\Psi}\rangle$ that is simulated in BULB, we need to simulate a full 4 or 6 dimensional state ket for 2 or 3 flavor simulations respectively. This means that we have to define a new multidimensional array, “psispm” that we will evolve with BULB. Here, a single multidimensional array is enough since the antineutrinos are now simply the right handed states of neutrinos.

The bulk of the modifications to BULB are contained in a newly written module “spincohere.F90”. This module contains the subroutine “hspinmsub” which is used to obtain the spin-flip Hamiltonian (the “m” in the name of this subroutine stands for Majorana), the subroutine “onestepspm” which takes a radial step in the neutrino spin and flavor evolution (as the name suggests, it is the analog to the subroutine “onestep” but for Majorana type spin transformations simulations), and the subroutine “exphcalsp” which can handle diagonalization of the (full $4 \times 4$ or $6 \times 6$) Hamiltonian for spin calculations. Diagonalization of $4 \times 4$ or $6 \times 6$ Hamiltonians is handled by the linear algebra package “zheev” which is the new package in LaPack (Linear algebra PACKage) for diagonalization of Hermitian matrices of arbitrary size.

In order to allow BULB to retain its original function and be able incorporate spin transformations, all modifications inside BULB are set inside if/else statements so that a single variable “spinco” (specified “bulb.in.new”) can be changed between 1 or 0 for making simulations with and without spin transformations respectively. Modifications to
the main code of BULB included changing the way the angle integrals are performed in order to accommodate a non-radial representative trajectory for neutrino simulation (for the reason why we must do this, see section 2.4), changing the error checking in BULB to accommodate the expanded dimensionality of the problem, and changing the way the relevant density matrix components (not all components are relevant for us, see section 2.3) are obtained from the new “psispm”. In addition, analogous subroutines to write information to the hard drive were necessary, these are the subroutines “wv_storespm”, “wrtdensspm”, and “wrtegyspm”.

Lastly, “helectron.F90” had to be changed. We needed to add back in the trace which was removed to all of the Hamiltonians (see sec 2.3 for details on why). Next, we had to be able to work with a variable electron fraction, so subroutine “varelec_init” was written in order to set up a way for us to read an electron fraction profile from a text file. A subroutine “linearsplint” was written to extrapolate the electron fraction in between data points in the electron fraction data file by simply piecewise fitting a linear function to the electron fraction profile (since our use of the variable electron fraction was quite crude, see chapter 2, it did not seem necessary to use a more complicated spline here).

All of these modifications combined to make BULB much more computationally intensive. A full three-flavor simulation with spin (assuming Majorana type neutrinos) can take upwards of 70 hours to complete (compare that to the typical one hour computation times for single angle simulations without spin).
2.1 Abstract

We present neutrino bulb model simulations of Majorana neutrino coherent spin transformation (i.e., neutrino-antineutrino transformation), coupled to neutrino flavor evolution, for conditions corresponding to the neutronization burst epoch of an Oxygen-Neon-Magnesium (O-Ne-Mg) core collapse supernova. Significant neutrino spin transformation in, for example, the neutronization burst, could alter the fluences of neutrinos and antineutrinos in a way which is potentially detectable for a Galactic core collapse supernova. Our calculations for the first time incorporate geometric dilution in the spin evolution of the neutrinos and combine two-flavor and three-flavor evolution with spin mixing physics. We find that significant spin transformations can occur, but only with a large neutrino luminosity and an electron fraction \( Y_e \) profile which facilitates adiabatic conditions for the spin-channel resonance. Using our adopted parameters of neutrino energy spectra, luminosity, density and \( Y_e \) profiles, our calculations require an
unrealistically large neutrino rest mass to sustain the spin transformation. It is an open question whether examining different density profiles or incorporating other sources of nonlinear feedback, such as $Y_e$ feedback, could mitigate this need. We find that spin transformations are not sensitive to the flavor structure of neutrinos, i.e., the spin transformations occur regardless of whether we simulate two- or three-flavor transformations. In the two-flavor case, spin transformations were insensitive to the choice of solar or atmospheric mass-squared splitting as well as the choice of the Majorana phase. Importantly, our three-flavor simulations, as well as our two-flavor simulations done with the atmospheric mass-squared splitting, show that the inclusion of spin degrees of freedom can significantly and qualitatively alter neutrino flavor evolution.

2.2 Introduction

In this paper we study new aspects of how neutrino flavor and spin physics could play out in the core collapse supernova environment. Neutrino flavor transformation in astrophysical environments can be a complicated, nonlinear phenomenon [3–13, 16–49]. In addition, there have been several studies of neutrino spin (or helicity) transformation as a consequence of an external magnetic field acting on a large neutrino magnetic moment, some of which are in the context of supernovae [50–76]. However, it has been discovered recently, via examination of the quantum kinetic equations (QKEs), that neutrinos may undergo this spin conversion from left-handed helicity states to right-handed helicity states purely kinetically in the presence of an asymmetric matter and neutrino flow (as would be present in a supernova environment), even in the absence of a magnetic field or a large magnetic moment [77–89]. In this paper we study spin conversions arising from purely kinetic effects.

In vacuum, active neutrinos are in left-handed helicity states and active antineu-
trinos are in right-handed helicity states. If neutrinos are Majorana in nature, spin transformations are equivalent to transformations of neutrinos into antineutrinos and vice versa. If neutrinos are Dirac in nature, this spin transformation would produce sterile states from the active neutrino species. In this paper, we assume neutrinos are Majorana in nature and examine the prospects for coherent neutrino-antineutrino transformation during the neutronization burst epoch of an O-Ne-Mg core collapse supernova.

In medium, the propagation states of neutrinos can be superpositions of left-handed and right-handed helicity states. As was first shown in Ref. [77], it is possible to find a “resonance”, akin to a Mikheyev-Smirnov-Wolfenstein (MSW) resonance [90, 91], through which adiabatic propagation gives nearly complete helicity flip. However, this spin resonance is narrow, i.e., the instantaneous neutrino energy eigenstates are nearly degenerate through resonance, implying that achieving the conditions required for adiabatic spin transformation is problematic. An outstanding question is whether nonlinear feedback from spin transformation can augment the adiabaticity in a core collapse supernova environment. In this paper we investigate this issue, with the new features here being coupled spin and flavor evolution and a more realistic geometry.

In seeking the optimal environment for neutrino spin degrees of freedom to affect neutrino evolution, we focus on the core collapse supernova neutronization burst. As a massive star reaches the end of its life, its core becomes dynamically unstable. If the core of the star is sufficiently massive, i.e., over the Chandrasekhar limit, electron degeneracy pressure is overcome by gravity and the core will catastrophically collapse until it reaches nuclear densities [92]. As the core collapses, it “neutronizes” via charged current electron capture on protons in heavy nuclei. An inner, homologous, core “bounces” at nuclear density and serves as a piston, driving a shock into the outer part of the core [92–94]. When this shock comes through the “neutrino sphere” (roughly coincident with the outer edge of the core), where the material becomes more or less transparent to neutrinos, we get
a “neutronization burst” [21,95]. This shock breakout, lasting $\approx 10\text{ms}$, is accompanied by a spike in the neutrino luminosity of order $10^{53}$ to $10^{54} \text{erg s}^{-1}$. Moreover, the flavor content of this neutronization burst is overwhelmingly electron type neutrinos, $\nu_e$ [95].

In this paper we examine the prospects for neutrino spin transformations specifically in the neutronization burst epoch for two main reasons. First, since the neutronization burst neutrino luminosities are extremely high [95], there can be a larger contribution to the $\nu_e \leftrightarrow \bar{\nu}_e$ transformation channel in the Hamiltonian during the neutronization burst than during other epochs. This may lead to conditions which are the most favorable for coherent spin transformation. Second, since the neutronization burst produces an overabundance of electron neutrinos over all other flavor and spin states [95], spin transformations, if they occur, can drastically change the ratio of left-handed neutrinos to right-handed antineutrinos coming out of the supernova. This therefore makes spin transformations during the neutronization burst a potentially measurable event. Detection of a neutronization burst in a terrestrial detector, e.g., the Deep Underground Neutrino Experiment (DUNE) or Hyper-Kamiokande (Hyper-K), could provide, in principle, a unique way to probe neutrino absolute masses and Majorana phases complementary to neutrinoless double beta decay experiments. In a hypothetical example, suppose Hyper-K detects a significant antineutrino content in the neutronization burst of a future galactic core-collapse supernova. What would that imply for parameters such as the neutrino absolute rest-mass scale, or matter density and electron fraction profiles in the envelope? What would that mean for models of neutrino heating or nucleosynthesis? Answering these questions requires detailed calculations.

For this paper, we conducted astrophysically simplistic, albeit computationally sophisticated, surveys of what neutrino flavor and spin transformations might occur, by simulating neutrino spin and flavor evolution using a variety of potential supernova electron fraction profiles, absolute neutrino masses, and neutrino luminosities. In this paper,
we present the results corresponding to one example set of parameters that led to large, measurable spin transformations. We look at the prospect for these spin transformations in both two- and three-flavor—coupled with two spin states—simulations carried out using a single angle neutrino bulb geometry (see Sec. 2.3) with the correct geometric dilution of neutrino fluxes.

In Sec. 2.3 of this paper, we discuss the Hamiltonian used in both the flavor and spin evolution of the neutrinos as well as the geometry of the neutrino bulb model. In Sec. 2.4 we present the results of our simulations, we discuss them in Sec. 2.5, and we conclude in Sec. 2.6.

2.3 Hamiltonian

In this paper we consider the coherent evolution of neutrinos undergoing forward scattering on a matter background and a background of other neutrinos in a neutrino bulb model [3, 4, 13]. Electron neutrinos are assumed to be emitted isotropically from the surface of a central neutrino sphere (or “bulb”) of radius $R_\nu \approx 60\text{km}$ (see Fig. 2.1), with a Fermi-Dirac blackbody-shaped distribution of energies

$$f(E_\nu) = \frac{1}{F_2(\eta_\nu)T_\nu^3} \frac{E_\nu^2}{e^{E_\nu/T_\nu} - \eta_\nu + 1},$$

(2.1)

where $\eta_\nu$ is the degeneracy parameter, and

$$F_k(\eta_\nu) = \int_0^\infty \frac{z^k}{e^{z-\eta_\nu} + 1} dz,$$

(2.2)

so that the distribution is normalized,

$$\int_0^\infty f(E_\nu)dE_\nu = 1.$$

(2.3)
Neutrinos are emitted isotropically from the surface of a central neutrino sphere with radius $R_\nu$, and subsequently interact with the matter background in the envelope and other neutrinos coming from this neutrino sphere.

We first consider a two-flavor neutrino example. These considerations are generalizable to the three-flavor case in obvious fashion. Since we are considering coherent flavor and spin evolution, the neutrinos can be described as pure states in a four-component ket, with radius and neutrino energy, i.e., $(r, E_\nu)$, arguments suppressed for brevity [20, 22, 96]:

$$
\langle \Psi_{\nu e} \rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \Psi_{\nu_x} \rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},
$$

$$
\langle \Psi_{\bar{\nu}_e} \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle \Psi_{\bar{\nu}_x} \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.
$$

(2.4)

In this paper, when dealing with two-flavor situations, we will use the symbol “$x$”, in place of “$\mu$” or “$\tau$” flavors, to refer to the second flavor state (besides the electron flavor). The $\nu_x$ refers to a particular linear combination of the nearly maximally mixed
\( \nu_{\mu} \) and \( \nu_{\tau} \)-flavor states \([97, 98]\). The neutrinos obey a Schrödinger-like equation, which for a neutrino of energy \( E_\nu \) is \([5–7, 20, 22, 38, 96]\):

\[
i \hbar \frac{\partial}{\partial r} |\Psi_\nu\rangle = H(r, E_\nu)|\Psi_\nu\rangle, \tag{2.5}
\]

where the Hamiltonian is now a \( 4 \times 4 \) matrix which encodes all the flavor and spin evolution of the neutrino states. In future discussion, we will also suppress the \( (r, E_\nu) \) arguments in the Hamiltonian for brevity. For convenience of discussion, we break up the Hamiltonian into \( 2 \times 2 \) blocks:

\[
H = \begin{pmatrix}
H_{\text{vac}} + H_m + H_{\nu\nu} & H^{sf} \\
(H^{sf})^\dagger & (H_{\text{vac}} - H_m - H_{\nu\nu})^T
\end{pmatrix}.
\tag{2.6}
\]

### 2.3.1 Diagonal Hamiltonian

The diagonal blocks of the total Hamiltonian are familiar from normal flavor evolution physics, with the caveat that the diagonal entries of \( H_m \) and \( H_{\nu\nu} \) now have to be defined relative to the vacuum rather than relative to other flavors. Another way to state this is to say that the traces which were removed from \( H_m \) and \( H_{\nu\nu} \) in usual studies of flavor evolution now have to be restored.

First we look at the vacuum term \( H_{\text{vac}} \) which is the Hamiltonian arising merely from the fact that neutrino mass eigenstates are not coincident with neutrino flavor eigenstates \([96]\). The vacuum Hamiltonian for both the neutrino sector and the antineutrino sector are the same since neutrinos and antineutrinos have the same mass \([6, 96]\):

\[
H_{\text{vac}} = \frac{\delta m^2}{4E_\nu} U \begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix} U^\dagger.
\tag{2.7}
\]
Note that here we can still use the traceless version of the vacuum Hamiltonian in this case. Here \( \delta m^2 = m_{\nu,2}^2 - m_{\nu,1}^2 \) is the mass-squared splitting of the two neutrino species, which we have taken to be either the solar splitting \( \delta m^2 = \delta m_{\odot}^2 = 7.6 \times 10^{-5} \text{eV}^2 \) or the atmospheric splitting \( \delta m^2 = \delta m_{\text{atm}}^2 = 2.4 \times 10^{-3} \text{eV}^2 \) \[96\]. \( U \) is the two-flavor version of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix \[96\]:

\[
U = \begin{bmatrix}
\cos \theta_V & \sin \theta_V \\
-\sin \theta_V & \cos \theta_V
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & e^{i \alpha / 2}
\end{bmatrix}.
\] (2.8)

In this matrix, \( \alpha \) is the Majorana phase which we have set to \( \alpha = 0 \) (we find that the results are insensitive to \( \alpha \), which is discussed in Sec. 2.5.4), and \( \theta_V \) is the mixing angle which we have taken to be \( \theta_V = 8.7^\circ \) for two-flavor simulations. The three-flavor version of the PMNS matrix will have three mixing angles, a \( CP \) violating Dirac phase, and two \( CP \) violating Majorana phases. Note that, even if \( \alpha \neq 0 \) here, the matrix multiplication in equation 2.7 will result in the cancellation of the Majorana phase terms. Equation 2.7 will be unchanged by a change in the Majorana phase, and so, as expected, flavor transformations are not affected by a Majorana phase.

The diagonal block matter term \( H_m \) is the same term familiar from neutrino flavor transformation physics, except that, as mentioned, the Hamiltonian must now be defined with respect to the vacuum. Therefore we must also include contributions from the neutral current scattering of neutrinos as well as charged current scattering \[6, 83, 96\]:

\[
H_m = \sqrt{2} G_F (1 - V_{\text{out}} \cos \beta) \begin{bmatrix}
n_e - n_n / 2 & 0 \\
0 & -n_n / 2
\end{bmatrix},
\] (2.9)

where \( G_F \) is the Fermi weak coupling constant, \( n_e \) is the local net electron number density, \( n_e \equiv n_{e^-} - n_{e^+} \), and \( n_n \) is the local neutron number density. \( V_{\text{out}} \) is the local
outflow velocity of matter and $\beta$ is the angle the neutrino makes with the matter outflow. Due to net charge neutrality, we can express this Hamiltonian in terms of the baryon number density $n_b$ and the electron fraction $Y_e \equiv n_e/n_b$ \cite{6, 83, 96}:

$$H_m = \frac{G_F n_b}{\sqrt{2}} (1 - V_{out} \cos \beta) \begin{bmatrix} 3Y_e - 1 & 0 \\ 0 & Y_e - 1 \end{bmatrix}. \quad (2.10)$$

The diagonal block neutrino-neutrino Hamiltonian $H_{\nu\nu}$ is more complicated and will depend on the geometry of the neutrino trajectories. Again, we have to define this Hamiltonian with respect to the vacuum. For a bulb model, the neutrino-neutrino Hamiltonian is \cite{6, 83}:

$$H_{\nu\nu} = \frac{\sqrt{2} G_F}{2 \pi R^2} \sum_{\kappa} \int_0^{\infty} \int_0^{\theta_{ns}} \frac{L_{\nu,\kappa}}{\langle E_{\nu,\kappa} \rangle} (1 - \cos \vartheta \cos \vartheta') \Lambda_{\nu,\kappa}(E', \vartheta') f_{\nu,\kappa}(E') \sin \vartheta' d\vartheta' dE'. \quad (2.11)$$

Here, the index $\kappa$ refers to the flavor and spin state of neutrinos at the point of emission, i.e., at the neutrino sphere surface. $\kappa$ runs over all four of the flavor and spin states; i.e., $\kappa = 1$ is an electron neutrino, $\kappa = 2$ is an $x$-neutrino, $\kappa = 3$ is an electron antineutrino, and $\kappa = 4$ is an $x$-antineutrino. The assumption that neutrinos are emitted in flavor and spin eigenstates is predicated on neutrino decoupling being instantaneous at the neutrino sphere, which is a reasonable approximation given the steep density profile. $\theta_{ns}$ is the maximum angle that the neutrino sphere subtends at the location of the neutrino which we are tracking, and thus $\sin \theta_{ns} = R_{\nu}/r$. $L_{\nu,\kappa}$ is the luminosity of the $\kappa$ state neutrinos emitted at the neutrino sphere, and $\langle E_{\nu,\kappa} \rangle$ is the average energy of those neutrinos. The angle $\vartheta$ is the angle that the test neutrino makes with the radial direction at the interaction site and we have to integrate over all the other neutrinos. Finally, $\Lambda_{\nu,\kappa}(E', \vartheta')$ is a two-by-two matrix:
\( \Lambda_{\nu,\kappa}(E', \vartheta') = \begin{bmatrix} 2\rho_{ee,\kappa} + \rho_{xx,\kappa} & \rho_{ex,\kappa} \\ \rho_{ex,\kappa}^* & \rho_{ee,\kappa} + 2\rho_{xx,\kappa} \end{bmatrix} (E', \vartheta') \)

\( - \begin{bmatrix} 2\rho_{\bar{e}e,\kappa} + \rho_{\bar{x}x,\kappa} & \rho_{\bar{e}x,\kappa} \\ \rho_{\bar{x}x,\kappa}^* & \rho_{\bar{e}e,\kappa} + 2\rho_{\bar{x}x,\kappa} \end{bmatrix} (E', \vartheta') \).

(2.12)

The density matrix elements in this equation are defined from the pure state kets as follows:

\[ \rho_{ij,\kappa}(r) = \Psi_{\nu,\kappa}^*(r) \Psi_{\nu,\kappa}(r). \]  

(2.13)

Here, \( \Psi_{\nu,\kappa} \) is the \( i \)th component of the state ket of the neutrino which started out at the neutrino sphere in the \( \kappa \) state. Here, the index \( i \) runs over the same flavor/spin basis states as the index \( \kappa \). Finally, since we are performing single angle calculations in this paper, the angle integrals can be evaluated analytically (for a spherical geometry). Since the single angle approximation entails that all neutrinos on all trajectories are assumed to evolve in the same way as a neutrino on the test trajectory, the density matrices are assumed to be not angle dependent, i.e., \( \Lambda_{\nu,\kappa}(E', \vartheta') = \Lambda_{\nu,\kappa}(E') \). Therefore, we find [6]

\[ H_{\nu\nu} = \frac{\sqrt{2}G_F}{2\pi R_\nu^2} \sum_\kappa \int_0^\infty L_{\nu,\kappa} \langle E_{\nu,\kappa} \rangle (A(r) - B(r) \cos \vartheta) \]

\[ \times \Lambda_{\nu,\kappa}(E') f_{\nu,\kappa}(E') dE'. \]

(2.14)

Here we have defined

\[ A(r) = 1 - \sqrt{1 - \frac{R_\nu^2}{r^2}}, \quad B(r) = \frac{1}{2} \frac{R_\nu^2}{r^2}. \]

(2.15)
2.3.2 Off-diagonal Hamiltonian

In this subsection we discuss the off-diagonal block, the spin-flip Hamiltonian $H_{sf}$. This Hamiltonian consists of two parts, one due to a matter background $H_{sf}^m$ and one due to the other background neutrinos $H_{sf}^{\nu\nu}$. The total spin-flip Hamiltonian is [79,82–84]:

$$H_{sf} = (H_{sf}^m + H_{sf}^{\nu\nu}) \frac{m^*}{E} + \frac{m^*}{E} (H_{sf}^m + H_{sf}^{\nu\nu})^T$$

(2.16)

The spin-flip Hamiltonian, unlike the diagonal flavor evolution parts of the total Hamiltonian, depends on the absolute mass of the neutrino. The mass matrix $m$ is

$$m = U^* \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} U^\dagger$$

(2.17)

and due to the presence of the $U^*$ instead of $U$ in this transformation, the Majorana phase can have an effect on this mass term, and therefore on spin transformations. As one can clearly see, the $m^*/E$ term will tend to make the spin-flip Hamiltonian much smaller than the diagonal block matter and neutrino-neutrino Hamiltonians. The matter and neutrino-neutrino parts of the spin-flip Hamiltonian are [79,82–84]

$$H_{sf}^m = -\frac{G_F n_b}{2\sqrt{2}} V_{out} \sin\beta \begin{bmatrix} 3Y_e - 1 & 0 \\ 0 & Y_e - 1 \end{bmatrix}$$

(2.18)

$$H_{sf}^{\nu\nu} = -\frac{\sqrt{2}G_F}{2\pi R_\psi^2} \sum_\kappa \int_0^\infty \int_0^{\theta_{\text{max}}} \frac{L_{\nu,\kappa}}{\langle E_{\nu,\kappa} \rangle} \sin\vartheta \cos\vartheta'$$

$$\times \Lambda_{\nu,\kappa}(E', \vartheta') f_{\nu,\kappa}(E') \sin\vartheta' d\vartheta' dE'$$

(2.19)
and again, in the single angle approximation we can perform the $\vartheta'$ integral in the last equation to obtain [83]:

$$
H_{\nu \nu}^{sf} = \frac{\sqrt{2} G_F}{2 \pi R^2_\nu} \sum_\kappa \int_0^\infty \frac{L_{\nu, \kappa}}{\langle E_{\nu, \kappa} \rangle} B(r) \sin \vartheta 
\times \Lambda_{\nu, \kappa}(E') f_{\nu, \kappa}(E') dE'.
$$

(2.20)

A nonzero spin-flip potential means that propagating neutrinos are, in general, coherent superpositions of left-handed and right-handed states. In other words, a neutrino’s instantaneous energy eigenstates are not coincident with the neutrino’s helicity eigenstates. As a neutrino propagates through the supernova environment its spin can rotate, for example, from an initial left-handed neutrino into a right-handed antineutrino.

### 2.4 Results

In this study, we ran several single angle simulations with a variety of initial conditions and neutrino parameters. An example set of conditions and parameters which fostered significant spin-flip transformations are outlined in Table 2.1, and the corresponding results are presented below. We used a version of the flavor evolution code developed by the authors in Refs. [3–13], but extensively modified to incorporate the spin degrees of freedom described above. We used conditions which are similar to those found during the neutronization burst epoch of a supernova: very high electron neutrino luminosities and no other flavor or spin states (i.e. no antineutrinos) present. The luminosity we used is toward the higher end of possible luminosities even for the neutronization burst, but this high luminosity was necessary to obtain significant spin transformations. As the neutronization burst neutrinos were emitted from the core before the shock front had traversed the material in the envelope of the star, matter speeds were subsonic and therefore we took the outflow velocity to be zero.
Table 2.1: Parameters used in single angle simulations with spin-flip. The parameters are chosen to highlight the spin-flip effect and also to match, as much as possible, the neutronization burst epoch of a O-Ne-Mg supernova. For two-flavor simulations we used the mixing angle $\theta_V = \theta_{13}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\nu,e}$</td>
<td>$1.8 \times 10^{54}$ erg/s</td>
</tr>
<tr>
<td>$L_{\nu,x,\bar{e},\bar{x}}$</td>
<td>0 erg/s</td>
</tr>
<tr>
<td>$\langle E_{\nu,e} \rangle$</td>
<td>11 MeV</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>0 m/s</td>
</tr>
<tr>
<td>$m_1$</td>
<td>10 eV</td>
</tr>
<tr>
<td>$\delta m^2_{\odot}$</td>
<td>$7.6 \times 10^{-5}$ eV$^2$</td>
</tr>
<tr>
<td>$\delta m^2_{\text{atm}}$</td>
<td>$2.4 \times 10^{-3}$ eV$^2$</td>
</tr>
<tr>
<td>$\vartheta_0$</td>
<td>60°</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>34.4°</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>8.7°</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>45°</td>
</tr>
<tr>
<td>$\delta_{cp}$</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
</tr>
</tbody>
</table>

The spin-flip potential experienced by a test neutrino is proportional to the component of the matter and neutrino currents transverse to its momentum. In a spherically symmetric model, for a radially directed test neutrino, the transverse background neutrino current has to add up to zero, just by symmetry (as is reflected in the $\sin \vartheta$ dependence in equation 2.19). Therefore, in the absence of convective currents or asymmetric matter outflows, a radially directed neutrino would experience no spin-flip potential. Consequently, we have chosen to track a neutrino which is emitted at 60 deg ($\vartheta_0 = 60^\circ$) with respect to the normal (radial direction) of the neutrino sphere. In a realistic supernova model, the presence of transverse currents and asymmetries in the neutrino outflow could, in principle, give rise to a spin-flip effect in even radially directed neutrinos.

Figure 2.2 shows the baryon density, $\rho_b$, and electron fraction, $Y_e$, profile we used for our simulations. The electron fraction was set to hover close to $Y_e \approx 1/3$ relatively close in to the neutrinosphere so as to best facilitate the spin transformations (see section
2.5.1 for details on why this is). Electron fraction profiles which were not flattened near $Y_e \approx 1/3$ or which go through $Y_e \approx 1/3$ significantly farther out did not produce a significant spin transformation effect. It should be noted that, even though the electron fraction profile we used in this study is artificial, it does, however, conform to the general expectation that the electron fraction is lower closer to the neutron rich material in the core and grows as we move out into the envelope.

![Figure 2.2](image)

Figure 2.2: The baryon density (blue) and electron fraction (red) profiles that we used in our simulations. The density profile is that of an O-Ne-Mg supernova taken from [37, 99], while the electron fraction profile was created artificially so as to increase the chances of inducing significant spin transformations. Notice that the density profile is extremely centrally concentrated, with a steep dropoff at $r \approx 1000$ km.

### 2.4.1 Solar Splitting

Two-flavor simulations using the solar splitting, $\delta m^2$, were carried out first in order to get a feeling for the spin transformations. Two-flavor simulations using the solar splitting are significantly faster to run than ones which use the atmospheric splitting. With a larger mass-squared splitting like the atmospheric one, the natural flavor oscillation wavelength is much shorter and as such, the step sizes used in simulations become much smaller. Full three-flavor simulations are quite computationally intensive and take upward of eight hours or more to run. Solar splitting results are also much simpler in
terms of the flavor evolution, and it is therefore easier to concentrate on the spin degrees of freedom. As such, most of the parameter space in terms of luminosities, electron fraction density profiles, etc., was explored using the solar splitting simulations. Only after finding significant spin transformations do we then run atmospheric splitting and three-flavor simulations in order to gauge any effect the spin transformations have on flavor transformations or vice versa.

**Figure 2.3:** The left-hand graph shows the probabilities for a neutrino which started out in the electron neutrino state to be in any of the four possible states as a function of radius. As spin-flip resonance occurs, beginning around a radius of $\approx 500\,\text{km}$, a large percentage of electron neutrinos are converted into electron antineutrinos. The neutrino flavor states stay stable for a few hundred kilometers before flavor evolution begins at a radius of $\approx 1100\,\text{km}$. The right-hand graph shows the normalized final neutrino energy spectral distribution functions. The normalization we employed here is the same as the normalization employed in [6]. The area under the magenta initial curve and, therefore the sum of the areas under the other four colored curves are equal to 1.

Figure 2.3 shows the energy averaged probability evolution history and the final spectral distribution of the initial electron neutrinos propagating out from the supernova to a final simulation radius of 5000 km. The final simulation distance of 5000 km was chosen to be quite far out so that we could see both the interesting flavor and spin transformations. We can see that significant spin transformation did occur. Approximately 45% of initial electron neutrinos were converted into electron antineutrinos. Once the spin-flip transformation ends at a radius of about $r \approx 800\,\text{km}$, the ratio of neutrinos
Figure 2.4: These are the probability evolution and spectral graphs for a solar splitting simulation where the spin-flip term has been turned off. All other parameters are the same as those used to produce the simulation in figure 2.3
to antineutrinos stays constant for a couple hundred kilometers, after which flavor transformations take over. The flavor evolution appears to go into a collective oscillation mode where essentially all of the neutrinos at all energies oscillate in step.

From the final spectral distribution we can see that the spin transformations converted preferentially lower energy neutrinos into antineutrinos, while leaving the very high energy neutrinos intact. This is to be expected simply due to the \( \frac{m}{E_{\nu}} \) factor in the spin-flip Hamiltonian which suppresses the spin-flip for high energy neutrinos. However, the fact that the spin transformation was not limited to simply the lower energy bins, but affected the mean energy neutrinos as well, is an interesting result. The spin transformation converted neutrinos into antineutrinos, and then the flavor transformation gave rise to $\nu$ and $\bar{\nu}$ neutrinos. Consequently, although we started out with all electron neutrinos, by the end of our simulation, we had neutrinos of every flavor and spin.

Figure 2.4 shows the same graphs as Fig. 2.3, for a simulation using the exact same parameters but with the spin coherence term turned off. The flavor transformations in Figs. 2.3 and 2.4 are qualitatively similar in the sense that, in both cases, beyond $r \approx 1000$ km, the neutrinos undergo synchronized flavor oscillations with a small amplitude,
thereby largely preserving their flavor composition through the process. The frequency of synchronized oscillations, $\Omega_{\text{sync}}$, is higher in the presence of antineutrinos (shown in figure 2.3), as is expected [13, 35]. Note that synchronized oscillations with the solar neutrino mass-squared splitting are still in effect at our final radius of $r = 5000\,\text{km}$ due to the smallness of the solar neutrino mass-squared splitting and therefore the vacuum Hamiltonian.

After running several simulations using the solar neutrino mass-squared splitting, we found that the spin transformations are very sensitive to the initial conditions inside the supernova. For example, our simulations have shown that, keeping everything else constant, raising or lowering the neutrino luminosity by more than 20%-30% from our adopted value will essentially destroy spin transformations. Additionally, if the electron fraction profile was made to go through $Y_e \lesssim 1/3$ more quickly, or if the neutrino rest mass was set to significantly less than the unrealistically large $[100–10^5]\,10\,\text{eV}$ value, no significant spin transformations occurred.

This behavior can be explained as follows: in order to achieve significant spin transformation, nonlinear effects must take hold to keep the neutrino Hamiltonian near resonance (the so-called tracking behavior; see section 2.5.3 for details). Therefore, it is possible that a small to moderate change in initial conditions will drastically affect the spin transformations. If the neutrino Hamiltonian is not kept near resonance, and no tracking behavior develops, the conversion of neutrinos into antineutrinos becomes entirely negligible in terms of a potential terrestrial detection (usually of order one part in 10 billion).

If these spin transformations happened in a real supernova, the effects could be detectable. Without spin transformations we expect not to see a significant antineutrino content coming from a neutronization burst signal. Thus, just from the solar splitting simulations, we would now expect a significant antineutrino content which would be
robust to any flavor transformation physics.

### 2.4.2 Atmospheric Splitting

![Figure 2.5](image)

**Figure 2.5**: As with figure 2.3, the left-hand graph shows the evolution of a neutrino which started out in the electron neutrino state and the right-hand graph shows the normalized final neutrino energy spectral distribution functions. Here we see that the spin transformations, $\nu_e \rightarrow \bar{\nu}_e$, were not changed from the simulations with the solar mass-squared splittings.

Simulations performed with the atmospheric neutrino mass-squared splitting and full three-flavor simulations showed essentially the same spin-transformation phenomena as the simulations with the solar splitting given in the previous section. In a broad brush, only the flavor transformations differ among the different simulations. This makes sense because the absolute neutrino mass that we chose to analyze is several orders of magnitude larger than the mass splittings.

Figure 2.5 shows the results obtained from two-flavor simulations using the atmospheric mass-squared splittings instead of the solar ones. As we can see, the spin transformations proceeded essentially identically to the solar splitting results. Again, approximately 45% of neutrinos were converted into antineutrinos and the spectrum of transformed neutrinos is the same as before. The spin-flip preferentially transformed lower energy neutrinos into antineutrinos. It is not surprising that in these simulations
Figure 2.6: These are the probability evolution and spectral graphs for an atmospheric splitting simulation where the spin-flip term has been turned off preventing any possibility for the spin-flip. All other parameters are the same as those used to produce the simulation in figure 2.5.

the spin transformations were not affected. The spin transformations occurred prior to any flavor transformations (for an examination of why, see section 2.5.5). The spin transformations began at a radius of $r \approx 500 \text{km}$, and all spin conversion was finished at a radius of $r \approx 800$ to the 55%–45% ratio of neutrinos to antineutrinos we see in the final spectrum. The flavor transformations did not set in until a radius outside of $r \gtrsim 1000 \text{km}$; this is in agreement with previous studies of flavor transformation in the neutronization epoch of an oxygen-neon-magnesium supernova [9, 12, 37]. Therefore, for these simulations, flavor transformations do not have a chance to feed back on the spin transformations.

The converse statement, however, is not true. Spin transformations in our simulations can have an effect on flavor transformations since they happen first. A transformation of 45% of neutrinos into antineutrinos affects the diagonal blocks of the Hamiltonian significantly and can change the subsequent flavor evolution. Figure 2.6 shows the results of a simulation where the spin coherence term has switched off. No spin flip was allowed to occur and only flavor transformations were possible. Unlike the solar mass-squared splitting case, the flavor evolution in the atmospheric mass-squared splitting simula-
tions were qualitatively affected by the spin transformations. These results presented in figure 2.6 differ qualitatively from those in figure 2.5. The flavor evolution here is qualitatively quite similar to previous studies of the flavor evolution for neutronization burst neutrinos in O-Ne-Mg supernovae [9, 12]. Even though we have used a quite high neutrino luminosity, we still get significant flavor transformation from the electron neutrino state to the $x$-neutrino state, $\nu_e \rightarrow \nu_x$, for neutrinos with energies less than approximately $E_l \lesssim 20\,\text{MeV}$, qualitatively similar to previous single angle and multiangle simulations. Almost all the low energy neutrinos have been converted by the so called “neutrino-background-enhanced MSW-like flavor transformation” [9]. By comparison, the results given in 2.5 show a much lower threshold energy, $E_l \approx 9\,\text{MeV}$, for the $\nu_e \rightarrow \nu_x$ flavor transformation channel. The presence of antineutrinos in the neutrino spectrum has affected the flavor transformations in such a dominant way. Without spin transformations, as much as approximately 90% of neutrinos were transformed into the $x$-neutrino state, whereas with spin transformations, only about 20% of the leftover neutrinos (those not transformed into antineutrinos) were converted into the $x$-neutrino state.

### 2.4.3 Three Flavor

Finally, a full three-flavor simulation was run to see if any qualitative differences can be found in the spin coherence between a full three-flavor simulation and two-flavor simulations. For this three-flavor simulation, the $CP$ violating Dirac phase, $\delta_{CP}$, and both possible Majorana phases, $\alpha_1, \alpha_2$, were set to zero. The results are presented in figure 2.7 for parameters which matched those used in figures 2.3 and 2.5. Here we see again, not surprisingly, that the spin coherence has not been essentially changed at all from either of the two-flavor results. The flavor evolution appears to be a superposition of the two different mass-squared splitting results. We still have neutrinos of energy less than $E_l \approx 9\,\text{MeV}$ being transformed into $\mu$ or $\tau$ neutrinos and we still have what appear to be
collective neutrino oscillations like in the solar mass-squared splitting case although these oscillations are much messier here. The fact that three-flavor evolution is a superposition of the two two-flavor results is also consistent with previous three-flavor studies of the ONeMg neutronization burst [9].

A three-flavor simulation with the spin coherence turned off was also run in order to compare the flavor transformation results. Results are presented in figure 2.8. We see here that the flavor transformation is essentially still a superposition of the two different mass-squared splitting results. The qualitative difference in how the flavor transformation between a simulation with and without spin coherence arises is again a superposition of the differences we found in the two different mass-squared splitting cases. By turning on the spin coherence, the swap energy, $E_{l}$, moved from $\approx 20\text{MeV}$ to $\approx 9\text{MeV}$ just like in the atmospheric splitting case. The synchronized oscillation frequency, $\Omega_{\text{sync}}$, in the solar mass-squared splitting regime grew in the presence of antineutrinos just like in the two-flavor simulations using the solar mass-squared splitting.
Figure 2.8: The same as figure 2.7 but for a simulation without spin coherence. Again, the three-flavor results are roughly a superposition of the two separate two-flavor results where the $\mu, \tau$ flavors are maximally mixed and collectively act like the second $\alpha$ flavor in the two-flavor simulations.

Since the three-flavor results are a superposition of the two different two-flavor results, and since the $\mu$ and $\tau$ flavors are essentially maximally mixed, this lends credence to our separate two-flavor analyses with solar and atmospheric mass-squared splittings. For clarity and simplicity, then, we can choose to focus our discussions on the two-flavor simulations. The three-flavor simulations do not present any phenomenon that was not present in the two-flavor simulations.

2.5 Discussion

2.5.1 Spin Resonance Conditions

In order for spin coherence to have a significant effect, the neutrinos must go through a resonance between left- and right-handed states [77]. For our discussion here, we will restrict ourselves to the two-flavor case since three-flavor simulations did not differ in the spin evolution of the neutrinos from two-flavor simulations and since two-flavor neutrino evolution is much simpler and more intuitive. In flavor evolution, a
MSW resonance occurs when the diagonal components of the Hamiltonian equal each other, i.e., when $H_{11} = H_{22}$ [96]. For clarity, we note that some sources may simply state the resonance condition as $H_{11} = 0$ for two-flavor (only) evolution due to the fact that removing the trace from a $2 \times 2$ matrix means that $H_{11} = -H_{22}$ and so the two conditions are equivalent for a traceless $2 \times 2$ Hamiltonian. Similarly, a resonance [77,83] for the $\nu_e \rightleftharpoons \bar{\nu}_e$ channel happens when the $\nu\nu$-component (the 1-1 component) of the Hamiltonian is equal to the $\bar{\nu}\bar{\nu}$-component (the 3-3 component) of the Hamiltonian ($H_{11} = H_{33}$):

$$ (H_{\text{vac}} + H_m + H_{\nu\nu})_{11} = (H_{\text{vac}} - H_m - H_{\nu\nu})_{11}. \quad (2.21) $$

This is taken directly from equation 2.6. We can see as a consequence of the fact that neutrinos and antineutrinos have identical mass-squared splittings, i.e., they have the same rest mass, that the only way for this resonance condition to hold is if the 1-1 component of the matter Hamiltonian cancels out the 1-1 component of the neutrino-neutrino Hamiltonian:

$$ \frac{G_F n_b}{\sqrt{2}} (3Y_e - 1) + (H_{\nu\nu})_{11} = 0. \quad (2.22) $$

An immediate consequence of this resonance condition is that, unlike the classic MSW resonance [90, 91, 96], it is not dependent on any neutrino energy. Neutrinos of all energies will go through this spin coherence resonance together. Close to the neutrino sphere, we expect the density to be so high that the matter term, neglecting the $(3Y_e - 1)$ part, would dominate over the neutrino-neutrino term even with the extremely high neutrino luminosities seen during the neutronization burst. However, because the neutral current terms now contribute to an energy splitting between neutrinos and antineutrinos, into the Hamiltonian, we can see that $(H_m)_{11}$ will now be negative if $Y_e < 1/3$ and positive if $Y_e > 1/3$. Therefore, if $Y_e \approx 1/3$ the matter Hamiltonian can be suppressed relative to the neutrino-neutrino Hamiltonian. Indeed, we find that passing near $Y_e \lesssim 1/3$
is in fact necessary for resonance to occur close to the neutrino sphere. We need the electron fraction to be less than $1/3$ because the neutrino-neutrino Hamiltonian will be positive due to fact that there are not antineutrinos initially. The condition for resonance will be satisfied, then, as long as $Y_e(r = R_\nu) < 1/3$ and then $Y_e$ passes through $1/3$ at some larger radius in our simulation. As discussed in Ref. [77], the feedback physics in the spin resonance channel we discuss here is quite similar to the matter-neutrino resonance [106–111].

### 2.5.2 Adiabaticity

Although it seems quite likely that the Hamiltonian will pass through spin coherence resonance at some point, another extremely important aspect of the spin transformations, which is the same for flavor transformations in the MSW effect, is whether the system goes through resonance adiabatically or not. If $H_{11}$ goes through zero very quickly, very nonadiabatically, then one would expect no significant spin transformations will occur even though there is a resonance [3, 47, 107]. As a consequence, not only do we have to examine $H_{11}$, we of course also have to look at the spin-flip Hamiltonian itself. For the $\nu_e \leftrightarrow \bar{\nu}_e$ channel, the relevant term to examine is $(H^{sf})_{11}$. Looking at this problem through the eyes of the MSW effect, we can define an adiabaticity parameter [3, 47, 77, 96, 107]:

$$\gamma \equiv \left( \frac{2|(H^{sf})_{11}|^2}{H_{11}} \right)_{\text{res}},$$

where the subscript “res” indicates that the quantities on the right-hand side are being evaluated as the system is passing through resonance. The adiabaticity parameter must satisfy $\gamma \gg 1$ in order for the Hamiltonian to be considered adiabatic as far as spin transformations are concerned [47]. In other words, we want the spin-flip Hamiltonian to
be large compared to the rate of change of the diagonal Hamiltonian term at resonance. We can immediately see, however, that due to the $m/E_\nu$ term in the spin-flip Hamiltonian, this adiabaticity condition will be hard to meet for the quickly changing conditions inside a O-Ne-Mg supernova. The extremely steep density dropoff and the geometric dilution of the neutrino fluxes make it especially hard for spin-flip transformations to be significant. Indeed, our simulations have so far been unsuccessful in generating large spin-flip transformations for neutrino masses $m_\nu \ll 10\text{eV}$. An iron core collapse supernova density profile would not be so centrally concentrated and might be better in terms of adiabaticity. Perhaps with an iron core collapse density profile, we could have gotten significant spin transformations for a smaller neutrino rest mass. For this paper, however, we chose to use the O-Ne-Mg supernova profile so that we could compare our flavor transformation results for the neutronization burst with previous studies like in [9, 12].

Of course, this neutrino mass of $\approx 10\text{eV}$ is unrealistically high. However, equation 2.23 shows that the adiabaticity parameter would be increased by decreasing $\dot{H}_{11}$, the rate at which the diagonal Hamiltonian changes. A sufficiently flattened matter potential $\propto n_b(Y_e - 1/3)$ could make spin transformations possible for more realistic neutrino masses, e.g. for $m_\nu \approx 0.1\text{eV}$. It must be noted here that although we did not artificially flatten the O-Ne-Mg supernova density profile for our simulations, we did use an electron fraction profile which hovered near $Y_e \lesssim 1/3$ for several hundred kilometers. In addition, as the neutrino-neutrino contribution to the Hamiltonian includes geometric dilution, that part of the Hamiltonian cannot be flattened.

As we move farther from the neutrino sphere, the $B(r)$ term in $H_{\nu\nu}^{\nu\nu}$, which encapsulates the integral over $\cos \vartheta'$, in the spin-flip Hamiltonian obviously drops by a factor of $r^2$. On top of that, the $\sin \vartheta$ term will drop as well for all emission angles $\vartheta_0$ as
we move out from the neutrino sphere [3,6]:

\[
\sin \vartheta = \frac{R_\nu \sin \vartheta_0}{r}
\]  

(2.24)

As a consequence, geometric dilution means \( H_{\nu\nu}^{sf} \approx 1/r^3 \). Notice that in equation 2.23, the adiabaticity parameter has an \( |(H^{sf})_{11}|^2 \) term in the numerator which will drop as six powers of the radius. Since this term drops so drastically as we get farther from the neutrinosphere, and it started out very small in the first place, it will be harder at large radius for the spin coherence to be adiabatic unless \( \dot{H}_{11} \) is extremely flat. Even if \( (H_m)_{11} \) is extremely flat far from the neutrinosphere, \( \langle \dot{H}_{\nu\nu} \rangle_{11} \) is determined simply by the geometric dilution of neutrinos. This term will certainly not decrease as six powers of the radius. Notice simulations have so far only been successful in obtaining significant spin transformations fairly close to the neutrino sphere as in the results presented in figure 2.3. A simple order of magnitude estimate from equation 2.23 however, suggests that adiabaticity is unlikely to ever hold for spin transformations. Clearly nonlinear effects are needed in order to obtain a large spin transformation (see the following subsection).

The \( m/E_\nu \) in the spin-flip Hamiltonian has an additional effect in that it makes lower energy neutrinos go through resonance marginally more adiabatically than high energy neutrinos. Thus, if spin transformations do occur, we might expect that lower energy neutrinos are more preferentially transformed into antineutrinos than higher energy neutrinos. As mentioned earlier, all neutrinos will go through the spin resonance together, and it is only the adiabaticity of the resonance that changes between neutrinos of different energies. This fact could help explain why in figure 2.3 the antineutrino spectrum arising from the spin coherence effect appears to be smooth and shows no sharp or jagged cutoffs in energies.
2.5.3 Non-Linear Effects

Figure 2.9: $(H_m)_{11}$, and $(H_{\nu\nu})_{11}$ as a function of radius. The green line is the neutrino-neutrino part of the Hamiltonian, the blue line is the matter part of the Hamiltonian, and the red line is the sum. Spin-flip resonance in the $\nu_e \leftrightarrow \bar{\nu}_e$ channel occurs when the sum of these two elements of the Hamiltonian, the red line “sum”, is zero. On the left-hand figure, this begins around a radius of $r \approx 500\text{ km}$ and ends around a radius of $r \approx 800\text{ km}$ which corresponds exactly to when the spin transformations in figure 2.3 began and ended. Due to nonlinear effects, as we can see, this element of the Hamiltonian tracks $H_{11} \approx 0\text{ MeV}$ for several hundred kilometers. On the right-hand figure, which is for a simulation which did not produce significant spin transformation, the Hamiltonian does not appear to track $H_{11} \approx 0$. The right-hand simulation used all the same parameters as the left-hand simulation but with a steeper electron fraction profile.

Nonlinear effects could strengthen spin transformations. Indeed, the simulations have shown that under specific circumstances, a large spin transformation effect can occur even if the transformation is not expected to be adiabatic for all but the most low energy neutrinos (as discussed above, our simulations require an unrealistically high neutrino rest mass and a specifically tailored electron fraction profile to obtain significant spin transformations). Naive linear reasoning, like that in section 2.5.2 would lead us to the conclusion that even with the highly flattened electron fraction profile we used, no significant spin transformations should occur. However, as the neutrinos move through the resonance (for a growing electron fraction profile this will correspond to $(H_m)_{11} + (H_{\nu\nu})_{11}$ passing from negative to positive values), if some low energy neutrinos
do transform into antineutrinos \((H_{\nu\nu})_{11}\) will be driven to lower values. If the rate of change of this effect is large enough, it can counteract the steeply rising matter potential, thus driving the sum of \((H_{\nu\nu})_{11}\) and \((H_m)_{11}\) back near zero. As discussed in [77], this nonlinear feedback of the neutrino-neutrino interaction tends to keep the sum of \((H_{\nu\nu})_{11}\) and \((H_m)_{11}\) near zero. This nonlinear feedback forcing \((H_m)_{11} + (H_{\nu\nu})_{11} \approx 0\) for an extended length scale is what we mean by tracking behavior.

The left-hand graph in figure 2.9 shows \((H_m)_{11}, (H_{\nu\nu})_{11}\), as well as the sum \((H_m)_{11} + (H_{\nu\nu})_{11}\), for a neutrino as it evolves with radius in the simulation shown in figure 2.3 (i.e., a simulation that produced a large spin-transformation effect). The right-hand graph in figure 2.9 shows the same thing but for a simulation which showed no significant spin transformations. In that simulation, the electron fraction profile was not made to be extremely flat through resonance. All other parameters we kept the same. As such, an extreme lack of adiabaticity in the \(\nu_e \leftrightarrow \bar{\nu}_e\) transformation channel precluded even the nonlinear tracking from beginning and no significant spin transformations occurred. We can see that the diagonal Hamiltonian has been made to track \((H_m)_{11} + (H_{\nu\nu})_{11} \approx 0\) for a few hundred kilometers for the simulation which produced significant spin transformations, but for the simulation which did not produce significant spin transformations, \((H_m)_{11} + (H_{\nu\nu})_{11}\) simply passed through zero smoothly.

We note that since \((H_m)_{11} + (H_{\nu\nu})_{11}\) did pass through zero even for the simulation shown in the right-hand graph in figure 2.9, the neutrinos did go through the spin resonance. The neutrinos simply did not go through the resonance adiabatically enough even for low energy neutrinos, and no tracking behavior was initiated, and thus no significant spin transformations occurred. We note that, although the electron fraction was set to hover near \(Y_e \lesssim 1/3\) for the simulation shown in figure 2.3, that alone is not enough to force the Hamiltonian to track near zero for so long as we see in the left-hand graph in figure 2.9. The electron fraction was flattened but was not finely tuned in order to exactly cancel
out the neutrino-neutrino potential. The tracking must be introduced by nonlinear effects in the neutrino evolution. We found over all simulations that this tracking behavior we just described is necessary in order to produce significant spin transformations. The tracking behavior was difficult to attain for various choices of parameter values. It seems likely that additional sources for feedback phenomena, e.g. electron fraction feed back mechanisms, might be necessary to increase the likelihood of getting into the tracking regime, and thereby produce a large spin-flip effect more robustly.

2.5.4 Majorana Phase

Spin transformation calculations like ours with two or more neutrino flavors involve Majorana phases in a nontrivial way. For a two-flavor system of neutrinos, there can be one Majorana phase which can affect the spin-flip Hamiltonian. For three-flavors, there can be two Majorana phases. Several two-flavor simulations were run with Majorana phases different from zero. It was found, given our chosen initial conditions and parameters, that the Majorana phase affected the spin transformations only negligibly. There are perhaps two reasons that the Majorana phase would not significantly affect our spin transformations. First, we note that equation 2.17 can be multiplied out and rewritten, with $c_\theta \equiv \cos \theta_V$ and $s_\theta \equiv \sin \theta_V$ for brevity, as [83, 84]:

\[
m = \frac{m_1 + m_2}{2} \begin{pmatrix}
  c_\theta^2 + e^{-i\alpha} s_\theta^2 & (e^{-i\alpha} - 1) s_\theta c_\theta \\
  (e^{-i\alpha} - 1) s_\theta c_\theta & s_\theta^2 + e^{-i\alpha} c_\theta^2
\end{pmatrix}
\]

\[
+ \frac{m_2^2 - m_1^2}{2(m_1 + m_2)} \begin{pmatrix}
  e^{-i\alpha} s_\theta^2 - c_\theta^2 & (e^{-i\alpha} + 1) s_\theta c_\theta \\
  (e^{-i\alpha} + 1) s_\theta c_\theta & e^{-i\alpha} c_\theta^2 - s_\theta^2
\end{pmatrix}.
\]

(2.25)

Notice that if $m_1 + m_2 \gg m_2 - m_1$, as was the case in all of our simulations, the first term in equation 2.25 will dominate. Furthermore, if the spin transformations occur before significant flavor transformations, then for the $\nu_e \rightarrow \bar{\nu}_e$ transformation channel it is
really only the top left term in the mass matrix (the $\frac{m_1+m_2}{2}(c_0^2+e^{-i\alpha}s_0^2)$ term) that matters. This is because $H_{\nu\nu}^{sf}$ will be diagonal if the neutrinos are all in flavor eigenstates and so $(H_{\nu\nu}^{sf})_{11}$ will only have contributions from this one mass matrix term. The first reason the Majorana phase only negligibly affected our simulations is that for the above-mentioned term in the mass matrix the Majorana phase, $e^{-i\alpha}$, multiplies a $\sin^2\theta_V$. Since we take $\theta_V = 8.7^\circ$ in our analysis, the relevant mass matrix term involving the Majorana phase will be very small: $\sin^2\theta_V \approx .023$ vs $\cos^2\theta_V \approx .98$. As a result, the Majorana phase, even if set to $\pi$, cannot significantly affect the pertinent term in the mass matrix for the given parameters used in our simulations.

The second potential reason the Majorana phase did not affect our simulations significantly is that the Majorana phase can only affect the adiabaticity parameter and not the spin resonance condition itself. The Majorana phase does not produce a vacuum splitting in the energy between neutrinos and antineutrinos, i.e. it only appears in $H_{\nu\nu}^{sf}$, not in $H_{\nu\nu}^{vac}$, $H_m$ or $H_{\nu\nu}$. The spin transformations are really set by nonlinear effects keeping the neutrinos near resonance. Nonlinear feedback keeping spin transformation adiabatic and “tracking” over a range of densities is a key feature of three-flavor, two-flavor and one-flavor calculations [77]. The ratio of electron neutrinos to antineutrinos is set by the fact that the neutrino-neutrino term in the diagonal Hamiltonian, $H_{11}$, has to cancel out the matter term in order for there to be such tracking. The matter potential that we used was the same for different simulations; therefore, as long as the neutrinos were made to track $H_{11} \approx 0$ over the same physical interval, the neutrino to antineutrino ratio had to remain roughly the same no matter what the Majorana phase was. Our simulations show that the slight change in adiabaticity introduced by changing the Majorana phase was not enough to significantly affect the tracking behavior, and therefore did not affect the spin transformations.
2.5.5 Onset of Transformations

An important aspect of spin and flavor transformations that we have been able to probe with our code is the locations, relative and absolute, where these spin and flavor transformations are most pronounced. Due to the nature of the spin transformation’s resonance conditions, spin transformations have so far been found to occur only in locations where $Y_e \lesssim 1/3$. In addition, simulations in which the electron fraction profile was set to $Y_e \approx 1/3$ far from the neutrino sphere, all else being the same, were not able to produce significant spin transformations. This is simply due to the fact that as we get farther from the neutrino sphere the geometric dilution will necessarily dilute the spin-flip Hamiltonian $H^{sf}$. If the spin-flip Hamiltonian is too small, then, by equation 2.23 spin transformations will be highly nonadiabatic at resonance and it will be difficult for significant spin transformations to set in.

Significant flavor transformations in our simulations have so far always occurred after spin transformations have ceased (see figures 2.3, 2.5, and 2.7). As the flavor transformations do depend on the neutrino spin content, i.e. on the ratio of neutrinos to antineutrinos (see equations 2.11,2.12) the spin transformations have the potential to affect flavor transformations. Vice versa, if flavor transformations were to occur prior to the onset of spin transformations, it is also possible that spin transformations could be affected. However, as the spin transformations appear to require an extremely large neutrino flux to be significant - the vacuum Hamiltonian does not contribute to spin-flip - it appears that significant spin transformations, if they do occur, are likely to occur closer to the neutrino sphere than flavor transformations.
2.5.6 Flavor Transformations

As was discussed earlier, flavor transformations have not been able to feed back into spin transformations in any way in our simulations since spin transformations begin and end before flavor transformations even start. In the solar mass-squared splitting case, spin transformations did not qualitatively change the flavor transformations. However, for the atmospheric mass-squared splitting and for three-flavor simulations, the spin transformations did significantly impact the flavor transformations. The process by which spin transformations change flavor transformations is simply through the production of antineutrinos. From equations 2.11 and 2.12 we can see that a flux of antineutrinos affects the energy splitting between the two-flavor states.

The simulation with no spin coherence and pure flavor transformations essentially reproduced previous results found in [9] and [12]. Due to the extreme neutrino fluxes found during the neutronization burst, the neutrinos all go through a lepton number \( (n_{\nu_e} - n_{\bar{\nu}_e}) \approx n_{\nu_e}, \) very few antineutrinos are present) nonconserving flavor resonance together in a “neutrino background assisted MSW-like resonance”. Basically, at the radius where a representative energy neutrino would go through the MSW resonance, the neutrino self-coupling locks neutrinos of all energies together so that all neutrinos go through the resonance together. This produces a neutrino spectrum which is overwhelmingly in the \( x \)-neutrino state. Immediately after going through this MSW-like resonance, the neutrinos are locked into collective oscillations and finally when those collective oscillations die out a lepton number conserving swap is formed at energy \( E_l \approx 20 \text{MeV}. \) Note that the conserved lepton number is actually the “mass basis lepton number” which would be \( L = (n_{\nu_1} - n_{\bar{\nu}_1}) - (n_{\nu_2} - n_{\bar{\nu}_2}) \) where \( \nu_1 \) and \( \nu_2 \) are the vacuum mass eigenstates. However, we used a small mixing angle for two-flavor simulations so that the flavor lepton number is approximately conserved. For qualitative discussion, we need not make the distinction. The swap energy \( E_l \) is determined by a conservation of the lepton number immediately
after the MSW-like resonance (mostly $x$-neutrinos). The swap energy is therefore high because the MSW-like resonance was quite efficient at destroying $n_{\nu_e}$.

The simulation with spin coherence was qualitatively and quantitatively different. Due to the presence of antineutrinos, the flavor transformations are able to undergo a classic spectral swap without first undergoing the neutrino background assisted MSW-like resonance. Also, due to the presence of a large number of antineutrinos, the MSW-like resonance may not be nearly as strong as for when there are no antineutrinos because the neutrino-neutrino Hamiltonian is suppressed by the presence of antineutrinos. Indeed, if $n_{\nu_e} = n_{\bar{\nu}_e}$, then we can see from equations 2.11 and 2.12 that $H_{\nu\nu} = 0$. The neutrinos and antineutrinos alike are locked into collective oscillation modes, but the spectral swap that develops conserves a lepton number which was not significantly affected by the MSW-like resonance. As such, the swap energy $E_l \approx 9\text{ MeV}$ was much lower because the MSW-like resonance was not able to convert the vast majority of electron neutrinos into the $x$-neutrino state.

### 2.6 Conclusions

This paper presents the first multiflavor simulations of coherent neutrino spin transformations using a neutrino bulb geometry. We explored a variety of initial conditions and parameters using an O-Ne-Mg supernova density profile and have presented results for those initial conditions and parameters which produced significant spin transformation. We found that it is likely, given the nature of the spin-flip Hamiltonian, that the spin transformations, if they occur, would occur prior to the onset of significant flavor transformations. As a result, the spin transformations (more precisely, the neutrino to antineutrino ratio produced by spin transformations) are not affected by the flavor structure of neutrinos (i.e., mass splittings, mixing angles, and number of flavors). How-
ever, there is potential for spin coherence to change the nature of the subsequent flavor transformations.

Our simulations found that, for significant spin transformations to occur, an unrealistically massive neutrino ($m_\nu = 10\text{eV}$), a large neutrino luminosity, and an electron fraction profile which hovered near $Y_e \lesssim 1/3$ for several hundred kilometers were required. For the parameters and initial conditions considered in our two-flavor simulations, a Majorana phase did not significantly alter the spin transformations. Changing the neutrino-to-antineutrino ratio requires changing the so-called tracking behavior $((H_m)_{11} + (H_{\nu\nu})_{11} \approx 0)$ of the neutrino Hamiltonian, which a change in the Majorana phase fails to affect.

Geometric dilution of the neutrinos, and the steep density dropoff combine to make tracking difficult, even with the extreme neutrino fluxes encountered during the neutronization burst. However, our simulations have not yet included other potential feedback loops such as the $Y_e$ feedback [77, 112, 113]. Perhaps the inclusion of other feedback mechanisms could enable spin transformations to occur with more realistic values of neutrino rest masses. If such feedback mechanisms help initiate tracking, then, as we have shown, a significant spin transformation in the neutrino population could significantly and qualitatively change the subsequent flavor evolution of these neutrinos. Unless it can be proven that no such mechanism exists in the supernova environment, which would make the spin coherence resonance adiabatic enough to engage the tracking behavior, spin degrees of freedom would necessarily have to be considered when considering neutrino flavor transformations. As neutrinos of different flavors interact with matter differently, changing the neutrino content could lead to ramifications on the nucleosynthesis ($r$-process) of elements during core collapse supernovae [22, 114] and the reheating of the initial supernova shock [115–117].
2.7 Acknowledgement

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Chapter 2, in full, is a reprint of the material as it appears in J. Y. Tian, A. V. Patwardhan, and G. M. Fuller. Prospects for neutrino spin coherence in supernovae. *Phys. Rev. D*, 95(6):063004, March 2017. This dissertation author was the primary investigator and author of this paper.
Chapter 3

Neutrino Flavor Evolution in Neutron Star Mergers

3.1 Abstract

We examine the flavor evolution of neutrinos emitted from the disk-like remnant (hereafter called “neutrino disk”) of a binary neutron star (BNS) merger. We specifically follow the neutrinos emitted from the center of the disk, along the polar axis perpendicular to the equatorial plane. We carried out two-flavor simulations using a variety of different possible initial neutrino luminosities and energy spectra, and for comparison, three-flavor simulations in specific cases. In all simulations, the normal neutrino mass hierarchy was used. The flavor evolution was found to be highly dependent on the initial neutrino luminosities and energy spectra; in particular, we found two broad classes of results depending on the sign of the initial net electron neutrino lepton number (i.e., the number of neutrinos minus the number of antineutrinos). In the antineutrino dominated case, we found that the Matter-Neutrino Resonance (MNR) effect dominates, consistent with previous results, whereas in the neutrino dominated case, a bipolar spectral swap develops.
The neutrino dominated conditions required for this latter result have been realized, e.g., in a BNS merger simulation that employs the “DD2” equation of state for neutron star matter [1]. For this case, in addition to the swap at low energies, a collective Mikheyev-Smirnov-Wolfenstein (MSW) mechanism generates a high-energy electron neutrino tail. The enhanced population of high-energy electron neutrinos in this scenario could have implications for the prospects of $r$-process nucleosynthesis in the material ejected outside the plane of the neutrino disk.

### 3.2 Introduction

In this paper we explore neutrino flavor transformation in the binary neutron star (BNS) merger environment, for axially directed neutrinos, and with an emphasis on scenarios with a higher number luminosity of neutrinos over antineutrinos. Depending on the BNS merger rate and on the amount of ejected material, BNS merger events could be a potential candidate site for the origin of nuclei heavier than $^{56}\text{Fe}$ via the $r$-process [118–132]. These cataclysmic events are accompanied by prodigious fluxes of neutrinos [1, 118, 124, 132, 133]. Flavor-dependent charged current neutrino capture reactions could influence the neutron content of these ejecta [45], depending on the material outflow speed and geometry, and on the neutrino luminosities, energy spectra, and emission geometry. Though many of these ingredient quantities have not yet been unambiguously determined by simulations, the importance and urgency of the $r$-process origin problem warrants an exploration of neutrino flavor physics in this environment.

There are roughly three potential sources of $r$-process material in BNS mergers: (1) the “tidal tails” of neutron-rich nuclear matter tidally stripped from the neutron stars during the in-spiral event before the stars touch; (2) the material ejected in the equatorial disk formed when the stars have merged; and (3) the material driven off
by either magneto-hydrodynamic (MHD) mechanisms or intense neutrino radiation, in directions outside the equatorial plane (e.g., along the polar axis). Of these, only the material in the first will not be accompanied by significant neutrino and antineutrino radiation exposure.

In order to evaluate the efficacy of BNS mergers in producing the observed $r$-process abundances, it is essential to have good estimates of merger rates over cosmic history, along with the average $r$-process yield per merger. Current BNS merger rate estimates are extremely primitive, as they are based on a very small sample size of observed binary pulsars in the Milky Way [134–141]. However, recent direct detections of gravitational waves from binary black-hole merger events by the Laser Interferometer Gravitational-wave Observatory (LIGO) [142, 143] have opened up an entirely new channel for exploring the universe. The current estimated upper limit on the BNS merger rate from LIGO is $12,600 \text{ Gpc}^{-3}\text{yr}^{-1}$ (at 90% C.L.), based on not having observed the gravitational wave signal from a BNS merger event yet [144]. This limit is consistent with the current binary pulsar based estimates. As LIGO begins to reach towards its ultimate design sensitivity within the next few years, it is hoped that it will enable us to obtain much better estimates, or at least more stringent upper bounds, on the rates of BNS merger events in the present-day universe [145].

While we wait for LIGO to give us a better observational handle on the merger rate, we can examine the other aspect of the problem by looking at the $r$-process yields of individual BNS merger events. Previous work has attempted to quantify the amount of $r$-process yields in the ejecta of neutron star mergers [118, 120–122, 124, 125, 130–132]. However, the neutrino physics that goes into these simulations is primitive at best, and in particular does not include a treatment of flavor conversion in these environments. Neutrinos are emitted with very high luminosities (on the order of $10^{53} \text{erg s}^{-1}$) from the disk-like (temporary) remnant of the neutron star merger. As long as the weak interactions...
are not fully decoupled, these neutrinos, via charged-current capture on nucleons, can
determine the electron fraction ($Y_e$) of the material they interact with. This, in turn, is
a major factor in evaluating the feasibility of these events as $r$-process producers. The
neutrinos affect the electron fraction via the following neutrino and antineutrino capture
reactions:

$$\nu_e + n \rightleftharpoons p + e^-, \quad (3.1a)$$
$$\bar{\nu}_e + p \rightleftharpoons n + e^+. \quad (3.1b)$$

Since the neutrino charged-current capture processes are flavor-asymmetric at
typical energies ($\sim 10$ MeV), i.e., only (anti-)neutrinos in the electron flavor state can
participate, it is essential that we know the flavor histories of these neutrinos as they
stream out of the merger site. A large change in the flavor content of neutrinos or even
just in their energy spectra (since the capture rates are energy dependent) could have a
correspondingly large effect on the electron fraction and therefore on the efficacy of the
$r$-process in these environments. Therefore, detailed analysis of neutrino flavor evolution
is necessary in order to better understand the potential these environments have for being
the main sites of the $r$-process.

Indeed, recent explorations of neutrino flavor evolution have found collective
phenomena in certain regions of BNS merger outflow [2, 2, 86, 108, 111, 146, 147].
Most of these consider initial conditions that exhibit an overall anti-neutrino num-
ber dominance over neutrinos. The various types of flavor transformation phenomena
found in these calculations include symmetric and standard Matter-Neutrino-Resonances
(MNR) [2, 106–111, 146], fast pairwise neutrino conversion [147], and the effects of
spin (helicity) coherence [86]. Ref. [2] discusses the trajectory-dependence of flavor
transformation in these environments, and finds a variety of behaviors on different trajectories in antineutrino-dominated conditions, including MNR, synchronized MSW (Mikheyev-Smirnov-Wolfenstein) flavor transformation, and bipolar oscillations (for the inverted mass hierarchy). Here, we do a complementary study involving flavor evolution along a different trajectory (axial), and with different choices of parameters such as luminosities, spectra, and matter densities. Specifically, we focus on conditions where the total number luminosity (integrated number flux) of electron neutrinos is higher than that of electron antineutrinos, although we present results for antineutrino-dominated cases as well.

The geometry of a disk-like neutrino source, a “neutrino disk”, is mathematically more difficult to implement than a spherical source, i.e., a “neutrino sphere”. A disk-like geometry admits fewer symmetries than a spherical geometry, and thereby increases the degrees of freedom (by two, see Sec. 3.3.2) that one needs to keep track of in order to fully self-consistently treat neutrino interactions in these environments. In order to keep the calculations tractable with the current technology available to us, we chose to run all simulations using a “single-angle approximation” (see Sec. 3.3.2), and track the flavor evolution of neutrinos which stream out along the polar axis of the neutrino disk. Along this trajectory there is an azimuthal symmetry which we can exploit to make calculations simpler. Since we are tracking the flavor-histories of only the polar-axis directed neutrinos, any conclusions on $r$-process effects will apply only to the last of the three aforementioned $r$-process contributions in BNS mergers, i.e., the wind-like ejecta outside the equatorial disk plane.

Merger simulations that use different neutron star equations of state result in different initial conditions in terms of neutrino number luminosities and energy spectra. These differing initial conditions can then lead to qualitatively different flavor transformation phenomena, which can have implications for observables such as the amount
and composition of ejecta, or the properties of the final remnant. An investigation of flavor transformation phenomena for these diverse initial conditions associated with different equations of state is therefore essential for accurately assessing the various possibilities across this landscape. Our results can be broadly separated into two classes, the matter-neutrino resonance (MNR) results for initial conditions where antineutrinos have higher number luminosities than neutrinos, and the bipolar spectral swap results for the neutrino-dominated number luminosities. We present results for both cases; however, we focus our discussion of flavor-transformation physics on the bipolar spectral swap results, since the MNR phenomenon has already been discussed extensively in this context [2, 108, 111, 146]. An example scenario where the neutrinos outnumber the antineutrinos can be found in Ref. [1], in a merger simulation that uses the “DD2” equation of state for neutron star matter [148, 149]. One aspect in which the DD2 equation of state differs from the other ones considered in Ref. [1], is a higher degree of stiffness, and the associated high maximum cold neutron star mass limit, leading to a stable neutron star remnant even after spin-down.

In Sec. 3.3 we detail the method we used, both the mathematical model and the computational methods, to obtain our results. In Sec. 3.4 we present our results. Sec. 3.5 contains a discussion of the underlying physics of the flavor transformations, as well as of the likely implications of our results for the nucleosynthesis and other physics in these environments. We state our conclusions in Sec. 3.6.

3.3 Methodology

Neutrinos propagating in dense matter can change their flavors through both scattering-induced decoherence and through coherent, forward-scattering processes. This behavior is generally described by the quantum kinetic equations, essentially generaliza-
tions of the Boltzmann equation, but including quantum mechanical phases [19, 78, 79, 82, 84, 89, 150–152]. However, in the regions where we see interesting flavor transformation effects, i.e., far above the BNS merger neutrino disk, treating only coherent forward scattering will likely be a good approximation. In fact, the conditions in these regions of interest along the polar axis resemble those of the supernova late-time neutrino-driven wind, which has been shown to be safe from neutrino halo effects [18].

### 3.3.1 Hamiltonian

Here we treat only the coherent evolution of neutrino flavor. In this limit, neutrinos undergo forward scattering on background matter and on other neutrinos. The flavor state of a neutrino of energy $E_\nu$, at a location $r$ (for the axially directed trajectory, $r = z$) can be described by a two or three dimensional (depending on the number of flavors considered) state vector $|\Psi_\nu\rangle$, which evolves according to the Schrödinger-like equation [34, 90, 91, 153, 154]:

$$i\hbar \frac{\partial}{\partial r} |\Psi_\nu(r, E_\nu)\rangle = H(r, E_\nu) |\Psi_\nu(r, E_\nu)\rangle.$$  

(3.2)

The Hamiltonian governing the evolution of these neutrinos is quite similar to those used in many previous simulations of collective neutrino flavor evolution [3–13, 16–49, 155–157]. In particular, we used a version of the Hamiltonian from the “neutrino bulb” model used in supernova neutrino flavor evolution studies [6, 13], modified to suit a BNS merger disk geometry. For the two-flavor case, easily generalizable to three flavors, the Hamiltonian is [4, 6, 13, 20, 96]:

$$H(r, E_\nu) =$$

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$$H(r, E_\nu) =$$
\[
H(\mathbf{r}, E_\nu) = \frac{\delta m^2}{4E_\nu} U \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U^\dagger + \sqrt{2} G_F n_e(\mathbf{r}) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} 
\]

\[
+ \sqrt{2} G_F \sum_\alpha \int d\nu_{\alpha}(\mathbf{p}') |\Psi_{\nu,\alpha}(\mathbf{p}')\rangle \langle \Psi_{\nu,\alpha}(\mathbf{p}')| (1 - \hat{p} \cdot \hat{p}') 
\]

\[
- \sqrt{2} G_F \sum_\alpha \int \bar{\nu}_{\alpha}(\mathbf{p}') |\Psi_{\bar{\nu},\alpha}(\mathbf{p}')\rangle \langle \Psi_{\bar{\nu},\alpha}(\mathbf{p}')| (1 - \hat{p} \cdot \hat{p}') , 
\]

(3.3)

where \( U \) is the 2 \times 2 version of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix:

\[
U = \begin{pmatrix} \cos \theta_\nu & \sin \theta_\nu \\ -\sin \theta_\nu & \cos \theta_\nu \end{pmatrix}, 
\]

(3.4)

with \( \theta_\nu = 8.7^\circ \) the mixing angle in vacuum (we have used the \( \theta_{13} \approx 8.7^\circ \) [158] mixing angle in our 2 \times 2 simulations). Here \( \delta m^2 = 2.4 \times 10^{-3} \text{eV}^2 \) is the mass-squared splitting (we have used the atmospheric splitting), \( G_F \) is the Fermi weak coupling constant, \( n_e \) is the net electron number density (\( n_e = n_{e^-} - n_{e^+} \)), and \( \mathbf{p}, \mathbf{p}' \) are the momenta of the test and background neutrinos, respectively (\( E_\nu = \sqrt{\mathbf{p}^2 + m_\nu^2} \)). We integrate over all of the background neutrinos encountered by our test neutrino, so that \( dn_{\nu,\alpha}(\mathbf{p}') \) is the local number density of neutrinos in state \( |\Psi_{\nu,\alpha}(\mathbf{p}')\rangle \). Here, the index \( \alpha \) refers to the initial flavor in which the neutrino was emitted at the neutrino disk.

The three terms of the Hamiltonian in Eq. (3.3) are written in the order of the vacuum Hamiltonian (\( H_{\text{vac}} \)), the matter Hamiltonian (\( H_m \)), and the neutrino-neutrino “self coupling” Hamiltonian (\( H_{\nu\nu} \)). The vacuum term in Eq. (3.3), \( H_{\text{vac}} \) arises merely from the fact that neutrinos have mass, and that the mass eigenstates are not coincident with the neutrino flavor eigenstates. The matter term, \( H_m \), arises from the neutrino forward scattering via the charged current interactions on the background matter (the potential from neutral current interactions contributes equally to all flavors of neutrinos).
and therefore does not need to be considered here). This matter Hamiltonian depends on the electron number density \( n_e \), which can be written in terms of the baryon number density \( n_b \) and the electron fraction \( Y_e \):

\[
n_e = Y_e n_b.
\]  

(3.5)

We chose the baryon density profile to have an inverse cubic relation to the radius (distance from the disk):

\[
n_b = n_{b,0} \left( \frac{r_0}{r} \right)^3
\]  

(3.6)

where \( n_{b,0} \) is the initial baryon density at the initial radius \( r_0 \). This relation will hold true if the material is in hydrostatic equilibrium and the entropy is mostly carried by relativistic particles (see Ref. [6, 159]). The last term in the Hamiltonian, \( H_{\nu\nu} \), in Eq. (3.3) arises from the test neutrino forward-scattering on other background neutrinos. This is the term which depends on the geometry which we chose, and requires careful consideration.

First, by exploiting the azimuthal symmetry of our chosen trajectory, we can rewrite the expression \( 1 - \hat{p} \cdot \hat{p}' \) in a convenient form:

\[
1 - \hat{p} \cdot \hat{p}' = 1 - \cos \theta',
\]  

(3.7)

where the test neutrino trajectory is taken to be along the \( z \)-direction, and therefore, the intersection angle between the test and background neutrino trajectories is simply the polar angle \( \theta' \) of the latter.

Second, the neutrino states can be enumerated in terms of energies and the pencil of solid angle (in the direction of \( p' \)), subtended at our test neutrino’s location, in which
the neutrinos are streaming. Or in more concrete mathematical terms:

\[ dn_{\nu,\alpha} = \frac{N_{\nu,\alpha}}{2\pi^2} \left( \frac{d\Omega_{\nu}}{4\pi} \right) f_{\nu,\alpha}(E_{\nu}) dE_{\nu}. \] (3.8)

Here, \( N_{\nu,\alpha} \) is a factor that normalizes the number density to the energy luminosity \( L_{\nu,\alpha} \) in the respective neutrino flavor. \( f_{\nu,\alpha}(E_{\nu}) \) is the (non-normalized) energy distribution of neutrinos initially emitted in state \( \alpha \) and \( d\Omega_{\nu} \) is a differential solid angle. We assume that neutrinos are emitted from the surface of the neutrino disk with a Fermi-Dirac black body-shaped distribution of energies so that:

\[ f_{\nu,\alpha}(E_{\nu}) = \frac{E_{\nu}^2}{e^{E_{\nu}/T_{\nu,\alpha} - \eta_{\nu,\alpha}} + 1}, \] (3.9)

where \( T_{\nu,\alpha} \) and \( \eta_{\nu,\alpha} \) are the temperature and degeneracy parameter, respectively, of the initial \( \nu_{\alpha} \) distribution. To normalize the differential number density \( dn_{\nu,\alpha} \) with respect to the luminosity \( L_{\nu,\alpha} \), we first calculate the neutrino energy flux \( F_{\nu,\alpha} \) at the disk surface:

\[ F_{\nu,\alpha} = \int_{\mathcal{P}} dn_{\nu,\alpha} E_{\nu} \cos \theta \]
\[ = \frac{N_{\nu,\alpha}}{2\pi^2} \int_0^1 \int_0^\infty \int_0^\infty E_{\nu} f_{\nu,\alpha}(E_{\nu}) dE_{\nu} \int_0^1 \cos \theta \frac{d\cos \theta d\phi}{4\pi} dE_{\nu}, \] (3.10)

where the neutrino speed is taken to be the speed of light \( c = 1 \), and the angle integration is performed over half the sky, i.e., over all neutrino unit momenta on one side of the disk. Now, we can introduce the normalized energy distribution function \( \tilde{f}_{\nu,\alpha}(E_{\nu}) \) defined as:

\[ \tilde{f}_{\nu,\alpha}(E_{\nu}) = \frac{1}{T_{\nu,\alpha}^3 F_2(\eta_{\nu,\alpha})} \frac{E_{\nu}^2}{e^{E_{\nu}/T_{\nu,\alpha} - \eta_{\nu,\alpha}} + 1}, \] (3.11)

so that \( \int_0^\infty \tilde{f}_{\nu,\alpha}(E_{\nu}) dE_{\nu} = 1 \), where \( F_2(\eta_{\nu}) \) is the complete Fermi-Dirac integral of order
2. With this, we can then evaluate the above integral to obtain

\[ F_{\nu,\alpha} = \frac{N_{\nu,\alpha}}{8\pi^2} \langle E_{\nu,\alpha} \rangle T_{\nu,\alpha}^3 F_2(\eta_{\nu,\alpha}) , \]  

(3.12)

where \( \langle E_{\nu,\alpha} \rangle \) is the average neutrino energy over the distribution \( \tilde{f}_{\nu,\alpha} \). We can now fix \( N_{\nu,\alpha} \) by relating the flux to the luminosity using \( F_{\nu,\alpha} = L_{\nu,\alpha} / (2\pi R_{\nu}^2) \), giving us

\[ dn_{\nu,\alpha} = \frac{L_{\nu,\alpha}}{2\pi^2 R_{\nu}^2 \langle E_{\nu,\alpha} \rangle} \tilde{f}_{\nu,\alpha}(E_{\nu,\alpha}) d\Omega dE_{\nu} . \]  

(3.13)

Note that this differs by a factor of two from the normalization for a spherical emission geometry. With this normalization, we can write the neutrino-neutrino Hamiltonian as an explicit integral:

\[ H_{\nu\nu} = \frac{\sqrt{2} G_F}{\pi R_{\nu}^2} \sum_{\alpha} \int_0^{\infty} \int_0^{\theta_m} \left[ \frac{L_{\nu,\alpha}}{\langle E_{\nu,\alpha} \rangle} \tilde{f}_{\nu,\alpha}(E_{\nu}) |\Psi_{\nu,\alpha}\rangle \langle \Psi_{\nu,\alpha}| 
\right. \\
- \left. \frac{L_{\bar{\nu},\alpha}}{\langle E_{\bar{\nu},\alpha} \rangle} \tilde{f}_{\bar{\nu},\alpha}(E_{\nu}) |\Psi_{\bar{\nu},\alpha}\rangle \langle \Psi_{\bar{\nu},\alpha}| \right] (1 - \cos \theta') \sin \theta' d\theta' dE_{\nu} . \]  

(3.14)

Here \( \theta_m \) is the maximum half-angle the neutrino disk subtends at the test neutrino’s location which with simple trigonometry we know to be: \( \tan \theta_m = R_{\nu} / r \). We have already performed the \( \phi' \) integration in Eq. (3.14) as that integral is trivially equal to \( 2\pi \) because of azimuthal symmetry. This is the final version of the Hamiltonian which we use in our calculations. We can see that the main difference between this Hamiltonian and the one used in [6] will be in how \( \theta_m \) differs between the spherical geometry case and the disk geometry case. As \( \theta_m \) is different between the two cases, the geometric dilution of neutrinos as we move farther from the source will be different.
3.3.2 Simulations

Here we chose to model the neutron star merger neutrino source as a flat circular disk, with neutrinos streaming from the two faces. We assumed that all neutrinos of all flavors are emitted from the same surface, i.e., that there are not multiple neutrino disks for different neutrino flavors. Neither of these assumptions are quite true for an actual neutron star merger; different neutrino flavors and types have different decoupling surface disks, and these have relative spacings of order tens of km, at most. Differences in neutrino decoupling surfaces for different neutrino types has been shown to be important [147,160], and that could be the case here as well, especially close to the neutrino disk. However, as we shall see, most of the collective flavor oscillations we find occur at distances (∼ a few hundred km) which are large compared to the neutrino disk separations. The effects of having separate disks are therefore unlikely to be significant at these distances.

We chose a disk radius of $R_\nu = 60$ km (see, e.g., Fig. 16 in Ref. [1]), and assumed that neutrinos are emitted isotropically from each point on the surface. Moreover, as mentioned earlier, we chose to follow the flavor evolution of neutrinos emitted from the center of the disk along the polar axis, perpendicular to the equatorial plane.

Simulations were performed using the neutrino BULB code, developed by the authors of Refs. [3–13]. The BULB code was modified to use the new geometry as discussed in Sec. 3.3.1. No major modifications to the underlying architecture of the code were necessary. We note that Eq. (3.14) was written in such a way as to leave the angle dependence of $|\Psi_{\nu,\alpha}\rangle$ ambiguous. In a fully self-consistent study of a merger geometry, this state vector would of course depend on the trajectory of the background neutrino we are integrating over. Indeed, even in a spherical “neutrino bulb” geometry, this state vector is emission angle (angle with respect to the normal of the neutrino sphere at which a particular neutrino is emitted) dependent. So called multi-angle simulations of the neutrino bulb model account for this fact. However, with the disk geometry, there are two
additional degrees of freedom, beyond multi-angle bulb simulations, that we must account for if we want to fully self consistently treat the neutrino trajectories. As the disk is not spherically symmetric, we must account for the emission location on the disk (one degree of freedom due to cylindrical symmetry). In addition, multi-angle bulb simulations only require one polar emission angle whereas a disk geometry would require two emission angles (polar and azimuthal) to track all of the neutrinos. Complicating matters further, as off-angle trajectories (from the central polar axis) do not exhibit azimuthal symmetry, the relation in Eq. (3.7) no longer holds. Also, for off-axis trajectories, the integral over the solid angle would depend on the polar angle $\theta$ in addition to $r$, and the separation into $\theta'$ and $\phi'$ integrals is nontrivial. As such, the underlying architecture of the BULB code would have to be modified to accommodate these extra degrees of freedom.

To avoid these complexities, simulations were run for this paper in the so-called “single-angle mode”. We assume, for simplicity, that all neutrinos on all other trajectories that encounter our test neutrino evolve in flavor exactly the same way as our test neutrino. It is known from simulations of supernova neutrino flavor evolution that multi-angle simulations incorporate the correct phase-averaging over different trajectories, implying that they can do a better job at predicting the locations of the onset of collective flavor transformations [161]. Nevertheless, single-angle simulations are known to capture many of the qualitative features that are present in multi-angle simulations, especially at locations sufficiently far from the source. We also note that previous studies of flavor evolution in BNS merger environments have also employed this approximation.

Since we are performing single-angle simulations, the state vectors $|\Psi_{\nu,\alpha}\rangle$ are not emission location or angle dependent. They are, however, still energy dependent. As such, we can perform the angle integration in Eq. (3.14) to obtain:
\[ H_{\nu \nu} = \frac{\sqrt{2} G_F}{\pi R_\nu^2} \left[ 1 - \frac{r}{\sqrt{r^2 + R_\nu^2}} \right]^2 \sum_{\alpha} \int_0^\infty \left[ \frac{L_{\nu,\alpha}}{\langle E_{\nu,\alpha} \rangle} \tilde{f}_{\nu,\alpha}(E_{\nu}) |\Psi_{\nu,\alpha}\rangle \langle \Psi_{\nu,\alpha}| - \frac{L_{\bar{\nu},\alpha}}{\langle E_{\bar{\nu},\alpha} \rangle} \tilde{f}_{\bar{\nu},\alpha}(E_{\nu}) |\Psi_{\bar{\nu},\alpha}\rangle \langle \Psi_{\bar{\nu},\alpha}| \right] dE_{\nu}. \] (3.15)

### 3.4 Results

#### 3.4.1 Initial Conditions

As the merger environment itself is extremely complex, the neutrino emission’s initial conditions are, not surprisingly, equally complex. The major regions of neutrino emission differ for the different neutrino flavors, and among the neutrino and the antineutrino sector. Most importantly with regards to the flavor transformations, neutrinos are emitted primarily from the polar regions of the merger while antineutrinos are mostly emitted from the hot shocked regions of the disk [1]. This means that different simulations giving differing temperatures for the polar regions versus the shocked regions of the disk, would give similarly different results in neutrino versus antineutrino emission. Most nuclear matter equations of state in use in BNS merger simulations result in a higher luminosity and number flux of antineutrinos over neutrinos being emitted from the neutrino disk [1, 124, 132, 133]. However, a particular simulation from Ref. [1], one that used the “DD2” equation of state for neutron matter, did produce a total number luminosity abundance of neutrinos over antineutrinos (although, due to the high average energy of the antineutrinos, the energy luminosity was still dominated by antineutrinos).

We do not include all of the intricacies of neutrino emission from the neutrino disk. Instead, we chose different sets of initial neutrino luminosities and energy spectra in order to try to capture the qualitative differences in flavor evolution which arise from differences in initial conditions. As most simulations of neutrino emission have
antineutrino dominance, studies of flavor evolution in merger environments up to now have focused on the “matter-neutrino resonance (MNR)” effect [2, 108, 111, 146]. This effect requires a cancellation in the total Hamiltonian between the matter term and the neutrino-neutrino term. Such a cancellation can only arise if the neutrino-neutrino term is negative, i.e., if the neutrinos are dominated by antineutrinos. To corroborate this, we ran a simulation with the same neutrino luminosities and spectra as found in Ref. [2], with an antineutrino abundance over neutrinos, and found that the MNR effect was indeed the dominant feature of collective neutrino oscillations. However, if neutrinos dominate over antineutrinos in number flux, the matter-neutrino resonance cannot easily occur. In Secs. 3.4.3–3.4.5, we highlight a different possible outcome of collective flavor oscillations in a merger environment, namely, the occurrence of a bipolar spectral swaps for neutrino-dominated number luminosities.

The simulations, the results of which are described below, all utilize the normal neutrino mass hierarchy. We take \( n_{b,0} = 10^8 \text{ g/cm}^3 \) (at \( r_0 = 20 \text{ km} \)), which is the same as in Ref. [1], in all our simulations except for the low luminosity/low density simulation of Sec. 3.4.5 which uses \( n_{b,0} = 2.5 \times 10^6 \text{ g/cm}^3 \). In addition, we chose a constant electron fraction \( Y_e = 0.4 \) in our calculations.

### 3.4.2 MNR Results

As outlined above, we will have the requisite conditions for MNR when:

\[
\left( \frac{L_{\nu,e}}{\langle E_{\nu,e} \rangle} \right) / \left( \frac{L_{\bar{\nu},e}}{\langle E_{\bar{\nu},e} \rangle} \right) < 1.
\]  

To get a MNR in such a scenario, one would need other mechanisms to first convert some of the electron neutrino lepton number excess, either into other flavors (e.g., via background-assisted MSW effect), or into anti-neutrinos (e.g., \( \nu_e \rightarrow \bar{\nu}_e \) via spin-coherence effects [77,86,162]).
In MNR, the neutrino-neutrino part of the Hamiltonian in Eq. (3.3) interacts with the matter and vacuum parts of the Hamiltonian to create an MSW-like resonance [2,47,106–111,146]. A resonance between the two flavors of neutrinos occurs when the diagonal elements of the total Hamiltonian equal each other, i.e. when $H_{11} = H_{22}$. As is standard in neutrino flavor evolution analyses, we use a traceless Hamiltonian in our simulations by removing the total trace from Eq. (3.3). For a traceless $2 \times 2$ Hamiltonian, the resonance condition is then simply $H_{11} = -H_{22} = 0$. A MNR therefore occurs when $(H_{\nu\nu})_{11}$ nearly cancels $(H_{m})_{11}$. Usually, MNR is augmented by nonlinear feedback in the neutrino flavor evolution which helps sustain this cancellation over a longer distance. Generally speaking, if the neutrinos move through this MNR adiabatically, then large scale flavor transformations can occur from the electron flavor state to the “$x$”- flavor state and vice versa. Here, the $x$-flavor refers to the other flavor besides $\nu_e$ in $2 \times 2$ calculations and is taken to be a particular linear combination of the nearly maximally mixed $\nu_\mu$ and $\nu_\tau$ flavor states [97,98].

In the normal mass hierarchy, the vacuum Hamiltonian matrix element $(H_{\text{vac}})_{11}$ is an energy dependent negative quantity. However, since $H_m$ and $H_{\nu\nu}$ are not energy dependent, the MNR cannot simultaneously satisfy $H_{11} = 0$ for neutrinos of all energies. The diagonal Hamiltonian can therefore vanish only for one specific energy, and it can be close to zero only for neutrinos with energies close to that energy. In other words, not all neutrinos of all energies may necessarily be affected by the MNR. This general observation is borne out in our simulations.

Table 3.1 shows the parameters we used in order to simulate neutrino flavor evolution using an antineutrino dominated neutrino number luminosity. These parameters for the neutrino luminosities and average energies are quite similar to those used in [2]. Notice that here $((L_{\nu,e}/\langle E_{\nu,e} \rangle)/(L_{\bar{\nu},e}/\langle E_{\bar{\nu},e} \rangle)) \approx 0.7$ which means there is a preponderance of antineutrinos over neutrinos and therefore the possibility for a MNR.
Table 3.1: Parameters used our two flavor simulation that produced MNR. The luminosities and average energies used here are taken from [2].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\nu,e}$</td>
<td>$1.5 \times 10^{52}$ erg/s</td>
</tr>
<tr>
<td>$L_{\bar{\nu},e}$</td>
<td>$3.0 \times 10^{52}$ erg/s</td>
</tr>
<tr>
<td>$L_{\nu,\bar{\nu},e}$</td>
<td>$1.6 \times 10^{52}$ erg/s</td>
</tr>
<tr>
<td>$\langle E_{\nu,e} \rangle$</td>
<td>10.6 MeV</td>
</tr>
<tr>
<td>$\langle E_{\bar{\nu},e} \rangle$</td>
<td>15.3 MeV</td>
</tr>
<tr>
<td>$\langle E_{\nu,\bar{\nu},e} \rangle$</td>
<td>17.3 MeV</td>
</tr>
<tr>
<td>$\delta m^2_{\text{atm}}$</td>
<td>$2.4 \times 10^{-3}$ eV$^2$</td>
</tr>
<tr>
<td>$\theta_V$</td>
<td>$8.7^\circ$</td>
</tr>
</tbody>
</table>

Figure 3.1 shows the energy spectra for neutrinos and antineutrinos at a final simulation radius of 5000 km, along with the initial spectra at the point of emission. These neutrinos moved through an MNR as shown in Fig. 3.4. We can see here that only the high energy neutrinos converted from one flavor to the other, while low energy neutrinos did not change flavors. This stems from the fact that the MNR set $H_{11} \approx 0$ only for these high energy neutrinos.

**Figure 3.1:** These plots show the initial (magenta and black) and final (blue and green) energy spectra for neutrinos (left) and antineutrinos (right), for the simulation with parameters as described in Table 3.1. The final spectra were plotted at a distance $r = 5000$ km along the polar axis. As is evident, the MNR affected primarily the high-energy neutrinos. Neutrinos with energies below $E_{\nu} \lesssim 16$ MeV and antineutrinos were not significantly affected.
For a neutrino of energy $E_\nu$ emitted initially in the $\alpha$ flavor state, the probability of being in the $\beta$ flavor state at a distance $r$ is $P_{\alpha\beta}(r, E_\nu) = |\langle \nu_\beta | \Psi_{\nu,\alpha}(r, E_\nu) \rangle|^2$. This can then be integrated over neutrino energies, weighted by the normalized distribution functions $\tilde{f}_{\nu,\alpha}(E_\nu)$, to obtain the energy-averaged survival ($\alpha = \beta$) or conversion ($\alpha \neq \beta$) probability as a function of distance:

$$P_{\alpha\beta}^{\text{avg}}(r) = \int \tilde{f}_{\nu,\alpha}(E_\nu) P_{\alpha\beta}(r, E_\nu) dE_\nu,$$

(3.17)

Figure 3.2 shows the energy-averaged flavor evolution probabilities for a neutrino and antineutrino which begin initially in the electron flavor state. As is evident, electron neutrinos began to convert into $x$-flavor neutrinos beginning quite close to the neutrino disk, at a distance of $\approx 200\text{km}$. The collective flavor transformation ended by about $\approx 1200\text{km}$. Figure 3.3 shows the energy-averaged flavor evolution of neutrinos and antineutrinos which begin in the $x$-flavor state. As can be concluded from Figs. 3.2 and 3.3 the antineutrinos did not significantly change flavors and only the neutrinos were affected by the MNR.

Figure 3.4 shows the 1-1 component of the matter and neutrino-neutrino Hamiltonians and the sum of the two for the simulation presented here. We can see that the nonlinear feedback from the MNR forced $(H_m)_{11} + (H_{\nu\nu})_{11} \approx 0$ over roughly a thousand kilometers ($r \approx 200$–$1200\text{km}$). The radius at which the MNR is reached essentially corresponds to the radius at which the neutrinos begin to transform their flavor, as seen from Figs. 3.2 and 3.3, establishing that the MNR was indeed the mechanism driving flavor transformation in this simulation.

For comparison, we also performed a three-flavor calculation for this anti-neutrino dominated case with the same initial conditions as those in Table 3.1, with the “$x$”-flavor luminosity equally split between the $\mu$ and $\tau$ flavors. Figure 3.5 shows plots of initial and
Figure 3.2: These figures show the energy-averaged flavor evolution of a neutrino (left) and antineutrino (right), initially in the electron flavor state, for the simulation with parameters as described in Table 3.1. It is evident that the MNR begins early on at a distance of about $\approx 200$ km and stabilizes at about $\approx 1200$ km. In each of these energy-averaged flavor evolution plots, the lines corresponding to different flavors (e.g., the blue and green lines in the left panel) sum to unity at each radius.

final neutrino spectra (left), as well as the energy-averaged flavor evolution of neutrinos starting out in the electron flavor state. As can be seen from the figure, the excursions in flavor space for the three-flavor calculation are bigger compared to the two-flavor case, and the collective effects do not die down completely, even by $r = 5000$ km, an effect that can be attributed to the influence of oscillations driven by the solar mass-squared splitting.

3.4.3 Two Flavor Bipolar Swap Results

The results of the simulations where the initial number luminosities are dominated by neutrinos rather than antineutrinos can be quite different. Our choice of parameters that correspond to such a neutrino dominance over antineutrinos is motivated by Foucart et al.’s DD2 equation of state simulation [1].

We performed two- and three-flavor simulations in which we observed the bipolar spectral swap phenomenon. Table 3.2 outlines the parameters used in the two flavor
Figure 3.3: Same as Fig. 3.2, but for a neutrino (left) and antineutrino (right) initially in the x flavor state. Mirroring the results of the initially electron flavor neutrinos, most of the flavor transformation occurs for the neutrinos, while the antineutrinos remain largely unaffected by the MNR.

simulation discussed in this section. The parameters chosen here represent an example set of neutrino luminosities and spectra that one might expect in these environments, based on physical insight. For instance, in a neutron-rich environment one would expect a pronounced hierarchy between the average energies of $\nu_e$ and $\bar{\nu}_e$, as well as those of $\nu_e$ and $\nu_x$. This is because only the electron neutrinos would experience significant charged-current interactions, and would therefore be expected to decouple further out where the temperatures are cooler. In addition, the parameter set that we have chosen here also has a prominent hierarchy between $\bar{\nu}_e$ and $\nu_x$ average energies.

The rationale behind this choice was to explore a scenario wherein flavor transformations could significantly affect the nucleosynthesis prospects. This is discussed in further detail in Sec. 3.5.2. Another justification is that there do exist simulations where such a hierarchy between $\bar{\nu}_e$ and $\nu_x$ average energies has been exhibited. For instance, neutrino emission from the “SFHo” equation of state simulation from Ref. [1] has average energies $\langle E_{\bar{\nu}_e} \rangle = 19.1$ MeV and $\langle E_{\nu_x} \rangle = 26.4$ MeV, although that particular simulation also had an overall antineutrino domination over neutrinos. By comparison, the hierarchy of neutrino energies in the DD2 equation of state simulation is less pronounced:
Figure 3.4: Shown here is the 1-1 component of the matter, neutrino-neutrino, and the sum of the matter and neutrino-neutrino Hamiltonian experienced by our test neutrinos, for the simulation with parameters as described in Table 3.1. As we can see, the MNR develops early on at a radius of $\approx 200\,\text{km}$. The nonlinear feedback of neutrino flavor transformations keep the total Hamiltonian near 0 for several hundred kilometers, thus giving rise to the MNR. In order to calculate the total Hamiltonian, an energy dependent $(H_{\text{vac}})_{11}$ would have to be added. This vacuum term would manifest as an energy dependent vertical offset (in the negative direction). Thus, not all neutrinos of all energies will go through the MNR, explaining the energy dependence of the MNR effect seen in Fig. 3.1.

$\langle E_{\bar{\nu},e} \rangle = 18.2\,\text{MeV}$ and $E_{\nu,x} = 21.9\,\text{MeV}$.

Table 3.2: Parameters used for one of our two-flavor simulations that exhibited the bipolar spectral swap.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\nu,e}$</td>
<td>$1.5 \times 10^{53},\text{erg/s}$</td>
</tr>
<tr>
<td>$L_{\bar{\nu},e}, L_{\nu,\bar{\nu},x}$</td>
<td>$2 \times 10^{53},\text{erg/s}$</td>
</tr>
<tr>
<td>$\langle E_{\nu,e} \rangle$</td>
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<td>18 MeV</td>
</tr>
<tr>
<td>$\langle E_{\nu,\bar{\nu},x} \rangle$</td>
<td>25 MeV</td>
</tr>
<tr>
<td>$\delta m^2_{\text{atm}}$</td>
<td>$2.4 \times 10^{-3},\text{eV}^2$</td>
</tr>
<tr>
<td>$\theta_\nu$</td>
<td>$8.7^\circ$</td>
</tr>
</tbody>
</table>

Figure 3.6 shows the initial and final neutrino energy spectra for $\nu_e$ and $\nu_x$ flavors, along the chosen trajectory described earlier. The final spectra were taken from our results at distance of 5000 km from the neutrino disk, by which point the collective

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$^2$The average energies were calculated from the RMS energies given in Ref. [1], assuming a neutrino degeneracy parameter $\eta_{\nu,e} = 3$ for all neutrino types.
oscillations have stabilized. The spectra were normalized in the same way as in [6]. The first feature that is readily apparent is a stepwise flavor swap which occurred at a critical energy $E_C \approx 8$ MeV. Electron and $x$-neutrinos with energies below this swap energy mostly converted into each other. This is consistent with previous studies of supernova neutrino flavor evolution in the normal mass hierarchy [9, 12]. Separately from the flavor swap at low energies, at energies greater than a threshold energy of about $E_H \approx 20$ MeV, a secondary flavor swap occurred and a significant portion of $x$-neutrinos were converted into electron neutrinos and vice versa. Neutrinos of intermediate energies, i.e., in between the critical and threshold energies, $E_C \lesssim E_V \lesssim E_H$, mostly remained in their initial states. As there are many more high energy $x$-neutrinos than electron neutrinos in the initial state, this secondary swap at energies greater than $E_H$ means that a net excess of high energy electron neutrinos develops in the tail as compared to the initial distribution.

Figure 3.7 shows the initial and final energy spectra in the antineutrino sector. As is evident from this figure, a spectral swap occurs in the antineutrino sector at energy $E_C$, though it is not as pronounced as the swap in the neutrino sector. However, the second swap at high energies did not occur in the antineutrino sector. As a result, no high energy
Figure 3.6: Shown here are the initial $\nu_e$ (magenta) and $\nu_x$ (black) energy spectra, as well as the final $\nu_e$ (blue) and $\nu_x$ (green) spectra, at a distance of 5000 km from the neutrino disk along the polar-axis trajectory, for the simulation with parameters as described in table 3.2. A flavor swap is seen to occur at an energy of approximately 8 MeV, and a high energy electron neutrino tail is also seen to develop. This tail of high energy electron neutrinos could potentially affect the electron fraction significantly.

electron-antineutrino tail developed in this case.

The plots in Fig. 3.8 show the energy-averaged probabilities for a (anti-)neutrino that started out in the electron flavor state to be in the electron or $x$ flavor states as a function of distance. It is evident from these figures that, in both the neutrino and antineutrino sectors, collective flavor evolution phenomena set in at a radius of approximately 500–600 km. Minimal flavor transformation occurred closer to the neutrino disk; however, significant large scale flavor conversion does not set in until farther out. The neutrino flavors oscillate rapidly with distance for approximately 1500 km and then stabilize around the final values at a radius of approximately 2000 km.

The plots in Fig. 3.9 show the energy-averaged probabilities for a (anti-)neutrino that started out in the $x$ flavor state to be in the electron or $x$ flavor states as a function of distance. These plots and the two previous plots demonstrate that although both the neutrino and antineutrino sectors go through rapid flavor oscillations, the antineutrino sector did not sustain significant overall flavor transformation while the neutrino sector did. As much as 40% of $x$-neutrinos were converted into electron neutrinos after the
Figure 3.7: Shown here are the initial $\bar{\nu}_e$ (magenta) and $\bar{\nu}_x$ (black) energy spectra, as well as the final $\bar{\nu}_e$ (blue) and $\bar{\nu}_x$ (green) spectra at a distance of 5000 km from the neutrino disk, along the polar-axis trajectory, for the same simulation as Fig. 3.6. Antineutrinos did not significantly convert from one flavor to another.

oscillations stabilized, while only a very small percentage of anti-$x$-neutrinos were converted into anti-electron-neutrinos. Most interestingly, as can be seen from the neutrino flavor evolution plots, while approximately 40% of initial $x$-neutrinos converted into electron neutrinos, only approximately 20% of initial electron neutrinos converted into $x$-neutrinos. Considering that the total initial luminosity of $x$-neutrinos was higher than that for electron neutrinos, and noting that preferentially higher energy $\nu_x$ were converted to $\nu_e$, while lower energy $\nu_e$ were converted to $\nu_x$ (see Fig. 3.6), it can be concluded that there is a net excess of electron neutrino energy flux resulting from this transformation. This phenomenon could potentially have a negative effect on the neutron excess, and thereby, on the prospects for $r$-process nucleosynthesis in ejecta moving out along this direction.

For completeness, we have also included an abbreviated set of plots (Fig. 3.10) showing the results of a flavor evolution calculation using the exact luminosities and spectra from the DD2 equation of state simulation in Ref. [1] (Table III from this reference). The left panel shows the initial and final ($r = 5000$ km) spectra for $\nu_e$ and $\nu_x$, whereas the right panel shows the energy-averaged flavor evolution probabilities for
Figure 3.8: These plots show the energy-averaged neutrino flavor evolution for a neutrino (left) and an antineutrino (right) initially in the electron flavor state, for the simulation with the parameters listed in table 3.2. We can see that significant flavor transformations begin to occur at a radius of approximately 600km and these flavor oscillations stabilize at a radius of approximately 2000km.

a neutrino initially in the electron flavor state. Qualitatively, these results can be seen to be almost identical to the corresponding plots from Figs. 3.6 and 3.8, even though this particular set of initial conditions does not exhibit as strong of an energy hierarchy between $\bar{\nu}_e$ and $\nu_x$ as the parameters in Table 3.2, as discussed earlier in this section.

### 3.4.4 Three Flavor Bipolar Swap Results

Table 3.3 shows the parameters we used in our three-flavor oscillation simulations. Luminosities in the $x$-neutrino sector used in our two-flavor simulations (Table 3.2) were split evenly among the $\mu$ and $\tau$ flavors in three-flavor simulations in order to keep constant the total neutrino luminosity among all flavors. Three-flavor neutrino mixing will involve both the atmospheric neutrino mass-squared splitting $\delta m^2_{\text{atm}}$ and the solar neutrino mass-squared splitting $\delta m^2_{\odot}$ as well as three total mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and an as yet unknown CP-violating phase $\delta_{CP}$. In our calculations, the $CP$-violating phase was set to 0.

Figure 3.11 shows the final energy distribution spectra for the electron neutrino,
Figure 3.9: This is the energy-averaged evolution of neutrino flavors for a neutrino (left) and antineutrino (right) initially in the $x$-flavor state.

muon neutrino, and tau neutrino in our three flavor simulation. As in the two-flavor simulation, a high-energy electron neutrino tail develops in this case. However, because of the presence of possible transformations into a third flavor, the high energy electron neutrino tail is less pronounced than in the two-flavor cases, particularly in the energy range of roughly 20–30 MeV. Moreover, at energies of approximately 8–20 MeV, the electron neutrinos significantly transform into other flavors of neutrinos, which was not the case in the two-flavor simulations. As a result, three-flavor simulations indicate fewer total electron neutrinos present at large distance than do two-flavor simulations.

Figure 3.12 shows the final energy distribution spectra for the antineutrino sector in our three-flavor simulations. Relatively more collective flavor conversion occurred in the antineutrino sector for three flavor simulations than for two flavor simulations. However, the flavor transformation in the antineutrino sector is nevertheless not as significant as that in the neutrino sector. Specifically, no high-energy electron antineutrino tail develops.

Figure 3.13 shows all the plots for the energy-averaged neutrino probability evolution in a three-flavor simulation. The collective neutrino transformation begins at a radius of 600 km. This is same as in the two-flavor case. The mu and tau neutrinos
Figure 3.10: Results for a calculation run using the luminosities and average energies adopted from the DD2 equation of state simulation in Ref. [1]. This calculation also demonstrates a bipolar spectral swap, qualitatively very similar to the one shown in Fig. 3.6. (Left) Initial and final ($r = 5000$ km) neutrino energy spectra. (Right) Energy-averaged flavor evolution of a neutrino initially emitted in the electron flavor state.

and antineutrinos remain nearly maximally mixed throughout the simulation. Since these neutrinos are nearly maximally mixed in vacuum, and since they experience nearly identical interactions in medium, they evolve nearly identically in our simulations.

3.4.5 Low Luminosity and Low Density Results

As mentioned before, neutron star merger environments can manifest a multitude of different initial conditions. Different simulations using different equations of state, or initial configurations of the neutron stars, produce different density profiles, neutrino luminosities, and neutrino spectra. In order to explore alternative initial conditions, we ran a simulation where the initial baryon mass density at the neutrino disk was lowered from $n_{b,0} = 10^8$ g/cm$^3$ to $n_{b,0} = 2.5 \times 10^6$ g/cm$^3$. This density is closer to the initial density found in the simulations in Ref. [2]. The neutrino spectral shape parameters ($\langle E_{\nu,\alpha} \rangle$ and $\eta_{\nu,\alpha}$) were kept the same as those used in Sec. 3.4.3, but the luminosities were lowered by an order of magnitude across the board: $L_{\nu,e} = 1.5 \times 10^{52}$ erg/s, $L_{\nu,\bar{e},\bar{x},\bar{x}} = 2 \times 10^{52}$ erg/s, closer to the luminosities found in Ref. [2].
Table 3.3: Parameters used in our three flavor simulation. Between the two flavor and three flavor case, the $\nu$-neutrino luminosity was split equally among the $\mu$ and $\tau$ flavors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\nu,e}$</td>
<td>$1.5 \times 10^{53}$ erg/s</td>
</tr>
<tr>
<td>$L_{\bar{\nu},e}$</td>
<td>$2 \times 10^{53}$ erg/s</td>
</tr>
<tr>
<td>$L_{\nu,\bar{\nu},\mu,\tau}$</td>
<td>$1 \times 10^{53}$ erg/s</td>
</tr>
<tr>
<td>$\langle E_{\nu,e} \rangle$</td>
<td>11 MeV</td>
</tr>
<tr>
<td>$\langle E_{\bar{\nu},e} \rangle$</td>
<td>18 MeV</td>
</tr>
<tr>
<td>$\langle E_{\nu,\bar{\nu},\mu,\tau} \rangle$</td>
<td>25 MeV</td>
</tr>
<tr>
<td>$\delta m_{\odot}^2$</td>
<td>$2.4 \times 10^{-3}$ eV$^2$</td>
</tr>
<tr>
<td>$\delta m_{\odot}^2$</td>
<td>$7.6 \times 10^{-5}$ eV$^2$</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>34.4°</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>8.7°</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>45°</td>
</tr>
<tr>
<td>$\delta_{\text{CP}}$</td>
<td>0°</td>
</tr>
</tbody>
</table>

Figure 3.14 shows the final spectra for neutrino and antineutrinos. This result is the clearest example of a bipolar spectral swap that was found in our simulations. There is a very sharp swap at low energies $E_C \approx 8$ MeV. In addition, the high energy electron neutrino tail is still present. However, the energy at which the tail manifests is higher, $\approx 38$ MeV. The transition to the high energy electron neutrino tail also is much more pronounced and sharp than in the simulations described in Secs. 3.4.3 and 3.4.4. Likewise, in the antineutrino sector, there are analogous, although less pronounced, effects of spectral swaps at low energies $\lesssim 4$ MeV and high energies $\gtrsim 51$ MeV. All spectral features in both the neutrino and antineutrino sectors are more pronounced and sharper in this simulation compared to the ones in Secs. 3.4.3 and 3.4.4.

Figures 3.15 and 3.16 show the energy-averaged neutrino and antineutrino flavor evolution for an initially electron flavor and initially $x$ flavor neutrino, respectively. These reveal that significant neutrino flavor evolution begins much closer to the neutrino disk, at a radius of $\approx 100$ km, than does the analogous flavor transformation in the simulations discussed in Secs. 3.4.3 and 3.4.4. As we will discuss in Sec. 3.5, this is to be expected. It is also easier and clearer to see here that the neutrino flavor evolution begins as
Figure 3.11: This is the final energy distribution spectra of neutrinos for a three flavor simulation. Initial \( \mu \) and \( \tau \) neutrinos have the same energy spectra and so overlap on this graph. The red line represents both flavors.

Figure 3.12: Same as Fig. 3.11, but for anti-neutrinos.

synchronized oscillations, before eventually settling down into a bipolar spectral swap.

3.5 Discussion

Collective neutrino oscillations driven by the nonlinear aspects of \( H_{\nu\nu} \) can occur in both the core-collapse supernova and BNS merger environments. This is unsurprising at some level, because both these astrophysical venues are characterized by prodigious neutrino fluxes. A particularly interesting collective neutrino flavor oscillation feature, the bipolar spectral swap, can appear in both environments as well. Ref. [2] showed that
Figure 3.13: Plots of the energy-averaged flavor evolution of neutrinos which start in the various initial states. We can see here that still interesting neutrino flavor transformations seem to occur at a radius of approximately 600km. However, here, the neutrino flavor transformations do not seem to stabilize as much as in the two flavor case. Although the antineutrinos mix more in the three flavor case than the two-flavor case, it still does not convert as many $\mu$ and $\tau$ flavor antineutrinos into the electron flavor as in the neutrino sector.
Figure 3.14: These figures show the final energy spectra of neutrinos and antineutrinos, respectively, run with lowered luminosity and density conditions from those simulations shown in Secs. 3.4.3 and 3.4.4. In the neutrino sector, we can see a very clear bipolar swap at an energy of $\approx 8\text{MeV}$. This is perhaps the clearest bipolar swap result found in our simulations. In addition, the high energy electron neutrino tail for neutrinos of energy $\gtrsim 38\text{MeV}$ is very pronounced.

Bipolar collective oscillations can occur in the BNS merger environment along oblique trajectories between the polar axis and the neutrino disk, in the inverted neutrino mass hierarchy and in antineutrino dominated conditions. The calculations presented here show that bipolar collective oscillations, along with ensuing spectral swaps, can also occur in neutrino-dominated conditions in the normal mass hierarchy. This can have potentially interesting implications, as we will discuss in Sec. 3.5.2.

3.5.1 Flavor Evolution

As discussed above, our simulations with the normal neutrino mass hierarchy show bipolar collective flavor oscillations which produce spectral swaps. In fact, we find a two-tiered stepwise spectral swap which gives rise to not only the usual swapping of electron and $\nu_x$-neutrinos at low energies, but also a secondary partial swap of flavors at high energies, resulting in an enhanced high energy electron neutrino tail. At low radii ($r \lesssim 100\text{km}$ for the low density and low luminosity simulations, and $r \lesssim 500\text{km}$...
Figure 3.15: These figures show the energy-averaged flavor evolution of a neutrino and antineutrino, respectively, which started out in the electron flavor state for a simulation with a lowered neutrino luminosity and initial density. We can see that flavor evolution sets in much closer to the neutrino disk than the simulations shown with a higher luminosity and density in Secs. 3.4.3 and 3.4.4. Here the flavor evolution begins around a radius of \( \approx 100 \text{ km} \).

for the high density and high luminosity simulations), the large matter and neutrino potentials keep the neutrinos mostly locked in their initial flavor states. This is a consequence of the instantaneous in-medium mass eigenstates being driven apart, thereby suppressing the corresponding in-medium flavor mixing parameters. At intermediate radii \( (r \approx 100–500 \text{ km} \text{ and } r \approx 500–800 \text{ km} \text{ for the aforementioned two cases}) \), the neutrinos undergo synchronized oscillations. Eventually \( (at r \gtrsim 500 \text{ km} \text{ and } r > 800 \text{ km}) \) bipolar spectral swaps develop.

Qualitatively speaking, this is similar to the behavior exhibited in flavor transformation simulations in core-collapse supernova environments. For instance, the final spectrum that we see in our simulations looks qualitatively similar to the final spectrum presented for the normal neutrino mass hierarchy in [6] (see Fig. 7 (a) in that paper). As such, it is likely that the same physical phenomenon which guided the flavor evolution in the simulations presented in that paper guides the flavor evolution here. The neutrino spectra that we used were quite similar to those in Ref. [6]; however, it bears noting that
Figure 3.16: This is the evolution of neutrino flavors for a neutrino and antineutrino initially in the x flavor state in a simulation with a lowered luminosity and initial density.

the characteristic radii at which collective neutrino transformations begin and end are quite different between our simulations discussed in Secs. 3.4.3 and 3.4.4, and those in Ref. [6]. This results from the much higher baryon density (by about two orders of magnitude) that we used in these simulations, as compared to the supernova environment analyzed in Ref. [6], as well as the high luminosity. If the luminosity and density are lowered, significant neutrino flavor conversion does occur closer to the neutrino disk, as observed in our low luminosity and low density simulation.

Refs. [6, 31, 163, 164] demonstrate how a geometric picture of flavor evolution can be developed in the two-flavor case by mapping the neutrino modes (described as $SU(2)$ spinors) to their equivalent $SO(3)$ representations, termed either “Neutrino flavor iso-spins (NFIS)” or “Polarization vectors”. As shown in these references, this can be used to explain the bipolar spectral swap at low energies $E_\nu \lesssim E_C$ using an analytic analysis of neutrino flavor evolution. However, at neutrino energies above the threshold energy $E_H$, the neutrinos may not be locked into the collective bipolar modes. These neutrinos may be converted via a background-assisted MSW mechanism to form the high energy electron neutrino tail that we see in both the high luminosity, high density and low luminosity, low density simulations.
The major difference between the physical conditions employed in our simulations and those in supernova simulations of neutrino flavor evolution is the geometric dilution of neutrinos in the two venues, i.e., a spherical neutrino source in the supernova case versus a disk-like source in the BNS merger case. Differences in neutrino luminosity and baryon number density mostly serve to change the relative locations of the onset of collective neutrino flavor evolution. The mechanisms through which the neutrino flavors transform, however, are not sensitive to this difference in geometric dilution. The bipolar flavor swap requires only that (1) the neutrino Hamiltonian dominate at some point to bring the neutrinos into a synchronized oscillation mode, and (2) the neutrino Hamiltonian must then gradually decrease with increasing radius in order for the flavor conversion to remain in the adiabatic regime. If these conditions are met, the bipolar spectral swap phenomenon seen in results of our simulations is robust to the details of the neutrino geometric dilution.

3.5.2 Electron Fraction Ramifications

A potentially important question is whether the collective oscillation induced modification of the neutrino and antineutrino energy spectra could affect the material composition, i.e., the electron fraction $Y_e \equiv n_e/n_b$, defined as the ratio of the net electron number density to the baryon number density. The electron fraction can be important in determining $r$-process yields, being directly related to the neutron-to-proton ratio, $n/p = (1/Y_e) - 1$. If we follow a fluid element as it leaves the merger environment, the local electron fraction would be determined by the interplay between the neutron-proton interconversion, via the weak capture processes of Eq. (3.1), and the matter outflow rate. We can label the rates (in units $s^{-1}$) for the reactions in Eq. (3.1) as $\lambda_{\nu_e n}, \lambda_{e^- p}, \lambda_{\bar{\nu}_e p}$, and $\lambda_{e^+ n}$ where the subscripts refer to particles entering each reaction. Two of these rates destroy neutrons and the other two produce neutrons. The electron fraction then evolves
according to these rates, as a competition between neutron production and destruction (where \(Y_p\) and \(Y_n\) are the proton and neutron fractions respectively):

\[
\frac{d}{dt} Y_e = (\lambda_{\nu_e n} + \lambda_{e+n}) Y_n - (\lambda_{\bar{\nu}_e p} + \lambda_{e^- p}) Y_p. \tag{3.18}
\]

Imposing charge neutrality, and assuming for our purposes that baryons are composed purely of neutrons and protons, we have \(Y_p = Y_e\), and \(Y_n = 1 - Y_e\). We can then turn Eq. (3.18) into an ordinary differential equation in \(Y_e\):

\[
\frac{d}{dt} Y_e = \lambda_1 - \lambda_2 Y_e, \tag{3.19}
\]

where we have defined \(\lambda_1 \equiv \lambda_{\nu_e n} + \lambda_{e+n}\) and \(\lambda_2 \equiv \lambda_{\nu_e n} + \lambda_{e^- p} + \lambda_{\bar{\nu}_e p} + \lambda_{e^+ n}\) [45].

Knowing the weak interaction rate and outflow velocity history for a given fluid element will then tell us about the way it evolves in \(Y_e\). Since our interest lies in evaluating the effects of neutrino flavor transformations on the weak processes, the rates we have chosen to focus on in what follows are the rates of neutron destruction and production via neutrino capture processes, \(\lambda_{\nu_e n}\) and \(\lambda_{\bar{\nu}_e p}\). These rates depend on the neutrino and antineutrino fluxes and distribution functions, and on the interaction cross-sections. Generically, dropping the subscripts so that the quantities may represent either of the two processes, these rates can be expressed as:

\[
\lambda(r) = \int_p \sigma(p) dn_{\nu_e}(p,r). \tag{3.20}
\]

Here \(\sigma\) is the appropriate neutrino interaction cross-section, and \(dn_{\nu} = dn_{\nu_e}\) or \(dn_{\bar{\nu}_e}\) is the differential number flux of \(\nu_e\) or \(\bar{\nu}_e\) at the interaction location. These number fluxes can be expressed in terms of \(dn_{\nu_\alpha}(E_\nu)\), i.e., the number density of neutrinos at energy \(E_\nu\) with initial flavor \(\alpha\) (discussed in Sec. 3.3.1), and the energy-dependent flavor
conversion/survival probabilities $P_{\alpha e}(r, E_\nu)$, as follows:

$$dn_{\nu_e}(r, E_\nu) = \sum_\alpha dn_{\nu_e}(E_\nu) P_{\alpha e}(r, E_\nu),$$

(3.21)

and similarly for $\bar{\nu}_e$. Note that, within the single-angle approximation, we can replace the neutrino momentum labels $p$ with just the energy $E_\nu$, since the neutrino fluxes/distributions are taken to be independent of emission trajectory.

In particular, to ascertain the conditions (e.g., outflow speeds) that may be required in order to preserve the neutron excess, we shall estimate the neutrino capture rate $\lambda_{\nu_e n}$ for some of our simulated environments. The rationale behind choosing to focus on $\lambda_{\nu_e n}$ is that one expects the material surrounding the BNS merger disk to be neutron rich to begin with (i.e., $Y_n > Y_p$), making the rate $\lambda_{\nu_e n}$ more important in the rate equation [Eq. (3.18)] compared to $\lambda_{\bar{\nu}_e p}$. Moreover, $\nu_e$ capture on $n$ does not have a threshold, unlike $\bar{\nu}_e$ on $p$, although the effect of this threshold is small at the typical energies in these environments. Borrowing the expression for $dn_{\nu_e \alpha}$ from Eq. (3.13), and integrating over angles, we can write the expression for the rate $\lambda_{\nu_e n}$ as

$$\lambda_{\nu_e n}(r) = \int_0^\infty \sum_\alpha \frac{L_{\nu_e \alpha}}{2 \pi^2 R_\nu^2 \langle E_{\nu, \alpha} \rangle} \tilde{f}_{\nu_e \alpha}(E_\nu) P_{\alpha e}(r, E_\nu) \cdot 2\pi \left(1 - \frac{r}{\sqrt{r^2 + R_\nu^2}}\right) \sigma(E_\nu) dE_\nu.$$

(3.22)

The appropriate neutrino capture cross-section in the low momentum-transfer limit is given by [165–170]:

$$\sigma(E_\nu) = \frac{2 \pi^2 (hc)^3 \ln 2}{c} \frac{\langle G \rangle}{\langle ft \rangle (m_e c^2)^2} (E_\nu + Q)^2 \equiv C (E_\nu + Q)^2,$$

(3.23)

where $\langle G \rangle$ is the average Coulomb correction factor, $\langle ft \rangle$ contains the pertinent (scatter-
ing) matrix elements, and $Q = (m_i - m_f)c^2$ is the $Q$-value of the reaction, i.e., the net rest-mass energy differential between the initial and final constituents. Using $\langle ft \rangle = 10^{3.035}$ s and $\langle G \rangle = 1$, one can calculate the pre-factor $C$ to be approximately $9.3 \times 10^{-44}$ cm$^2$/MeV$^2$. Note that we have explicitly written all the $c$ and $\hbar$ symbols in Eq. (3.23) to facilitate calculating the cross-section in cm$^2$, rather than in energy units.

For simplicity, we assume here that the constituents of the charged-current neutrino capture processes are the proton, neutron, the electron (or positron), and the nearly massless neutrino (i.e., no heavier nuclei). Therefore, $Q = +0.782$ MeV and $-1.804$ MeV for processes (3.1a) and (3.1b), respectively. In particular, if an antineutrino does not have enough energy to turn a proton into a neutron plus a positron, then that reaction will not proceed, and therefore, the $\bar{\nu}_e$ on $p$ cross-section is zero for $E_{\nu} < 1.804$ MeV. For $\nu_e$ on $n$, however, there is no threshold, and therefore the cross-section is always positive definite. The important thing to note about Eq. (3.23) is the dependence on $E_{\nu}^2$, implying that higher energy neutrinos would have a stronger effect on the electron fraction. Consequently, the high energy electron neutrino tail which develops in both two- and three-flavor simulations in the bipolar spectral swap case could have a non-negligible effect on the electron fraction.

Using Eqs. (3.22) and (3.23), and taking the far-field limit ($r \gg R_\nu$), one can write

$$\lambda_{\nu_e n}(r) \approx \frac{C}{2\pi r^2} \left( \langle E_{\nu_e}^2(r) \rangle + 2Q \langle E_{\nu_e}(r) \rangle + Q^2 \right) \mathcal{N}_{\nu_e}(r),$$

(3.24)

where the averages $\langle E_{\nu_e}(r) \rangle$ and $\langle E_{\nu_e}^2(r) \rangle$ have been calculated with respect to the weighting function

$$f'_{\nu_e}(r, E_{\nu}) \equiv \sum_\alpha \frac{L_{\nu, \alpha}}{\langle E_{\nu, \alpha} \rangle} \tilde{f}_{\nu, \alpha}(E_{\nu}) P_{\alpha e}(r, E_{\nu}),$$

(3.25)

which can be recognized as the effective electron-flavor neutrino distribution function
(non-normalized) at a radius $r$, with

$$\mathcal{N}_{\nu_e}(r) \equiv \int_0^\infty \sum_\alpha \frac{L_{\nu_e \alpha}}{\langle E_{\nu_e \alpha} \rangle} \tilde{f}_{\nu_e \alpha}(E_{\nu_e}) P_{\alpha e}(r, E_{\nu}) \, dE_{\nu}$$

(3.26)

being the effective number luminosity of electron-flavor neutrinos at a radius $r$. For instance, the expression for average electron neutrino energy-squared at a radius $r$ can be calculated as

$$\langle E^2_{\nu_e}(r) \rangle = \frac{1}{\mathcal{N}_{\nu_e}(r)} \int_0^\infty E^2_{\nu_e} f'_{\nu_e}(r, E_{\nu}) \, dE_{\nu}$$

(3.27)

Armed with this, we can calculate the effective electron neutrino number luminosities and average energies and thereby get an idea of whether, or under what circumstances (e.g., outflow speeds), the neutrinos can have an effect on the electron fraction at different radii within the envelope. Tables 3.4 and 3.5 list the values of quantities $\langle E_{\nu_e} \rangle$, $\langle E^2_{\nu_e} \rangle$, and $\mathcal{N}_{\nu_e}$, along with the calculated $\lambda_{\nu_e n}$ capture rates, for the bipolar spectral swap simulations with high and low luminosities/matter densities, as discussed in Secs. 3.4.3 and 3.4.5, at radii of $r = 2000$ km and $r = 1200$ km, respectively. In each table, for comparison we also present a second set of values calculated at these radii, but using the unaltered initial neutrino energy spectra, i.e., assuming that no flavor evolution occurred in the interim (taking $P_{\alpha e}(r, E_{\nu}) = \delta_{\alpha e}$). The choices of radii were based on the points at which collective flavor oscillations more-or-less ended in the respective simulations.

For comparison, the rate calculations for the simulation that used the DD2 equation of state luminosities and average energies from Ref. [1] are presented in Table 3.6. Even though this simulation exhibited qualitatively similar flavor transformation features, including a bipolar spectral swap at low energies, and a high-energy electron neutrino tail, the lower initial $\nu_x$ luminosity and the relatively weaker energy hierarchy between $\nu_e$ and $\nu_x$ rendered the resulting high-energy electron neutrino tail less potent, both in terms of energy and number. The $\sim 40\%$ change in $\lambda_{\nu_e n}$, although less drastic than for
Table 3.4: Table showing values of average energy, average energy-squared, and effective number luminosity in the electron-flavor, along with the calculated charged-current capture rate $\lambda_{\nu_e,n}$, in the two-flavor bipolar spectral swap simulation with high luminosity and matter density (Table 3.2). The numbers presented above are evaluated at a simulation radius of $r = 2000$ km. The numbers in the second column are calculated assuming no neutrino flavor evolution occurs in the interim, whereas those in the third column reflect the changes due to the flavor evolution, corresponding to the results of that simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No oscillations</th>
<th>With oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle E_{\nu_e}(r) \rangle$</td>
<td>11 MeV</td>
<td>16.6 MeV</td>
</tr>
<tr>
<td>$\langle E_{\nu_e}^2(r) \rangle$</td>
<td>145.6 MeV$^2$</td>
<td>387.6 MeV$^2$</td>
</tr>
<tr>
<td>$\mathcal{N}_{\nu_e}(r)$</td>
<td>$8.5 \times 10^{57}$ s$^{-1}$</td>
<td>$8.5 \times 10^{57}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{\nu_e,n}(r)$</td>
<td>0.51 s$^{-1}$</td>
<td>1.3 s$^{-1}$</td>
</tr>
</tbody>
</table>

Table 3.5: Same as table 3.4, but for the bipolar spectral swap simulation with low luminosity and matter density (Sec. 3.4.5), at a radius $r = 1200$ km.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No oscillations</th>
<th>With oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle E_{\nu_e}(r) \rangle$</td>
<td>11 MeV</td>
<td>15.5 MeV</td>
</tr>
<tr>
<td>$\langle E_{\nu_e}^2(r) \rangle$</td>
<td>145.6 MeV$^2$</td>
<td>354.8 MeV$^2$</td>
</tr>
<tr>
<td>$\mathcal{N}_{\nu_e}(r)$</td>
<td>$8.5 \times 10^{56}$ s$^{-1}$</td>
<td>$7.6 \times 10^{56}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{\nu_e,n}(r)$</td>
<td>0.14 s$^{-1}$</td>
<td>0.3 s$^{-1}$</td>
</tr>
</tbody>
</table>

the case presented in Table 3.4, can nevertheless be significant for determining $Y_e$ and the corresponding effects on nucleosynthesis.

Table 3.7 shows the corresponding rates for a three-flavor calculation exhibiting the bipolar spectral swap (Sec. 3.4.4). In this case, the enhancement of the $\nu_e$ capture rate stemming from flavor transformations is less drastic as compared to the result in the corresponding two-flavor case (Table 3.4). However, the effect of the high-energy tail still stands out: despite the effective number luminosity $\mathcal{N}_{\nu_e}$ of electron neutrinos dropping by almost a factor of two from the initial luminosity, the total rate is nevertheless enhanced by about 30%. This can be attributed to the presence of the high-energy tail and the strong energy dependence of the weak capture cross sections.

Lastly, the capture rate calculations corresponding to the MNR simulation in
Table 3.6: Same as table 3.4, but for the two-flavor bipolar spectral swap simulation run using the initial luminosities and spectra adopted from the DD2 equation of state simulation in Ref. [1]. The numbers presented above are evaluated at a simulation radius of \( r = 2000 \) km.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No oscillations</th>
<th>With oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle E_{\nu_e}(r) \rangle )</td>
<td>11.9 MeV</td>
<td>14.4 MeV</td>
</tr>
<tr>
<td>( \langle E_{\nu_e}^2(r) \rangle )</td>
<td>169.3 MeV (^2)</td>
<td>266.3 MeV (^2)</td>
</tr>
<tr>
<td>( \mathcal{N}_{\nu_e}(r) )</td>
<td>( 8.4 \times 10^{57} ) s(^{-1})</td>
<td>( 7.6 \times 10^{57} ) s(^{-1})</td>
</tr>
<tr>
<td>( \lambda_{\nu_e n}(r) )</td>
<td>0.59 s(^{-1})</td>
<td>0.82 s(^{-1})</td>
</tr>
</tbody>
</table>

Table 3.7: Same as table 3.4, but for the three-flavor bipolar spectral swap simulation (Sec. 3.4.4), at a radius \( r = 2000 \) km.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No oscillations</th>
<th>With oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle E_{\nu_e}(r) \rangle )</td>
<td>11 MeV</td>
<td>16.5 MeV</td>
</tr>
<tr>
<td>( \langle E_{\nu_e}^2(r) \rangle )</td>
<td>145.6 MeV (^2)</td>
<td>377.3 MeV (^2)</td>
</tr>
<tr>
<td>( \mathcal{N}_{\nu_e}(r) )</td>
<td>( 8.5 \times 10^{57} ) s(^{-1})</td>
<td>( 4.4 \times 10^{57} ) s(^{-1})</td>
</tr>
<tr>
<td>( \lambda_{\nu_e n}(r) )</td>
<td>0.51 s(^{-1})</td>
<td>0.65 s(^{-1})</td>
</tr>
</tbody>
</table>

Sec. 3.4.2 are presented in table 3.8. In this case, neutrino flavor evolution boosts the rate \( \lambda_{\nu_e n} \) not only through the high-energy electron neutrinos in the tail, but also through a net increase in the total number luminosity of electron neutrinos. Nevertheless, because of the weaker hierarchy in average neutrino energies in this case, the effect is still less drastic compared to that shown in table 3.4.

Table 3.8: Same as table 3.4, but for the calculation that exhibits the matter-neutrino resonance (Table 3.1), at a radius \( r = 2000 \) km.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No oscillations</th>
<th>With oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle E_{\nu_e}(r) \rangle )</td>
<td>10.6 MeV</td>
<td>13.2 MeV</td>
</tr>
<tr>
<td>( \langle E_{\nu_e}^2(r) \rangle )</td>
<td>135.2 MeV (^2)</td>
<td>232.4 MeV (^2)</td>
</tr>
<tr>
<td>( \mathcal{N}_{\nu_e}(r) )</td>
<td>( 8.8 \times 10^{56} ) s(^{-1})</td>
<td>( 1 \times 10^{57} ) s(^{-1})</td>
</tr>
<tr>
<td>( \lambda_{\nu_e n}(r) )</td>
<td>0.05 s(^{-1})</td>
<td>0.09 s(^{-1})</td>
</tr>
</tbody>
</table>

In both the high and low luminosity/matter density simulations (Tables 3.4 and 3.5), we see that the rates \( \lambda_{\nu_e n} \) calculated using our observed flavor transformation are
greater than those with no flavor transformation, by factors of two to three. This is not surprising, considering that in both simulations, a strong high-energy electron neutrino tail develops, which skews the average energy and energy-squared towards higher values. To determine whether these neutrinos actually have any purchase on the electron fraction, one must know the local outflow rate of the material in the envelope. Conversely, we can use our neutrino capture rate to estimate what the local outflow velocity $v_{\text{out}}$ would have to be at any radius along our trajectory in order to effectively decouple the neutrinos, so that the neutron excess can be preserved to facilitate the $r$-process. Neutrino decoupling necessarily requires

$$\frac{v_{\text{out}}}{r} \gg \lambda_{\nu_e n}.$$  \hspace{1cm} (3.28)

This implies that, to completely decouple the neutron excess from the neutrinos, the outflow velocities would have to be much greater than $v_{\text{out}} \approx 2600$ km/s and $v_{\text{out}} \approx 360$ km/s, for the rates presented in the right-hand columns of tables 3.4 and 3.5, respectively. Therefore, as long as the outflow velocities are comparable to these numbers or smaller, the neutrinos would likely be coupled to the electron fraction in the matter, rendering neutrino flavor evolution potentially important in determining the $r$-process production feasibility for the wind-like ejecta outside the neutrino disk plane.

We can see from Eq. (3.19) that an increase in the cross section for neutrino capture, and therefore and increase in the rate $\lambda_{\nu_e n}$, would tend to raise the electron fraction $Y_e$. Even though this rate appears in both the positive and negative parts of the differential equation, the negative part is multiplied by $Y_e$ itself which must be less than 1. The net effect of increasing this rate, then, would be to increase $Y_e$ towards 1. This makes sense as this rate is a rate for a reaction which destroys neutrons and creates protons. If the high energy electron neutrino tail would cause the $Y_e$ to rise above the level that current simulations without neutrino flavor evolution account for, then this would generally hurt the efficiency of the $r$-process. For a robust $r$-process, there must
be a sufficiently large ratio of neutrons to seed nuclei, usually implying the necessity of a low electron fraction.

### 3.6 Conclusions

We have investigated flavor transformation phenomena for polar-axis directed neutrinos streaming out from a BNS merger neutrino disk. In cases where the total number luminosity of neutrinos is higher than antineutrinos, we have seen that neutrino flavor transformations in a BNS merger neutrino-driven wind may give rise to a bipolar spectral swap at low energies, along with a high energy electron neutrino tail, in the normal mass hierarchy. Such a scenario (neutrino number dominated) can arise in merger simulations with the DD2 neutron star equation of state. The bipolar spectral swaps found in our results are qualitatively similar to those obtained from flavor transformation simulations in supernova environments, demonstrating the robustness of the mechanism underlying the swap to the geometric differences between the two cases. In our calculations, this phenomenon was observed in simulations with varying luminosities and matter densities, as long as the total number luminosity of electron neutrinos was higher than that of electron antineutrinos. In fact, bipolar oscillations in BNS merger environments were also found in Ref. [2] for anti-neutrino dominated spectra on certain trajectories. However, those calculations used the inverted mass hierarchy. For the case with a higher electron antineutrino number luminosity, we were able to qualitatively reproduce the matter-neutrino-resonance (MNR) that was observed in previous studies of the binary neutron star merger environment. In both cases, the high energy tail which develops in the electron neutrino spectrum, with the absence of an analogous phenomenon in the antineutrino sector, serves to enhance the charged-current neutrino capture rate on neutrons. In the absence of rapid matter outflows, this increase in the capture rate could lead to reduction
in the neutron fraction, and thereby a less efficient $r$-process than would be expected if neutrino flavor evolution were not taken into account.

It is intriguing that aspects of the hot, neutron matter equation of state which determine the emergent neutrino energy spectra and fluxes, also may qualitatively influence the nature and outcome of collective neutrino oscillations and consequently, the outflow composition in some cases.

### 3.7 Acknowledgement

This work was supported in part by NSF Grants PHY-1307372 and PHY-1614864 at UCSD. We also acknowledge a grant from the University of California Office of the President.

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Chapter 4

Final Conclusions

From our spin coherence simulations we have seen that although it is very hard to initiate the spin coherence (one must use high electron neutrino luminosities, an unrealistically large neutrino rest mass, and a relatively flat electron fraction profile which hovers near resonance), once spin coherence does happen and “tracking” is initiated, a significant portion of neutrinos may be transformed into antineutrinos. This neutrino to antineutrino transformation can then feedback into the flavor transformations and qualitatively and quantitatively alter them from cases where spin coherence was not considered.

In order to initiate spin coherence with more realistic values of the neutrino rest mass (which was by far the most unrealistic parameter that we had to use) we may need to consider additional sources of nonlinear feedback such as the electron fraction feedback mechanism. If these mechanisms can initiate tracking in the neutrino-neutrino Hamiltonian, then it is conceivable that a significant spin transformation can occur. More research and study is needed to see whether these feedback mechanisms can indeed initiate the tracking behavior.

Additional study into different supernova environments might also increase the
chances of finding significant spin transformations. The steep density drop off in the ONeMg type supernovae that we considered in this thesis does not help to foster a strong spin coherence effect (but here we chose this environment so that we could compare our results with previous studies of neutrino flavor transformation). It might be easier to have a strong spin coherence effect in an iron core-collapse supernova with a shallower density gradient. These are only a couple of directions that future research into spin coherence could take. It seems that in order to fully understand the way neutrinos evolve in a supernova environment, a great deal more attention has to be made with respect to the neutrinos’ spin degrees of freedom.

Our binary neutron star merger simulations showed that both MNR and bipolar flavor swaps can occur inside a BNS merger environment. Either of these effects, if they do happen, could potentially have a significant effect on the proton to neutron ratio inside of these environments and therefore on the robustness of the $r$-process and their yields. We have shown that unless outflow speeds in a BNS merger environment are so extraordinarily fast that the neutrinos are totally decoupled from these environments after only a few hundred kilometers, we must consider neutrino flavor evolution in order to obtain the correct electron fraction and therefore correctly calculate $r$-process yields. A simulation where neutrinos are considered to not evolve in flavor space as they stream out of these environments may not be adequate in obtaining correct $r$-process yields.
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