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Universality and $m_X$ cut effects in $B \to X_s \ell^+ \ell^-$

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The most precise comparison between theory and experiment for the $B \to X_s \ell^+ \ell^-$ rate is in the $q^2 < 6 \text{GeV}^2$ region. The hadronic uncertainties associated with an experimentally required cut on $m_X$ potentially spoil the extraction of short distance flavor-changing neutral current couplings. We compute the $m_X$ cut dependence of $d \Gamma(B \to X_s \ell^+ \ell^-)/dq^2$ using the $B \to X_s \gamma$ shape function, and show that the effect is universal for all short distance contributions in the limit $m_X^2 \ll m_B^2$. This universality is not spoiled by realistic values of the $m_X$ cut, nor by $\alpha_s$ corrections. Alternatively, normalizing the $B \to X_s \ell^+ \ell^-$ rate to $B \to X_s \ell \bar \nu$ with the same cuts removes the main uncertainties. We find that the forward-backward asymmetry vanishes near $q^2 = 3 \text{GeV}^2$.

I. INTRODUCTION

In the standard model (SM) the flavor-changing neutral current process $B \to X_s \ell^+ \ell^-$ does not occur at tree level, and is a sensitive probe of new physics. Predicting its rate involves integrating out the $W$, $Z$, and $t$ at a scale of order $m_W$ by matching on to the Hamiltonian 

$$H_W = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1}^{6} C_i O_i + \frac{1}{4\pi^2} \sum_{i=7}^{10} C_i O_i \right],$$

(1)
evolving to $\mu = m_b$, and computing matrix elements of $H_W$. Here $O_1 - O_6$ are four-quark operators and

$$O_7 = m_b \bar s \gamma_\mu \epsilon F^{\mu\nu} P_R b, \quad O_8 = m_b \bar s \gamma_\mu \gamma_5 gG^{\mu\nu} P_R b, \quad O_9 = e^2 (\bar s \gamma_\mu P_L b) (\bar \epsilon \gamma_\nu \epsilon), \quad O_{10} = e^2 (\bar s \gamma_\mu P_R b) (\bar \epsilon \gamma_\nu \gamma_5 \epsilon),$$

(2)
where $P_{L,R} = (1 \mp \gamma_5)/2$. The dilepton invariant mass spectrum, $q^2 = (p_{X}\pm p_\ell^\pm)^2$, can be calculated in an operator product expansion (OPE), and the leading nonperturbative corrections are suppressed by $\Lambda_{\text{QCD}}^2/m_b^2$. The matching and anomalous dimension calculations for $C_i$ are known at next-to-next-to-leading order (NNLL) order, as are the largest perturbative QCD corrections to the matrix elements of $O_i$.

An important complication in $B \to X_s \ell^+ \ell^-$ compared to $B \to X_s \gamma$ is that the long distance contributions, $B \to J/\psi X_s$ and $\psi' X_s$ followed by $J/\psi, \psi' \to \ell^+ \ell^-$, are of order magnitude larger than the short distance prediction, a fact which is not well-understood. Therefore, either theory and data are both interpolated, or the short distance calculation is compared with the data for $q^2 < m_{J/\psi}^2$ or $q^2 > m_{\psi'}^2$. The low $q^2$ region, $q^2 < 6 \text{GeV}^2$, allows the most precise comparison with the SM, but requires a cut on the invariant mass of the hadronic final state, $m_X < m_{\ell^+ \ell^-}$. In the latest Belle analysis $m_X^{\text{cut}} = 2 \text{GeV}$, while Babar uses $m_X^{\text{cut}} = 1.8 \text{GeV}$. These cuts are to remove backgrounds, and will likely be required for quite some time.

In this letter we investigate the effects of the $m_X$ cut on predictions for $B \to X_s \ell^+ \ell^-$ decay in the low $q^2$ region. This was previously studied in the Fermi-motion model in Ref. [14]. For $(m_X^{\text{cut}})^2 = O(\Lambda_{\text{QCD}} m_b)$, the local OPE breaks down, and is replaced by an OPE involving nonlocal operators, whose matrix elements are $b$ quark distribution functions in the $B$ meson. We define

$$\Gamma_{ij} = \int q_{1i}^2 dq_{1i} \int_0^{m_{\ell^+ \ell^-}} dm_X \Re (c_i c_j^*) \frac{d^4 \Gamma_{ij}^{\text{cut}}}{dq^2 dm_X},$$

(3)
where $ij = \{77, 99, 00, 79\}$ label contributions of time-ordered products $T(O_i^\dagger O_j)$. The $\eta_{ij}$'s contain the effects of the $m_X$ cut, and the short distance coefficients $c_7, c_9, c_{10}$ track the $c_7, c_9, c_{10}$ dependence in Eq. [14]. Here $c_7 = C_7^{\text{mix}}(q^2)$, $c_9 = C_9^{\text{mix}}(q^2)$, and $c_{10} = C_{10}$ can be obtained from local OPE calculations at each order, as discussed in Ref. [15]. The functions $G_{99,00} = (2q^2 + m_b^2), G_{77} = 4m_b^2 (1 + 2m_b^2/q^2), G_{79} = 12m_b m_s$ arise from kinematics, where $m_b$ is a short distance mass, such as $m_b^{\text{S}}$. Here and below, Finally,

$$\Gamma_0 = G_2^2 m_b^5 \frac{\alpha_s}{192\pi^2} \frac{m_s}{4\pi^2} |V_{tb}V_{ts}|^2.$$  

(4)
We also study $\eta_{ij}(p_{\ell^+}^{\text{cut}}, q_1^2, q_2^2)$, which are defined by replacing $m_X$ in Eq. (3) with $m_X^\pm = E_\ell - |p_\ell^\pm|$. The total rate for $B \to X_s \ell^+ \ell^-$ with cuts is $\Gamma_{\text{cut}} = \sum_{ij} \Gamma_{ij}$.

At leading order in $\Lambda_{\text{QCD}}/m_b$ and $\alpha_s$, $\eta_{ij} = 1$ for $m_X^{\text{cut}} = m_B$, and therefore $\eta_{ij}$ give the fraction of events with $m_X < m_{\ell^+ \ell^-}$. This is altered at subleading order by perturbative corrections, but $\eta_{ij}$ still determine the rate. In principle, $\eta_{ij}$ depend in a nontrivial way on $ij$ (and $q_1^2$ and $q_2^2$) due to different dependence on kinematic variables, $\alpha_s$ corrections, etc. Working to leading order in $\Lambda_{\text{QCD}}/m_b$, we demonstrate that $\eta_{ij}$ are independent of the choice of $ij$, which we call "universality". We first show this formally at leading order in $p_{\ell^+}^2/m_B \ll 1$ for the $p_{\ell^+}^2$ cut, $q_1^2$, and then numerically for the experimentally relevant $m_X^{\text{cut}}$, $\eta$, including the $\alpha_s$ corrections and all phase space effects. Since the same shape function occurs in $B \to X_s \ell^+ \ell^-$, $X_s \ell \bar \nu$, and $X_s \gamma$, the $m_X^{\text{cut}}$ or $p_{\ell^+}^2$ cut dependence in one can be determined from the others.
II. $m_X$ CUT EFFECTS AT LEADING ORDER

For simplicity, consider the kinematics in the $B$ meson’s rest frame. Since $q = p_B - p_X$,

$$2m_B E_X = m_B^2 + m_X^2 - q^2.$$  

(5)

If $m_X^2 \ll m_B^2$ and $q^2$ is not near $m_b^2$, then $E_X = \mathcal{O}(m_B)$. Since $E_X^2 \gg m_X^2$, $p_X$ is near the light-cone, with $p_X^+ = E_X - |p_X| = \mathcal{O}(\Lambda_{QCD})$ and $p_X^- = E_X + |p_X| = \mathcal{O}(m_B)$. Of the variables symmetric in $p_+^+ - p_+^-$, only two are independent, and we work with $q^2$ and $p_X^\pm$ or $m_X$. The phase space cuts are shown in Fig. 1.

For the $p_X^+ \ll p_X^-$ region, factorization of the form $d\Gamma = HJ \otimes f(0)$ has been proven for semileptonic and radiative $B$ decays [14], where $H$ contains perturbative physics at $\mu_b \sim m_b$, $J$ at $\mu \sim \sqrt{\Lambda_{QCD} m_b}$, and $f(0)(\omega)$ is a universal nonperturbative shape function. This factorization also applies for $B \to X_s \ell^+ \ell^-$ with the same universal $f(0)$, as long as $q^2$ is not parametrically small [13].

In the $q^2 < 6\, GeV^2$ region, $|C_9^{\text{mix}}(\mu_0 = 4.8\, GeV)| = 4.52$ to better than 1%, and can be taken to be constant. We neglect $\alpha_s$ corrections in this section and find

$$\frac{d\Gamma}{dp_X^+ dq^2} = \int f(0)(p_X^+) \, \Gamma_0 \left[ (m_B - p_X^+)^2 - q^2 \right]^2 \times \left\{ \left(C_9^{\text{mix}} + C_{10}\right)[2q^2 + (m_B - p_X^+)^2] + 4m_B^2 C_7^{\text{mix}} \left[ 2 + \frac{2(m_B - p_X^+)^2}{q^2} \right] + 12m_B \Re(C_7^{\text{mix}} C_9^{\text{mix*}}) (m_B - p_X^+) \right\},$$

(6)

where $f(0)(\omega)$ has support in $\omega \in [0, \infty)$. As a function of $p_X^+$, the kinematic terms in Eq. (5) vary only on a scale $m_B$, while $f(0)(p_X^+)$ varies on a scale $\Lambda_{QCD}$. Writing $m_X = m_b + \Lambda$ and expanding in $(p_X^- - \Lambda)/m_B$, decouples the $p_X^+$ and $q^2$ dependences in Eq. (6), and gives the local OPE prefactors, $(m_b^2 - q^2)^2 G_{ij}(q^2)$, in Eq. (6). For $\eta_j(p_X^+; q_1^2, q_2^2)$ the $p_X^+$ integration is over a rectangle in Fig. 1, whose boundaries do not couple $p_X^+$ and $q^2$. Thus, $\eta_j = \int dp_X^+ f(0)(p_X^+)$, independent of $ij$ and $q_1^2, q_2^2$. While the $m_X$ cut retains more events than the $p_X^+$ cut, the latter may give theoretically cleaner constraints on short distance physics when statistical errors become small.

The effect of the $m_X$ cut is $q^2$ dependent, because the upper limit of the $p_X^+$ integration is $q^2$ dependent, as shown in Fig. 1. Including the full $p_X^+$ dependence in Eq. (6), the universality of $\eta_j(m_X^{\text{cut}}, q_1^2, q_2^2)$ is maintained to better than 3% for $1 \, GeV^2 \leq q_1^2, q_2^2 \leq 2 \, GeV^2$, $5 \, GeV^2 \leq q_2^2 \leq 7 \, GeV^2$, and $m_X^{\text{cut}} \geq 1.7 \, GeV$, because the region where the $p_X^+$ and $q^2$ integration limits are coupled has a small effect on the $ij$ dependence. This is exhibited in Fig. 2, where the solid curves show $\eta_{ij}(m_X^{\text{cut}}, 1 \, GeV^2, 6 \, GeV^2)$ with the shape function set to model-1 of [16] with $m_0^2 = 4.68 \, GeV^2$ and $\lambda_1$ from [17].

(Taking $q_2^2 = 1 \, GeV^2$ instead of $4m_b^2$ increases the sensitivity to $C_{9,10}$, but one may be concerned by local duality/resonances near $q^2 = 1 \, GeV^2$. To estimate this uncertainty, assume the $\phi$ is just below the cut and $B(B \to X_s \phi) \approx 10 \times B(B \to K^{(*)}\phi)$. Then $B \to X_s \phi \to X_s \ell^+ \ell^-$ is $\sim 2\%$ of the $X_s \ell^+ \ell^-$ rate.)

The local OPE results for $\eta_{ij}(m_X^{\text{cut}}, q_1^2, q_2^2)$ are obtained by replacing $f(0)(p_X^+)$ by $\delta(\Lambda - p_X^-)$ in Eq. (6). Performing the $p_X^+$ integral sets $(m_B - p_X^+) = m_b$ and implies $m_X^2 > \Lambda(m_b - q^2/m_b)$. This makes the lower limit on $q^2$ equal max{$q_1^2, m_b(m_b - (m_X^{\text{cut}})^2/\Lambda)$}, and so the $\eta_j$’s depend on the shape of $d\Gamma$. In Fig. 2, the local OPE results are shown by dashed lines, and clearly $\eta_{77} \neq \eta_{99}$. However, the local OPE is not applicable for $p_X^- \sim \Lambda_{QCD}$.

The universality of $\eta_{ij}$ can be broken by $\alpha_s$ corrections in the hard and jet functions, or by renormalization group evolution, since these effects couple $p_X^+$ and $q^2$ and have been neglected so far. We consider these next.

III. CALCULATION AND RESULTS AT $\mathcal{O}(\alpha_s)$

A complication in calculating $B \to X_s \ell^+ \ell^-$ compared to $B \to X_u \ell \nu$ is that, in the evolution of the effective Hamiltonian down to $m_b$, $C_b(\mu)$ receives a $\ln(m_{\ell^\pm}^2/m_b^2)$
enhanced contribution from the mixing of \( O_2 \). Thus, formally, \( C_9 \sim O(1/\alpha_s) \), and conventionally one expands the amplitude in \( \alpha_s \), treating \( \alpha_s \ln m^2_{q_T}/m^2_T = O(1) \) \[2\].

In the local OPE this is reasonable, since the nonperturbative corrections are small, and at next-to-leading log (NLL) all dominant terms in the rate are included. However, in the shape function region nonperturbative effects are \( O(1) \) and only the rate is calculable. With the traditional counting the \( C^2_9 \) contribution to the rate would be needed to \( O(\alpha^2_s) \) before the \( C^2_{10} \) terms could be included.

This would be a bad way to organize the perturbative corrections (numerically \( |C_9(m_b)| \approx |C_{10}| \)). It can be circumvented by using a “split matching” procedure to decouple the perturbation series above and below the scale \( m_b \) \[13\]. This allows us to consider the short distance coefficients \( C^\text{mix}_9, C^\text{mix}_9, \) and \( C_{10} \) as \( O(1) \) numbers when organizing the perturbation theory at \( m^2_B \) and \( m^2_{\Lambda_{QCD}} \).

The rate and the forward-backward asymmetry are

\[
\begin{align*}
\frac{d^2T}{dq^2dp^+_X} &= \frac{\Gamma_0}{m^2_B} H(q^2, p^+_X) F^{(0)}(p^+_X, p^-), \\
\frac{d^2A_{FB}}{dq^2dp^+_X} &= \frac{\Gamma_0}{m^2_B} K(q^2, p^+_X) F^{(0)}(p^+_X, p^-),
\end{align*}
\]  

where \( p^- = m_b - q^2/(m_b - p^+_X) \). The hard functions \( H \) and \( K \) were computed in Ref. \[13\] using SCET \[8 \ 19\] and split matching, which factorizes the dependence on scales above and below \( m_b \) as \( H_1(\mu_0) H_2(\mu_b) \). Here, to the order one is working at, \( H_1 \) is \( \mu_0 \) independent, the \( \mu_b \) dependence in \( H_2 \) and \( F^{(0)} \) cancels, and \( F^{(0)} \) is \( \mu_i \) independent. The shape function model is specified at \( \mu_A \). The convolution of jet and shape functions at NLL including \( O(\alpha_s) \) corrections is

\[
\begin{align*}
F^{(0)}(p^+_X, p^-) &= U_H(p^+, \mu_i, m_b) \left( \hat{f}^{(0)}(p^+_X, \mu_i) + \frac{\alpha_s(\mu_i) C_F}{4\pi} \left\{ 2 \ln \frac{\hat{p}^+_X p^-}{\mu_i^2} - 3 \ln \frac{\hat{p}^+_X p^-}{\mu_i^2} + 7 - \pi^2 \right\} \hat{f}^{(0)}(p^+_X, \mu_i) \right) \\
&\quad + \int_1^0 \frac{dx}{x} \left\{ 4 \ln \frac{2\hat{p}^+_X p^-}{\mu_i^2} - 3 \left[ \hat{f}^{(0)}(p^+_X(1-z), \mu_i) - \hat{f}^{(0)}(p^+_X, \mu_i) \right] \right\}, \\
\hat{f}^{(0)}(\omega, \mu_i) &= e^{V_S(\mu_i, \mu_A)} \left( \frac{\omega}{\mu_A} \right)^{\eta} \int_0^1 dt \hat{f}^{(0)}(\omega(1-t^{1/\eta}), \mu_A),
\end{align*}
\]  

where \( U_H \) was computed in Ref. \[13\], the one-loop jet function in Ref. \[20, 21\], and the shape function evolution up to \( \mu_i \) in Refs. \[18, 21\] (for earlier calculations, see Refs. \[13, 22\]). The \( H \) and \( K \) are

\[
\begin{align*}
H(q^2, p^+_X) &= \frac{[1 - \hat{p}^+_X]^2 - \hat{q}^2]^2}{(1 - \hat{p}^+_X)^3} \left\{ \left[ C^\text{mix}_9(s, \mu_b) + C^\text{mix}_{10}(s, \mu_b) \right] + 8 q^2 \Omega^2_A(s, \mu_b) \right\} \\
&\quad + 4 |C^\text{mix}_7(\mu_b)|^2 \left[ \Omega^2_A(s, \mu_b) + 2 \frac{(1 - \hat{p}^+_X)^2}{q^2} \Omega^2_D(s, \mu_b) \right] + 12 \text{Re} \left\{ C^\text{mix}_7(\mu_b) C^\text{mix}_{10}(s, \mu_b) \right\} (1 - \hat{p}^+_X) \Omega_E(s, \mu_b),
\end{align*}
\]  

\[
\begin{align*}
K(q^2, p^+_X) &= -\frac{3q^2(1 - \hat{p}^+_X)^2 - \hat{q}^4}{(1 - \hat{p}^+_X)^3} \Omega_A(s, \mu_b) \text{Re} \left\{ C^\text{mix}_9(s, \mu_b) \Omega_A(s, \mu_b) + 2 \frac{(1 - \hat{p}^+_X)}{q^2} C^\text{mix}_7(\mu_b) \Omega_D(s, \mu_b) \right\},
\end{align*}
\]  

where \( s = q^2/m^2_B, \hat{q}^2 = q^2/m^2_B, \hat{p}^+_X = p^+_X/m_B \), and

\[
\begin{align*}
\Omega_A &= 1 + \frac{\alpha_s}{\pi} \omega^V(s, \mu_b), \\
\Omega_C &= 1 + \frac{\alpha_s}{\pi} \omega^T(s, \mu_b), \\
\Omega_B &= 1 + \frac{\alpha_s}{\pi} \left[ \omega^V(s, \mu_b) + \frac{(1 - \hat{p}^+_X)^2 - \hat{q}^2}{2(1 - \hat{p}^+_X)^2} \omega^T(s, \mu_b) \right], \\
\Omega_D &= 1 + \frac{\alpha_s}{\pi} \left[ \omega^T(s, \mu_b) - \omega^V(s, \mu_b) \right], \\
\Omega_E &= (2 \Omega_A \Omega_D + \Omega_B \Omega_C)/3.
\end{align*}
\]  

Here \( \alpha_s = \alpha_s(\mu_b) \) and \( \omega^V, \omega^T \) are defined in Ref. \[13\].

In Fig. \[3\] we plot \( \eta_{00}(\alpha_s, 1 \text{GeV}^2, 6 \text{GeV}^2) \), including the \( C_i \) corrections. For each \( \hat{f}^{(0)} \), the deviations of the \( \eta_{ij} \)'s from being universal is still below 3%. We use five different models for the shape function, constructed to obey the known constraints on its moments \[21\]. The orange, green and purple (medium, light, dark) curves correspond to \( m^\text{cut}_{10} = 4.68 \text{ GeV}, 4.63 \text{ GeV}, \) and 4.73 GeV, respectively, using the central values \( \mu_0 = \mu_b = 4.8 \text{ GeV} \) and \( \mu_i = 2.5 \text{ GeV} \). For \( m^\text{cut}_{10} = 2 \text{ GeV} \), varying \( \mu_b \) in the range 3.5 GeV < \( \mu_b < 7.5 \text{ GeV} \) changes \( \eta_{00} \) by ±6%. We find a ±5% variation for 2 GeV < \( \mu_b < 3 \text{ GeV} \). The curves with slightly lower [higher] values of \( \eta_{00} \) at large \( m^\text{cut} \) correspond to \( \mu_A = 1.5 \text{ GeV} \) [2 GeV].

The \( \mu_b \) dependence of the rate is similar to that in the local OPE, and will be reduced by including the known NNLL corrections \[13 \ 16 \ 19\]. We did not study it here.

Using the \( c_i \)'s at NLL, for 1 GeV^2 < \( q^2 < 6 \text{ GeV}^2 \) and \( m^\text{cut} = 1.8 \) and 2.0 GeV, we obtain \( \Gamma^\text{cut} \tau_B = (1.20 \pm 0.15) \times 10^{-6} \) and \( (1.48 \pm 0.14) \times 10^{-6}, \) respectively.

The largest uncertainty in the rate and the largest source of universality breaking in the \( \eta_{ij} \)'s are due to sub-
leading shape functions, which affect the rate by \( \approx 5\% \) for \( m_{\chi}^{cut} = 2 \) GeV and by \( \approx 10\% \) for \( m_{\chi}^{cut} = 1.8 \) GeV [23].

If the \( m_{\chi}^{cut} \) dependence were not universal, it would modify the zero of the forward-backward asymmetry, \( A_{FB}(q_0^2) = 0 \). For \( m_{\chi}^{cut} = 2 \) GeV we find at NLL \( \Delta q_0^2 \approx -0.04 \) GeV\(^2\), much below the higher order uncertainties [9]. However, we obtain \( q_0^2 = 2.8 \) GeV\(^2\), lower than earlier results [9]. In the local OPE limit we get \( q_0^2 = 2 m_b \bar{m}_b(\mu) C^\text{eff}(\mu)/\text{Re}[C^\text{eff}(\mu)] \). Here \( m_b \) can be taken to be \( m_b^{pole} \) or expanded about \( m_b^{S} \), but to ensure that the \( \mu \) dependence cancels at the order we are working, we cannot reexpand \( \bar{m}_b(\mu) \) in terms of \( m_b^{pole} \).

In conclusion, we pointed out that the experimentally used upper cut on \( m_X \) makes the observed \( B \to X_\ell \ell^- \) rate in the low \( q^2 \) region sensitive to the shape function. In this region there is an OPE only for the decay rate and not for the amplitude, necessitating a reorganization of the usual perturbation expansion. Since one can use the shape function measured in other processes, the sensitivity to new physics is not reduced. We found that the \( q^2 \)'s for the different operators' contributions are universal to a good approximation. The theoretical uncertainties are reduced by raising the \( m_X^{cut} \). Another possibility is to keep \( m_X^{cut} < m_D \) and measure with the same cuts

\[
R = \Gamma^{cut}(B \to X_\ell \ell^-)/\Gamma^{cut}(B \to X_{\nu} \ell \bar{\nu}),
\]

since the effect of \( m_X^{cut} \), as well as the \( m_b \) dependence, are drastically reduced in this ratio. These results also apply for \( B \to X_d \ell^+ \ell^- \), which may be studied at a higher luminosity \( B \) factory. Subleading \( \Lambda_{QCD}/m_b \) as well as NNLL corrections to the rate and the forward-backward asymmetry will be studied in a separate publication [23].

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