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Abstract of the Thesis

Design, Modeling, and Control of a Hydrostatic Actuator for MRI

by

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Master of Science in Mechanical Engineering
University of California, Los Angeles, 2014
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This thesis explores the design and characterization of a hydrostatic actuator for use in magnetic imaging resonance (MRI) environments. Using a simple fluid flow architecture, the thesis proves the feasibility of using hydrostatic actuators for MRI actuation and lays the theoretical groundwork for further development of hydrostatic actuation for use in creating more complicated hydrostatic systems.
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University of California, Los Angeles
2014
To Eftitan Akam, the light that guides me forward and on the straight path
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Chapter 1

Introduction

1.1 Problem Overview

In the medical field, Magnetic Resonance Imaging (MRI) has come to be the preferred imaging method for many physicians for diagnostic and interventional purposes. The ability of MR images to transmit detailed tissue information from multiple scanning planes in real time makes it a very useful tool for physicians in a broad category of procedures. Its unique features distinguish it from other imaging modalities like X-Ray Computed Tomography (CT) or ultrasound, which are not able to take the high-resolution, real-time images in multiple planes with sufficient ability to examine specific tissue qualities which MRI is able to. Furthermore, MRI does not use any potentially harmful radiation like CT so does not require additional safety precautions like the lead vests worn by those interacting with X-Ray machines.

While MRI has a number of very useful advantages in terms of providing benefit in diagnostic capability and image clarity, it comes with a number of drawbacks. Access to the patient is limited while in the scanner because the doctor is unable to reach inside the imaging bore, simply due to the physical geometry of the scanning setup. To be in the strongest part of the magnetic field, the patient will be too far into the bore to be reached
without assistance. Because of this, real-time imaging cannot be used while performing any procedures, unlike X-ray fluoroscopy or ultrasound where the doctor has direct access to the patient during imaging. Additionally, any assistive devices or actuators that are to be placed within the imaging bore, or even the room generally, must not interfere with the strong magnetic field generated by the imaging equipment. If proper precautions are not taken, the equipment can cause severe degradation of the image or even be a source of danger to the patient if magnetized materials fly into the bore.

Because of the benefits provided by allowing physicians access to patients undergoing MRI and the simultaneous restrictions present in MR environments, many researchers have attempted to create actuation systems to allow physicians to perform procedures while simultaneously imaging. Some of these projects will be elaborated on in the prior art section of this thesis.

The particular system that this project explores is a new actuation architecture involving low-pressure, water-based hydrostatic force transmission. This is realized with a master-slave actuation system using a simple hydrostatic transmission concept. By connecting two pistons via closed fluid channels, force and displacement can be transmitted from outside the imaging bore to inside the bore. The current project focuses on linear
transmission, though rotational transmission is also possible.

The system benefits from its relative simplicity and the complete lack of electric or ferrous materials to be put inside the imaging bore that would disrupt the imaging process.

1.2 Motivation

A good way to explain the usefulness of actuation within MRI is to give an example of a potential end-use. Biopsy of the liver is a complicated task that requires image guidance in order to be accurate in locating a potential site of cancerous tissue. Since the liver is fairly deep in tissue, especially for overweight or obese patients, ultrasound guidance cannot always provide the necessary image quality for a successful biopsy. Additionally, because the liver moves with patient respiration, real-time imaging is preferred so that the needle’s trajectory and its desired final location can be tracked during needle insertion. In current practice for MRI or CT imaging, the patient is put into the bore for imaging to find where the needle should go. Then the patient is removed from the bore and only the tip of a needle is inserted into the patient at an insertion site determined by the first image. The patient is then moved into the bore to confirm that the needle is going in at the proper direction. Then the patient must be moved out of the bore to advance the needle and make any corrections needed in the needle trajectory. This proceeds in stages until the needle reaches its final target. This process separates periods of imaging and needle movement, which not only adds a great deal of time to the procedure, but also can increase physician error in the procedure. Optimally, a needle would be inserted while the patient is being imaged with MRI simultaneously. This cannot be done manually because of the distance involved, nor can traditional motors be used for a positioning system because of the restrictions due to the use of strong magnets in the imaging. Developing an alternative actuation system would solve these challenges and allow physicians access inside the imaging bore to perform a liver biopsy which maintaining real-time imaging to guide the insertion.
This example illustrates the core issues in procedures done under MRI guidance. Because of both material constraints and the geometry of MRI bores, imaging of the patient cannot be done at the same time as a procedure needing physical physician input. This means moving the patient in and out of the imaging bore repeatedly, which not only increases the time of the procedure, but prevents the accuracy that could be afforded to a physician using full image guidance. While MRI’s capacity for real-time imaging makes it a very useful diagnostic tool, as an interventional tool it suffers because the imaging is decoupled from any desired actuation. Solving this decoupling in an intuitive and easy-to-implement way that can reduce clinician error would greatly help in carrying out a variety of procedures.

1.3 Prior Art

Because the need for actuation in MRI has been known for a number of years, different researchers and even companies have created technologies that allow for actuation in MRI environments. The field has been developed utilizing a number of different design approaches and architectures. A straightforward grouping method of the different approaches is to classify them is by their actuating mechanisms. While the physical architecture of any given system may vary, the actuation technology is one of the most important pieces of the design in order to improve MR-compatibility. The distinguishing factor of most designs is the type of actuation architecture that is used, so the work presented here is broken up into three sections: pneumatic, hydrostatic, and other architectures. The hydrostatic work is presented last as it is most relevant to the system being worked on.
1.3.1 Pneumatic

A number of very well-developed systems have been created using pneumatic systems for actuation. These systems include some of the most highly explored robots in the field. Among the most notable is the INNOMOTION system developed in Germany that has been licensed in Europe. The system is currently in use in a number of countries in the continent, having obtained the CE mark in 2005. It consists of a five degree of freedom pneumatic positioning system with 2 more degrees of manual motion. The INNOMOTION system is currently used clinically in back operations, biopsies, and other procedures. The system is also compatible with CT systems. [14]

Another system was developed at John’s Hopkins University, dubbed ”MRBot”. This system is based on accessing the prostate and uses a novel kind of pneumatic stepper motor for high-precision movement. Researchers working on the project have also developed optical encoders to detect displacement in the MR bore. [19]

Other research developed a pneumatically-powered needle insertion robot for the prostate with a single-needle method, focusing on reducing movement of the prostate. The author called this the Single Needle Implant Device (SNID). The device focused on needle insertion to the prostate that would also allow for tapping the needle in addition to pushing it, in order to increase the cutting force and reduce movement of the prostate during needle insertion. [11]

Another group from Johns Hopkins developed a Pneumatic MRI robot for prostate needle placement. This relied on a number of piezo-controlled servovalves in a box inside the MR room providing the control signals to the pneumatic system. The system employed a number of linkages in order to achieve the desired degrees of freedom derived from the pneumatic cylinders. [6]
1.3.2 Other motors

Abdelaziz et al developed a cable-driven robot for short-bore MRI in order to perform prostate interventions. The cable system was made to transmit forces into the MR bore for needle positioning under image guidance and used optical encoders to better determine the position of the actuators.[1]

A number of other projects have been developed relying on piezomotors. While often cited as a source of image degradation, many have tried to work around this and limit the source of image artefacts to the area directly around the motor. An Access to the Prostate Robot was developed using this technology. Multiple iterations of the design have been made, most recently, Generation IV (APT-IV) which uses piezomotors to control a three degree of freedom prostate access device.[3] At WPI others developed a general purpose robot using piezomotors for needle placement. This robot, like others in the field, relied on fiberoptics to create force sensors. [21]

1.3.3 Hydrostatic

Three particular institutions have attempted similar hydrostatic actuators to that proposed in this thesis, though none in fully the same way as this project, and with significant differences between each project as well.

ETH performed many analyses of a hydrostatic actuator that used classical high-pressure oil-based hydraulic actuators. The system was able to track slow sine waves in a single dimension and the authors developed an accurate system model. [7]. Further sensors were developed including optical force and position sensing [8].

Students at MIT for two master’s theses also developed a hydrostatic actuator for use in MRI, specifically for wrist rehabilitation. While the system was able to achieve fairly accurate modeling and control, there were limitations on the architecture due to the lack of seals used within the rotary vane apparatus. This caused internal leakage in the system, allowing for the system to become more and more decoupled from input to output with use.
The system used a combination of water and antifreeze to increase viscosity and reduce leakage, which alleviated the problem, but was not the optimal choice.[9][15]

While not used for positioning in an MR-setting, the "P-Arm" was developed for holding a laparoscope during minimally invasive surgery. The idea is very similar to that employed in this thesis: water was transferred from a control unit to the robot which was in this case a 6-DOF stewart platform. The platform was able to hold a laparoscope and could be remotely adjusted via a joystick during a surgery. [17]

1.4 Opportunity for New Development

Given the wide range of prior art in the field of MRI-compatible robotics, there needs to be some compelling reason to introduce a new or refined technology. A hydrostatic actuation system should be able to resolve many of the issues present in other design approaches.

The simplicity of the architecture is one of the most attractive pieces of the design. Being pumpless, it means that it does not have to rely on more complicated valved controls. It also does not need any complicated setup - merely the connection and filling of the fluid lines.

Unlike a number of other approaches, the design is also inherently MR-compatible. While the electronics involved in pneumatic switches or piezomotors must be shielded, this architecture does not require any sort of shielding and is able to be adopted in any given MR setting without additional setup required.

The simplicity also makes it an excellent platform for development because it reduces the cost both in the experimental phase and in implementation.

While the three works cited above have already developed some important features, this current project attempts to go beyond them in a few key areas. First, the design uses pure water, low pressure transmission, which is different from the approach by the researchers at ETH which used high pressure hydraulic fluid. This simplified architecture is
not only easier to implement, but also benefits from more ready adoption within the medical setting. The architecture is similar to that of Hanumara and Mendelowitz, but uses a sealed connection for linear motion instead of a seal-less rotary actuator. This allows for better tracking over repeated runs because there will be no leakage from one fluid path to the other. The physical form of the water-based actuators is very similar to that of the P-arm, but the intended use is very different. Because the P-Arm was only required for positioning without any insertion required, it only relied on single-ended cylinders which would not be able to perform for higher-force applications. Additionally, as the P-Arm was intended for positioning in laparascopic surgery, the dynamic characteristics of the actuators were not characterized, nor was the design intended for MRI compatibility.

A final advantage of this system is the ability to be completely actuated by hand. If the losses within the fluid lines are small enough, the input piston can be moved in a desired movement profile to create the same movement profile at the output. This would allow a physician to operate the system manually by simply moving a piston that is out of the MR bore in a master-slave system. While mounting the input piston to an electric motor would require long fluid lines to outside the MRI room or to a shielded box within the room, a manual system would only require fluid lines long enough to reach outside the imaging bore where the physician could manually manipulate the actuators. This usage case motivates the reduction of pressure loss from input to output as much as possible, because this would also allow for haptic feedback to the operating physician. If the pressure loss is low enough, a master-slave system would not only be able to accurately transmit position from a master actuator outside the bore to a slave manipulator inside the imaging bore but also lend force feedback to the operator. This is especially important in procedures like needle insertion, where feeling the tissue response to the needle’s travel is very important in keeping a safe insertion profile.

Given these unfilled needs which are met by this actuator architecture, there is an opportunity for a low-pressure, water-based hydrostatic actuator for usage within MRI with
its dynamics characterized and controlled and input-output losses minimized.
Chapter 2

Preliminary Design Concept: Orienting Mechanism

As a proof of concept in order to show how fluid actuators could be used to control a positioning system within an MR environment, a 2 degree of freedom (DOF) orienting mechanism was constructed using basic fluid actuators (syringes) to move a central platform, emulating a basic MR-compatible needle positioning system.

2.1 Orienting Mechanism Prior Art

The main need for a needle orienting mechanism is in the ability for a physician to guide a needle during insertion into the body. This requires an system that has two angular DOFs in order to reach different areas of the workspace within the body as well as a system that keeps the insertion point immobile so that when the needle angle is adjusted, the needle does not tear the skin unnecessarily. In order to achieve this, a number of spherical orienting mechanisms were considered for the needle positioning prototype, as a spherical mechanism fulfills these requirements and many spherical mechanisms were developed for this purpose in the first place. For this application, those employing linear actuators are preferred in order to make physical realization with simple syringe-based actuators.
relatively straightforward.

A number of previously developed orienting mechanisms were investigated to explore what may be potential architectures for the orienting mechanism. An oft-cited spherical mechanism that would be useful for orienting around an insertion point is the "Agile Eye"[2]. This is a parallel RRR RRR mechanism that uses two kinematic chains of revolute joints to achieve two degree of freedom movement around a single point. A similar in structure orienting mechanism was developed in a patent by Maillard, creating two degrees of angular freedom around a central point. [13]

There are also a number of other mechanisms that have been developed based on the Stewart-Gough Platform. The Stewart Platform is able to give general motion in six degrees of freedom, but therefore lacks the constraints built into many spherical mechanisms. Additionally, depending on the system architecture, it can suffer from singular points within the workspace. Its design, involving six universal and six spherical joints can also be difficult to manufacture. However, there are a few similar platform types based on the stewart platform that have been developed. One potential mechanism is that proposed in [12]. This paper proposes a constrained stewart platform with only three instead of six actuators and a passive link that connects the base to the platform with a spherical joint. This constrains the mechanism to only revolve around that connection point. A similar architecture achieves four degrees of freedom with a similar kinematic structure, but with the base and platform reversed and the passive link replaced with a prismatic joint.[5] This allows for spherical movement around a point as well as a fourth degree of freedom for insertion toward the point. This is an excellent structure for needle insertion, as it was developed for that purpose.

However, due to the complex nature of fabricating most of these designs, they were not used as the basis for the development of this orienting mechanism and instead a kinematic structure based on the work of Carricato [4] was used. This design is for generalized two degree of freedom spherical movement. It benefits from its parallel structure as well as a
decoupling of the control of the angle of the end-effector. Each actuator controls one of two euler angles independently, allowing for simpler kinematics and design. The actuators are also designed as prismatic joints, which readily adapts to the syringes that were to be used in this prototype.

2.2 Orienting Mechanism Design, Fabrication, Results

After evaluating the mechanisms described above, a solution similar to that developed by Carricato[4] was used. This would allow both syringe-actuators to remain stationary and also to be flat and close to the ground, which saves space. It also is readily adapted to fit the constraints on using linear actuators, which was necessary for being compatible with the syringe-actuators.

A prototype was made out of predominately polypropylene parts. Following the general idea of Carricato, a system utilizing two inverted cranks was used. A few changes to the architecture were made for space and complexity savings, as seen in figure 2.2. Due to the long length of the syringes to be used, it was desired to put them in different planes in order to save space by having some of the syringes overlap. Otherwise, with two perpendicular syringes making the outer edges of the system, the floor-space of the system would be much
too large for clinical applications. For this reason, the two pairs of syringes were located in parallel planes - one above the other. This change, in addition to saving space, made the system suited for a changed kinematic structure as well. While the originally proposed wrist prototype used two symmetric kinematic chains, the authors note that this symmetry is not needed for the kinematic structure of the system, which was another advantage of using this type of parallel mechanism. Thus, for this application, one of the degrees of freedom was changed from a crank mechanism to a pin-in-slot mechanism, which is a mechanically simpler way to transmit linear to rotary motion, involving fewer moving parts.

The polypropylene was manufactured mainly using a waterjet cutter in order to accommodate the variety of shapes required for the design. A mill was used to incorporate any final holes needed for attaching different pieces of the system together. The final result of this was the actuator seen in figure 2.3.

The design was made in order to allow for at least 90 degrees of motion in each degree of freedom to allow a wide workspace to allow for a range of positioning angles. To this end, based on the structure, and the possible travel ranges of the syringes for both degrees

Figure 2.2: Design of a 2DOF decoupled orienting mechanism
Figure 2.3: Prototype 2DOF decoupled orienting mechanism

of freedom, the length of all relevant links was calculated. In the end, this target was not met, due to interference of some of the components with one another - e-clips pushed into pieces of the stage and disallowed movement for the full desired range.

An accelerometer was mounted on the positioning platform to measure an accurate reading of the final angular range of the device. The device allowed for 20 degrees of movement on each side from the vertical direction in one degree of freedom and about 25 degrees of movement from the vertical in the other. This was able to be repeatedly generated - a given syringe position related to the same angular measurement from the accelerometer. Both degrees of freedom allowed for an inch of linear movement to traverse the whole range of angular movement.

As a proof-of-concept model, the orienting system was actuated manually by pushing the syringes that were connected to the actuators. Even still, with relative ease the stage could be positioned within the noise level of the accelerometer repeatably.
Chapter 3

Design of a One DOF Actuator

With a demonstrated orienting mechanism created with syringes, the next step was to create a custom-made actuator to be able to characterize the performance of hydrostatic linear motion over different distances and to be used in the development of more complicated robotic systems. This custom-made piston will be used as a test system for design and control in order to see what applications such an actuator may have and over what distances the system can usefully transmit force and displacement.

3.1 Functional Requirements/Specifications

The prototype system was made to analyze the hydraulic actuation architecture for use within clinical settings; thus clinical needs were considered, but put secondary to the full realization of a reliable system that can be used for testing purposes that can later be adapted for specific clinical purposes.

It was desired to continue with the same basic setup as syringes have, using a linear motion profile. As was shown in the orienting prototype, linear motion can be intelligently converted into appropriate rotary motion for most positioning needs within an MR context, as well as can be used for a variety of other purposes in the creation of linear positioning or needle insertion stages. While linear motion for use in needle insertion can be as large as
15cm, the actuator used for the testing system (to be elaborated in Chapter 5) has a full range of 1 in, so it was decided to have a system with at least 2 inches of travel and no more than 4. This keeps the prototype portable and still allows for some manual manipulation for testing purposes. In the future, the range can be lengthened on an as-needed basis. Because of the forces involved in a needle insertion process, the actuator was made to be able to transmit at least 20 lbs of force reliably.

The device must also be fully compatible with its use in a medical setting and also in an MRI setting. This puts two separate sets of constraints on the design. In terms of medical use, all components must be sterilizable for use in an operating theater. Also, any fluids used must be safe for use close to patients or even touching them in case of catastrophic failure and spillage of the working fluid. To this end, traditional hydraulic fluid was eschewed in favor of water as a transmission fluid which does not carry with it any of the contamination concerns of the heavy oils normally used in hydraulic actuators. This provides an added benefit for the force transmission characteristics of the system because of the higher bulk modulus of water when compared to hydraulic oils.

MRI compatibility is an even more important challenge than the medical setting. Materials used in the whole system must be MRI-safe and not contribute to negative imaging artifacts. The optimal case for this level of compatibility is that the actuator is completely MR-transparent and does not show up in MR scans nor interfere with imaging of nearby structures with image artifacts. The basic level required for the system to be used in an MR setting is that the system does not have any physically MR unsafe characteristics that could cause damage to the MRI system, the patient, or the device.

3.2 Linear Actuator Architecture

Having decided to create a single DOF system, the next decision was what sort of architecture would make the most sense. A piston setup is the most natural choice for a
linear hydrostatic actuator, but more specifics would need to be determined to best fit this project’s purpose.

In the end, a double-ended single rod pistons were chosen for this application, as shown in figure 3.2. The double-ended architecture was chosen in order to not have any force limitations on movement in both directions while maintaining as small a profile as possible. The two other main possible piston architectures are shown in figure 3.1. A single-ended piston could be used to push from input to output, but in the reverse direction, there would be a maximum pull force that would be related to the air pressure on the piston. If the actuator was used for needle insertion or similar other uses involving movement within a patient, a low maximum force in one direction would not be desirable. This design criterion leads to the decision to use a double-ended piston.

The piston was also made to be single rod-ended, mainly for space saving. While the piston exhibits higher symmetry as a double rod-ended setup, the fundamental dynamics would not change in a single-rod system, and the system would have a reduced number of sealing interface for potential leakage which is an added benefit. Because saving space is especially important for MRI applications, effectively halving the space taken up by the actuator by using a single rod instead of double rod was deemed the most appropriate piston architecture.
The piston bore was to be made of smooth circular material which would be mounted in two end blocks to allow for stability and mounting of the piston. The piston bore was designed to be externally threaded on both ends to screw into the end blocks in order to create a reliable seal at both ends that could still be disassembled for alteration and maintenance. The piston blocks allowed for connection between the piston bore and the fluid connectors to the tubing of the system.

In order to allow for the piston rod not to cause a leak in the system at its exit of the piston, a seal was required in end block around the exit. The natural placement for this is on the exterior of the end block around the exit hole, however an additional plate is required to hold the seal in place, so an additional face plate is screwed into the front of the end block to hold the rod seal in place. The result of this design can be seen in the rendered image in figure 3.3.

3.3 Dynamic Response: Informing Design

The piston size was determined by some preliminary work into the fluid dynamics of the system. While a fuller system model was developed, this was not done prior to the original
Figure 3.3: Render of the design of a single piston setup

design process. However, some preliminary exploration of the system’s dynamics helped evaluate key factors of the design to make informed decisions while designing the system.

If fully developed flow is assumed, the fluid will behave according to Bernoulli’s equation, in equation 3.1. Fully developed flow cannot be taken as a given for this sort of dynamic system, but can be used to begin characterizing the system and get a feel for what the dynamics should be.

\[
\text{fluid head} = z(x) + \frac{p(x)}{\rho g} + \frac{V(x)^2}{2g}
\]  

(3.1)

From Bernoulli’s equation, the relationship between pressure loss at two points along a pipe can be derived as follows:

\[
p_b = p_a - \rho g (\Delta z + f \frac{LV^2}{2Dg})
\]  

(3.2)

Ignoring the height term:

\[
\Delta p = -\rho f \frac{LV^2}{2D}
\]  

(3.3)

Where \( p \) is pressure, \( \rho \) is density, \( z \) is height, \( L \) is the length of tube, \( V \) is fluid velocity,
g is the gravitational constant, and f is the friction factor. In order to create a system that transmits force and displacement most reliably, it is desired to have the least amount of pressure loss. The z term can be neglected as the height of the input and output actuators is not a fundamental component of the system design. This leaves the term with L, V, D, and f as parameters that can be adjusted. As is intuitively expected, L is directly proportional to the pressure loss, meaning that the length of the transmission tube should be minimized in order to create the best transmission. Different lengths of tube will be tested, but this is not a parameter that informs directly the design of the piston. The diameter of the tube, D has an inversely proportional effect to the pressure loss - so the tube should be as large as possible. Overly large tubing would result in a bulkier system than desired, as two fluid transmission lines must be used for each actuator of a system, so while minuscule lines will not be considered for the pressure loss they would incur, overly large lines are also undesirable for practical concerns. In the end, 1/4” OD tubing was chosen as a suitable compromise that also is compatible with a large variety of connectors, which is an extremely important practical consideration.

The other terms in the equation are f and $V^2$. f is a friction factor that is related to a number of factors, including pipe roughness and the reynolds number. If laminar flow is presumed, which relates to a Reynolds number lower than 2000, then there is a very clear tradeoff between the two terms. In laminar flow, f can be approximated as $\frac{64}{Re}$. And the Reynolds number is calculated as $Re = \frac{\rho V D}{\mu}$. So f is inversely proportional to velocity, which means increasing velocity will decrease friction and reduce the pressure drop. However, from the original equation, note that there is also a $V^2$ term, so the total pressure loss is approximately proportional to V. This would encourage the smallest fluid flow velocity in the pipeline.

While the input forces and desired motion profiles will determine the actuator velocity, the design can still be optimized to reduce fluid flow velocity. Since the desired use of the actuator, not the design, determines the input force and velocity parameters, these can be
taken as a constant in the design process. However, while there is a given velocity and
desired total displacement in the actuator, this velocity is not the same as the fluid velocity.
If the actuator piston diameter is much larger than the tubing’s diameter, then the fluid
velocity will be much greater than that of the actuator. Conversely, if the actuator
diameter is much smaller than that of the tubing, then the fluid velocity will be much
smaller. Generally, due volume continuity, the velocity is proportional to the ratio of the
actuator and tubing areas. Because of this fact, the actuator size should be minimized in
order to reduce velocity and improve the transmission behavior of the system to decrease
pressure loss in the line.

Above, laminar flow was assumed for ease in calculation. However, it remains to be
seen whether this is an accurate characterization. There will certainly be times in actuator
use when it will operate in the laminar region for slow velocities, but in faster operation,
will the actuator also enter the turbulent region? In some preliminary calculations, the
answer is yes, it is possible. For pure water, its density is 1000 $kg/m^3$, viscosity is .001 Pa-s,
and the tubing diameter was set to .25 in. For these parameters, the crossover velocity is
about 12.5 in/s for fluid velocity. While this seems a little fast, if the actuator has a
diameter double that of the tube, which is non unreasonable, the input velocity is only a
quarter of that, or only about 3 in/s, which is certainly possible. So it can be assumed that
the system may move in and out of the turbulent region.

Fortunately, this does not have a great deal of effects on the actuator characteristics
and the presumptions made above. The most important effect is on the friction factor,
whose effects actually start to decrease as the reynolds number increases, as can be seen in
the Moody Chart shown in figure 3.4. This still leaves the most important factor in
reducing pressure loss in keeping the actuator to tubing area ratio low.
3.4 Materials and Seals

Material selection was important in order to maintain MR compatibility while not compromising system performance. A number of options were considered in the design process. A combination of PET and polypropylene were chosen as plastic parts for the housing of the piston as well as the piston itself. These were chosen for the low water-absorbency of both materials, so that they would not be subject to swelling on being immersed in water.

The rod of the piston was originally nonmagnetic copper, but was in the end replaced with nonmagnetic aluminum due to manufacturing considerations.

Acrylic was chosen as cylinder material due to its clarity so that the position of the pistons could be readily observed and any air pockets could be detected during operation.

Nylon tubing and connectors were selected for their cheap cost, MR compatibility, FDA certification, and certification of reliability up to at least 100 psi.

In order to prevent leakage at any of the important fluid interfaces, rubber seals were employed in key locations of the design. O-rings were used for their ready availability and low cost. Glands were sized according to normal sizes for reciprocating hydraulic applications.
O rings were chosen to simplify design and fabrication considerations. As mentioned above, the piston bore diameter should be close in size to the tubing diameter, so around 0.25 inches. The rod diameter must be smaller than that to allow for the effective piston diameter on that side of the piston to not decrease too much, but must be large enough to keep the rod rigid under normal operation. It would also be useful to decrease the number of different seals. To satisfy all of these constraints, a 0.375” diameter piston and cylinder were chosen with a 0.1875” rod. The piston was slightly undersized in order to allow for smooth sliding - the o ring would take care of the sealing. A dash number 106 o-ring was chosen to seal both the piston and the rod, as it was made to match both that particular rod outer diameter and piston size.

3.5 Fluid Connections

The connections between the pistons and their placement was an important part of ensuring that the design would not only operate as planned but could be easily assembled and disassembled as well as made readily able to accommodate changes in working fluid.

While the basic architecture for the system is a simple connection between one side of each piston and the corresponding side of the opposite piston, some slight additions are necessary to allow for complete filling. This is shown in figure 3.5. This architecture - with a T connection and a valve at each end of each piston - allows for filling and draining the system with ease. However, there is still one difficulty incurred during the filling process: the space between the piston and the T connection can trap air bubbles. Especially in tests with syringes held horizontally, this can prove to be a difficult challenge to overcome. While movement of the actuators back and forth can remove some air, it is not always sufficient to remove all of it. Usually, it was necessary to pre-fill the actuator and the lines leading directly from it in order to circumvent this issue.

During the testing process, an additional connection was added between the two lines
3.6 Fabrication

The realization of the single degree-of-freedom design was more difficult than the relatively simple design would suggest. The first roadblock was in the cylinder of the piston. The acrylic cylinder was chosen for its low water absorbency and clarity so that the piston could be observed in the cylinder and any air pockets could be readily identified. A hollow acrylic cylinder was purchased to fit the dimensions required for the o-rings that had been sized according to standard sealing practice, as mentioned above. The first problem came with trying to fit the acrylic cylinder into the side blocks of the actuator. While designed for a threaded fit, the acrylic would repeatedly crack or shatter while cutting the threads due to its brittle nature and the depth of the required threads. An example of what this did to one cylinder is seen in figure 3.6. While both single-point threading and using a die were
Figure 3.6: Broken acrylic cylinder due to acrylic’s brittle nature and depth of cuts needed for thread forming

attempted, in the end this was unsuccessful. To accommodate this, the acrylic was left at its original size and the threaded connection was replaced with a bonding agent to both seal the cylinder and keep it in place.

A second problem was how to create a satisfying seal in the acrylic that fit well and would not at the same time result in extremely high static friction. The original designed sizes for the piston, cylinder, and seal would not fit together - the cylinder’s inner diameter was too small for the other parts. The first attempted solution was to reduce the piston size to better accommodate the acrylic. This was unsuccessful in reducing the static friction as the parts still only barely fit together. The next attempt was to increase the bore diameter of the acrylic, which also was difficult as the acrylic was not made for post-processing and frequently suffered from overheating or melting. Due to these impediments, acrylic was put aside as a potential cylinder material and a replacement material was selected.

The acrylic was chosen for its clarity in order to see the inside of the piston, but since it was more important to have a cylinder that moved smoothly, plastics were eschewed in order to have a cylinder that could be machined to a precise finish. Brass was originally tested, and then aluminum was selected as the final material as it is lightweight, easy to machine, cheap, and MR compatible.

While numerous tests were undertaken with different materials and changes in the
diameter of the cylinder and piston in order to reduce friction without affecting the sealing properties of the piston, it was difficult to meet both results with a pure water system. Since o-rings are normally made for use in oil-based systems, this formed a problem - even with a seal that was loose enough to be leaky, the piston would not slide smoothly. Without the o-ring, the piston would slide smoothly in the bore without lubrication due to its precise sizing, but the o-ring was desired for guarantee of a good seal. With lubrication from teflon spray, the o ring would also slide smoothly, but when water was added, it would again seize up. In order to reduce this problem, propylene glycol was added to the system. Propylene glycol is a common additive for water hydraulic systems and is considered by the FDA to be generally safe to humans.[22] The glycol’s increased viscosity allowed for better lubrication of the o rings, leading to a smoother response.

Reducing leakage in the system was also an important issue. Because of manufacturing problems with threading the acrylic, the design was changed to accommodate a close fit sealed with bond and or sealant. However, this proved to be continually plagued by small leaks, so the design was reverted to the original idea of threaded connections between the cylinder and the end blocks. These connections were also reinforced by sealant to make sure that there would be no leakage. Even the face plate where the piston rod exited the cylinder that had an o-ring in place required additional sealant to remove leaks.

After all these fabrication issues were dealt with, the system was ready for testing.

![Assembled hydraulic pistons](image)

Figure 3.7: Assembled hydraulic pistons
Chapter 4

Preliminary System Model

Before the test system had been fully created and ready to take data, a physics-based, white-box model was developed in order to have a baseline understanding of the dynamics of the system for further modeling after collecting data. While grounded in work by Ganesh and Hanumara, a number of different choices were made to streamline and improve the system model, while also accounting for the particulars of this system. The basic model was motivated by modeling the fluid lines as springs with associated stiffness and damping. Each input piston is considered as a mass connected by a spring and damper to represent the dynamics of the fluid connections.

4.1 Fluid modeling

Since a double-ended cylinder architecture is used in this application, there is an issue of indeterminate geometry. However, this indeterminate geometry can simply be solved using one path with double the resistance/mass/friction.

The way by which this substitution is reasonable is analogous to a circular spring-mass system. The forces are exactly the same as those of a 2-mass spring system with 2 springs (or dashpots) in parallel. This can be seen in figure 4.1. A displacement of one piston by \( dx \) results in both pistons receiving a force of \( 2K \, dx \), which is equivalent to putting two springs
between the first and second piston in parallel. Because of this congruence, the double acting system is represented like a single fluid-flow system with doubled mass, spring constant, and damping constant.

The first piece of the model to be derived was the spring constant, which pertains mostly to the fluid bulk modulus, which is a function of how the fluid’s displacement changes under pressure. This can be derived as follows:

Generally, pressure $P$, force $F$, and area $A$ can be described as:

$$ P = \frac{F}{A} \quad (4.1) $$

$$ F = A \quad (4.2) $$

Taking a derivative, as force changes, so does pressure. The area, described by the piston, does not change because it is a physical parameter.

$$ dp = \frac{df}{A} \quad (4.3) $$

Working from the volume continuity side, the change in the x locations of the pistons relates to a change in volume:

$$ dV = A(dx_0 - dx_i) \quad (4.4) $$
And from the definition of bulk modulus,

\[ B = \frac{dp}{dV} \] (4.5)

\[ B \, dV = V \, dP \] (4.6)

With these relationships defined, we can substitute equation 4.3 and 4.4 into equation 4.6 and solve in terms of dF:

\[ V \frac{dF}{A} = B \, A(dx_0 - dx_i) \] (4.7)

\[ dF = \frac{BA^2}{V}(dx_0 - dx_i) \] (4.8)

\[ dF = kdx \] (4.9)

This is a spring equation with:

\[ k = \frac{BA^2}{V} \] (4.10)

### 4.2 Fluid Damping

The damping coefficient is determined from the fluid friction equation. The Darcy-Weisbach equation gives a good approximation to use, that fluid head loss is:

\[ h_f = \frac{fL\dot{V}^2}{2Dg} \] (4.11)

In this equation V is fluid velocity, proportional to \( \dot{x} \) but scaled by \( \frac{A_{piston}}{A_{tubing}} \). It is also known that pressure loss can be related to head loss by:

\[ \Delta P = \rho g h_f \] (4.12)
These equations can be combined so that:

\[
\Delta F = \frac{fL \rho V^2 A}{2D} = C_{\text{effective}}(V^2)
\]  

(4.13)

This is similar to the general form of a damper in a mass spring damper system, but with a \( V^2 \) term instead of \( \dot{x} \). And so, there is an effective damping constant:

\[
C_{\text{effective}} = \frac{fL \rho A}{2D}
\]  

(4.14)

In the damping term, there is a velocity-dependent friction constant, \( f \):

\( f \) is determined by the Reynolds number, which is in turn dependent on the fluid velocity, so must be calculated separately in different time steps.

For laminar flow, \( f \) is \( f = \frac{64}{Re} \) For turbulent flow, \( f \) can be approximated as:

\[
f_0 = 0.25 \left[ \log \left( \frac{e/D}{3.7} + \frac{5.74}{Re^{0.5}} \right) \right]^{-2}
\]  

(4.15)

\[
f = \left[ -2 \log \left( \frac{e/D}{3.7} + \frac{2.51}{Re f_0^{0.5}} \right) \right]^{-2}
\]  

(4.16)

(4.17)

To be conservative, in some applications, \( f \) can be estimated to be 0.1 because it never exceeds that value, and so this gives an upper bound on the fluid damping.

### 4.3 Fluid Mass

While a traditional spring-mass-damper system ignores the mass of the spring, the opposite should be true for this system. The fluid is very massive compared to the pistons, so its mass must be included in the model. To this end, the traditional massless spring is replaced with a "massed" one.
This does not change the spring constant, so the total potential energy of the system remains unchanged, but the kinetic energy is very different once the spring is considered to also have mass. Simple force relationships no longer can fully characterize the motion, so the euler-lagrange equation is used to find equations of motion:

\[ L = T - V \]  \hspace{1cm} (4.18)

\[ V = 1/2k(x_2 - x_1)^2 \]  \hspace{1cm} (4.19)

\[ T = 1/2M_1\dot{x}_1^2 + 1/2M_2\dot{x}_2^2 + \int 1/2dm(V_{\text{segment}})^2 \]  \hspace{1cm} (4.20)

\[ (4.21) \]

Presuming a uniform spring mass,

\[ dm = \frac{m}{L}dy \]  \hspace{1cm} (4.22)

\[ V_{\text{segment}} = Va + \frac{V_b - V_a}{L}y \]  \hspace{1cm} (4.23)

Plugging into the equation for \( L \) and doing the math:

\[ L = mx_1^2/6 + mx_2^2/6 + mx_1\dot{x}_2/6 + 1/2M_1\dot{x}_1^2 \]

\[ + 1/2M_2\dot{x}_2^2 - k/2x_1^2 - k/2x_2^2 + kx_1x_2 \]  \hspace{1cm} (4.24)

\[ (4.25) \]

Then, using the euler-lagrange method

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} + \sum F_x \]  \hspace{1cm} (4.26)
This gives the equations of motion

\[ m\ddot{x}_1/3 + m\ddot{x}_2/3 + M_1\ddot{x}_1 = k(x_2 - x_1) + C(\dot{x}_2 - \dot{x}_1) + \sum F_1 \]

\[ m\ddot{x}_2/3 + m\ddot{x}_1/3 + M_2\ddot{x}_2 = k(x_1 - x_2) + C(\dot{x}_1 - \dot{x}_2) + \sum F_2 \]

Solve for \( \ddot{x}_1 \) and \( \ddot{x}_2 \) shown:

\[ (m/3 + M_2 - \frac{m^2}{36(m/3 + M_1)})\ddot{x}_2 = \frac{mk(x_1 - x_2)}{6(m/3 + M_1)} + \frac{mC(\dot{x}_1 - \dot{x}_2)}{6(m/3 + M_1)} \]

\[ - \frac{m\sum F_1}{6(m/3 + M_1)} + k(x_1 - x_2) \]

\[ + C(\dot{x}_1 - \dot{x}_2) + \sum F_2 \]  

Combine terms, simplify and define new constants:

\[ \ddot{x}_1 = (M_\beta + 1)k(x_2 - x_1) + (M_\beta + 1)C(\dot{x}_2 - \dot{x}_1) - M_\beta \sum F_2 + \sum F_1 \]  \hspace{1cm} (4.28)

\[ \ddot{x}_2 = (M_\beta + 1)k(x_1 - x_2) + (M_\beta + 1)C(\dot{x}_1 - \dot{x}_2) - M_\beta \sum F_1 + \sum F_2 \]  \hspace{1cm} (4.29)

### 4.4 Seal Friction

In addition to fluid friction, the seals on the pistons/rods present additional friction, modeled as coulomb friction. Estimates have been made from o-ring properties to estimate the friction present. In simulation, a "locking" model has been used to have a more accurate model of the effects of stiction. This is discussed more in section 4.6.

### 4.5 Linear Model

Especially for developing a controller, a linear SISO plant model in state-space form is desired. The more complicated model is linearized for control design. Because static
friction is inherently nonlinear, it will be removed from this model. Additionally, the fluid damping will be modeled as proportional to velocity instead of velocity squared with a constant friction factor, instead of one that varies with fluid velocity.

From our earlier equation 4.29, the dynamics can be represented neatly as a second order spring-damper system when the damping term is declared as proportional to velocity. From this, a state-space realization of the linearized model can be derived.

\[
\begin{bmatrix}
\dot{x}_1 \\
\ddot{x}_1 \\
\dot{x}_2 \\
\ddot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
\frac{-2k(a+1)}{A} & \frac{-2C(a+1)}{A} & \frac{2k(a+1)}{A} & \frac{2C(a+1)}{A} \\
0 & 0 & 0 & 1 \\
\frac{2k(a+1)}{A} & \frac{2C(a+1)}{A} & \frac{-2k(a+1)}{A} & \frac{-2C(a+1)}{A}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\frac{1}{A} \\
\frac{\alpha}{A}
\end{bmatrix} \text{\ input}
\] (4.30)

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\dot{x}_1 \\
x_2 \\
\dot{x}_2
\end{bmatrix}
\] (4.31)

For control, it is also desired to examine the transfer functions from input to both outputs. Looking at equation 4.29 and using the laplace transform, these can be found:

\[
\ddot{x}_1 = (M_\beta + 1)k(x_2 - x_1) + (M_\beta + 1)C(\dot{x}_2 - \dot{x}_1) - M_\beta \sum F_2 + \sum F_1
\]
\[
\ddot{x}_2 = (M_\beta + 1)k(x_1 - x_2) + (M_\beta + 1)C(\dot{x}_1 - \dot{x}_2) - M_\beta \sum F_1 + \sum F_2
\]

Taking the laplace transform:

\[
s^2X_1(s) = K_{ef}(X_2(s) - X_1(s)) + C_{ef}s(X_2(s) - X_1(s)) + F_1(s)/A
\]
\[
s^2X_2(s) = K_{ef}(X_1(s) - X_2(s)) + C_{ef}s(X_1(s) - X_2(s)) + \alpha F_1(s)/A
\]
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Modulus (water)</td>
<td>3.12x10^{15}</td>
<td>psi</td>
</tr>
<tr>
<td>Viscosity (water)</td>
<td>1.45x10^{-7}</td>
<td>psi s</td>
</tr>
<tr>
<td>Density (water)</td>
<td>0.036</td>
<td>lb/in^3</td>
</tr>
<tr>
<td>Length (tubing)</td>
<td>48</td>
<td>in</td>
</tr>
<tr>
<td>Diameter (piston)</td>
<td>0.375</td>
<td>in</td>
</tr>
<tr>
<td>Diameter (tube)</td>
<td>0.19</td>
<td>in</td>
</tr>
<tr>
<td>Mass (piston)</td>
<td>0.02</td>
<td>lb</td>
</tr>
<tr>
<td>Roughness</td>
<td>0.00016</td>
<td>unitless</td>
</tr>
</tbody>
</table>

Table 4.1: System Values for Simulation

We can now solve for the transfer functions from our input force to the two outputs.

\[
\frac{X_1(s)}{F_1(s)} = \frac{s^2 + s(\alpha C_{ef} + C_{ef}) + K_{ef} + \alpha K_{ef}}{A s^2 [s^2 + 2sC_{ef} + 2K_{ef}]} \\
\frac{X_2(s)}{F} = \frac{1}{A} \frac{s^2 \alpha + Cs(\alpha + 1) + K(\alpha + 1)}{s^2 [s^2 + 2sC_{ef} + 2K_{ef}]} 
\]

4.6 Original Parameter Estimates

As stated above, \( K = \frac{BA^2}{V} \) and \( C = \frac{LLpA}{2D} \). Both these values must then be doubled to account for the indeterminate nature of the system. Using the values in table 4.1, this comes to an effective spring constant of 805 lbs/in, and a damping of 0.26 lb s^2/in^2. The total mass of the water - found by multiplying the tubing length by its area and doubling for both lines is also calculated.

An estimate of the coulomb friction was calculated based on o ring parameters from the Parker O-Ring Handbook[18]. The total coulomb friction is the sum of the friction components from both the piston and rod seals. These have approximate friction forces (respectively) of: \( F_{piston} = f_c * L_p + f_h A_p \) and \( F_{rod} = f_c + f_h * A_r \), where \( f_c \) is a frictional component from the handbook tables, \( f_h \) is the pressure, and \( L_p, A_p, \) and \( A_r \) are other geometric values again based on the geometry of the o-ring’s size. For the o-ring used in this application, \( f_c=1, f_h=10, L_p=\pi * A_{piston} in^2, A_p=0.374 \ in^2, \) and \( A_r=0.186 \ in^2 \). This results in a total coulomb friction of about 3.3 lbs.
4.7 Preliminary Model Results

Using Simulink software, a full nonlinear model was created to simulate the results of the white-box model with the original calculated parameters. The model used a locking friction model that used each piston’s velocity as a measure of whether it should be subject to static or kinematic friction. If the absolute value of the velocity was below a small threshold and no force was applied to keep it moving, it would ”lock” which would then only be able to be ”unlocked” by a force above the static friction threshold. In the unlocked state, a basic coulomb friction model was used.

In order to simulate the system response, some open loop force commands were sent into the input piston and the output of both pistons was observed. Force signals at low speeds results in close following by both pistons in tandem, as seen in figure 4.2. Zooming in on the dynamics shows the errors a little more - while a square wave input shows the curves generally following, the pistons can be seen to be out of phase with an error hovering around 0.02” with the whole input moving 2” peak to peak.

![Figure 4.2: Open loop square input.](image)

A low speed sinusoidal input to a closed loop simulation also show the effects of the stiction from the piston seals. The feedback loop was closed with unity feedback in order to look at the preliminary tracking ability of the system. As shown in figure 4.3, both piston
profiles were flattened out by the static friction in the system.

![Closed Loop Sin Input](image)

**Figure 4.3: Closed Loop Sinusoidal Simulation Results**

As a test to validate how the model responds to changes in the architecture, if the bulk modulus is reduced, the two pistons become desynchronized, as is intuitively expected. This is shown in figure 4.4 - in the leftmost figure, the pistons are almost completely in sync, whereas in the rightmost figure, the discrepancy between the input and output becomes more pronounced. If the coupling is desired to be maintained for the final experimental apparatus, this shows the importance of keeping air out of the fluid system in order to keep the bulk modulus as high as possible.

![Comparison of lowering bulk modulus in simulation](image)

**Figure 4.4: Comparison of lowering bulk modulus in simulation**
Chapter 5

Experimental Setup and First Trials

5.1 Testing Procedure and Devices

A test setup for the one degree of freedom actuator was created to characterize the response of the system to force input on one of the pistons. In order to fully capture the dynamics of the system, an actuator was attached to one of the pistons, designated the input piston, and displacement sensors were used to measure the position of both the input and output pistons. The motor used was a voice coil actuator made by BEI Motion Systems Company, model LA40-92-0012. The actuator takes in a current command which was given from a Motion Science Inc servo amplifier. The current input command is directly proportional to the output force from the motor, with a constant gain of 58 N/A. The actuator has a full stroke length of one inch, with no built-in sensing.

Both displacement sensors are Optodyne laser displacement sensors. Each sensor has a resolution of 25 microinches with a range of 2,000 in and are capable of measuring speeds up to 160 in/s, all of which are more than sufficient for the system under examination. Retroreflectors were mounted to the input and output pistons in order to have a clean reflective surface for the lasers. The laser sensors transmit position data in encoder counts, so are used for relative positioning only. Because the lasers do not have the capacity for
absolute measurement and the motor has a short stroke length with no method of identifying where the piston position is within that stroke, a limit switch was mounted to the top of the motor to detect when the motor has neared one edge of its movement, and software was used to set another limit near the opposite side of the full range of movement. A diagram of this setup and a photo of the actual system are seen in figure 5.1.

Data acquisition was performed using a National Instruments myRIO device, running labview software. The measurement and control for all experiments was run on the myRIO’s processor at a sampling rate of 1 kHz, while the myRIO’s built-in field programmable gate array managed the faster sampling needed for analog to digital conversion and encoder counting.

(a) Diagram of test setup.  
(b) Photo of test setup.

Figure 5.1: Diagram and Photo of Test Setup for Dynamic Response Testing

The pistons were affixed to a solid mounting plate in order to create a consistent set of coordinates for measurement and to make sure that they did not move during operation. Since one piston is rigidly attached to the motor, its position is labeled in all diagrams as ”Motor”, while the opposite position receiving forces through the fluid transmission lines is labeled as ”Output”. For closed loop experiments, ”Reference” refers to the input reference to the closed loop system, while ”Motor Command” refers to the voltage command sent from the myRIO device through the amplifier to the motor. This signal is proportional to the force output from the motor onto the input piston.
5.2 Preliminary Linear Actuator Results

With a single degree of freedom actuator constructed, tests were carried out to evaluate its response. While the actuator was able to transmit movement from the input to the output, there were a number of issues that brought into question the long-term viability of the prototype.

One immediate concern was that the high static friction in the input and output actuators led to a "deadband" zone every time the actuator motion direction was reversed, as seen in figure 5.2. While the input piston is able to move almost 0.15 in, the output actuator did not move at all. While larger step sizes would result in more movement, a deadband of this size was not only detrimental to the system response, but also made experiments difficult due to the actuator’s one inch stroke limit. While some degree of error will always be expected, the sort of decoupling exhibited by the system is much larger than simulation or preliminary estimates would suggest. The wide gap between input and output pistons is not optimal and was a main cause for eventual actuator revision.

Figure 5.2: Example of Deadband Response from Custom-Made Piston

Especially due to the high static friction in the system, the minimization of
compressibility in the system was also very important. With high friction and high compressibility, the pistons can become more and more decoupled. This manifests itself clearly in the formation of bubbles within the test system when operated. The fluid line that was in tension would have large bubbles form, being pulled out of the water-glycol solution because the pistons had become decoupled and a vacuum was created on that fluid line. After longer use, the air bubbles would form pockets that would further reduce compressibility and lead to worse performance of the system.

Luckily, it was determined that the formation of air bubbles was lessened when there was no glycol in the system. While this further increased the static friction, it allowed for operation of the system for longer periods of time without the need to flush the air out of the system continually to get reasonable levels of movement transferred from one end of the system to the other.

An additional problem was that while great lengths had been taken in order to eliminate leakage, there was still evidence of slight leakage after repeated operation. While the system was sealed under static conditions, small leaks occurred during operation due to the dynamic pressures within the system. These resulted in the addition of air to the lines of the system, which increased the compressibility of the fluid lines as well as was not desirable for operation in a clean medical setting.

As the custom-made actuator proved to be sub-optimal for the reasons listed above, it was in the end discarded in favor of a different actuator system using syringes as the main driving pistons. While this proved bulkier in space, it was able to reduce friction and provide a system response that relied less on fabrication specifics and more on the nature of the fluid connections.

Using the same PID control structure that is be elaborated more below, some preliminary step tracking was attempted as seen in figure 5.3. While the system was able to show response in the shape of the reference, results were overall poor with steady state error on the order of 0.1 in and input-to-output decoupling at times as large at 0.2 in.
Figure 5.3: Output step tracking

While the performance could be improved with more time in tuning gains, the system’s flaws, especially in the large decoupling between input and output, were deemed too major to try to spend unnecessary time making a better controller for a sub-par system.

5.3 Friction Characteristics

While the system was to be replaced, it was still important to characterize some of its response characteristics, especially the frictional response of the system to characterize how much static friction was present in the system. It was noted that this friction was still very high even when using high-percentage glycol solutions, so obtaining an understanding of how much an effect that would have on the system was very important.

Tests were taken of the input actuator’s response to a variety of open loop step signals. Since the motor output force is proportional to input command, these signals can be used to generate a plot of the velocity versus the force input, which can then be used to derive the static friction of the system.

In the original tests, there was an unexpected response found in the data. There was
Figure 5.4: Output data showing frictional effects on the prototype linear actuator

an inflection point in the same place in a number of the tests, as shown in figure 5.4. This is indicative of there being a physical difference between the piston/bore before and after that location. While the cylinder had been machined with great care and attention, this indicates that the smoothness was uneven and contained at least two separate regions with different frictional characteristics. In order to not deal with this added nonlinearity, the piston location was biased to only operate beyond that separating point and more friction responses were evaluated.

The results of the friction tests with the new location are located in figure 5.5. This shows the average velocity of the input piston in response to a range of input commands. For each step input, a least squares fit of a line to the response was used to approximate the velocity of the response for each input.

This shows that the actuator’s approximate static friction is related to the motor output at 1 V. In absolute terms, this value is currently unidentified because the motor amplifier had to be replaced during testing and the voltage-to-amps conversion factor has not yet been re-identified.
5.4 Testing in MR Environment

In order to see if the actuator would work well inside an MR environment, the custom-made actuator prototype was taken to the research MRI scanner at UCLA to evaluate its preliminary performance. Since the motor that was used to drive the system was not MR compatible, and the system was not ready for tests with very long fluid lines, the system was used to allow access from the bottom of the MR bed to the inside of the imaging bore. This is emulates a potential end use of the system which would allow a physician to reach from outside the bore to the inside using the actuator under manual control. As seen in figure 5.6, the operator can stand outside the imaging bore with the actuator while the output side moves inside the MRI bore.

Figure 5.7 shows the system with the bed retracted. Sandbags were used in order to keep the pistons in place during insertion. Short, four-toot tubing was long enough to reach from the center of the imaging bore to the end of the bed which sticks out of the bore allowing for manual operation. The target for the needle, a grapefruit for the first test, was surrounded with water bottles in order to create some context for the image.
The tests verified two basic roadmarks: the compatibility of the system with MR imaging, and the ability of the system to control a needle with manual operation.

The first phantom accessed was a grapefruit. The piston was easily able to transmit enough force to puncture the fruit’s skin, although had to be guided slightly to account for bending of the needle. From a starting point with the needle inside the fruit’s skin, the bed was moved inside the bore with the slave actuator and fruit being imaged while the master actuator was at the base of the table where it could be accessed to drive the system. The
master actuator was moved as per verbal commands given from the command room where the images were being processed. These images also were projected inside the imaging room, so a physician standing at the base of the bed would be able to perform any desired movements under image guidance as desired. There were few notable structures to target within the grapefruit or the gel phantom used later, so no tests were undertaken on the accuracy of the system.

The system, in addition to being MR safe, did not create any apparent image artifacts, which was a very important fulfillment of system requirements. It was actually somewhat difficult to find the proper plane to image the needle coming out of the prototype because there was no easy reference to find from the piston.

The use of this actuator with MRI guidance is a significant proof of concept for the ability of this technology to be used for interventions in the future. Using real-time imaging feedback, a physician would be able to use the system to position a needle or other device within MRI without having to be inside the bore. While the accuracy of the prototype under manual guidance was not measured accurately during this trial, it can be assumed that the use of continuous imaging feedback would result in not only improved accuracy of movement when compared to repeated small movements outside the bore with periodic
imaging feedback, but would also result in time saving.

5.5 Actuator Revision

While the custom-made piston was able to give some heartening preliminary responses, it was desired to create a better system that was not as tied to the particularities of the actuator’s design and fabrication to characterize the response due to the fluid connections, not the piston itself.

This new design was used to better characterize the fluid transmission system for different tubing lengths. The new test setup used a similar syringe setup as the orienting mechanism used early in the design process, which had been proved able to transmit displacement fairly losslessly.

The system was composed of two linear guides, each made from a simple-to-construct two rail structure. This linear bearing architecture is prone to jamming if not carefully designed and fabricated, so care was taken at all steps in the process to make sure that the system would not fall into problems down the line. The design can be seen in figure 5.9. As with the original test system, the input actuator was attached rigidly to the motor, and the output was allowed to move freely with both positions being tracked by laser displacement sensors.

Each actuator, like those used in the orienting mechanism, is composed of two syringes in order to create a double-ended piston architecture. The syringes are held in place with a close fitting hole that the bore of the syringe is pressed into. While the hole was sized so that the syringe must be pressed into place, in order to make sure that the syringes would not be pushed out of place during operation of the test system, each syringe was also pushed into place with an L bracket facing the front of the syringe and pushing up against its face so that it could not slide axially out of its resting position.

For ease of manufacture, aluminum L extrusions were used to constrain the guide rails
and to form the base of the system. These were held down by the optical table which acted as the reference plane.

The motor side and output side of the test system were set up with slightly different carriage setups. Because a rigid connection was needed to translate the motion of the motor to the input syringe actuator pair, it was decided to have the rods themselves be moving, being guided by a single pair of linear bearings. This allowed for simple translation of the motor’s movement to the syringes without extra linear guidance necessary. The output system uses a carriage-on-rails system. As mentioned above, the two rail guiding system is prone to jamming - due to this factor, the output carriage was made to be slightly longer than the space between the rails, which is a common method to reduce or prevent jamming[20]. While a carriage multiple times the distance would be preferable in order to completely avoid jamming, this was unfeasible due to space considerations. Thus, the carriage was simply made to have a greater length of bearing than the characteristic dimension of the sliding surface. Due to the importance of smooth bearing motion in reducing friction, brass linear bearings were used on all sliding surfaces.

While most of the test setup was made from aluminum for its cheap and abundant
nature, the carriages were made from polypropylene which is lighter and so would be less likely to create uneven loading on the guide rails.

Like the previous testing setup, retroreflectors were mounted to the actuators in order to collect position information using two lasers.
Chapter 6

Experimental Results

With the actuators replaced and a new system created, a new round of tests were undertaken. The first round of tests used relatively short tubing - four feet for each connection from one piston to the other. After those tests, a second round of tests was carried out for 17.5-foot tubing to determine the effects of long fluid lines.

Figure 6.1: Final test system
6.1 Effect of Motor Dynamics on Model

The voice coil actuator used in running the experiment has its own set of dynamics that must be accounted for in creating a full model, and decoupling the response of the testing apparatus from the hydrostatic system itself. In order to make the white-box model more able to fit the form of the data taken from the system, this would need to be added to the dynamic model.

Returning to the two-mass-spring problem from the system model section of this thesis, the damping of the motor can be thought of as adding damping to the input mass. This representation can be seen in figure 6.2.

For the linearized model, which will ignore the nonlinear frictional qualities of the motor, the addition of this damping only slightly changes the system’s nature. The fundamental structure is not changed, and only one of the entries in the model is slightly revised, as seen below:

Figure 6.2: Spring-Mass model is repeated, with additional damping from the motor
\[
\begin{bmatrix}
\dot{x}_1 \\
\ddot{x}_1 \\
\dot{x}_2 \\
\ddot{x}_2
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 & 0 & 0 \\
\frac{-2k(\alpha+1)}{A} & \frac{-2C(\alpha+1) - C_{\text{motor}}}{A} & \frac{2k(\alpha+1)}{A} & \frac{2C(\alpha+1)}{A} \\
0 & 0 & 0 & 1 \\
\frac{2k(\alpha+1)}{A} & \frac{2C(\alpha+1)}{A} & \frac{-2k(\alpha+1)}{A} & \frac{-2C(\alpha+1)}{A}
\end{bmatrix} 
\begin{bmatrix}
x_1 \\
x_2 \\
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
\frac{1}{A} \\
\frac{\alpha}{A}
\end{bmatrix} \cdot F_{\text{input}} \quad (6.1)
\]

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} 
\begin{bmatrix}
x_1 \\
x_2 \\
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} \quad (6.2)
\]

6.2 Time Domain Response and Control

Using basic proportional, integral, derivative (PID) control, a number of tests were undertaken for both the short and long tube length systems. Gains were tuned for each test independently in order to give a best-case scenario for each situation.

The PID controller was implemented as a discrete controller. The form of the controller was:

\[
u(k) = K_p \cdot e(k) + K_i T \cdot \sum_{j=0}^{k} e(j) + K_d \cdot \frac{1}{2T} \cdot [e(k) - e(k-1)] \quad (6.3)
\]

In the z domain, this controller can be formulated as:

\[
C(z) = K_p + K_i T \frac{z + 1}{z - 1} + K_d \frac{z - 1}{T z^n} \quad (6.4)
\]

For a number of the tests, PI control was used in place of PID as it was sufficient to control the system within desired bounds, and even small D values created fast oscillations at the motor with little to no difference in performance. For all tests unless stated otherwise, the system was run using continual measurements of the output piston to track a
reference at the output.

The most important time-domain specification was to keep steady state error less than 0.02 in for any sort of step input. This allows for error tracking to the level of the imaging resolution of an MRI scanner. MRI is able to have resolution on the sub-millimeter level, and an error of 0.02 in corresponds to about half a millimeter, which, based on conversations with clinicians, is sufficient for most positioning tasks in MRI. At the same time, the system is desired not to exhibit significant overshoot or oscillation in order to reduce unnecessary tissue damage for needle insertion tasks.

6.2.1 Short Tubing

The first set of tests were run on the system with short, four-foot tubing. The first test was to test the ability of the system to follow a series of steps. Because of nonlinearities in the system and the effects of the position of the input piston on the output, a single step response is insufficient to characterize the system’s response. As can be seen in figure 6.3, the rise time, steady state error, and overshoot are different for each step input. This test was carried out with $K_p=50$, $K_d=1$, and $K_i=25$ and the motor output saturated at 3 volts.
The average absolute steady state error was 0.004 in, with a maximum of 0.0075 in. There were two steps with overshoot which had values 0.0066 in and 0.0088 in above the step level. Since these values are less than the resolution of an MRI scanner, they were deemed within the bounds of acceptable values. For all steps, settling time was less than 0.25 s, which is more than fast enough for the relatively slow motion required for most clinical purposes.

In addition to the step response, using PI control, the ability of the system to follow a trapezoidal displacement profile was tested. While the step response of the system is useful for characterizing the dynamics of the system, the actual use of the system will be more similar to a trapezoidal displacement profile - a constant velocity insertion phase followed by a hold at the final insertion depth, and then the withdrawal of the needle. The results of this test can be seen in figure 6.4. This particular experiment was performed with PI control, with a gain Kp of 20 and Ki of 40.

![Figure 6.4: Trapezoidal Response for Short Tubing](image)

(a) Short Tubing Trapezoid Response    (b) Top of Trapezoid Response

In this test, there is a delay of 0.35 seconds before the output begins to move. Once the output has risen to meet the inclined reference, the average steady-state error between the output and the reference is 0.0031 in. After less than 0.5 s, the output settles to a steady state error of 0.014 in. However, near the end of the flat portion of the reference, the error falls to 0.001 in.

At first, this was believed to be due to the decoupling of the input and output pistons,
but with a response delayed by multiple seconds, this seemed unlikely. When looking at the motor command, the reason for this behavior becomes apparent, as seen in figure 6.5. The red box highlights the motor command given during the time that the reference is flat at the top of the trapezoidal profile - the controller is having problems with integrator windup, causing the response to have a long delayed response when the reference changes its nature.

![Figure 6.5: Short Tubing Trapezoidal Response - Windup](image)

6.2.2 Long Tubing

The same tests that were undertaken for the short tube length system were repeated for tubes with length of 17.5 ft. Again, the first test was the ability of the system to follow a series of step inputs. Unfortunately, for this test, the laser displacement sensor measuring the motor position ceased to record complete data. An internal failure in the laser prevented more data from being taken of the input piston displacement, so only the output measurement is shown for this data set and for figure 6.9.

As seen in figure 6.6, the rise time, steady state error, and overshoot are different for each step input. This test was carried out with Kp=20, Kd=0, and Ki=45. The output was saturated at 5 volts. The average absolute steady state error was 0.005 in, with a maximum
of 0.0104 in. Most of the steps contained overshoot, with the notable first step overshoot greater than 0.1 in. This is undesirable for something like needle insertion in a medical setting, but a step input would not be the preferred contour in that context. While a number of different gains were used to attempt to reduce this overshoot, none were able to fully eliminate it. The large static friction in the system accounted for a large amount of this error: in order to be able to escape the friction, the gains would have to be tuned too high to prevent overshoot from occurring.

In addition to the step response, the ability of the system to follow a trapezoidal displacement profile was tested using PI control. The results of this test can be seen in figure 6.7. This particular experiment was performed with a gain Kp of 45 and Ki of 60.
In this test, there is a delay of 0.5 seconds before the output begins to move. Once the output has risen to meet the inclined reference, the average steady-state error between the output and the reference during ramp tracking is 0.0022 in. After less than 0.15 s, the output settles to a steady state error of 0.0065 in. At the top of the trapezoid, the steady state error is about 0.011. The output settles to within 1% of this final value for this part of the curve within 0.35 seconds after the ramp command is changed to a flat command.

In figure 6.8, the same reference signal was used to track the motor position instead of the output piston. This simulates the use case of not relying on imaging feedback for the positioning of the system and simply controlling the input piston displacement and relying
on the system’s inherent coupling to make sure that the output piston mimics the input within the error bounds of the final application.

As in the experiments where the output was tracked, the motor experienced some overshoot - in this case with a steady state error of about 0.51 in. However, the output piston, which was not being tracked, had a steady state error of only 0.001 in compared to the reference command. In this particular case, the motor overshoot accounted for almost the exact amount of decoupling between the input and output piston displacements. While this particular case gives an excellent response in terms of output error, it cannot be assumed for any general case that the error in motor tracking will compensate for the decoupling of the input and output pistons. It could easily increase the error if the reference was in the opposite direction. With a steady state difference between the two pistons of 0.05 in, a perfect tracking by the input would result in enough difference in the output to be concerned for the accuracy of the system. For gross movement, a 0.05 in error should be accommodated easily for a wide range of positioning tasks, but for precise movement such as targeting a location for biopsy, output tracking will be needed to supplement input positioning at tubing of this length. Conditionally, this proves the feasibility of tracking motion only measuring the displacement of the input piston. However, for fine adjustment, additional measurement will need to be taken on the output as well.

This sort of control architecture, tracking only the input without measuring the output would make for simple implementation of the hydraulic system because traditional actuators and sensors could be used to measure and control the input piston for gross movement of the system, and fine adjustments could be performed using image guidance. Since even real-time MRI scanning only has a sampling rate at most on the order of single-digit Hz, the use of sensing outside the MR bore would make for more reliable control.

The original trapezoidal test was repeated tracking the output position for a slightly altered controller, with results seen in figure 6.9. The gains were increased to improve the transient performance, both Kp and Ki were increased to 70, but more importantly, a reset
was added to the integrator in accordance with the change in input signal. On changing from the ramp input to flat, the integrator in the controller was reset, which improved the response at the top of the trapezoid, resulting in a final error less than 0.001 in. Also, due to the changed gains, the ramp average error improved to only 0.003 in.

### 6.3 Static Friction

The second test setup was also tested for static friction in order to better characterize the system response. Like the first setup, a series of open loop step commands were given to the motor and the positions were recorded.

The point where the velocity become nonzero is around 1.1 V, again with a corresponding force unknown due to new equipment with the actuator. This is an interesting point because it would suggest that the static friction of this system is slightly larger than that of the custom-made piston system, which does not feel like the case for manual positioning. It is presumed that when coupled with the mass of the fluid and the fluid damping, the additional effects must increase the effective static friction of the system.
6.4 Frequency Domain Response

A number of trials were conducted evaluating the frequency response of the system. A dynamic signal analyzer was used to pass a varying-frequency sine wave to the system to evaluate the response. The frequency response of the system was analyzed both at the input to the system, the displacement of the piston attached to the motor, as well as at the output where the final piston was connected.

Because of the continued large static friction in the data, fitting a transfer function model to the frequency response of the system was not useful - the fundamental system is too ruled by frictional nonlinearity to create a linear model that makes sense. While it was attempted, no models could match the actual data from the system. However, the frequency response of the system is still an important piece of the characterizing the system’s response and can also help characterize the behavior of the system and the robustness of the controllers.
The first data set and its response can be seen in figure 6.11. This shows the frequency response of the system from motor input in volts to output piston displacement in inches. The same setup was used for all frequency response data. For the long tubing, the result is shown in 6.12. The two show very similar shapes overall, with a few extra spikes and valleys in both the magnitude and phase plots of the long-tubing system.
Using a combination of the frequency response data and the controller used for these experiments, some stability bounds can be calculated. The discrete PI controller can be represented by a continuous time controller as:

\[ C(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \]  \hspace{1cm} (6.5)

Using this approximation, the frequency response of the loop gain can be studied in order to better understand the performance and robustness of the controllers used above.

As seen in figure 6.13, for a unity feedback system, the gain margin is 52.8 dB, and the phase margin is at least 79 degrees. Note that because the frequency data was not taken below 1 Hz, the exact phase margin cannot be determined because the gain crossover frequency is below 1 Hz. However, it is not expected that the phase would decrease for smaller input frequencies, so a lower bound on the phase margin is that taken at 1 Hz. These plots can also be used to set stability bounds on the gains of the system: for a proportional gain controller, this sets a stability limit that \( K_p \) must be less than 435, where the gain margin becomes zero.

Figure 6.13: Long Tubing Unity Feedback Stability margins
Figure 6.14 shows the stability margins for a PI controller with gains $K_p=70$ and $K_i=70$. This system, which provided good control for a trapezoidal profile, has a phase margin of 67 degrees and gain margin of 17 dB, which represents a fairly stable control architecture.

Looking at the Nyquist plot, seen in figure 6.15 it is clear that the controller does not come overly close to encircling the -1 point.
These plots can be continued to be used for evaluating stability in further control development in working on future controllers if different performance is desired.
Chapter 7

Conclusion

7.1 Project Summary

This thesis has presented the design, modeling, and control of a low-pressure, water-based hydrostatic actuation system for MRI. A two degree of freedom orienting mechanism was developed as well as work on the development of a single degree of freedom linear actuator. A white box model was proposed to characterize the system model, which could not provide full explanation of the system dynamics, but lays groundwork for further work in creating a physics-motivated model that can be applied to a variety of line lengths, piston diameters, and other similar system characteristics.

With consistent output measurement, the system is able to repeatably achieve steady state errors on the order of .02 in and less for trapezoidal and step input displacement profiles. Moreover, with intelligently performed input motion, the input and output pistons can be made to have a decoupling on the order of only 0.05 in, which proves the ability of the system to be used without output measurement for gross movement and even fairly precise positioning even with long tubing. This provides a baseline for the fundamental work to develop further hydrostatic actuators within the MRI environment and shows the feasibility of using such actuators for more complicated robotic systems.
7.2 Future Work

While a number of milestones have been accomplished, there is an entire field of future work that is possible based on the actuator architecture proposed in this thesis. There are also a few specific improvements and adjustments that can be undertaken in order to more fully characterize the current single degree of freedom actuator.

As shown in the section on system identification, a suitable full model was unable to be developed for this system. The problem should be approached from both ends: the physics motivated model should be re-evaluated to see if any improvements in the fundamental assumptions of the model can be changed. At the same time, improvements to the physical system in terms of reducing friction can also be attempted to make the system more well-behaved. Even with an adjusted system, the static friction dominated the dynamics, so improving the system’s frictional characteristics will be important if a better system model is to be developed.

With a better system model, better control strategies can be attempted. Of greatest use to this application would be the use of the model to propagate an estimate of the output position between imaging samples. While these experiments were taken with full information of the output displacement at a one kHz sampling rate, tests within an MR environment will have to rely on image-based measurements of the displacement that will at most have sampling rates on the order of single-digit Hz. Using a fuller model of the system with periodic measurements of the output and consistent measurements of the input piston would allow the actuator to function better for future development.

However, the close following shown in the trapezoidal displacement tests, even with longer tubing, shows that the ability of the output to track the input motion for slow velocities may allow the system to be run presuming the input and output pistons are rigidly coupled. This eliminates the need for continual measurement of the output piston, as the input and output pistons remain closely in sync at slow velocities. For this strategy, the output only needs to be measured for fine adjustments. This control strategy should be
run experimentally for a variety of displacement profiles in order to see how close the pistons remain in sync for different velocities.

The use of a prefabricated piston and bore system may be more useful for future developments of the hydrostatic actuation system. While syringes were useful in creating prototypes and in system characterization, a more permanent solution will rely on professionally crafted reciprocating seals. Any further development of the system will need to include an improved piston system.
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