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A SYSTEMATIC LIFTING OF EXCHANGE-DEGENERACY THAT CLARIFIES THE
RELATIONSHIP BETWEEN POMERON, REGGEONS AND SU\textsubscript{2}-SYMMETRY VIOLATION\textsuperscript{*}

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ABSTRACT

The cylinder correction to the planar S matrix splits \( I = 0 \) from \( I = 1 \), lifts exchange degeneracy of \( I = 0 \) states and induces deviations from ideal mixing. All three effects are approximately described by one parameter which gives the shift of the \( \omega \) intercept below the \( \rho \), the couplings of \( f \), \( f' \), \( \omega \), \( \phi \), the shift of the \( f \) above the \( \rho \), and its tendency to become an SU\textsubscript{2} singlet. The splitting apparently decreases as \( t \) increases so that the \( J = 1^-, 2^+ \) particles are nearly ideal (planar) nonets. A smooth connection between \( t = 0 \) and these physical particles suggests that the phenomenological pomeron is the upward shifted \( f \) and thereby explains the small pomeron slope.

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Veneziano\textsuperscript{1} has proposed a new topological expansion which promises to facilitate the understanding of hadrons. The starting point is a "planar" S matrix possessing nonexotic Regge poles but no Regge cuts and no pomeron. There is exchange-degeneracy between the poles of even and odd charge conjugation (\( C = \pm \)). The next component of this systematic expansion has the properties of a cylinder and communicates (along its axis) only with states of zero additive quantum numbers; it contains \( C = \mp \) but does not maintain exchange degeneracy. The highest lying new Regge trajectory has \( C = + \) and at \( t = 0 \) lies above the highest planar trajectories. It has been supposed that this trajectory is to be identified as a "bare pomeron" whose properties will be modified by subsequent components of the expansion but which should bear a resemblance to the phenomenological pomeron.

Our investigation\textsuperscript{2} (details will be reported elsewhere) shows that the cylinder shifts planar poles upward in \( J \) for \( C = + \) and downward for \( C = - \) but probably produces no new poles. Identifying our leading trajectory as the pomeron, we find a concrete realization of the "\( f \)-dominated pomeron\textsuperscript{3,4}" with SU\textsubscript{2} breaking as proposed by Carlitz, Green, and Zee.\textsuperscript{3} We go further than these authors by giving, without additional parameters, the shifts of all four leading trajectories (\( f \), \( \omega \), \( f' \), \( \phi' \)) and the modifications of their couplings, by clarifying the status of the four planar trajectories and by pointing out the connection with the violation of the Ishida-Okubo-Zweig\textsuperscript{5} rule. Our one parameter (at each value of \( t \)) can in principle be calculated from a knowledge of the planar S matrix but we have not yet done so.

Although new poles may arise, together with Regge branch points, in higher-order components of the topological expansion, at the cylinder level the pomeron and \( f \) correspond to the same trajectory, in contrast with standard dual resonance models where a new family of reggeon singularities appears at the level of the nonplanar loop.

Lest the reader lose interest at this point, on the grounds that two high-lying but separate positive-charge-conjugation trajectories are unequivocally required by experiment, we call attention to the work of Dash and collaborators.\textsuperscript{6} They showed how a single such trajectory with intercept near 0.85 could satisfy experimental requirements for laboratory energies \( \leq 30 \text{ GeV} \). Since terms beyond the cylinder become more important at higher energies, there is no cause
for despair. On the contrary, reducing the number of trajectories needed to represent moderate-energy data should be welcome.

Our analysis has been carried out in the J plane for the t channel, in a Hilbert space of those (J,t) planar poles communicating with the cylinder. Each pole belongs to a family characterized by a single quark index and definite C. The relevant Regge-behaved planar duality diagram is displayed in Fig. 1a; the dashed line indicating that we are considering the s discontinuity. The cylinder correction corresponds to the sum of lb, lc, etc., and may be expressed as the solution of a linear integral equation whose kernel can in principle be calculated from the residues of the planar poles. We formally write the integral equation for the sum of the planar term and its cylinder corrections, with the external legs amputated, as

\[ A = P + FC_1P + FC_1FC_1P + \ldots = P(1 - C_1P)^{-1}. \] (1)

\( P \) is the planar reggeon propagator, \( C_1 \) is the cylinder operator with structure shown in the box of Fig. 1c. The kernel has the same magnitude but opposite sign for \( C = + \) or \( - \), and at \( t = 0 \) the even \( C_1 \) kernel is positive definite.

Assuming the kernel to be Fredholm it is easy to show that the I = 0 planar poles of (1) are cancelled in the complete sum while the I \( \neq 0 \) poles are left untouched by the cylinder correction. In a simple approximate calculation we find that new I = 0 poles appear at shifted locations but in one-to-one correspondence with the original planar poles.

It is usually supposed that for \( t = 0 \) and \( 0 \leq j \leq 1 \) the planar spectrum contains three poles of each C. For simplicity we confine our attention to these, each of which is associated with a different quark index. With SU(2) the four planar poles associated with nonstrange quarks are degenerate; we label their common trajectory by \( c_0(t) \). The two poles associated with the strange quark are degenerate; these we refer to as \( c_3(t) \). If SU(3) symmetry were valid, then \( c_3 = c_0 \), but it appears that \( c_3(t) \) in fact lies substantially lower. As is standard, we break the symmetry only by this mass shift.

We split the Hilbert space according to total isospin, one state carrying \( I = 1 \) and two carrying \( I = 0 \). Only the latter are affected by the cylinder, so we use the observed \( I = 1 \) \( \rho \) and \( A_2 \) trajectories to fix \( c_0 \) and the associated planar residues. The choice of \( c_3 \) is less direct but in practice relatively unambiguous. Our concern here is with the position and residues of the four \( I = 0 \) trajectories \( f, f', \omega, \phi \), which emerge from the integral equation, given \( c_0, c_3, \) and a kernel built from SU(3)-symmetric planar residues. The simplified integral (Eq. 1), specified by the choice

\[ P = \begin{pmatrix} 1/(J - c_0) & 0 & 0 \\ 0 & 1/(J - c_0) & 0 \\ 0 & 0 & 1/(J - c_3) \end{pmatrix}, \]

\( C_1 = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \)

has the solution

\[ A = \sum \frac{1}{J - \alpha_x} C^x, \] (3)

where \( x \) is the pole label \( (f, \omega, f', \phi) \); \( C^x \) is the coupling matrix \( \alpha_{ij} = g_{1i}^x g_{1j}^x \); and \( g_{1i}^x \) the coupling of the \( x \)-th pole to quark type \( i \).

The two leading output poles, presumed to be the \( f \) and \( \omega \) are located at

\[ a_{F, \omega} = \frac{1}{2} \left\{ c_0 + c_3 \pm 3k + \left[ (c_0 - c_3 \pm k)^2 + 8k^2 \right]^{1/2} \right\} \] (4)
where the plus (minus) goes with the even (odd) \( f(m) \). The two remaining poles \( f' \) and \( \phi' \) are also given by Formula (4) but with a negative sign before the square root. In the limit of exact \( SU_3 \) the \( f \) and \( \phi \) become singlets (see couplings below) and are shifted while the \( \omega \) and \( f' \) are unshifted, being members of octets that do not communicate with the cylinder. Even without \( SU_3 \) symmetry the upward shift of the \( \omega \) is larger than the downward shift of the \( \omega \), the latter being bounded below by \( \frac{1}{2}(\alpha_0 + 2\alpha_3) \) while the former is unbounded above.

For \( t \gtrsim 0.5 \text{ GeV}^2 \) the small \( \rho - \omega \) and \( f - A_2 \) mass differences allow use of the small \( k \) approximation to Formula (4):

\[
\alpha_{f,\omega} \approx \alpha_0 \mp 2k, \quad \text{and} \quad \alpha_{f',\phi}' = \alpha_3 \pm k.
\]

The trajectories are shown in Fig. 2. The coupling matrix for the \( f \) is expressed in terms of a mixing angle \( \theta^+ \) (the rotation toward \( 0 \) from the ideal planar mixing angle \( \cot^{-1} \sqrt{2} \)):

\[
\tan 2 \theta^+ = \frac{1}{2} \frac{\sqrt{6} k}{\alpha_0 - \alpha_3} \quad \text{and} \quad \varepsilon_{1,2}^f = \frac{\cos \theta^+}{\sqrt{2}}, \quad \varepsilon_3^f = \sin \theta^+.
\]

(5)

Couplings for the \( \omega \) are given by the interchange \( \theta^+ \rightarrow \theta^- \), while those for the \( f' \), \( \phi' \) are given by \( \theta^+ \rightarrow \theta^- + \frac{\pi}{2} \). The connection with the COZ prescription is exhibited by the equivalent alternative formula:

\[
\tan \theta^+ = \frac{\alpha_2 - \alpha_0}{\sqrt{2}(\alpha_1 - \alpha_2)} = \frac{\sqrt{2}(\alpha_1 - \alpha_3)}{\alpha_2 - \alpha_0},
\]

(6)

with a similar equation for \( \theta^- \) if \( f \rightarrow \omega \). The \( SU_3 \)-symmetric limit corresponds to \( k \gg \alpha_0 - \alpha_3 \) (\( \theta^+ = 35^\circ \), \( \theta^- = 55^\circ \)) while \( k \ll \alpha_0 - \alpha_3 \) (\( \theta^+ = \theta^- = 0 \)) leaves the original "ideal" mixing of singlet and octet. Physical couplings at \( t = 0 \) are roughly midway between these limits. For an illustrative fit it is natural to fix as many parameters as possible at the planar level. The choice \( \alpha_0(0) = \alpha_0(0) = 0.57 \) is straightforward, and we choose \( \alpha_3(0) = 0.2 \), corresponding to roughly parallel \( f' - \phi \) and \( \rho - A_2 \) (planar) trajectories (see Fig. 2). The remaining parameter, \( k \), is fixed by a characteristic nonplanar effect. Choosing this to be the downward shift of the \( \omega \) intercept from \( \alpha_0(0) \) to \( \alpha_0(0) = 0.15 \), we find \( k = 0.10 \), \( \theta^- = 25^\circ \), \( \theta^+ = 16^\circ \), \( \alpha_1 = 0.81 \), \( \alpha_2 = 0.26 \), \( \alpha_3 = 0.04 \), --a substantial deviation from ideal (planar) mixing. The upward shift of the \( f \) to an intercept greater than 0.8 allows identification with the "bare pomeron" of Dash. We do not want or need another high-lying trajectory; a second vacuum trajectory with intercept > 0.8 being superfluous.

Following COZ, we may calculate various ratios of total cross sections, e.g., \( \sigma_{\text{KN}}/\sigma_{\text{KN}} = 0.70 \), \( \sigma_{\text{PNN}}/\sigma_{\text{NN}} = 0.39 \) given by our "f-dominated" pomeron couplings. Had we fixed \( k \) by requiring \( \alpha_1 \) to be close to 1 we would have found \( \omega(0) = 0.39 \), \( \sigma_{\text{KN}}/\sigma_{\text{KN}} = 0.77 \), \( \sigma_{\text{PNN}}/\sigma_{\text{NN}} = 0.53 \), showing the relative stability of the parameterization. Our simple model cannot be expected to make predictions of great accuracy, but it qualitatively correlates the various manifestations of nonplanarity and broken \( SU_3 \) revealed by Regge pole analysis at moderate energy.

How is the IOZ rule related to the topological expansion? At the planar level this rule is exact, but nonplanar corrections lead to violation. For example, \( \varepsilon_{\text{KNN}}/\varepsilon_{\text{KK}} = \tan \theta^- (t = m_\phi^2) \), so to calculate the partial width for \( \phi \) decay into \( np \) we need \( k(t = m_\phi^2) \) as well as \( \alpha_0 - \alpha_2 \approx \alpha_0 - \alpha_\phi \) at \( t = m_\phi^2 \). An estimate of 0.4 for the latter difference is reasonable (see Fig. 2), while \( k \) may be estimated...
as $\sim 0.01$ from the mass difference between $\rho$ and $\omega$. We arrive at an estimate of $\theta^*$ at $t = m_\rho^2$ consistent in order of magnitude with the experimental value of $g_{\rho\rho(\rho\omega)}$. If the $\psi$ is considered as an SU$_4$ analog of the $\phi$, our model is easily extended to estimate the $\psi N$ total cross section as well as the partial width for $\psi$ decay into noncharmed hadrons. These couplings are predicted to be smaller than the corresponding $\phi$ couplings to the extent that $\alpha_\rho - \alpha_\phi < \alpha_\rho - \alpha_\psi$.

Experimental facts require the cylinder shift for $t > m_\rho^2$ to be much smaller than at $t = 0$. As already emphasized, the cylinder generator $C_1$ can in principle be calculated from planar residues so this sharp variation stands as a challenge to the theory. If the $f$ trajectory is to pass close to $1$ (the planar) value at $t = m_\rho^2$, but has a $t = 0$ intercept $1 - \epsilon$, then the slope at $t = 0$ must be $\leq \epsilon / m_\rho^2$. Explaining the small slope of the "pomeron" is thus an equivalent, not a separate challenge.

As this note was being prepared we received a preprint discussing some of the points considered here. These authors discovered the odd-C poles in the cylinder and the extinction of the planar poles. However, they did not consider symmetry breaking and thus did not arrive at our physical interpretation of the cylinder poles.

**FOOTNOTES AND REFERENCES**

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2. G. F. Chew and C. Rosenzweig, to be published in Nucl. Phys. B.


7. Such an equation has recently been studied by Chan H. M., J. E. Paton, Tsou S. T., and Ng S. W., Rutherford Lab., RL-74-149 T99. These authors deal exclusively with the leading trajectory and ignore symmetry breaking effects. They implicitly calculate $k(0)$ and find a value compatible in order of magnitude with the experimental $f - \rho$ splitting.


**FIGURE CAPTIONS**

1. (a) The planar amplitude communicating with zero additive quantum numbers in $t$ channels.

(b) The corresponding single-twist cylinder amplitude.

(c) The double-twist cylinder amplitude, showing the cylinder kernel that generates the entire series.

2. The leading trajectory pattern after the cylinder correction has displaced the $I = 0$ states. The scale of the $t = 0$ splitting shown here is fixed by the choice $\alpha_0 - \alpha_3 = 0.37, \alpha_0 - \alpha_\omega = 0.14$. 
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