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EVIDENCE FOR A NEW REGION OF DEFORMATION WITH LESS THAN EIGHTY-TWO NEUTRONS

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Berkeley, California
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EVIDENCE FOR A NEW REGION OF DEFORMATION WITH LESS THAN EIGHTY-TWO NEUTRONS

Richard N. Chanda
(Ph.D. Thesis)
April 29, 1963
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EVIDENCE FOR A NEW REGION OF DEFORMATION
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Richard N. Chanda
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Berkeley, California
April 29, 1963

ABSTRACT

Experimental evidence for a new region of nuclear deformation in the neutron-deficient rare earths has been obtained through the use of reactions of the type (heavy ion,xn) leading to several new neutron-deficient lanthanum isotopes. Three previously unreported lanthanum isotopes have been identified: $^{126}$La, $^{128}$La, and $^{130}$La, with half-lives of $1.0 \pm 0.3$, $6.1 \pm 0.6$ and $9.0 \pm 1.0$ min respectively.

The decays of these odd-odd lanthanum isotopes have been investigated using $r - r$ coincidence spectroscopy and partial-level schemes are proposed for the respective even-even daughter barium isotopes. Observed energy levels (in keV) with their proposed spins and parities are:

\[
\begin{array}{cccc}
\text{Ba}^{126} & 256(2^+) & 716(4^+) & 1341(6^+) \\
\text{Ba}^{128} & 279(2^+) & 764(4^+) & 1385(6^+) \\
\text{Ba}^{130} & 360(2^+) & 900(1-,2^+) \\
\end{array}
\]

Systematics of deformed nuclei are discussed in general and the barium nuclei compared with other deformed nuclei from other regions of deformation. Evidence for the existence of deformation is derived both from the energies of the first excited states and from the ratios of energies expected and observed in rotational bands in even-even nuclei.
I. INTRODUCTION

With the development of the collective model of nuclear structure by Bohr and Mottelson in 1953 it became possible to describe the nuclear properties of not only the spherical nuclei, for which the shell model has been so successful, but also the properties of the spheroidal or so-called deformed nuclei. At present there are three or possibly four regions of the nuclear periodic table where nuclei have been experimentally shown to possess stable spheroidal configurations, namely $A \approx 25, 150 \leq A \leq 190, A > 222$, and possibly $A \approx 8$. The great success of the collective model in describing the various properties of nuclides in these regions has prompted spectroscopists to devote considerable time to studying these deformed nuclei.

A schematic representation of the nuclear periodic table is presented in Fig. 1, showing neutron and proton closed shells as vertical and horizontal solid lines respectively. Previously observed regions of deformation are indicated by cross hatching. The thin banana-shaped curve encloses approximately those nuclei that have been experimentally studied. The line of beta stability runs approximately down the center of this area and, it will be noted, embraces those nuclei with $N = Z$ up to $A = 40$. Therefore the first limited regions of deformation, labeled 5 and 6 in Fig. 1, have $N = Z$. As the proton number is increased Coulombic repulsion among the positive protons of the nucleus is also increased. Thus the locus of stable nuclei is shifted from the line $N = Z$ toward nuclei with $N > Z$. This neutron excess then implies that the closings of neutron and proton shells occur for different nuclei. Since it is necessary that both neutron and proton numbers be far removed from closed shells in order to achieve a stable spheroidal shape, the next regions of deformation do not occur until the regions $150 \leq A \leq 190$ and $A \geq 222$, where again the neutron and proton numbers are "in phase." The regions labeled 2 and 4 in Fig. 1 are the well-known rare earth and actinide deformed regions. However, nuclei in
Fig. 1. A schematic representation of the nuclear periodic table showing the neutron (vertical solid lines) and proton (horizontal solid lines) closed shells. The dashed vertical and horizontal lines represent semi-closed shells, which have an effect on regions of deformation. Due to the complexity of the diagram the semi-closed shells of 6 protons and 6 neutrons are not shown. The thin banana-shaped curve approximately encloses those nuclei that have been experimentally studied. Regions where nuclei have been experimentally observed to be deformed are indicated with cross-hatching. Additional regions where it is reasonable to expect to find deformed nuclei are labeled 1 and 3. This research is concerned with region 1. This figure, patterned after one suggested by E. K. Hyde, is taken from Sheline et al., ref. 3.
the regions labeled 1 and 3 in Fig. 1, although highly neutron-deficient and correspondingly unstable, should also contain deformed nuclei.\textsuperscript{3,4}

This work is concerned with establishing experimental evidence for deformed nuclei in region 1 where N and Z vary from 50 to 82.

There are several methods which may be used to show that nuclei are deformed. To obtain some insight into this problem let us first examine briefly what changes are taking place in the nucleus as we proceed from spherical nuclei near closed shells into the spheroidal region. The addition of nucleons outside the closed-shell core may be thought of as exerting a centrifugal pressure on the walls of the nucleus, tending to deform or polarize the nuclear surface.\textsuperscript{5} However, two different actions are opposing the polarizing effects of these extra-core nucleons.

The particles in the closed-shell core strongly prefer a spherical nuclear shape. In addition, pairing forces between nucleons outside the closed-shell core tend to couple two equivalent nucleons to a state of zero angular momentum and thus of spherical symmetry. The overall effects of these forces can be readily seen by inspection of Fig. 2. The curves illustrate schematically the potential energy of the ground state of even-even nuclei as one moves away from closed shells.\textsuperscript{6} As the curves indicate, nuclei at the closed shell strongly resist any change in shape, as evidenced by the steep potential-energy curve for small deformations. As more nucleons are added outside the core the nucleus becomes more elastic, or less resistant to changes in shape, although still preserving a spherical stable configuration. Eventually the combined effects of many nucleons in unfilled shells exert a strong enough polarizing force to overcome the resistance to deformation described above, and the nucleus assumes a stable nonspherical or spheroidal shape shown as a potential-energy minimum in Fig. 2.
Fig. 2. Potential energy as a function of deformation for the ground state of even-even nuclei near closed shells, a few nucleons removed from closed shells, and far removed from closed shells where the nucleus is stabilized in a non-spherical shape. Although the details of the figure have no quantitative significance, the qualitative trends shown are suggested by rather simple considerations (see text). This figure is reproduced from Hyde, ref. 6.
These deformed nuclei possess several familiar distinguishing characteristics that can be experimentally verified. For these nuclei, the collective spectrum separates into excitations of vibrational and rotational types; the former corresponds to oscillations about the equilibrium shape for fixed orientation of the nucleus, whereas the rotational type implies a collective motion that rotates the nuclear orientation while preserving the shape. The nuclear deformation implies that rotational motion is associated with a large mass transport. This motion can thus occur with small frequency and consequently not affect the nuclear shape (or intrinsic structure). Consequently, low-energy excitations of strongly deformed nuclei are predominantly rotational, while the vibrational levels occur normally around 1 to 2 MeV. For even-even strongly deformed nuclei, for which the ground state has spin and parity $0^+$, an especially simple type of rotational spectrum occurs built on the ground state. This spectrum has to first order the form

$$E_I = \frac{\hbar^2}{2I} \frac{I(I+1)}{J(2J)},$$

where $E_I$ represents the energy of a state with spin $I$ and effective moment of inertia $\mathcal{I}$. Further, symmetry considerations impose the restrictions that only even values of $I$ and positive parity are allowed. In addition to possessing the $I(I+1)$ energy dependence, these rotational spectra of even-even nuclei de-excite by a cascade of $E2$ transitions with no cross-over transitions.

As noted above, the addition of extra-core nucleons tends to make the nucleus less resistant to shape changes. This manifests itself in a lowering of the first-excited-state energy, $E_{2^+}$, of even-even nuclei as we proceed away from closed shells. First-excited-state energies are especially small for regions of large deformations and, is discussed further in Section IV, rotational spectra are expected only in nuclei for which $E_{2^+}$ is less than a certain critical value. There are
of course many other characteristics of deformed nuclei, such as enhanced E2 transition probabilities, large electric quadrupole moments, etc. However, for the study reported herein the two most important are the especially low values expected for E2+ and the existence of a cascade of γ transitions with energy levels corresponding to the I(I+1) relationship.

In choosing the particular nucleus or series of nuclei in which to look for a region of deformation several factors were considered. First of all, even-even nuclei were chosen, since — as pointed out above — these nuclei possess the simplest form of rotational spectra. The nuclides of radium (Z = 88) and osmium (Z = 76), both of which are six protons removed from the closed proton shell of 82, are generally considered to lie at the edge of the actinide and rare earth regions of deformation respectively. Thus it might be expected that one would find a similar situation in the neutron-deficient rare earths. Thus, neutron-deficient barium (Z = 56) nuclei, six protons above the closed shell of 50 protons, might be expected to have characteristics similar to those of radium and osmium. Since the bariums are populated from the decay of lanthanum isotopes, and since lanthanum (Z = 57) is the first member of the rare earths, a simple radiochemical procedure isolates lanthanum parents from reaction products of Z ≤ 56.

For these reasons it was decided to study the levels in the even-even barium isotopes, in particular mass numbers 130, 128, and 126. By use of the Berkeley heavy-ion linear accelerator (Hilac) the previously unreported neutron-deficient lanthanum parents were prepared and identified as to mass number and half-life. Gamma-gamma coincidence spectroscopy was used to study the levels in the three barium daughter nuclides, and partial-level schemes are proposed. By comparing the barium energy-level data with nuclear models and with other deformed nuclei from other regions of deformation it appears that Ba126 is similar to 0+90 and to Ra224 and Ra226; that is, Ba126 appears to lie at the edge of a region of deformation where Z and N are varying from 50 to 82.
II. EXPERIMENTAL METHODS

A. Bombardment Procedures

The three isotopes $^{126}$La, $^{128}$La, and $^{130}$La were produced in the Berkeley heavy-ion linear accelerator (Hilac) by use of beams of $^{12}$C and $^{0}$O on antimony and indium targets respectively (Table I).

A bombarding energy was chosen that would give as near maximum yield of a desired isotope as possible and at the same time minimize interfering radiations from neighboring isotopes. This bombarding energy was estimated by using Cameron's mass table, assuming that each evaporated neutron carries off 6 MeV of excitation energy as neutron kinetic energy. In the case of $^{126}$La, a high $^{0}$O bombarding energy (120 MeV) was purposely chosen to minimize the simultaneous production of $^{128}$La, for reasons given in Section III-A.

Table I. Nuclear reactions used to produce the nuclides $^{126}$La, $^{128}$La, and $^{130}$La

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Nuclear reactions</th>
<th>Heavy-ion bombarding energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{126}$La</td>
<td>$^{115}$In($^{0}$O, 5n)$^{126}$La</td>
<td>120</td>
</tr>
<tr>
<td>$^{128}$La</td>
<td>$^{115}$In($^{0}$O, 3n)$^{128}$La</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>$^{121}$Sb($^{12}$C, 5n)$^{128}$La</td>
<td>90</td>
</tr>
<tr>
<td>$^{130}$La</td>
<td>$^{121}$Sb($^{12}$C, 3n)$^{130}$La</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>$^{123}$Sb($^{12}$C, 5n)$^{130}$La</td>
<td>90</td>
</tr>
</tbody>
</table>
Since the half-lives were expected to be relatively short, a target chamber was designed that would allow for rapid removal of the activities from the bombardment area at the end of bombardment. For this reason the target chamber (Fig. 3) was at atmospheric pressure.

The heavy-ion beam enters the chamber through a 7.1-mg/cm$^2$ nickel vacuum foil, after which it is collimated by a 3/8-in.-diameter graphite collimator. The beam then passes through the first of two ten-position wheels which serve to hold the absorbers and targets respectively. Weighed aluminum foils were used as absorbers to degrade the beam to the desired value according to the range-energy curves of Hubbard. Target materials consisted of thin films of natural indium and antimony metals, and were either self-supporting or vaporized onto a 2.7-mg/cm$^2$ nickel backing. Target thicknesses varied from 1 to 2 mg/cm$^2$.

Reaction products recoiling from the target were stopped in a 1.8-mg/cm$^2$ palladium catcher foil placed 0.5 cm in back of the target. The palladium foil was spot-welded between two thin metal holders which were then attached to a plunger. This plunger could be quickly removed through the top of the chamber at the end of bombardment.

After passing through the catcher foil the beam was stopped in a Faraday cup and the collected charge measured by an electrometer. Electron scattering into or out of the Faraday cup was inhibited by a permanent magnet placed around the cup. Typical beam currents used were 0.10 to 0.25 μA.

A small negative pressure was maintained on the target chamber to prevent any activities produced in the air in the beam path from escaping into the room.

*Other catcher foils including aluminum are also suitable. Palladium was chosen originally since thin foils were readily available at the time.
Fig. 3. Target chamber used to produce the lanthanum nuclides. Assembled (top view) showing right to left, nickel vacuum foil, graphite collimator, absorber wheel, target wheel, plunger with catcher foil attached, and Faraday cup and magnet. Disassembled (bottom view), showing the internal parts listed above.
B. Chemistry

After irradiation the palladium catcher foil was removed from the metal rings with forceps and dissolved in a mixture of hot concentrated nitric and hydrochloric acids on a small ceramic planchet. This solution was transferred to a heated 3-ml Lusteroid cone containing approx 1 mg each of lanthanum, barium, and cesium carriers. Upon the addition of concentrated ammonium hydroxide the lanthanum activities were brought down as the hydroxide and washed once with hot water. The activity was then dissolved in 3 drops of concentrated nitric acid, barium carrier was again added, and the addition of ammonium hydroxide repeated. Following a hot water wash the lanthanum hydroxide was again taken up in nitric acid, hot concentrated hydrofluoric acid added, and the lanthanum activities precipitated as lanthanum fluoride. The precipitate was washed once with hot dilute hydrofluoric acid and the lanthanum activities counted in the same Lusteroid cone used in the chemistry. With the above chemical procedure counting was begun approximately 6 minutes after the end of bombardment.

For the work on $1.0 \pm 0.3$-min La$^{126}$ a more rapid chemical procedure was adopted. A single lanthanum fluoride precipitation with dilute hydrofluoric acid wash was used in this case. This procedure took about 1.7 minutes from the end of bombardment to the start of counting.

Several methods were used to verify the radiochemical purity of the lanthanum activities, particularly in the case of the single lanthanum fluoride precipitation. When the same targets were bombarded by ions that produce only elements below lanthanum in atomic number, a single lanthanum fluoride precipitation separated only a small fraction of the activity. Further tests made with combined barium and lanthanum tracers confirmed the above test. Tests with barium tracer also showed that the separation factor for barium approached $10^7$ when two hydroxide precipitations preceded the fluoride precipitation. In
addition, separation of the barium activities as barium sulfate from
the lanthanum fraction showed no γ-ray transitions corresponding to
those assigned to the lanthanum nuclides. Finally, a separation of
carrier-free lanthanum from Dowex-50 cation-exchange resin eluted with
0.481 M α-hydroxy isobutyrate buffered to pH=4.8 gave a spectrum iden-
tical to that obtained by using the above procedures.9

C. Instruments

Singles γ-ray spectra were studied by using a 3-in.-thick
× 3-in.-diameter NaI(Tl) crystal integrally mounted to a DuMont 6363
photomultiplier tube;10 the spectra were displayed on either a 100-
channel Penco or a 400-channel Victoreen pulse-height analyzer.

For the gamma-gamma coincidence work the coincidence circuitry
described in Section C-1 below was used. The system included three
crystals, two 3-in.-thick × 3-in.-diameter NaI(Tl) crystals mounted on
DuMont 6363 phototubes and one 2×2-in. crystal mounted on an RCA 6342A
phototube.10 The following procedure was used. A conventional fast-
slow coincidence was performed at 90°, using the 2×2-in. "gate"
crystal (I) and one 3×3-in. crystal (II) (see Fig. 4). Slow-coincidence pulses
from the two 3×3-in. crystals were then placed in anticoincidence with
the fast-slow pulses. In this manner, coincidence events in which
photons strike all three crystals within τc, the resolving time of the
circuit, were not accepted.

Since a positron annihilating at rest gives rise to two 511-keV
quanta at 180°, the third crystal (III) was positioned at 180° to crystal
II in which the coincidence spectrum was recorded. If a I-II fast-
slow coincidence was recorded in which a 511-keV annihilation photon
was detected in crystal II, then its counterpart should theoretically,
if the positron annihilates at rest and in the source, be detected in
crystal III. Thus placing the slow II-III coincidence pulses in anti-
coincidence should reduce the intensity of the 511-keV annihilation
Fig. 4. Crystal arrangement used for the $\gamma - \gamma$ coincidence measurements with "source" in place. Each 3- by 3-in. crystal is housed in an Al "can" for support and mounted on an adjustable "lab-jack"; the 2- by 2-in. crystal is mounted on a brass support. All three crystal mountings are such as to allow radial as well as angular movement about the center "source" holder.
radiation in the coincidence spectrum. With such an arrangement a reduction of 50 to 70% in the annihilation radiation peak was obtained.

Usual source-to-crystal distance was 8 cm, with lead scattering shields between crystals I and II and I and III. These lead shields were covered with cadmium and copper to absorb the lead x rays.

1. Coincidence Circuitry

The $\gamma-\gamma$ coincidence measurements were performed with a coincidence analyzer assembled earlier in a study of the decay of Ce$^{135}$. This analyzer was designed for dual-channel operation with data being recorded directly onto magnetic tape for computer processing. In the lanthanum decay studies described here, the circuitry was used primarily in "single channel" operation, although some dual-channel work was done also. The description of equipment which follows applies to single-channel operation, although only slight modifications are necessary to make the circuitry usable in the dual-channel mode. (see Figs. 5 and 6).

Fast system

Fast pulses were taken from the anodes of phototubes I and II, and after passing through a preamplifier each pulse was led through a cascade of four two-stage noninverting distributed amplifiers (Hewlett-Packard 460A) which gave an output pulse of up to -20v. In order to minimize the secondary overshoot the first two units of each cascade were modified to include three 10-µf coupling capacitors, inserted one each at the input, between stages one and two, and at the output. After amplification the pulses were fed into a tube-type fast-coincidence unit equipped with a 12.5-ft RG/63 125-Ω clipping line. Coincidence output pulses were then fed into a transistorized "slow" coincidence unit (resolving time $\tau_r \approx 10^{-6}$ sec), Tranco "B."
Fig. 5. Block diagram of crystal arrangement and electronics used in the $\gamma - \gamma$ coincidence measurements. Pulses entering the Tranco and fast coincidence units are labeled "A" and "C" to represent anti-coincidence and coincidence modes respectively (see text).
Fig. 6. Coincidence electronics as outlined in Fig. 5, showing the fast system at the lower left and slow system at the right.
Linear system

A slow pulse was taken from the ninth dynode of each of the three 10-stage phototubes, fed through a preamplifier which stretches and shapes the pulse, and then into a DD2 linear amplifier. The output of the DD2 receiving the pulse from crystal II (DD2-II) was sent through a single-channel analyzer (mounted in the DD2 chassis), and the +18v output in turn was sent to the Tranco "B" coincidence unit. This pulse served to select the particular γ transition whose coincidence spectrum was to be studied. Outputs from amplifiers I and III were fed through transistorized single-channel analyzers with no energy discrimination—that is, "wide open"--to obtain square positive 20-volt pulses, and thence into Tranco "A." In addition, the output of DD2-I was sent through 40 ft of HH 2000 176/U delay cable, a cathode follower, and finally to the pulse-height analyzer.

Coincidence system

The coincidence output of Tranco "A" was fed into Tranco "B" in the anticoincidence mode, along with the fast-coincidence unit output and the slow "gate" from DD2-II. Thus events in which a I-III coincidence was detected along with a I-II coincidence were not recorded, as explained above. The output of Tranco "B" was then used as the trigger pulse for the pulse-height analyzer receiving the slow pulse from DD2-I. Variable delay and gate units were inserted in appropriate places for proper time matching of the pulses. These are indicated schematically in Fig. 5. Linearity is required for pulse-height analysis, and for this reason a standard delay and gate unit is not suitable for the slow pulse. Thus, the slow pulse from DD2-I was time-matched with the coincidence trigger by use of the HH 2000 delay cable referred to above.

Prior to each coincidence run several tests were made. In addition to checking the shape and amplitude of all fast and slow pulses, a delay curve was run on the fast-coincidence unit. The
coincidence counting rate is recorded as a function of delay inserted into one or the other incoming fast pulses. Such a curve should exhibit a relatively flat plateau whose width is approximately twice the transit time of the clip line on the coincidence unit. If a plateau is not obtained then a longer clip line should be used.

In order to insure 100% coincidence-counting efficiency through the fast-coincidence unit, delay curves were run for various lengths of clip line. The shorter the clip line, the narrower the delay curve, but as long as the peak counting rate on the plateau remains constant, 100% coincidence efficiency is obtained. Normally a 12.5-ft 125-Ω clip line with a transit time of 15 nsec was used.
III. ISOTOPE RESULTS

A. Half-Lives and Mass Assignments

At the time this study was begun the most neutron-deficient lanthanum isotope that had been identified was La$^{131}$. Therefore, it was necessary to establish half-lives and mass assignments for La$^{130}$, La$^{128}$, and La$^{126}$.

Preliminary bombardments over a range of heavy-ion energies sufficient to produce selectively all the lanthanum isotopes of approx $124 < A < 131$ indicated only three peaks in the gamma spectra in the energy range 200 to 510 keV. Typical spectra for this energy interval are shown in Figs. 7, 8, and 9. The half-life of each of these $\gamma$ rays was determined by following its intensity decay on a 3×3-in. NaI(Tl) crystal. The $\gamma$-ray energy, half-life, and mass assignment are shown in Table II. No evidence was seen for a transition due to La$^{124}$, indicating that the half-life for this isotope must be considerably less than 1 minute. As mentioned briefly in Section II-A, a relatively high $\alpha$ bombarding energy was purposely chosen for the production of La$^{126}$ in order to minimize as far as possible the amount of La$^{128}$ produced. Since La$^{126}$ and La$^{128}$ have rather close-lying transitions at 256 and 279 keV respectively, the high bombarding energy assured that the contribution of La$^{128}$ was negligible and thus the 256-keV photopeak could be considered as due entirely to La$^{126}$.

Several arguments may be used in defense of the above mass assignments. La$^{130}$ emits a strong 360-keV $\gamma$ ray. Fagg, in the Coulomb excitation of Ba$^{130}$, observed a 359-keV $\gamma$ ray de-exciting the 2+ level. In addition, Yaffe et al. produced La$^{130}$ by the Ba$^{130}$ (p,n) reaction, using 12-MeV protons. The gamma spectrum showed a prominent 360-keV $\gamma$ ray which decayed with a half-life of $8.7 \pm 0.1$ min, which agrees with the value of $9.0 \pm 1.0$ min reported here. In addition, targets of naturally occurring barium and of barium enriched in Ba$^{130}$ were used,
Fig. 7. Low-energy La$^{126}$ gamma ray singles spectrum showing the prominent 256 keV transition. The peak at 142 keV is due to Ce$^{141}$ which was added as a tracer in the chemistry.
Fig. 8. Low-energy $^{128}$La gamma ray singles spectrum showing the prominent 279 keV transition. The peak at 142 keV is due to $^{141}$Ce which was added as a tracer in the chemistry.
Fig. 9. Low-energy $^{130}$La gamma ray singles spectrum showing the prominent 360 keV transition. The transition at 279 keV is due to $^{128}$La which is produced simultaneously with $^{130}$La in the Sb$^{121,123}$($^{12}$C,xn) reaction.
Table II. Measured half-lives for La\textsuperscript{126}, La\textsuperscript{128}, and La\textsuperscript{130}

<table>
<thead>
<tr>
<th>Gamma-ray energy (keV)</th>
<th>Half-life (min.)</th>
<th>La mass Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Value selected</td>
</tr>
<tr>
<td>256</td>
<td>1.1</td>
<td>1.0 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>279</td>
<td>6.2</td>
<td>6.1 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>5.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td>360</td>
<td>8.7</td>
<td>9.0 ± 1.0</td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.0</td>
<td></td>
</tr>
</tbody>
</table>
and the amount of the 360-keV γ ray present in the lanthanum fractions was proportional to the ratio of the Ba\(^{130}\) content in the Ba targets.\(^{14}\)

Finally, the cross-bombardments utilizing the two isotopes of natural Sb again help to confirm the mass assignment.

Cross-bombardments using the In\(^{115}\) + O\(^{16}\) and Sb\(^{121,123}\) + C\(^{12}\) reactions show that the 279-keV γ ray should be assigned to \(\text{La}^{128}\). The assignment of the 256-keV transition to \(\text{La}^{126}\) is based upon bombarding energy and the observation of the growth of the Ba\(^{126}\) γ spectrum.\(^{15}\)

In addition, rather crude excitation functions were measured for each of the above three transitions. The photopeak yields were taken as the area above an estimated background. Chemical yields were standardized by adding 20 \(\lambda\) of a Ce\(^{141}\) tracer solution to each sample and assaying the 142-keV γ ray of Ce\(^{141}\) 48 hours after the end of bombardment. The curves so obtained are shown in the upper portions of Figs. 10 and 11. For comparison, the excitation functions calculated from the statistical model proposed by Jackson\(^{16}\) are included in the lower portions of these curves. These are plotted as \(P(E^*, x_n)\), the probability that a nucleus excited to an energy \(E^*\) will emit \(X\) neutrons with the assumption of 6 MeV average kinetic energy per emitted neutron, that is, a nuclear temperature of 3 MeV (see Fig. 5 of reference 17).

The curve for the 360-keV γ peaks at a position consistent with a \((\text{C}^{12}, 5n)\) reaction as calculated from the Jackson model and this γ ray is assigned to \(\text{La}^{130}\) as discussed above. Since the natural antimony target used contains two isotopes in essentially equal abundance, it would be expected that another peak would appear at the \((\text{C}^{12}, 3n)\) position for lower bombarding energies. The 360-keV peak was indeed seen again at these lower energies; however, its yield was not quantitatively determined. Similarly, the 279-keV γ-ray curve in Fig. 10 when compared to the Jackson-model curves, corresponds quite closely to a \((\text{C}^{12}, 5n)\) reaction, and is assigned to \(\text{La}^{128}\). This same isotope is also produced from the \((\text{C}^{12}, 7n)\) reaction on Sb\(^{123}\) and thus probably accounts
Fig. 10. Excitation functions for the 279 and 360 keV gamma-rays for the reaction Sb\textsubscript{121,123} + Cl\textsubscript{2}. Included in the lower portion of the curve are the calculated patterns expected from the Jackson model\textsuperscript{15} for the Sb\textsubscript{121} + Cl\textsubscript{2} reaction assuming a nuclear temperature of 3 MeV (see text). Experimental uncertainties may be as large as ± 50\%. 

\textit{E}_{\text{lab}} \text{ (MeV)}
Fig. 11. Excitation function for the 256 keV gamma-ray for the reaction In$^{115}$ + O$^{16}$. Included in the lower portion of the curve are the calculated patterns expected from the Jackson model$^{15}$ assuming a nuclear temperature of 3 MeV (see text).
for the observation that the excitation function does not drop off rapidly at the high-energy side. Since La$^{128}$ cannot be produced by a $(0^{12}, 3n)$ reaction the upturn at 70 MeV is not real, but is due to experimental uncertainty. In Fig. 11, the excitation function for the 256-keV $\gamma$ ray is noticeably broad, and could fit either the $(0^{16}, 4n)$ or $(0^{16}, 5n)$ reaction. However, the 279-keV $\gamma$ from the In$^{115}(0^{16}, 3n)$La$^{128}$ reaction may be partially responsible for the low-energy broadening. The yield of the 279-keV $\gamma$ of La$^{128}$ was not measured quantitatively although, like the 360-keV $\gamma$ of La$^{130}$, it definitely appears at about the $(0^{16}, 3n)$ position.

Thus although we have assigned the 256-keV $\gamma$ to La$^{128}$, i.e., an $(0^{16}, 5n)$ reaction—the excitation function does not rule out completely the possibility of an $(0^{16}, 4n)$ reaction giving rise to the 256-keV $\gamma$. Although experimental uncertainties in the $\gamma$-ray yields may be as large as $\pm 50\%$, the mass assignments on the basis of the excitation functions appear to be unique with the exception of the 256-keV $\gamma$, which may be in error by not more than 1 amu.

Possibly the most convincing evidence for the mass assignments comes from the $\gamma-\gamma$ coincidence work. The patterns of $\gamma$ de-excitation observed in the Ba daughters are characteristic of the decay of odd-odd lanthanum nuclei and not of the neighboring odd-even nuclei. (These are discussed more fully below.) A simple cascade relationship is observed which is not expected except in the de-excitation of even-even nuclei. Thus the combination of the observed $\gamma$ spectra and the excitation function alone make the mass assignments for all three La isotopes unique.

Yaffe et al.$^{14}$ and Preiss et al.$^{18}$ have recently measured the half-lives of these same lanthanum isotopes, including the odd-even isotopes La$^{127}$ and La$^{129}$, by "milking" out the respective barium daughters and cesium granddaughters and also from $\gamma$-ray decay. A summary of the data is presented in Table III. Included is the
approximate 11-min half-life of the 110-keV γ observed in the singles spectra of La$^{130}$ (Fig. 9). This transition may be tentatively assigned to La$^{129}$ on the basis of the half-life reported by Yaffe et al.\textsuperscript{14} It is seen that the half-lives for La$^{126}$ and La$^{130}$ agree quite well, while the two values of Ba$^{128}$ obtained from milking experiments are slightly lower than the value obtained in this work. The difference is not large, but some possible explanations may be offered. In general, one would believe that the half-life obtained from the decay of the 279-keV γ would be more accurate than values obtained from milking experiments. There are no sources of error in the γ-decay determination other than those from background subtractions, while milking experiments may be in error from several sources, including carrying through of the parent, differences in chemical yield, resolution of β decay curves, etc. In addition, it should be pointed out that if the parent does carry through the milking chemistry, then the resultant half-life obtained for the parent is too low. In addition, there is a rather large discrepancy of almost 3 min in the half-life for La$^{129}$ as reported by Preiss and by Yaffe. Yaffe’s result of 10.0 ± 0.5 min was obtained, not by milking, but from following the decay of two photons, the 110-keV γ and the 511-keV annihilation radiation, from a (p,2n) reaction on enriched Ba$^{130}$. Milking experiments performed by Yaffe gave half-lives varying from 5.8 to 10.6 min for this same isotope. Thus it appears that the discrepancy in the half-life for La$^{128}$ may not be real, and indeed may be due to differences in the experimental procedures used.
Table III. Half-lives of lanthanum isotopes

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>This work</th>
<th>Preiss et al.</th>
<th>Yaffe et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>La$^{126}$</td>
<td>1.0 ± 0.3</td>
<td>1.0 ± 0.3</td>
<td>—</td>
</tr>
<tr>
<td>La$^{127}$</td>
<td>—</td>
<td>3.8 ± 0.5</td>
<td>3.5</td>
</tr>
<tr>
<td>La$^{128}$</td>
<td>6.1 ± 0.6</td>
<td>4.2 ± 0.5</td>
<td>4.6 ± 0.6</td>
</tr>
<tr>
<td>La$^{129}$</td>
<td>∼ 11.0</td>
<td>7.2 ± 0.5</td>
<td>10.0 ± 0.5$^a$</td>
</tr>
<tr>
<td>La$^{130}$</td>
<td>9.0 ± 1.0</td>
<td>—</td>
<td>8.7 ± 0.1$^a$</td>
</tr>
</tbody>
</table>

$^a$These half-life values were obtained from γ-ray decay and not from milking.
B. Lanthanum-126

 Singles spectra of La$^{126}$ (Figs. 7 and 12) show only one prominent peak, other than annihilation radiation. This is the 256-keV transition. Coincidence measurements taken with use of a 256-keV "gate" show transitions at 460 and approx 625 keV (Fig. 13). With use of a standard "peak-stripping" procedure starting with the 625-keV peak, successive subtractions of standard gamma spectra show a weak transition at approx 340 keV in coincidence with the 256-keV transition. In coincidences obtained when the gate was set at 405 to 475 keV the 340-keV peak does not appear (Fig. 14), but it again appears when the gate is set at 625 keV (Fig. 15). The 256-, 460-, and 625-keV transitions are all in coincidence (Table IV), and thus probably form a cascade as shown in Fig. 16. These three transitions have been assigned to the rotational band built on the 0+ ground state of the even-even nuclide Ba$^{126}$ (Fig. 16). This level scheme is discussed further in Section III-E.

 It will be noted that the $\gamma$ transitions are superimposed on a rather large background. One of the major contributions to this background is probably bremsstrahlung radiation from the stopping or deceleration of electrons in the sample holder, etc. In addition, the high positron end-point energy of about 8 MeV allows high-energy positrons to penetrate the NaI crystal, adding to the background continuum. Further, since the excitation functions for heavy-ion reactions overlap at a given energy (see Figs. 10 and 11) the La$^{126}$ $\gamma$ spectrum also contains $\gamma$ transitions from the neighboring odd-even isotopes. Also, since the chemical procedure had to be carried out quickly and involved only a single fluoride precipitation, some barium carried through the chemistry and contributes to the $\gamma$ spectrum (see Section II-B). (Similar statements concerning the background also apply to La$^{128}$ and La$^{130}$).

 The placement of the 340-keV gamma ray in the decay scheme is uncertain. Its appearance in coincidence with the 256-keV transition and not the 460-keV transition suggests that it may arise from the deexcit-
Fig. 12. $^{126}$La gamma ray singles spectrum for the energy range 0-1000 keV.
Fig. 13. A coincidence spectrum of La$^{126}$ showing the gamma rays in coincidence with the 256 keV gamma ray transition.
Fig. 14. A coincidence spectrum of La$^{126}$ showing the gamma rays in coincidence with the 460 keV gamma ray transition.
Fig. 15. A coincidence spectrum of $^{126}$La showing the gamma rays in coincidence with the 625 keV gamma ray transition.
Table IV. \( \gamma-\gamma \) coincidence relationships for La\(^{136}\)

<table>
<thead>
<tr>
<th>Coincidence gate (keV)</th>
<th>Transitions in coincidence with gate (keV)</th>
<th>Relative intensities ( I(256)/I(340)/I(460)/I(625) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>256</td>
<td>340</td>
</tr>
<tr>
<td>256</td>
<td>-</td>
<td>Y</td>
</tr>
<tr>
<td>460</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>625</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

\( Y \) - indicates a coincidence

\( N \) - indicates no coincidence
Fig. 16. Partial level scheme for \( {\text{Ba}}^{126} \) populated from the decay of \( {\text{La}}^{126} \). Spin and parity assignments in parentheses indicate probable assignments made from systematics of the expected rotational bands.
ation of a second excited 2+ level at 596 keV. However, if this were the case then the 625-keV transition should not appear in coincidence with the 340. Let us assume there exists a 2+ state at 596 keV and a 4+ state at 716 keV. Using the non-axially-symmetric rotor theory of Davydov and Filippov\textsuperscript{19} and the ratios of the energies of the above states to that of the first 2+ state at 256 keV, we can obtain a value of the parameter $\gamma$ that characterizes the eccentricity of the nuclear shape. In both cases a value of $\gamma=26.1^\circ$ is obtained.*

However, if this 340-keV transition is truly in coincidence with the 625-keV transition, as indicated by Fig. 15, and further, if (as assumed) the 625-keV transition is the third member of the ground-state rotational band, then a 2+ assignment for a 596-keV level is not correct. However, the possibility that there are two transitions of approx 625 keV, one of which is feeding a (2+) level at 596 keV, cannot be ruled out. For these reasons no level at 596 keV is indicated in the decay scheme.

Uncertainties in energy determinations are of the order of 1 to 2% with the exception of the 340-keV transition, which may be as large as 4 to 5%. Since the half-life of $^{126}$La is only 1.0 min, many separate runs had to be summed in order to minimize statistical errors. "Good" statistics were usually obtained by summing the results of 12 to 18 runs, depending upon the gate setting, chemical yield, etc. Typically, coincidence counting was begun approximately 1$\frac{1}{2}$ half-lives after the end of bombardment and continued for 3 more half-lives.

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*The excellent agreement is somewhat fortuitous, since there is a large uncertainty (about 10 to 15 keV) in the energy of the weak 340-keV transition.
C. Lanthanum-128

The gamma singles spectra shown in Figs. 8 and 17 show three transitions (279, approx 620, and approx 1070 keV), and possibly a fourth transition (about 1300 keV), that have been assigned to La$^{128}$. The spectrum in coincidence with the 279-keV transition (Fig. 18) shows the 620- and 1070-keV transitions as well as a transition at 485 keV not observed in the singles spectrum. When a 620-keV gate was used (Fig. 19) the coincidence spectrum showed the 279-, 485-, and 1070-keV transitions. When the coincidence gate was placed on the 1070-keV transition (Fig. 20) the coincidence spectrum again included the 279-, 485-, and 620-keV transitions. In no case does the 1300-keV transition appear. Table V summarizes the γ-γ coincidence data discussed above. No coincidence measurements were made with the 1300-keV transition used as a gate. Although this transition has a half-life approximating that of La$^{128}$, the fact that it does not appear in coincidence with any of the other transitions makes its assignment to La$^{128}$ doubtful.

The coincidence data then indicate a cascade with no cross-over transitions, as one would expect in a rotational band. The 279-, 485-, and 620-keV transitions have been placed in a rotational band built on the ground state of 0+ of the even-even nucleus Ba$^{128}$ as shown in Fig. 21.

The 1070-keV transition does not appear to be a cross-over, since it is in coincidence with the other three transitions; but neither does it fit an assignment to the 8+ → 6+ transition of the ground-state rotational band. On a plot of energy level vs the rotational spin dependence I(I+1), the assignment of an 8+ level at approx 2455 keV is not consistent with the other members of the rotational band (see Fig. 30 of Section IV-C). The intensity of the 1070-keV transition is higher than the upper two members of the ground-state band (Table V), and thus it could not be feeding the top of the rotational band alone. It is
Fig. 17. La\textsuperscript{128} gamma ray singles spectrum for the energy range 0-1350 keV. La\textsuperscript{128} was produced by the In\textsuperscript{115} (O\textsuperscript{16}, 5n) reaction.
Fig. 18. A coincidence spectrum of La$^{128}$ showing the gamma rays in coincidence with the 279 keV gamma ray transition.
Fig. 19. A coincidence spectrum of La$^{128}$ showing the gamma rays in coincidence with the 620 keV gamma ray transition.
Fig. 20. A coincidence spectrum of La$^{128}$ showing the gamma rays in coincidence with the 1070 keV gamma ray transition.
Table V. $\gamma-\gamma$ coincidence relationships for $\text{La}^{128}$.

<table>
<thead>
<tr>
<th>Coincidence gate (keV)</th>
<th>Transitions in coincidence with gate (keV)</th>
<th>Relative intensities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>279</td>
<td>485</td>
</tr>
<tr>
<td>279</td>
<td>-</td>
<td>Y</td>
</tr>
<tr>
<td>620</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>1070</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Y-indicates a coincidence
N-indicates no coincidence
Fig. 21. Partial level scheme for Ba$^{128}$ populated from the decay of La$^{128}$. Spin and parity assignments in parentheses indicate probable assignments made from systematics of the expected rotational bands. The double parentheses around the 6+ assignment is used to indicate that both the collective and asymmetric models do not satisfactorily reproduce this level (See Section IV-C, Table VIII).
possible, then, that the 1070-keV peak is complex and involves two transitions, one of which enters the ground-state band at the 6+ level and the other at the 4+ level.

Errors in the assigned energies of the transitions are as follows: the 279-keV transition is believed accurate to at least ±1 keV since this transition could be compared to the 279-keV γ of Hg$^{203}$. The other three transitions should be considered accurate to approx ±1%. Adequate statistics were usually obtained by summing 6 to 10 separate runs, beginning approximately 1 half-life after the end of bombardment, and recording coincidences for another 2 half-lives.

D. Lanthanum-130

The singles spectra of La$^{130}$ are complex and show several transitions (see Figs. 9 and 22). La$^{130}$ was made by bombardment of natural Sb, which contains two isotopes; thus the spectrum is further complicated by the presence of La$^{128}$. However, γ-γ coincidence studies revealed five transitions in the energy region up to approx 1200 keV which have been assigned to La$^{130}$ (see Table VI.). Coincidence measurements gating on the 360-keV γ ray showed it to be in coincidence with transitions of 540, approx 680, and approx 1015 keV (Fig. 23). The peak that appears at 1070 keV in Fig. 23 also appears in the singles and coincidence spectra of La$^{128}$. Since La$^{128}$ is present in the La$^{130}$ sample, setting the gate at 360 keV will allow some Compton events from the 511-keV annihilation radiation of La$^{128}$ to be recorded in the gate, which may explain the presence of the peak at approx 1070 keV. In addition, this peak does not appear in the coincidence spectra when "gates" above 511 keV are used. Thus no transition of approx 1100 keV was assigned to La$^{130}$.

Coincidence measurements were also made by gating on the 680-keV transition. In this case, in addition to the previously observed transitions, a transition of 900 keV that appears in high intensity in
La$^{130}$ singles spectrum for the energy range 0-1350 keV. The spectrum also includes La$^{128}$ produced simultaneously with La$^{130}$ in the Sb$^{121,123}$ (Cl$^2$,xn) reaction.
Table VI. $\gamma$-$\gamma$ coincidence relationships for La$^{130}$.

<table>
<thead>
<tr>
<th>Coincidence gate (keV)</th>
<th>$360$</th>
<th>$540$</th>
<th>$680$</th>
<th>$900$</th>
<th>$1015$</th>
<th>$I(360)/I(540)/I(680)/I(1015)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$360$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$- / 1.0 / 0.10 / 0.30$</td>
</tr>
<tr>
<td>$680$</td>
<td></td>
<td></td>
<td></td>
<td>$Y$</td>
<td>$Y$</td>
<td></td>
</tr>
<tr>
<td>$900$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$Y$-indicates a coincidence
$N$-indicates no coincidence
$?$-indicates uncertainty
$W$-indicates a weak coincidence
Fig. 23. A coincidence spectrum of $^{130}\text{La}$ showing the gamma rays in coincidence with the 360 keV gamma ray transition. The peak at 279 keV and probably also the peak at 1070 keV are due to $^{128}\text{La}$ (see text).
The singles spectrum is in coincidence with the 660-keV transition. A run was also made in which the gate was set on the lower side of the distribution ranging from 900 to 1015 keV, however, the results were inconclusive.

Thus only three transitions are placed in a partial decay scheme for La\textsuperscript{130}, as is shown in Fig. 24. The 360-keV level is assigned the spin 2+ on the basis of the Coulomb excitation of Ba\textsuperscript{130}.\textsuperscript{13} Since the 900-keV transition is not in coincidence with the 360-keV transition, the former is probably de-exciting a level at 900 keV to ground. It appears, then, that a level at 900 keV de-excites directly to ground and also by a cascade of two gammas, 540 and 360 keV. Thus, the 900-keV level may have spin and parity 1-, in which case the 540- and 900-keV transitions are E1's, or the 900-keV level may be a 2+ level which de-excites by two E2 transitions.

The experimental intensity ratio \( \frac{I(900)}{I(540)} \) is approx 1 to 2, as estimated from the La\textsuperscript{130} singles spectrum (Fig. 22). This would indicate a ratio of the reduced transition probabilities \( \frac{B(E2; \text{cross over})}{B(E2; \text{cascade})} \) of approx 0.1, which is similar to the values (0.12 to 0.15) experimentally observed in the Os\textsuperscript{190} region,\textsuperscript{20} where deformations are comparable. However, this ratio would be expected to be changing rapidly in this region, increasing with the trend toward more spheroidal nuclei and decreasing with the trend toward the more vibrational nuclei [see Fig. 1 of reference 18 where the B(E2) ratio is plotted vs the energy ratio E(2')/E(2)]. Experimental B(E2) ratios are seen to scatter in the Os\textsuperscript{190} region. Also, theory would predict a B(E2) ratio of 2.33 for similar de-excitations of a 1- level. Therefore, a spin assignment for the 900-keV level on the basis of \( \gamma \) intensities would hardly be valid.
Fig. 24. Partial level scheme for $^{130}\text{Ba}$ populated from the decay of $^{130}\text{La}$. The spin and parity of the first 2+ level have been assigned from the coulomb excitation of $^{130}\text{Ba}$. The spin and parity assignment of the (1-,2+) level is uncertain as indicated by the double parentheses.
E. **Summary**

Although the data on the levels of the barium nuclides does not include direct spin measurements of levels nor multipolarities of transitions between these levels, the evidence clearly indicates a cascade pattern of de-excitation with no cross-overs in Ba\textsuperscript{126} and Ba\textsuperscript{128}. In addition, the intensities of the presumed cascade transitions fall off with increasing energy (see Tables IV and V). Such a pattern is consistent only with an ordering in which the lower-energy transitions lie lower in the cascade, as they do in a rotational band; also it is necessary to assume that there are gamma (or beta) transitions feeding into the cascade at all the different levels and not entirely at the top. Such \( \gamma \) (or \( \beta \)) feeding from upper levels is not unusual and, in fact, is seen quite often in other deformed even-even nuclei.

One question that might be asked is why the spin-6 levels seem to be populated to such a high degree. Obviously this requires that the lanthanum parents must have spins of at least 5 in order for \( \beta \) decay to populate such high spin levels. A strict spherical shell-model picture would estimate the 57th proton to have a \( \frac{5}{2}^+ \) assignment and the 71st neutron \( \frac{1}{2}^- \). Nordheim's rules for odd-odd nuclei would then predict a coupling of these two orbitals to the lowest value and a resultant ground-state spin and parity of 2- for La\textsuperscript{128}. A state of \( \frac{11}{2}^- + \frac{7}{2}^- = 9^- \) may be expected to appear as an excited state. Similarly La\textsuperscript{126}, assuming the 69th neutron to be in the \( \frac{3}{2}^- \) (or \( \frac{1}{2}^- \)) orbital, would be predicted to have a ground-state spin and parity of 5- (or 3-) with an excited state of 2- (or 4-). Thus there are ample high-spin states available according to the spherical shell-model picture. However, since these nuclides are seven protons and ten to twelve neutrons removed from closed shells, one would not expect the shell model to give necessarily valid predictions.

A look at the level diagrams by Mottelson and Nilsson\textsuperscript{21} may provide a more realistic estimate of the lanthanum spin and parities. Several single-particle levels are available for the 57th proton in
the region of deformation $\beta = 0.20$ to 0.25 (see Section IV-B, including Table VII, for a discussion of deformation), among them $9/2^+ [404]$, $3/2^+ [422]$, $3/2^- [541]$ and $1/2^+ [420]$. Similarly, from a neutron level diagram, the 69th and 71st neutrons in this same range of deformation have several possible level assignments available to them: $5/2^- [532]$, $5/2^+ [402]$ and $7/2^+ [404]$. Thus there is a variety of ways in which to create high spin levels in odd-odd nuclei in this region. For example, consider La$^{128}$ with a proton assignment $9/2^+ [404]$ and 71st neutron $5/2^- [532]$. According to the coupling rules of Gallagher and Moszkowski one would expect a ground state $7^-$ with a $2^-$ level lying higher. A $7^-$ state in La$^{128}$ could then be expected to populate the observed $6^+$ level in Ba$^{128}$. It must be pointed out that no Nilsson level assignments for odd-$A$ nuclei have been experimentally made in this region and thus any discussion of probable assignments is speculative. However, the fact that high spin states in lanthanum can be easily postulated lends credence to such an argument.

On the basis of the large decay energies available for these lanthanum nuclei (6.80 MeV for La$^{130}$ to 9.26 MeV for La$^{126}$), one might expect (for allowed transitions) much shorter half-lives than those observed. However, the measured half-life and decay energy give log $ft$ values of 6.0 to 6.5 even for transitions proceeding to ground or first excited states. Such log $ft$ values are in the range observed for first-forbidden (nonunique) transitions in the spherical nuclei or for an (allowed hindered) or lu (first-forbidden unhindered) transitions in spheroidal nuclei. Thus it is not necessary to postulate high spins for the lanthanum parents on the basis of half-life and decay energy alone. However, the population of higher-spin members of a rotational band in barium does require high lanthanum spin assignments, as discussed above.
IV. DISCUSSION AND INTERPRETATION OF RESULTS

With the experimental methods used here one could expect only to penetrate the edge of the region of deformation labeled l in Fig. 1; thus it was not expected at the outset that very strongly deformed nuclei would be produced. Consequently, then, one would not expect to experimentally find that the energy levels in the three barium nuclides studied here correspond rigorously, for example, to the simple \( I(I+1) \) relationship of the strong coupling model. In the absence of a direct measurement of the nuclear deformation—such as could be obtained from a lifetime measurement of the first \( 2^+ \) level, for example—other methods must be used to estimate the deformation of these barium nuclei. Included in the following section is a review of recent theoretical calculations of deformations expected in this region by Marshalek, Person, and Sheline. In addition, three methods of using the experimental first-excited-state energy of even-even nuclei as a criterion for deformation are discussed. Included are a discussion of (a) an \( (E_2^+)_{\text{crit}} \) suggested by Alder et al., 25 (b) a determination of an empirical deformation, \( \beta_{\text{emp}} \), from a relationship 26,27 between \( s/3 \) and \( \beta \), and (c) a comparison of a theoretical deformation, \( \beta_{\text{Mig}} \), calculated by Migdal. 28 Finally, the collective-model description by Bohr and Mottelson 1 and the asymmetric rotor model by Davydov and Filippov 19 are used to discuss the data on the higher-lying excited states observed in the gamma-gamma coincidence work.
A. Calculated Deformations

Recent calculations of deformations to be expected in the several regions of spheroidal nuclei have been made by Marshalek, Person, and Sheline. By summing Nilsson's single-particle energies, without a pairing force or Coulomb corrections, as a function of deformation, an estimate of the total nuclear energy can be obtained. Thus, for each value of the deformation parameter one chooses the configuration that minimizes the sum of the single-particle energies. The equilibrium deformation for a nuclide then corresponds to that value of the deformation for which the energy summation is a minimum. These calculations show that, as expected, an extended region of deformation exists in the neutron-deficient rare earths. A few of the major points are discussed in more detail.

The authors have plotted their data as contour maps of deformation and energy of deformation against neutron and proton number. Figure 25 shows such a plot of deformations to be expected for the neutron-deficient rare earths under consideration here. It will be noted that the deformation contours rise sharply to a maximum soon after nuclear deformation is achieved and then decrease gradually with increasing neutron and proton number. These sharp rises are in the proton particle–neutron particle configuration (lower left corner of Fig. 25) and also in the proton particle–neutron hole type of configuration (lower right portion of Fig. 25), which includes the barium nuclei studied here. Similar patterns were obtained for the neutron-excess rare earths and for the neutron-deficient and neutron-excess actinides, although the latter two are less dramatic. Experimentally this same type of trend is observed in the rare earths, which is the only region for which experimental data are available for both the "beginning" and "end" of an extensive deformed region. Figure 26 shows more clearly the trends in deformation calculated by Marshalek et al. for the even-even barium isotopes. The maximum deformation calculated by Marshalek for this region is indicated by the arrow at the left for
Fig. 25. A contour map of the deformation vs. neutron and proton number in the region where the protons and neutrons both go from 50 to 82. The values on the contour lines are for $\epsilon \approx 0.95\, \beta$ where $\beta$ is the deformation parameter of Bohr and Mottelson (ref. 1). Note the sharp rise in deformation in going from 74 to 70 neutrons for $Z=56$ (see text and Fig. 26). This figure is reproduced from ref. 4.
Fig. 26. Deformation of barium nuclei (Z=56) plotted vs. mass number. Values of deformation are taken from Fig. 25. The arrow on the ordinate indicates the maximum deformation calculated for this region where Z and N both go from 50 to 82.
comparison. From these calculations one would expect that the three barium nuclei studied here would encompass this region of sharp rise in deformation and in the case of Ba$^{128}$ and Ba$^{126}$ have deformations near the maximum for this region.

It must be remembered, however, that several approximations were made in these calculations, including the neglect of a pairing force. Thus, as the authors point out, it is the trends that are important here rather than absolute values. It is important to emphasize further that the pairing force is of particular importance in determining the boundaries of a deformed region. Since this force is not included here, the region of sharp rise of deformation may actually occur for more neutron-deficient species than Figs. 25 and 26 would indicate.

However, it is certainly reasonable to assume on the basis of these calculations that an extended region of deformation in the neutron-deficient rare earths does exist even though the boundaries may not be well defined.

**B. Deformations Estimated From First-Excited-State Energies**

a. The energies of the first excited states of even-even nuclei show a rather smooth dependence upon atomic number and decrease as one moves away from closed shells. This smooth trend may be noted in Fig. 27, where the $E_{2+}$ of even-even nuclides are plotted as a function of neutron number $N$ and proton number $Z$. The regions of large deformations are characterized by particularly small values of this excitation energy, $E_{2+}$ and, according to a model proposed by Bohr and Mottelson, $^{30}$ rotational spectra are expected for only those nuclei whose $E_{2+}$ lie below a certain critical value. The dotted curve in Fig. 27 shows this critical value for the stable-mass region.

This estimate of the separation between spherical and spheroidal nuclei is based on a two-nucleon model in which all the nucleons outside of closed shells are represented by two interacting nucleons in p
Fig. 27. Energy systematics of first excited \(2^+\) states in even-even nuclei. The energies of the first excited \(2^+\) states, \(E_{2^+}\), are plotted as a function of neutron number \(N\) and proton number \(Z\). Note the strong maxima near closed shell positions. The rotational spectra (spheroidal nuclei) occur in the regions furthest from closed shells where the excitation energies, \(E_{2^+}\), are lowest; in other regions the excitations have the character of collective vibrations (see refs. 6 and 25). The separation between these two regions is approximately given by 2, which is represented by the dotted curve following the stable mass region. Thus the rotational spectra are found in the regions where the observed first excited states have energies less than this separation line. (See text.) The above figure is reproduced from Alder et al., ref. 25.
states. The strength of the interaction between the nucleons outside closed shells is represented by a parameter, \( v \), which in the harmonic oscillator field chosen for the model is given by

\[
v = \frac{U}{\hbar \omega}
\]

where \( U \) is the energy difference between the \( J=0 \) and \( J=2 \) states of two nucleons. An estimate is then made of the moment of inertia needed to produce a rotational band in an even-even nucleus.

It has been shown\(^{30,31} \) that if the intrinsic nuclear structure of strongly deformed nuclei could be described in terms of many nucleons moving independently within the nucleus, then the moment of inertia would approach that of a rigid rotator given by

\[
\mathcal{I}_{\text{rig}} = \frac{2}{5} \frac{A}{\hbar^2} \left(1 + 0.3 \beta\right).
\]

However, if one includes effects due to the residual interactions of particles in unfilled shells, the moment of inertia is reduced from that of a rigid rotator. If these residual interactions were to become so large as to break down the shell structure, the moment would then approach that of the irrotational flow of an incompressible fluid,

\[
\mathcal{I}_{\text{irrot}} = \frac{2}{5} \frac{A}{\hbar^2}.
\]

Experimentally, moments of inertia indicate interactions that are only one-third to one-half this limit.

Thus an estimate of the transition between rotational and vibrational types of spectra can be made by estimating the smallest moment of inertia that would allow a rotational spectrum. The two-nucleon model predicts that this should occur when the deformation becomes comparable to \( 0.6 v \) (with the interaction parameter \( v=1.8A^{-1/3} \))
and the moment of inertia $= 0.23 \hbar^2$. This would imply that rotational spectra should occur in even-even nuclei only when the energy of the first excited $2^+$ state is less than a critical value given by

$$(E_{2+})_{\text{crit}} \approx \frac{3\hbar^2}{0.23 \hbar^2} = \frac{13h^2}{3\hbar^2}.$$  \hspace{1cm} (2)$$

This relationship is borne out in the heavier-element regions ($150 \leq A \leq 190$ and $A \geq 222$) and in the light elements of $A \approx 8$ and $A \approx 25$ for first excited $2^+$ states. Rotational levels have been positively identified in the former, and the latter are probable. It should be noted that no dependence upon $A$ is included in the coefficient; this may or may not alter this critical value. Nonetheless, this model can serve as an indication of the approach to a region of deformation.

Table VII compares the data on the barium isotopes with data for similar nuclei from other deformed regions (only the first three columns of Table VII concern us in this section). Column three gives the values of $(E_{2+})_{\text{crit}}$ calculated from the above model, Eq. (2), using (1) with $\beta = 1.08 A^{-1/3}$ and $R_0 = 1.2 \times 10^{-13} A^{1/3} \text{ cm}$. Since the energies of these first excited states in barium lie quite close to $(E_{2+})_{\text{crit}}$ (this is shown graphically in Fig. 28), and owing to the approximate nature of the model used, one would expect these barium nuclei to be quite close to if not at the edge of a new region of deformation.

b. It is also possible to estimate the deformation $\beta$, from an empirical relationship shown by Elbek. From the experimentally measured reduced transition probabilities, $B(E2)$, to the first excited state of even-even nuclei the intrinsic electric quadrupole moment, $Q^0$, may be calculated from the relation

$$B(E2; I_i \rightarrow I_f) = \frac{5}{16} e^2 Q^2 \langle I_i^2 K_0 | I_f^2 I_f K \rangle^2$$  \hspace{1cm} (3)$$
Table VII. Comparisons between the nuclei $^{126}$Ba, $^{128}$Ba, and $^{130}$Ba and nuclei from other regions of deformation.\(^{a}\)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_{2+}$ (keV)</th>
<th>$(E_{2+})_{\text{crit.}}$ (keV)</th>
<th>$S/S_{\text{rig.}}$</th>
<th>$\beta_{\text{exp}}$</th>
<th>$\beta_{\text{Mig}}$</th>
<th>$\beta_{\text{exp}}$</th>
<th>Ratios of energies in the rotational band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ba$^{126}$</td>
<td>256</td>
<td>279</td>
<td>0.251</td>
<td>0.20</td>
<td>0.16</td>
<td>1.0:2.80:5.24</td>
<td></td>
</tr>
<tr>
<td>Ba$^{128}$</td>
<td>279</td>
<td>272</td>
<td>0.225</td>
<td>0.18</td>
<td>0.15</td>
<td>1.0:2.73:4.95</td>
<td></td>
</tr>
<tr>
<td>Ba$^{130}$</td>
<td>360</td>
<td>265</td>
<td>0.172</td>
<td>0.16</td>
<td>0.12</td>
<td>1.0:2.73:4.95</td>
<td></td>
</tr>
<tr>
<td>Gd$^{157}$</td>
<td>123</td>
<td>200</td>
<td>0.462</td>
<td>0.29</td>
<td>0.25</td>
<td>1.0:0.30:5.84</td>
<td></td>
</tr>
<tr>
<td>Dy$^{156}$</td>
<td>138</td>
<td>196</td>
<td>0.333</td>
<td>0.23</td>
<td>0.19</td>
<td>1.0:2.93:5.59</td>
<td></td>
</tr>
<tr>
<td>W$^{186}$</td>
<td>124</td>
<td>147</td>
<td>0.273</td>
<td>0.20</td>
<td>0.17</td>
<td>1.0:3.28</td>
<td></td>
</tr>
<tr>
<td>Os$^{186}$</td>
<td>137</td>
<td>147</td>
<td>0.247</td>
<td>0.19</td>
<td>0.15</td>
<td>1.0:3.19:6.33</td>
<td></td>
</tr>
<tr>
<td>Os$^{188}$</td>
<td>155</td>
<td>144</td>
<td>0.215</td>
<td>0.18</td>
<td>0.14</td>
<td>1.0:3.09</td>
<td></td>
</tr>
<tr>
<td>Os$^{190}$</td>
<td>188</td>
<td>142</td>
<td>0.174</td>
<td>0.16</td>
<td>0.12</td>
<td>1.0:2.93:5.60:8.89</td>
<td></td>
</tr>
<tr>
<td>Ra$^{222}$</td>
<td>111</td>
<td>110</td>
<td>0.228</td>
<td>0.19</td>
<td>0.15</td>
<td>1.0:2.79:5.78</td>
<td></td>
</tr>
<tr>
<td>Ra$^{224}$</td>
<td>84.5</td>
<td>106</td>
<td>0.295</td>
<td>0.21</td>
<td>0.18</td>
<td>1.0:2.98</td>
<td></td>
</tr>
<tr>
<td>Th$^{226}$</td>
<td>72.1</td>
<td>106</td>
<td>0.341</td>
<td>0.23</td>
<td>0.20</td>
<td>1.0:3.14:6.33</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)This table is taken from Sheline, Sikkeland, and Chanda (reference 3) with the following additions: rotational band energy ratios for Ba$^{126}$ and Ba$^{128}$ and the value of 360 keV for $E_{2+}$ for Ba$^{130}$ (reported herein).

\(^{b},^{d},^{e}\)See text for explanation.

\(^{c}\)Ratio of $S = 3n^2/E_{2+}$ to $S_{\text{rig.}}$ (Eq. 1).

\(^{f}\)Experimental deformations calculated from available values of the transition probabilities for Coulomb excitation.
Fig. 28. The first excited states of even-even barium isotopes. The dashed line shown represents \((E2+)_{\text{crit}}\), the approximate dividing point between spherical and spheroidal nuclei.
for an E2 transition from a state \((I_1, K)\) to another state of the same rotational band \((I_r, K)\). The deformation \(\beta\) is then obtained by equating the intrinsic quadrupole moment to that of a homogeneously charged spheroid,

\[
Q_0 = \frac{3}{(5\pi)^{1/2}} \frac{Z R^2}{R_0} \beta (1+0.16\beta+\ldots),
\]

where \(Z\) is the nuclear charge number and \(R_0 = 1.2 \times 10^{-13} \text{ A}^{1/3} \text{ cm}\).

The moment of inertia, as calculated from the observed rotational-band energies, plotted against the deformation \(\beta\) (calculated from 3 and 4) shows that the experimental points follow approximately a straight-line relationship for the rare earth and actinide regions, with the points for the latter region lying definitely higher.\(^{26}\) A calculation of the deformation from the quadrupole moment based on the particle structure of the nucleus, which attempts to avoid the more or less arbitrary assumption that the nucleus is a homogeneously charged spheroid, has been carried out by Glendenning and Sawicki.\(^{27}\) This calculation gives deformations that essentially bring the points for the lighter and the heavier nuclei together on a common curve when \(\beta\) is plotted as a function of \(\beta_{\text{rig}}\). Column five of Table VII compares the deformations taken from such a curve for the barium nuclei (using the experimentally determined \(\beta_{\text{rig}}\)) with other deformed nuclei in the rare earth and actinide regions.

From such a comparison it would appear that \(^{126}\text{Ba}\) is similar to \(^{186}\text{W}\), \(^{186}\text{Os}\), and \(^{224}\text{Ra}\) in deformation; likewise \(^{128}\text{Ba}\) compares with \(^{188}\text{Os}\) and \(^{222}\text{Ra}\). Also, the trend is toward increasing deformation with decreasing mass number, as expected for this region.

c. A method presented by Migdal\(^{28}\) permits one to study superfluidity for finite-size systems. It is argued that direct experimental proof that nuclear matter is in a superfluid state is provided by the fact that experimental moments of inertia are only one-third to one-half those
calculated from the formula for the moment of inertia of rigid bodies (1). Therefore, Migdal calculated moments of inertia of nuclei based upon the present theory of superfluidity of Fermi systems. The general relationship obtained is

\[ S = \frac{2}{5} A M R_0^2 \times (2.33 B_{\text{Mig.}} - 0.1) \]  

(5)

Deformations calculated by using (5) and moments of inertia \( S = 3h^2/E_{2^+} \) based on the energy of the first excited state, \( E_{2^+} \), are shown in column six of Table VII. As in (1) and (4), \( R_0 = 1.2 \times 10^{-13} \text{ } \text{A}^{1/3} \text{ cm} \).

Again the trend is toward higher deformation for the barium nuclei as we proceed away from the closed shell of 82 neutrons.

C. Collective or Symmetric Rotor Model

The collective model of Bohr and Mottelson predicts to first order the energy levels of deformed nuclei as functions of \( I \) from the relation

\[ E_I = E_0 + \frac{\hbar^2}{2I} \left[ I(I+1) + a(-1)^{I+1/2}(I+1/2) \delta_{K,1/2} \right] \]

in which \( E_0 \) is a constant depending only on the intrinsic structure. The last term involving \( a \), the decoupling constant, is present only for the special case in which \( K = 1/2 \).

For even-even nuclei in which the ground state has \( I = K = 0 \) the energy-level pattern is especially simple. For this special case the rotational spectrum is given by

\[ E_I = \frac{\hbar^2}{2I} I(I+1), \]  

(6)

with only even spin states and even parity allowed. Thus Eq. (6) would
predict a rotational band with the energy ratios

\[ \frac{E_4}{E_2} = 3.33; \frac{E_6}{E_2} = 7.00; \text{ etc.} \]

In the very strongly deformed regions these relationships hold very well and provide one of the strong confirmations of the collective model.

However, deviations from the strong-coupling limit—that is, incomplete separation of rotational from vibrational and intrinsic motion—imply corrections to the rotational spectrum (6). Some of these corrections have the same \( I \) dependence as shown above, and thus affect only the moment of inertia \( \beta \). Others, however, are similar to the rotation-vibration interaction in molecules and involve higher powers of \( I \). This tends to distort the rotational spectrum downward, resulting in a lowering of the energy ratios predicted above. As in the molecular case, where this distortion involves a centrifugal stretching, so also in nuclei it implies that \( \beta \) increases somewhat with \( I \) because of a similar stretching. This effect has been accounted for by introducing a second term in the above equation for \( E_I \), giving

\[ E_I = \frac{h^2}{2I} I(I+1) + B I^2 (I+1)^2, \]

where \( B \) is a constant to be evaluated empirically from the energies of the first three members of the rotational band. \( B \) is always negative for ground-state rotational bands of even-even nuclei, and in the strongly deformed actinides produces a correction of \( \leq 1\% \) to the energy spectrum.

Table VIII compares the experimentally observed energy levels in barium with those calculated by use of the above formalism. It is obvious that the collective model by Bohr and Mottelson, even with the rotation-vibration interaction term included, is completely incapable of reproducing the level systematics of Ba\(^{126}\) and Ba\(^{128}\). Some even-
Table VIII. Comparison of experimental energy levels with predictions of the symmetric and asymmetric rotor models. (all values in keV)

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Experimental</th>
<th>Theoretical</th>
<th>Rigid asymmetric rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rotational band</td>
<td>Vibrational levels</td>
<td>Symmetric rotor with vibration-rotation correction term</td>
</tr>
<tr>
<td></td>
<td>2+ 4+ 6+ 8+ 2+</td>
<td>2+ 4+ 6+ 8+ ( N ) R</td>
<td>2+ 4+ 6+ 8+ 2+ R</td>
</tr>
<tr>
<td>Ba(_{136})</td>
<td>856 716 1341 - -</td>
<td>(256) (716) 1053 -</td>
<td>45.6 0.469</td>
</tr>
<tr>
<td>Ba(_{138})</td>
<td>279 764 1384 - -</td>
<td>(279) (764) 1060 -</td>
<td>50.1 0.592</td>
</tr>
<tr>
<td>Os(_{186(a)})</td>
<td>137.2 437 869 - 758i</td>
<td>(1372) (437) 965.7 -</td>
<td>23.3 0.073</td>
</tr>
<tr>
<td>Os(_{188(a)})</td>
<td>255 478 - - 533</td>
<td>(155) (478) - -</td>
<td>26.7 0.136</td>
</tr>
<tr>
<td>Os(_{190(b)})</td>
<td>196.7 547.9 1048 1670 558.1</td>
<td>(196.7) (547.9) 905.0 977.4 38.71 0.3698</td>
<td>(196.7) (547.9) 1042 1673 531 23.3°</td>
</tr>
<tr>
<td>Ra(_{222(a)})</td>
<td>121 310 642 - -</td>
<td>(121) (310) 455 -</td>
<td>19.8 0.224</td>
</tr>
<tr>
<td>Ra(_{224(c)})</td>
<td>94.5 233 - -</td>
<td>(94.5) (233) - -</td>
<td>14.7 0.1076</td>
</tr>
<tr>
<td>Ra(_{226(c)})</td>
<td>67.76 210 446 - -</td>
<td>(67.76) (210) 309 -</td>
<td>11.64 0.057</td>
</tr>
</tbody>
</table>

(a) Experimental data taken from R. K. Sheline, reference 34.
(b) Experimental data taken from R. Chanda et al. reference 35.
(c) Experimental data taken from E. K. Hyde, reference 6.
even nuclei of osmium and radium, which like the bariums are six protons removed from a closed shell, are included for comparison. These nuclei are considered to be at the edges of the regions of deformation $150 < A < 190$ and $A > 222$ respectively. It is noted that equally poor fits are obtained with these nuclei as with the bariums.

Figure 29 presents the data in the form of the ratio $E_1/E_{2+}$ (the ratio of the energy of a state of spin I to that of the first excited $2^+$ level) as a function of neutron number. Several distinguishing features are evident. The sharp rise in deformation that occurs at 88 to 90 neutrons in the rare earth region is shown clearly; the $E_4/E_{2+}$ ratios for $\text{Ba}^{126}$ and $\text{Ba}^{128}$ lie in this same range. Also, Ra$^{222}$ and $\text{Ba}^{126}$ have equal $E_4/E_{2+}$ ratios. Recent data$^{36}$ show that the xenon isotopes lie much lower than the bariums, as expected (see also Fig. 30). A relatively large increase in the energy ratio occurs in going from xenon ($Z = 54$) to barium ($Z = 56$); it is similar to that for the transition from $Z = 78$ to $Z = 76$ in the platinum-osmium nuclides. In both cases $Z$ is changing from 4 to 6 protons removed from a closed shell and thus into the beginning of a deformed region.

D. Asymmetric Rotor Model

Although it was once thought that the collective or symmetric rotor model predicted the energy levels for strongly deformed nuclei—i.e. $E_{4+}/E_{2+} > 3.27$—within experimental error$^{37}$ by use of Eq. (7), it has recently been shown$^{38}$ in several cases that greater deviations from (7) occur even for $E_{4+}/E_{2+} > 3.27$. These discrepancies have been partially explained with a model which does not require axial symmetry as does the Bohr-Mottelson theory.

This asymmetric rotor theory, first proposed by Davydov and Filippov,$^{19}$ considers every even-even nucleus to have an ellipsoidally deformed shape and rotations are performed while holding the shape fixed, i.e. no internal motions are considered. It is shown that
Fig. 29. Energy level ratios $E_1/E_2$ (ratio of the energy of a state of spin I to that of the first excited 2+ level) of even-even nuclei above the neutron closed shell of 50 and above proton number 54. The vertical dashed lines indicate the neutron shell closures of 50, 82, and 126. Not all even-even nuclei in this region have been included; the symbols for those nuclei of special interest here are indicated for each of the three regions separated by the closed neutron shells. References: most of the data are taken from Nuclear Data Sheets (National Academy of Sciences-National Research Council, Washington, D.C.). In addition, Rn$^{218}$ is taken from F. Stephens, Jr., F. Asaro, and I. Perlman, Phys. Rev. 119, 796 (1960); the data for the Xe nuclei is from N. Lark and H. Morinaga (ref. 36).
Fig. 30. Energy levels of even-even Xe and Ba nuclei plotted vs. $I(I+1)$. The Xe data is from Lark and Morinaga (ref. 33) obtained from direct observation of the gamma rays emitted in He$^+$ bombardments on various enriched tellurium isotopes. Note that the levels of Ba$^{126}$ and Ba$^{128}$ lie on smooth curves and fall below the Xe levels.
while violation of axial symmetry only slightly affects the rotational spectrum of the axial nucleus some additional rotational states appear. These have spin and parity $2^+, 3^+, 4^+, 5^+,$ ... and are just those levels which in the symmetric rotor theory are interpreted as members of the rotational band built on the $2^+$ gamma vibrational level.

The two parameters, $\beta$ and $\gamma$, of Bohr and Mottelson$^1$ are again used to characterize the shape of the nucleus. In the limit of the non-axiality parameter $\gamma \to 0^\circ$ the model predicts spheroidal nuclei whose energy levels correspond approximately to an $I(I+1)$ relationship. As $\gamma \to 30^\circ$ predictions of the model correspond to nuclei in the vibrational region of Scharff-Goldhaber and Weneser.$^3$ In between the above values the spectra vary smoothly with $\gamma$ and embody fairly well the observed spectra in the transitional regions. Thus determination of a value of $\gamma$ from experimental energy levels can provide an estimate of the degree of deformation of a nucleus when compared with the $\gamma$ of nuclei with known deformations.

The usual estimates of gamma are made from the ratio of the energy of the second $2^+$ state to that of the first $2^+$ state. However, in the absence of an experimental determination of the second $2^+$ level, the $4^+/2^+$ energy ratio may be used. This latter ratio is not as sensitive to variations in $\gamma$ as the former (see Fig. 1, reference 19), but in this case where experimental uncertainties are comparatively large the small difference obtained in the two cases is insignificant.

Table VIII shows the predictions of the rigid asymmetric rotor theory for Ba$^{126}$ and Ba$^{128}$ based on the $4^+/2^+$ energy ratios. Other nuclei are included for comparison. Values of $\gamma$ are taken from the tables of Day, Klema, and Mallmann$^4$ with the non-adiabaticity parameter $\mu=0$, i.e., no correction is made for the beta-vibration-rotation interaction. A value of $\gamma=26.1^\circ$ reproduces relatively well the level spectrum of Ba$^{126}$; the fit for Ba$^{128}$ is only fair at best using $\gamma=28.1^\circ$. 
It should be noted that using the generalized theory of Davydov and Chaban, where the beta-vibration-rotation interaction is taken into account and thus three parameters, $\beta$, $\gamma$, and $\mu$, are available, that one needs a second excited $2^+$ level in order to obtain a unique fit to the model. We can attempt to estimate a value of $\gamma$ and $\mu$ for $^{126}$Ba and $^{128}$Ba by assuming that these bariums will be comparable to $^{190}$Os in deformation. Even using the $4^+/2^+$ and $2^+/2^+$ energy ratios for $^{190}$Os does not provide a unique $\gamma$ and $\mu$ for both ratios. For the former $\mu=0.32$ at $\gamma=20^\circ$ and $\mu=0.49$ at $\gamma=20^\circ$. For $\gamma=20^\circ$, $^{126}$Ba is fit with $\mu=0.38$ for the $4^+/2^+$ ratio; similarly for $\gamma=20^\circ$, $^{128}$Ba is fit with $\mu=0.42$. Thus although the second excited $2^+$ level in these bariums is lacking and a quantitative comparison with the Davydov-Chaban model is not possible, the different values of $\mu$ for $^{126}$Ba and $^{128}$Ba obtained from a comparison with $^{190}$Os show qualitatively the expected trend. That is, as we move further from the neutron closed shell of 82 the vibration-rotation-interaction is becoming smaller. In addition, the rigid rotor model indicates that as seen from other arguments $^{126}$Ba is similar in deformation to Ra$^{222}$ and Os$^{190}$.

E. Summary

Calculation of deformations expected in the neutron deficient rare earth region show definitely that an extended region of deformation does exist. Further, comparisons based on the first excited state energies of the barium nuclei would indicate that these nuclei, in particular $^{126}$Ba, are similar to $^{190}$Os, Ra$^{222}$, and Ra$^{224}$ which are generally considered to represent the beginning of the regions of deformation $150 < A < 190$ and $A > 222$ respectively. The higher rotational band energy ratios also indicate that $^{126}$Ba and $^{128}$Ba are similar in deformation to the region of 83 to 90 neutrons in the rare earths where a rather abrupt change from spherical to spheroidal nuclei occurs; a similar comparison may be made with Ra$^{222}$. 
Swiatecki has recently proposed a semi-empirical mass formula based on the liquid drop model from which deformations may be calculated. Although the formula employs but three parameters and was designed to predict mainly overall trends rather than accurate deformations of particular nuclides it is interesting to note that results predicted by this formula show rather good agreement with other data. In its present form this mass formula predicts that the barium isotopes should be deformed beginning with mass $130$. In addition, $Xe_{124}$ and $Rn_{220}$ are predicted to be deformed, although radon, for example, is generally considered to be transitional in character. With a modification, this apparent anomaly can possibly be removed without affecting the predictions for the higher mass regions.

It thus appears from the several experimental and theoretical arguments presented that the barium nuclei, in particular $Ba_{126}$, do indeed represent the beginning of a new region of deformation.
V. CONCLUSION

The experiments leading to evidence for the existence of this new region of deformation should be just the beginning of observations in this region. Not only are measurements needed on spins and parities of levels reported here but many more nuclei are amenable to observation using methods similar to those used here. In addition to studying other even-even isotopes of the neutron deficient-rare earths identification of energy levels in odd-A nuclei, although somewhat more difficult experimentally, would be particularly informative. By fitting these levels to S. G. Nilsson orbitals direct estimates of deformation would be possible.

Severe limitations are imposed on the number of nuclei available for study by requiring a time-consuming radiochemical separation prior to counting. In order to study the regions of very large deformation which also will have very short half-lives techniques will be required in which measurements are either made simultaneously with the production of nuclei in the accelerator or very shortly (seconds or less) thereafter.

The ideal arrangement of course would be to measure conversion electron spectra in a beta-spectrometer in which the target serves as the spectrometer source. Such a method has been used for example by Diamond and Stephens in a study of the light thallium isotopes. Thus not only can transition energies be measured very accurately but multipo larities of transitions can be obtained from the various conversion electron sub-shell ratios. Since measurements can be made with the accelerator beam on, theoretically all nuclides could be studied that are capable of being produced in the accelerator.

However, other less sophisticated techniques can also be used to great advantages. For example, a method similar to one used by Macfarlane and Griffioen to study α-emitters with half-lives as short as 10 μsec might prove useful. In this method the recoil products are stopped in helium gas and pumped through a collimator onto a collection plate which is then,
turned 180° to face a solid state α-counter. Backgrounds in a NaI(Tl) crystal would be much too high if used near the bombarding area; however, after collection of the recoils the collection plate could be rapidly moved via mechanical means to a low background counting area for γ-analysis. Such a system, or a modification thereof, should allow gamma ray studies into areas where half-lives are the order of a few seconds.

Since the work described in this dissertation has only "touched" the edge of this new region of deformation it is hoped that similar studies will continue utilizing possibly techniques similar to those briefly mentioned above. As experimental data obtained from studying nuclides in this new region of deformation accumulates, it will be interesting to continually compare this data with that of the other deformed regions and to note the differences and similarities.
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REFERENCES

10. The crystals and photomultipliers were obtained as integral units from Harshaw Chemical Company, Cleveland, Ohio.
12. For further details concerning the electronics described below see Radiation Laboratory Counting Handbook, UCRL-3307 (Rev.), May 1961.


36. N. Lark and H. Morinaga, private communication from N. Lark (Raymond College, Stockton, California).


44. W. J. Swiatecki (Lawrence Radiation Laboratory), private communication.


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