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requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

Angelo Christopher Limnios

June 2015

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Table of Contents

List of Figures vi

List of Tables x

Acknowledgments xii

1 Introduction 1

2 Can frictions in the housing market help explain frictions in the labor market? 3
  2.1 Introduction ............................................. 3
  2.2 The model economy ....................................... 9
    2.2.1 Value functions .................................... 12
    2.2.2 The surplus sharing rules ............................ 20
    2.2.3 The equilibrium wage rate ........................... 21
    2.2.4 The equilibrium rental rate ......................... 23
    2.2.5 The endogenous commute cutoff ..................... 23
    2.2.6 The agent’s law(s) of motion ....................... 28
    2.2.7 Labor market tightness ............................. 30
    2.2.8 Rental market tightness ............................. 30
    2.2.9 Matching functions ................................ 31
    2.2.10 Ancillaries ........................................ 33
  2.3 Empirical analysis ...................................... 35
    2.3.1 U.S. labor and housing market facts ............... 35
    2.3.2 Parameterization .................................... 51
    2.3.3 Steady-state values ................................ 53
  2.4 Numerical analysis ..................................... 55
    2.4.1 Numerical Result #1: Labor Productivity & Renter Bargaining Power ............................................. 58
    2.4.2 Numerical Result #2: Labor Separation & Renter Bargaining Power ........................... 59
    2.4.3 Numerical Result #3: Labor Bargaining Share .................. 61
    2.4.4 Robustness check: On the time-invariance of the bargaining shares 63
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.3.1</td>
<td>Parameterization of unemployment benefits $b$</td>
<td>171</td>
</tr>
<tr>
<td>A.3.2</td>
<td>Parameterization of renter separation $\sigma^H$</td>
<td>172</td>
</tr>
<tr>
<td>A.3.3</td>
<td>Rent-to-income ratio</td>
<td>174</td>
</tr>
<tr>
<td>A.3.4</td>
<td>Persistence parameters on the processes for $\hat{y}$, $\sigma^E$ and $\bar{v}$</td>
<td>174</td>
</tr>
<tr>
<td>A.3.5</td>
<td>Rental vacancy rate</td>
<td>177</td>
</tr>
<tr>
<td>A.4</td>
<td>Numerical appendix</td>
<td>178</td>
</tr>
<tr>
<td>A.5</td>
<td>Estimation appendix</td>
<td>178</td>
</tr>
<tr>
<td>B</td>
<td>Appendix to chapter 3</td>
<td>182</td>
</tr>
<tr>
<td>B.1</td>
<td>Linear approximation of the model</td>
<td>182</td>
</tr>
<tr>
<td>C</td>
<td>Appendix to chapter 4</td>
<td>188</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Illustration of the flows of agents, firms and landlords in the economy. The dashed line is there to indicate the endogenous entry of both firms and landlords. Unemployed agents immediately enter the labor market and either match and become employed without a residence, or become unemployed again. Agents who are employed without a residence attempt to find a landlord in the rental market. If successful, they move on to being employed and with residence type agents. At all times, agents who are matched with landlords may separate from their homes, and/or separate from their employer. If agents separate from employment in any state, they start the next period as unemployed.  

2.2 Source: Seasonally adjusted series from U.S. Department of Labor: Bureau of Labor Statistics. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.  

2.3 Source: Seasonally adjusted series from U.S. Department of Labor: Bureau of Labor Statistics. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.  

2.4 Source: Series for short-term unemployed and unemployed come from U.S. Department of Labor: Bureau of Labor Statistics. Job-finding rate constructed from those two series using equation (E1). Trend line is an HP filter of the series with smoothing parameter $\lambda = 10^5$.  

2.5 Source for vacancy-unemployment ratio data: U.S. Department of Labor: Bureau of Labor Statistics; Job-finding rate taken from equation (E1). Both labor market tightness and the job finding rate have been detrended using an HP filter with smoothing parameter $\lambda = 10^5$.  

2.6 This series was put together using equation (E2); source for the data used is U.S. Department of Labor: Bureau of Labor Statistics. Trendline constructed using an HP filter with smoothing parameter $\lambda = 10^5$.  

11 36 37 40 41 43
2.7 The series is real output per person in the U.S. manufacturing sector against its HP filter trend with smoothing parameter $\lambda = 10^5$. The series is given as an index. Prior to filtering, a cubic spline algorithm (see footnote #36) was used to expand the original quarterly series to a monthly frequency.

2.8 Both series expressed as log deviations from HP filter trend with smoothing parameter $\lambda = 10^5$. Source for both series is U.S. Department of Labor: Bureau of Labor Statistics. Note that there are two axes for both series to account for the large difference in their volatilities.

2.9 Source for the data used is Zillow. Trendline constructed using an HP filter with smoothing parameter $\lambda = 10^5$. Data for this series is available from October 2010.

2.10 Privately Owned Rental Housing Starts in the United States (Purpose of Construction: Built for Rent Two or More Units, Thousands of Units, Quarterly) and Real Output Per Hour of All Persons in the Manufacturing Sector (Index 2009=100, Quarterly). In this graph, the housing series is lagged by 2 quarters. Both series are expressed as log-deviations from trend (trend was constructed using an HP filter with smoothing parameter $\lambda = 10^5$).

2.11 Impulse responses for unemployment, vacancies, tightness and the job-finding rate following a negative labor productivity shock (the standard error of the innovation to $y$ is 0.01, a value I use throughout the numerical simulations in order to match the empirical standard deviation and autocorrelation as closely as possible). The black impulse responses are indicative of constant labor bargaining shares, while blue impulse responses illustrate a situation with an endogenous labor bargaining weight.

2.12 Empirical distribution for the errors of the AR(1) model for the log-deviation of output.

2.13 Empirical distribution for the errors of the AR(1) model for the log-deviation of labor market separations.

2.14 Empirical distribution for the errors of the AR(1) model for the log-deviation of renter bargaining power.

2.15 Impulse responses illustrating a similar relationship between Wheaton’s results with the model of this chapter. Plots show the impulse responses for the price of housing (rental rate) and the time-to-rent (Wheaton’s version of “time-to-sell” - the inverse of the housing matching rate) as a result of a shock to $\sigma^H$.

3.1 Impulse response functions resulting from a one-time shock to aggregate technology; Impulse responses indicated with blue circles result from a persistence level of $\rho_z = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_z = 0.90$. 

vii
3.2 Impulse response functions resulting from a one-time shock to aggregate technology; Impulse responses indicated with blue circles result from a persistence level of $\rho_z = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_z = 0.90$.

3.3 Impulse response functions resulting from a one-time shock to the exogenous rate of credit match destruction; Impulse responses indicated with blue circles result from a persistence level of $\rho_\delta = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_\delta = 0.90$.

3.4 Impulse response functions resulting from a one-time shock to the exogenous rate of credit match destruction; Impulse responses indicated with blue circles result from a persistence level of $\rho_\delta = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_\delta = 0.90$.

4.1 The time path of the regression coefficient for Okun’s law resulting by “rolling” a sliding interval of 100 observations. 95% confidence bands included.

4.2 The time path for the residuals from the Okun’s Law regression. Note the pattern of residual trajectories during all of the NBER recessions and how the pattern was reversed during the Great Recession.

4.3 Source: Authors regression residuals. Data series for the civilian unemployment rate and the long-term and short-term natural rate of unemployment come from FRED. GDP data comes from FRED, while potential GDP is the Hodrick-Prescott filtered series of the same GDP data with standard $\lambda = 1600$.

4.4 Source: Same as figure 2.

4.5 Implied series (in percentage deviation) for the firm’s implicit cost to posting a vacancy.

4.6 The series for the St. Louis Financial Stress Index and our series for v-hat. Source: FRED.

A.1 Source: U.S. Census Bureau, Current Population Survey. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.

A.2 Time series of the series for the log-deviation of output and the model fit for this process. Line widths for the series have been lightened to illustrate the extent of the fit.

A.3 Time series of the log-deviation of labor match separations and the model fit for this process.

A.4 Time series of the empirical log-deviation of the renter’s bargaining weight and the model fit for this process.
A.5  Source: U.S. Department of Commerce: Census Bureau. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.  

A.6  Impulse responses for key labor market variables following an orthogonalized shock to labor productivity. Red (dashed) indicates a model response without housing, while black indicates a model response with housing.  

A.7  Impulse responses for key labor market variables following an orthogonalized shock to labor match separations. Red (dashed) indicates a model response without housing, while black indicates a model response with housing.  

A.8  Impulse responses for key labor market variables following an orthogonalized shock to labor bargaining weight. Red (dashed) indicates a model response without housing, while black indicates a model response with housing.  

A.9  Prior and estimated posterior distributions for parameters which underwent the Bayesian estimation.  

A.10 Prior and estimated posterior distribution for persistence on the bargaining power of labor.
List of Tables

2.1 Labor Market Summary Statistics, Monthly U.S. Data, 2000 - 2014. The data for unemployment $u$, vacancies $v$, and productivity $y$ are taken from the U.S. Department of Labor: Bureau of Labor Statistics. The job-finding rate was constructed using equation (E1), while separations $\sigma^E$ was constructed using equation (E2). All variables are expressed as log deviations from an HP filter trend with smoothing parameter $\lambda = 10^5$. 48

2.2 Parameter values set. Descriptions with an asterisk indicate parameterizations without an empirical justification. 53

2.3 Steady-state values for endogenous variables. 55

2.4 Simulation results for the labor model under shocks to productivity and renter’s bargaining power. Values in blue (top row) are the empirical values from Table 1 reproduced for convenience of comparison, values in red (middle row) indicate the values for when the housing side of the model is turned off (becomes a basic, stochastic version of DMP), while values in black (bottom row) are the values for when the housing side of the model is active. Since housing is absent from the baseline model, the N/As indicate that the renter’s bargaining power would be non-applicable. 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments. 70

2.5 Simulation results for the labor model under shocks to labor match separation and rental surplus bargaining. Values in blue (top row) are the empirical values from Table 1 reproduced for convenience of comparison, values in red (middle row) indicate the values for when the housing side of the model is turned off (becomes a basic, stochastic version of DMP), while values in black (bottom row) are the values for when the housing side of the model is active. Since housing is absent from the baseline model, the N/As indicate that the renter’s bargaining power would be non-applicable. 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments. 71
2.6 Simulation results for the labor model under shocks to labor bargaining. Values in blue (top row) are the empirical values from Table 1 reproduced for convenience of comparison, values in red (middle row) indicate the values for when the housing side of the model is turned off (becomes a basic, stochastic version of DMP), while values in black (bottom row) are the values for when the housing side of the model is active. The N/As indicate that there is no empirical data available for the standard deviation, monthly autocorrelation and empirical correlation with other variables for the labor bargaining weight. 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments.

2.7 This table reports the prior and posterior distribution of the estimated parameters of the structural parameters and parameters of the exogenous shock processes. IGamma refers to the Inverse-gamma distribution.

3.1 Parameter values.

3.2 Steady-state targets.

A.1 Labor Market Summary Statistics, Monthly U.S. Data, 2000 - 2014, using aggregate output $y$ instead of output from the manufacturing sector. All variables are expressed as log deviations from an HP filter trend with smoothing parameter $\lambda = 10^5$.

A.2 Labor Market Summary Statistics sourced from Shimer (2005). All variables are expressed as log deviations from an HP filter trend with smoothing parameter $\lambda = 10^5$.


A.4 Examples of unemployment benefit - wage ratios for various states.

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Chapter 1

Introduction

The focus of this dissertation is in the application of three variations of the canonical Diamond-Mortensen-Pissarides\(^1\) (DMP) search and matching model of the labor market to answer questions pertaining to the economics of labor and housing markets (Ch. 2), credit markets (Ch. 3), and to the dynamics of labor productivity (Ch. 4).

In chapter 2, I augment the (DMP) search and matching model of the labor market with a rental housing market characterized by search and matching frictions, integrating both markets into a single, coherent macroeconomic model. The resulting framework is then used to study how frictions in the housing market spill over into the labor market. Agents search and match with employers and once an employment match is established, turn to the rental market in order to secure housing. The chapter concludes with three numerical simulations of the augmented model, including shocks

\(^{1}\text{Diamond (1982), Mortensen (1982), Pissarides (1985); this framework has emerged as the benchmark method to model labor markets.}\)
to labor productivity, the rate at which labor match separate, and the labor bargaining weight, demonstrate a substantial increase in the ability of the housing-augmented model to match recent (2000 - 2014) empirical labor market data over a model with labor frictions alone.

Chapter 3 studies a channel system for implementing monetary policy when bank lending is subject to frictions. These frictions affect the spread between the policy rate and the loan rate. We show how the width of the channel and the nature of random payment flows in the interbank market also affect the spread and therefore the transmission of monetary policy to credit markets. Simulation exercises are used to conduct a series of policy experiments.

The fourth chapter of this dissertation implements a variation of the DMP model in order to analyze the implied productivity of existing and new employment matches during the wake of both the early 2000s recession and the Great Recession. We find that surviving and new employment matches during the Great Recession exhibit a productivity level 2.16 times higher than those during the early 2000s recession. We show that the increased idiosyncratic productivity threshold in the newly-formed matches was largely the result of the increase in the labor financing costs facing firms originating from disruptions to credit markets during the Great Recession.
Chapter 2

Can frictions in the housing market help explain frictions in the labor market?

2.1 Introduction

In this chapter, I augment the basic DMP labor search model with a housing market, which is also characterized by search and matching frictions. This results in the relaxation of the assumption that labor markets are friction-less in the spatial dimension, so that distance-to-work becomes an important characteristic of employment to a prospective worker. This chapter provides an answer to the research question of how and to what extent can frictions in the housing market spill over into the labor market. Under empirically calibrated shocks to labor productivity, separation of labor matches and bargaining in the housing market, the augmented model is able to account for higher levels of volatility in key labor market variables over the baseline model.
without housing.

In the model, once an unemployed agent secures a labor match, they turn to a rental housing market to secure a housing unit to rent.\textsuperscript{1} When an unemployed agent matches with a firm, the wage is derived via Nash bargaining between counterparties over the match surplus. Symmetrically, when the now-employed agent is looking for a rental unit, they search in a decentralized housing market, match with a landlord, and bargain over the (match) surplus to determine the rental rate. A new feature in the housing market of this model is that housing units differ only in their distance from the renter’s place of employment, with more proximal units (units with a shorter commute) garnering a higher match value to the agent; since the renter’s distance to work cannot surpass an endogenously determined commute cutoff, this implies that only a subset of housing matches will progress to the bargaining stage. An additional key, new result of this model is that when a firm and worker bargain over the surplus of a new employment match, a portion of the employment match surplus is made up of the agent’s anticipated housing match surplus (since the next step for the new employee is to locate a rental unit). This implies that the firm then takes expected future housing market conditions into consideration before posting a job vacancy - this new driving factor is contingent on the extent, via the bargaining weight, to which the firm can extract anticipated housing market value from the agent during the wage-bargaining process. Since the

\textsuperscript{1}This specification for the timing follows urban models such as Alonso (1960) where employment locations were predetermined and assumed to be located in a central business district, resulting in a “negative rent gradient”. As I ignore saving and/or credit in the model presented in this chapter, the timing relies on the assumption that agents need to secure employment prior to housing since the wage will be used to pay their housing costs. This timing assumption greatly increases the tractability of the model.
labor side of the model anticipates/takes expectations over what occurs on the housing side, shocks/movements to the housing market manifest themselves as real labor market effects.

This chapter provides two contributions. The first is an integration of a labor market with a housing market, both characterized by search and matching frictions, into a single, coherent macroeconomic model which combines three branches of the literature: the labor search literature, as highlighted by this model’s decentralized labor market; the housing search literature, as highlighted by this model’s decentralized housing market; and the urban economics literature, as highlighted by this model’s endogenous commute cutoff delineating where agents choose to allocate themselves spatially as a direct result of what occurs in the other two markets.

The second contribution is that the integration of the housing market into the labor market increases the ability of the resulting model to match the empirical moments of recent labor market data. In a stochastic simulation of the model undergoing shocks to labor productivity, the simulated standard deviations for unemployment, vacancies, labor market tightness, and the job finding rate come much closer to matching their empirical counterparts (Table 1, pg. 28) than the baseline model without housing. In a stochastic simulation of the model undergoing shocks to labor match separations, the simulated standard deviations for labor market tightness, and the job finding rate come much closer to matching their empirical counterparts than the baseline model without housing. Additionally, in the simulation under labor separation shocks, the housing-augmented model provides a correct coefficient (negative) on the correlation between
unemployment and vacancies\textsuperscript{2}, a fairly important relationship the baseline model fails to produce.

The timing of the labor-housing model presented in this chapter\textsuperscript{3} follows early urban models such as Alonso (1960). The Alonso model assumes a predetermined workplace and provides a framework which explains the “negative rent gradient”: the inverse relationship between rents paid by households and commute distance (accessibility to the central business district). Coulson and Engle (1987) and Tse and Chan (2003) tested whether or not a rent gradient exists using housing and commute data with both papers concluding that increases in transportation costs raise the price of centrally-located housing.

This chapter relates to many previous attempts to improve the fit of the canonical DMP model of the labor market following Shimer’s (2005) famous critique. Shimer (2005) outlined the empirical shortcoming of the DMP labor market search model by demonstrating that in order for the model to match the empirical volatilities in unemployment, job vacancies, and labor market tightness, it would have to be subjected to shocks which are empirically implausible. Specifically, shocks to labor market productivity, separation, and the bargaining weight would have to be unrealistically high. Silva and Toledo (2009) augment the DMP model with labor turnover costs and endogenous destruction of matches. Similar to the model of this chapter, Silva and Toledo assume 3

\textsuperscript{2}This is a well-known shortcoming of the baseline labor model; Shimer 2005 highlighted how the correlation coefficient on unemployment and vacancies following a separation shock in the baseline model is positive, which has never occurred in the empirical data

\textsuperscript{3}Timing here refers to residential search following an employment search or predetermined employment location. For a theory of workplace location with respect to a predetermined place of residence, see Siegel (1975) and Simpson (1980).
types of agents: unemployed, entrant, and incumbent. Incumbants are more productive than entrants, and if a firm were to lay off an incumbent, they face a turnover cost. While these extensions produce fluctuations in vacancies and labor market tightness of more than 2 times what the standard model is able to generate, the volatility of labor market tightness still falls short of what is seen empirically in the data.

The structure of the model of this chapter is motivated by models such as those in Wasmer and Weil (AER 2004) and Petrosky-Nadeu and Wasmer (2013) where agents, namely entrepreneurs, have three “stages” they are in at any one point: a credit stage to seek to match with a bank, followed by a labor stage to seek to match with a worker, followed by a production stage. The agents in the model of this chapter also progress between three states/stages, however, in both Wasmer and Weil (2004) and Petrosky-Nadeu and Wasmer (2013), the tightness of their credit and labor markets was constant, whereas in the model in this chapter, tightness in both labor and housing markets fluctuates to reflect relative supply and demand conditions in each market.

Search models much in the flavor of DMP have also been used to model housing markets as early as Wheaton (1990). In that paper, the author uses a simple search and matching model (housing search exclusively) to explain the existence of structural vacancy in the housing market. Wheaton models an owner-occupied housing market, while this chapter models a rental market. In his model, agents search for better housing matches when their current housing no longer meets their requirements. Agents secure another home (some households end up holding onto two homes at once) and then sell their mismatched home. Since the research question involved explaining the structural
vacancy rate in housing, the labor market is ignored. Rupert and Wasmer (2012) integrates a DMP model of the labor market with housing by assuming each job offer to an agent comes with a commute distance drawn from a distribution and by relaxing the assumption that agents are indifferent between employment locations. The authors focus on the trade-off between commuting time and locational decisions and use that model to explain the mobility patterns of US workers and their European counterparts. Ruppert and Wasmer assume that the employment locations themselves serve as housing units and agents incur no specific housing costs except for one related to commute; while the labor market is modelled using DMP, the housing market is essentially ignored. In the model of this chapter, the housing market is modelled symmetrically to the labor market with the addition that an endogenous commute threshold emerges which eliminates some of the homes searching agents find as too distant from their employment.

A discussion towards the end of the numerical section outlines how my model provides new interpretations of some of the main results/model propositions\(^4\) of Wheaton.

The model is constructed in the following section, an in-depth empirical analysis of the labor market and housing market, including the calibration of the model, is carried out in section 3, section 4 reports the numerical results, a robustness check, and estimation results, and section 5 concludes.

\(^4\)My model is consistent with each of the important propositions, even though Wheaton models a housing market exclusively.
2.2 The model economy

The economy is populated by a representative agent (henceforth referred to as the agent), a representative firm (henceforth referred to as the firm), and a representative landlord (henceforth referred to as the landlord). The agent will at any point in time be in one of three states: unemployed without residence, employed without residence, and employed with residence. If an employment match separates, the agent immediately commences search for a new match. The timing is as follows:

1. The unemployed agent searches for a job.

2. Once the unemployed agent matches with a searching firm, they bargain over the surplus to the employment match (and implicitly the anticipated surplus to the agent’s future housing match; this is a new feature of this model) and redistribute the surplus according to an equilibrium wage.

3. Once the agent secures employment, the employed agent now searches for a residence.

4. Once the agent matches with a searching landlord, the agent draws a commute from a distribution - the shorter (longer) the commute, the higher (lower) the value of this surplus. The only difference in rental units is the commute distance to the renter’s employment location.

5. The agent and landlord then bargain over the surplus to the residence match and redistribute the surplus according to an equilibrium rental rate.
6. An endogenous commute “cutoff” marks the point where the surplus to the residence match net of the opportunity cost of other potential residences\(^5\) is non-negative. This implies that the portion of the residential matches which also satisfy the endogenous commute cutoff progress to the bargaining stage.

Once an employment match is formed, the firm and the agent always face the possibility that the match will exogenously terminate. The same holds true of the match between the (employed) agent and the landlord, implying two potentially different separation rates; if an agent happens to be renting and exogenously separates from their employment, they also separate from their residence match - the exogenous separation of employment by an agent which is also currently renting is one which is said to “destroy all specificity”. This modelling assumption follows Wasmer and Weil (2004) and Petrosky-Nadeu and Wasmer (2013)\(^6\). The agents in this economy do not save, thus if there is an employment separation, this constitutes an interruption in income which implies the agent (while unemployed) cannot meet their housing costs, resulting in a separation from their residential match. This assumption allows the model to remain very tractable while ignoring complex issues such as optimal saving and credit.

\(^5\)In this framework, the survival of the housing match not only relies on the housing separation rate, but also on the agent’s employment match not exogenously terminating. Thus the landlord and the (renting) agent need to take employment into consideration, because while renting (and employed), the agent gives up the opportunity to switch to a closer residence if conditions in the housing market improve. The landlord also gives up the opportunity to rent to a different agent which happens to come along with a shorter commute than the current one (and thus a higher housing value to contribute to the joint surplus); this opportunity cost to matching is a new feature of this model, as well.

\(^6\)Both papers study search and matching frictions in financial and labor markets; in their models, if a firm is matched with a lender and a laborer, and the match with the lender exogenously destroys itself, the match with the laborer ends up separating as well.
I will assume that the firms (landlords) endogenously post employment (residence) vacancies at constant cost. Additionally, I will assume that while firms and landlords endogenously enter their respective markets, there is a time-invariant unit mass of representative agents. Finally, I will assume that landlords do not have the option to sell their rentals and agents currently renting do not have the option to sub-lease to other agents searching for housing.

A schematic of the model is given in Figure 1 below.

Figure 2.1: Illustration of the flows of agents, firms and landlords in the economy. The dashed line is there to indicate the endogenous entry of both firms and landlords. Unemployed agents immediately enter the labor market and either match and become employed without a residence, or become unemployed again. Agents who are employed without a residence attempt to find a landlord in the rental market. If successful, they move on to being employed and with residence type agents. At all times, agents who are matched with landlords may separate from their homes, and/or separate from their employer. If agents separate from employment in any state, they start the next period as unemployed.
2.2.1 Value functions

Both the firms and the landlords are either matched or unmatched, hence there will be two value functions for each of these agents. The representative agent will be in one of three states, implying there will be three value functions for this agent - one for each state. The surpluses for the firms and the landlord will be the difference in value of each state. The agent, however, will have a set of 2 surpluses, each of which will be used in bargaining with the respective counterparty. Thus, there will be an agent “employment surplus” ("residential surplus") which will contribute towards the joint surplus of the match between the agent and the firm (landlord). Since the agent, the firm, and the landlord have access to the same information set, when the firm is bargaining with the agent for a wage, this bargaining internalizes (takes into consideration) the agent’s anticipated housing match surplus\(^7\). So, for example, if a firm (and the newly matched employee; both the agent and the firm share the same information set) expects a new employee’s future (next period’s) housing prospects to be high, the firm use this as a bargaining chip against the employee when negotiating the wage\(^8\). The corollary to this is that if an employee, going into wage negotiations, knows that their future housing prospects will be high, they won’t be as demanding regarding the wage.

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\(^7\)The agent’s anticipated housing match surplus is contingent on the tightness of the housing market and the distance of the agent’s housing match to their employment match (the agent’s commute, which is drawn from a distribution).

\(^8\)Some employers in San Francisco, for example, may pay their new employees a wage which is lower than the market wage but will “substitute” the difference by paying for the employees’ bridge toll or BART tickets or even provide a commute shuttle (Google Bus).
2.2.1.1 The firm’s value functions

An unmatched firm which has entered the labor market pays $\gamma$ to post a job vacancy, and matches with a worker with probability $p^F$. If the match then survives the exogenous separation probability $\sigma^E$, the firm enters the following period as a matched firm which receives value $V^{FM}$. If the new match separates, or if the firm fails to locate a worker, the firm enters the following period as an unmatched firm. The value function which describes this sequence of events is

$$V^{FU}_t = -\gamma + p^E_t \beta E_t \left( (1 - \sigma^E_t) V^{FM}_{t+1} + \sigma^E_t V^{FU}_{t+1} \right) + (1 - p^E_t) \beta E_t V^{FU}_{t+1} \quad \text{M1}$$

where $V^{FU}$ is the value the unmatched and searching firm receives.

A matched firm receives the output $y$ produced from the employment match and pays the agent an endogenous wage $w$. If the match survives the exogenous separation, then the firm is once again matched the following period and receives value $V^{FM}$. If the match exogenously terminates, the firm becomes unmatched and receives $V^{FU}$. The value function describing this scenario is

$$V^{FM}_t = y_t - w_t + \beta E_t \left( (1 - \sigma^E_t) V^{FM}_{t+1} + \sigma^E_t V^{FU}_{t+1} \right) \quad \text{M2}$$

The surplus to a firm is the difference in value of being matched over being unmatched. Denoting this surplus as $V^{FS}$,

$$V^{FS}_t = V^{FM}_t - V^{FU}_t$$

$$= y_t - w_t + \gamma + (1 - p^E_t) \beta (1 - \sigma^E_t) E_t V^{FS}_{t+1} \quad \text{M3}$$
The firm’s net surplus is the revenue net of labor costs plus the vacancy posting costs saved and the expected value of the net surplus in the following period.

Free entry in the labor market facing the firm implies that $V^{FU} = 0$. From (M1), the resulting entry condition is

$$\frac{\gamma_f}{p^F_t} = \beta \left( 1 - \sigma^E \right) E_t V^{F \Sigma}_{t+1},$$

(M4)

where the left-hand side represents the expected cost of a match (the vacancy posting cost scaled by the expected length of search) and the right-hand side represents the expected benefit to a match (the expected discounted value of the firm’s net surplus).

2.2.1.2 The landlord’s value functions

An unmatched landlord who has entered the housing market pays $\xi$ to post the vacancy, and matches with a searching (employed) agent with probability $p^L$. If the match then survives both the exogenous probability that the agent’s new housing match and employment match separates ($\sigma^H$ and $\sigma^E$, respectively), the agent draws a commute $\kappa$ from a distribution $G$ which has support $[\kappa, \bar{\kappa}]$. If this commute is below (above) the cutoff, the landlord enters the following period as a matched (unmatched) landlord who receives value $V^{LM}$ ($V^{LU}$). If the new match separates, or if the landlord fails to locate a searching (employed) agent, the landlord enters the following period unmatched. The

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9Separation here includes the agent surviving the housing (employment) separation, but suffering the employment (housing) separation, or suffering both.
value function which describes this sequence of events is

\[ V_{LU}^t = -\xi + p_L t \beta E_t \left\{ \left( 1 - \sigma^E \right) \left( 1 - \sigma^H \right) \int_0^\infty V_L^t (\kappa_{t+1}) \, dG (\kappa) + \left[ (1 - \sigma^E)\sigma^H + \sigma^E \right] V_{LU}^{t+1} \right\} + (1 - p_L t) \beta E_t V_{LU}^{t+1}. \]  

(M5)

A landlord who is matched receives an endogenous rental rate \( r(\kappa_t) \) and pays a fixed cost \( \lambda \) of running the rental\(^{10}\). If the match survives the agent’s exogenous employment and housing separation, then the landlord is once again matched the following period with the same commute and receives value \( V_{LM}^t (\kappa_t) \). If the agent’s employment (housing) match exogenously terminates, the landlord becomes unmatched and enters the following period receiving value \( V_{LU}^t \). The value function describing this scenario is

\[ V_{LM}^t (\kappa_t) = r(\kappa_t) - \lambda + \beta E_t \left\{ \left( 1 - \sigma^E \right) \left( 1 - \sigma^H \right) V_{LM}^t (\kappa_t) + \left[ (1 - \sigma^E)\sigma^H + \sigma^E \right] V_{LU}^{t+1} \right\}. \]  

(M6)

The surplus to being a landlord is the difference in value of being matched over being unmatched. Denoting this surplus as \( V_{LS} \),

\[ V_{LS}^t (\kappa_t) = V_{LM}^t (\kappa_t) - V_{LU}^t \]

\[ = r(\kappa_t) - (\lambda - \xi) + \beta (1 - \sigma^E)(1 - \sigma^H)E_t \left\{ \left[ V_{LM}^t (\kappa_t) - V_{LU}^{t+1} \right] - p_L t \left[ \int_0^\infty V_L^t (\kappa_{t+1}) \, dG (\kappa) - V_{LU}^{t+1} \right] \right\} \]  

(M7)

The landlord’s net surplus is the rental revenue net of operating costs\(^{11}\) plus the (expected) difference between the value facing the landlord if the agent’s employment and

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\(^{10}\)I assume this cost is the associated property management fee. More details in the calibration section.

\(^{11}\)Operating costs here are net: the value of the cost \( \lambda \) over the savings of not having to pay the vacancy posting.
housing matches survive termination and the value the landlord is giving up (scaled by the matching probability $p^L$) as a result; i.e., the continuation value is the expected benefit of carrying the current match into the future net of the expected opportunity cost of that match. I will refer to the term $E_t \left[ V^{LM}(\kappa_t) - V^{LU}_{t+1} \right]$ as the landlord’s “retention surplus” and the term $E_t \left[ \int_{\kappa} V^L(\kappa_{t+1}) dG(\kappa) - V^{LU}_{t+1} \right]$ as the landlord’s “new-renter surplus”\(^\text{12}\).

While a housing unit is unoccupied, a landlord may face upkeep costs (replacement of carpets, roof maintenance, etcetra) or may have the option to sell the unit at a capital gain/loss. I am assuming that landlords do not have the option to sell their rentals\(^\text{13}\), and I will also assume that upkeep costs and other options facing the landlord are outside of the scope of this model. These assumptions allow a free entry condition

\[ V^{L_{\Sigma}}(\kappa_t) = \frac{r(\kappa_t) - (\lambda - \xi)}{1 - \beta(1 - \sigma^E)(1 - \sigma^H)} - p^L \left( \frac{\beta(1 - \sigma^E)(1 - \sigma^H)}{1 - \beta(1 - \sigma^E)(1 - \sigma^H)} \right) E_t \int_{\kappa} V^{L_{\Sigma}}(\kappa_{t+1}) dG(\kappa), \quad (M9) \]

which, similar to (M7), states that a landlord’s net surplus to a current match with an agent with commute $\kappa_t$ is the expected duration of net renter income (the rental rate net of operating costs divided by an expression which normalizes this amount for the discount rate and the renter’s expected employment tenure). The expression to the right of the minus sign outlines the opportunity cost facing the landlord who is “stuck” with this same tenant (as a result of the agent not losing their job) - the present discounted landlord surplus associated with a new tenant (which would have a new commute draw) normalized by the same expression as the landlord’s net rental income benefit to the current match. A very nice feature of this specification is that landlords take into consideration all of the future commute draws which contribute to various surplus amounts, resulting in an optimal “trigger” strategy when settling on a particular agent which presents themselves with a commute draw. A somewhat similar feature is studied in Crane (1996) where residents take into consideration all future employment prospects when optimally picking a home; thus a whole sequence of expected future potential commutes need to be taken into account (on top of the current one), resulting in a flatter rent gradient. Equation (M9) displays a similar “flattening”, where shorter expected commutes in the future decrease the surplus associated with the current commute which then results in a decrease in the equilibrium rent during the bargaining stage.

\(^{12}\)The same entry condition can be imposed on (M7) to alternatively express the landlord’s net surplus as

\[ V^{L_{\Sigma}}(\kappa_t) = \frac{r(\kappa_t) - (\lambda - \xi)}{1 - \beta(1 - \sigma^E)(1 - \sigma^H)} - p^L \left( \frac{\beta(1 - \sigma^E)(1 - \sigma^H)}{1 - \beta(1 - \sigma^E)(1 - \sigma^H)} \right) E_t \int_{\kappa} V^{L_{\Sigma}}(\kappa_{t+1}) dG(\kappa), \]

\(^{13}\)Recall from the introduction to section 2, that the other restriction I have assumed/imposed is that agents which are currently renting do not have the option to sub-lease to other agents searching for housing.
in the housing market (symmetric to the labor market) implying landlords will post
costs until $V_{LU} = 0$. From (M5), the resulting entry condition is
\[
\frac{\xi_{p}}{p_{t}^{L}} = \beta \left(1 - \sigma^{E}\right) \left(1 - \sigma^{H}\right) E_{t} \int_{\kappa_{t+1}}^{\tilde{R}_{t+1}} V^{L_{E}}(\kappa_{t+1})dG(\kappa),
\]
where the left-hand side represents the expected cost of a match (the vacancy posting
cost scaled by the expected length of search) and the right-hand side represents the
expected benefit to a match (the expected discounted value of the landlord’s net surplus).

2.2.1.3 The agent’s value functions

Recall that the agent is always in one of three states: unemployed and without
residence, employed and without residence, and employed with residence. I denote the
value functions for each of these states as $V^{AU}$ (agent-unemployed), $V^{AEHU}$ (agent-
employed, housing-unmatched), and $V^{AEHM}$ (agent-employed, housing-matched). An
unemployed agent receives unemployment benefit $b$, and with probability $p^{J}$ matches
with a searching firm. If the match survives the exogenous separation rate $\sigma^{E}$, the
agent then enters the following period as employed and without residence. If the match
exogenously terminates, or if the agent fails to match with a firm, the agent enters the
following period as unemployed. The value function which describes this state is
\[
V_{t}^{AU} = b + \beta E_{t} \left\{ p^{J}_{t} \left[ (1 - \sigma^{E}) V^{AEHU}_{t+1} + \sigma^{E} V^{AU}_{t+1} \right] + (1 - p^{J}_{t}) V^{AU}_{t+1} \right\} .
\]

The agent who has secured employment and is now searching for a residence receives
a wage $w$ and, surviving the exogenous employment separation rate, matches with
a searching landlord with probability $p^{H}$. If this new residential match survives the
exogenous housing separation $\sigma^H$, the agent then draws a commute associated with the new residence; if the commute is below (above) the cutoff, the agent becomes employed with (without) residence the following period. If the agent matches with a landlord, survives the employment termination, but doesn’t survive the housing termination, they enter the following period as employed without residence. If the agent survives the exogenous employment separation rate but fails to match with a landlord, the agent enters the following period as employed without residence. If the agent’s employment match exogenously terminates, the agent enters the following period as unemployed.

The value function which describes this state is

$$V_{t}^{AEHU} = w_t + \beta E_t \left( (1 - \sigma^E) \left\{ p_t^H \left[ (1 - \sigma^H) \int \sigma^H V_{t+1}^{AEH} dG(\kappa) + \sigma^H V_{t+1}^{AEHU} \right] + (1 - p_t^H) V_{t+1}^{AEHU} \right\} + \sigma^E V_{t+1}^{AU} \right).$$

(M11)

The agent who is employed and matched with residence enjoys value $V_{t}^{AEHM}$ which is made up of the agent’s utility value associated with the residence (inversely related to the commute distance) net of the rent paid$^{14}$, plus the wage received from the employer, plus the discounted continuation value. If the agent survives the exogenous employment and housing separations (agent keeps the job and keeps the rental) the agent enters the following period matched with the same rental (and hence the same commute). If the

---

$^{14}$The motivation for modelling the value function in manner follows Albrecht et al. (2007). In their model, a home buyer would realize value $x - p$, where $x$ represents a (constant) utility flow value and $p$ represents the cost of housing. Wheaton (1980) implements a similar value function structure. I assume basic properties on $u$:

$$\frac{du}{d\kappa} < 0, \quad \frac{d^2 u}{d\kappa^2} > 0.$$
agent survives the employment separation, but suffers the housing match separation, they enter the following period as employed without residence. If the agent’s employment match exogenously separates, the agent enters the following period as unemployed. The value function associated with this state is

\[ V_{AEHM}(\kappa_t) = u(\kappa_t) - r(\kappa_t) + w_t + \beta E_t \{ (1 - \sigma^E) [V_{AEHM}(\kappa_t) + \sigma^H V_{AEHU}(\kappa_t + 1)] + \sigma^E V_{AU}(\kappa_t + 1) \}. \]  \hspace{1cm} (M12)

The value of the agent’s net employment surplus \( V_{AE\Sigma} \) is the difference between the value of the agent in the employed and without residence state and the unemployed and without residence state. Or,

\[ V_{t AE\Sigma} = V_{t AEHU} - V_{t AU} = w_t - b + (1 - p_t^H) \beta (1 - \sigma^E) E_t V_{t+1 AE\Sigma} + p_t^H \beta (1 - \sigma^E)(1 - \sigma^H) E_t \left[ \int_{\mathcal{K}} V^{AEH}(\kappa_{t+1})dG(\kappa) - V_{t+1 AEHU} \right]. \]  \hspace{1cm} (M13)

The agent’s net employment surplus is the wage net of the unemployment benefit plus the expected value of the net employment surplus in the following period plus a term representing the expected housing surplus in the following period, scaled by the probability of a housing match \( p^H \).

This is a key result of the framework I have set forth - (M13) illustrates quantitatively how the agent’s employment surplus also includes their future expected housing surplus. The reason for this is that in order for the agent to participate in the housing market, they have to have employment secured in order to pay the rental rate. Thus,
one of the benefits of being matched with an employer is that employment now affords the agent an additional expected benefit.

The value of the agent’s net housing surplus $V^{AHS}$ is the difference between the value of the agent in the employed with residence state and the employed and without residence state. Or,

$$V^{AHS}(\kappa_t) = V^{AEHM}(\kappa_t) - V^{AEHU}_t$$

$$= u(\kappa_t) - r(\kappa_t) +$$

$$+ \beta (1 - \sigma^E) (1 - \sigma^H) E_t \left\{ V^{AEHM}(\kappa_t) - V^{AEHU}_{t+1} \right\} -$$

$$- p^H_t \left[ \int_{\mathcal{K}}^{\pi(t+1)} V^{AEH}(\kappa_{t+1})dG(\kappa) - V^{AEHU}_{t+1} \right].$$

(M14)

The agent’s net housing surplus is the utility value associated with the residence (the commute) net of the rental rate plus the (expected) difference between the value facing the renter (agent) if the renter’s employment and housing arrangements do not terminate and the value the renter is giving up (scaled by the matching probability $p^H$) as a result. I will refer to the term $E_t \left[ V^{AEHM}(\kappa_t) - V^{AEHU}_{t+1} \right]$ as the renter’s “retention surplus” and the term $E_t \left[ \int_{\mathcal{K}}^{\pi(t+1)} V^{AEH}(\kappa_{t+1})dG(\kappa) - V^{AEHU}_{t+1} \right]$ as the renter’s “new residence surplus”.

2.2.2 The surplus sharing rules

In accordance with Nash bargaining, the joint surplus to the match is redistributed to both counterparties via the equilibrium wage (rental rate) in accordance with each counterparty’s relative bargaining weight. For the bargaining between the firm and
the agent to result in an equilibrium wage, if $1 - \eta$ represents the bargaining power of the firm and $\eta$ represents the bargaining power of the worker (agent), optimization leads to the sharing rule

$$\eta V^{FS} = (1 - \eta)V^{AES}.$$  \hspace{1cm} \text{(M15)}

For the bargaining between the agent and the landlord, if $1 - \bar{\eta}$ is the bargaining weight of the landlord while $\bar{\eta}$ represents the bargaining weight of the agent, then the sharing rule for the housing surplus is

$$\bar{\eta} V^{LS} = (1 - \bar{\eta})V^{AHS}.$$  \hspace{1cm} \text{(M16)}

### 2.2.3 The equilibrium wage rate

Substituting the corresponding surpluses into the sharing rule described in (M15) and isolating the equilibrium wage rate leads to

$$\eta \left\{ y_t - w_t + \gamma + (1 - p_t^F) \beta (1 - \sigma^E) E_t V^{FS}_{t+1} \right\}$$

$$= (1 - \eta) \left\{ w_t - b + (1 - p_t^J) \beta (1 - \sigma^E) E_t V^{AES}_{t+1} +$$

$$+ p_t^H \beta (1 - \sigma^E) (1 - \sigma^H) E_t \left[ \int_{\kappa} V^{AEH}(\kappa_{t+1})dG(\kappa) - V^{AEH}_{t+1} \right] \right\}$$

Re-introducing the sharing rule (M15) to express $V^{AES}$ in terms of $V^{FS}$ and then using the firm’s free entry condition in (M4) results in

$$w_t = (1 - \eta) b + \eta \left\{ y_t + \gamma \left( \frac{p_t^J}{p_t^F} \right) \right\} - (1 - \eta) p_t^H \beta (1 - \sigma^E) (1 - \sigma^H) E_t V^{AHS}_{t+1},$$

\text{15} The optimization problem is fleshed out in the model appendix.
which illustrates how a high level of expected housing surplus for the agent in the following period (favorable commuting conditions, for example) exerts downward pressure on the wage. Since the firm and agent both share the exact same information set, the firm can leverage the agent’s expected housing benefit in the following period against it when bargaining over the wage, in essence *extracting* a portion of the agent’s expected housing surplus. As shown, the extent to which the firm can extract this surplus is contingent on the firm’s bargaining power \(1 - \eta\).

Using the sharing rule in (M16) and the landlord’s entry condition from (M8), results in the wage equation

\[
 w_t = (1 - \eta) \left[ b - \left( \frac{\eta}{1 - \eta} \right) \xi \left( \frac{p^H_t}{p^L_t} \right) \right] + \eta \left[ y_t + \gamma \left( \frac{p^J_t}{p^F_t} \right) \right],
\]

which again shows that either an increase in the renter’s bargaining power \(\eta\), or an increase in the ratio of rentals to rental-seekers (equivalent to an increase in \(p^H / p^L\); i.e., more favorable housing conditions facing the renter) aids in stabilizing the wage during an exogenous increase in labor productivity \(y_t\). An increase in the stability of the wage during a positive productivity shock results in the firm receiving a greater proportion of the joint surplus to the employment match, motivating an increase in the posting of job vacancies, amplifying the volatility of labor market tightness.

\[^{16}\text{Intuition for this result is discussed in footnote number 10.}\]
\[^{17}\text{See Hall (2005) for a treatment of the DMP model with equilibrium wage stickiness.}\]
2.2.4 The equilibrium rental rate

Substituting the housing surpluses of the landlord and the agent from (M7) and (M14) into the sharing equation (M16) results in

\[ \eta \left( r(\kappa_t) - (\lambda - \xi) + \beta (1 - \sigma^E) (1 - \sigma^H) E_t \right \} \left[ V^{LM}(\kappa_t) - V^{LU}_{t+1} \right] - \]

\[ - p^L_t \left[ \int_{\kappa}^{\tau} V^L (\kappa_{t+1}) dG(\kappa) - V^{LU}_{t+1} \right] \right) \]

\[ = (1 - \eta) \left( u(\kappa_t) - r(\kappa_t) + \beta (1 - \sigma^E) (1 - \sigma^H) E_t \right \} \left[ V^{AEHM}(\kappa_t) - V^{AEHU}_{t+1} \right] - \]

\[ - p^H_t \left[ \int_{\kappa}^{\tau} V^{AEH} (\kappa_{t+1}) dG(\kappa) - V^{AEHU}_{t+1} \right] \right) \].

Incorporating the sharing rule (M16) into this expression results in an elimination of the “retention surpluses” and allows \( V^{AH\Sigma} \) to be written in terms of \( V^{LS\Sigma} \). The landlord entry condition from (M8) can then be used to write the rental rate as

\[ r(\kappa_t) = (1 - \eta) u(\kappa_t) + \eta \left( \lambda - \xi \left( \frac{p^H_{t+1}}{p^L_t} \right) \right). \]  \( \text{(M18)} \)

2.2.5 The endogenous commute cutoff

The cutoff commute \( \tilde{\kappa} \) is defined as the point where the joint surplus associated with the housing match, net of the opportunity cost of a potentially higher quality match (in terms of a better commute draw) the following period is equal to zero. The
endogenous commute cutoff $\tilde{\kappa}_t$ satisfies

$$V^{HJS}(\tilde{\kappa}_t) - \beta \left(1 - \sigma^E \right) \left(1 - \sigma^H \right) \left[ V^{HJS}(\tilde{\kappa}_t) - \int_{\tilde{\kappa}}^\kappa V^{HJS}(\kappa_{t+1}) dG(\kappa) \right] = 0. \quad (M19)$$

This condition is new to this type of model, as a result of the integration of both labor and housing markets. If jobs terminated every period (i.e., $\sigma^E = 1$), then the cutoff would simply satisfy $V^{HJS}(\tilde{\kappa}_t) = 0$ as a commute would be drawn each period\(^{18}\). However, employment on average lasts $\frac{1}{\sigma^E}$ months and, as a result has to be taken into consideration (along with the housing separation rate) when calculating the commute cutoff. Since the only way a housing match terminates is if the renter exogenously loses their job or if the renter exogenously separates from the home, both the renter and landlord understand that they will be matched with each other for an expected period of $\frac{1}{(1-\sigma^E)\sigma^H+\sigma^E}$ months, implying that both will be giving up the opportunity to match with a more favorable counterparty (renter could potentially match with a closer home which increases utility value of home; landlord could potentially match with a more local renter which increases the joint surplus of the housing match). This opportunity cost (and “optimal stopping time strategy” for picking a cutoff commute) is captured by equation (M19). In solving the model, I will be assuming $G$, the distribution the agent draws their commute from is a uniform distribution.

The joint surplus is the sum of the agent’s and landlord’s housing surpluses

\(^{18}\)This period-by-period draw would follow the branch of the search literature where the productivity of a labor match (Walsh 2005, Mortensen-Pissarides 1994, Pissarides 1985) or credit match (Bebrun-Diant Tripier 2010) is drawn each period.
given by (M7) and (M14) and is given by

\[ V_{HJ} \equiv V_{L} + V_{AH} = r(\kappa_t) - (\lambda - \xi) + \beta(1 - \sigma^E)(1 - \sigma^H) E_t \left\{ [V^{LM}(\kappa_t) - V^{LU}_{t+1}] - \right. \\
- p_t^L \left[ \int_{\Xi} V^L(\kappa_{t+1}) dG(\kappa) - V^{LU}_{t+1} \right] \right\} \\
+ u(\kappa_t) - r(\kappa_t) + \\
\left( + \beta (1 - \sigma^E) (1 - \sigma^H) E_t \left[ V^{AEHM}(\kappa_t) - V^{AEH}\right] - \\
- p_t^H \left[ \int_{\Xi} V^{AEH}(\kappa_{t+1}) dG(\kappa) - V^{AEH}_{t+1} \right] \right). \]

Using the sharing rule (M16) to write this equation in terms of the landlord’s surpluses results in

\[ V_{HJ} = u(\kappa_t) - (\lambda - \xi) + \frac{1}{1 - \eta} \beta (1 - \sigma^E) (1 - \sigma^H) E_t [V^{LM}(\kappa_t) - V^{LU}_{t+1}] - \\
- \left[ p_t^L + \left( \frac{\eta}{1 - \eta} \right) \right] \beta (1 - \sigma^E) (1 - \sigma^H) E_t V^{LS}_{t+1}. \]

Incorporating the entry condition for the landlord given in (M8) results in

\[ V_{HJ} = u(\kappa_t) - \lambda + \frac{\beta (1 - \sigma^E) (1 - \sigma^H)}{1 - \eta} E_t V^{LS}(\kappa_t) - \left( \frac{\eta}{1 - \eta} \right) \xi \left( \frac{p_t^H}{p_t^L} \right) \]

Taking the landlord’s surplus from (M7) and using the entry condition from (M8), along with the expression for the rental rate from (M18) to eliminate \( V^{LS}(\kappa_t) \) from the above results in

\[ V_{HJ} = \frac{1}{1 - \beta (1 - \sigma^E) (1 - \sigma^H)} \left[ u(\kappa_t) - \lambda - \left( \frac{\eta}{1 - \eta} \right) \xi \left( \frac{p_t^H}{p_t^L} \right) \right]. \]
The cut-off commute delineates where this joint surplus satisfies (M19) and the utility function
\[ u(\kappa_t) \equiv \frac{\zeta}{\kappa^\omega} . \]

The resulting threshold commute is given by
\[ \tilde{\kappa} = \left[ \frac{\zeta (1 - \eta) p_t^l}{\lambda (1 - \eta) p_t^l - \xi (1 - \eta p_H^l)} \right]^{\frac{1}{\omega}} . \tag{M20} \]

Equation (M20) illustrates how the threshold commute is:

- **increasing** in \( \zeta \): If the utility parameter \( \zeta \) increases, this implies the flow value to the renter increases for all commute draws, resulting in an increase in the joint surplus, implying the threshold cutoff increases. An increase in the utility flow value for all commute draws would result from an increase in housing amenities exogenous to this model. For example, a decrease in the crime rate, or an increase in the quality of the school district may offset some of the disutility of commuting.

- **decreasing** in \( \omega \): If the utility parameter \( \omega \) increases, this implies that the renter is less tolerant to commuting, resulting in a decrease in the flow value, eroding the joint surplus, resulting in a fall in the threshold commute\(^{19} \).

\(^{19}\) This increase in the “intolerance” of commuting implies agents will be more particular of their commute draw. As commutes are drawn from a uniform distribution, this implies that the expected duration of time until a housing match forms increases. Taking the logarithmic total derivative shows that this is indeed the case:
\[ \frac{dp^H}{d\omega} = -\frac{\ln (\tilde{\kappa}) \left[ \lambda (1 - \eta) p^H - \xi (1 - \eta p^H) \right]}{\xi \eta} , \]
which, for the calibrated parameter values, is negative. This implies \( p^H \), the probability that an employed agent searching for a home meets a landlord, is decreasing in \( \omega \), resulting in a longer expected duration of search \( \frac{1}{p^H} \).
• *decreasing* in $\lambda$: An increase in management costs facing the landlord will necessarily be transferred to the renter in the form of a higher rental rate, driving down the cutoff commute; in other words, it would take a very short commute to justify paying the increase in rent resulting from the increased costs facing the landlord.

• *increasing* in $\xi$: An increase in the cost of posting the vacancy/vetting renters implies that turning away from a potential rental match carries a larger cost, which in turn motivates the renter and landlord to be less demanding of a shorter commute draw.

• *increasing* in $p_L^t$: An increase in $p_L^t$ is indicative of a relative increase in the ratio of rental-seekers to rentals, implying the increased level of competition between rent seekers drives their willingness to commute up. In the model, if there is an economic expansion (increase in labor productivity $y$), this would result in more employment vacancies, which would then increase the number of employment matches formed. This would result in an increase in rental-seekers, which for a given number of rentals, would drive up the willingness of renters to commute\textsuperscript{20}.

• *decreasing* in $p_H^t$: If there is an increase in the probability of a rental-seeker successfully matching with a rental, they can afford to wait for a more local rental, resulting in a decrease in the commute cutoff.

\textsuperscript{20}This comparative static result is consistent with the increase in traffic and average commute times experienced by Silicon Valley residents during the dot-com expansion. See Maignan, Ottaviano, and Pinelli (2003)
2.2.6 The agent’s law(s) of motion

I will annotate the proportion of the unit mass of the agent in each state \( i \) as \( A^i \), where \( i \) is the collection of states \( AU \) unemployed and without residence, \( AEHU \) employed and without residence, and \( AEHM \) employed with residence. Since there is a total mass of one for the agent then

\[
A^AU_t + A^AEHU_t + A^AEHM_t = 1.
\]

The law of motion for the agents which are unemployed and without residence is

\[
A^AU_{t+1} = (1 - p^J_t)A^AU_t + p^J_t \sigma^F_t A^AU_t + \sigma^E_t A^AEHU_t + \sigma^E_t (1 - \sigma^H_t) A^AEHM_t + \sigma^E_t \sigma^H_t A^AEHM_t,
\]

which states that the unemployed agents in the following period are the ones which failed to find a match, plus the ones which did match with a firm but were exogenously separated, plus the employed without residence agents which exogenously separated plus the employed with residence agents which survived the housing match termination, but didn’t survive the employment match termination, plus the employed with residence agents which didn’t survive the housing and employment termination. This law of motion can be written

\[
A^AU_{t+1} = [1 - p^J_t (1 - \sigma^F_t)] A^AU_t + \sigma^E_t (A^AEHU_t + A^AEHM_t).
\]

The law of motion for the agents which are employed and without residence is

\[
A^AEHU_{t+1} = (1 - \sigma^E_t) \left\{ \left[ 1 - (1 - \sigma^H_t) \int_{\kappa_t}^\pi p^H_t \kappa dG(\kappa) \right] A^AEHU_t + p^J_t A^AU_t + \sigma^H_t A^AEHM_t \right\},
\]

which states that next period’s employed without residence agents are made up of the current employed without residence agents which survived the employment separation
and failed to match with a landlord net of the ones which matched with a landlord
while surviving both separations and drew a commute within the threshold, plus the
newly matched unemployed which survived the employment separation, plus the agents
which are employed and housing matched which survive the employment match, but
exogenously separated from their housing.

The law of motion for the employed with residence agents is given by

\[ A_{t+1}^{AEHM} = (1 - \sigma^E) (1 - \sigma^H) \left[ A_t^{AEHM} + \int_{\kappa_t}^{\tilde{\kappa}_t} p_t^H A_t^{AEHU} dG(\kappa) \right] \]

which states that the following period’s employed with residence agents are composed
of the current employed with residence agents which survived the exogenous separation,
in addition to the employed without residence agents which matched with a landlord
and, survived both exogenous separations and drew a commute which was within the
threshold.

I will summarize the laws of motion for the agent with the following turnover
block\textsuperscript{21}:

\[ A_t^{AU} = 1 - \left( A_t^{AEHU} + A_t^{AEHM} \right) \quad (M21) \]

\[ A_{t+1}^{AEHU} = (1 - \sigma^E) \left\{ 1 - (1 - \sigma^H) \int_{\kappa_t}^{\pi} p_t^H dG(\kappa) \right\} \left[ A_t^{AEHU} + p_t^J A_t^{AU} + \sigma^H A_t^{AEHM} \right] \quad (M22) \]

\[ A_{t+1}^{AEHM} = (1 - \sigma^E) (1 - \sigma^H) \left[ A_t^{AEHM} + \int_{\kappa_t}^{\tilde{\kappa}_t} p_t^H A_t^{AEHU} dG(\kappa) \right] \quad (M23) \]

\textsuperscript{21}Adding all three laws of motion results in

\[ A_{t+1}^{AU} + A_{t+1}^{AEHU} + A_{t+1}^{AEHM} = A_t^{AU} + A_t^{AEHU} + A_t^{AEHM} \equiv 1, \]

which illustrates the invariance of the mass of agents.
2.2.7 Labor market tightness

Recall the entry condition for the firm
\[
\gamma p_t = \beta (1 - \sigma^E) E_t V_{t+1}^{FS}.
\]
Taking the net surplus equation for the firm
\[
V_t^{FS} = y_t - w_t + \gamma + (1 - p_t^F) \beta (1 - \sigma^E) E_t V_{t+1}^{FS},
\]
and incorporating the entry condition results in
\[
V_t^{FS} = y_t - w_t + \gamma p_t^F.
\]
Advancing this one period and substituting the result back into the firm’s entry condition yields
\[
\frac{\gamma p_t^F}{p_t} = \beta (1 - \sigma^E) E_t \left( y_{t+1} - w_{t+1} + \gamma p_{t+1}^F \right).
\]
Eliminating \( w_{t+1} \) using (M17) results in
\[
\frac{\gamma p_t^F}{p_t} = \beta (1 - \sigma^E) E_t \left[ (1 - \eta) (y_{t+1} - b) + \eta \left( \frac{1 - \eta}{1 - \bar{\eta}} \right) \xi \left( \frac{p_{t+1}^H}{p_{t+1}^L} \right) + (1 - \eta \eta p_{t+1}^J) \left( \frac{\gamma p_t^F}{p_{t+1}^F} \right) \right].
\]

2.2.8 Rental market tightness

Combining the landlord’s net surplus equation (M7) with the free entry condition (M8) and the equation for the rental rate (M18) results in the expression
\[
V^{LS}(\kappa_t) = \frac{1}{1 - \beta (1 - \sigma^E)(1 - \sigma^H)} \left\{ (1 - \eta) \left[ u(\kappa_t) - \lambda \right] - \eta \xi \left( \frac{p_t^H}{p_t^L} \right) \right\}.
\]
This expression can be advanced one period and substituted into the landlord’s entry condition to get

$$\frac{\xi}{p^L_t} = \frac{\beta (1 - \sigma) (1 - \sigma_H)}{1 - \beta (1 - \sigma) (1 - \sigma_H)} E_t \int_{\tilde{\kappa}}^{\tilde{\kappa} + 1} \left\{ (1 - \eta) u(\kappa_{t+1}) - \lambda - \eta_\xi \left( \frac{p^H_{t+1}}{p^{E}_{t+1}} \right) \right\} dG(\kappa).$$

Incorporating the formulation for the utility function, the uniform distribution $G$, and performing a change of variables on this integral results in

$$\frac{\xi}{p^L_t} = \frac{\beta (1 - \sigma) (1 - \sigma_H)}{1 - \beta (1 - \sigma) (1 - \sigma_H)} E_t \int_{\tilde{\kappa}}^{\tilde{\kappa} + 1} \left\{ (1 - \eta) \left( \phi^{1-\omega} - (1 - \eta) \lambda - \eta_\xi \left( \frac{p^H_{t+1}}{p^{E}_{t+1}} \right) \right) \right\} \left( \frac{1}{\bar{\kappa} - \kappa} \right) d\phi.$$

Evaluating the integral yields

$$\frac{\xi}{p^L_t} = \frac{\beta (1 - \sigma) (1 - \sigma_H)}{1 - \beta (1 - \sigma) (1 - \sigma_H)} E_t \left\{ (1 - \eta) \left[ \frac{\zeta}{(1 - \omega)} \left( \frac{\tilde{\kappa}^{1-\omega} - \bar{\kappa}^{1-\omega}}{\tilde{\kappa} - \bar{\kappa}} \right) \right] - \lambda \right\} - \eta_\xi \left( \frac{p^H_{t+1}}{p^{E}_{t+1}} \right) \left( \frac{\xi}{p^{L}_{t+1}} \right).$$

### 2.2.9 Matching functions

If $A^AU_t$ represents the mass of unemployed agents and $V_t$ represents the mass of job openings, then matches in the labor market will be given by

$$m^E_t = M \left( V_t, A^AU_t \right) = \mu^E V^\epsilon_t \left( A^AU_t \right)^{1-\epsilon},$$

where I have assumed a Cobb-Douglas form for the matching function as is common in the literature. $\mu^E$ and $\epsilon$ are the level and matching parameter for the function, respectively. The Cobb-Douglas form results in the following matching probabilities for
the unemployed agent and the searching firm:

\[
p_t^J = \frac{m_t^E}{A_t^{AU}} = \frac{\mu^E V_t^e (A_t^{AU})^{1-\epsilon}}{A_t^{AU}} = \mu^E (\tau_t^E)^\epsilon, \tag{M26}
\]

\[
p_t^F = \frac{m_t^E}{V_t} = \frac{\mu^E V_t^e (A_t^{AU})^{1-\epsilon}}{V_t} = \mu^E (\tau_t^E)^{\epsilon-1}, \tag{M27}
\]

where \(\tau_t^E = \frac{V_t}{A_t^{AU}}\) represents the tightness of the labor market.

Turning to the rental market, if \(A_t^{AEHU}\) represents the mass of agents which are employed, but searching for a rental and \(L_t^U\) represents the mass of unmatched landlords, then matches will be modelled similar to that of the labor market

\[
m_t^H = M (L_t^u, A_t^{AEHU}) = \mu^H (L_t^u)^\chi (A_t^{AEHU})^{1-\chi},
\]

where I have also assumed a Cobb-Douglas function with parameters \(\mu^H\) and \(\chi\). The corresponding matching probabilities for the prospective renter and searching landlord are

\[
p_t^H = \frac{m_t^H}{A_t^{AEHU}} = \frac{\mu^H (L_t^u)^\chi (A_t^{AEHU})^{1-\chi}}{A_t^{AEHU}} = \mu^H (\tau_t^H)^\chi, \tag{M28}
\]

\[
p_t^L = \frac{m_t^H}{L_t^U} = \frac{\mu^H (L_t^u)^\chi (A_t^{AEHU})^{1-\chi}}{L_t^U} = \mu^H (\tau_t^H)^{\chi-1}, \tag{M29}
\]

where the rental market tightness is given by \(\tau_t^H = \frac{L_t^U}{A_t^{AEHU}}\).
2.2.10 Ancillaries

I model labor productivity, the labor match separation rate, the housing bargaining weight and the labor bargaining weight as

\[
y_t = y_{t-1} + \epsilon^y_t \sim N(0, \sigma_y),
\]

\[
\sigma^E_t = \sigma^E_{t-1} + \epsilon^E_t \sim N(0, \sigma^E),
\]

\[
\eta_t = \eta_{t-1} + \epsilon^\eta_t \sim N(0, \sigma^\eta),
\]

Since commutes are drawn from a uniform distribution over the interval \([\kappa, \bar{\kappa}]\), then

\[
\int_{\kappa}^{\bar{\kappa}} dG(\kappa) = \frac{\bar{\kappa} - \kappa}{\bar{\kappa} - \kappa}
\]

Incorporating the matching functions (M26) - (M29), the utility functional

\[
u(\kappa_t) = \frac{\zeta}{\kappa_t}
\]

\[<sup>22</sup>To a first-order approximation, these become the AR(1) processes

\[
y_t = \phi_y y_{t-1} + \epsilon^y_t,
\]

\[
\sigma^E_t = \phi^E_{t-1} \sigma^E_{t-1} + \epsilon^E_t,
\]

\[
\eta_t = \phi^\eta_{t-1} \eta_{t-1} + \epsilon^\eta_t,
\]

\[
\bar{\eta}_t = \phi^\eta_{t-1} \bar{\eta}_{t-1} + \epsilon^\eta_t.
\]
and the threshold/cutoff commute into the model’s equilibrium equations results in the following system

\[
\begin{align*}
    w_t &= (1 - \eta)b + \eta \left( y_t + \gamma_t^E \right) - \eta \left( \frac{1 - \eta}{1 - \eta} \right) \xi_t^H \\
    r(\kappa_t) &= (1 - \eta) \frac{\zeta}{\kappa_t} + \eta (\lambda - \xi_t^H) \\
    \left[ \lambda (1 - \eta) \mu^H (\tau_t^H)^{\chi - 1} + \bar{\eta} \xi^H (\tau_t^H)^{\chi} \right] \tilde{\kappa}_t &= \zeta (1 - \eta) \mu^H (\tau_t^H)^{\chi - 1} \\
    A_t^{AU} &= 1 - (A_t^{AEHU} + A_t^{AEHM}) \\
    A_{t+1}^{AEHU} &= (1 - \sigma^E) \left\{ 1 - (1 - \sigma^H) \left( \frac{\tilde{\kappa}_t - \frac{\kappa}{\kappa}}{\bar{\kappa} - \frac{\kappa}{\kappa}} \right) \mu^H (\tau_t^H)^{\chi} \right\} A_t^{AEHU} + \\
    &+ \mu^E (\tau_t^E) \epsilon A_t^{AU} + \sigma^H A_t^{AEHM} \\
    A_{t+1}^{AEHM} &= (1 - \sigma^E) (1 - \sigma^H) \left[ A_t^{AEHM} + \left( \frac{\tilde{\kappa}_t - \frac{\kappa}{\kappa}}{\bar{\kappa} - \frac{\kappa}{\kappa}} \right) \mu^H (\tau_t^H)^{\chi} A_t^{AEHU} \right] \\
    \frac{\gamma}{\mu^E} (\tau_t^E)^{1 - \epsilon} &= \beta (1 - \sigma^E) E_t \left\{ (1 - \eta) (y_{t+1} - b) + \eta \left( \frac{1 - \eta}{1 - \eta} \right) \xi_{t+1}^H + \\
    &+ \left[ 1 - \eta \mu^E (\tau_{t+1}^E) \right] \left[ \frac{\gamma}{\mu^E} (\tau_{t+1}^E)^{1 - \epsilon} \right] \right\} \\
    \frac{\xi}{\mu^H} (\tau_t^H)^{1 - \chi} &= \beta \left( 1 - \sigma^E \right) \left( 1 - \sigma^H \right) E_t \left\{ (1 - \eta) \left[ \frac{\zeta}{(1 - \omega)} \left( \frac{\tilde{\kappa}_{t+1} - \frac{\kappa}{\kappa}}{\bar{\kappa} - \frac{\kappa}{\kappa}} \right) - \lambda \right] - \\
    &- \bar{\eta} \mu^H (\tau_{t+1}^H)^{\chi} \left( \frac{\xi}{\mu^H} (\tau_{t+1}^H)^{1 - \chi} \right) \right\} \\
    ur_t &= A_t^{AU}, \\
    vr_t &= 1 - \frac{A_t^{AEHM}}{A_{t+1}^{AEHU} + A_t^{AEHM}},
\end{align*}
\]

where I have defined \( ur_t \) to be the unemployment rate; given that I have assumed a constant unit mass of agents, this is simply the agents searching for an employer. \( vr_t \) is
the rental vacancy rate and it derives from

\[
\frac{\text{unrented homes}}{\text{total stock}} + \frac{\text{rented homes}}{\text{total stock}} = 1 \\
\frac{\text{unrented homes}}{\text{total stock}} = 1 - \frac{\text{rented homes}}{\text{total stock}} \\
\Rightarrow vr = 1 - \frac{A_{AEHM}}{L^U + A_{AEHM}}.
\]

2.3 Empirical analysis

The first part of this section studies the empirical behavior of the key labor and housing market variables. The second part of this section will outline the calibration of the model, which is based on the empirical results. The data used in this section to perform all of the empirical work come from monthly data spanning December of 2000 through March of 2014\textsuperscript{23}.

2.3.1 U.S. labor and housing market facts

This subsection discusses the labor and housing market data the model attempts to match. Important variables include unemployment, job vacancies, the job-finding rate (this is the rate at which unemployed workers match with job vacancies), the separation rate (the rate at which employment matches terminate), labor productivity, and bargaining in the housing market. The statistical methodologies will closely follow Shimer (2005)\textsuperscript{24}.

\textsuperscript{23}More specific details are given in the data appendix.
\textsuperscript{24}I look at monthly data sourced from the JOLTS dataset covering the time period 12/2000 - 3/2014 for a total of 160 observations.
2.3.1.1 Unemployment

The one aggregate variable often seen as the “pulse” of the labor market is the level/rate of unemployment. Figure 2 illustrates the time series behavior of unemployment in the U.S. over the 2000 - 2014 period, where a trend has been added.

Figure 2: Source: Seasonally adjusted series from U.S. Department of Labor: Bureau of Labor Statistics. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.

One aspect of the time series which stands out is the surge in unemployment following the onset of the financial crisis and ensuing Great Recession. The log-deviation of unemployment from trend exhibits a standard deviation of 0.133 and a fairly persistent monthly autocorrelation coefficient of 0.972.

Figure 2 does not include those workers who choose to not participate in the labor force. For example, between 2004 and early 2008, the labor force participation

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25 Following Shimer (2005), I work with unemployment in levels rather than its rate.
26 Following Shimer (2005), the trend was estimated using a Hodrick-Prescott filter with smoothing parameter set to $\lambda = 100,000$. 

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36
rate was stable at approximately 66%; however starting November of 2008, it has fallen to a low of 62.8% in December of 2013 and has been steadily rising since.\textsuperscript{27}

2.3.1.2 Vacancies

When a firm has an economic need for an employee, the firm posts a vacancy - the firm’s equivalent to unemployment. The time series behavior of job vacancies in the United States is shown in figure 3, presented using a similar framework as in figure 2.

Figure 2.3: Source: Seasonally adjusted series from U.S. Department of Labor: Bureau of Labor Statistics. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.

Interestingly, the log-deviation of vacancies from trend shares a similar standard deviation as that of unemployment ($\sigma_{\text{vac.}} = 0.132$). In Shimer (2005), the standard deviation of unemployment and vacancies over the 1951 - 2003 period were 0.190 and 0.202, respectively. The decreased volatility reported here may be suggestive of the large part the Great Moderation had in lowering overall economic volatility, or even the

\textsuperscript{27}See C. Erceg and A. Levin (2013) for a general equilibrium treatment of labor force participation.
role the internet has played in bridging the gap between searching workers and firms, or perhaps both\textsuperscript{28}. One aspect of this graph worth noting is the near-mirror image the series for vacancies shares with the series for unemployment.

If vacancies are procyclical and unemployment is countercyclical, then the vacancy-unemployment ratio - the so-called “tightness” of the labor market - should exhibit strong pro-cyclicality\textsuperscript{29}. The standard deviation of this measure from its trend is $\sigma_{v/u} = 0.265$ and this is where Shimer (2005) pointed out that the DMP framework fell short in matching the high level of volatility in this extremely crucial labor market variable.

\subsection*{2.3.1.3 Job-finding rate}

As is typical in the search literature, new matches are formed via a matching function $m$ which takes the level of the unemployed $u$ and the level of the vacancies $v$ and combines them as

$$m_t = m(u_t, v_t).$$

\textsuperscript{28}Gali and van Rens (2010) suggest that while the internet may have played an important role in decreasing labor market volatility, other major changes within the U.S. labor market deserving of consideration include the introduction of wrongful discharge laws in many states, the increase in temporary help services and the decline of labor force unionization.

\textsuperscript{29}For example, in log-deviation, labor market tightness - here annotated as $\theta$ - is

$$\hat{\theta} = \hat{v} - \hat{u}.$$ 

If vacancies are pro-cyclical and unemployment is countercyclical, than $\hat{v} > 0$ and $\hat{u} < 0$, implying $\hat{\theta} > 0$. Of course, the degree of procyclicality of $\theta$ is contingent on the degree of procyclicality of both $v$ and $u$. 

38
The job-finding rate for workers, annotated $p^J$, is the number of matches per searching, unemployed worker, or

$$p^J_t = \frac{m_t}{u_t}.$$  

A candidate matching function would be

$$m_t = \mu v^*_t u_t^{1-\epsilon},$$

which implies that the job finding rate would then be

$$p^J_t = \frac{\mu v^*_t u_t^{1-\epsilon}}{u_t} = \mu \left( \frac{v_t}{u_t} \right)^\epsilon,$$

a function of the tightness of the labor market - the vacancy to unemployment ratio.

While I model the job-finding rate theoretically using the matching function approach, I construct the empirical data series using a similar decomposition as in Shimer (2005) which differentiates between the unemployed and the short-term unemployed. Specifically, if the total number of unemployed next period $u_{t+1}$ is made up of the unemployed this period who failed to find a job and the next period’s short-term unemployed $u^s_{t+1}$

$$u_{t+1} = u_t (1 - p^J_t) + u^s_{t+1},$$

then the job-finding rate is isolated

$$p^J_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t}. \tag{E1}$$

30 The Cobb-Douglas form for the matching function follows the vast majority of the literature. For an overview, see Petrongolo and Pissarides (2001).

31 In order to isolate the job finding rate empirically, I use the following two BLS series:

• $u_t$: UNEMPLOY; Unemployed, Thousands of Persons, Monthly, Seasonally Adjusted

• $u^s_{t}$: UEMPLOY; Number of Civilians Unemployed - Less Than 5 Weeks, Thousands of Persons, Monthly, Seasonally Adjusted
Figure 4 shows the time series behavior of the constructed job-finding rate series along with an HP filtered trend.

\[
\hat{p}_t^J = \epsilon \tau_t^E.
\]

The mean job-finding rate over the entire 2000 - 2014 time period is 0.294, while its standard deviation is \( \sigma_{p^J} = 0.120 \).

According to the definition of the job-finding rate, \( p^J \) is increasing in \( \frac{u_t}{u_t} \), and indeed the correlation between the log-deviations of the job-finding rate and tightness is 0.842. To get a better sense of the quantitative relationship between the job-finding rate and the vacancy-unemployment ratio, figure 5 displays the scatter plot of their log-deviations.

Defining the vacancy-unemployment ratio - labor market tightness - as \( \tau^E \), the job finding rate as defined using the Cobb-Douglas formulation in log-deviations is
Figure 2.5: Source for vacancy-unemployment ratio data: U.S. Department of Labor: Bureau of Labor Statistics; Job-finding rate taken from equation (E1). Both labor market tightness and the job finding rate have been detrended using an HP filter with smoothing parameter $\lambda = 10^5$.

There is a fairly large body of the literature pertaining to the estimation of equation such as this\(^{32}\). OLS estimation of this equation using the pertinent data yields $\epsilon = 0.38$, a number which is in line with the vast majority of the search and matching literature\(^{33}\).

### 2.3.1.4 Separation rate

The separation rate is the rate at which matches between employees and firms terminates\(^{34}\), where termination of employment is either initiated by the employee (a quit), or the employer (a lay-off). Once a job is exogenously destroyed, the former employee is considered unemployed (part of the short-term unemployed) approximately

\(^{32}\)See for example, Shimer (2005), Petrongolo-Pissarides (2001).

\(^{33}\)For this study, I will be using $\epsilon = 0.40$ to stay consistent with the literature.

\(^{34}\)This rate is also referred to as a job destruction rate.
two weeks after the job terminated. This then implies that the following period’s short-term unemployed can be written

\[ u_{t+1}^s = \sigma_t^E e_t \left( 1 - \frac{1}{2} p_t^j \right), \]

where \( \sigma_t^E \) is the labor match separation rate, and \( e_t \) is the number of employed individuals. Expressing the separation rate in this manner corrects for the bias in the separation rate. If one wants to express the separation rate as that rate which dictates the flow from employment to short-term unemployment, counting the separations which quickly become new jobs within two weeks would obviously over-state the extent of job separation. Using the above formulation corrects for this problem by isolating the actual rate of flow from employed to short-term unemployment. Solving for \( \sigma_t^E \) gives

\[ \sigma_t^E = \frac{u_{t+1}^s}{e_t \left( 1 - \frac{1}{2} p_t^j \right)}. \] (E2)

With the data for the short-term unemployed, the level of employed and the constructed series for the job-finding rate, I can construct the series for separations\(^{35}\), shown in figure 6 along with its trend.

What is clear from the picture is that while separations have been following a downward trajectory over the entire data set, during the financial crisis this series experienced a spike. Over the entire period, the average rate of employment separation is 0.024, or approximately 2.4%, a number which implies that on average, jobs over this period lasted \( \frac{1}{0.024} \approx 41.67 \) months, or 3.47 years. The volatility of this series is

\(^{35}\)While a data set such as the QWI may provide detailed firm characteristics including geography, industry, age, size and worker demographics information including sex, age, education, race, ethnicity, the frequency is quarterly. The series constructed here is monthly in order to match the frequency of the other labor market variables.
Figure 2.6: This series was put together using equation (E2); source for the data used is U.S. Department of Labor: Bureau of Labor Statistics. Trendline constructed using an HP filter with smoothing parameter $\lambda = 10^5$.

\[ \sigma_{FE} = 0.062, \] which is slightly greater than half that of the job-finding rate and less than a third of the volatility of the tightness measure, suggestive, as in Hall (2005), of the subordinate role labor separation volatility plays in the fluctuations of labor market flows.

### 2.3.1.5 Labor productivity

I measure average labor productivity as real output per person in the manufacturing sector\(^{36}\). Figure 7 shows this series along with its trend.

The series seems to follow its trend fairly closely, except for the time around the financial crisis and Great Recession. The standard deviation of the series from its trend is $\sigma_y = 0.020$. Figure 8 plots the log-deviation of output against the log-deviation

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\(^{36}\)Series number PRS30006163 from the FRED economic database. Following the same methodology used by the Fed in constructing the monthly Industrial Production Index from a quarterly series, a cubic spline algorithm was implemented to expand the quarterly labor productivity series to monthly frequency.
Figure 2.7: The series is real output per person in the U.S. manufacturing sector against its HP filter trend with smoothing parameter $\lambda = 10^5$. The series is given as an index. Prior to filtering, a cubic spline algorithm (see footnote #36) was used to expand the original quarterly series to a monthly frequency.

As was mentioned earlier, figure 8 confirms the high procyclicality of labor market tightness. Although the cyclical components of labor productivity and tightness correlate very well (the correlation coefficient is 0.461.), there is a stark difference in their individual volatilities\(^{37}\) ($\sigma_{v/u} = 0.265$ while $\sigma_y = 0.020$).

### 2.3.1.6 Bargaining in the housing market

Following Diaz and Jerez (2013) I define a home-buyer’s bargaining power as the inverse of the ratio of the sales price to the listing price for a specific home. Hence, if a buyer is able to negotiate a decrease in the sales price from the price for which the home is listed, this would result in an increase in the ratio. Merlo and Ortalo-Magne (2004) provide some evidence for the presence of ex-post renegotiation/bargaining in

\(^{37}\)This empirical finding was also noted in Shimer (2005).
the U.K. housing market by showing that in their sample, bargaining can reduce the final sale price of a home by 4-5% of the original listing price - a number which is on par with the estimate the National Association of Realtors provides for the U.S. housing market. As I am focusing on the rental market and not the owner-occupied market, the ratio of list-to-sales price will have to be normalized by the rent-price ratio in order to construct a series to serve as a proxy for the bargaining power of renters. This implies that the renter’s bargaining power $\bar{\eta}$ can be written as

$$\bar{\eta}_t = \alpha \sum_i \bar{w}_{i,t}s_{i,t}\rho_{i,t},$$

(E3)

where $\alpha$ is a level parameter, $\bar{w}_{i,t}$ is the ratio of the number of home sales in state $i$ during month $t$ to total home sales during month $t$, $s_{i,t}$ is the median of the ratio between the list price and the sales price for all homes in state $i$ during month $t$, and $\rho_{i,t}$ is the rent-to-price ratio in state $i$ during month $t$. The online real estate database Zillow provides
the state-level data\textsuperscript{38} I used to construct (E3). Figure 9 plots the monthly series for the bargaining power of renters along with its trend. Even though the data series for this figure is short compared to all of the other labor market variables analyzed, to remain consistent, I use the same HP filter parameter.

Figure 2.9: Source for the data used is Zillow. Trendline constructed using an HP filter with smoothing parameter $\lambda = 10^5$. Data for this series is available from October 2010.

A cross-correlation exercise was performed and the highest level of correlation between renter bargaining power and labor productivity occurred when bargaining power was lagged 4 months, resulting in a correlation coefficient of 0.233. The interpretation is that an increase in the productivity of labor manifests itself in more favorable bargaining conditions for renters 4 months later. This would then imply that while owner-occupied housing \textit{leads} the business cycle (Kydland et al. 2012, Leamer 2007), the rental market \textit{lags} the business cycle.

In order to buttress this assumption further, figure 10 shows the quarterly series for rental housing starts and labor productivity from 1987 to the present, both

\textsuperscript{38}See Zillow’s data page.
Figure 2.10: Privately Owned Rental Housing Starts in the United States (Purpose of Construction: Built for Rent Two or More Units, Thousands of Units, Quarterly) and Real Output Per Hour of All Persons in the Manufacturing Sector (Index 2009=100, Quarterly). In this graph, the housing series is lagged by 2 quarters. Both series are expressed as log-deviations from trend (trend was constructed using an HP filter with smoothing parameter $\lambda = 10^5$).

in log-deviation from trend. Performing a similar cross-correlation exercise with this much longer series yields the same result; the highest level of correlation between the cyclical components, 0.246, is reached when housing starts were lagged by 2 quarters\(^{39}\), providing further empirical support to the assumption that the rental market, while procyclical, lags the labor market.

2.3.1.7 Summary of labor market and housing market statistics

Table 1 summarizes important aspects of all the empirical measures in this subsection including the volatility, monthly autocorrelation, and correlation matrix of unemployment, vacancies, tightness, the job-finding rate, separations, and labor pro-

\(^{39}\)Omitting the Great Recession period increases the correlation coefficient to 0.481.
ductivity. The signs on the correlations between the variables seems in line with prior intuition. The numbers in this table will serve as the benchmark for the overall performance of the new model.

Table 1 excludes the series for the agent’s bargaining power in the housing market given the limited data availability (compared to the JOLTS). However, as I am assuming that movements in labor productivity \( \hat{y} \), separations \( \hat{\sigma}^E \), and bargaining in the housing market \( \hat{\eta} \) are exogenous features of the model, I have included a separate correlation matrix below (Correlation Matrix 1) for the cyclical components of each in order to provide a sense for how bargaining in the housing market behaves relative to labor productivity and separation. As expected, productivity and the lagged series for

<table>
<thead>
<tr>
<th>Correlation Matrix 1</th>
<th>( u )</th>
<th>( v )</th>
<th>( v/u )</th>
<th>( p^J )</th>
<th>( \sigma^E )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.133</td>
<td>0.132</td>
<td>0.265</td>
<td>0.120</td>
<td>0.062</td>
<td>0.020</td>
</tr>
<tr>
<td>Monthly autocorrelation</td>
<td>0.972</td>
<td>0.908</td>
<td>0.963</td>
<td>0.711</td>
<td>0.447</td>
<td>0.979</td>
</tr>
<tr>
<td>( u )</td>
<td>1</td>
<td>-0.877</td>
<td>-0.915</td>
<td>-0.795</td>
<td>0.301</td>
<td>-0.308</td>
</tr>
<tr>
<td>( v )</td>
<td>—</td>
<td>1</td>
<td>0.976</td>
<td>0.816</td>
<td>-0.401</td>
<td>0.481</td>
</tr>
<tr>
<td>( v/u )</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>0.842</td>
<td>-0.398</td>
<td>0.461</td>
</tr>
<tr>
<td>( p^J )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>-0.158</td>
<td>0.316</td>
</tr>
<tr>
<td>( \sigma^E )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>-0.643</td>
</tr>
<tr>
<td>( y )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.1: Labor Market Summary Statistics, Monthly U.S. Data, 2000 - 2014. The data for unemployment \( u \), vacancies \( v \), and productivity \( y \) are taken from the U.S. Department of Labor: Bureau of Labor Statistics. The job-finding rate was constructed using equation (E1), while separations \( \sigma^E \) was constructed using equation (E2). All variables are expressed as log deviations from an HP filter trend with smoothing parameter \( \lambda = 10^5 \).

For comparison, I have included Shimer’s version of this table in the first section of the online appendix entitled “Introduction appendix”.

For comparison, I have included Shimer’s version of this table in the first section of the online appendix entitled “Introduction appendix”.

48
Correlation Matrix 1: Standard deviation and correlation coefficients, 2010:11 - 2014:03

<table>
<thead>
<tr>
<th></th>
<th>( \hat{y} )</th>
<th>( \hat{\sigma}^E )</th>
<th>( \hat{\eta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Deviation</td>
<td>0.003</td>
<td>0.048</td>
<td>0.035</td>
</tr>
<tr>
<td>1.000</td>
<td>-0.255</td>
<td>0.234</td>
<td>( \hat{y} )</td>
</tr>
<tr>
<td>Corr. Matrix</td>
<td>1.000</td>
<td>0.108</td>
<td>( \hat{\sigma}^E )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.000</td>
<td>( \hat{\eta} )</td>
</tr>
</tbody>
</table>

negatively. The positive comovement between separations and renter bargaining power is counterintuitive; this is most likely due to the restricted amount of data, as the \( p \)-value is 0.524 under the null hypothesis of no correlation.

To provide additional intuition, Correlation Matrix 2 shows the quarterly correlation coefficients for the cyclical components of labor productivity \( \hat{y} \), Shimer’s series for the quarterly labor separation rate \( \hat{\sigma}^E \) and the same lagged rental housing start series from figure 9, here annotated as \( \hat{\eta}^* \). While \( \hat{y} \) is sourced from the same data as that in Table 1 and Correlation Matrix 1, separations and bargaining are not which is why those variable are annotated with an asterisk. The benefit Correlation Matrix 2 provides is that the data used to construct it span a much longer length of time. The data set used for this correlation matrix spans 1987 - 2007 since the series for housing starts begins in 1987 and Shimer’s quarterly series for the labor separation rate ends in 2007. As before, labor productivity and rental housing comove positively while labor productivity and separations comove negatively. Although weak, the sign on the relationship between separations and rental housing is negative, which is more in line with intuition.

In summary, here are the stylized facts that the model will aim to capture:
**Correlation Matrix 2:** Quarterly standard deviation and correlation coefficients, incorporating Shimer’s quarterly separation series and (lagged) quarterly rental housing starts, 1987:1 - 2007:1

<table>
<thead>
<tr>
<th></th>
<th>( \hat{y} )</th>
<th>( \hat{\sigma}^E )</th>
<th>( \hat{\eta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Deviation</td>
<td>0.035</td>
<td>0.038</td>
<td>0.313</td>
</tr>
<tr>
<td>Corr. Matrix</td>
<td>1.000</td>
<td>-0.072</td>
<td>0.401</td>
</tr>
</tbody>
</table>

- Following an increase in the productivity of labor and a lagged increase in renter bargaining power
  - unemployment falls
  - employment vacancies rise (firm posts more job vacancies)
  - labor market tightness (vacancy-to-unemployment ratio) increases
  - the job-finding rate (rate which unemployed match with jobs) increases

- Following an decrease in the productivity of labor and a lagged decrease in the renter bargaining power
  - unemployment rises
  - employment vacancies fall*
  - labor market tightness falls
  - the job-finding rate falls

While the baseline DMP model is consistent with most of these relationships, it comes up fairly short in generating the level of volatility seen in the labor market data. Ad-
ditionally, following an increase in labor match separations, the negative correlation between unemployment and vacancies is a relationship (asterisked above) the baseline DMP model fails to capture. Specifically, in the baseline model, following shocks to labor separation, unemployment and vacancies both rise in near perfect correlation, an event which has never been observed in the data. In the remainder of this chapter, I will show how integrating housing into the baseline model narrows the gap towards overcoming both of these deficiencies.

2.3.2 Parameterization

As is standard in the literature, the discount parameter is set to $\beta = 0.984$ from $\frac{1}{1+i}$, where $i$ is the average real interest rate from the monthly series for the 3-Month Treasury Bill.

The separation rate for employment matches is set to $\sigma_E = 0.024$, the average of the constructed series from equation (E2). Unemployment benefits are set to $b = 0.54\bar{w}$, where $\bar{w}$ is the target calibration for steady-state wages. The proportion of wages 0.54 is sourced from my own weighted average of unemployment benefit calculators for various states in the U.S. $\epsilon$, the matching function parameter, is set to 0.40; this is the value chosen by Walsh (2005), Blanchard and Diamond (1989) and further, is a value consistent with the literature. The job posting costs follow Petrosky-Nadeu and Wasmer (2013) and Silva and Toledo (2007) who assume this a cost related to recruiting a worker $\sim 3.6$ percent of the wage rate; hence I set $\gamma = 0.036\frac{\bar{w}}{\bar{w}}$.

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$^{41}$see Shimer 2005.

$^{42}$See the data appendix for details
The separation rate for housing matches is set to $\sigma^H = 0.020$, the average of the series titled “Geographical Mobility by Tenure” from the U.S. Census Bureau, Current Population Survey. The landlord’s rental management costs are set to $\lambda = 0.09\pi$, where $\pi$ is the target calibration for the steady-state rental rate. A sample of 12 residential property management firms\(^{43}\) was conducted and nine percent of monthly rentals is the industry average. Further, property management costs is an appropriate proxy for this parameter as all property management firms only charge their management fee when the unit managed is occupied; this matches the structure of the landlord’s value functions, where $\lambda$ only shows up when the landlord is matched with a renter. The limits on the uniform distribution $G$ are set to the unit interval so that $\kappa = 0$ and $\pi = 1$. There is no data available to parameterize the landlord vacancy posting cost $\xi$. I follow Petrosky-Nadeu and Wasmer (2013) and set $\xi$ equal to their loan vacancy posting cost. I am assuming that the costs a lender pays in vetting a potential borrower are similar to the costs a landlord pays in vetting a potential renter\(^{44}\). As is customary in the literature (Walsh 2005, Ravenna-Walsh 2011, Albrecht et al. 2007, Mortensen-Pissarides 1994, Rupert-Wasmer 2012), bargaining weights are assumed symmetric and set to $\eta = \eta = 0.5$.

The persistence parameters on the exogenous processes for output, labor match separations, and the renter’s bargaining weight are set to their empirical AR(1) coeffi-

\(^{43}\)See the data appendix for individual details. One large firm was contacted in each of the following cities: Los Angeles, CA; San Francisco, CA (2 firms); San Diego, CA; Dallas, TX; Oklahoma City, OK; Minneapolis, MN; Chicago, IL; Miami, FL; Charlotte, NC; Boston, MA; New York, NY. Additionally, there were a couple of firms which also offered a contract where an amount equivalent to one month’s rent of the property would satisfy a year’s worth of property management.

\(^{44}\)Similar costs involved in the vetting of both borrowers and renters include performing a credit check and verifying deposits at the borrower’s savings institution.
cient estimates\textsuperscript{45} for the cyclical components of each, hence $\phi_y = 0.976$, $\phi_{\sigma E} = 0.451$, and $\phi_\eta = 0.985$.

All other parameters with no empirical justification\textsuperscript{46} are set to “reasonable” values and are included in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount</td>
<td>0.984</td>
</tr>
<tr>
<td>$\sigma^E$</td>
<td>Labor match separation rate</td>
<td>0.024</td>
</tr>
<tr>
<td>$b$</td>
<td>Unemployment benefits</td>
<td>0.360</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Labor matching function parameter</td>
<td>0.400</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Job-posting/recruiting cost</td>
<td>0.883</td>
</tr>
<tr>
<td>$\sigma^H$</td>
<td>Housing match separation rate</td>
<td>0.020</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Property management costs</td>
<td>0.020</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Lower support of commute distribution</td>
<td>0.000</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Upper support of commute distribution</td>
<td>1.000</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Persistence parameter on output shock</td>
<td>0.976</td>
</tr>
<tr>
<td>$\phi_{\sigma E}$</td>
<td>Persistence parameter on labor separation shock</td>
<td>0.451</td>
</tr>
<tr>
<td>$\phi_\eta$</td>
<td>Persistence parameter on housing market bargaining shock</td>
<td>0.985</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Rental posting cost</td>
<td>1.580</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Bargaining weight over labor match surplus\textsuperscript{*}</td>
<td>0.500</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>Bargaining weight over housing match surplus\textsuperscript{*}</td>
<td>0.500</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Housing utility parameter\textsuperscript{*}</td>
<td>1.000</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Housing utility parameter\textsuperscript{*}</td>
<td>0.500</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Housing matching function parameter\textsuperscript{*}</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Table 2.2: Parameter values set. Descriptions with an asterisk indicate parameterizations without an empirical justification.

\subsection{2.3.3 Steady-state values}

Following Petrosky-Nadeu and Wasmer (2013) I normalize output $y$ to unity. The total mass of agents is set to unity, and thus the unemployment rate and level coincide. I set unemployment to $A^{AU} = 0.066$ to match the mean over the data period.

\footnotesize\textsuperscript{45}Refer to the calibration appendix for more details.\textsuperscript{46}These are parameters such as those involved in the housing utility function ($\zeta$ & $\omega$) and bargaining weights, where pertinent empirical data/references to the literature do not exist.
The job-finding rate is set to $p^J = 0.294$ to match the mean constructed series from equation (E1). The rental vacancy rate for the U.S. is set to $rv = 0.095$ to match the series from the U.S. Department of Commerce: Census Bureau. Those targets can be substituted into the steady-state versions of the agent’s law of motions between all three states to pin down calibrations for the measure of agents which are employed, but without a home $A^{AEHU} = 0.268$, the measure of agents which are employed and matched with a home $A^{AEHM} = 0.666$, the rate at which employed agents match with landlords $p^H = 0.559$, the rate at which landlords match with employed agents $^{47} p^J = 2.137$, and the resulting housing market tightness $\tau^H = 0.261$. Tightness in the labor market is set to match the empirical series of vacancies per unemployed $\tau^E = 0.412$, implying the rate that firms match with the unemployed is $\frac{p^J}{\tau^E} = 0.715$.

Following Petrosky-Nadeu and Wasmer (2013) and Golin (2002), I set $\frac{w}{y} = 2.3$. I set $\tilde{\kappa} = 0.34 \bar{w}$ to match the rent-income ratio from data provided by Zillow\textsuperscript{48}.

I target a value for the cutoff commute $\tilde{\kappa}$ of 0.2. In the “Journey to Work” portion of the American Community Survey questionnaire, the shortest commute time is 0 minutes, while the longest is 120. In the 1980 Census, mean travel time to work was 21.7 minutes, in the 1990 Census it was 22.4 minutes in the 2000 Census it was 25.5 minutes and in the 2009 Census it was 25.1. Since I am looking at data spanning 2000 - 2014, I take the linear average between the 2000 and 2009 Census and arrive at

\textsuperscript{47}In the labor search literature, a matching rate greater than one implies one job vacancy results in more than one individual hire. I am making a similar assumption here, as the housing market is modelled symmetrically to the labor market. Hence a rental filling rate greater than one implies that a landlord which posts a single vacancy can match with multiple agents: an individual landlord may own a multi-resident apartment complex, or a combination of single family rentals and complexes, for example.

\textsuperscript{48}See the data appendix for details
a mean travel time of approximately 25.3 minutes. Normalized for the unit measure of the distribution $\tilde{\kappa} = \frac{25.3}{120} \approx 0.2$.

All targets are summarized in the table below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>Output</td>
<td>1.000</td>
</tr>
<tr>
<td>$A^{AU}$</td>
<td>Unemployed agents/unemployment rate</td>
<td>0.066</td>
</tr>
<tr>
<td>$p^J$</td>
<td>Job-finding rate</td>
<td>0.294</td>
</tr>
<tr>
<td>$rv$</td>
<td>U.S. rental vacancy rate</td>
<td>0.095</td>
</tr>
<tr>
<td>$p^H$</td>
<td>Agent housing match rate</td>
<td>0.559</td>
</tr>
<tr>
<td>$p^L$</td>
<td>Rate which landlord matches with renter</td>
<td>2.137</td>
</tr>
<tr>
<td>$\tau^H$</td>
<td>Housing market tightness</td>
<td>0.261</td>
</tr>
<tr>
<td>$\tau^E$</td>
<td>Labor market tightness</td>
<td>0.412</td>
</tr>
<tr>
<td>$p^F$</td>
<td>Rate which firms match with unemployed</td>
<td>0.715</td>
</tr>
<tr>
<td>$w$</td>
<td>Wage</td>
<td>0.667</td>
</tr>
<tr>
<td>$r$</td>
<td>Rental rate</td>
<td>0.227</td>
</tr>
<tr>
<td>$\tilde{\kappa}$</td>
<td>Commute</td>
<td>0.200</td>
</tr>
</tbody>
</table>

Table 2.3: Steady-state values for endogenous variables.

2.4 Numerical analysis

NOTE: The tables illustrating the main results of this section are placed after the conclusion.

This section will discuss the results from the stochastic simulation of the model. The simulation procedure I follow is to approximate the non-linear equilibrium using a first-order approximation and once the model is parameterized/calibrated using the values from the calibration section (section 3), simulate the economy’s response under the various exogenous shocks, paying particular attention to the behavior of the key
labor market variables\textsuperscript{49}. To calculate the simulated moments in each experiments, 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments.

As discussed in the empirical section (section 3), the persistence parameters and standard deviations of the error terms for labor productivity, labor separations, and renter bargaining power are estimated so that the theoretical moments on the processes match their empirical counterparts. The persistence parameter $\phi$ and the standard deviation of the error term for labor productivity are

$$\phi_y = 0.976 \quad \sigma_y = 0.0094.$$  

For labor match separations

$$\phi_{\sigma E} = 0.451 \quad \sigma_{\sigma E} = 0.0572.$$  

Finally, for the renter’s bargaining weight

$$\phi_\pi = 0.985 \quad \sigma_\pi = 0.016.$$  

These values lead to an exact match for the standard deviations\textsuperscript{50} outlined in the empirical section.

For the worker’s bargaining weight, there is no plausible means to set the parameters on the bargaining process to match an empirical counterpart, as there is

\textsuperscript{49}Variables include unemployment, vacancies, labor market tightness and the job finding rate.  
\textsuperscript{50}In subsection 3.1.6, I discussed how I constructed the series the renter’s bargaining power by using monthly data from the online real estate database Zillow. This data is available monthly at the state level. However, the data for this series is available only from 10/2010, while the JOLTS data I used for the labor market variables is available from 12/2000.
no series of data which can stand in as a proxy to the bargaining weight during wage negotiations. The literature typically refers to shocks to the bargaining weight as “wage shocks”. I thus set the value on the persistence parameter to ensure a stable near-unit root process and I set the standard deviation on the shock so that it nearly matches that of output:

\[ \phi_\eta = 0.95 \quad \sigma_\eta = 0.0099. \]

As shown in the model section, an important new mechanism of the model is the firm’s ability to extract a portion of the agent’s anticipated housing match surplus (see equation M13), where the proportion accessible to the firm is contingent on the housing bargaining weight \( \eta \). Subsection 3.1.6 outlines exactly how bargaining in housing comoves with labor productivity and separations. I will thus make use of the empirical values in incorporating shocks to the renter’s bargaining power in all of the stochastic simulations except for the simulation resulting from shocks to the labor bargaining parameter \( \eta \). The reason for this is that there is simply no empirical evidence of how bargaining in housing and bargaining in labor markets should comove. Further, since a shock to labor bargaining is essentially a wage shock, the question of what exactly a wage shock is still remains. Even though a contemporaneous shock to the renter’s bargaining power is omitted from this last simulation exercise involving a shock to the worker’s bargaining power, the housing-augmented model still exceeds the baseline model without housing in its ability to account for the volatility in key labor

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51 Both of the parameters set on the process dictating the bargaining weight are estimated using Bayesian methods in subsection 4.5.

52 This exact issue is brought up in Shimer (2005).
market variables.

The remainder of the section will go over the 3 simulation exercises\textsuperscript{53}, a robustness check, the Bayesian estimation exercise, and finally a discussion regarding a series of my model’s new interpretations of the results from another prominent work in housing.

2.4.1 Numerical Result #1: Labor Productivity & Renter Bargaining Power

The first exercise conducted is a simulation of the model with labor productivity shocks and shocks to the bargaining power of renters. Table 4 displays the results, where for ease of comparison entries in blue (top row) represent the empirical numbers from Table 1, entries in red (middle row) are the results from omitting the housing side of the model, and entries in black (bottom row) represent numbers resulting from the model with housing. In the numerical appendix, I have included Shimer’s (2005) results for comparison, along with the IRFs from this exercise. As the table shows, incorporating housing into the model makes a significant difference to the response of the key labor market variables. Since output and the renter’s bargaining power are exogenous features of the model, they are simulated such that the standard deviations and correlation match their empirical counterparts precisely. Although the volatilities produced by the housing-augmented model still come up short when compared to the actual data, the improvement over the baseline model is significant. In all endogenous

\textsuperscript{53}Motivation for the specific numerical experiments and for implementing contemporaneous shocks follows Shimer (2005).
variables, the housing-augmented model produces a slightly greater than three-fold increase in the level of volatility. Consistent with equation (M13), the procyclical renter’s bargaining weight results in a greater proportion of the (anticipated) housing surplus to be redistributed to the agent, increasing the joint employment surplus. The increase in the surplus to employment matches motivates the firm to post more vacancies, enlarging the volatility in the tightness of the labor market.

In this experiment, (including) housing does not seem to have a significant impact on the correlations. Compared to their empirical counterparts, the model exhibits a level of correlation which is too high in all pairs except for the correlation between unemployment and vacancies. Although the theoretical correlations are too high, all correlations match their empirical counterparts in sign.

2.4.2 Numerical Result #2: Labor Separation & Renter Bargaining Power

This numerical exercise consists of a simulation of the model with shocks to the separation rate for labor matches $\sigma^E$ and shocks to the bargaining power of renters. Since labor separations are counter-cyclical, in this experiment, when incorporating the housing side into model, positive shocks to labor match separations are accompanied by countercyclical shocks to the renter’s bargaining weight. Similar to the experiment involving labor productivity, separations and renter bargaining are calibrated to precisely match their (exogenous) empirical counterparts. Table 5 displays the results.

Compared to the baseline model, the housing-augmented model generates a
similar level of volatility in unemployment and vacancies. There is a significant improvement to labor-market tightness and the job-finding rate. The housing-augmented model is able to generate a level of volatility in tightness (standard deviation of 0.111) significantly greater than that of the baseline model (standard deviation of 0.004). A similar magnitude of improvement is seen in the job-finding rate; the standard deviation produced by the baseline model is 0.002, while the housing-augmented model generates a standard deviation of 0.044. However, even with the housing market incorporated into the model, these variables still fall short from their empirical counterparts (empirical standard deviations for tightness and the job-finding rate are 0.265 and 0.120, respectively).

One interesting result and significant improvement worth discussing involves the correlations: incorporating the housing market results in the correct sign on the theoretical correlations. The model-implied correlation between unemployment and vacancies when housing is omitted is positive; however, the empirical correlation between unemployment and vacancies is negative. Incorporating housing delivers the correct relationship between the two variables. The exact same thing can be said regarding the model-implied correlations between vacancies and tightness, vacancies and the job finding rate, and vacancies and the separation rate. Table 4 in Shimer (2005) reveals a similar model-implied error on an identical set of correlations; for example, from Shimer (2005): “These (results from Table 4 in that paper) introduce an almost perfectly positive correlation between unemployment and vacancies, an event which has essentially never been observed in the United States at business cycle frequencies”. In the basic DMP
model without housing, in the midst of an exogenous increase in unemployment, the economic environment is such that there is still (positive expected) value in the firm posting a vacancy.

2.4.3 Numerical Result #3: Labor Bargaining Share

The last experiment I report the results for is a shock directly to the labor bargaining weight $\eta$. In the previous experiments, a procyclical movement in the renter’s bargaining power $\overline{\eta}$ altered the relative proportion of the future anticipated housing surplus the newly employment-matched representative agent brought to the employment joint surplus. An increase in the renter’s proportion of the housing joint surplus as a result of the increase in the renter’s bargaining weight “trickled” to the agent-firm bargaining stage.

This experiment differs in that the renter’s bargaining share is held constant while the model is subjected to shocks in the labor bargaining weight - a “wage shock”. Unlike labor productivity, separations and the bargaining power of renters, there is no proxy, empirical or otherwise, that one could use to estimate a ”wage shock”. I set the persistence parameter and the volatility of the shock so that the standard deviation of the movements in bargaining are of identical magnitude to those of labor productivity.

When omitting housing from the experiment, the model becomes a basic stochastic version of the DMP model, hence there is no anticipated housing surplus the agent brings to the employment joint surplus with the firm. Table 6 reports the results.
Although there is a significant difference in the response of the housing-augmented model to the response of the baseline model as a result of subjecting both to the same labor bargaining shocks, housing improves the model’s fit. The model with housing is able to generate a level of volatility in all variables which is more than double that of the model without housing. In the baseline model, an increase in the proportion of the employment surplus distributed to the agent decreases the firm’s incentive to post vacancies, increasing unemployment and decreasing the job-finding rate. Incorporating housing in the model exacerbates this, as an increase in the agent’s bargaining power allows them to keep a greater proportion of their (anticipated) housing match surplus, increasing the wage further than in a model without housing.\(^{54}\)

The signs on all of the entries in the correlation matrix are invariant to whether or not housing is included in the model. Further, the signs match those on Shimer’s correlation matrix for the same experiment.

2.4.3.1 Discussion

The simulation under labor productivity shocks and shocks to renter bargaining power demonstrated a significant increase in the volatility generated in the key labor market variables. The results from the simulation under labor separation shocks and shocks to renter bargaining power also demonstrated a significant increase in volatility, in addition to delivering empirically correct signs on several extremely important correlations (the correlation between unemployment and vacancies during a shock to labor

\(^{54}\)See equation (M17)
separations, for example). The question of which combination or “mix” of shocks is appropriate to recreate the combination of shocks which continually hit the economy still remains, however.

One persistent problem highlighted by these experiments is that the level of volatility generated in the endogenous variables still falls short of what is seen in the data. Adding an additional friction(s) such as a wage rigidity (as in Hall 2005) or a rent rigidity may further improve the performance of the model further. Despite these shortcomings, the integration of housing frictions into the basic labor search model has proven itself a worthwhile extension of the model. While this chapter is not an advocacy of housing as the “silver bullet” solution to the Shimer puzzle, it is advocating housing as a non-trivial dimension and source of fluctuations which have a potential to effect the labor market.

2.4.4 Robustness check: On the time-invariance of the bargaining shares

Following Diaz and Jerez (2013), an alternative modelling assumption regarding the bargaining shares $\eta$ and $\bar{\eta}$ is to relate them to the tightness level in their respective markets\textsuperscript{55}. For example, I can reformulate the labor bargaining parameter

\textsuperscript{55}Diaz and Jerez (2013) assume an urn-ball matching process

$$m(\theta) = 1 - e^{-\theta},$$

where $\theta$ is their measure of housing market tightness. They then define their bargaining shares as

$$\eta(\theta) = \frac{m'(\theta) \theta}{m(\theta)} = \frac{\theta e^{-\theta}}{1 - e^{-\theta}},$$

an implementation of the Hosios (1990) rule. This rule says that each period the buyer gets a share of the bilateral surplus that is equal to the elasticity of the seller’s matching probability.
as

$$\eta_t = \eta^* \left( \frac{\tau E_t}{\tau E^*} \right)^\delta, \quad \delta > 0,$$

where variables with an asterisk indicate steady-state equivalents. This expression shows how if vacancies increase relative to the unemployed, this increases $\eta_t$ according to the curvature parameter $\delta$, increasing the bargaining power of all matched workers. This would illustrate a so-called “buyer’s” or “seller’s” market, through an endogenous increase or decrease in the bargaining power, contingent on the relative supply or demand in the market. Figure 11 illustrates the response of two sets of key labor market variables in a simple stochastic version of DMP following a negative labor productivity shock - one set originates from constant labor bargaining shares, while the other originates from endogenous labor bargaining shares.

![Figure 2.11: Impulse responses for unemployment, vacancies, tightness and the job-finding rate following a negative labor productivity shock (the standard error of the innovation to $y$ is 0.01, a value I use throughout the numerical simulations in order to match the empirical standard deviation and autocorrelation as closely as possible). The black impulse responses are indicative of constant labor bargaining shares, while blue impulse responses illustrate a situation with an endogenous labor bargaining weight.](image)

While this may seem a logically attractive extension to the model, it actually results in a further deterioration of the model’s ability to match the data. For example, if
we start with the most basic, stochastic version of the DMP framework, but with time-varying labor bargaining shares, and the economy experiences a negative innovation to labor productivity, as figure 11 shows, the responses of unemployment, vacancies, tightness, and the rate of job-finding are muted.

The firm essentially internalizes the endogenous response of the bargaining shares, and hence understands that for each additional unemployed individual looking to match with the firm, the proportion of the labor match surplus the firm is redistributed is incrementally increased, motivating a weakened negative response from job vacancies. This implies that the job finding rate for an unemployed individual falls by less than a situation with constant shares, implying unemployment rises less sharply. Labor market tightness, a variable whose procyclicality was discussed in section 3.1.2, falls as a result of the movements of vacancies and unemployment, albeit by less than a situation where bargaining weights are constant.

2.4.5 Assessing the goodness-of-fit of the model: Bayesian estimation of parameters with no empirical justification

In this subsection, I will go over the results of estimating the model using Bayesian methods. As mentioned in the parameterization section, the integration of a decentralized housing market into the baseline DMP model was accompanied with the problem of choosing values for parameters which didn’t have standard values established in the literature, or any empirical data I could estimate them from. Bayesian estimation provides the ability to at least conclude if the prior distributions chosen for
these parameters depart significantly from their resulting posteriors. These parameters include the utility function parameters $\zeta$ and $\omega$, the housing market matching parameter $\chi$, the cost of posting a rental vacancy $\xi$, the bargaining weights $\eta$ and $\overline{\eta}$, the persistence parameters on the AR(1) processes for the labor bargaining weight $\phi_\eta$ and, and the standard error of the disturbance terms on the AR(1) processes dictating labor productivity, the labor match separation rate, the renter bargaining weight, and the labor bargaining weight.

2.4.5.1 Empirically-justified priors

There are two sets of priors for which there exists empirical justification; they include the persistence and disturbance terms on the processes for labor productivity, labor match separations, and the renter bargaining power. For the labor bargaining weight process (persistence parameter and standard error of the disturbance term) and all of the other asterisked parameters from the parameterization subsection, I set the prior distributions following the methods of Gertler, Sala & Trigari (2008).

For the process dictating labor productivity $y$, the persistence parameter $\phi_y$ is set to its empirical estimate\textsuperscript{56} and figure 12 shows the frequency distribution for the residuals of the fitted process.

The empirical distribution has a mean not different from zero and a standard deviation of 0.004. I will thus follow Gertler, Sala, and Trigari (2008) and set the prior distribution of the standard error of this disturbance to an Inverse-gamma distribution

\textsuperscript{56}For details, please see the calibration appendix.
Figure 2.12: Empirical distribution for the errors of the AR(1) model for the log-deviation of output.

with mean 0.004 and standard deviation 0.15.

Figure 13 illustrates the empirical frequency distribution the residuals for the separation series.

As in labor productivity, I model labor match separations using an AR(1) process\textsuperscript{57}. The standard deviation for this distribution is 0.056; again, following Gertler, Sala, and Trigari (2008), I will set the prior distribution for the standard error of the disturbance term to an Inverse-gamma with mean 0.056 and standard deviation 0.15.

The last set of priors I would like to discuss involve the series for the renter’s bargaining weight. This series is also modelled as an AR(1) process. Figure 14 shows the distribution of the errors of the model.

The standard deviation of the distribution of error terms is 0.013; I set the prior distribution for the standard error of the disturbance term to an Inverse-gamma

\textsuperscript{57}Details in the calibration appendix.
with mean 0.013 and standard deviation 0.15.

### 2.4.5.2 Remaining priors & estimation results

The remaining priors are set following the lead of Gertler, Sala, and Trigari (2008) and Sala, Soderstrom, and Trigari (2008) fairly closely. These include prior distributions for $\zeta$, $\omega$, $\chi$, $\eta$, $\eta_i$, and $\epsilon^\eta_t$. Observable variables used in the estimation include unemployment $\hat{u}$, labor market tightness $\hat{t}$, and the job-finding rate $\hat{p}^J$. Table 7 lists both the assumed priors and the estimated means and 5% and 95% percentiles of the posterior distribution. I have included plots for all priors and posteriors in the estimation appendix.
Figure 2.14: Empirical distribution for the errors of the AR(1) model for the log-deviation of renter bargaining power.
### Numerical Result #1: Labor Productivity & Renter Bargaining Power

<table>
<thead>
<tr>
<th></th>
<th>( u )</th>
<th>( v )</th>
<th>( v/u )</th>
<th>( p^J )</th>
<th>( y )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.133</strong></td>
<td>0.132</td>
<td>0.265</td>
<td>0.120</td>
<td>0.020</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td><strong>0.014</strong></td>
<td>0.034</td>
<td>0.044</td>
<td>0.018</td>
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<td>0.020</td>
<td>0.035</td>
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<tr>
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<td>0.963</td>
<td>0.711</td>
<td>0.979</td>
<td>0.899</td>
<td></td>
</tr>
<tr>
<td><strong>0.977</strong></td>
<td>0.820</td>
<td>0.898</td>
<td>0.898</td>
<td>0.898</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td><strong>0.979</strong></td>
<td>0.871</td>
<td>0.917</td>
<td>0.917</td>
<td>0.898</td>
<td>0.901</td>
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<tr>
<td><strong>-0.877</strong></td>
<td>-0.915</td>
<td>-0.795</td>
<td>-0.308</td>
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<td></td>
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<td><strong>u</strong></td>
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<td>-0.804</td>
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<tr>
<td></td>
<td></td>
<td>-0.834</td>
<td>-0.919</td>
<td>-0.486</td>
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<td>0.976</td>
<td>0.816</td>
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<tr>
<td><strong>v</strong></td>
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<td>0.969</td>
<td>0.969</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.984</td>
<td>0.984</td>
<td>0.531</td>
<td>0.937</td>
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</tr>
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<td><strong>Correlation matrix</strong></td>
<td><strong>v/u</strong></td>
<td><strong>v</strong></td>
<td><strong>u</strong></td>
<td><strong>v</strong></td>
<td><strong>w</strong></td>
<td><strong>x</strong></td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
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<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
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<td><strong>p^J</strong></td>
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<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.536</td>
<td>0.945</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y</strong></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.234</td>
<td>0.234</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>( \eta )</strong></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.4: Simulation results for the labor model under shocks to productivity and renter’s bargaining power. Values in blue (top row) are the empirical values from Table 1 reproduced for convenience of comparison, values in red (middle row) indicate the values for when the housing side of the model is turned off (becomes a basic, stochastic version of DMP), while values in black (bottom row) are the values for when the housing side of the model is active. Since housing is absent from the baseline model, the N/As indicate that the renter’s bargaining power would be non-applicable. 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments.
### Numerical Result #2: Labor Separation & Renter Bargaining Power

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>v/u</th>
<th>p(^J)</th>
<th>(\sigma^E)</th>
<th>(\eta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.133</td>
<td>0.132</td>
<td>0.265</td>
<td>0.120</td>
<td>0.062</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.040</td>
<td>0.039</td>
<td>0.004</td>
<td>0.002</td>
<td>0.062</td>
<td>N/A</td>
</tr>
<tr>
<td>0.059</td>
<td>0.058</td>
<td>0.111</td>
<td>0.044</td>
<td>0.062</td>
<td>0.035</td>
<td></td>
</tr>
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<td>0.908</td>
<td>0.963</td>
<td>0.711</td>
<td>0.447</td>
<td>0.899</td>
</tr>
<tr>
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<td>0.438</td>
<td>N/A</td>
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</tr>
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<td>0.907</td>
<td>0.963</td>
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<td>0.938</td>
<td>0.438</td>
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<td>0.795</td>
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<td>N/A</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>0.040</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>1</td>
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<td>-0.196</td>
<td>0.196</td>
<td>N/A</td>
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</tr>
<tr>
<td>0.422</td>
<td>N/A</td>
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<td></td>
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</tr>
<tr>
<td>v/u</td>
<td>1</td>
<td>0.842</td>
<td>-0.398</td>
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</tr>
<tr>
<td>0.816</td>
<td>N/A</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(^J)</td>
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</tr>
<tr>
<td>0.284</td>
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<tr>
<td>(\sigma^E)</td>
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<tr>
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<tr>
<td>(\eta)</td>
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</table>

Table 2.5: Simulation results for the labor model under shocks to labor match separation and rental surplus bargaining. Values in blue (top row) are the empirical values from Table 1 reproduced for convenience of comparison, values in red (middle row) indicate the values for when the housing side of the model is turned off (becomes a basic, stochastic version of DMP), while values in black (bottom row) are the values for when the housing side of the model is active. Since housing is absent from the baseline model, the N/As indicate that the renter’s bargaining power would be non-applicable. 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments.
### Table 2.6: Simulation results for the labor model under shocks to labor bargaining.

Values in blue (top row) are the empirical values from Table 1 reproduced for convenience of comparison, values in red (middle row) indicate the values for when the housing side of the model is turned off (becomes a basic, stochastic version of DMP), while values in black (bottom row) are the values for when the housing side of the model is active. The N/As indicate that there is no empirical data available for the standard deviation, monthly autocorrelation and empirical correlation with other variables for the labor bargaining weight. 10,000 draws were taken from each of the distributions of the shocks, with the first 1,000 draws dropped prior to the calculation of the moments.

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
<th>$p^J$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.133</td>
<td>0.132</td>
<td>0.265</td>
<td>0.120</td>
<td>0.020</td>
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<td>0.024</td>
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<td>0.908</td>
<td>0.963</td>
<td>0.711</td>
<td>N/A</td>
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<td>-0.795</td>
<td>N/A</td>
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</tr>
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<td>-0.807</td>
<td>-0.902</td>
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<tr>
<td>$v$</td>
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<td>0.976</td>
<td>0.816</td>
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<td></td>
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<td>0.983</td>
<td>0.983</td>
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<tr>
<td>$v/u$</td>
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</tr>
<tr>
<td>$p^J$</td>
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<td></td>
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<td>$\eta$</td>
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### Estimation Results

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<tr>
<th>parameter</th>
<th>prior mean</th>
<th>post mean</th>
<th>90% HPD interval</th>
<th>prior dist</th>
<th>prior std dev</th>
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<td>1.0016</td>
<td>0.8317 1.1476</td>
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<tr>
<td>Labor Bargaining $\eta$</td>
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<td>0.4774</td>
<td>0.2973 0.6725</td>
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<td>Std Err On $y$ Proces $e^y$</td>
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<td>0.1500</td>
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</table>

Table 2.7: This table reports the prior and posterior distribution of the estimated parameters of the structural parameters and parameters of the exogenous shock processes. IGamma refers to the Inverse-gamma distribution.
Overall, the model fits the data fairly well, in terms of the prior means falling within the 90% credible interval, aside from the parameters describing the labor separation shock $\epsilon^s$, the persistence of labor bargaining $\phi_\eta$ and the labor bargaining shock $\epsilon^\eta$. The estimated posteriors on the separation and rental bargaining shocks suggest that the prior I had set on separations was too low, while the prior on wage bargaining was too high. This is primarily a result of my setting the volatility on the wage shock to match that of labor productivity. If we consider shocks to the bargaining power of laborers as “wage shocks”, the fact that the posterior mean is roughly half that of the prior reinforces the fact that wages are less volatile than labor productivity. Further, the prior on the persistence parameter for the process dictating labor bargaining falls short of the credible interval. Upon inspection of the posterior distribution (see the estimation appendix) there is evidence of bi-modality, indicating that the bargaining power of labor may have experienced a “regime switch” as a result of the Great Recession.

Unlike the shock to labor bargaining, the posterior estimate for the shock to labor separations indicates that the prior to this parameter was set too low. Like the persistence parameter for labor bargaining, the prior falls short of the credible window.

2.4.6 Discussion: How do the results of this model relate to others?

In Wheaton (1990), the author pointed out three steady state comparative statics results I would like to highlight and discuss new interpretations my model provides for these comparative statics results. The first is proposition 3 which states that
the price of housing and the housing vacancies in the market are inversely related.\footnote{Page 1283 of that article} This is a standard result in search models, where prices are redistributed via bargaining over the joint surplus to a match. Recall equation (M18), the rental rate for housing

\[
r = (1 - \eta) u (\kappa t) + \eta \left( \lambda - \xi \tau_H \right),
\]

where \( \tau_H \) is the ratio of vacant units to employed individuals looking for a home to rent.

As in Wheaton, an increase in vacancies has a negative relationship to the equilibrium rental rate. However, the transmission mechanism for this result in the model of this chapter is sourced from the renter’s “outside option”.\footnote{During bargaining between the landlord and the renter, a high level of vacancies gives the renter more leverage than the landlord.}

The second comparative static result I would like to discuss my model’s interpretation of is what Wheaton refers to as “familial turnover”.\footnote{In Wheaton (1990) there are two types of households. Each household can exogenously change from one type to the other, however, when a change occurs, they are no longer matched to their home and they become “separated”. An example would be if a family expands; the home may no longer be accommodating to the larger family. This is what Wheaton refers to as “familial turnover”.} In the model constructed in this chapter, “familial turnover” is equivalent to housing match separation \( \sigma_H \). In Wheaton, proposition 6 states that the price of housing is increasing in familial turnover, a relationship the model in this chapter shares as shown in figure 15 below. I have also added the impulse response for the inverse of the matching rate, or what Wheaton refers to as “time-to-sell”. He couldn’t sign the comparative static in the paper as it was mathematically ambiguous if time-to-sell increases or decreases as a result of an increase in familial turnover. As figure 15 shows, time-to-rent in my model increases as a result of a shock to \( \sigma_H \). In the model constructed in this chapter, an increase in
Figure 2.15: Impulse responses illustrating a similar relationship between Wheaton’s results with the model of this chapter. Plots show the impulse responses for the price of housing (rental rate) and the time-to-rent (Wheaton’s version of “time-to-sell” - the inverse of the housing matching rate) as a result of a shock to $\sigma^H$.

separations motivates a decrease in the entry of landlords in the rental market, resulting in a fall in matching probability, implying an increase in the expected time it takes to form a match. Both impulses are the result of a 0.05 standard deviation shock to $\sigma^H$.

2.5 Conclusion

This chapter provides two contributions. The first is an integration of a labor market characterized by search and matching frictions with a housing market characterized by search and matching frictions into a single, coherent macroeconomic model which combines three branches of the literature: the labor search literature, as highlighted by this model’s decentralized labor market; the housing search literature, as highlighted by this model’s decentralized housing market; and the urban economics lit-
erature, as highlighted by this model’s endogenous commute cutoff delineating where agents choose to allocate themselves spatially as a direct result of what occurs in the other two markets.

The second contribution is that the integration of the housing market into the labor market increases the ability of the resulting model to match the empirical moments of recent labor market data, through a new channel established in the model where unanticipated changes in the housing market manifest themselves as real labor market effects.

Although this chapter is not proclaimed as a “silver bullet” solution to the Shimer (2005) puzzle, it provides a means to study how an alternative source of fluctuations such as those originating from the housing market may impact local labor markets.

An interesting extension to the model would be the incorporation of optimal saving by agents to safeguard against employment separations. This would allow an agent which separated from their employer to stay in their current rental (instead of automatically separating from their housing match), paying their housing costs with their savings until another job is secured. This extension would not come without cost, however, since once the agent would then match with a searching firm, the firm would then know the exact commute and this would then potentially motivate a strategic game of hidden information on the part of the unemployed agent. A second interesting extension to the model would be to incorporate some type of barrier to entry facing the landlords, resulting in an amplification of housing market responses to shocks originating
from the labor market. Finally, a third extension includes adding an owner-occupied market in order to investigate the potential role the home-ownership rate may play in labor migration. I leave these extensions for future work.
Chapter 3

Monetary policy operating procedures, lending frictions, and employment
- with David Florian-Hoyle
and Carl Walsh -

3.1 Introduction

How does the central bank’s operating procedure affect the transmission process of monetary policy? In the 20 years prior to the financial crisis beginning in 2007, this question was little examined. With major central banks directly targeting the interbank interest rate, this single interest rate was viewed as the sole link between actions of the central bank and the real economy. And how the central bank managed discount borrowing and whether it paid interest on reserves were implicitly deemed irrelevant to
understanding how changes in the target for the interbank rate affected real economic activity. This view was most explicit in standard new Keynesian models in which the policy interest rate was the sole interest rate appearing in the model and monetary aggregates, including bank reserves, could be ignored.

The financial crisis, the renewed recognition that financial markets are subject to frictions, the constraint imposed by the zero lower bound on the policy interest rate, and the adoption of new procedures for affecting reserve supply call for a reexamination of the links between the central banks operating procedures in the interbank market, the availability of credit, and the impact of monetary policy on the real economy.

In this chapter, we examine these links in a model in which banks hold reserves to meet random fluctuations in settlements, and the central bank pays interest on reserves, lends reserves at a penalty rate, and can independently affect the quantity of reserves and the level of interest rates. Banks make loans to firms in credit markets characterized by matching frictions, and interest rates on loans are set in bilateral bargaining between banks and firms.

The type of monetary policy operating procedure we analyze is often called a corridor or channel system of interest rate control. Such a system is employed by several central banks (e.g., the Reserve Bank of New Zealand) and is the type of system the U.S. Federal Reserve seems likely to employ when interest rates return to historically more normal levels. In a channel system, a central bank offers a lending facility, whereby commercial banks are permitted to borrow against collateral from the central bank at an interest rate that is above the target rate (the penalty or ceiling rate) and a
deposit facility, whereby banks can earn overnight interest on their excess reserves at a rate that is below the target rate (the floor rate). The ceiling and floor rates form an interest rate channel (or corridor). In reality, many central banks use what is known as a symmetric channel system for monetary policy implementation, in which the ceiling and floor rates are the same number of basis points (the width) above and below the target rate. The symmetric channel systems used by various central banks differ in many respects. For example, the Bank of England and the ECB institute a relatively wide channel framework with a spread of 100 basis points on each side of the target. Australia and Canada, in contrast, operate narrow channels with a spread of only 25 basis points above and below their targets.¹

There is a small existing literature on channel systems. Woodford (2000, 2001, 2003) discusses how to conduct monetary policy with a vanishing stock of money using the framework of a channel system. Whitesell (2006) evaluates reserves regimes versus channel systems.² Berentsen and Monnet (2007) develop a general equilibrium framework of a channel system and investigate optimal policy. Berentsen, Marchesiani, and Waller (2010) develop a general equilibrium model to show that a positive spread between the policy rate and the interest rate paid on reserves is optimal. The uncertainty facing banks in these papers arises from a Diamond-Dybvig environment in which depositors are revealed, ex post, to be either patient or impatient. Thus, banks must hold excess reserves to insure against a net payment drain from the entire banking system. In contrast, we assume uncertainty arises from the random distribution of payment flows.

¹ Australia and Canada have no reserve requirements.
² These models are also discussed in Walsh (2010).
among banks that result in some banks facing a net outflow while others experience a net inflow. However, the net flow aggregated across the banking system is always zero. Other work related to elements of channel systems include Gaspar, Quiros and Mendizabal (2004), Guthrie and Wright (2000), and Heller and Lengwiler (2003).

In contrast to these papers, we focus on the links between the implementation of monetary policy under a channel system of interest-rate control and credit spreads in the market for bank loans in the face of lending frictions. These lending frictions are captured by a simple search-and-matching framework, with lending interest rates determined by Nash bargaining between lenders (banks) and borrowers (firms). In this environment, the joint surplus to the bank and the firm depends, in part, on the structure of the interbank market as the structure of the interbank market affects the outside opportunity of the bank.

This chapter is also closely related to another stream of literature — frictions in credit markets. Most work on credit market frictions has rightly focused on issues related to informational asymmetry and moral hazard issues. The literature on this is large; early examples include Carlstrom-Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997), while more recent papers include Gertler and Kiyotaki (2010), Gertler and Karadi (2010), and del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

Another type of frictions in financial markets is what is referred to as “search and entry frictions” (Becsi et al, 2000), which represents the search or negotiating costs related to the initial participation of firms into credit markets. Recent empirical
evidence using U.S disaggregated bank-level data by Contessi and Francis (2011 and 2013), Craig and Haubrich (2006) Dell Ariccia and Garibaldi (2005) and Herrera, Kolar and Minetti (2007/2011) suggest that sizable gross credit flows coexist at the business cycle frequency. This literature emphasizes the existence of heterogeneous patterns of credit creation and contraction at any phase of the business cycle. For example, Dell Ariccia and Garibaldi (2005) find that in the United States, gross credit flows are by an order of magnitude more volatile than GDP and investment. The empirical dynamics patterns of credit flows found in this literature are consistent with those predicted by search models in which the interaction of shocks generates simultaneous occurrence of credit expansions and contractions.


In this chapter, we incorporate a search-and-matching process between borrowers (firms) and lenders (banks). The financial contract and the credit interest rate are an outcome of a Nash bargaining, and idiosyncratic shocks to the entrepreneurs productivity level determine the rate of endogenous match destruction. That is, banks with funds and firms with projects search for partners in the credit market. Banks
obtain funds to finance firms by raising retail deposits. To produce, an individual firm must be matched with a bank; to lend, an individual bank must be matched with a firm. Unmatched banks search for lending opportunities; unmatched firms search for a bank to finance their production. As in den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Beauburn-Diant and Tripier (2009), matched banks and firms decide whether to maintain or sever their credit relationship, depending on the productivity of the firm’s project. If the firm and the bank choose to cooperate, Nash bargaining determines the loan rate that determines how the joint surplus of the match is shared between the bank and the firm.

We extend the search and matching model of credit frictions to incorporating a second stage where banks operate in a centralized bond and interbank market. The nature of the interbank market the central banks use of a corridor system affects the bargaining process involving banks and rms and affects directly the equilibrium interest rate on loans. Thus, the model consists of an interbank market that involves banks and the monetary authority, and a loan market in which banks and firms participate. Banks need to meet their need for settlement balances in the interbank market and, besides interbank lending, banks can deposit excess reserves at the central bank or borrow reserves through a standing facility. The structure of the interbank market affects the lending decisions of banks in the loan market, and the resulting spread between the average lending rate and the central banks policy rate depends on this matching process, the nature of Nash bargaining, and structure of the interbank market and monetary policy operating procedures.
A further contribution of this chapter pertains to the cost channel of monetary policy (Ravenna and Walsh 2006) in which, because firms must finance wage payments in advance of production, the relevant cost of labor is affected by the interest rate firms pay on loans. However, when the loan rate is the outcome of a bargaining process, its role is to split the surplus between the borrower (the firm) and the lender (the bank) and thus is irrelevant for the firm’s employment decision which is made to maximize the joint surplus. There is still a cost channel but it depends on the opportunity cost of funds to the bank, not the interest rate charged on the loan, and therefore it too is dependent on the structure of the interbank market. Changes in the policy interest rate, the penalty for borrowing reserves from the central bank, the interest rate paid on reserve deposits at the central bank, the supply of bank reserves by the central bank, and the volatility of settlement payment flows all influence this outside opportunity and therefore affect the equilibrium spread between the average rate on bank loans and the policy interest rate.

Finally, by assuming individual firms are subject to stochastic productivity shocks, and the threshold productivity level below which the firm is unable to obtain financing depend on these same factors characterizing the interbank market. In particular, we show that a rise in interbank volatility increases the credit spread and, by raising the threshold productivity level the firm needs to obtain financing, reduces the number of firms able to obtain loans. Similarly, monetary policy has effects on employment and output on both extensive (the fraction of firms receiving loans) and intensive (the size of loans conditional on obtaining one) margins. The latter arises as a reduction in
the cost of funds for banks makes it optimal for firms with access to credit to expand employment. The former arises because the lower cost of finance will make it profitable for banks to lend to more firms.

The remainder of the chapter is arranged as follows. The next section briefly describes a channel system and some of the relevant literature. Section 3.2 presents the basic model setting, describing first the reserve market under a corridor system and then the loan market. The bargaining solution that determines the interest rate on loans, the evolution of the number of bank-firm matches, and the equilibrium are also discussed in this section. The results of a numerical analysis are described in section 3.3. Conclusions are given in section 3.4.

3.2 The Model

The model economy is populated by households, banks, firms, and a central bank. Households supply labor to firms, hold cash and bank deposits, and purchase final output in the goods market. Firms seek financing, hire labor financed by bank loans and produce output. Banks accept deposits, hold reserves and bonds, and finance the wage bill of firms. The central bank pays interest on reserve deposits and charges a penalty rate on lending to banks.

Three aspects of the model are of critical importance. First, it is assumed that households cannot lend directly to firms. While this type of market segmentation is taken as exogenous, one could easily motivate it by assuming informational asymmetries.
under which households are unable to monitor firms while banks are able to do so. This
asymmetry also forces firms to make up-front payments to workers to secure labor.
Second, lending activity involving firms and banks occurs in a decentralized market
characterized by random matching. And third, we assume all payment flows must
be settled at the end of each period and that individual banks face idiosyncratic and
uninsurable risk arising from random end-of-period settlement flows.

At the beginning of each period, aggregate shocks are realized and households
deposit funds with a bank. The market for deposits is competitive and all banks offer
the same interest rate on deposits. In the lending market, firms seek funding to make
wage payments. Firms are subject to aggregate and idiosyncratic productivity shocks
and these determine whether it is profitable for a firm to operate and, if it is, at what
scale. If a firm is not already matched with a bank, it must seek out a new lender.
Similarly, banks not already matched with a firm must search for borrowers. After the
loan market closes, firms and workers produce and households consume, while banks can
participate in the interbank market, investing deposits net of loans into risk-free bonds,
lending to or borrowing from other banks, and holding deposits with the central bank.
After these markets close, all net payment flows are settled. Banks receive repayment
from firms, but since firm receipts arise from households with deposits at different banks,
some banks experience a net payment outflow, others an inflow. Banks with a shortage
of funds must borrow from the central bank’s standing facility; those with an excess of
funds can deposit these with the central bank.

The market for bank loans is characterized by search and matching frictions.
While these frictions are more commonly employed in the analysis of labor markets (e.g., Mortensen and Pissarides 1994), this approach has also been used to model credit market frictions; examples include Diamond 1990, Acemoglu 2001, den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), Becsi, Li and Wang (2005), Beauburn-Diant and Tripier 2009 and Petrovsky-Nadeu and Wasmer (2013). In contrast to Beauburn-Diant and Tripier, we allow employment and loan size to vary across firms due to firm-specific idiosyncratic productivity shocks. Firms must finance their wage bill, which introduces a cost channel (Ravenna and Walsh 2006) for monetary policy. Wasmer and Weil (2004) and Petrovsky-Nadeau and Wasmer (2013) incorporate matching frictions in both credit and labor markets and impose a free-entry condition on both firms and banks. Since our focus is on the role of the interbank market in affecting loan supply, we assume a continuum of firms on the unit interval who are either producing or seeking finance and treat the labor market as competitive with firms taking the wage as given in deciding how much labor to employ. Beaubrun-Diant and Tripier, Wasmer and Weill, and Petrovsky-Nadeau and Wasmer all treat the opportunity cost of funds to the banks as an exogenous parameter. In contrast, we focus on the role of the interbank market and central bank policy implementation as affecting the cost of funds for banks, and we provide a complete general equilibrium model.
3.2.1 Households

Households consume final output and supply labor to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, 1 - N_{t+i}); \quad 0 < \beta < 1.$$  

The utility function has standard properties. The household enters the period with nominal assets $A_{t-1}$ consisting of the existing stock of government debt held by the private sector and holdings of high powered money, and these assets are allocated by the household between bank deposits and bond holdings:

$$A_{t-1} \equiv HP_{t-1} + B_{t-1}^p = D_t + B_t^h.$$ (3.1)

The household is subject to a cash-in-advance constraint that requires consumption to be purchased using initial deposit balances and current period wage receipts, or

$$D_t + w_t P_t N_t = A_{t-1} - B_t^h + w_t P_t N_t \geq P_t C_t,$$

where $w$ is the real wage and $P$ is the price level.\(^3\) In real terms,

$$d_t + w_t N_t - C_t \geq 0,$$ (3.2)

where $d_t = D_t / P_t$. Let $i^d_t$ be the nominal return on bank deposits and $i^b_t$ the nominal return on bonds. The household’s budget constraint in nominal terms is

$$A_t = A_{t-1} + i^d_t D_t + i^b_t B_t^h + w_t P_t N_t + \Pi_t^b + \Pi_t^d - P_t C_t - P_t T_t,$$  

\(^3\)This constraint could, if one felt it necessary, be motivated by assuming households are anonymous to firms so firms will not sell goods on credit to be repaid in future periods. This assumption, combined with the assumption that banks cannot track future deposit activity of households would suffice.
where $\Pi^i, i = b, f$ are bank and firm nominal profits and $P_t T_t$ are nominal lump-sum taxes or transfers. Define $a_t = A_t / P_t$, and $b^h_t = B^h_t / P_t$. In real terms, the budget constraint becomes

$$a_t = \left( \frac{1}{1 + \pi_t} \right) a_{t-1} + i_t^d d_t + i_t^h b^h_t + w_t N_t + \left( \frac{\Pi^b_t + \Pi^f_t}{P_t} \right) - C_t - T_t. \quad (3.3)$$

The value function for the representative household defined by

$$V \left( \frac{a_{t-1}}{1 + \pi_t} \right) = \max_{d_t, b^h_t, a_t, C_t, N_t} \left[ U(C_t, 1 - N_t) + \beta E_t V \left( \frac{a_t}{1 + \pi_{t+1}} \right) \right],$$

where $1 + \pi_t = P_t / P_{t-1}$ and the maximization is subject to (3.2), (3.3), and, from (3.1),

$$\left( \frac{1}{1 + \pi_t} \right) a_{t-1} - d_t - b^h_t = 0. \quad (3.4)$$

Let $\mu$ and $\lambda$ be the Lagrangian multipliers associated with the cash-in-advance and budget constraints. Let $\varphi$ be the Lagrangian multiplier on the constraint (3.4). Then the first order necessary conditions for the household’s problem of maximizing utility are

$$C_t : \quad U_C(C_t, 1 - N_t) = (\mu_t + \lambda_t)$$

$$N_t : \quad U_N(C_t, 1 - N_t) = w_t (\mu_t + \lambda_t)$$

$$a_t : \quad -\lambda_t + E_t \beta \left( \frac{1}{1 + \pi_{t+1}} \right) V' \left( \frac{a_t}{1 + \pi_{t+1}} \right) = 0$$

$$d_t : \quad \mu_t + i_t^d \lambda_t - \varphi_t = 0$$

$$b^h_t : \quad i_t^h \lambda_t - \varphi_t = 0 \Rightarrow \varphi_t = i_t^h \lambda_t$$

As is standard, the first two imply

$$\frac{U_N(C_t, 1 - N_t)}{U_C(C_t, 1 - N_t)} = w_t. \quad (3.5)$$
while the last two imply
\[ \mu_t = \varphi_t - i^d_t \lambda_t = \left( \frac{i^b_t}{i^d_t} - i^d_t \right) \lambda_t \]  
(3.6)
so that the excess yield of bonds over deposits measures the liquidity services provided by deposits. This in turn implies that
\[ U_C(C_t, 1 - N_t) = \mu_t + \lambda_t = \left( 1 + \frac{i^b_t}{1 + \pi_t} \right) \lambda_t. \]  
(3.7)

From the envelope theorem,
\[ V'(a_t - 1) = \lambda_t + \varphi_t = \left( 1 + \frac{i^b_t}{1 + \pi_t + 1} \right) \lambda_t, \]
and the first order condition for \( a_t \) can then be written as
\[ \lambda_t = \beta E_t \left( \frac{1}{1 + \pi_{t+1}} \right) V'(a_t - 1) = \beta E_t \left( \frac{1 + i^b_{t+1}}{1 + \pi_{t+1}} \right) \lambda_{t+1}. \]  
(3.8)

### 3.2.2 The loan market

We set up a non-Walrasian loan market by assuming that the process of finding a credit partner is costly in terms of time and resources, leading to the existence of sunk costs at the time of trading and a surplus to be shared between lenders (banks) and borrowers (firms) whom meet in a pairwise manner. Search and matching frictions prevent instantaneous trading in the loan market, implying that not all market participants will end up matched at a given point in time. We allow for both exogenous and endogenous destruction of credit matches, and as is common in the search and matching literature, we assume a matching technology that determines the flow of new credit relationships over time as a function of the relative number of lenders and borrowers.
searching for trading partners. Upon a successful match (i.e., a match that survives the exogenous and endogenous separation hazards), bilateral Nash bargaining between the parties determine the firms’ employment level and the way the match surplus is shared. The loan size is chosen to maximize the joint surplus to the lender and borrower, while the corresponding loan interest rate determines how the surplus is split between the two partners.

In this section we first present the search and matching process in the loan market as well as the evolution of credit relationships across time. We also define credit creation and destruction. Then we present the behavior of firms and banks as well as the Nash bargaining solution. Finally, we characterize the loan market equilibrium in terms of the tightness of the credit market and a threshold productivity level which below the firm is unable to obtain external funds.

3.2.2.1 The matching process

We assume a continuum of banks and firms with the number of banks seeking borrowers varying endogenously and being determined by a free entry condition to the loan market. We assume banks have a constant returns to scale technology for managing loans, so we can treat each loan as a separate match between a bank and a firm. Each firm is endowed with one project and is either searching for external funds or involved in an ongoing credit contract with a bank. If a firm is matched with a bank, then the bank extends the necessary funds to allow the firm to hire workers.

Firms searching for external funds, $f_t$, are matched to banks searching for
borrowers, $b_t^u$, according to the following constant return to scale matching function

$$m_t = \mu_t f_t^u (b_t^u)^{1-\varphi}$$

The function $m_t$ is strictly concave with constant returns to scale and determines the flow of new credit contracts during date $t$; $0 < \mu_t < 1$ measures the time-varying productivity of the matching function and $0 < \varphi < 1$ is the elasticity of match arrivals with respect to the mass of searching firms.

**Matching rates** The variable $\tau_t = f_t / b_t^u$ is a measure of credit market tightness, and corresponds to the standard measure of market tightness arising in search and matching models of the labor market: The probability that an entrepreneur with an unfunded project is matched with a bank seeking to lend at date $t$ is denoted by $p_t^f$ and is given by

$$p_t^f = \mu_t \tau_t^{\varphi-1}$$  \hfill (3.9)

Similarly, the probability that any bank seeking borrowers is matched with an unfunded entrepreneur at time $t$ is denoted by $p_t^b$ and is given by

$$p_t^b = \mu_t \tau_t^\varphi$$  \hfill (3.10)

Since $\tau_t = p_t^b / p_t^f$, a rise in $\tau_t$ implies it is easier for a bank to find a borrower relative to a firm finding a lender and so corresponds to a tighter credit market. An increase (decrease) in $\tau_t$ reduces the expected time a bank (firm) must search for a credit partner, lowering the bank’s (firm’s) expected pecuniary search costs. Since $\tau_t = f_t / b_t^u = p_t^b / p_t^f$, at any date $t$ the number of newly matched banks must equal the the number of newly matched firms: $p_t^b / b_t^u = p_t^f / f_t$. 

93
Separations and the evolution of loan contracts

(i.e., loan contracts) end for exogenous reasons with (time-varying) probability \( \delta_t \). Contractual parties engaged in a credit relationship that survive this exogenous separation hazard may also decide to dissolve the contract depending on the realization of the productivity of the firm’s project, taken to be \( z_t \omega_{it} \), where \( z_t \) is an aggregate productivity shock affecting all firms (projects) and \( \omega_{it} \) is a firm-specific idiosyncratic productivity shock with distribution function \( G(\omega_{it}) \). As shown below, the decision to endogenously dissolve a credit relationship is characterized by an optimal reservation policy with respect to \( \omega_{it} \) and denoted by \( \tilde{\omega}_{it} \). If the realization of the idiosyncratic productivity shock \( \omega_{it} \) is above the reservation firm-specific productivity, \( \omega_{it} > \tilde{\omega}_{it} \), both parties agree to continue the credit relationship and the entrepreneur is able to produce. On the contrary, if the realization of \( \omega_{it} \) is below \( \tilde{\omega}_{it} \), both parties choose to dissolve the credit relationship.

The probability of endogenous termination is \( \gamma_t = \text{prob}(\omega_{it} \leq \tilde{\omega}_{it}) = G(\tilde{\omega}_{it}) \). The overall separation rate is \( \delta_t + (1 - \delta_t)\gamma_t \). Existence and uniqueness of the optimal reservation policy \( \tilde{\omega}_{it} \) is shown in the appendix.

Of the firms surviving the exogenous hazard, of whom there are \( (1 - \delta_t) \bar{f}_{t-1}^m \), a fraction \( \gamma_t \) of these receive idiosyncratic productivity shocks that are less than \( \tilde{\omega}_{it} \) and so do not produce. The number of firms that actually produce in period \( t \), therefore, is \( (1 - \delta_t)(1 - \gamma_t) \bar{f}_{t-1}^m \). The number of firms in a credit relationship at the end of period \( t \), denoted by \( f_t^m \), is given by the number of firms producing during time \( t \) plus all the
new matches formed at time \( t \) so that the evolution of \( f_t^m \) is expressed as

\[
f_t^m = (1 - \delta_t) (1 - \gamma_t) f_{t-1}^m + m_t = (1 - \delta_t) (1 - \gamma_t) f_{t-1}^m + p_t f_t.
\]

We normalize the total number of firms in every time period to one and assume that if a credit relationship is exogenously separated at time \( t \), both parties will start searching immediately during the same loan market trading session. If the credit relationship survives the exogenous separation hazard but then endogenously separates, both parties must wait until the beginning of the next period’s loan market trading session in order to start searching for a credit partner again. This assumption implies that the number of firms seeking finance during period \( t \), which we have denoted by \( f_t \), is equal to the number of searching firms at the beginning of time \( t \), \((1 - f_{t-1}^m)\) plus the number of firms that started the period matched with a bank but were exogenously separated \( \delta_t f_{t-1}^m \).

Therefore,

\[
f_t = 1 - (1 - \delta_t) f_{t-1}^m.
\]  \hspace{1cm} (3.11)

Notice that there are still some firms that have been endogenously separated but cannot search in period \( t \). These firms are unmatched but waiting to start searching again next period. The number of new matches during the loan market trading session at time \( t \) can be written as

\[
m_t = \mu_t \tau_{t-1}^n [1 - (1 - \delta_t) f_{t-1}^m].
\]

Thus the evolution of \( f_t^m \) can be written as

\[
f_t^m = (1 - \delta_t) (1 - \gamma_t) f_{t-1}^m + \mu_t \tau_{t-1}^n [1 - (1 - \delta_t) f_{t-1}^m].
\]  \hspace{1cm} (3.12)
Credit creation and destruction

Our timing assumption implies that the fraction \( p_t^f \delta_t f_{t-1}^m \), of matched firms that were exogenously separated during time \( t \), are able to find a new credit relationship within the same period of time. Then, credit creation, \( CC_t \), is defined to be equal to the number of newly created credit relationships at the end of time \( t \) net of the number of exogenous credit separations that are successfully re-matched in a given period. That is

\[
CC_t = m_t - p_t^f \delta_t f_{t-1}^m.
\]

The credit creation rate, \( cc_t \), is

\[
cc_t = \frac{m_t}{f_{t-1}^m} - p_t^b \delta_t.
\] (3.13)

On the other hand, credit destruction, \( CD_t \), is defined as the total number of credit separations at the end of time \( t \), \( (1 - \varphi_t) b_t^m \) net of the number of exogenous credit separations that are successfully re-matched in a given period. Thus,

\[
CD_t = (1 - \varphi_t) f_{t-1}^m - p_t^b \delta_t f_{t-1}^m,
\]

where \( \varphi_t \) is the total continuation rate, \( \varphi_t = (1 - \delta_t)(1 - \gamma_t) \), and the credit destruction rate, \( cd_t \), is given by

\[
cd_t = (1 - \varphi_t) - p_t^b \delta_t.
\] (3.14)

Finally, net credit growth is defined as

\[
cg_t = cc_t - cd_t.
\] (3.15)
3.2.2.2 Firms and the loan market

In our setting, a credit relationship is a contract between a bank and a firm that allows the latter to operate an specific production technology, hire workers and pay their wage bill in advance of production. As long as the credit contract prevails, the firm will receive sufficient external funds to pay workers in advance of production every period of time. After selling its output, the firm will repay its debt with the bank and transfer all remaining profits to the household. Therefore, as in De Fiore and Tristani (2012) we abstract from the endogenous evolution of net worth by assuming firms do not accumulate internal funds after repaying their debt.

Value functions Firm $i$ is endowed with a production technology given by

$$y_{it} = z_t \omega_{it} N_{it}^\alpha, \quad 0 < \alpha \leq 1,$$

where $z_t$ is the aggregate productivity level with mean $\bar{z}$ and $\omega_{it}$ is the firm-specific idiosyncratic productivity level which is picked from a uniform distribution function $G(\omega)$ with support $[\omega, \bar{\omega}]$. Define

$$g(\omega) \equiv \frac{dG(\omega)}{d\omega} = \frac{1}{\bar{\omega} - \omega}, \quad \text{with} \quad \bar{\omega} > \omega > 0$$

and normalize $\bar{z}$ so that the unconditional expectation of $z_t \omega_{it}(i)$ is equal to one. If the firm obtains financing and produces, the firm’s instantaneous real profit flow is

$$\pi_t^F = z_t \omega_{it} N_{it}^\alpha - w_t R_{it}^L N_{it} - x^F$$

where $w_t$ is the real wage, $R_{it}^L$ is the firm-specific gross nominal loan interest rate and $x^F$ is a fixed cost of production. The labor market is competitive so all firms face the same
real wage. The loan principle is \( w_t N_{it} \) and the loan contract requires the repayment of the total debt with the bank \( w_t R_{it}^L N_{it} \) within the same period.

Firm profit \( \pi^f_t \) depends on the status of the firm, that is, if the firm is searching for external funds or if it is producing. A firm searching for external funds can not produce and obtains zero real profits \( \pi^f_t = 0 \). Assuming no search costs for an entrepreneur,

\[
\pi^f_t = \begin{cases} 
\pi^f_t (\omega_{it}) & \text{with external funds} \\
0 & \text{without external funds}
\end{cases}
\]

The firm is characterized by two value functions or Bellman equations: The value of being matched with a bank and able to produce at date \( t \), denoted by \( V^{FP}(\omega_{it}) \) and the value of searching for external funds at date \( t \), denoted by \( V^{FN}_t \), both measured in terms of current consumption of the final good. \( V^{FP}(\omega_{it}) \) is given by

\[
V^{FP}(\omega_{it}) = \pi^f (\omega_{it}) + E_t \Delta_{t,t+1} \left\{ \delta_t V^{FN}_{t+1} + (1 - \delta_t) \int_{\omega} \max \left\{ V^{FP}(\omega_{it+1}), V^{FN}_{t+1} \right\} dG(\omega) \right\}
\]

where \( \Delta_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \) is the stochastic discount factor. The value of producing is the flow value of current real profits (firms real cash flow) plus the expected continuation value. At the end of the period, the credit relationship is exogenously dissolved with probability \( \delta_t \), and the firm must seek new financing. With probability \( (1 - \delta_t) \), the firm survives the exogenous separation hazard. In the latter case, only firms receiving an idiosyncratic productivity realization \( \omega_{it+1} \geq \tilde{\omega}_{it+1} \) defined by \( V^{FP}(\omega_{it+1}) = V^{FN}_{t+1} \) will remain matched and produce. Firms with \( \omega_{it+1} < \tilde{\omega}_{it+1} \) endogenously separate from their bank and obtain \( V^{FN}_{t+1} \).
The value of searching for external funds $V_t^{FN}$ for a firm at date $t$ expressed in terms of current consumption is

$$V_t^{FN} = p_t^f E_t \Delta_{t,t+1} \left[ \delta_t V_{t+1}^{FN} + \left( 1 - \delta_t \right) \int_{\omega}^{\bar{\omega}} \max \{ V^{FP}(\omega_{it+1}), V_{t+1}^{FN} \} dG(\omega) \right] +$$

$$+ \left( 1 - p_t^f \right) E_t \Delta_{t,t+1} V_{t+1}^{FN}$$

where $p_t^f$ is the probability of matching with a bank. Notice that we assume matches made in period $t$ do not produce until $t+1$. With probability $(1 - p_t^f)$, the firm does not match and must continue searching for external funds during next periods loan market.

Under Nash bargaining, the reservation productivity level $\tilde{\omega}_{it}$ that triggers endogenous separation is determined by the point at with the joint surplus of the match is equal to zero. Thus, if $\omega_{it+1} < \tilde{\omega}_{it+1}$, both parties agree to end the relationship. The probability of endogenously separating is $\gamma_{t+1} = G(\tilde{\omega}_{it+1}) = \prob(\omega_{it+1} \leq \tilde{\omega}_{it+1})$. Given existence and uniqueness of $\tilde{\omega}_{it+1}$, the integral term on the expected continuation value is

$$\int_{\omega}^{\bar{\omega}} \max \{ V^{FP}(\omega_{it+1}), V_{t+1}^{FN} \} dG(\omega) =$$

$$= \gamma_{t+1} V_{t+1}^{FN} + (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{it+1}}^{\bar{\omega}} V^{FP}(\omega_{it+1}) dG(\omega) \frac{1}{1 - \gamma_{t+1}}.$$ 

Therefore, we have

$$V^{FP}(\omega_{it}) =$$

$$= \pi^f(\omega_{it}) + E_t \Delta_{t,t+1} \left\{ \delta_t V_{t+1}^{FN} + (1 - \delta_t) \left[ \gamma_{t+1} V_{t+1}^{FN} + \int_{\tilde{\omega}_{it+1}}^{\bar{\omega}} V^{FP}(\omega_{it+1}) dG(\omega) \right] \right\}$$
\[ V_t^{FN} = E_t \Delta_{t,t+1} \left\{ p_t^f (1 - \delta_t) \left[ \gamma_{t+1} V_t^{FN} + \int_{\omega} V^{FP} (\omega_{t+1}) dG (\omega) \right] + \left[ 1 - p_t^f (1 - \delta_t) \right] V_{t+1}^{FN} \right\} . \]

The surplus to a producing firm is \( V^{FS} (\omega_{it}) = V^{FP} (\omega_{it}) - V_t^{FN} \) and is given by

\[ V^{FS} (\omega_{it}) = \pi^f (\omega_{it}) + (1 - \delta_t) E_t \Delta_{t,t+1} \left( 1 - p_t^f \right) (1 - \gamma_{t+1}) \int_{\omega_{t+1}} V^{FS} (\omega_{it+1}) \frac{dG (\omega)}{1 - \gamma_{t+1}} . \]

### 3.2.2.3 Bank and the loan market

There is a continuum of banks with infinite mass that are owned but not controlled by the representative household. Banks operate in various centralized markets such as the interbank, bond and deposit market but also operate in the decentralized loan market. Banks’ activities in the centralized markets include: raise deposits from households, hold excess reserve balances with the central bank, borrow and lend reserves with other banks as part of the payments settlement system and hold government bonds. These activities are explained in more detail in the following section. On the other hand, the decentralized nature of the loan market and the existence of search and matching frictions imply that banks have to spend time and resources searching for firms before being able to extend loans. At any point in time, banks operating in the loan market may be involved in a credit contract with a particular firm or may be seeking a potential borrower. We assume that banks decide to enter the loan market to search for potential
borrowers until the expected cost of extending a loan is equal to its expected benefit. At this point, banks will be indifferent between searching for projects or only operating in the centralized markets of the economy.

**Value functions** Define $\pi_b^t$ as the bank’s real profits. Each period, when the loan market opens, a bank may be in a credit relationship with a firm or searching for potential borrowers. If a bank is matched with a firm whose idiosyncratic productivity realization exceeds $\tilde{\omega}_{it}$, bank profits will equal

$$\pi^b(\omega_{it}) = [R^L(\omega_{it}) - R_t] l(\omega_{it})$$

(3.21)

where $R^L(\omega_{it}) - R_t$ is the spread between the interest rate on the bank’s loan to a firm with idiosyncratic productivity $\omega_{it}$ and the bank’s opportunity cost of funds $R_t$. The determination of $R_t$ is explained below; it will be shown to be the same for all banks. The real loan size is $l(\omega_{it})$. A bank searching for a borrower will incur in a search cost $\kappa$, measured in current consumption units and will earn zero current profits in the loan market.

Under these assumptions the problem of a bank can be characterized by two value functions (Bellman equations). The value of lending to a firm at date $t$, denoted by $V^{BL}(\omega_{it})$ and the value of searching for a potential borrower at date $t$, denoted by $V^{BN}_t$. Both value functions are measured in terms of current consumption of the final
good and are given by

\[ V^{BL}(\omega_{it}) = \pi^b(\omega_{it}) + \]

\[ + E_t \Delta_{t,t+1} \left\{ \delta_t V^{BN}_{t+1} + (1 - \delta_t) \left[ \gamma_{t+1} V^{BN}_{t+1} + (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\bar{\omega}_{t+1}} V^{BL}(\omega_{it+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}} \right] \right\} \]

and

\[ V^{BN}_t = -\kappa + E_t \Delta_{t,t+1} \left\{ \delta_t V^{BN}_{t+1} + (1 - \delta_t) \left[ \gamma_{t+1} V^{BN}_{t+1} + \right. \right. \]

\[ + (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\bar{\omega}_{t+1}} V^{BL}(\omega_{it+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}} \right\} + \left[ 1 - p^b_t (1 - \delta_t) \right] V^{BN}_{t+1} \] (3.22)

The value of extending a loan (equation 3.22) is the current value of real profits plus the expected continuation value. A bank that extends a loan to a firm with idiosyncratic productivity \( \omega_{it} \) at date \( t \) will continue financing the same firm at time \( t + 1 \) with probability \( (1 - \delta_t)(1 - \gamma_{t+1}) \). In this event, the bank obtains the future expected value of lending conditional on having \( \omega_{it+1} > \tilde{\omega}_{it+1} \) given by \( \int_{\tilde{\omega}_{it+1}}^{\bar{\omega}_{it+1}} V^{BL}(\omega_{it+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}} \).

The credit relationship will be severed at time \( t + 1 \) with probability \( \delta_t + (1 - \delta_t) \gamma_{t+1} \) and the bank obtains a future value of \( V^{BN}_{t+1} \). On the other hand, the value of a bank searching for a borrower at date \( t \) (equation 3.22) is given by the flow value of the search costs plus the continuation value. A searching bank faces a probability \( 1 - p^b_t \) of not being matched during time \( t \), obtaining a future value of \( V^{BN}_{t+1} \) but a probability \( p^b_t \) of being matched. If a searching bank ends up being matched with a firm at time \( t \), then at the beginning of period \( t + 1 \) will face a probability of separation before extending the loan.

In equilibrium, free entry of banks into the loan market ensures that \( V^{BN}_t = 0 \).
Using this in (3.22), the free entry condition can be written as

\[
\frac{\kappa}{p^b_t} = E_t \Delta_{t,t+1} \left[ (1 - \delta_t) (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\omega_{t+1}} V^{BL}(\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}} \right].
\] (3.23)

Banks will enter the loan market until the expected cost of finding a borrower \( \kappa/p^b_t \) is equal to the expected benefit of extending a loan to a firm with idiosyncratic productivity \( \omega_{it+1} \geq \tilde{\omega}_{it+1} \). If the expected cost of extending a loan is lower than the corresponding expected benefits, banks will enter the loan market to search for borrowers and the probability that a searching bank finds a borrower will fall, up to the point where condition (3.23) is restored. Note that free entry of banks into the loan market modifies the value function \( V^{BL}(\omega_{it}) \) as follows

\[
V^{BL}(\omega_{it}) = \pi^b(\omega_{it}) + E_t \Delta_{t,t+1} \left[ (1 - \delta_t) (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\omega_{t+1}} V^{BL}(\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}} \right].
\] (3.24)

Using (3.22), the net surplus for bank extending a loan is

\[
V^{BS}(\omega_{it}) = \pi^b(\omega_{it}) + \frac{\kappa}{p^b_t}.
\] (3.25)

### 3.2.2.4 Employment and the loan contract: Nash bargaining

At any point in time, a matched firm and bank that survive the exogenous and endogenous separation hazards engage in bilateral bargaining over the interest rate and loan size to split the joint surplus that results from the match. This joint surplus
is defined as \( V^{JS}(\omega_{it}) = V^{FS}(\omega_{it}) + V^{BS}(\omega_{it}) \). Using (3.20), (3.24), (3.18) and (3.21),

\[
V^{JS}(\omega_{it}) = \left[ \pi^f(\omega_{it}) + \pi^b(\omega_{it}) \right] + \frac{\kappa}{p^f_t} + E_t \Delta_{t,t+1} \left( 1 - p^f_t \right) (1 - \delta_t) (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\pi} V^{FS}(\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}}
\]

\[
= z_t \omega_{it} N_{it}^\alpha - w_t R_t N_{it} - x^F + E_t \Delta_{t,t+1} \left( 1 - p^f_t \right) (1 - \delta_t) (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\pi} V^{FS}(\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}}
\]

\[
+ E_t \Delta_{t,t+1} \left( 1 - p^f_t \right) (1 - \delta_t) (1 - \gamma_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\pi} V^{FS}(\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}}.
\]

The firm’s demand for external funds is given by its wage bill: \( l(\omega_{it}) = w_t N_{it} \) for all \( \omega_{it} \geq \tilde{\omega}_{it} \). Thus, the joint surplus is

\[
V^{JS}(\omega_{it}) = z_t \omega_{it} N_{it}^\alpha - w_t R_t N_{it} - x^F + E_t \Delta_{t,t+1} \left( 1 - p^f_t \right) (1 - \delta_t) \int_{\tilde{\omega}_{t+1}}^{\pi} V^{FS}(\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}}.
\]

(3.26)

We assume Nash bargaining with fixed bargaining shares. Let \( \eta \) be the firm’s share of the joint surplus, and \( 1 - \eta \) the bank’s. Both parties have an incentive to maximize the joint surplus. Thus, the firm’s employment level will be chosen to solve

\[
\max_{N_{it}} \left[ z_t \omega_{it} N_{it}^\alpha - w_t R_t N_{it} - x^F \right].
\]

The first order condition sets the marginal product of labor equal to the marginal cost of labor inclusive of the (bank’) opportunity cost of funds:

\[
\alpha z_t \omega_{it} N_{it}^{\alpha - 1} = w_t R_t.
\]

(3.27)

The marginal cost of labor is independent of \( i \); employment at firm \( i \) is

\[
N^*(\omega_{it}) = \left( \frac{\alpha z_t \omega_{it}}{w_t R_t} \right)^{\frac{1}{1 - \alpha}}
\]
and the loan size is

$$l^* (\omega_{it}) = \left( \frac{\alpha z_i \omega_{it}}{w_t^\alpha R_t} \right)^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (3.28)

These results can be used to rewrite the free entry condition (3.23), the net surplus for a firm (3.20) and the joint surplus for a credit relationship (3.26) as

$$\frac{\kappa}{p^*_t} = (1 - \eta)(1 - \delta_t) E_t \Delta t_{t+1} \int_{\omega_{t+1}} V^{JS} (\omega_{t+1}) dG(\omega)$$  \hspace{1cm} (3.29)

$$V^{FS} (\omega_{it}) = \pi^f (\omega_{it}) + \eta \left( \frac{1 - p^*_t}{1 - \eta} \right) \frac{\kappa}{p^*_t}$$  \hspace{1cm} (3.30)

$$V^{JS} (\omega_{it}) = \pi^{f*} (\omega_{it}) + \pi^{b*} (\omega_{it}) = \left( \frac{1 - \eta p^*_t}{1 - \eta} \right) \frac{\kappa}{p^*_t}$$  \hspace{1cm} (3.31)

where

$$\pi^{f*} (\omega_{it}) + \pi^{b*} (\omega_{it}) = y^* (\omega_{it}) - R_t w_t N^* (\omega_{it}) - x^F$$

and $\pi^{f*} (\omega_{it})$ is given by (3.18) and $\pi^{b*} (\omega_{it})$ by equation (3.21) but evaluated at optimal level of employment $N^* (\omega_{it})$ and output $y^* (\omega_{it})$ chosen by the credit match. The optimal level of output produced by firm $i$ is

$$y^* (\omega_{it}) = (z_i \omega_{it}) \frac{1}{\alpha} \left( \frac{\alpha}{w_t R_t} \right)^{\frac{\alpha}{1-\alpha}}.$$  \hspace{1cm} (3.32)

The effect of the nominal interest rate on the cost of labor is generally referred to as the cost channel of monetary policy (Ravenna and Walsh 2006). Normally, the relevant interest rate is taken to be the interest rate the firm pays on loans taken to finance wage payments. Here, the loan interest rate simply ensures the joint surplus generated by a credit relationship is divided optimally between the firm and the bank, with the relevant interest rate capturing the cost channel being $R_t$, the bank’s opportunity cost.
of funds. As shown below, $R_t$ depends on the interest rate in the interbank market and
the marginal value of loans as collateral. Even though firms will face different interest
rates on bank loans, since the loan rate depends on the firm’s idiosyncratic productivity
realization, the interest cost relevant for labor demand is the same for all firms\(^4\).

Finally, since equation (3.32) is equivalent to $\alpha y^*(\omega_{it})/N^*(\omega_{it}) = w_t R_t$ the the
joint surplus of a credit relationship (3.31) can be written explicitly as a function of
the idiosyncratic productivity shock $\omega_{it}$ in order to facilitate the characterization of the
loan market equilibrium

$$V^{JS}(\omega_{it}) = \left[ (1 - \alpha) (z_t \omega_{it}) \frac{1}{1-\alpha} \left( \frac{\alpha}{w_t R_t} \right)^{\frac{\alpha}{1-\alpha}} - x^F \right] + \left( \frac{1 - \eta p_f}{1 - \eta} \right) \frac{\kappa}{p_t}$$

(3.33)

### 3.2.2.5 The optimal reserve policy: Endogenous separations

The optimal reservation policy with respect to the idiosyncratic productivity
shock is given by

- if $\omega_{it} \leq \tilde{\omega}_{it} \implies V^{JS}(\omega_{it}) \leq 0$
- if $\omega_{it} > \tilde{\omega}_{it} \implies V^{JS}(\omega_{it}) > 0$

Since the joint surplus is increasing in the firm’s idiosyncratic productivity, there exists
a unique threshold level $\tilde{\omega}_{it}$ defined by

$$V^{JS}(\tilde{\omega}_{it}) = 0$$

\(^4\)Details on the derivation of the interest rate on the loan are given in the appendix.
such that the joint surplus is negative for any firm facing an idiosyncratic productivity $\omega_{it} < \tilde{\omega}_{it}$. Using (3.33) we can solve for $\tilde{\omega}_{it}$ and obtain\(^5\)

$$\tilde{\omega}_{it} = \frac{(w_t R_t)^\alpha}{z_t} H_t$$  \hspace{1cm} (3.34)

where

$$H_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \left[ x^F - \left( \frac{1 - \bar{\mu}_t \tau_t^{-1}}{1 - \eta} \right) \frac{\kappa}{\bar{\mu}_t \gamma_t^\alpha} \right]^{1-\alpha}$$

Since $\tilde{\omega}_t$ is independent of $i$, the cutoff value is the same for all firms and banks. Moreover, it is decreasing in aggregate productivity $z_t$ so that a positive aggregate productivity shock means the number of credit matches that separate endogenously falls and more matched firms produce. The cutoff value is increasing in the cost of labor ($w_t R_t$) and the firms fixed cost ($x^F$).

The banks’ opportunity costs of funds $R_t$ influences the level of economic activity at both the extensive and intensive margins. From (3.34), a rise in $R_t$ increases the threshold level of the idiosyncratic productivity of firms that generate a positive joint surplus. As a consequence, fewer firms obtain financing and produce. This is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to $w_t R_t$, so a rise in $R_t$ reduces labor demand at each level of the real wage. This is the intensive margin effect. Both channels work to reduce aggregate output as $R_t$ rises.

In addition, credit market conditions reflected in $\tau_t$ directly affect the extensive margin; a rise in $\tau_t$ (a credit tightening) increases $\tilde{\omega}_t$ and fewer firms obtain credit. Both interest costs measured by $R$ and credit conditions measured by $\tau$ matter for employment and

\(^5\)See the appendix
3.2.2.6 Aggregation

Aggregate output is the number of producing firms times the expected output of each firm, conditional on its realization of $\omega_{it}$ exceeding $\tilde{\omega}_t$. Recall that the number of matched firms at the start of period $t$ is $f_{t-1}^m$ and that only a fraction $(1 - \delta_t)(1 - \gamma_t)$ of those firms survive both separation hazards and consequently end up producing. Aggregate output is then

$$Y_t = (1 - \delta_t)(1 - \gamma_t) f_{t-1}^m E_t [y^* (\omega_{it}) | \omega_{it} > \tilde{\omega}_t],$$

where

$$E_t [y^* (\omega_{it}) | \omega_{it} > \tilde{\omega}_t] = \int_{\tilde{\omega}_t}^{\infty} y^* (\omega_{it}) \frac{dG(\omega)}{1 - \gamma_t}.$$ 

Using the assumption that $\omega$ follows a uniform distribution with density $g(\omega) = dG(\omega) = 1/(\omega - \omega)$, the appendix shows that

$$Y_t = (1 - \delta_t) \alpha \left[ \frac{z}{(w_t R_t)\alpha} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{\omega^k - \tilde{\omega}_t^k}{k (\omega - \omega)} \right], \quad (3.35)$$

where $k \equiv (2 - \alpha)/(1 - \alpha) > 1$. Aggregate employment is

$$N_t = (1 - \delta_t) \alpha \left[ \frac{\omega}{w_t R_t} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{\omega^k - \tilde{\omega}_t^k}{k (\omega - \omega)} \right]. \quad (3.36)$$

Combining (3.35) and (3.36),

$$Y_t = z_t F_t^{1-\alpha} N_t^\alpha, \quad (3.37)$$

where

$$F_t \equiv (1 - \delta_t) \left[ \frac{\omega^k - \tilde{\omega}_t^k}{k (\omega - \omega)} \right] f_{t-1}^m.$$
Equation (3.37) is the effective aggregate production function for this economy and illustrates the way in which aggregate output depends on the aggregate productivity shock and employment but also on the number of producing firms and their average idiosyncratic productivity as reflected in $F_t$. Credit market disruptions that lead to an exogenous rise in match breakups (a shock to $\delta$, for example) act like a negative productivity shock. An increase in the cutoff productivity level $\tilde{\omega}_t$ reduces output (given $N$) by reducing the firms that actually produce.

Following the same steps, the appendix shows that the aggregate loans are

$$l_t = w_t N_t,$$

the average loan interest rate $R^L_t$ is given by

$$R^L_t = \left[1 - \eta (1 - \alpha)\right] \frac{Y_t}{l_t} - (1 - \delta_t) (1 - \gamma_t) f^m_{t-1} \left[\frac{(1 - \eta) x^F + \eta \left(\frac{\omega}{\gamma}\right)}{l_t}\right],$$

and the average credit spread $R^L_t - R_t$, defined as $R^p_t$, is

$$R^p_t \equiv R^L_t - R_t = \frac{(1 - \eta)(1 - \alpha)}{\alpha} R_t - \left[ (1 - \eta) x^F + \eta \frac{\kappa}{\tilde{\tau}_t} \left(\frac{w^p R_t}{\alpha z_t}\right) \left[\frac{\omega^g - \tilde{\omega}^g_t}{g(\omega - \tilde{\omega}_t)}\right]\right],$$

where $g = -\frac{\alpha}{1 - \alpha}$. Finally, aggregate profits for producing firms are

$$\pi^f_t = Y_t - R^L_t \left(\frac{L_t}{P_t}\right) - (1 - \delta_t) (1 - \gamma_t) f^m_{t-1} x^F,$$

where $L_t$ is the nominal aggregate level of loans, $P_t$ is the price level, and aggregate profits for banks extending loans are

$$\pi^b_t = (R^L_t - R_t) \left(\frac{L_t}{P_t}\right).$$
3.2.2.7 Characterization of the loan market equilibrium

In this section we characterize the loan market equilibrium in terms of two main equations. The first equation relates the cutoff idiosyncratic productivity level, \( \tilde{\omega}_t \), with our measure of credit market tightness \( \tau_t = f_t/b_t^\alpha \). The second equation is an Euler equation that describes the dynamics of credit market tightness as a function of the cutoff productivity level.

The threshold level for \( \tilde{\omega}_t \) is

\[
\tilde{\omega}_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \bar{z}_t} \left( \frac{w_t R_t}{\bar{z}_t} \right)^{\alpha} \left[ x^F - \left( \frac{1 - \eta \mu^\tau_{t-1}}{1 - \eta} \right) \frac{\kappa_{t-\tau}^{\varphi}}{\mu_{t-\tau}^{\varphi}} \right]^{1-\alpha}.
\]

Combining the free entry condition and the joint surplus of a credit relationship yields the following equation that characterizes the dynamics of \( \tau \):

\[
\frac{\kappa_{t-\tau}^{\varphi}}{\mu_{t-\tau}^{\varphi}} = (1 - \eta) E_{t+1} \Delta_{t+1} \left\{ (1 - \alpha) \frac{Y_{t+1}}{\bar{F}_t} - \varphi(\tilde{\omega}_{t+1}) \left[ x^F - \left( \frac{1 - \eta \mu^\tau_{t+1}}{1 - \eta} \right) \frac{\kappa_{t+1}^{\varphi}}{\mu_{t+1}^{\varphi}} \right] \right\}.
\]

where

\[
\varphi(\tilde{\omega}_t) = (1 - \delta_t) \left( \frac{\bar{\omega} - \tilde{\omega}_t}{\bar{\omega} - \bar{\omega}} \right)
\]

is the net survival rate for credit matches.

3.2.3 The interbank market

The interbank market involves the direct participation of banks and the central bank. Net payments between banks must be settled at the end of each period, after the interbank market has closed. The random nature of settlement payment flows from the perspective of an individual bank will generate a demand for excess reserves (reserves
in excess of any required reserves). The cost of holding a level of excess reserves that, ex post, is too high or too low will depend on the opportunity costs of, in the first case, holding reserves as deposits at the central bank and, in the second case, borrowing reserves from the central bank. The central bank sets the interest paid on reserves, the rate charged on borrowed reserves, the quantity of reserves, and the haircuts applied to bank assets posted as collateral when borrowing reserves. Not all these instruments can be set independently.

### 3.2.3.1 Banks

The balance sheet of bank $j$ in nominal terms is

$$L_t(j) + B^b_t(j) + I_t(j) + H_t(j) = (1 - \rho)D_t(j)$$  \hspace{1cm} (3.38)

where $L_t(j)$ are loans, $B^b_t(j)$ represent holdings of government bonds, $I_t(j)$ is (net) lending in the interbank market, $H_t(j)$ are excess reserve holdings, and $\rho$ is the fractional reserve requirement ratio.

During the period, banks make payments to and receive payments from other banks as part of the payments settlement system. Banks can trade reserve balances in a competitive interbank market at the market rate $i_t$. After the interbank market has closed, banks experience a net payment shock $\phi_t(j) = \varepsilon_t D_t(j)$, taken to be homogenous of degree one in the level of the bank’s deposit liabilities. The payment shock itself is assumed to be uniformly distributed over the interval $[-\varepsilon D_t(j), \varepsilon D_t(j)]$.\footnote{Ashcroft, et al 2011 also models unpredictable payment flows as drawn from a uniform distribution.} The density and cumulative distribution functions of this shock are $f(\phi) = 1/[2\varepsilon D_t(j)]$ and $F(\phi) = \frac{\phi}{\varepsilon D_t(j)}$.\footnote{Ashcroft, et al 2011 also models unpredictable payment flows as drawn from a uniform distribution.}
\( F(\varepsilon D) = (\varepsilon + \bar{\varepsilon})/2\bar{\varepsilon} \). Since \( E \phi = 0 \) and \( \text{var}(\phi) = \varepsilon^2 D^2_t(j)/3 \), an increase in \( \varepsilon \) represents a mean preserving spread in the distribution of payment shocks. If \( H_t(j) + \phi_t(j) < 0 \), the bank must borrow reserves from the central bank to meet its net payment outflow. If \( H_t(j) + \phi_t(j) > 0 \), the bank can earn interest on its net balances by depositing them with the central bank.

Assume the central bank sets a desired interest rate \( i^*_t \) and remunerates (required or excess) reserve balances at a rate \( i^*_t - s \) and lends reserves at a penalty rate \( i^*_t + s \) (see Woodford 2001, Whitesell 2003, 2006, or Walsh 2006, 2010). The rate paid on reserves places a floor on the interbank rate as no bank will lend to another at a rate less than \( i^*_t - s \). And, in the absence of a collateral constraint on borrowing from the central bank, the penalty rate places a ceiling on the interbank rate as no bank will borrow in the interbank market at a rate greater than \( i^*_t + s \). In this case, \( s \) is the symmetric width of the channel within which the interbank rate is contained. In practice central bank lending is collateralized while interbank lending is unsecured, though the traditional analysis of a channel system (Woodford 2001, Whitesell 2003, 2006) ignores collateral (but see Berentsen and Monnet 2008). We assume the central bank accepts both government bonds and commercial loans as collateral, applying a haircut to each but imposing a larger haircut on loans.\(^7\) If \( H_t(j) + \phi_t(j) < 0 \), the maximum a bank...

---

\(^7\)General equilibrium models with channel systems are developed in Berensten and Monnet (2006, 2008), and Berensten, Marchesiani, and Waller (2010). See also Friedman and Kuttner (2010).

\(^8\)In fact, during 2009-2013, the federal funds rate has been below the rate the Federal Reserve pays on reserves. Beck and Klee (2011) explain that this phenomena can arise because Government Sponsored Enterprises (GSE) hold reserves but cannot earn interest on them from the Federal Reserve. As Furfine (2011) points out, there must be limits to arbitrage that prevent banks from borrowing these fed funds from GSEs and depositing them in their own interest earning reserve accounts.

can borrow from the central bank is \( \xi_b B_t(j) + \xi_L L_t(j) \), where \( 0 < 1 - \xi_L < 1 - \xi_b < 1 \) are the haircuts on commercial loans and bonds posted as collateral. For example, the Federal Reserve currently sets \( \xi_b = 0.99 \) for U.S. bills and bonds with less than 5 years to maturity and \( \xi_L = 0.65 \) for zero coupon, normal risk-rated commercial loans of 5 years maturity.\(^{10}\) For simplicity, we assume banks hold collateralizable assets and reserves sufficient to meet all net settlement flows.\(^{11}\) This requires

\[
H_t(j) + \xi_b B_t(j) + \xi_L L_t(j) \geq \varepsilon D_t(j). \quad (3.39)
\]

Let \( i^l \) be the nominal interest rate on loans, and let \( x^l \) be the cost (per dollar) of servicing loans. Then the profits of bank \( j \) with household deposits \( D_t(j) \) and loans \( L_t(j) \) can be written as

\[
\Pi_b^j(t) = \left( i^l_t - i_t - x^l \right) L_t(j) + \left[ i_t(1 - \rho) + (i^*_t - s) \rho - i^d_t - x^d \right] D_t(j)
\]

\[
+ \max_{B_t^b, H_t} \left\{ \left( i^b_t - i_t \right) B_t^b(j) - i_t H_t(j) \right. \\
+ \int_{-\tau D_t(j)}^{\tau D_t(j)} (i^*_t - s) [H_t(j) - \phi_t(j)] f(\phi) d\phi \\
+ \int_{H_t(j)}^{\tau D_t(j)} (i^*_t + s) [H_t(j) - \phi_t(j)] f(\phi) d\phi \left\} , \quad (3.40)
\]

where (3.38) has been used to eliminate \( I_t(j) \) and the maximization is subject to (3.39).

The first two terms on the right in (3.40) represent the net interest income on loans and deposits, where \( x^l (x^d) \) is the cost of servicing loans (deposits), \( i^l \) is the loan interest

\(^{10}\)See the Fed’s Discount Window and Payment System Risk Collateral Margins Table, available at http://www.frbdiscountwindow.org/discountwindowbook.cfm?hdrID=14&dtlID=43

\(^{11}\)This avoids needing to specify the consequences if a bank is unable to meet an extremely large unexpected outflow.
rate, and $i_t(1 - \rho) + (i_t^* - s)\rho - x^d$ is the return on an additional dollar of deposits.

The next two terms represent the interest income on bond holdings and the opportunity cost of holding excess reserves or bonds rather than lending in the interbank market.

The first integral captures the outcomes where the net payment shock is such that the bank ends the period with positive excess reserves. These are held in deposits with the central bank and remunerated at rate $i_t^* - s$. The second integral captures the opposite situation, where the shock is larger than $H_t(j)$, leaving the bank with a negative net position that requires it to borrow through the central bank’s lending facility at the penalty rate $i_t^* + s$.

Let $h_t(j) \equiv H_t(j)/D_t(j)$. If $\chi_t(j)$ denotes the Lagrangian multiplier on the collateral constraint, the first order conditions for $h_t(j)$ and $B_b^t(j)$ are

$$h_t(j): \quad -i_t + (i_t^* - s)\left[\frac{h_t(j) + \bar{\varepsilon}}{2\bar{\varepsilon}}\right] + (i_t^* + s)\left\{1 - \left[\frac{h_t(j) + \bar{\varepsilon}}{2\bar{\varepsilon}}\right]\right\} + \chi_t(j) = 0 \quad (3.41)$$

and

$$B_b^t(j): \quad (i_t^* - i_b) + \xi_b\chi_t(j) = 0. \quad (3.42)$$

The optimal choice of excess reserves equates the opportunity cost of holding one more unit of reserves, $i_t$, with the weighted sum of the marginal costs in expected reserve deficiency, $(i_t^* + s)[1 - (h_t(j) + \bar{\varepsilon})/2\bar{\varepsilon}]$, and the marginal gains in expected interest income, $(i_t^* - s)(b + \bar{\varepsilon})/(2\bar{\varepsilon})$ from holding excess reserves and the collateral value of an extra dollar of reserve holdings $\chi_t(j)$. Equation (3.42) implies the interest rate on bonds plus their collateral value equals the interbank market rate, or $\chi_t(j) = (i_t - i_b^*)/\xi_b$. Hence, $\chi_t$ is independent of $j$. From (3.41), this also implies that $h_t$ is independent of $j$, and
the demand for excess reserves is given by

\[ h_t = \left( \frac{\bar{\varepsilon}}{s} \right) (i_t^* - i_t + \chi_t). \] (3.43)

Total excess reserve demand is increasing in the volatility of payment flows (measured by \( \bar{\varepsilon} \)). It is decreasing in the width of the channel \( s \) and increasing in the spread between the policy rate and the interbank rate \( i_t^* - i_t \) and the value of collateral.

Rewriting (3.41) as

\[ i_t = (i_t^* - s) \left( \frac{h_t + \bar{\varepsilon}}{2\bar{\varepsilon}} \right) + (i_t^* + s) \left[ 1 - \left( \frac{h_t + \bar{\varepsilon}}{2\bar{\varepsilon}} \right) \right] + \chi_t \]

shows that \( i_t \) equals a weighted average of the interest rate on central bank deposits \( i_t^* - s \) and the rate of borrowing reserves \( i_t^* + s \), adjusted for the marginal value of collateral \( \chi_t \). Thus,

\[ i_t^* - s \leq i_t - \chi_t \leq i_t^* + s. \]

If \( \chi_t = 0 \) so that collateral constraints do not bind, then the standard result that the interbank rate is bounded between the rate paid on reserves \( i_t^* - s \) and the rate charged on borrowing \( i_t^* + s \) is obtained. When collateral constraints bind, \( \chi_t > 0 \) and the rate paid on reserves provides a floor for the interbank rate, but \( i_t \) could exceed the penalty rate on reserve borrowings. These bounds on \( i_t \), imply

\[ i_t^* - i_t - \chi_t \]

\[ -s \leq i_t^* - i_t - \chi_t \leq s, \]

so from (3.43) reserve demand is also bounded:

\[ -\bar{\varepsilon} \leq h_t \leq \bar{\varepsilon}. \]

115
If \( i = i^* \) (the interbank rate equals the central bank’s policy rate), then

\[
h_t = \left( \frac{\bar{\varepsilon}}{s} \right) \chi_t \geq 0,
\]

and excess reserves are positive. In the absence of a collateral constraint, excess reserves would be zero.\(^\text{12}\) Equivalently, if the central bank provides a level of total reserves equal to required reserves so that excess reserves are zero,

\[
i_t = i^*_t + \chi_t \geq i^*_t.
\]

In this case, the interbank rate exceeds the policy rate (see Berensten and Monnet 2006).

The first order condition for deposits at bank \( j \) can be written as

\[
\frac{\partial \Pi_t(j)}{\partial D_t(j)} = \zeta_t D_t(j),
\]

where

\[
\zeta_t \equiv i_t (1 - \rho) + (i^*_t - s)\rho - i^d_t - x^d + f (i^*_t - i_t, \chi_t)
\]

and the function \( f (i^*_t - i_t, \chi_t) \) is defined as

\[
f (i^*_t - i_t, \chi_t) \equiv \left( \frac{1}{2} \right) \left( \frac{\bar{\varepsilon}}{s} \right) (i^*_t + \chi_t - i_t)^2 - \left( \chi_t + \frac{s}{2} \right) \bar{\varepsilon}.
\]

Competition for deposits among banks will ensure that \( \zeta_t = 0 \), implying

\[
i^d_t = i_t (1 - \rho) + (i^*_t - s)\rho - x^d + f (i^*_t - i_t, \chi_t).
\]

Hence, the deposit rate is a weighted average of the interbank rate and the rate earned on required reserves adjusted for the bank’s cost of providing deposits and the effect of

\(^{12}\)This is the case considered, for example, by Whitesell (2006). In the presence of a collateral constraint, the interest rate on unsecured borrowing can exceed the rate \( i^*_t + s \) on secured borrowing by the value of the collateral necessary to access the central bank’s secured funding.
deposits on the need for additional collateral and excess reserves. If excess reserves are zero, (3.43) implies \( i^* + \chi_t - i_t = 0 \) and

\[
i^d_t = i_t(1 - \rho) + (i^*_t - s)\rho - x^d - \left(\chi_t + \frac{s}{2}\right)\varepsilon.
\]

We can now rewrite the bank’s profit in real terms (3.40) using (3.39) to eliminate \( B^b_t \) and (3.44) to eliminate \( i^d_t \) as

\[
\pi^b_t(j) = \left(i^d_t - i_t + \xi_L \chi_t - x^l\right) l_t(j).
\] (3.45)

The difference between a bank that has made a loan and one that has not is, in real terms,

\[
\pi^b_t(l_t(j)) - \pi^b_t(0) = \left(i^d_t + \xi_L \chi_t - x^l - i_t\right) l_t(j).
\] (3.46)

This difference is simply equal to the marginal value of the loan \( i^d_t + \xi_L \chi_t - x^l \) (it’s interest income plus collateral value net of costs) minus its opportunity cost \( i_t \).

In discussing the loan market earlier, the surplus to the bank of making a loan was equal to \( [R^l(i) - R_t] l_t(i) \), where \( R^l \) and \( R_t \) were gross interest rates. Using (3.46), it follows that

\[
R_t = 1 + i_t - \xi_L \chi_t + x^l.
\] (3.47)

Ceteris peribus, an increase in the haircut applied to loans used as collateral with the central bank (a fall in \( \xi_L \)) increases the opportunity cost of lending. As a result, the effective cost of labor increases and the demand for labor falls. This negative effect on employment holds for a given interbank rate \( i_t \).
Equation (3.42) implies that the Lagrangian multiplier on the collateral constraint is equal to

$$\chi_t = \frac{i_t - i^b_t}{\xi_b}.$$  

Given an estimate of $\chi_t$, (3.47) can be used to obtain an estimate of the opportunity cost of funds for banks up to the unobserved constant $x^l$.

Combining (3.42) and (3.47) allows one to express the opportunity cost of funds for banks $R_t$ (up to the constant $x^l$) as a weighted average of the funds rate and the bond rate:

$$R_t = 1 + x^l + \left(1 - \frac{\xi_L}{\xi_b}\right) i_t + \left(\frac{\xi_L}{\xi_b}\right) i^b_t.$$  

3.2.4 The central bank

We assume the government sector is made up of the treasury/fiscal authority and the central bank. The budget constraint facing the central bank is given by

$$B^c_t B^c_{t-1} + (i^*_t - s) (\rho D_t + ER_t) + RBC_t = i^b_t B^c_t - (i^*_t + s) BR_t + HP^a_t - HP^a_{t-1}, \quad (3.48)$$

where the right-hand side represents the central bank’s revenue and the left-hand side represents the central bank’s expenditures. Revenue is composed of interest payments from the central bank’s holdings of government debt ($i^b_t B^c_t$), the change in holdings of high-powered money ($HP^a_t - HP^a_{t-1}$) and interest payments from bank $j$ on borrowed reserves\(^{13}\) ($i^*_t + s) BR_t$. Expenditures are made up of changes in the central bank’s government debt holdings ($B^c_t - B^c_{t-1}$), its interest payments to bank $j$ on excess reserves

\(^{13}\)Recall that borrowed reserves are negative, which is why there is a negative sign preceding this term.
\((i_t^* - s) (\rho D_t + E R_t)\), and the transfers of the central bank’s (net) receipts back to the treasury\(^{14}\) \((RCB_t)\).

As discussed earlier, the optimal excess reserve policy \(h\) was independent of \(j\) and hence all banks would pursue the same policy. This implies that aggregate excess and borrowed reserve, \(ER_t\) and \(BR_t\), are obtained by aggregating over the economy-wide mass of repayment shocks\(^{15}\) so that

\[
ER_t = \int D_t(j) \int_{-\tau}^{h_t} (h_t - \epsilon_j t) f(\epsilon) \, d\epsilon \, dj = \frac{(h_t + \tau)^2}{4\tau} D_t, \\
BR_t = \int D_t(j) \int_{h_t}^{\tau} (h_t - \epsilon_j t) f(\epsilon) \, d\epsilon \, dj = -\frac{(h_t - \tau)^2}{4\tau} D_t.
\]

The budget constraint facing the treasury is given by

\[
P_t T_t + B_t^T - B_{t-1}^T + RCB_t = P_t G_t + i_t B_t^T, \tag{3.49}
\]

where the left hand side represents the treasury’s revenue composed of taxes/transfers to/fro households \((T_t)\), new issuance of interest-bearing government debt \((B_t^T - B_{t-1}^T)\) as well as the central bank’s treasury-transfer receipts \((RCB_t)\). The right-hand side represents treasury expenditures made up of government spending on goods and services \((G_t)\) and interest payments on government debt \((i_t B_t^T)\).

In nominal terms, the consolidated government sector budget constraint results from combining the two constraints via receipts \(RCB_t\) and is given by

\[
T_t + B_t^P - B_{t-1}^P + H P_t - H P_{t-1} = G_t + X_t + i_t^P B_t^P \tag{3.50}
\]

\(^{14}\) We can think of this term as the central bank’s post-operation “retained earnings”.

\(^{15}\) We are technically aggregating over the difference between \(h\) and the repayment shock for each bank \(j\).
where
\[ X_t = (i^* - s)(\rho D_t + ER_t) - (i^* + s)BR_t. \] (3.51)

In real terms,
\[ (hp_t^e + b_T^e) - \left(\frac{1}{1 + \pi_t}\right)(hp_{t-1}^e + b_{T-1}^e) = \frac{G_t - T_t + X_t}{P_t} + t^e b_T^p \] (3.52)
and
\[ x_t \equiv \frac{X_t}{P_t} = (i_t^* - s)(\rho d_t + er_t) + (i_t^* + s)br_t, \] (3.53)
where \( b_T^e = b_T^T - b_T^{cb} \). We can rewrite (3.52) as
\[ \left(hp_t^e - b_t^{cb}\right) - \left(\frac{1}{1 + \pi_t}\right)\left(hp_{t-1}^e - b_{t-1}^{cb}\right) = \bar{f}_t + x_t, \] (3.54)
where
\[ \bar{f}_t = \frac{G_t - T_t}{P_t} + t^{cb} b_T^T - b_T^T + \left(\frac{1}{1 + \pi_t}\right) b_{T-1}^T \] (3.55)
will be treated as an exogenous fiscal variable. Given the policy rate \( i^* \) and private sector decisions that determine reserve holdings, \( x_t \) is not controlled directly by the central bank. The budget constraint (3.54) links high powered money and the central bank’s bond holdings via the effects of open market operations.

Central bank policy Equilibrium in the interbank market\(^{16} \) requires reserve demand and reserve supply to balance, so if \( HP_t^e \) is the total supply of high powered money set by the central bank, reserve demand is \( \rho D_t + H_t \) and aggregating

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\(^{16}\)Notice that there households have not yet accumulated financial assets at this moment of time. It is only at the end the period (after the overnight period) where households are able to hold high powered money as well as government bonds and carry them into the following period. So, the equilibrium interbank interest rate does not depend on the household’s demand of cash.
(3.43) over all banks implies

\[ i_t = i_t^* + \chi_t - \left( \frac{s}{\varepsilon} \right) (hp_t^s - \rho), \]  

(3.56)

where

\[ hp_t^s = \frac{HP_t^s}{D_t}. \]

Equation (3.56) illustrates that the central bank has multiple instruments for achieving a given interbank rate \( i_t \). For a given reserve supply and collateral value \( \chi_t \), \( i_t \) can be raised directly by raising the target policy rate \( i_t^* \) or by decreasing the width of the corridor. Holding \( i_t^* \), \( \chi_t \), and \( s \) constant, a decrease in reserve supply (relative to deposit liabilities of the banking sector) increases \( i_t \). A further implication of (3.56) is that if \( s > 0 \) the equilibrium interbank rate will equal the policy rate only when

\[ hp_t^s = \rho + \left( \frac{\varepsilon}{s} \right) \chi_t \geq \rho, \]

that is, only when the central bank supplies a level of reserves greater than the level of required reserves when \( \chi_t > 0 \).

While Berensten and Monnet (2006) emphasized that the evolution of the money stock is endogenous, depending on the net payment of interest on reserve balances and the net interest income from lending reserves to private banks, in our numerical simulation exercises we assume that the central bank supplies (nominal) reserves exogenously according to

\[ HP_t^s = (1 + \theta_t) HP_{t-1}^s, \]  

(3.57)

where \( \theta \) represents the growth rate of nominal reserves\(^{17} \).

\(^{17}\) Trend inflation in this model is pinned down via the steady state version of (3.57), so that if expressed in real terms, we get

\[ \tilde{hp}_t = \left( \frac{1 + \theta_t}{1 + \pi_t} \right) \tilde{hp}_{t-1}, \]

so that in steady-state, \( \theta = \pi \).
3.2.5 Aggregate market clearing

We can combine the government’s consolidated budget constraint, the household’s budget constraint, along with aggregate bank and firm profits to derive the economy’s aggregate resource constraint to get

\[ Y_t = C_t + G_t + \phi(\tilde{\omega}_t)f_{t-1}^mF_t + x^d t + x^d d_t + \kappa b^p_t, \]  

(3.58)

which shows that aggregate output in this economy is equal to the sum of aggregate consumption and government expenditures and total fixed costs of production, deposit management and the aggregate bank search costs.

3.2.6 Collecting equilibrium conditions

The following blocks of equations summarize the equilibrium conditions for the entire model.

3.2.6.1 Household block

\[ \lambda_t = \beta E_t \left( \frac{1 + \phi_{t+1}}{1 + \pi_{t+1}} \right) \lambda_{t+1}. \]

\[ \lambda_t = \frac{C_t^{-\sigma}}{1 + i^b_t - i^d_t}, \]

\[ d_t + w_t N_t = C_t. \]
\[ \Delta_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + i_b^t - i_d^t)}{(1 + i_b^{t+1} - i_d^{t+1})}. \]

\[ \Theta N_t^3 C_t^\sigma = w_t \]

3.2.6.2 Bank j’s block

\[ h_t = \frac{\xi_t}{s} (i_t^* - i_t + \chi_t) \]

\[ \chi_t = \frac{i_t - i_b^t}{\xi_b} \]

\[ h_t d_t + \xi_b b_t^b + \xi_L l_t = \bar{\varepsilon} d_t \]

\[ l_t + b_t^b + \bar{h}_t d_t = (1 - \rho) d_t \]

\[ i_t^d = (1 - \rho) i_t + \rho (i_t^* - s) - x^d + \left( \frac{1}{2} \right) \left( \frac{s}{\bar{\varepsilon}} \right) h_t^2 - \left( \chi_t + \frac{s}{2} \right) \tau \]
3.2.6.3 The decentralized block

\[
f_t = 1 - (1 - \delta)f^{m}_{t-1}
\]

\[
\phi(\tilde{\omega}_t) = (1 - \delta) \left( \frac{\bar{\omega} - \tilde{\omega}_t}{\bar{\omega} - \bar{\omega}} \right)
\]

\[
f^m_t = \phi(\tilde{\omega}_t)f^m_{t-1} + \mu \tau_t^{\phi-1} f_t
\]

\[
\tau = \frac{f_t}{b_t^{\mu}}
\]

\[
\eta \frac{\kappa \tau_t^\phi}{\mu} = (1 - \eta)E_t \Delta_{t,t+1} \left\{ (1 - \alpha) \frac{Y_{t+1}}{f^m_{t+1}} - \phi(\tilde{\omega}_{t+1}) \left[ x^F - \left( \frac{1 - \eta \mu \tau_t^{\phi-1}}{1 - \eta} \right) \frac{\kappa \tau_t^\phi}{\mu} \right] \right\}
\]

\[
\tilde{\omega}_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1 - \alpha}} \frac{(w_t R_t)^\alpha}{z_t} \left[ x^F - \left( \frac{1 - \eta \mu \tau_t^{\phi-1}}{1 - \eta} \right) \frac{\kappa \tau_t^\phi}{\mu} \right]^{1 - \alpha}
\]

3.2.6.4 Aggregate variable block

\[
Y_t = C_t + G_t + \phi(\tilde{\omega}_t)f^m_{t-1}x^F + x^L l_t + x^d d_t + \kappa b_t^{\mu}
\]

\[
Y_t = z_t (F_t)^{1 - \alpha} N_t^\alpha
\]

\[
N_t = \left( \frac{\alpha z_t}{w_t R_t} \right)^{\frac{1 - \alpha}{\alpha}} F_t
\]

\[
F_t = (1 - \delta_t) \left( \frac{\bar{\omega}^k - \tilde{\omega}_t^k}{k(\bar{\omega} - \bar{\omega})} \right) f^m_{t-1}
\]

\[
l_t = w_t N_t.
\]

\[
e r_t = \frac{(h_t + \tau_t)^2}{4r_t} d_t
\]

\[
 b r_t = - \frac{(h_t - \tau_t)^2}{4r_t} d_t
\]

124
3.2.6.5 Government block

\[
\left[ \tilde{h}_p^s - \left( \frac{1}{1 + \pi_t} \right) h_{p_{t-1}} \right] - \left[ b_t^h - \left( \frac{1}{1 + \pi_t} \right) b_{t-1}^h \right] + i_t^h b_t^h = f_t + x_t
\]

\[
x_t = (i_t^s - s)(\rho d_t + er_t) + (i_t^s + s)br_t
\]

\[
\tilde{h}_p^s = h_t d_t + \rho d_t
\]

3.2.6.6 Interest rates

\[
R_t = 1 + i_t - \xi L \chi_t + x^l
\]

\[
R_t^p = \frac{(1 - \eta)(1 - \alpha)}{\alpha} R_t - \left( 1 - \alpha \right) x^F + \eta^F \left( \frac{w_t^\kappa}{\alpha z_t} \right) \left[ \frac{\omega^g - \tilde{\omega}^g}{g(\omega - \tilde{\omega})} \right]
\]

3.2.6.7 Policy block

\[
HP_t^s = (1 + \theta_t) HP_{t-1}^s
\]

3.3 Numerical Analysis

In this section, a numerical analysis is carried out to clarify and describe the properties and dynamic behavior of the interest rates, credit market conditions and the other aggregate variables.
3.3.1 Calibration

To calibrate the model, we adopt the following strategy. We target conventional steady state values for the supply of labor and labor’s share of output: \( N = \frac{1}{3} \), and \( \frac{wN}{r} = \frac{2}{3} \) with the technology parameter \( \xi \) chosen to normalize the steady-state level of output to unity \( Y = 1 \). Additionally, we target the steady-state values for the gross continuation rate of credit matches following the empirical work in Chodorow-Reich (2014) and the steady-state credit destruction rate following the empirical work in Contessi and Francis (2013) and thus set \( \phi(\tilde{\omega}) = 0.70 \) and \( cd = 0.029 \), respectively. The steady state values for loans, excess reserves, and central bank bond holdings - all as fractions of deposits - are taken directly from their empirical values over the Great Moderation period\(^{18} \) and so we target \( \frac{l}{d} = 0.600 \), \( h^* = \frac{H}{D} = 0.015 \), and \( \frac{v}{d} = 0.382 \). We choose savings deposits and commercial and industrial loans for all commercial banks as our measure of deposits and loans respectively. As our measure for excess reserves, we use reserve balances with Federal Reserve Banks and federal debt held by Federal Reserve Banks as central bank bond holdings. The steady-state values for the interest rates on bonds and interbank lending along with the rate of inflation are also taken from the Great Moderation period and so we target \( i^b = 0.015 \), \( i = 0.016 \), and \( \pi = 0.0050 \). Finally, we assume a steady state value for the fixed cost share of GDP equal to 10%.

We parametrize the haircuts on bonds and loans that serve as the banks’ collateral to \( \xi_b = 0.99 \), and \( \xi_L = 0.65 \), which are values taken directly from the Federal Reserve’s guidelines, while the parameter value for the required reserve ratio which is

\(^{18}\)This period covers 1985Q1 through 2007Q1.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>Production function normalizing parameter</td>
<td>1.831</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Utility parameter</td>
<td>7.042</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Production function parameter</td>
<td>0.677</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Level parameter on matching function</td>
<td>4.059</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Rate of exogenous match separation</td>
<td>0.244</td>
</tr>
<tr>
<td>$\xi^{cia}$</td>
<td>CIA constraint parameter</td>
<td>0.926</td>
</tr>
<tr>
<td>$\xi^{bs}$</td>
<td>Bank balance sheet parameter</td>
<td>0.675</td>
</tr>
<tr>
<td>$f^g$</td>
<td>Exogenous fiscal constant</td>
<td>0.004</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount rate</td>
<td>0.990</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Repayment shock parameter</td>
<td>0.019</td>
</tr>
<tr>
<td>$x_f$</td>
<td>Fixed cost of production</td>
<td>0.147</td>
</tr>
<tr>
<td>$s$</td>
<td>Corridor spread</td>
<td>0.0013</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Utility parameter</td>
<td>1.000</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Utility parameter</td>
<td>1.000</td>
</tr>
<tr>
<td>$\xi_b$</td>
<td>Haircut on bonds</td>
<td>0.990</td>
</tr>
<tr>
<td>$\xi_L$</td>
<td>Haircut on loans</td>
<td>0.650</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Reserve requirement ratio</td>
<td>0.094</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Bank search costs</td>
<td>1.580</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Matching function elasticity parameter</td>
<td>0.400</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>Bargaining weight in lending market</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Table 3.1: Parameter values.

consistent with the steady state of the model is $\rho = 0.094$. We normalize the level of aggregate technology in the steady state to be $z = 1$, and set the minimum and maximum of the firm-specific idiosyncratic productivity to $\underline{\omega} = 0.00$ and $\overline{\omega} = 2.00$, respectively.

We assume that the household’s utility function is logarithmic in consumption and labor and set $\sigma = \eta = 1.00$. Finally, following Petrosky-Nadeau and Wasmer (2013), we set the banks’ search cost for equal to $\kappa = 1.58$, and set the symmetric floor and ceiling rate, the Nash bargaining parameter and the elasticity of the matching function to standard\(^{19}\) values so that $s = 0.0013$, $\bar{\eta} = 0.50$, and $\phi = 0.40$, respectively.

\(^{19}\)Note that here we are not satisfying the Hosios condition; see Petrosky-Nadeau and Wasmer (2013) for an example of a model of credit with search and matching friction which does satisfy the Hosios...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>Trend inflation</td>
<td>0.02/4</td>
</tr>
<tr>
<td>$i^b$</td>
<td>Bond return</td>
<td>0.015</td>
</tr>
<tr>
<td>$i$</td>
<td>Fed funds rate</td>
<td>0.016</td>
</tr>
<tr>
<td>$i^*$</td>
<td>Fed funds target</td>
<td>0.016</td>
</tr>
<tr>
<td>$z$</td>
<td>Aggregate productivity</td>
<td>1.000</td>
</tr>
<tr>
<td>$N$</td>
<td>Labor</td>
<td>0.333</td>
</tr>
<tr>
<td>$Y$</td>
<td>Aggregate output</td>
<td>1.000</td>
</tr>
<tr>
<td>$w$</td>
<td>Wages</td>
<td>2.000</td>
</tr>
<tr>
<td>$\varphi(\tilde{\omega})$</td>
<td>Credit match survival rate</td>
<td>0.700</td>
</tr>
<tr>
<td>$h$</td>
<td>Demand for excess reserves/deposits</td>
<td>0.015</td>
</tr>
<tr>
<td>$b^{cb}/d$</td>
<td>Central bank bond holdings/deposits</td>
<td>0.382</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>Consumption/Output</td>
<td>0.852</td>
</tr>
</tbody>
</table>

Table 3.2: Steady-state targets.

We choose values for the 11 remaining parameters, listed in table 3.1, so that conditional on the above parameterization, the model satisfies the steady state targets defined above. The remaining steady state variables are listed in table 3.2.

### 3.3.2 Policy experiments

We seek to evaluate the model’s response to an aggregate shock to technology ($z$) and a shock to the rate at which credit matches exogenously destroy themselves ($\delta$). One important response of the model is the endogenous adjustment of the commercial banks’ mix of bond, loan and excess reserve holdings in response to these shocks as this has implications for the transmission of monetary policy. For example, Kashyap and Stein (1999) find that the impact of monetary policy on lending behavior is stronger for banks with lower ratios of securities to assets\(^{20}\).

\(^{20}\)See also Kashyap and Stein (1997).
3.3.2.1 Technology

The first numerical experiment conducted will be a one-time shock to aggregate technology $z$. Deviations to technology are assumed to follow

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_{z,t} \quad 0 < \rho_z < 1.$$ 

In figure 3.1 and figure 3.2 below, the impulse responses for several important variables are shown, resulting from a one-time 0.004 standard deviation shock to $\epsilon_z$ under two levels of persistence. There are two sets of responses - figure 3.1 displays the model’s response from the “financial side”, while figure 3.2 displays the model’s response from the “real side”.

Figure 3.1: Impulse response functions resulting from a one-time shock to aggregate technology; Impulse responses indicated with blue circles result from a persistence level of $\rho_z = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_z = 0.90$.

In both figures, the blue impulse responses indicated with circles show the
Figure 3.2: Impulse response functions resulting from a one-time shock to aggregate technology; Impulse responses indicated with blue circles result from a persistence level of $\rho_z = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_z = 0.90$. On the financial side, the positive technology shock induces banks to generate more lending to firms as the increase in productivity resulting from the shock has increased the return to lending. The increase in the demand for excess reserves is due to two factors: from (3.43), the equation for $h_t$, the demand for excess reserves is increasing in the spread between the interbank and policy rate and increasing in the value of the collateral constraint. In all of our simulation exercises, we have assumed that $i = i^*$, and that the value of the multiplier is positive, as we are assuming a steady-state where the collateral constraint is binding. Thus, in a situation where the return
to lending is increasing relative to holding bonds, banks will shift more resources away from holding bonds (as shown) in favor of holding loans, exerting downward pressure on $i^b$ in relation to $i$, resulting in an increase in the value of collateral $\chi$ and the demand for excess reserves $h$. The second reason for the increase in the demand for excess reserves is that banks are seeking additional financial capital to turn around and lend out to firms seeking financing; given that we have assumed they can freely borrow from the interbank market at rate $i$, the increase in the demand for federal funds exerts further pressure for the interbank rate to rise, increasing the spread between $i$ and $i^b$, resulting in a further increase in the value of collateral $\chi$ and thus $h$. As deposits are an additional source for financial capital, the increase in demand for deposits induces a rise in the deposit rate.

Turning to the real side of the economy, the shock to aggregate productivity increases the output of firms currently matched with lenders also making potential matches more attractive, motivating an influx of lending, which generates more matches, resulting in an increase in aggregate loans and output. The increase in the joint surplus to matches as a result of the increase in productivity motivates a fall in the threshold productivity cutoff, resulting in a proportional increase in “productive” matches as well as increase in the net rate of match survival $\phi(\tilde{\omega})$ and a decrease in the rate of credit destruction.
### 3.3.2.2 Match destruction

This experiment studies the model’s response to a one-time shock to the rate at which credit matches exogenously terminate themselves. Recall that we have set this steady-state rate to approximately $\delta = 0.24$, but are now allowing it to vary with time according to

$$\hat{\delta}_t = \rho \hat{\delta}_{t-1} + \epsilon_{\delta,t}, \quad 0 < \rho \delta < 1.$$ 

An exogenous increase in the destruction of credit matches can be thought of originating from a financial crisis where in a situation in which lending is contingent on the value of collateralizable assets, financial shocks which have a measurable impact on the value of assets may magnify themselves through the economy’s financial leveraging. Lenders may have no way of assessing the value of collateral or there may be a level of uncertainty which renders credit relationships insolvent, thus leading to their destruction. We model this scenario as an increase in $\delta$.

The increase in the exogenous probability of match termination decreases the return to lending activities relative to holding securities, thus banks are motivated to shift a greater proportion of their portfolios away from loans and into bonds. This adjustment places downward pressure on the relative spread between the interbank rate and the return to bonds, decreasing the value of collateral $\chi$, resulting in a fall in the demand for excess reserves $h$. As banks are now demanding less financial capital to create loans with, this results in a fall in the interbank rate, exerting further downward pressure on the value of collateral (the multiplier $\chi$) and the demand for excess reserves.
The decrease in demand also spills over into the deposit market, as illustrated by the fall in the deposit rate.

Turning to the real side, the increase in exogenous match break-up results in an immediate fall in output due to the proportion of productive matches which cease to exist, as well as making lending a less attractive activity for banks (even in the face of a fall in the opportunity cost of lending brought on by a decrease in the interbank rate), decreasing aggregate loans, resulting in a (further) fall in aggregate output. The productivity threshold rises as a result of the increase in destruction, which indicates that the only credit matches feasible to proceed to the funding stage are the matches with a high idiosyncratic productivity component. The combination of the increase in the threshold and the exogenous increase in $\delta$ decrease the net rate of match survival.
Figure 3.4: Impulse response functions resulting from a one-time shock to the exogenous rate of credit match destruction; Impulse responses indicated with blue circles result from a persistence level of $\rho_\delta = 0.30$ on the technology shock while impulse responses indicated with red squares result from $\rho_\delta = 0.90$.

$\phi(\omega)$ and result in an increase in the credit destruction rate.

3.4 Concluding Remarks

We have constructed a framework to study a channel system for the implementation of monetary policy when bank lending is subject to frictions. We show how the width of the channel and the nature of random payment flows in the interbank market also affect the spread and therefore the transmission of monetary policy to credit markets. The two simulation exercises illustrate how the endogenous adjustment in the composition of the bank’s balance sheet is motivated by changes in the (relative) returns to each asset. The relative returns are largely contingent on the nature of the specific
shock(s) which hit the economy, hence policies such as the payment of interest on reserves and the implementation of standing lending facilities such as in a channel system (which have the potential to alter the composition of bank balance sheets) may serve as an effective stabilization tool since balance sheet composition is an important factor in the transmission of monetary policy. Extending the current framework to account for the effectiveness of different policy prescriptions is left for future work.
Chapter 4

A comparative assessment of labor productivity during the Great Recession and the early 2000s recession: How choosier have employers become?

- with David Florian-Hoyle -

4.1 Introduction

One potential hypothesis explaining the so-called “jobless” recovery following the Great Recession is the increase in the productivity of labor in new matches formed and those which survived [McGrattan and Prescott (2012), Schaal (2011), Mulligan (2011)]. A natural place to begin the search for evidence of a jobless recovery is Okun’s
law, the long-standing statistical relationship between the unemployment gap and the output gap\(^1\), since according to this statistical “rule-of-thumb”, a 2% increase in output corresponds to a 1% decrease in the unemployment rate; hence, we may call a recovery in output following a recession *jobless* if it is not met with the corresponding fall in the unemployment rate in accordance with Okun’s law.

In order to get a sense for this empirical relationship, we regress the unemployment gap on the output gap. We construct the series for the unemployment gap by taking the difference in the quarterly civilian unemployment rate (FRED series UNEMPLOY) and the natural rate of unemployment (FRED series NROU) while we construct the series for the output gap by taking the log-deviation of quarterly real GDP (FRED series GDPC1) from its HP filtered trend component (with filtering parameter \(\lambda = 1600\)) and multiply the result by 100 to convert it to percent. Using U.S. data from 1949Q1 through 2014Q4, the resulting regression is

\[
\hat{U}_t = -0.608 \hat{y}_t
\]

where \(\hat{y}_t\) is the output gap and \(\hat{U}_t\) is the unemployment gap defined earlier. The interpretation of this estimation is that a one percent decrease in output from trend (quarterly) would be associated with an \(\approx 0.61\) percentage point increase in the unemployment rate

\(^1\)The unemployment gap is defined as the difference between the actual unemployment rate and its natural level, while the output gap is defined as the percentage difference between the economy’s current level of output and its potential level.
above the natural rate.

Of particular interest is in observing the relative stability of the statistical relationship over time, since a break in the statistical relationship congruent with a “jobless recovery” of the Great Recession would manifest itself as a deviation of the estimation given in the above regression. Figure 1 illustrates the time series for the Okun’s Law regression coefficient resulting from the deployment of a “rolling regression” using a sliding window of 100 observations.

Figure 4.1: The time path of the regression coefficient for Okun’s law resulting by “rolling” a sliding interval of 100 observations. 95% confidence bands included.

The first data point corresponds to the regression coefficient resulting from the first 100 observations (1949Q1 - 1973Q4; this window allows for 25 years worth of data per estimation point); the window is then advanced to the next quarter and the estimation is repeated until the last observation is used. Also included is the 95% confidence interval around this estimation point (coeff ±1.96×std error). The figure demonstrates the relative stability of Okun’s Law up until 2003. What follows is a fairly
abrupt fall in the time path (low value of approximately -0.75 during 2006Q3) followed
by a large rise in the coefficient (high value of approximately -0.36 during 2008Q4). The
abrupt rise in the regression coefficient implies that the relationship has adjusted to one
where a given increase in the output gap is now associated with a smaller decrease in
the unemployment gap, providing support for the existence of a jobless recovery.

Using the estimate from the Okun’s law regression, the residual $\epsilon$ is defined by

$$\hat{\epsilon}_t = \hat{U}_t + 0.608 \hat{y}_t.$$

Figure 2 illustrates the time series of the residual\(^2\) from the regression for Okun’s Law.

Figure 4.2: The time path for the residuals from the Okun’s Law regression. Note the
pattern of residual trajectories during all of the NBER recessions and how the pattern
was reversed during the Great Recession.

Even though the simple Okun’s Law regression specified above actually \textit{fails}
the Bruesch-Godfrey autocorrelation test\(^3\), the residual series from the regression reveals

\(^2\)This series is constructed by taking the actual data for $\hat{U}$ and $\hat{y}$ and feeding it into the definition
of $\hat{\epsilon}$ to get the residuals.

\(^3\)A regression of the residual upon one lag of itself (an autoregression) reveals a fairly statistically
a relatively interesting and potentially important observation; of the 10 NBER recession prior to the Great Recession, the pattern of the residual is to fall during the recession and to rise abruptly during the recovery. As figure 2 shows, this pattern is perfectly reversed (and of a relatively larger degree) during the Great Recession. The interpretation would be that before the Great Recession, the typical timing of events during an economic recovery would be a recovery in labor, closely followed by a recovery in output - a pattern which reversed itself during the Great Recession.

Figure 3 illustrates the distribution of this residual during the recovery of the Early 2000s recession.

Figure 4.3: **Source:** Authors regression residuals. Data series for the civilian unemployment rate and the long-term and short-term natural rate of unemployment come from FRED. GDP data comes from FRED, while potential GDP is the Hodrick-Prescott filtered series of the same GDP data with standard $\lambda = 1600$.

As expected, during an economic recovery from the trough of an output gap significant relationship. Authors have corrected this by incorporating lags of the independent variable into the regression; see Ball et al. (2012), for example.
through the point where $y = y^{potential}$, the mass of the residuals of the Okun regression should occupy the negative portion of the support, as during this portion of the business cycle, $\hat{y} < 0$ and $\hat{U} > 0$. Figure 4 shows the same depiction for the Great Recession.

![Figure 4.4: Okun's low residuals during the recovery of the Great Recession (2009.05 - present)](image)

In contrast with the Early 2000s recession, the Okun’s residuals for the (ongoing) recovery of the Great Recession are vastly positive, contradicting what we would expect from an economic “recovery”. Figure 4 shows evidence of $\hat{U}_t > -0.608 \tilde{y}_t$, which offers some support to the idea that the recovery from the Great Recession has been a “jobless” one, as $\hat{U} > 0$ during this period.

We assess the validity of the theory that, consistent with a jobless recovery, existing and new employment relationships experienced a surge in productivity using a modified version of the Mortensen-Pissarides framework. Specifically, we formulate a method to quantitatively assess the productivity “cutoff” of firm-worker matches.

---

4The “cutoff” level of productivity refers to the theoretical level where the joint surplus to the match
This allows us to compare the level of productivity of firm-employee matches during the recovery of the 2008 financial crisis in relation to the recovery of the early 2000s recession.

The two results of this chapter are:

1. During the Great Recession the productivity “spread” was 104% greater than that of the Early 2000s recession. This supports the theory of a higher level of productivity in new and existing matches contributing towards the jobless recovery.

2. Allowing the job maintenance cost to fluctuate endogenously, we show that the implied series exhibits a “pulse” during both the Great Recession and early 2000s recession; during the Great Recession the series exhibited a jump of 15% while during the early 2000s recession the jump was 5%; we interpret this difference to be largely the result of the financial turmoil during the Great Recession and subsequent increase in the difficulty of many firms to secure financing. Our implied job maintenance cost series shares a correlation coefficient of 0.52 with the St. Louis financial stress index, which we see as lending credence to our interpretation of this series as encapsulating (relative) employment financing costs.

We construct the model of the labor market in section 2 and bring the model to the data in section 3.

---

5 “Spread” refers to the difference between the highest and lowest theoretical value for productivity during the specific recession.

6 In the DMP model, the job maintenance cost is a cost paid by employers to “maintain a job”. We interpret this series to not only reflect the costs of recruiting a worker, but also the implicit costs facing an employer of financing their labor.
4.2 Model of the labor market

We assume that the market is populated by workers and firms. Firms endogenously decide to enter the labor market by choosing to post job openings/vacancies. Both populations are either matched or not. Once a worker matches with a firm, the worker draws a productivity $\pi$ from an unspecified distribution $G$ which has direct implications for the productivity of the match. We assume that $G$ has support $[\bar{\pi}, \pi]$, where the productivity cut-off $\bar{\pi}$ marks the lower bound of a match with non-negative joint surplus\(^7\). If the worker is currently employed but draws a productivity which is below the cutoff, the match terminates itself. At that point, the firm decides to endogenously enter the labor market and search again for an employee. The (now) unmatched laborer costlessly awaits its next probable match. In this framework, the cutoff productivity level $\bar{\pi}$ is endogenously determined.

4.2.1 Value functions

Workers and firms are either matched or unmatched, where each state has a corresponding value function. For each agent, their individual economic surplus is the difference in the value of being matched over unmatched. Once the agents match with each other, the joint surplus of the match is the sum of the individual surpluses. The productivity cutoff delineates a level of productivity which would result in a positive joint surplus.

\(^7\)Each agents’ economic surplus is the difference in value between being matched and unmatched. The joint surplus to a match is the sum of the firm and laborer’s individual surpluses.
4.2.1.1 The firm value functions

The value of a firm which is matched (annotated $FM$) with a worker which drew productivity above the cutoff is given by

$$V^{FM}(\pi_t, \pi_{t+1}) = y_t - w(\pi_t) + \beta E_t \int_{\pi}^{\bar{\pi}} [V^{FM}(\pi_{t+1}) + V^{FS}_{t+1}] dG(\pi_{t+1}),$$

where $w(\pi_t)$ is the wage, $\beta$ is the discount parameter, and we have assumed that the output of a match is multiplicatively linear in the product of the aggregate level of labor productivity $\bar{z}_t$ and the worker’s idiosyncratic productivity level $\pi_t$ so that

$$y_t = \bar{z}_t \pi_t.$$

This function expresses that the value of being a matched firm is the output net of costs plus the expected value of being a firm in the following period, which is made up of the expected value of continuing as a matched firm $V^{FM}$, or having the match endogenously separate and becoming a searching firm $V^{FS}$, contingent on the productivity level of the match in the following period.

The value function of a firm which has posted a job vacancy and is thus searching for a worker (annotated $FS$) is given by

$$V^{FS}_t = -v + p^F_t \beta \int_{\pi}^{\bar{\pi}} [V^{FM}(\pi_{t+1}) + V^{FS}_{t+1}] dG(\pi_{t+1}) + (1 - p^F_t) \beta V^{FS}_{t+1},$$

where $v$ is the cost of posting the vacancy and $p^F_t$ is the probability a firm matches with an unemployed searching worker. This function states that a firm pays $v$ to post a vacancy and with probability $p^F_t$ matches with an unemployed worker. The following
period, a productivity draw is made and if this probability is above the cutoff, a pro-
ductive match is established. If the productivity draw is below the cutoff, the match
terminates itself. With probability \((1 - p^F_t)\), the firm doesn’t match with an unemployed
worker and is once again a searching firm the following period.

As the firm is endogenously entering the market for employment by choosing
whether or not to post a job vacancy, the free entry condition is the result of assuming
that enough firms would enter such that the market would be saturated to the point
that the value of being a searching firm is zero. Imposing this on the value function for
searching firms results in

\[
v = p^F_t \beta \int_{\tilde{\pi}}^{\pi} \mathbb{P}^M_t(\pi_{t+1}) dG(\pi_{t+1}).
\]  
(entry condition)

The left-hand side of this expression is the per-period cost of the search and the right-
hand side is the (probabilistic) expected benefit of the search.

### 4.2.1.2 The worker value functions

The value function of a worker which is matched with a firm (annotated \(WM\))
is given by

\[
\mathbb{V}^{WM}(\pi_t, \pi_{t+1}) = w(\pi_t) + \beta \int_{\pi_t}^{\pi} [\mathbb{V}^{WM}(\pi_{t+1}) + \mathbb{V}^{WU}_{t+1}] dG(\pi_{t+1}),
\]

This function expresses that the value of being an employed worker is the wage plus the
expected value of continuing on as a worker in the following period \(V^{WM}\), or having
the productivity drawn in the following period fall short of the cutoff, resulting in an
endogenous match termination, and thus unemployment, which carries value \(V^{WU}\).
The value of an unemployed worker currently searching for a firm (annotated \(WU\)) is given by

\[
V_{t}^{WU} = b + p_{t}^{W} \beta \int_{\tilde{\pi}}^{\pi} \left[ V_{t+1}^{WM}(\pi_{t+1}) + V_{t+1}^{WU} \right] dG(\pi_{t+1}) + (1 - p_{t}^{W}) \beta V_{t+1}^{WU},
\]

where \(b\) is the unemployment benefit a searching worker receives, and \(p_{t}^{W}\) is the probability that an unemployed worker meets a firm. This equation states that the value of being an unemployed worker is the unemployment benefit a searching worker receives, plus the probability of matching and the expected value of that match - contingent on the productivity draw - plus the expected value of not finding a match and starting the following period as an unemployed worker.

### 4.2.1.3 Firm and worker surpluses

We denote the firm’s surplus as \(\Psi^{FS}(\pi_{t}, \pi \geq \tilde{\pi})\) and it is simply the difference in the firm’s value functions, so that

\[
\Psi^{FS}(\pi_{t}, \pi \geq \tilde{\pi}) = y_{t} - w(\pi_{t}) + \beta \int_{\tilde{\pi}}^{\pi} \Psi^{F}(\pi_{t+1})dG(\pi_{t+1}) + v - p_{t}^{F} \beta \int_{\tilde{\pi}}^{\pi} \Psi^{F}(\pi_{t+1})dG(\pi_{t+1}) - (1 - p_{t}^{F}) \beta \Psi_{t+1}^{FS} - (1 - p_{t}^{F}) \beta \Psi_{t+1}^{FS} - (1 - p_{t}^{F}) \beta \Psi_{t+1}^{FS} - (1 - p_{t}^{F}) \beta \Psi_{t+1}^{FS} - (1 - p_{t}^{F}) \beta \Psi_{t+1}^{FS}.
\]

Incorporating the firm’s entry condition and the formulation for output \(y_{t}\) into this last expression gives

\[
\Psi^{FS}(\pi_{t}, \pi \geq \tilde{\pi}) = z_{t} - w(\pi_{t}) + \beta \int_{\tilde{\pi}}^{\pi} \Psi^{FS}(\pi_{t+1})dG(\pi_{t+1}).
\]
Finally, the entry condition can be used once again to arrive at

$$\Psi_{F}^{\Sigma}(\pi_{t, \pi \geq \tilde{\pi}}) = z_{t} \pi_{t} - w(\pi_{t}) + \frac{v}{p_{t}}. \quad \text{(firm surplus)}$$

Likewise, the worker’s surplus is denoted as $$\Psi_{W}^{\Sigma}(\pi_{t, \pi \geq \tilde{\pi}})$$ and is also defined as the difference in the worker’s value functions, so that

$$\Psi_{W}^{\Sigma}(\pi_{t, \pi \geq \tilde{\pi}}) = w(\pi_{t}) + \beta \int_{\pi}^{\Pi} \Psi_{W}^{a}(\pi_{t+1})dG(\pi_{t+1}) - b \quad - p_{t} \beta \int_{\pi}^{\Pi} \Psi_{W}^{a}(\pi_{t+1})dG(\pi_{t+1}) - (1 - p_{t})\beta \Psi_{U}^{WU}$$

$$= w(\pi_{t}) - b + (1 - p_{t})\beta \int_{\pi}^{\Pi} \Psi_{W}^{a}(\pi_{t+1})dG(\pi_{t+1}) - (1 - p_{t})\beta \Psi_{U}^{WU}$$

$$= w(\pi_{t}) - b + (1 - p_{t})\beta \int_{\pi}^{\Pi} [\Psi_{W}^{a}(\pi_{t+1}) - \Psi_{U}^{WU}]dG(\pi_{t+1}). \quad \text{(worker surplus)}$$

### 4.2.1.4 The surplus sharing rule

In accordance with Nash bargaining, the joint surplus to the match is redistributed to both counterparties via the equilibrium wage in accordance with each counterparty’s relative bargaining weight. Specifically, let $$\eta$$ represent the bargaining power of the firm and $$(1 - \eta)$$ represent the bargaining power of the worker. The optimization problem is

$$\max_{w^{*}} \left( \Psi_{F}^{\Sigma} \right)^{\eta} \left( \Psi_{W}^{\Sigma} \right)^{(1-\eta)}$$

The first order condition for this problem is

$$\eta \left( \Psi_{F}^{\Sigma} \right)^{\eta-1} \frac{\partial \Psi_{F}^{\Sigma}}{\partial w} \left( \Psi_{W}^{\Sigma} \right)^{(1-\eta)} + \left( \Psi_{F}^{\Sigma} \right)^{\eta} (1 - \eta) \left( \Psi_{W}^{\Sigma} \right)^{-\eta} \frac{\partial \Psi_{W}^{\Sigma}}{\partial w} = 0.$$
Rearranging this expression and substituting in the partial derivatives leads to the traditional sharing rule

\[(1 - \eta)\mathcal{V}^F \mathcal{V} = \eta \mathcal{V}^W \mathcal{V}.\]  

(sharing rule)

### 4.2.1.5 The productivity cutoff

The cutoff productivity level \(\tilde{\pi}\) is defined as the point where the joint surplus associated with the match is equal to zero. The joint surplus to an employment match is defined as the sum of the worker and firm surpluses and is written

\[\mathcal{V}^J \mathcal{V}^\Sigma = \mathcal{V}^W \mathcal{V}^\Sigma + \mathcal{V}^F \mathcal{V}^W = \int_{\mathcal{V}} \frac{v}{p_t} \mathcal{V}^W (\pi_t) \ dG(\pi_t)\]

The sharing rule can be used to write this expression as

\[\mathcal{V}^J \mathcal{V}^\Sigma = \int_{\mathcal{V}} \frac{v}{p_t} \mathcal{V}^W (\pi_t) \ dG(\pi_t)\]

Incorporating the entry condition for the firm results in

\[\mathcal{V}^{J\Sigma} = \mathcal{V}^W \mathcal{V}^\Sigma + \mathcal{V}^F \mathcal{V}^W = \int_{\mathcal{V}} \frac{v}{p_t} \mathcal{V}^W (\pi_t) \ dG(\pi_t)\]

(joint surplus)
Setting this equal to zero to isolate the cutoff productivity, we get

\[ v^J = \bar{z}_t \bar{\pi}_t - b + \left[ \frac{1 - (1 - \eta)p_t^W}{\eta} \right] \left( \frac{v}{p_t^F} \right) = 0 \]

or

\[ \bar{\pi}_t = b - \left[ \frac{1 - (1 - \eta)p_t^W}{\eta} \right] \left( \frac{v}{p_t^F} \right) \]

or

\[ \bar{\pi}_t = \frac{b \eta p_t^F - v \left[ 1 - (1 - \eta)p_t^W \right]}{\bar{z}_t \eta p_t^F} \] (productivity cutoff)

### 4.3 Empirical implementation

The empirical approach we take entails implementing standard parameter values used throughout the labor search literature along with the pertinent time series data into the productivity cutoff equation in order to construct a series for the idiosyncratic productivity cutoff which is consistent with the variation of the DMP model we have used. We use the resulting model and data-consistent series for the idiosyncratic productivity as a proxy for the level of employer “choosiness”. It is of particular interest to us to see how the dynamics of this implied series has evolved during both the early 2000s recession and the Great Recession in answering our research question.

The probability of a firm matching with a searching worker \( p_t^F \) can be written as

\[ p_t^F = \frac{m_t}{V_t}, \]

where \( m_t \) are the numbers of matches and \( V_t \) are the number of open job vacancies.
(firms) searching for a worker. Assuming the same specification for $p_t^W$,

$$\frac{p_t^W}{p_t^F} = \frac{m_t}{V_t} = \frac{V_t}{U_t} = \tau_t,$$

where $\tau$ represents tightness in the labor market.

Following the vast majority of the literature, we assume that the matching function takes on the Cobb-Douglas structure in vacancies and unemployed and is written

$$m_t = \psi U_t^\alpha V_t^{1-\alpha},$$

where $\psi$ and $\alpha$ are parameters. This functional form then implies that

$$p_t^F = \frac{m_t}{V_t} = \frac{\psi U_t^\alpha V_t^{1-\alpha}}{V_t} = \psi U_t^\alpha V_t^{1-\alpha} = \psi \left(\frac{V_t}{U_t}\right) = \psi \tau_t^{1-\alpha},$$

$$p_t^W = \frac{m_t}{U_t} = \frac{\psi U_t^\alpha V_t^{1-\alpha}}{U_t} = \psi U_t^{\alpha-1} V_t^{1-\alpha} = \psi \left(\frac{V_t}{U_t}\right)^{1-\alpha} = \psi \tau_t^{1-\alpha}.$$

Substituting these probabilities into the productivity-cutoff equation results in

$$\frac{\pi_t}{\eta \psi \tau_t^{-\alpha}} = \frac{b \eta \psi \tau_t^{-\alpha} - v \left[1 - (1 - \eta)\psi \tau_t^{1-\alpha}\right]}{z_t \eta \psi \tau_t^{-\alpha}}.$$

### 4.3.1 Parameterization

Unemployment benefits are set to $b = 0.54 \bar{w}$, where $\bar{w}$ represents steady-state wages. The proportion of wages 0.54 is sourced from a weighted average of unemployment benefit calculators for various states in the U.S. The job posting/vacancy costs $v$ follow Petrosky-Nadeu and Wasmer (2013) and Silva and Toledo (2007) who estimate this as the cost to recruiting a worker and thus $v \sim 3.6$ percent of the wage rate; hence
we set \( v = 0.036 \frac{\tau}{U} \) where \( U \) and \( \tau \) are the steady-state unemployment rate\(^8\), and labor market tightness, respectively.

The steady-state wage \( \bar{w} \) is set equal to aggregate nominal compensation as a fraction of aggregate output, or 0.438. The steady-state unemployment rate \( U \) and labor market tightness \( \tau \) are set to their empirical equivalents, and thus \( U = 0.054 \) and \( \tau = 0.478 \). This implies the unemployment benefit \( b = 0.54(0.438) = 0.237 \) and vacancy posting cost is \( v = 0.036 \frac{0.443}{0.478 \times 0.054} = 0.618 \).

We follow Walsh (2005) and Blanchard & Diamond (1989) and set \( \alpha = 0.4 \). Following Den Haan et al. (2000) and Petrosky-Nadeau and Wasmer (2013), based on the estimates for the U.S., we target a job filling rate \( p_F = 0.4 \). This then implies that the level parameter in the matching function \( \psi = 0.4(0.478)^{0.4} = 0.298 \). Finally, we follow the vast majority of the literature and set the bargaining weight \( \eta \) symmetrically to 0.50.

Finally, for the labor market tightness, we use the ratio of total nonfarm job openings to total unemployed\(^9\) and for the series for aggregate labor productivity, we use the ULC total labor productivity series\(^10\).

\(^8\) Assuming a unit mass labor force implies that the unemployment rate is equivalent to the unemployment level.

\(^9\) FRED series JTSJOL/UNEMPLOY which are Job Openings: Total Nonfarm, 1000s (Level), Monthly, S.A. divided by Unemployed, 1000s, Monthly, S.A..

\(^10\) FRED series ULQELP01USQ661S which is ULC Indicators: Total Labor Productivity for the United States, Quarterly, Seasonally Adjusted
4.3.2 Results

Incorporating the parameters and the empirical steady-state values into the productivity cutoff equation results in an expression we can use to extract the model and data-consistent time series for the idiosyncratic productivity level of the labor matches formed from December of 2000 through December of 2014

\[ \tilde{\pi}_t = \frac{0.036\tau_t^{-0.4} - 0.618 \left[ 1 - 0.149\tau_t^{0.6} \right]}{\tau_t^{0.149}\tau_t^{-0.4}}. \]

The time series for \( \tilde{\pi} \) is displayed in figure 5 below.

![Time series for model-implied level of \( \tilde{\pi} \).](image)

Figure 4.5: Time series of the model-implied level of \( \tilde{\pi} \).

One thing to notice which is very much in line with the jobless recovery hypothesis of an increase in the productivity of existing matches following the onset of the recession. The other is the numerical degree of difference in \( \tilde{\pi} \) during the latest recession and that of the late 2000s. At their most extreme (for the data available) the following
are some numbers:

\[ \tilde{\pi}_{2001:3} = 0.124 \]

\[ \tilde{\pi}_{2001:11} = 0.143, \]

where consistent with the NBER bands for the early 2000s recession, March of 2001 was commencement and November of 2001 was termination. The equivalent values for the Great recession include

\[ \tilde{\pi}_{2007:12} = 0.134 \]

\[ \tilde{\pi}_{2009:06} = 0.175 \]

The “spread” of the cutoff in the early 2000s recession is 0.124 - 0.143 = 0.019 and the spread in the latest recession is 0.175 - 0.134 = 0.041. The difference is suggestive of a more than doubling of the cutoff. More specifically,

\[ \frac{\text{late 2000s spread} - \text{early 2000s spread}}{\text{early 2000s spread}} = \frac{0.041 - 0.019}{0.019} = 1.158 \]

or a 116% difference in the idiosyncratic productivity of existing and newly-created matches of the latest recession compared to the early 2000s recession.

4.3.3 Explanation of the results: What has changed?

The productivity cutoff equation is an equilibrium condition, and thus if we are to arrive at a convincing answer as to how the underlying behavior of agents changed during both recessions, we need to isolate what piece of this equation changed and to what magnitude. While the early 2000s recession had the dot-com bubble collapse in
combination with the September 11th terrorist attacks as its originating factors, the 
Great Recession was largely brought on by a collapse in the housing market followed by 
deep financial turmoil and panic in credit markets, making borrowing/seeking finance 
an especially difficult endeavor. The differences in their origins provides us with a clue 
as to what may have contributed to the more than doubling of the implied idiosyncratic 
productivity cutoff during the Great Recession.

Typically, small businesses finance their labor costs using some form of re-
volving credit. Thus, if credit markets are disrupted on par with a financial panic, we 
would expect to see the implicit costs of financing labor increase by a measurable degree. 
Turning to our variation of the DMP model, we can test this hypothesis by analyzing 
how the vacancy cost parameter would behave if it were allowed to vary with time.

The equilibrium wage is the solution to the Nash sharing rule and is given by

\[ w_t = \eta b + (1 - \eta) \left[ y_t + v_t \left( \frac{p^W_t}{p^F_t} \right) \right], \]

where we have allowed the job opening cost to vary with time. The intuition behind this 
equation is that the equilibrium wage is a weighted sum of the unemployment benefit \( b \) 
and the sum of output and (weighted) savings the match provides the firm; for example 
if \( v \) is the cost of the match, and \( p^W \) were to increase relative to \( p^F \), this implies that 
securing a future match would be more expensive for the firm and thus the current 
match the firm is in becomes more valuable, implicitly. The linear weights reflect the 
bargaining power of the employee and firm.

154
This equation can be solved for \( y_t \) to get
\[
y_t = \frac{w_t - \eta b}{1 - \eta} - v_t \left( \frac{p^W_t}{p^F_t} \right).
\]

We can use this expression to eliminate \( y_t \) from the productivity cutoff to get
\[
\frac{w_t - \eta b}{1 - \eta} - v_t \left( \frac{p^W_t}{p^F_t} \right) = b - \left[ \frac{1 - (1 - \eta)p^W_t}{\eta} \right] v_t p^F_t,
\]
or, more compactly,
\[
v_t = \left( \frac{w_t - b}{1 - \eta} \right) \left( \frac{\eta p^F_t}{p^W_t - 1} \right).
\]

Finally, log-linearizing this equation about its steady-state results in
\[
\hat{v}_t = \left( \frac{w}{w - b} \right) \hat{w}_t - \left[ \alpha + \left( \frac{p^W_t}{p^W - 1} \right) (1 - \alpha) \right] \hat{\tau}_t.
\]

We can now incorporate the same parameter and steady-state values from earlier along with the appropriately transformed empirical data on the wage and labor market tightness to extract a model and data-consistent time series for the implied labor financing costs facing firms.

We assume that the data for the equilibrium wage comes from the ratio of\(^{11}\) the aggregate compensation of all employees (wages and salaries, in billions) to total GDP (billions). We take the constructed wage series, apply a HP filter to it with smoothing parameter\(^{12}\) \( \lambda = 14, 400 \), and then take the log-difference between the actual series and the smoothed series to isolate \( \hat{w} \). A similar procedure\(^{13}\) is used in isolating \( \hat{\tau} \).

---

\(^{11}\)Total compensation comes from the FRED series A576RC1Q027SBEA Compensation of employees: Wages and salaries, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate and GDP comes from the FRED series GDP Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate. We assume that the equilibrium wage is the ratio of the two time series.

\(^{12}\)While we used a smoothing parameter of \( \lambda = 1600 \) earlier when we regressed Okun’s law, that was because we were using quarterly data. We are now at the monthly frequency and have thus updated our smoothing parameter appropriately.

\(^{13}\)This is following Shimer (2005).
Incorporating the parameter and steady-state values into the job vacancy cost equation gives

$$ \hat{v}_t = (2.179) \hat{w}_t - (0.258) \hat{\tau}_t. $$

The implied series is given in figure 6 below.

![Time series for model and data-consistent percentage change in vacancy posting cost](image)

Figure 4.6: Implied series (in percentage deviation) for the firm’s implicit cost to posting a vacancy.

We can draw three conclusions from observing figure 6. First, outside of recessions, the cost to posting jobs seems (relatively) stable and that during recessions, this parameter exhibits behavior which parallels a regime shift. Second, the cost facing firms to financing a job opening increased during the onset of the Great Recession to a max of 15% at its conclusion. This is a value 3 times as great as the increase in the implied vacancy posting cost during the early 2000s recession. Last, figure 6 exhibits a striking similarity with the many different “financial stress” indices provided by various Federal Reserve banks. For example, figure 7 displays the financial stress index constructed by the St. Louis Fed laid upon our series for v-hat, which may offer some credence to the
linkage between the increase in the idiosyncratic productivity level of new labor matches resulting from increased “scrutiny” brought on by (increasingly) risk-averse firms facing heightened levels of financial stress negatively impacting their ability to finance labor costs. The two series share a correlation coefficient of 0.52 (p-value of 0.000 under the null of no correlation). If we lag the St. Louis financial stress index by 5 months, the correlation coefficient jumps to 0.75 (p-value of 0.000 under the null of no correlation).

4.4 Conclusion

Using a simple variation of the canonical DMP model of the labor market, we have illustrated two primary results. The first is that consistent with the model and empirical data, the idiosyncratic threshold productivity levels for the surviving and newly-created employment matches increased to a much greater degree (slightly more than double) during the Great Recession in comparison to the early 2000s recession.
The second is that the increase in the “scrutiny” imposed by hiring firms is largely the result of the increase in risk aversion originating from the large disruptions to credit markets during the wake of the Great Recession.

This chapter can be extended in a couple of ways in order to provide more support for our hypothesis. First, the aggregate labor productivity series used can be normalized using TFP data in order to control for fluctuations in capital intensity. Second, it would be worth exploring various ways to distinguish between a shift in the vacancy posting cost $v$ and movements in the level parameter $\psi$ of the matching function. We leave these extensions for future work.
Bibliography


163


Appendix A

Appendix to chapter 2

A.1 Empirical appendix

A.1.1 Data sources

The data for this section of the chapter are easily accessible from the FRED database\(^1\) and come from the following series:

- \(u\): Unemployment (level); series ID UNEMPLOY; Unemployed, Thousands of Persons, Monthly, Seasonally Adjusted

- \(v\): Vacancies (level); series ID JTSJOL; Job Openings: Total Nonfarm, Level in Thousands, Monthly, Seasonally Adjusted

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\(^1\)The FRED database is a centralized source for many various data series; for example, many of these series come from the Bureau of Labor Statistics.
• $e$: Employees (level); series ID PAYEMS; All Employees: Total nonfarm, Thousands of Persons, Monthly, Seasonally Adjusted

• $u^a$: Short-term unemployed (level); series ID UEMPLT5; Number of Civilians Unemployed - Less Than 5 Weeks, Thousands of Persons, Monthly, Seasonally Adjusted

• $y$: Aggregate average labor productivity; series ID PRS85006163; Nonfarm Business Sector: Real Output Per Person, Index 2009=100, Quarterly, Seasonally Adjusted

• $y^m$: Average labor productivity in the manufacturing sector; series ID PRS30006163; Manufacturing Sector: Real Output Per Person, Index 2009=100, Quarterly, Seasonally Adjusted

The series for the job-finding and separation rates are constructed using the above series and equations (I1) and (I2).

The series for the renter’s bargaining power was constructed using the following data files, available from Zillow:

• Sales-to-list price ratio:

  http://files.zillowstatic.com/research/public/State/State_SalePriceToListRatio_AllHomes.csv

• Price-to-rent ratio:
A.1.2 Correlation tables: why use manufacturing output instead of aggregate non-business output

In Shimer (2005), the series for aggregate output \( y \) is used as part of the empirical analysis in order to incorporate labor productivity as one of the important labor market variables. Using this same series under the JOLTS dates leads to the correlation table below.

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<th></th>
<th>( u )</th>
<th>( v )</th>
<th>( v/u )</th>
<th>( p' )</th>
<th>( \sigma^E )</th>
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<td>0.132</td>
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<td>0.120</td>
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<td>0.301</td>
<td>0.466</td>
</tr>
<tr>
<td>( v )</td>
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<td>0.976</td>
<td>0.816</td>
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</tr>
<tr>
<td>Correlation matrix</td>
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<td>—</td>
<td>1</td>
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<tr>
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Table A.1: Labor Market Summary Statistics, Monthly U.S. Data, 2000 - 2014, using aggregate output \( y \) instead of output from the manufacturing sector. All variables are expressed as log deviations from an HP filter trend with smoothing parameter \( \lambda = 10^5 \).

A fairly significant problem posed here is the correlation between output and all other important labor market variables, excluding separations. The signs on these correlations not only go against economic intuition, but the mechanics of the DMP framework, the model that this chapter is trying to build on. For comparison, below is Shimer’s version of the same table.

Here we see that labor productivity/output comoves correctly (in line with
### Table A.2: Labor Market Summary Statistics sourced from Shimer (2005). All variables are expressed as log deviations from an HP filter trend with smoothing parameter $\lambda = 10^5$.

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The table is a much closer match to Shimer’s benchmark not only in sign, but in volatility. For this reason, I have chosen to use the series for output from the manufacturing sector rather than the aggregate series.
A.2  Model appendix I: Derivation of surplus sharing rules

The optimization problem for the bargaining over the joint surplus to labor matches is

$$\max_{w^*} (V^F)^{1-\eta} (V^W)^{\eta}.$$ 

The first order condition for this problem is

$$(1-\eta)(V^F)^{-\eta} \frac{\partial V^F}{\partial w} (V^W)^{\eta} + (V^F)^{1-\eta} \eta (V^W)^{\eta-1} \frac{\partial V^W}{\partial w} = 0.$$ 

Rearranging this expression and substituting in the partial derivatives leads to the traditional sharing rule

$$\eta V^F = (1-\eta) V^{AE}.$$ \hspace{1cm} (M15) 

For the bargaining between the agent and the landlord, $1-\bar{\eta}$ will be the bargaining weight of the landlord while $\bar{\eta}$ will represent the bargaining weight of the agent. Then, following the same steps used to derive (M15), the sharing rule for the housing surplus is

$$\bar{\eta} V^L = (1-\bar{\eta}) V^{AH}.$$ \hspace{1cm} (M16) 

A.3  Model tuning appendix

A.3.1  Parameterization of unemployment benefits $b$

Many of the labor departments for each state of the union contain an unemployment benefits calculator which is posted to the state’s .gov page. If one wants to know what unemployment benefits they would receive, you enter the highest salary you
were receiving within the year prior to becoming unemployed, and the calculator gives you a weekly benefit amount you will receive. Thus if an individual was earning $50,000 per annum prior to becoming unemployed, you enter the quarterly amount into the calculator ($12,500) and the calculator would give you a weekly amount in benefit. To get the unemployment benefit ratio, I would then take the weekly amount and multiply by 52 and divide this total by the yearly salary the hypothetical agent was earning prior to unemployment. The table below provides some of the examples I used.

<table>
<thead>
<tr>
<th>State</th>
<th>Salary &amp; b #1</th>
<th>Salary &amp; b #2</th>
<th>Benefit ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>$40K; $20,020</td>
<td>$30K; $15,015</td>
<td>0.5005</td>
</tr>
<tr>
<td>Texas</td>
<td>$40K; $20,800</td>
<td>$28K; $14,560</td>
<td>0.5200</td>
</tr>
<tr>
<td>New Jersey</td>
<td>$50K; $29,952</td>
<td>$25K; $14,976</td>
<td>0.5990</td>
</tr>
<tr>
<td>Colorado</td>
<td>$30K; $17,992</td>
<td>$20K; $11,960</td>
<td>0.5997</td>
</tr>
<tr>
<td>Illinois</td>
<td>$40K; $18,720</td>
<td>$20K; $9,360</td>
<td>0.4680</td>
</tr>
</tbody>
</table>

Table A.4: Examples of unemployment benefit - wage ratios for various states.

A.3.2 Parameterization of renter separation $\sigma^H$

The data for the parameterization of $\sigma^H$ were taken from “Table A-4. Geographical Mobility by Tenure: 1988-2013” sourced from U.S. Census Bureau, Current Population Survey; Internet Release Date: November 2013. The table gives a historical time series for renters which were separated from their rentals during the year, but which had resided in that specific rental unit for at least one year prior to the separation to control for high mobility renters. This series then had to be corrected to take out the proportion of movers who moved for employment reasons, in order to isolate housing separations, solely. In the model, a labor separation $\sigma^E$ results in an auto-
matic housing separation, thus leaving housing separations due to employment reasons in the series for $\sigma^H$ would constitute double-counting. “Table A-5 Reason for Move (All Categories): 1999-2012” sourced from U.S. Census Bureau, Current Population Survey; Internet Release Date: November 2013 was used to isolate housing separations due to employment.

Figure 1 below plots the behavior of the resultant series for $\sigma^H$ over the 1998 - 2012 period.

![Figure A.1](image)

Figure A.1: Source: U.S. Census Bureau, Current Population Survey. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.

What we can notice from the figure is the fairly steady, downward trend of rental separations over the entire period. The annual average over the 1998 - 2012 period is 24.28736688%, which when adjusted to monthly results in $\sigma^H = 0.020239472$. Alternatively, this can be taken as a monthly compounded rate, so that

$$\sigma^H = \left[ \left(1 + \frac{24.3}{100} \right)^{\frac{1}{12}} - 1 \right] \times 100 \approx 1.83\%$$

In the numerical analysis, I choose the linear monthly proportion.
A.3.3 Rent-to-income ratio

The following table lists the median rent as a share of median income in a sample of 20 large and dispersed U.S. cities.

<table>
<thead>
<tr>
<th>City</th>
<th>Rent:Income 2000</th>
<th>Rent:Income 2013</th>
<th>Linear average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles</td>
<td>34.1</td>
<td>47</td>
<td>40.55</td>
</tr>
<tr>
<td>Miami</td>
<td>26.5</td>
<td>43.2</td>
<td>34.85</td>
</tr>
<tr>
<td>College Station, TX</td>
<td>24.4</td>
<td>41.8</td>
<td>33.1</td>
</tr>
<tr>
<td>Santa Cruz, CA</td>
<td>26.2</td>
<td>41.6</td>
<td>33.9</td>
</tr>
<tr>
<td>San Diego</td>
<td>30.9</td>
<td>41.4</td>
<td>36.15</td>
</tr>
<tr>
<td>San Francisco</td>
<td>24.7</td>
<td>40.7</td>
<td>32.7</td>
</tr>
<tr>
<td>Salinas, CA</td>
<td>29.8</td>
<td>40.6</td>
<td>35.2</td>
</tr>
<tr>
<td>San Luis Obispo, CA</td>
<td>34.1</td>
<td>40.5</td>
<td>37.3</td>
</tr>
<tr>
<td>Santa Rosa, CA</td>
<td>24.1</td>
<td>39.8</td>
<td>31.95</td>
</tr>
<tr>
<td>New York</td>
<td>23.7</td>
<td>39.5</td>
<td>31.6</td>
</tr>
<tr>
<td>Ithaca, New York</td>
<td>34.3</td>
<td>38.6</td>
<td>36.45</td>
</tr>
<tr>
<td>Napa, CA</td>
<td>24.8</td>
<td>38.5</td>
<td>31.65</td>
</tr>
<tr>
<td>Flagstaff, AZ</td>
<td>27.6</td>
<td>37.8</td>
<td>32.7</td>
</tr>
<tr>
<td>Punta Gorda, FL</td>
<td>26.2</td>
<td>37.7</td>
<td>31.95</td>
</tr>
<tr>
<td>Boulder, CO</td>
<td>22.6</td>
<td>37.2</td>
<td>29.9</td>
</tr>
<tr>
<td>Honolulu, HI</td>
<td>27.6</td>
<td>36.9</td>
<td>32.25</td>
</tr>
<tr>
<td>Santa Barbara, CA</td>
<td>32.9</td>
<td>36.9</td>
<td>34.9</td>
</tr>
<tr>
<td>Prescott, AZ</td>
<td>27.7</td>
<td>36.7</td>
<td>32.2</td>
</tr>
<tr>
<td>Ocean City, NJ</td>
<td>33.3</td>
<td>35.9</td>
<td>34.6</td>
</tr>
<tr>
<td>Hattiesburg, MS</td>
<td>19.9</td>
<td>35.8</td>
<td>27.85</td>
</tr>
</tbody>
</table>


A.3.4 Persistence parameters on the processes for $\hat{y}$, $\hat{\sigma}^E$ and $\hat{\eta}$

Following much of the literature\(^2\) I model the process for $\hat{y}_t$ as an AR(1) process. The estimate for $\hat{y}_t$ is below.

AR(1) model for exogenous output, using observations 2000:12–2014:03 ($T = 160$)

\(^2\)For example, see Petrosky-Nadeu & Wasmer (2013)
Dependent variable: $\hat{y}$

Standard errors based on Hessian

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>$z$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_y$</td>
<td>0.976209</td>
<td>0.0142428</td>
<td>68.5405</td>
</tr>
</tbody>
</table>

Mean dependent var $-0.000296$ S.D. dependent var $0.020175$
Mean of innovations $-0.000046$ S.D. of innovations $0.003992$
Log-likelihood $655.1826$ Akaike criterion $-1306.365$
Schwarz criterion $-1300.215$ Hannan–Quinn $-1303.868$

<table>
<thead>
<tr>
<th>AR</th>
<th>Real</th>
<th>Imaginary</th>
<th>Modulus</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root 1</td>
<td>1.0244</td>
<td>0.0000</td>
<td>1.0244</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The result is a near unit-root process. Figure 2 shows a plot of the the model fit versus the actual data, illustrating that an AR(1) model is parsimonious.

Figure A.2: Time series of the series for the log-deviation of output and the model fit for this process. Line widths for the series have been lightened to illustrate the extent of the fit.

The second persistence parameter I will empirically justify is the process dictating labor match separations. Once again, I will model the process which dictates the log-deviation of labor separations as an AR(1) process. The estimation for the process is below
AR(1) model for exogenous labor match separations, using observations

2000:12–2014:03 ($T = 160$)

Dependent variable: $\hat{\sigma}^E$

Standard errors based on Hessian

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{\sigma^E}$</td>
<td>0.451278</td>
<td>0.0707398</td>
<td>6.3794</td>
</tr>
</tbody>
</table>

Mean dependent var $-0.002344$  S.D. dependent var $0.062349$

Mean of innovations $-0.001247$  S.D. of innovations $0.055518$

Log-likelihood $235.4241$  Akaike criterion $-466.8482$

Schwarz criterion $-460.6979$  Hannan–Quinn $-464.3508$

<table>
<thead>
<tr>
<th>AR</th>
<th>Real</th>
<th>Imaginary</th>
<th>Modulus</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root 1</td>
<td>2.2159</td>
<td>0.0000</td>
<td>2.2159</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Modelling this process as an AR(1) results in a powerful estimate for the coefficient.

Figure 3 shows a plot of the the model fit versus the actual data, illustrating that the AR(1) model captures much of the series.

![Figure A.3: Time series of the log-deviation of labor match separations and the model fit for this process.](image-url)
Finally, the third persistence parameter I will empirically justify is the process dictating the bargaining weight of the renter. As before, I model the process which dictates the log-deviation of renter bargaining weight as an AR(1) process. The estimation for the process is below:

AR(1) model for renter’s bargaining weight, using observations 2010:11–2014:03 ($T = 41$)

Dependent variable: $\hat{\eta}$

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>$z$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.985016</td>
<td>0.0191744</td>
<td>51.3715</td>
</tr>
<tr>
<td>Mean dependent var</td>
<td>−0.000659</td>
<td>S.D. dependent var</td>
<td>0.034842</td>
</tr>
<tr>
<td>Mean of innovations</td>
<td>−0.001245</td>
<td>S.D. of innovations</td>
<td>0.008738</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>134.40999</td>
<td>Akaike criterion</td>
<td>−264.8198</td>
</tr>
<tr>
<td>Schwarz criterion</td>
<td>−261.3927</td>
<td>Hannan–Quinn</td>
<td>−263.5718</td>
</tr>
</tbody>
</table>

Real  | Imaginary | Modulus  | Frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AR Root 1</td>
<td>1.0152</td>
<td>0.0000</td>
<td>1.0152</td>
</tr>
</tbody>
</table>

Figure 4 shows a plot of the model fit versus the actual data.

### A.3.5 Rental vacancy rate

The rental vacancy rate is defined as the stock of rental units which are vacant and ready to be occupied divided by the total stock of rentals. Figure 5 below gives the time series behavior of the rental vacancy rate at a quarterly frequency beginning in the first quarter of 1990 and spanning to the first quarter of 2014.

For the steady state target $r_{\nu} = 0.095$, I take the average rental vacancy rate.
Figure A.4: Time series of the empirical log-deviation of the renter’s bargaining weight and the model fit for this process over the 2000Q3 through 2014Q1 period.

A.4 Numerical appendix

Figures 6, 7, and 8 show the impulse response functions for the three experiments carried out in the numerical section. Additionally, the blue impulse responses show the model’s response with housing, while the black impulse responses show the model’s response with housing shut off.

A.5 Estimation appendix

Figures 9 and 10 show the priors and estimated posteriors for the parameters discussed in the section on Bayesian estimation.
Figure A.5: Source: U.S. Department of Commerce: Census Bureau. Trend is an HP filter with smoothing parameter $\lambda = 10^5$.

Figure A.6: Impulse responses for key labor market variables following an orthogonalized shock to labor productivity. Red (dashed) indicates a model response without housing, while black indicates a model response with housing.
Figure A.7: Impulse responses for key labor market variables following an orthogonalized shock to labor match separations. Red (dashed) indicates a model response without housing, while black indicates a model response with housing.

Figure A.8: Impulse responses for key labor market variables following an orthogonalized shock to labor bargaining weight. Red (dashed) indicates a model response without housing, while black indicates a model response with housing.
Figure A.9: Prior and estimated posterior distributions for parameters which underwent the Bayesian estimation

Figure A.10: Prior and estimated posterior distribution for persistence on the bargaining power of labor
Appendix B

Appendix to chapter 3

B.1 Linear approximation of the model

1. Real supply of reserves

\[ \tilde{h}^s_p = \tilde{h}^s_p - 1 + \tilde{\theta}_t - \tilde{\pi}_t \]  \hspace{1cm} (D1)

2. Policy rate

\[ \hat{i}_t = \hat{i}_t^s \]  \hspace{1cm} (D2)

3. Euler equation

\[ \tilde{\lambda}_t = \tilde{\lambda}_{t+1} + \beta_{t+1} - \hat{\pi}_{t+1} \]  \hspace{1cm} (D3)

4. Marginal utility of income

\[ \tilde{\lambda}_t = -\sigma \tilde{C}_t + \left( \frac{1}{1 + \hat{i}_t^b - \hat{i}_t^d} \right) \left( \hat{\pi}_t^d - \hat{\pi}_t^b \right) \]  \hspace{1cm} (D4)
5. The collateral constraint multiplier

\[ \hat{\alpha}_t = \hat{\beta}_t^b + \xi_b \hat{\chi}_t \] (D5)

6. Interest rate on deposits

\[ \hat{\gamma}^d_t = (1 - \rho) \hat{\gamma}_t + \rho \hat{\gamma}_t^s - \rho \hat{s}_t - \hat{\chi} \sigma_t + \frac{(h^s)^2}{2\pi} \hat{s}_t + s \left( \frac{(h^s)^2}{2\pi} \right) \left( 2\hat{\rho}_t - \frac{\sigma}{2} \right) \hat{\gamma}_t - \frac{\sigma}{2} \hat{s}_t - \frac{s \sigma}{2} \] (D6)

7. Interbank market equilibrium

\[ \hat{h}^s_t = \left( \frac{h^s d}{h^p} \right) \hat{\alpha}_t + \hat{d}_t \] (D7)

8. The demand for excess reserves

\[ \hat{h}^s_t = \hat{z}_t - \hat{s}_t + \left( \frac{1}{i^* - i + \chi} \right) \left( \hat{\gamma}_t - \hat{\gamma}_t^s + \hat{\chi}_t \right) \] (D8)

9. Consolidated gov. budget constraint

\[ \hat{h}^s_t = \left( \frac{1}{1 + \pi} \right) \left( \hat{h}^s_{t-1} - \frac{\hat{s}_t}{s} \right) \left( \frac{b^c}{h^p} \right) \left( \hat{\gamma}_t^c - \left( \frac{1}{1 + \pi} \right) \left( \hat{b}^c_{t-1} - \hat{\pi}_t \right) \right) + \left( \frac{b^c}{h^p} \right) \hat{\gamma}_t^c + \left( \frac{b^c}{h^p} \right) \hat{b}^c_t = \left( \frac{x}{h^p} \right) \hat{x}_t \] (D9)

10. Central bank’s net payment of interest on reserves

\[ \hat{x}_t = \left( \frac{\rho d + e r}{x} \right) \left( \hat{\gamma}_t - \hat{s}_t \right) + \left( \frac{\rho d (i^* - s)}{x} \right) \hat{d}_t + \left( \frac{e r (i^* - s)}{x} \right) \hat{e}_t + \left( \frac{b r}{x} \right) \left( \hat{\gamma}_t^c + \hat{s}_t + (i^* + s) \hat{\beta}_t \right) \] (D10)

11. Aggregate excess reserves

\[ \frac{e r}{d} \hat{e}_t = \frac{e r}{d} \hat{d}_t + \frac{h^s}{2} \left( \frac{h^s + \sigma}{\sigma} \right) \hat{h}_t^s + \frac{1}{4} \left( \frac{\sigma - (h^s)^2}{\sigma} \right) \hat{x}_t \] (D11)
12. Aggregate borrowed reserves

\[ \frac{br}{d} \tilde{b}t = \frac{br}{d} \tilde{d}t - h^* \left( \frac{h^* - \varepsilon}{\varepsilon} \right) \tilde{h}^*_t + \left( \frac{(h^*)^2 - \varepsilon^2}{4\varepsilon} \right) \tilde{\varepsilon}_t \]  

(D12)

13. Aggregate collateral constraint

\[ h^* \tilde{h}^*_t + \left( \xi b \right) \tilde{b}^*_t + \left( \xi L \right) \tilde{l}_t = \varepsilon \tilde{\varepsilon}_t + \left( \varepsilon + \xi b^s - h^* \right) \tilde{d}_t \]  

(D13)

14. Aggregate banks balance sheet

\[ \left( \frac{l}{d} \right) \tilde{d}_t + \left( \frac{b}{d} \right) \tilde{b}^*_t + h^* \tilde{h}^*_t = (1 - \rho - h^*) \tilde{d}_t \]  

(D14)

15. The CIA constraint

\[ \left( \frac{C}{d} \right) \tilde{C}_t = \tilde{d}_t + \left( \frac{wN}{d} \right) \left( \tilde{w}_t + \tilde{N}_t \right) \]  

(D15)

16. Labor supply

\[ \eta \tilde{N}_t + \sigma \tilde{C}_t = \tilde{w}_t \]  

(D16)

17. Aggregate loans

\[ \tilde{l}_t = \tilde{w}_t + \tilde{N}_t \]  

(D17)

18. Resource constraint

\[ \tilde{Y}_t = \left( \frac{C}{Y} \right) \tilde{C}_t + \left( \frac{\varphi(\tilde{\omega}) f^{m} x^{1}}{Y} \right) \left( \tilde{\varphi}_t (\tilde{\omega}_t) + \tilde{f}^{m}_{t-1} \right) + \left( \frac{\kappa b^u}{Y} \right) \tilde{b}^u_t \]  

(D18)

19. Opportunity cost of lending

\[ \tilde{R}_t = \tilde{t}_t - \xi L \tilde{\lambda}_t \]  

(D19)
20. Aggregate employment

\[ \tilde{N}_t = \frac{1}{1-\alpha} \left( \tilde{z}_t - \tilde{\omega}_t - \tilde{R}_t \right) + \tilde{F}_t \]  
\[ \text{(D20)} \]

21. Aggregate output

\[ \tilde{Y}_t = \tilde{z}_t + (1-\alpha) \tilde{F}_t + \alpha \tilde{N}_t \]  
\[ \text{(D21)} \]

22. Credit market tightness

\[ \tilde{\tau}_t = \tilde{f}_t - \tilde{b}_t \]  
\[ \text{(D22)} \]

23. Number of firms in a credit relationship

\[ \tilde{f}_m = \varphi(\tilde{\omega}) \left( \tilde{\varphi}_t (\tilde{\omega}_t) + \tilde{f}_{m-1} \right) + \left( \frac{p^{lf}}{\tilde{f}_m} \right) \left( \tilde{p}^l + \tilde{f}_t \right) \]  
\[ \text{(D23)} \]

24. Number of firms searching for workers

\[ \tilde{f}_t = - \left( \frac{(1-\delta) f_m}{f} \right) \tilde{f}_{t-1} + \left( \frac{\delta f_m}{f} \right) \tilde{\delta}_t \]  
\[ \text{(D24)} \]

25. Continuation rate

\[ \varphi(\tilde{\omega}) \tilde{\varphi}_t (\tilde{\omega}_t) = - \left( \frac{(1-\delta) \tilde{\omega}}{\tilde{\omega} - \omega} \right) \tilde{\omega}_t - \delta \left( \frac{\tilde{\omega} - \tilde{\omega}}{\tilde{\omega} - \omega} \right) \tilde{\delta}_t \]  
\[ \text{(D25)} \]

26. \( F_t \)

\[ \tilde{F}_t = \tilde{f}_m - \left( \frac{\delta}{1-\delta} \right) \tilde{\delta}_t - \left( \frac{k (\tilde{\omega})^k}{(\tilde{\omega})^k - (\tilde{\omega})^k} \right) \tilde{\omega}_t \]  
\[ \text{(D26)} \]

27. Cut-off productivity level

\[ \alpha^{\frac{1}{\gamma}} \tilde{\omega}^{\frac{1}{\gamma}} \left( \frac{z \tilde{\omega}}{(wR)^\alpha} \right)^{\frac{1}{\gamma}} \left( \tilde{z}_t + \tilde{\omega}_t - \alpha \tilde{\omega}_t \tilde{R}_t \right) = \]  
\[ = \left( \frac{1}{1 - \eta p^f} \right) \left( 1 - \eta p^f \right) \tilde{p}_t + \eta p^f \tilde{p}_t \]  
\[ \text{(D27)} \]
28. Credit market tightness

\[-\frac{\kappa}{p_b} p_b^b - \frac{\beta \kappa \varphi (\bar{\omega})}{p_b} E_t \left( \bar{\Delta}_{t,t+1} + \bar{\varphi}_{t+1} (\bar{\omega}_{t+1}) - \bar{p}_{t+1}^b \right) + \]

\[+ \frac{\eta \beta \kappa \varphi (\bar{\omega})}{p_b} p_f^f E_t \left( \frac{\bar{p}_{t+1}^f}{p_b} \left( \bar{\Delta}_{t,t+1} + \bar{\varphi}_{t+1} (\bar{\omega}_{t+1}) + \bar{p}_{t+1}^f - \bar{p}_{t+1}^b \right) \right) \]

\[= (1 - \eta) (1 - \alpha) \beta Y_{ft} E_t \left( \bar{\Delta}_{t,t+1} + \bar{\varphi}_{t+1} (\bar{\omega}_{t+1}) \right) \]  

\[- \frac{\eta \beta \kappa \varphi (\bar{\omega})}{p_b} p_f^f E_t \left( \bar{p}_{t+1}^f + \bar{\varphi}_{t+1} (\bar{\omega}_{t+1}) \right) \]  

(D28)

29. Stochastic discount factor

\[\bar{\Delta}_{t,t+1} = \bar{\lambda}_{t+1} - \bar{\lambda}_t \]  

(D29)

30. Credit destruction rate

\[\bar{c}_d_t = -\left( \frac{\varphi (\bar{\omega})}{cd} \right) \bar{\varphi}_t (\bar{\omega}_t) - \left( \frac{p_f \delta}{cd} \right) \left( \bar{p}_t^f + \bar{\delta}_t \right) \]  

(D30)

31. Matching rates

\[\bar{p}_t^f = \bar{\mu}_t + (\varphi - 1) \bar{\tau}_t \]  

(D31)

and

\[\bar{p}_t^b = \bar{\mu}_t + \varphi \bar{\tau}_t \]  

(D32)

32. Aggregate productivity

\[\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_t^z \]  

(D33)

33. Exogenous separation rate

\[\tilde{\delta}_t = \rho_\delta \tilde{\delta}_{t-1} + \epsilon_t^\delta \]  

(D34)
34. Growth rate of nominal supply of reserves

\[ \tilde{\theta}_t - \theta = \rho_\theta \tilde{\theta}_{t-1} + \epsilon^\theta_t \]  
(D35)

35. Payment shock

\[ \tilde{\varepsilon}_t = \rho_{\varepsilon} \tilde{\varepsilon}_{t-1} + \tilde{\varepsilon}_t \]  
(D36)

36. Matching efficiency shock

\[ \tilde{\mu}_t = \rho_{\mu} \tilde{\mu}_{t-1} + \epsilon^\mu_t \]  
(D37)

37. Width of the corridor

\[ \tilde{s}_t = \rho_s \tilde{s}_{t-1} + \epsilon^s_t \]  
(D38)
Appendix C

Appendix to chapter 4

C.0.0.1 Derivation of the equilibrium wage rate

The sharing rule is

$$(1 - \eta)^{\mathcal{V}^F \Sigma} = \eta^{\mathcal{V}^W \Sigma}.$$  

Substituting in the surpluses,

$$(1 - \eta) \left( y_t - w_t + \frac{v}{p_t^F} \right) = \eta \left[ w_t - b + (1 - p_t^W) \left( \frac{1 - \eta}{\eta} \right) \frac{v}{p_t^F} \right].$$

Solving for $w_t$ gives the result

$$w_t = \eta b + (1 - \eta) \left[ y_t + v \left( \frac{p_t^W}{p_t^F} \right) \right].$$