Title
Performance analysis of multi-antenna OFDM systems with phase noise and imperfect channel estimation

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Performance Analysis of Multi-Antenna OFDM Systems with Phase Noise and Imperfect Channel Estimation

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy

in

Electrical Engineering
(Communication Theory and Systems)

by

Mohamed Jalloh

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Professor Rene Cruz
Professor Philip Gill
Professor William Hodkgiss

2009
The dissertation of Mohamed Jalloh is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Co-Chair

Chair

University of California, San Diego

2009
To my entire family
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<td>Adaptive Modulation and Coding</td>
</tr>
<tr>
<td>AMPS</td>
<td>Advanced Mobile Phone Services</td>
</tr>
<tr>
<td>AAS</td>
<td>Adaptive Antenna Selection</td>
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<tr>
<td>ARQ</td>
<td>Automatic repeat Request</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<tr>
<td>BLAST</td>
<td>Bell Labs Layered Space-Time</td>
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<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<tr>
<td>BTC</td>
<td>Block Turbo Coding</td>
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<tr>
<td>BS</td>
<td>base Station</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CEE</td>
<td>Channel Estimation Error</td>
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<tr>
<td>CIR</td>
<td>Channel Impulse response</td>
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<tr>
<td>CLT</td>
<td>Central Limit Theorem</td>
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<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
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<tr>
<td>CPE</td>
<td>Common Phase Error</td>
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<tr>
<td>CSI</td>
<td>Channel State Information</td>
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<tr>
<td>CTC</td>
<td>Convolutional Turbo Coding</td>
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<tr>
<td>D-AMPS</td>
<td>Digital Advanced Mobile Phone Services</td>
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<tr>
<td>DAB</td>
<td>Digital Audio Broadcast</td>
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<tr>
<td>D-BLAST</td>
<td>Diagonal-Bell Labs Layered Space-Time</td>
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<tr>
<td>DFS</td>
<td>Dynamic Frequency Selection</td>
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<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<td>DL</td>
<td>DownLink</td>
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<tr>
<td>DSL</td>
<td>Digital Subscriber Line</td>
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<td>DVB-T</td>
<td>Terrestrial Digital Video Broadcast</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>EGC</td>
<td>Equal Gain Combining</td>
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<tr>
<td>EVDO</td>
<td>Evolution-Data Optimized</td>
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<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
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<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
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<tr>
<td>FDD</td>
<td>Frequency Division Duplexing</td>
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<tr>
<td>FEC</td>
<td>Forward Error Correction</td>
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<tr>
<td>FFT</td>
<td>fast Fourier Transform</td>
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<tr>
<td>FLASH-OFDM</td>
<td>fast Low Latency Access with Seamless Handoff - OFDM</td>
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<tr>
<td>GSM</td>
<td>Global System for Mobile Communications</td>
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<tr>
<td>GCOD</td>
<td>Generalized Complex Orthogonal Designs</td>
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<td>GROD</td>
<td>Generalized Real Orthogonal Designs</td>
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<td>3GPP</td>
<td>Third Generation Partnership Project</td>
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<td>HSDPA</td>
<td>High Speed Downlink Packet Access</td>
</tr>
<tr>
<td>HO</td>
<td>Handover</td>
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<tr>
<td>ICE</td>
<td>Imperfect Channel estimation</td>
</tr>
<tr>
<td>ICI</td>
<td>Intercarrier Interference</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronic Engineers</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fourier Transform</td>
</tr>
<tr>
<td>IP</td>
<td>Internet Protocol</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-symbol Interference</td>
</tr>
<tr>
<td>Kbps</td>
<td>Kilobits per second</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-Density Parity Check Codes</td>
</tr>
<tr>
<td>LMMSE</td>
<td>Linear Minimum Mean Square Error</td>
</tr>
<tr>
<td>LPF</td>
<td>Low Pass Filter</td>
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<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
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<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<tr>
<td>LoS</td>
<td>Line of Sight</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>MAC</td>
<td>Media Access Control</td>
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<tr>
<td>MANet</td>
<td>Mobile Adhoc Network</td>
</tr>
<tr>
<td>MBWA</td>
<td>Mobile Broadband Wireless Access</td>
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<tr>
<td>MC</td>
<td>Multi-Carrier</td>
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<tr>
<td>MIMO</td>
<td>Multiple Input, Multiple Output</td>
</tr>
<tr>
<td>Mbps</td>
<td>Megabits per second</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple Input, Single Output</td>
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<tr>
<td>MRC</td>
<td>Maximal Ratio Combining</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>MSH</td>
<td>Mesh Network Configuration</td>
</tr>
<tr>
<td>NLoS</td>
<td>Non-Line of Sight</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<tr>
<td>OSTBC</td>
<td>Orthogonal Space Time Block Codes</td>
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<tr>
<td>PD</td>
<td>Phase Detector</td>
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<tr>
<td>PDC</td>
<td>Pacific Digital Cellular</td>
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<tr>
<td>PHY</td>
<td>Physical Layer</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>PN</td>
<td>Phase Noise</td>
</tr>
<tr>
<td>PSC</td>
<td>Power Saving Class</td>
</tr>
<tr>
<td>Q</td>
<td>Quality Factor</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RX</td>
<td>Receiver</td>
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</table>
Sc Single carrier
SC Selection Combining
SDMA Spatial Division Multiple Access
SER Symbol Error Rate
SIMO Single Input, Multiple Output
SINR Signal to Interference plus Noise Ratio
SNR Signal to Noise Ratio
STBC Space Time Block Codes
STC Space Time Codes
STTC Space Time Trellis Codes

TDMA Time Division Multiple Access
TDD Time Division Duplexing
TX Transmitter

UL UpLink
UMTS Universal Mobile Telecommunication Systems
UWB Ultra Wideband

V-BLAST Vertical Bell Labs Layered Space-Time
VCO Voltage Controlled Oscillator

WCDMA Wideband Code Division Multiple Access
WiBro Wireless Broadband
WiMAX Worldwide Interoperability for Microwave Access
WiFi Wireless Fidelity
WLAN Wireless Local Area Network
WMAN Wireless Metropolitan Area Network
WPAN Wireless Personal Area Network
LIST OF SYMBOLS

\( (\cdot)^* \) Complex conjugate Operator
\( (\cdot)^H \) Hermitian Operator
\( [A] \) Largest integer less than \( A \)

\( \alpha(t) \) Random Time Shift

\( \beta \) Intercarrier Interference (ICI)
\( \tilde{\beta} \) ICI after Channel Normalization
\( \tilde{\beta}^e \) ICI with Channel Estimation Error
\( B(t) \) Wiener Process

\( C \) Capacity of system

\( D \) Desired Signal Component
\( \hat{D} \) Desired Signal Component after Channel Normalization
\( \delta \) Dirac Delta Function
\( \varepsilon \) Error in Channel Estimate
\( \Delta f_{3dB} \) 3dB Bandwidth of Phase Noise
\( \Delta f_{carr} \) Subcarrier Spacing
$f_{\text{corner}}$  Corner Frequency of Oscillator
$f_{c}$  Carrier Frequency

$\Gamma$  Gamma Function; a Generalization of the Factorial Notation
$\gamma$  Signal to Noise Ratio (SNR)
$N(0, 1)$  Zero-mean Unit Variance Gaussian Random Process

$h$  Channel Impulse Response
$H$  Channel Frequency Response
$\hat{H}$  Estimate of $H$
$\mathbf{H}$  Channel Matrix
$\mathbf{H}^H$  Hermitian of $\mathbf{H}$

$I_{N \times N}$  $N \times N$ Identity Matrix
$I_0(\alpha)$  Zeroth-order Bessel Function of 1st kind

$L$  Inductance of Inductor
$LC$  Inductor-Capacitor

$\nu$  Degrees of Freedom for Chi Square Distributed Random Variable
$n_R$  Number of Receive Antennas
$n_T$  Number of Transmit Antennas

$\Omega$  Covariance Matrix
$\omega_0$  Oscillator Operating Frequency
$\Delta \omega$  Offset Frequency from Operating Frequency

$\Pi$  Product Operator
$\pi$  3.14285714285714 ...
$P_{SB}$  Power Spectral Density
$P_{\text{sig}}$  Signal Power
$P_e$  Probability of Error
$P_e^c$  Probability of Error with Channel Estimation error
Quality Factor

Quality Factor of Capacitor

Quality Factor of Inductor

Resistance of Resistor

Noise term due to Channel Estimation Error

Noise term after Channel Normalization

Summation Operator

Variance of $\alpha$

Variance of $\theta$

Variance of $|H|$

Variance of $W$

Variance of $\beta$

Useful OFDM symbol time

OFDM symbol guard time (cyclic prefix)

Total OFDM symbol time

Phase Noise

Common Phase Error Term (CPE)

Combiner weighting Factor on $i^{th}$ MRC Branch

Additive Gaussian Noise (AWGN)

Frequency domain Gaussian Noise

Vector-valued Gaussian Noise

$W$ with Phase Noise

$W$ with Phase Noise and Channel Estimation Error

Hermitian of $W$

Transmitted Symbol

Complex conjugate of $X$

Detected Symbol

Detected Symbol with Channel Estimation Error

Impedance of a component or circuit
ACKNOWLEDGEMENTS

I wish to express my deepest and sincere gratitude to my advisor, Prof. Pankaj Das, for his continuous encouragement and guidance, and for believing in me. Both technical and personal discussions with him facilitated the successful completion of my research.

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The combination of all the above serves as a recipe for the much anticipated and successful outcome of "Mission Accomplished!"

Chapter 4, in full, is a reprint of the material as it appears in "Performance of OFDM systems in Rayleigh fading channels with Phase Noise and Channel Estimation Errors", M. Jalloh, M. Al-Gharabally and P. Das, Proc. IEEE Military
Communications Conference, pp. 1-7, Oct 2006. The dissertation author was the primary researcher and author, and the co-authors listed in this publication directed and supervised the research which forms the basis for the chapter.

Chapter 5, in full, is an extension of the material that appeared in ”BER Analysis of MRC-OFDM Receiver with RF Impairments and Imperfect Channel State Information”, M. Jalloh and P. Das, IEEE Personal Indoor and Mobile Radio Conference’ pp. 1-6, Sep 2008. The dissertation author was the primary researcher and author, and the co-author listed in this publication directed and supervised the research which forms the basis for the chapter.

Chapter 6, in full, is a reprint of the material as it appears in the following publications: ”Effect of Phase Noise and Imperfect Channel Estimation on Coded OFDM Transmit Diversity with Application to IEEE 802.16 ”, M. Jalloh and P. Das, Proc. IEEE Military Communications Conference’ pp. 1-5, Nov 2008. ”Performance Analysis of STBC-OFDM with RF Impairments and Channel Estimation Errors “, IEEE Vehicular Technology Conference’, M. Jalloh and P. Das, VTC Spring 2009, pp. 11-15 , Apr 2009. The dissertation author was the primary researcher and author, and the co-author listed in these publications directed and supervised the research which forms the basis for the chapter.
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PUBLICATIONS


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ABSTRACT OF THE DISSERTATION

Performance Analysis of Multi-Antenna OFDM Systems with Phase Noise and Imperfect Channel Estimation

by

Mohamed Jalloh
Doctor of Philosophy in Electrical Engineering (Communication Theory and Systems)
University of California San Diego, 2009

Professor Pankaj Das, Chair
Professor Lawrence Milstein, Co-Chair

The age of wireless technologies and the associated convenience that portable wireless products and services provide, continues to drive the need for more advanced products and broadband wireless services that demand carefully selected spectrally efficient modulation schemes to support the desired higher data rates. The development of multicarrier systems has paved the way to meeting the system requirements and fulfilling these needs.

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation technique that has provided effective means for achieving high data rates and spectral efficiency requirements in wireless communication systems, while making use of relatively simple receiver designs.

In wireless communications, one of the major implementation challenges is system sensitivity to synchronization issues, which is even more pronounced in multicarrier systems such as OFDM. The use of OFDM with a high number of subcarriers, to achieve the high data rates it provides, makes it more susceptible to these non-idealities such as phase noise. Another critically limiting factor of system performance is the introduction of channel estimation errors as a consequence of imperfect channel estimation at the receiver. Accurate channel estimates are very essential in ensuring
correct detection of the transmitted signal at the receiver. A thorough understanding of system operation therefore requires a detailed analysis of the effects of all the impairments.

This dissertation addresses these system impairments by investigating the effects analytically on OFDM systems, deriving bit error rate (BER) expressions for systems that employ a single antenna at both the transmitter and receiver. The investigation is extended to also include OFDM systems that employ multi-antenna configurations at the receiver followed by a similar multi-antenna configuration at the transmitter. The impact of these negative effects is further investigated when applied to a practical system, comparing the analytical results with simulations that make use of system parameters of the IEEE 802.16 standards.
1

Introduction

The experienced growth in the utilization of digital networks has led to the need for the design of new communication networks with higher capacity. The telecommunication industry is also continuously evolving, with a demand for a greater range of services, such as video conferencing, or applications with rich multimedia contents. The increased reliance on computer networking and the Internet has resulted in a wider demand for connectivity to be provided *any where, any time*, leading to a rise in the requirements for higher capacity and high reliability broadband wireless telecommunication systems. Broadband availability brings high performance connectivity to over a billion users worldwide, thus developing new wireless broadband standards and deploying the technologies that will rapidly span the needed wireless coverage. Wireless digital communications are an emerging field that has experienced an unprecedented growth over the last decade. Moreover, the huge uptake rate of cellular technology, WLAN (Wireless Local Area Network) and the exponential growth of Internet have resulted in a significant increase in demand for new methods of obtaining high capacity wireless networks [1], [2], [3], [4], [5].

Cellular communication systems are often categorized as different generations depending on the services offered. The *first generation* comprises the analog Frequency-Division Multiple Access (FDMA) systems such as the Advanced Mobile Phone Services (AMPS). The *second generation* consists of the first digital cellular communication systems such as the Time Division Multiple Access (TDMA) based GSM (Global System for Mobile Communication), D-AMPS (Digital AMPS) PDC
(Pacific Digital Cellular) and the Code Division Multiple Access (CDMA) based system IS-95. These systems offer mainly speech/voice communication, but with data communication limited to very low transmission rates. The *third generation* is characterized by enhanced data communication capabilities providing data rates of at least 384 kbits/s over a wide area and includes the Wideband-CDMA (WCDMA) system and the CDMA-2000 system.

Currently, there is an active ongoing worldwide effort to develop the requirements for the next generation mobile wireless systems, also known as forth-generation (4G).

Among the many technologies being proposed for (4G) currently under development, there is uncertainty in the technology of choice that will emerge and celebrate the wide scale commercial deployment. However, recent projections seem to indicate that the following technologies are among the top contenders:

WirelessMAN: The IEEE 802.16 wireless metropolitan area network (WirelessMAN) standard, also known as WiMAX (*Worldwide Interoperability for Microwave Access*) is designed to deliver high speed wireless data over distances that exceeds the 802.11 WiFi standard, and to be able to support different network configurations such as point to point links as well as cellular networks.

Flash-OFDM: Fast low-latency access with seamless handoff orthogonal frequency division multiplexing (FLASH-OFDM) is developed by Qualcomm-Flarion to provide high data rate mobile wireless internet services to consumers using an IP (internet protocol) based mobile cellular network configuration.

3GPP LTE: 3GPP long term evolution (3GPP LTE) is a project that is intended to upgrade third generation UMTS (universal mobile telecommunications system) networks to forth generation (4G).

MBWA: The IEEE 802.20 mobile broadband wireless access (MBWA) standard, also known as MobileFi aims to provide a packet-based air interface that is designed for IP-based services, and is being designed to support full mobile wireless broadband network configurations. It is expected that WMAN will support mobile speeds up to 250 km/hr.

*Worldwide Interoperability for Microwave Access*, known as WiMAX, is a wireless networking standard which aims to address interoperability across IEEE
802.16 standard-based products. WiMAX defines a Wireless Metropolitan Area Network (WMAN), a kind of huge hot-spot that provides interoperable broadband wireless connectivity to fixed, portable, and mobile users. It facilitates communications in non-Line of site environments, emerging as an attractive connection alternative for cable, DSL, and T1/E1 systems, as well as a possible transport network for Wi-Fi hot-spots. It is thus becoming a solution to develop broadband industry platforms. Furthermore, products based on WiMAX technology can be integrated with other technologies to offer broadband access in many other possible scenarios.

WiMAX will likely substitute other broadband technologies competing in the same segment, and will become an excellent solution for the deployment of the well-known last mile infrastructures in markets very difficult to access with other technologies, such as cable or DSL, and where the costs of deployment and maintenance of such technologies will not be profitable. As such, WiMAX will connect rural areas in developing countries as well as underserved metropolitan areas. It can also be deployed to deliver backhaul for carrier structures, enterprise campus, and Wi-Fi hot-spots. WiMAX offers a good solution to these challenges because it provides a cost-effective, rapidly deployable solution.

Additionally, WiMAX potentially represents a very credible competitor to 3G cellular systems as high speed mobile data applications will be achieved with the IEEE 802.16e specification.

The diagram in Fig-1.1 attempts to capture the evolution and convergence of wireless communications over the last two decades, including cellular networks, Wireless Local Area Networks (WLANs), and emerging Wireless Personal Area Networks (WPANs). It is apparent that the future of wireless communications is leading to the seamless interoperability of once disjoint technologies.

### 1.1 Digital Communication Systems

A digital communication system is often divided into several functional blocks as shown in the transmitter and receiver blocks of Figure-1.2 and Figure-1.3 respectively.

The task of the source encoder is to represent the analog or digital information
Figure 1.1 Evolution of Wireless Communication

Figure 1.2 Transmitter Block Diagram for Digital Communication System
by bits in an efficient way. The bits are then fed to the channel encoder, which add extra bits in a predefined pattern to enable detection and correction of transmission errors. The bits from the encoder are grouped and mapped to certain symbols (or waveforms) by the modulator, and these waveforms are combined with a carrier signal to obtain a desired signal suitable for transmission through the transmission medium, typically referred in the literature as channel.

At the receiver, the reverse functions occur as shown in Figure-1.3. The received symbols are demodulated and the soft (or hard) values of the corresponding bits are passed to the decoder. The decoder analyzes the structure of the received bit pattern and tries to detect or correct errors. Finally, the corrected bits are fed to the source decoder that is used to reconstruct the speech signal or data input.

These transmitter and receiver structures described above constitute the fundamental building blocks of wireless communication systems. Systems that are designed for broadband applications with high data rates requirements, such as multicarrier communication systems, make use of the appropriate arrangement of these blocks to achieve the design criteria. OFDM, a multicarrier communication system, is an example that employs a parallel combination of this transmitter and receiver to achieve the high data rates it offers.

1.2 OFDM in Digital Communication Systems

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique that has become very popular in broadband multicarrier communication systems. The principles of Orthogonal Frequency Division Multiplexing (OFDM) modulation have been in existence since the mid 1960’s, when Chang published his paper on
the synthesis of bandlimited signals for multichannel transmission [6]. He presented a principle for transmitting messages simultaneously through a linear bandlimited channel without interchannel interference (ICI) and intersymbol interference (ISI). Shortly after Chang’s work, Saltzberg presented his work [7] on system performance where he concluded that for an efficient parallel system design, the emphasis should be placed on reducing crosstalk (or ICI) between adjacent channels as it is the dominant factor in system performance. A major contribution to OFDM was presented in 1971 by Weinstein and Ebert [8], who used the discrete fourier transform (DFT) to perform baseband modulation and demodulation, thereby introducing efficient processing, and thus eliminating the need for banks of subcarrier oscillators. To combat ISI and ICI they used both a guard space between the symbols and raised-cosine windowing in the time domain. Another significant contribution was due to Peled and Ruiz in 1980 [9], who introduced the cyclic prefix, solving the subcarrier orthogonality problem.

In the last several years, OFDM modulation has emerged as a key modulation technique of commercial high speed communication systems. The principal reason for this increasing interest is due to its capability to provide high-speed data rate transmissions with low implementation complexity and to combat the effects of intersymbol interference (ISI) introduced by the dispersive nature of the wireless channels. For this reason OFDM modulation has been adopted by several digital wireline and wireless communication standards, such as the European digital audio and video broadcasting standards, as well as local area networks such as IEEE802.11, possible next generation cellular system IEEE802.16, IEEE802.20. Nevertheless, the implementation of OFDM systems with a large number of subcarriers has some significant disadvantages. A major drawback is its high sensitivity to synchronization nonidealities between the transmitter and receiver oscillators such as carrier frequency offset and phase noise [3]-[8]. Specifically, incorrect timing synchronization can cause interference between successive symbols and, if not perfectly compensated before the equalization process, can lead to a severe system performance degradation. In addition, these effects induce an amplitude reduction of the useful signal and resulting in intercarrier interference (ICI) between adjacent subcarriers [10], [11], [12], [13].

Several studies have investigated the estimation of system parameter for OFDM
systems, such as channel quality, carrier frequency offset for signal detection. These estimates are frequently based on data-aided and non data-aided (blind) techniques in order to cancel or minimize the ICI caused by the effects of synchronization errors [14], [15], [16], [17], [18], [19]. The data-aided case requires the transmission of known sequences or the use of a training symbol with a known structure, and is very popular due to its relative ease of implementation.

1.3 Non-Idealities in OFDM

With all these advantages notwithstanding, the performance of OFDM systems is limited by synchronization issues such as carrier frequency offset and hardware imperfections such as phase noise. Frequency offset is typically arises due to a frequency deviation between the transmitter and receiver, while phase noise results from time-variations of the inherent noise and losses in oscillator circuits. The phase noise destroys the orthogonality between subcarriers, leading to the introduction of intercarrier interference. An OFDM system that is free of non-ideal effects maintains the orthogonality of subcarriers as illustrated in Figure-(1.4A). However, due to practical limitations in design or manufacturing process, the phase noise renders the system imperfect, leading to ICI as shown in Figure-(1.4B).

The performance of the system is further plagued by design limitations such as Peak to Average Power Ratio (PAPR) at the transmitter. A peak in the transmitted signal power occurs when all or most of the subcarriers are aligned in phase and combine constructively. This implies the larger the number of subcarriers, the higher the PAPR due to the constructive addition of the transmitted signal power of all the subcarriers. Another performance limiting factor in OFDM is the effect of doppler shifts under mobility. The channel is rapidly changing in mobile conditions, and as a result, effective channel estimation becomes more challenging. A permutation of these effects can significantly degrade OFDM system performance.

In this thesis, the effects of phase noise and channel estimation errors, due to imperfect channel estimation, in OFDM systems is further examined. We first consider the effect of phase noise and channel estimation errors in single antenna systems succeeded by configurations employing multi-antennas at both the transmitter
and/or the receiver.

The presented analysis with numerical simulation results, illustrates that in multipath channels, the effects of these impairments present significant performance challenges that need to be addressed for robust system design and implementation.
1.4 Thesis Organization

The organization of this thesis is as follows:

• Chapter 2 provides a general review of the IEEE802.16 standard.

• In Chapter 3, an overview of the theory and characterization of phase noise is presented, including phase noise models and the corresponding distributions of these models.

• Chapter 4 describes the effect of phase noise in single-Transmit, single-Receive antenna OFDM systems, with imperfect channel estimation.

• Chapter 5 describes the effect of phase noise in single-Transmit and multi-Receive antenna OFDM systems employing Maximal Ratio Combining, in the presence of imperfect channel estimation.

• Chapter 6 describes the effect of phase noise in multi-Transmit and single-Receive antenna OFDM systems employing Space-Time Block codes, with imperfect channel estimation.

• Finally, the concluding remarks and technical contributions are summarized in Chapter 7.
2

IEEE 802.16 Standard

2.1 Introduction

In the midst of the explosive growth in the demand for high data rate wireless communication, high speed and content-rich multimedia applications and internet access in the residential sector, coupled with an equally fast paced growth in the last-mile wireless access technologies, Broadband Wireless Access (BWA) has emerged as a promising solution. This increasing demand has driven a rapid growth in communications technologies, and gave birth to different wireless technologies that, with no doubt, continues to make a tremendous impact on our day to day lives. The healthy growth in broadband technology is expected to continue steadily over the coming years, providing wireless communications operators the incentives to offer broadband services to populations that would otherwise find themselves out of the broadband loop.

Some of the most prominent wireless technologies include the digital audio broadcasting (DAB), terrestrial digital video broadcasting (DVB-T) and wireless local area networks (WLAN). All of these systems use orthogonal frequency division multiplexing (OFDM) as their modulation format.

Currently, there is an active ongoing worldwide effort to develop the requirements for the next generation cellular wireless systems, also known as forth-generation (4G). Since 4G is currently under development, it is not quite evident which of the
many proposed technologies, will emerge and continue to dominate wide scale commercial deployment. However, recent projections signal that WirelessMAN, Flash-OFDM, 3GPP LTE, and MBWA technologies described in Chapter-1, are among the top competitors.

Regardless of which one of the above systems will prevail, a common theme in all the above technologies is that they use orthogonal frequency division multiplexing (OFDM) as the standard modulation scheme.

Due to the above compelling reasons of OFDM’s unanimous acceptance and its potential ubiquitous deployment, this thesis is concerned with system performance evaluation, and the impact of the negative effects that plague OFDM systems. In addition to the effect of imperfect channel estimation, we consider the impact of RF impairments such as carrier frequency offset and phase noise, on the performance of OFDM systems.

2.1.1 Brief History of OFDM

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation scheme that is best suited for high data rate transmission over time dispersive (frequency selective) channels [20], [21], [22].

OFDM converts a high rate data stream into a number of low rate streams that are transmitted over parallel, frequency flat channels. This gives OFDM the ability to effectively combat the effects of channel frequency selectivity such as intersymbol interference (ISI).

The concept of using parallel data transmission and frequency division multiplexing was developed as early as the mid-1960s [6]. A U.S. patent for OFDM was filed on November 14, 1966, and the patent was issued on January 6, 1970 [23], [24]. In 1971, Weinstein and Ebert were the first to apply the discrete Fourier transform (to replace the bank of oscillators) to parallel data transmission systems as part of the modulation and demodulation process [8]. In 1985, Cimini was the first to suggest
using OFDM for wireless communications [25]. But it was only in the early 1990s that advances in hardware for digital signal processing made OFDM a realistic option for wireless systems, and the digital audio broadcasting (DAB) standard became the first commercial OFDM based wireless system in 1995. The following list summarizes the evolution of OFDM in chronological order [26]

- 1958: Kineplex a military multicarrier high frequency communication system [27].
- 1966: R. W. Chang at Bell labs described the concept of using parallel data transmission and a U.S. patent was filed.
- 1970: Patent was issued.
- 1971: Weinstein and Ebert were the first to apply the discrete Fourier transform to parallel data transmission systems as part of the modulation and demodulation process.
- 1985: Cimini described the use of OFDM for mobile communications [25].
- 2002: IEEE 802.11g standard for wireless LAN.
- 2005: IEEE 802.16e standard for Mobile WMANs called Mobile WiMAX
2.2 Overview of the IEEE 802.16 Standard

The IEEE 802.16 standard for wireless metropolitan area networks (WirelessMAN), also known as WiMAX (Worldwide Interoperability for Microwave Access) is a wireless transmission system that falls into the category known as broadband wireless access (BWA) [30]. The objective of WiMAX is to provide high speed wireless connectivity between the consumer premises and the network backhaul, and can be considered as a cost effective solution to the last mile problem currently being addressed by cable or digital subscriber line (DSL) solutions that suffer from time-consuming deployment and correspondingly high maintenance costs.

In order for WiMAX to be able to serve larger residential and commercial areas where LoS is not always guaranteed, an amendment to the 802.16 standard was made. The IEEE 802.16 standard was initially designed to address communications with direct visibility in the frequency band from 10 to 66 GHz. Due to the fact that non-line-of-sight transmissions are difficult when communicating at high frequencies, the amendment 802.16a was specified for working in a lower frequency band, between 2 and 11 GHz. The IEEE 802.16d specification is a variation of the fixed standard (IEEE 802.16a) with the main advantage of optimizing the power consumption of the cellular products. The subsequent revision of this specification is better known as IEEE 802.16-2004 [3].

In 2004, the final version of the WiMAX standard was published [29]. The IEEE 802.16-2004 standard document included both the specifications of the 802.16 and the 802.16a documents, i.e., both the single carrier physical layer operating in the 10-66 GHz band with Line of site (LoS) and the multicarrier (OFDM) physical layer operating in the 2-11 GHz band with non-Line of site (NLoS) were included in the final document. The OFDM physical layer is also divided into two modes of operation, namely, the single-user 256 subcarriers OFDM and the multi-user 1024, 2048 subcarriers Orthogonal Frequency Division Multiple Access (OFDMA).

1Most of the discussion in Section 2.2 is obtained from the IEEE 802.16-2004 standard document [29]
On the other hand, the IEEE 802.16e standard is an amendment to the 802.16-2004 base specification with the aim of targeting the mobile market by adding portability. WiMAX standard-based products are designed to work not only with IEEE 802.16-2004 but also with the IEEE 802.16e specification. While the 802.16-2004 is primarily intended for stationary transmission, the 802.16e is oriented to both stationary and mobile deployments.

Figure 2.1 illustrates the evolution of the development of the IEEE 802.16 standard.

![Figure 2.1 Evolution of the IEEE 802.16 Standard](image)

**2.3 IEEE 802.16-2004**

A electromagnetic wave in the 10-66 GHz frequency range is a focused line-of-sight (LoS) beam, which theoretically can cover long distances through LoS propagation. Designers deemed that single-carrier modulation was a sufficient choice and the physical layer standard version of this band is called WirelessMAN-SC (single carrier). WirelessMAN-SC can support frequency division duplex (FDD) and time division duplex (TDD) modes. However, operation in the 2-11 GHz band required changes in the physical layer specification to support NLoS propagation.

Mainly, three new PHYsical layer (PHY) specifications were introduced to meet this requirement. A single-carrier PHY, a 256-point FFT OFDM PHY, and the
The IEEE 802.16-2004 standard defines three different PHYs that can be used in conjunction with the MAC layer to provide a reliable end-to-end link. These PHY specifications are:

- A single carrier (SC) modulated air interface.
- A 256-point FFT OFDM multiplexing scheme.
- A 1024-point FFT, 2048-point FFT OFDMA schemes (10MHz, 20MHz bandwidth respectively).

While the SC air interface is used for line-of-sight (LoS) transmissions, the two OFDM-based systems are more suitable for non line-of-sight (NLoS) operations due to the simplicity of the equalization process for multicarrier signals. The fixed WiMAX standard defines system profiles using the 256-point FFT OFDM PHY layer specification. Furthermore, fixed WiMAX systems provide up to 5 km of service area allowing transmissions with a maximum data rate up to 70 Mbps in a 20 MHz channel bandwidth, and offer users a broadband connectivity without needing a direct line-of-sight to the base station.

### 2.3.1 Description and Features

The main features of the mentioned fixed WiMAX include:

- Use of an OFDM modulation scheme, which allows the simultaneous transmission of multiple signals using different subcarriers simultaneously. Because the OFDM waveform is composed of multiple narrowband orthogonal carriers, selective fading is localized to a subset of carriers that are relatively easy to equalize.

- Design of an adaptive modulation and coding mechanism that depends on channel and interference conditions. It adjusts the modulation method almost in-
stantaneously for optimum data transfer, thus making a most efficient use of the bandwidth.

- Support of both time and frequency division duplexing formats, FDD and TDD, allowing the system to be adapted to the regulations in different countries.

- Robust FEC (Forward Error Correction) techniques, used to detect and correct errors in order to improve throughput. The FEC scheme is implemented with a Reed-Solomon encoder concatenated with a convolutional code, and followed by an interleaver. Optional support of block turbo coding (BTC) and convolutional turbo coding (CTC) can be implemented.

- Use of flexible channel bandwidths, comprised from 1.25MHz to 20 MHz, thus providing the necessary flexibility to operate in many different frequency bands with varying channel requirements around the world. This flexibility facilitates transmissions over longer distances and from different types of subscriber platforms. In addition, it is also crucial for cell planning, especially in the licensed spectrum.

- Optional support of both transmit and receive diversity to enhance performance in fading environments through spatial diversity, allowing the system to increase capacity. The transmitter implements Space-Time coding (STC) to provide transmit source independence, reducing the fade margin requirement and combating interference. The receiver however, uses Maximal Ratio Combining (MRC) techniques to maximize the received SNR, and improve overall system performance.

- Design of a dynamic frequency selection (DFS) mechanism to minimize signal interference. With this feature, the transmission frequency can be chosen based on predicted channel conditions.

- Optional support of smart antennas, whose beams can be steered in desired directions of the receiver. Consequently, it avoids interference between adjacent channels, and increases the spectral density and the SNR. There are two basic types of smart antennas, those with multiple beam (directional antennas), and
those known as adaptive antenna systems (AAS). The first ones can use either a fixed number of beams choosing the most suitable for the transmission or a steering beam to the desired antenna. The second type works with multi-element antennas with a varying beam pattern. These smart antennas are becoming a good alternative for BWA deployments.

- Implementation of channel quality measurements which help in the selection and assignment of the adaptive antenna profiles.

- Support of both time and frequency division multiplexing formats (TDM and FDM), to allow interoperability between cellular systems working with TDM, and wireless systems that use FDM.

The single-carrier PHY, designated as WirelessMAN-SCa, is based on the Wireless MAN-SC specification. The second and third PHY specifications employ orthogonal frequency division multiplexing (OFDM) which is a multicarrier transmission technique suitable for high-speed NLoS. The key difference between Wireless MAN-SC and OFDM is that OFDM is more resilient to the multipath effect. OFDM has higher bandwidth (spectral) efficiency since it allows neighboring subcarriers to overlap. For instance, OFDM provides a spectral efficiency of 3.6 bps/Hz (WiMAX Forum, 2004) with modulated data rate of 72 Mbps over a channel bandwidth of 20 MHz.

Orthogonal frequency division multiple access (OFDMA) is a 2048 subcarrier OFDM scheme. The difference between OFDM and OFDMA is that OFDMA organizes the time (i.e., the symbols) and the frequency (i.e., subcarriers) resources into subchannels for allocation to individual receivers, which allows for multiple access. Thus, OFDMA operates over two dimensions, time and frequency. There are two types of subcarrier permutations, namely: subchannel-diversity and contiguous-diversity. The former permutation draws subcarriers pseudorandomly to form a subchannel. The contiguous permutation groups a block of contiguous subcarriers to form a subchannel. OFDM PHY is common in the IEEE802.16 because of its frequency synchronization requirements and efficient use of faster Fast Fourier Transform (FFT) calculation. Consequently, WiMAX Forum focuses on 256-carrier OFDM PHY in all
Table 2.1 802.16-2004 PHY features.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>256-point FFT OFDM</td>
<td>Simple equalization of multipath channels in LoS and NLoS environments</td>
</tr>
<tr>
<td>Adaptive modulation and variable error correction encoding per radio frequency (RF) burst</td>
<td>Ensures a robust RF link while maximizing number of bits per second per subscriber unit</td>
</tr>
<tr>
<td>TDD and FDD duplexing support</td>
<td>Addresses varying worldwide regulations when one or both may be allowed</td>
</tr>
<tr>
<td>Flexible channel sizes (1.25MHz to 20MHz)</td>
<td>Provides necessary flexibility to operate in many different frequency bands with varying requirements around the world</td>
</tr>
<tr>
<td>DFS support</td>
<td>Minimizes interference between adjacent channels</td>
</tr>
<tr>
<td>TDM and FDM support</td>
<td>Allows interoperability between cellular systems (TDM) and wireless systems (FDM)</td>
</tr>
<tr>
<td>Designed to support Multiple-Input Multiple-Output (MIMO) schemes</td>
<td>Implemented in DL to increase diversity and capacity. STC algorithms at transmitter, MRC at receiver</td>
</tr>
</tbody>
</table>
its profiles, instead of the single-carrier modulated air-interface. Furthermore, OFDM and OFDMA support NLoS performance making maximum use of the available spectrum.

The PHY layer also has other features, some of which are mandatory while the others are optional. These features empower the performance of the technology to provide robust performance over a wide range of frequencies and under different channel conditions.

**Adaptive Antenna System**

The Adaptive antenna system uses multiple antennas at both the receiver and the transmitter to form a *Multi Input Multi Output* (MIMO) system. This increases channel capacity by steering the antenna beams toward multiple users to achieve in-cell frequency reuse. MIMO system is also beneficial in increasing the signal-to-interference plus noise (SINR) ratio through coherently combining multiple signals [31]. Another benefit of AAS is the reduction in the required power due to utilization of beams formed by the adaptive antennas.

**Adaptive modulation**

IEEE 802.16-2004 allows for different modulation schemes in the downlink and the uplink of the communication system, i.e., BPSK, QPSK, 16QAM, 64QAM, and 256QAM. The 802.16 standard defines different combinations of the aforementioned modulation schemes and coding rates, providing for a wide range of trade-offs of data rate and robustness depending on channel conditions.

The IEEE802.16 standard employs Reed-Solomon block code with an inner convolution code or Turbo coding. The latter is left as an optional feature.

**Space-Time coding**

Space-Time coding is an optional feature of IEEE802.16 that can be used in downlink communication to provide for Space Transmit diversity. Space-Time coding assumes that the base station is using two transmit antennas and the subscriber station uses one transmit antenna.
2.4 IEEE 802.16e-2005

Due to the inevitable need for mobility support in BWA, a study group called IEEE 802.16 Mobile WirelessMAN Task Group was initiated to produce an amendment covering the PHY and MAC layers for combined, fixed, and mobile operations in the licensed band range. The amendment was approved in December 2005 and the new standard called IEEE 802.16e-2005 was published in 2006. The scope of this standard is to provide mobility enhancement support for users moving at the vehicular speed, in addition to corrections to 802.16-2004 fixed operation that was developed as IEEE 802.16-2004 and published along with IEEE 802.16e-2005. IEEE 802.16e introduces some changes to PHY and MAC layer protocols owing to mobility support, which required addressing new issues that were not required in 802.16-2004, such as handoff and power management, better support for Quality of Service (QoS) and the use of scalable OFDMA, and is sometimes called ”Mobile Wimax”.

2.4.1 Description and Features

The mobile WiMAX (IEEE 802.16e) uses the 2048-point FFT OFDMA PHY specification. It provides a service area coverage from 1.6 to 5 km, allowing transmission rates of 5 Mbps in a 5 MHz channel bandwidth, and with a user maximum speed below 100 km/h. It presents the same features as those of the fixed WiMAX specification that have been already mentioned. However, other features such as handoffs and power-saving mechanisms are added to offer a reliable communication. Battery life and handoff are two critical issues for mobile applications. On one hand, maximizing battery life implies minimizing the mobile station (MS) power usage. On the other hand, handoff and handovers are necessary to enable the MS to switch from one BS to another at vehicular speeds without interrupting the connection. The main features of the initial IEEE 802.16 standard, and those of the so-called fixed and mobile WiMAX, 802.16-2004 and 802.16e respectively, are summarized in Table 2.2.
Table 2.2 The IEEE 802.16 standards Family

<table>
<thead>
<tr>
<th></th>
<th>802.16</th>
<th>802.16-2004</th>
<th>802.16e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrum</td>
<td>10-66GHz</td>
<td>2-11GHz</td>
<td>2-6GHz</td>
</tr>
<tr>
<td>Maximum Data Rate</td>
<td>32-134Mbps</td>
<td>up to 70Mbps</td>
<td>up to 15Mbps</td>
</tr>
<tr>
<td>(28MHz channel)</td>
<td>(20MHz channel)</td>
<td>(5 MHz channel)</td>
<td></td>
</tr>
<tr>
<td>Alignment</td>
<td>LoS</td>
<td>LoS and NLoS</td>
<td>LoS and NLoS</td>
</tr>
<tr>
<td>Coverage Range</td>
<td>2-5 km approx.</td>
<td>5-10 km approx.</td>
<td>2-5 km approx.</td>
</tr>
<tr>
<td>Channel Bandwidth</td>
<td>20, 25 and 28 MHz</td>
<td>Flexible, from 1.25 up to 20 MHz</td>
<td>Equal to 802.16-2004</td>
</tr>
<tr>
<td>Modulation</td>
<td>2-PAM, 4-QAM, 16-QAM, and 64-QAM</td>
<td>OFDM with 256 subcarriers, 2-PAM, 4-QAM, 16-QAM, 64-QAM</td>
<td>OFDMA with 1024, 2048 subcarriers, 2-PAM, 4-QAM, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td>Mobility</td>
<td>Fixed</td>
<td>Fixed and Pedestrian</td>
<td>Vehicular (20-100 km/h)</td>
</tr>
</tbody>
</table>

2.4.2 Changes and Additional Features

The discussion below includes the changes to the PHY layer introduced by IEEE 802.16e-2005:

- IEEE802.16e operation is limited to licensed bands suitable for mobility below 6 GHz. This may introduce a compatibility problem between 802.16-2004 and 802.16e, since the available licensed spectrum may need to be split between the two technologies.

- IEEE802.16e defines a new PHY air interface called scalable-OFDMA (SOFDMA). Besides those defined by IEEE802.16-2004, S-OFDMA uses FFT size of 128, 512, 1024, or 2048 subcarriers. S-OFDMA employs these numbers of subcarriers to provide the ability to scale system bandwidth while at the same time keeping the subcarrier separation and symbol duration constant as the system bandwidth changes. The base station (BS) determines the subcarrier(s) used to adapt to the varying channel conditions.
• The AAS, Space-Time code, and closed-loop MIMO modes are enhanced in IEEE802.16e to improve coverage and data transmission rate. Additionally, support for coordinated spatial division multiple access (SDMA) is introduced.

• IEEE802.16e includes an additional advanced low complexity coding option method, low-density parity check (LDPC) to provide for more flexible encoding. LDPC codes 6 bits for every 5 data bits with a rate of 5/6. This provides higher-performance coding technique than the methods included in 802.16-2004 that provide 3/4 code rate.

The IEEE 802.16e system called Mobile WiMAX is standardized to add user mobility to the original IEEE802.16 system (WiMAX). Since mobility causes a number of problems and results in added design requirements in wireless systems, a mobile station (MS) and a base station (BS) in the mobile WiMAX system need to introduce several mobility-supporting functions to the existing WiMAX system in order to support user mobility. Terminals in mobile environments rely on portable power sources such as batteries. It is therefore very crucial for mobile terminals to be equipped with efficient power-saving features. On this basis, the IEEE 802.16e system also provides a sleep mode operation that provides efficient power-saving mechanisms and take into account the traffic attributes of various application services. Furthermore, the IEEE802.16e supports seamless handover (HO) because an MS may move out of the coverage range of the current BS due to its mobility. Hence, to maintain a seamless service connection, the handover (HO) function enables the MS to have unlimited mobility and continuity of service. It is therefore, a very important function in wireless cellular networks.

The IEEE802.16e system provides not only a basic HO function to support MS mobility, but also various techniques that enhance HO performance.

The system also supports location-based updates by broadcasting a paging message at pre-determined times. The location updates enables mobile stations (MSs) to inform the network of their location.

The wireless system also supports paging scheme that allows MSs to operate in two modes: active mode and idle mode. If there is no traffic to or from an MS for a given period, the MS is allowed to change its mode to idle. In idle mode, the MS
does not have to maintain the connection with the network and performs location update less frequently, since there is no need for the location of the MS to be traced precisely. Therefore, the MS can reduce its battery power consumption and radio resources significantly, and the BS can eliminate unnecessary air interface and HO traffic as well.

To allow networks to take advantage of the benefits of paging and location updates, the IEEE802.16e system also makes use of the MSs idle mode operation to support such function. The main mobility functions of IEEE802.16e are captured in the MAC layer of the standard. These functions include paging and location updates, handover operation, and a power-saving mechanism.

**Power-Saving Mechanism**

The power-saving mechanism of IEEE802.16e enables mobile stations (MS)s to operate in one of the two operational modes: active mode and sleep mode. In active mode, MSs always power-up to communicate with their serving base station (BS), while in sleep mode they can power down to conserve energy during pre-negotiated intervals. In sleep mode, there are two operational time intervals: sleep window and listening window. MSs in sleep mode basically switch between the two windows. During a sleep window, MSs turn off most of the hardware to minimize power consumption. As a result, data cannot be received or transmitted. During a listening window, the MS synchronizes with the serving BS to receive traffic indication message.

The IEEE802.16e standard provides three kinds of power-saving mechanisms also referred to as power-saving class (PSC), which operate according to the characteristics of the traffic for various types of services (voice, data, video, etc). Each PSC uses a different operational mechanism and parameter set appropriate to the traffic characteristics. If an MS has multiple simultaneous service connections, each with different traffic characteristics, it can utilize several PSCs simultaneously. Each of the PSCs is appropriate for a different service connection.

The three types of power-saving mechanisms are:

PSC-I: used for nonreal-time services and provides a truncated binary exponential algorithm to decide the size of the sleepwindow, which is suitable for services
with burst traffic attribute.

PSC-II: used for real-time services and provides periodic sleep and listening intervals, taking into account the traffic characteristics of real-time services.

PSC-III: used for multicast or management message transmission and provides an efficient sleep mechanism for aperiodic and continuous services.

**Handover**

The IEEE802.16e system provides an HO function to support the mobility of MS across all coverage areas. When the signal quality of the current BS degrades due to channel fading or interference due to mobility, the MS hands over to another BS that provides better signal quality and quality of service (QoS).

**Paging and Location Update**

The IEEE 802.16e system defines an MS idle mode to provide paging and location update mechanisms. The MS can be in idle mode when there is no traffic to/from the MS for a given period. Idle mode allows an MS to become periodically available for DL broadcast traffic messaging without registering with a specific BS while it traverses an air link environment consisting of multiple BSs. An MS in idle mode does not have to perform HO and can suspend all normal operation requirements. Hence, it can conserve power.

In summary, the main mobility functions defined in the IEEE 802.16e standard are power-saving mechanism, HO operation, and paging and location update.

- Power-saving mechanisms as noted above.
- Basic HO operation and enhanced mechanisms for fast and seamless HO.
- Paging and location update operations.

Figure 2.2 shows the building blocks that constitute the basic WiMAX transmitter. The transmitter consists of three major blocks, namely, the channel coding block, pilot and data modulation block and the inverse fast Fourier transform (IFFT) and guard insertion block. In Section 2.5, some basic parameters and definitions
regarding the physical layer are introduced. The channel coding and pilots/data modulation blocks are explained in Sections 2.5.1 and 2.5.3 respectively.

### 2.5 Physical Layer Parameters

The time domain OFDM symbol is simply defined as the output of the IFFT block. The duration of the time domain OFDM symbol is given by $T_b$ and is known as the useful symbol duration. In order to eliminate intersymbol interference (ISI) caused by the time dispersion of the channel, few samples from the end of the useful symbol are copied back to the beginning of the useful symbol to form a guard interval. The duration of the guard interval, denoted by $T_g$, is a fraction of the useful symbol duration and is usually chosen to be no less than the channel delay spread. The IEEE802.16 standard document specifies $1/4$, $1/8$, $1/16$ and $1/32$ for the fraction of the guard duration to the useful symbol duration. The overall OFDM symbol duration is then equal to the sum of the useful duration and the guard duration, i.e, $T_s = T_b + T_g$.

Because the guard interval constitutes an overhead, the transmitter energy increases with the length of the guard interval while the receiver energy remains the same (the guard interval is discarded at the receiver), so there is a $10 \log_{10}(1 - T_g/(T_b + T_g))$ dB loss in signal to noise ratio (SNR). Using a cyclic extension as guard interval also provides tolerance for symbol time synchronization errors as well as multipath immunity.

Figure-2.3 illustrates the structure of the time-domain OFDM symbol. In the frequency domain, an OFDM symbol is made up of many subcarriers, the number of subcarriers in each OFDM symbol will determine the size of the IFFT/FFT block.
There are three types of subcarriers in each OFDM symbol, namely

- Data subcarriers: subcarriers used to transmit user data.
- Pilot subcarriers: subcarriers used for channel estimation purposes.
- Null carriers: empty subcarriers for guard band and DC.

Figure-2.4 illustrates the distribution of the subcarriers in each OFDM symbol. In each OFDM symbol consisting of 256 subcarriers, a total of 55 subcarriers are designated as guard bands. There are 28 subcarriers in the lower frequency guard band and 27 subcarrier in the upper frequency guard band. The purpose of the guard bands (lower and upper frequency) is to enable the signal to naturally decay smoothly and create the FFT brick Wall shaping.

The IEEE802.16 standard document specified 8 subcarriers to be used for channel estimation purposes. These subcarriers are known as pilot subcarriers. If the index of the first, middle and last subcarriers in the OFDM symbol shown in
Figure-2.4 is -128, 0 and 128 respectively, then the index of the 8 pilot subcarriers is specified by the standard to be -84, -60, -36, -12, 12, 36, 60 and 84, with each subcarrier modulating a binary phase shift keying (BPSK) symbol.

In some cases, only part of the data subcarriers may be used by the transmitter, the other subcarriers of which may be intended for different (groups of) receivers. A set of subcarriers intended for one (group of) receiver(s) is termed a subchannel. The carriers forming one subchannel may, but need not be adjacent. This type of configuration is called orthogonal frequency division multiple access (OFDMA) and will not be considered in the thesis. Table 2.3 summarize the OFDM physical layer parameters as given by the standard [29]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFFT/FFT length</td>
<td>256</td>
</tr>
<tr>
<td>Number of used subcarriers (pilot+data)</td>
<td>200</td>
</tr>
<tr>
<td>$T_g/T_b$</td>
<td>1/4, 1/8, 1/16, 1/32</td>
</tr>
<tr>
<td>Lower frequency guard subcarriers</td>
<td>28 subcarriers</td>
</tr>
<tr>
<td>Upper frequency guard subcarriers</td>
<td>27 subcarriers</td>
</tr>
<tr>
<td>Index of lower guard subcarriers</td>
<td>-128, -127, ..., -101</td>
</tr>
<tr>
<td>Index of upper guard subcarriers</td>
<td>+101, +102, ..., +127</td>
</tr>
<tr>
<td>Index of pilot subcarriers</td>
<td>-84, -60, -36, -12, 12, 36, 60, 84</td>
</tr>
</tbody>
</table>

### 2.5.1 Channel Coding

Channel coding is composed of three steps: randomizer (scrambler), forward error correction (FEC) and interleaving. The steps are applied in this order at transmission. The complementary operations are similarly applied in reverse order at reception. A model of the channel coding block is shown in Figure-2.2

The randomization is performed on each transmitted OFDM symbol inde-
pendently. If the amount of data to transmit does not fit exactly the amount of data allocated (subcarriers in the frequency domain or samples in the time domain), padding of ones shall be added to the end of the transmission block, up to the amount of data allocated. The shift-register of the randomizer shall be initialized for each new allocation or for every 1250 bytes passed through (if the allocation is larger than 1250 bytes). The pseudo random binary sequence (PRBS) generator shall be $1 + X^{14} + X^{15}$.

Each data byte to be transmitted shall enter sequentially into the randomizer, most significant bit (MSB) first. The structure of the PRBS generator is shown in Figure 2.5. The bits issued from the randomizer shall be sent to the forward error correction block.

Forward error correction (FEC) consist of the concatenation of a Reed-Solomon (RS) outer code and a rate-compatible convolutional inner code. Support of block turbo codes (BTC) and convolutional turbo codes (CTC) is optional. The encoding is performed by first passing the data in block format through the RS encoder and then passing it through a zero-terminating convolutional encoder. The Reed Solomon encoding shall be derived from a systematic RS ($N=255$, $K=239$, $T=8$) code using $GF(2^8)$, where

- $N$: number of overall bytes after encoding.
- $K$: number of data bytes before encoding.
- $T$: number of data bytes which can be corrected.
The following polynomials are used for the systematic code

- Code generator polynomial: \( g(x) = (x + \lambda^0)(x + \lambda^1) \ldots (x + \lambda^{2^7-1}) \) \( \lambda = 0.2_{\text{hex}} \).
- Field generator polynomial: \( p(x) = x^8 + x^4 + x^3 + x^2 + x \).

Data bits generated by the RS encoder are fed to the convolutional encoder shown in Figure 2.6.

![Convolutional Encoder Block Diagram](image)

Figure 2.6 Convolutional Encoder Block Diagram

The convolutional code shall have a native rate of 1/2, a constraint length of 7. Puncturing patterns and serialization order that are used to realize different code rates are defined in Table 2.4 and in Table 2.5, the block sizes and the code rates for different modulations are listed. In table 2.4, a “1” means a transmitted bit and “0” denotes a removed bit, whereas X and Y are the outputs of the encoder.

### Table 2.4 The inner Convolutional code with puncturing configuration

<table>
<thead>
<tr>
<th>Rate</th>
<th>1/2</th>
<th>2/3</th>
<th>3/4</th>
<th>5/6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{free} )</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>X</td>
<td>1</td>
<td>10</td>
<td>101</td>
<td>10101</td>
</tr>
<tr>
<td>Y</td>
<td>1</td>
<td>11</td>
<td>110</td>
<td>11010</td>
</tr>
<tr>
<td>XY</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All encoded data bits shall be interleaved by a block interleaver with a block size corresponding to the number of coded bits, \( N_{cbps} \). The interleaver is defined by a two step permutation. The first ensures that adjacent coded bits are mapped onto nonadjacent carriers. The second permutation ensures that adjacent coded bits are mapped alternately onto less or more significant bits of the constellation, thus avoiding long runs of lowly reliable bits.


Table 2.5  Data block sizes, Coding Rates & Modulation in WiMAX

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Uncoded block size(byte)</th>
<th>Coded block size(byte)</th>
<th>Overall coding rate</th>
<th>RS code</th>
<th>CC code rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>12</td>
<td>24</td>
<td>1/2</td>
<td>(12,12,0)</td>
<td>1/2</td>
</tr>
<tr>
<td>QPSK</td>
<td>24</td>
<td>48</td>
<td>1/2</td>
<td>(32,24,4)</td>
<td>2/3</td>
</tr>
<tr>
<td>QPSK</td>
<td>36</td>
<td>48</td>
<td>3/4</td>
<td>(40,36,2)</td>
<td>5/6</td>
</tr>
<tr>
<td>16-QAM</td>
<td>48</td>
<td>96</td>
<td>1/2</td>
<td>(64,48,8)</td>
<td>2/3</td>
</tr>
<tr>
<td>16-QAM</td>
<td>72</td>
<td>96</td>
<td>3/4</td>
<td>(80,72,4)</td>
<td>5/6</td>
</tr>
<tr>
<td>64-QAM</td>
<td>96</td>
<td>144</td>
<td>2/3</td>
<td>(108,96,6)</td>
<td>3/4</td>
</tr>
<tr>
<td>64-QAM</td>
<td>108</td>
<td>144</td>
<td>3/4</td>
<td>(120,108,6)</td>
<td>5/6</td>
</tr>
</tbody>
</table>

Let $N_{cbc}$ be the number of coded bits per carrier, i.e., 2, 4 or 6 for QPSK, 16-QAM or 64-QAM, respectively. Let $s = N_{cpe}/2$, $k$ be the index of the coded bit before the first permutation at transmission, $m$ be the index after the first and before the second permutation and $j$ be the index after the second permutation just prior to modulation mapping. The first permutation is defined by the following rule

$$m = \left( \frac{N_{cbps}}{16} \right) \cdot k_{mod(16)} + \text{floor}\left( \frac{k}{16} \right) \quad k = 0, 1, \ldots, N_{cbps} - 1 \quad (2.1)$$

and the second permutation is

$$j = s \cdot \text{floor}\left( \frac{m}{s} \right) + (m + N_{cbps} - \text{floor}\left( \frac{16m}{N_{cbps}} \right))_{mod(s)} \quad m = 0, 1, \ldots, N_{cbps} - 1 \quad (2.2)$$

### 2.5.2 Interleaver

Data interleaving is generally used to scatter error bursts and thus, reduce the error concentration to be corrected with the purpose of increasing the efficiency of FEC by spreading burst errors introduced by the transmission channel over a longer time. Interleaving is normally implemented by using a two-dimensional array buffer, such that the data enters the buffer in rows, which specify the number of interleaving levels, and then, it is read out in columns. The result is that a burst of errors in the channel after interleaving becomes in few scarcely spaced single symbol errors, which are more easily correctable. WiMAX uses an interleaver that combines data using 12 interleaving levels. The effect of this process can be understood as a spreading of the bits of the different symbols, which are combined to get new symbols, with the same size but with rearranged bits.
To achieve equal average symbol power, the constellations described above are normalized by multiplying all of its points by an appropriate factor $C_m$. Values for this factor $C_m$ are given in Table 2.6.

Table 2.6 Modulation Normalization Factors

<table>
<thead>
<tr>
<th>Modulation scheme</th>
<th>Normalization constant for unit Average Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>$C_m = 1$</td>
</tr>
<tr>
<td>QPSK/4QAM</td>
<td>$C_m = 1/\sqrt{2}$</td>
</tr>
<tr>
<td>16QAM</td>
<td>$C_m = 1/\sqrt{10}$</td>
</tr>
<tr>
<td>64QAM</td>
<td>$C_m = 1/\sqrt{42}$</td>
</tr>
</tbody>
</table>

Furthermore, an adaptive modulation and coding mechanism is supported in the downlink with the purpose of allowing the number of transmitted bits per symbol to be varied depending on the channel conditions.

Table 2.7 Modulation Alphabet for Constellation Map

<table>
<thead>
<tr>
<th>Modulation Scheme</th>
<th>Symbol Alphabet</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>$A_{sym} = (1, -1)$</td>
</tr>
<tr>
<td>4QAM</td>
<td>$A_{sym} = (1+j, 1-j, -1+j, -1-j)$</td>
</tr>
<tr>
<td>16QAM</td>
<td>$A = (j, 3j, -j, -3j)$  $A_{sym} = (A+1, A+3, A-1, A-3)$</td>
</tr>
</tbody>
</table>

2.5.3 Modulation

Once the signal has been coded, it enters the modulation block. All wireless communication systems use a modulation scheme to map coded bits to a form that can be effectively transmitted over the communication channel. Thus, the bits are mapped to a subcarrier amplitude and phase, which is represented by a complex in-phase and quadrature-phase (IQ) vector. The IQ plot for a modulation scheme shows the transmitted vector for all data word combinations. Gray coding is a method for this allocation so that adjacent points in the constellation only differ by a single bit.
This coding helps to minimize the overall bit error rate as it reduces the chance of multiple bit errors occurring from a single symbol error.

The modulation format used for data subcarriers is QPSK, 16-QAM or optionally 64-QAM, a block diagram illustration is in Figure-2.7. These modulations are used adaptively in the downlink and the uplink in order to achieve the maximum throughput for each link. The modulation on the downlink can be changed for each allocation, to best fit the modulation for specific user/users. For the uplink, each user is allocated a modulation scheme, which is best suited for his needs. The pilot carriers for the uplink and downlink are mapped using a BPSK modulation. The data bits entering the mapper after bit interleaving are entered serially to the mapper. The constellation-mapped data are subsequently modulated onto the allocated data subcarriers.

**Adaptive Modulation and Coding**

An adaptive modulation and coding mechanism is supported in the downlink with the purpose of allowing the number of transmitted bits per symbol to be varied depending on the channel conditions.

The growing demand of all types of services, including voice, data and content-rich multimedia services, successfully drives the design of increasingly more intelligent and agile communication systems, capable of providing spectrally efficient and flex-
ible data rate access. These systems are capable of adapting and hence adjust the transmission parameters based on the transmission link quality, improving the spectrum efficiency of the system, thereby achieving the capacity limits of the underlying wireless channel. Link adaptation techniques, often referred to as adaptive modulation and coding (AMC), are very successful methods for achieving the desired system requirements. AMC techniques are designed to track the channel variations, thus changing the modulation and coding scheme to yield a higher throughput by transmitting with high information rates under favorable channel conditions and reducing the information rate in response to channel degradation.

The available radio spectrum for wireless communications is extremely scarce with the rapid growth in the demand of services for commercially portable and user-friendly wireless devices. As such, the overall system supporting these services become ever more complex, with the increased need of spectrally efficient transmission schemes supporting higher information rates.

In traditional communication systems, the transmission is designed for a worst case wireless channel scenario thus coping with the channel variations and still delivering an error rate below a specific limit. Adaptive transmission schemes, however, are designed to track the channel quality by adapting the channel throughput to the actual channel state. These techniques take advantage of the time-varying nature of the wireless channel to vary the transmitted power level, symbol rate, coding scheme, constellation size, or any combination of these parameters, with the purpose of improving the link average spectral efficiency, i.e. the number of information bits transmitted per second per Hz bandwidth used.

Adaptive modulation and coding (AMC) is a promising tool for increasing the spectral efficiency of time-varying wireless channels while maintaining a predictable BER [28]. In AMC, both the modulation order and the FEC scheme are varied by adjusting the code rate to the variations in the communication channel. For example, in periods of deep channel fade when the channel is in a poor state, i.e. low SNR, the signal constellation size is reduced in order to improve system performance, lowering the effective SNR to make transmission more robust. Conversely, in good channel conditions or high SNR, the signal constellation size is increased in order to allow higher data rate modulation schemes to be employed with low probability of error,
thus improving the instantaneous SNR.

In AMC-enabled system, as the range increases, the system steps down to a lower modulation, but as closer to the base station, higher order modulations can be used for increased throughput. The choice of the appropriate modulation and coding combination to be used in the next transmission is made by the transmitter, based on the prediction of the channel conditions for the next time interval. An SNR threshold $BER_0$ such that it guarantees a BER below the target BER, is defined by the system for each scheme whenever the SNR is above the SNR threshold.

Each of the schemes is assigned to operate within a particular SNR region. When the SNR falls within the desired region, the associated channel state information is sent back to the transmitter. The transmitter then adapts its transmission rate together with the coding and modulation schemes by transmitting with a modulation scheme such that it guarantees a BER below $BER_0$. This enables the system to transmit with high spectral efficiency when the SNR is high, and to reduce the spectral efficiency as the SNR decreases.

### 2.5.4 Pilot symbols

Pilot symbols are used in the estimation of unknown system parameters at the receiver. They can also be used to perform frequency offset compensation at the receiver. Additionally, they are used for channel estimation in both in time-variant and time-invariant channels. Pilot symbols are allocated specific subcarriers in all OFDM data symbols. These pilots are obtained by a pseudo-random binary sequence (PRBS) generator that is based on the polynomial $x^{11} + x^9 + 1$. The pilots are typically PBSK modulated. The PRBS requires an initialization sequence, which may vary depending on the direction of transmission, i.e. the downlink or the uplink. A sequence of all $\text{ones}$ is used in the downlink while a sequence of alternated $\text{ones}$ and $\text{zeros}$, with the first bit set to $\text{one}$, is used in the uplink.

### 2.5.5 Training Sequences

In WiMAX systems, preambles are composed of training sequences both in the Downlink (DL) and the Uplink (UL). Although three types of training sequences
are specified, they are derived from the same sequence in the frequency domain. This sequence has a length of 201 subcarriers and is called $P_{ALL}$. An in-depth definition can be found in the standards document. For DL transmissions, the first preamble as well as the initial ranging preamble consists of two consecutive OFDM symbols. The first symbol is a short training sequence, $P_{SHORT}$, used for synchronization and given by (2.3). It is a sequence which uses only the subcarriers of $P_{ALL}$ whose indices are a multiple of 4, filling the other subcarriers with zeros. The second OFDM symbol uses a long training sequence, required in the receiver for channel estimation. As with the first OFDM symbol of the preamble, the long training sequence is also constructed using a subset of the subcarriers of the 201 subcarriers of $P_{ALL}$ above. The long training sequence utilizes only even subcarriers, and is therefore called $P_{EVEN}$. Equation 2.4 defines the sequence for this long training.

$$P_{SHORT}(k) = \begin{cases} \sqrt{2} \sqrt{2} \text{conj}(P_{ALL}(k)), & k \mod 4 = 0 \\ 0, & k \mod 4 \neq 0 \end{cases} \quad (2.3)$$

$$P_{EVEN}(k) = \begin{cases} \sqrt{2} \text{conj}(P_{ALL}(k)), & k \mod 2 = 0 \\ 0, & k \mod 2 \neq 0 \end{cases} \quad (2.4)$$

In both (2.3) and (2.4), the factor of $\sqrt{2}$ represents a boost of 3dB. Furthermore, the additional factor of $\sqrt{2}$ in $P_{SHORT}$ aims to equate the root-mean-square (RMS) power with the power of the data symbols.

Another training sequence is used when transmitting Space-Time Coded (STC) downlink bursts. Because the STC scheme achieves diversity by transmitting with multiple antennas, a preamble has to be transmitted from all transmit antennas simultaneously. In the case of a $2T \times, 1R \times$ configuration, the first transmit antenna transmits a preamble using $P_{EVEN}$ and the preamble transmitted from the second antenna is set according to the sequence $P_{ODD}$. Similar to $P_{EVEN}$, it is derived from the sequence $P_{ALL}$, but using a subset of odd subcarriers.
\[ P_{ODD}(k) = \begin{cases} 0, & k \mod 2 = 0 \\ \sqrt{2} \text{conj}(P_{ALL}(k)), & k \mod 2 \neq 0 \end{cases} \] (2.5)

Furthermore, the long preamble is followed by a Header Control Frame, which contains decoded information for the subscriber station, i.e. information about the modulation type and the FEC (Forward Error Correction) code length for each burst profile in the Downlink (DL) and Uplink (UL).

## 2.5.6 Assembler

WiMAX specifications for the 256-point FFT OFDM PHY layer define three types of subcarriers; Data, Pilot and Null, as shown in Figure-2.4.

200 of the total 256 subcarriers are used as data and pilot subcarriers, 8 of which are pilots permanently spaced throughout the OFDM spectrum. The remaining 192 active carriers take up the data subcarriers. The rest of the potential carriers are nulled and set aside for guard bands and removal of the center frequency subcarrier.

In order to construct an OFDM symbol, a process to rearrange these carriers is needed. With this purpose, the assembler block is inserted in the simulator. It performs this operation in two steps by first inserting the pilot tones and the zero DC subcarrier between data with a process of vertical concatenation, and then appending the training symbols at the beginning of each burst in an horizontal sequence.

## 2.5.7 Guard Bands

The standard specifies transmission with 256 frequency subcarriers. The total amount of subcarriers to be used is determined by the number of points needed to perform the FFT/IFFT. After the assembling process described in the previous section, only 201 of the total 256 subcarriers are used. The remaining 55 carriers are zero subcarriers appended at the end, and act as guard bands with the purpose of ensuring the natural decay of the signal. These guard bands are used to decrease emissions in adjacent frequency channels.
2.5.8 Inverse Fast Fourier Transform

The IFFT is used to produce a time domain signal. Each of the discrete samples prior to applying the IFFT algorithm corresponds to an individual subcarrier. Besides ensuring the orthogonality of the OFDM subcarriers, the IFFT represents also a rapid way for modulating these subcarriers in parallel. Thus, the use of multiple modulators and demodulators that consume significant amount of time and resources to perform this operation, is avoided or considerably minimized.

The FFT (or IFFT) should be of length $2^r$ (where $r$ is an integer) to facilitate the realization of the algorithm. For this reason, the FFT length is given by

$$N_{FFT} = 2^{\lceil \log_2(N_{data}) \rceil} \quad (2.6)$$

where $\lceil X \rceil$ represents the largest integer less than $X$.

2.5.9 Cyclic Prefix

The robust nature of OFDM transmission against multipath delay spread is achieved by having a long symbol period with the purpose of minimizing the inter-symbol interference (ISI). Figure-2.8 depicts one approach used to perform the long symbol period, creating a cyclically extended guard interval where each OFDM symbol is preceded by a periodic extension of the signal itself. This guard interval is a copy of the last portion of the data symbol, and is known as the cyclic prefix (CP).
Appending the end of a symbol to the beginning leads to a longer symbol time. Thus, the resulting total length of the symbol is

\[ T_s = T_b + T_g, \]  

and \( G \) is defined as

\[ G = \frac{T_g}{T_b} \]  

where:

- \( T_s \) is the OFDM total symbol time,
- \( T_b \) is the OFDM useful symbol time, and
- \( T_g \) represents the CP time, and serves as guard interval.

The parameter \( G \) defines the ratio of the CP length to the useful symbol time. When eliminating ISI, it should be noted that the CP must be longer than the dispersion of the channel. Moreover, the CP should be as small as possible to minimize power consumption at the transmitter. For these reasons, \( G \) is usually less than \( 1/4 \) [21].

### 2.5.10 De-interleaver

The deinterleaver rearranges the bits from each burst in the correct order by rearranging them consecutively as before the interleaving process. It consists of two blocks, a general block deinterleaver and a matrix deinterleaver. These blocks work similarly as the ones used in the interleaver, but complimentary. The general block deinterleaver rearranges the elements of its input according to an index vector. The matrix deinterleaver performs block deinterleaving by filling a matrix with the input symbols column by column, and then, sending its contents to the output row by row. The parameters used in both blocks are the same as those ones used in the interleaving process.
2.5.11 Physical Layer Summary

Table-2.8 summarizes additional Physical Layer parameters and specifications of the IEEE 802.16 Standards.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Frequency Bands</th>
<th>MAC Layer</th>
<th>Duplexing</th>
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</thead>
<tbody>
<tr>
<td>WirelessMAN-SC</td>
<td>10-66 GHz Licensed</td>
<td>Basic</td>
<td>TDD, FDD, HFDD</td>
</tr>
<tr>
<td>WirelessMAN-SC</td>
<td>2-11 GHz Licensed</td>
<td>Basic, (STC), (AAS), (ARQ)</td>
<td>TDD, FDD</td>
</tr>
<tr>
<td>WirelessMAN-OFDM</td>
<td>2-11 GHz Licensed</td>
<td>Basic, (STC), (AAS), (ARQ)</td>
<td>TDD, FDD</td>
</tr>
<tr>
<td>WirelessMAN-OFDM</td>
<td>2-11 GHz Licensed-exempt</td>
<td>Basic, (STC), (AAS), (ARQ), (DFS), (MSH)</td>
<td>TDD</td>
</tr>
<tr>
<td>WirelessMAN-OFDMA</td>
<td>2-11 GHz Licensed</td>
<td>Basic, (STC), (AAS), (ARQ)</td>
<td>TDD, FDD</td>
</tr>
<tr>
<td>WirelessMAN-OFDMA</td>
<td>2-11 GHz Licensed-exempt</td>
<td>Basic, (STC), (AAS), (ARQ), (DFS), (MSH)</td>
<td>TDD</td>
</tr>
</tbody>
</table>

2.6 Multi-Antenna Communication

The IEEE 802.16 standard document includes support for optional multi-antenna configuration. In this section, and for the sake of completeness, a general overview of multi-antenna technologies is presented. The overview is not intended to be comprehensive, and will only cover the most common multi-antenna technologies used to date.

A multi-antenna system can be defined as a wireless communication system for which the transmission end and/or the receiving end are equipped with multiple antenna elements. The idea behind multi-antenna systems is that the signals on the transmit antennas at one end and the receive antennas at the other end are combined
in such a way that the quality of service (bit error probability) or the transmission rate are improved. A diagram of a general multi-antenna system is shown in Figure-2.9

![Diagram of MIMO Wireless Communication System](image)

**Figure 2.9 Diagram of MIMO Wireless Communication System**

Applying multiple antennas at both ends of a communication system can not only greatly improve the capacity and the throughput of a wireless link in flat-fading channels but also substantially improve the performance in frequency-selective fading channels, specially when the environment provides rich scattering [32], [33], [20]. Multiple-input multiple-output systems, also known as MIMO, have multi-element antenna arrays at both transmit and receive sides. High data rates are achieved when implementing such structures without increasing neither the bandwidth nor the total transmission power. Additionally, the use of multiple antennas at both transmitter and receiver provides a diversity advantage, with the significant increase in capacity, i.e. improvement in SNR and hence in BER at the receiver [34], [35], [36], [37].

When communicating through a wireless channel, transmitted signals suffer from attenuation and fading due to multipath in the channel, thus making it difficult for the receiver to determine these signals. Diversity techniques take advantage of the multipath propagation characteristics to improve receiver sensitivity. MIMO systems utilize antenna diversity to obtain the mentioned improvement and hence combat fading [38], [39], [40], [31], [37].

The main advantages of MIMO channels over traditional SISO channels are the array gain, the diversity gain, and the multiplexing gain [41], [31], [42]. Array gain and diversity gain are not exclusive of MIMO channels and also exist in SIMO
and MISO channels. Multiplexing gain, however, is a unique characteristic of MIMO channels. Array gain is the improvement in SINR (Signal to Interference plus Noise Ratio) obtained by coherently combining the signals on multiple transmit or multiple receive dimensions and is easily characterized as a shift of the BER curve due to the gain in SINR. Diversity gain is the improvement in link reliability obtained by receiving replicas of the information signal through independently fading links, branches, or dimensions. It is characterized by an steeper slope of the BER curve in the low BER region. Diversity can be defined as a means to insure that different copies (replicas) of the transmitted signal are available at the receiver. In fading channels, this means that it is less likely for all the different copies to be in deep fade if they are fading independently, hence, the receiver is more likely to be able to estimate the transmitted signal from the received copies [33].

In Figure 2.9, the input bits are first sent to the channel encoder, which may be similar to the channel encoder described in Section 2.5.1. The data blocks at the output of the channel encoder are then modulated and mapped to different constellation points. The constellation points, also known as transmitted symbols are then sent to the multi-antenna multiplexer. The multiplexer distributes the transmitted symbols to the transmit antennas according to the type of multi-antenna transmission technique required. In Figure 2.9, the space time encoder is given as an example, where the transmitted symbols are multiplexed in both space and time. The transmitted symbols may also be multiplexed in space and frequency such as in space frequency multi-antenna transmission systems or just in space such as spatial multiplexing and receive diversity systems.

The added complexity and cost of employing multi-antennas for data transmission is considered a small price to pay compared with the advantages that multi-antenna systems can provide. There are many advantages to employing multi-antennas. The two most important are the increase in transmission rate and the improvement of the quality of service (lowering the bit error probability) of the system.

A notable fraction of multi-antenna configurations are designed to increase
the transmission rate. Some are designed to improve the performance (bit error probability) without increasing the transmission rate, while others are designed to achieve certain levels of both. It is also common to combine two systems to maximize the potential gain.

In Figure 2.10 referenced from [24], different multi-antenna technologies are divided into two categories depending on the primary design objective, whether increasing the transmission rate or improving the system performance. Note that although some technologies may provide both, they are specified according to the most obvious design objective.

![Multi-Antenna Technology Diagram]

Figure 2.10 Multi-Antenna Technology

Increasing the transmission rate is achieved by a technique known as spatial multi-
plexing. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel [43]. Therefore, the throughput is $N_t$ symbols per channel use for a system with $N_t$ transmit antennas [33].

From an information theoretic point of view, the increase in transmission rate can be observed by investigating the channel capacity when multiple antennas are employed and comparing it with the channel capacity for the case when single transmit and receive antennas are used. As an example, consider the following signal model for a system with single transmit and receive antennas

$$r = h x + w$$  \hspace{1cm} (2.9)

where $r$ is the received signal, $h$ is the channel gain, $x$ is the transmitted signal and $w$ is the additive noise which is modeled as a complex Gaussian random variable with zero mean and variance $\sigma_w^2$ per dimension. The capacity of the system in (2.9) is well known, and is given by [44]

$$C_1 = \log_2(1 + \gamma |h|^2) \quad \text{bits/sec/Hz}$$  \hspace{1cm} (2.10)

where $\gamma$ is the signal to noise ratio (SNR). If the system is equipped with a single transmit antenna and $n_R$ receive antennas (receive diversity), then the capacity can be shown to be

$$C_2 = \log_2(1 + \gamma \sum_{i=1}^{n_R} |h_i|^2) \quad \text{bits/sec/Hz}$$  \hspace{1cm} (2.11)

where $h_i$ is the channel gain from the transmit antenna to the $i_{th}$ receive antenna. For a system equipped with $n_T$ transmit antennas and a single receive antenna (transmit diversity), the capacity when the transmitter does not have knowledge of the channel is given by

$$C_3 = \log_2(1 + \frac{\gamma}{n_T} \sum_{i=1}^{n_T} |h_i|^2) \quad \text{bits/sec/Hz}$$  \hspace{1cm} (2.12)

where the normalization by $n_T$ ensures a fixed total transmitter power. Now consider the multi-antenna system shown in Figure-2.9, the input-output relationship for the

---

2Equations 2.10, 2.11, 2.12 and 2.14 are obtained from Reference [44].
above system can be given by the following model

\[ \mathbf{r} = \mathbf{H} \mathbf{x} + \mathbf{w} \]  

(2.13)

where \( \mathbf{r} \) is the \( n_R \times 1 \) received signal vector, \( \mathbf{x} \) is the \( n_T \times 1 \) transmitted signal vector, \( \mathbf{w} \) is an \( n_R \times 1 \) vector of additive noise terms and \( \mathbf{H} \) is an \( n_R \times n_T \) channel matrix. The capacity for the signal model in (2.13) when assuming uncorrelated sources is given by

\[ C_4 = \log_2 \left[ \det \left( \mathbf{I}_{n_R} + \frac{\gamma}{n_T} \mathbf{H} \mathbf{H}^H \right) \right] \text{ bits/sec/Hz} \]  

(2.14)

where \( (\cdot)^H \) is the transpose conjugate operation. The increase in capacity from \( C_1 \) to \( C_2 \) and \( C_3 \) when multiple antennas are used either in the transmitter side or the receiver side is obvious. A very important observation here is that increasing the value of \( n_T \) in (2.12) or \( n_R \) in (2.11) only results in a logarithmic increase in average capacity compared to the capacity expression given in (2.10) [44]. On the other-hand, it has been shown in [45] and [46] that the capacity \( C_4 \) in (2.14) grows linearly with \( \min(n_T, n_R) \) rather than logarithmically.

The first multi-antenna technology that is intended to achieve such a linear increase in capacity is the one proposed by Fochini in [47], and is known as the Bell Laboratories Layered Space-Time (BLAST). There are different flavors of BLAST that are extensively used in nowadays applications, two examples are shown in Figure 2.10, namely, the vertical BLAST (V-BLAST) and the diagonal BLAST (D-BLAST).

Improving the bit error probability performance with multi-antenna transmission is achieved primarily by two methods. The first method aims to reduce interference, and hence improve the system performance via a technique known as beamforming. In beamforming, the radiation pattern of the antenna array (transmitter, receiver or both) is properly shaped and steered such that the phases of the desired signals are added constructively, and the interfering signals are nulled.

The second method improves the system performance by means of increasing the diversity gain via exploiting the spatial, temporal or frequency diversity using
multiple antenna elements and proper encoding/combining techniques. A combination of spatial-temporal, spatial-frequency or spatial-temporal-frequency diversity may also be used to increase the diversity gain and improve system performance.

Diversity can be defined as a means to insure that different copies (replicas) of the transmitted signal are available at the receiver. In fading channels, this means that it is less likely for all the different copies to be in deep fade if they are fading independently, hence, the receiver is more likely to be able to estimate the transmitted signal from the received copies [33].

In spatial diversity, the different copies of the transmitted signal are generated by transmitting/receiving the signal from different antennas. These multiple copies are processed at the receiver through combining to improve the system performance. Examples of spatial diversity techniques are Maximal Ratio Combining (MRC), Equal Gain Combining (EGC) and Selection Combining (SC). In temporal diversity, the signal is transmitted in different time slots, and signals from the different time slots (copies) are processed at the receiver in such a way that the system performance is improved. Spatial-temporal diversity is achieved by transmitting the signals using different transmit antennas and different time slots. An example of spatial-temporal diversity is the well known space time coding technique. The signal can also be transmitted using different antennas and different frequencies. This technique is known as space frequency coding. Finally, the signal can also be transmitted from different antennas, different time slots and different frequencies. This is known as space time frequency coding. The different techniques discussed above are shown in Figure 2.10

In this thesis, we investigate a special class of Space-Time Codes known as Orthogonal Space-Time Block Codes (OSTBC) which includes the Alamouti STBC, and extensions to higher number of transmit antennas. Space-Time Codes can be divided into three categories, namely, Space Time Block Codes (STBCs), Space Time Trellis Codes (STTCs) and other Space Time Codes. STBCs act on a block of data at once (similarly to block codes) and provide only diversity gain [48], while STTCs
distribute a trellis code over multiple antennas and multiple time-slots and provide both coding gain and diversity gain [49]. STBCs can also be separated into three types, they are orthogonal space time block codes (OSTBCs), quasi-orthogonal space time block (QOSTBCs) codes and non-orthogonal space time block codes. Examples of the OSTBCs are the Alamouti OSTBC [50], codes from generalized real orthogonal designs (GROD) [48], codes from generalized complex orthogonal designs (GCOD) and pseudo OSTBCs. The different categories of space time codes along with their associated codes are shown in Figure 2.11, which is referenced from [24].

![Figure 2.11 Space-Time Coding Tree Diagram](image)

**2.7 MIMO Communications in Wireless Standards**

MIMO systems have found their way into several communication standards due to the great promises they offer in wireless communications systems, especially wireless local area networks and cellular networks [28], [29], [51], [52].

Examples of these standards include IEEE802.11, IEEE802.16, and the 3rd
Group Partnership Project (3GPP). The IEEE802.11 has been developed for Wi-Fi, which is normally used for public wireless access. A version of this standard (IEEE802.11a) supports data rates up to 11 Mbps, whereas the IEEE802.11g version supports data rates up to 54 Mbps. The latest version (IEEE802.11n) incorporates MIMO communications. MIMO communications has been incorporated as an option in the IEEE 802.16. In some cases, the multiple antennas are used to deliver high data rates, and in others such as cellular networks, the multiple antennas are used for beamforming to improve the overall network capacity, i.e., number of supported users in the network.

The 3GPP technology also known as wideband code division multiple access (W-CDMA) is an extension of CDMA technology. MIMO has been incorporated in this standard, with $2 \times 1$, $2 \times 2$, $4 \times 2$, and $4 \times 4$ MIMO configurations employing STBC.

MIMO is also considered in IEEE802.20 and IEEE802.22 standards. The latter standard aims at constructing wireless regional area networks utilizing channels that are not used within the already allocated frequency spectrum.

\section{Conclusion}

In this Chapter, a description of the IEEE 802.16 family of Standards is presented, including the evolution and motivation for the evolution of the Standard. The inter-operability that WiMAX introduces is also discussed, followed by a description of system functional blocks and Physical Layer parameters. In conjunction with the system performance degradation that phase noise and imperfect channel estimation introduce, these parameters are used in the simulations of a practical wireless communication system and compared with the corresponding analytical results presented in the following Chapters.
3

Phase Noise in Oscillators: Theory and Characterization

3.1 Introduction

In order to adequately utilize spectral resources, wireless communication systems are allocated specific frequency spectrum for efficient communication with compatible technologies in order to avoid unintentional signal interference with other wireless systems during transmission. The rapid growth in wireless communication and its inevitability is driving emerging broadband wireless technologies to move towards available higher radio frequencies (RF) in the range of tens of GHz.

Electronic blocks in wireless devices called oscillators are used to accomplish the task of translating (up-convertin) information signals at the transmitter from baseband frequency to the allocated frequency band before transmission through the wireless channel. They are also used at the receiver to down-convert the received signal for processing at baseband frequency. Oscillators are ubiquitous in physical systems, especially electronic and optical ones. Oscillators are also present in digital electronic systems such as computers, clock signals and radios, which require a time reference in order to synchronize operations.

To perfectly accomplish the tasks of upconverting/downconverting signals, an ideal oscillator is required. The desired ideal oscillator is a lossless circuit that produces pure localized tones (i.e. harmonics) at the carrier frequencies of interest. Due
to the behavior of practical oscillators however, the signal produced by the oscillator exhibits time-variations due to inherent noise and losses within the circuit. The time variations of the signal produced by the oscillator is also manifested in the form of noise in the frequency spectrum. A perfect oscillator would have localized tones at discrete frequencies, but any corrupting noise spreads these perfect tones, resulting in high power levels at neighboring frequencies [53]. Figure-3.1 shows a diagram of the desired ideal oscillator spectrum in (A) and that of a practical oscillator in (B).

Noise in oscillators is a major concern, because the contribution of small amounts of noise in an oscillator can lead to dramatic changes in the frequency spectrum and time domain properties. Phase noise is a major performance limiter in wireless communication systems. This is even more so in multicarrier systems such as in OFDM [18], [54], [55] [56], [57], [58], [59], [60], [61], [62].

The effect of phase noise is the major contributor to undesired phenomena such as phase error in wireless communication. Phase error is known to cause rotation on the signal constellation of the transmitted signal, and hence the likelihood of more errors during signal detection. This effect of constellation rotation is illustrated in Figure-3.2. Each point on the constellation corresponds to a different symbol which typically represent multiple bits of transmitted data.

A communication system is typically power constrained. As a result, it can be seen that as the number of transmitted symbols on the constellation increases, the distance between neighboring points on the signal constellation decreases correspondingly. The system becomes more susceptible to non-idealities including phase error, which leads to an increase in the bit error rate. The effect of the impairment is even
more pronounced in higher order constellations such as 16QAM, 64QAM.

In addition to phase error and constellation rotation, inter-carrier interference (ICI) is also introduced in multi-carrier communication systems. The ICI results from loss of orthogonality among overlapping but orthogonal subcarriers, as illustrated in Figure-3.3. This added interference leads to more noise in the system, and hence an additional increase in the bit error rate (BER) [54], [61], [18], [62].

Just as phase noise is a frequency domain viewpoint, the time variations exhibited by signals from oscillators can also be understood from a time domain perspective. In other words, phase noise and jitter are two related quantities associated with a noisy oscillator. Phase noise is a frequency-domain view of the noise spectrum around the oscillator’s output signal, while jitter is a time domain measure of the timing (in)accuracy of the period of the oscillator’s signal [55], [57], [56]. Phase
noise is a frequency domain specification that characterizes the spectral purity of the oscillator. The description of the noise phenomenon as jitter is more prevalent in the field of computers, clocks and sampled-data systems where time domain analysis and timing description are extensively utilized.

Figure-3.4 and Figure-3.5 illustrates the effect of jitter, which is the random time variations of a signal with reference to the zero-crossings of the corresponding ideal signal. In RF applications, RF engineers frequently work with power spectra of signals and generally speak of the phase noise of an oscillator. On the other hand, digital system engineers characterize the noise in the time domain and speak of the jitter in the signal since they work mostly with clock circuits and timing issues in electronics and communication systems.

Oscillators are frequently described based on their operating frequency. An oscillator with a low output frequency is referred to as a low frequency oscillator. Similarly, an oscillator with a high output frequency is described accordingly as a high frequency oscillator. In RF applications, frequencies above 1 GHz are typically
considered high operating frequency. It is known that a low frequency oscillator possess a superior phase noise performance to that of a higher frequency oscillator [63], [64], [56]. It has also been reported in [62] that the effect of noisy oscillator can grow quadratically with the carrier frequency. Therefore, it is desirable to choose a lower frequency oscillator and utilize a frequency multiplier to obtain the desired higher output frequency whenever possible.

Unlike the signal of a practical oscillator or clock circuit, that of an ideal oscillator is periodic. The uncertainties in the periodicity of the signal of a practical oscillator caused by the phase noise lead to synchronization issues in communication systems. The effect of oscillators phase noise is so significant that oscillators are sometimes separately classified in the literature due to the impact of the negative effects they introduce. A thorough characterization of the noise and the associated impairments in oscillators is therefore crucial for all practical applications. Furthermore, effective approaches to combat the effects of phase noise is considered very valuable, both in wireless communication systems and clock applications as well.

In the next section, we describe the basic building block of the oscillator and its theory of operation. We next look at the causes of phase noise in the oscillator, followed by different models used to characterize the phase noise that results due to imperfect realization of the oscillator.

### 3.2 The Oscillator

An oscillator is a circuit that produces an electronic signal, often a sine wave or a square wave. Oscillators are important in many types of electronic equipment, including clocks, radios, telephones, and computers. The main components of oscillators are inductor and capacitor. The frequency produced by oscillators is determined by the combination of capacitor and inductor. This combination is used to determine the operating frequency of the oscillator, and is frequently referred to as a tuned circuit or tank circuit. Oscillators are implemented either as LC (Inductor-Capacitor) tank circuit or as crystal oscillators. They are called LC oscillators or tank oscillators when implemented using inductors and capacitors.

A diagram of the LC realization of an oscillator circuit is shown in Figure-3.6.
For ideal operation, the frequency of an oscillator is required to be stable, but due to non-idealities and temperature variations, this may not be so.

An ideal oscillator has no associated circuit losses, with infinite signal amplitude swing, at precisely the desired operating frequency. The presence of losses results in a finite signal swing over a relative narrow spread of frequencies. The quality of an oscillator is measured by the combined quality of the individual components of the underlying tank circuit. This measure of quality is expressed as the *quality factor* \( Q \) of the tank circuit. One definition of quality factor or \( Q \) is expressed as

\[
Q = \frac{\omega_0}{\omega_2 - \omega_1} \tag{3.1}
\]

where \( \omega_0 \) is the oscillator operating frequency, \( \omega_1 \) and \( \omega_2 \) are the lower and upper frequencies respectively at the 3dB points (or equivalently, the half power points) on the frequency spectrum.

Another definition of \( Q \) is expressed as a function of the individual \( Q \)'s of the inductor and the capacitor of the tank circuit, and is written as

\[
\frac{1}{Q} = \frac{1}{Q_C} + \frac{1}{Q_L} \]

\[
Q = \frac{Q_C Q_L}{Q_C Q_L} \tag{3.2}
\]

where \( Q_L \) and \( Q_C \) are the respective quality factors of the inductor and the capacitor.

The component quality factor \( Q_L \) of the inductor is given by

\[
Q_L = \frac{\omega_0 L}{R} \tag{3.3}
\]
where $L$ is the value of the inductance expressed in Henrys ($H$), and $R$ is the inductor’s associated series resistance in Ohms ($\Omega$).

The component quality factor $Q_C$ of the capacitor is given by

$$Q_C = \frac{1}{R\omega_0 C} \quad (3.4)$$

where $C$ is the value of the capacitance expressed in Farads ($F$), and $R$ is the associated series resistance of the capacitor.

At the resonance frequency, commonly known as the operating frequency, a pure LC tank with no losses presents infinite impedance (i.e. $Z_{load} = \infty$) to an applied signal across the tank. This implies that the output signal across the tank will be infinite if the tank is driven by a source with infinite impedance.

If the tank has real losses (as is typically the case), however, the losses can be modeled as if the lossless tank had a finite impedance load, i.e. $Z_{load} \neq \infty$. In practice, it is always desirable to have as large of a $Z_{load}$ as possible for a good oscillator, because the finite impedance load $Z_{load}$ will diminish the $Q$ of the tank resulting in a finite output signal.

Often, capacitors have much higher component $Q$ than inductors such that

$$Q_L \ll Q_C \quad (3.5)$$

Then, from

$$Q = \frac{Q_C Q_L}{Q_C Q_L}$$

$$\approx \frac{Q_C Q_L}{Q_C}$$

$$\approx Q_L \quad (3.6)$$

it is observed that $Q_L$ determines the quality factor of the lossy tank. Generally over a reasonable range of inductors, the maximum $Q$ possible is more or less independent of the value of inductance. Furthermore, it is also almost independent of frequency over a fairly large frequency range.

Oscillators are usually controlled by an applied voltage. As a result, they are called voltage-controlled oscillators (VCOs). VCOs are generally designed for minimum phase noise under the constraints of power dissipation, tuning range, and
output signal swing. The tuning range of a VCO is required to be in excess of a certain minimum percentage of the center frequency, \( \omega_0 \). The LC tank is made tunable by implementing the \( C \) of the LC tank using a variable capacitor. Parasitic resistances increase the losses in the VCO.

Parasitic capacitances can combine with the tank capacitance, \( C \), degrading the quality factor of the tank and affecting the oscillation frequency. Therefore the capacitance of the tank must be decreased to allow for these parasitic capacitances. Parasitic resistances will also contribute thermal noise increasing oscillator phase noise.

The noise in oscillators is injected by the devices that constitute the oscillator itself, including active transistors and passive elements such as resistors, and will disturb both the amplitude and frequency of oscillation. Amplitude noise is usually not considered critical because non-linearities that limit the amplitude of oscillation also limit and stabilize the amplitude noise. Phase noise, on the other hand, is essentially a random deviation in frequency which can also be viewed as a random variation in the zero crossing points of the time-dependent oscillator waveform, as shown in Figure-3.1.

To see this mathematically, consider a signal \( x(t) \) given by

\[
x(t) = A \cos(\omega_0 t + \theta(t))
\]  

(3.7)

where \( A \) is the noiseless oscillator amplitude, \( \omega_0 \) is the oscillator frequency, \( \theta(t) \) is the phase noise, \( x(t) \) is the oscillator output signal.

For small phase noise \( \theta(t) \), it is known that \( \cos(\theta(t)) \approx \cos(0) = 1 \), and \( \sin(\theta(t)) \approx \theta(t) \).

We then obtain

\[
x(t) = A \cos(\omega_0 t + \theta(t))
\]

\[
= A \left[ \cos(\omega_0 t) \cos(\theta(t)) - \sin(\omega_0 t) \sin(\theta(t)) \right]
\]

\[
\approx A \cos(\omega_0 t) - A \theta(t) \sin(\omega_0 t)
\]  

(3.8)

phase noise translation
Therefore, the spectrum of the phase noise is effectively translated to the oscillation frequency $\omega_0$. This implies the phase noise signal is amplitude modulating a sinusoid signal of frequency $\omega_0$ that is in turn superimposed on the ideal oscillator itself. An oscillator, however, is a frequency selective (or narrow band, high quality factor) circuit and will tend to reject out of band signals, i.e. signals at frequency offsets $\Delta \omega$ from $\omega_0$, to some degree. This rejection increases at larger offsets from $\omega_0$. As a result, the effect of the noise sources will produce skirts on either side of the ideal spectrum\(^1\) of the oscillator in the frequency domain, as shown in 3.1.

To measure or quantify phase noise, one considers a unit bandwidth at an offset frequency $\Delta \omega$ with respect to $\omega_0$, calculates the noise power in this bandwidth, and divide the result by the average carrier power.

Phase noise of an LC oscillator depends on the Q of the LC tank circuit. For an LC tank, Q is an indication of how much of the energy is lost as it is transferred from the capacitor to the inductor and vice versa.

Q is also defined as the sharpness of the magnitude of the frequency response. The phase noise of an oscillator is known to be proportional to $1/Q^2$. Therefore, for minimum phase noise, it is required by design to make Q as high as possible.

### 3.3 Theory of Phase Noise

A vast collection of literature is available on the phase noise phenomenon. Most investigations of oscillators aim to provide insight into the frequency-domain properties of phase noise in order to develop rules for designing practical oscillators and improve system performance. Some of the well-known references include, but not limited to, [56], [57], [59], [60], [63], [65], [64], [62], [55], [61], [18], [54].

The spectrum of a practical oscillator is frequency-dependent. It typically exhibits a profile that is approximately inversely proportional to the frequency [63], [65], [56], [57], [58], [59], [60] and will tend to follow an inverse power law over frequency. It has been noted that the primary cause of phase noise in oscillators is the inadequate accountability of the $1/f$ noise (sometimes referred to as flicker noise [63], [64]).

\(^1\)The ideal spectrum of an oscillator is an impulse function
It is shown in [63] that a combination of $1/f$ noise and white noise may lead to a spectrum as in Fig-3.7. At low frequencies, the $1/f$-noise component dominates, while at high frequencies only the white noise is apparent. The boundary between both regions, where both contributions are equal, is designated as the corner frequency $f_{\text{corner}}$. The corner frequencies of noisy electronics components in wireless devices can range from 1KHz up to approximately 100MHz [63], [64], [56], [57], [59], [60].

It has been shown that for linear applications of electronics components such as small signal amplifiers, the $1/f$ noise is not a major concern if the frequency range of interest lies above the corner frequency [56], [57], [59], [60]. The situation is significantly different and quite severe when non-linear effects have to be taken into account as in wireless applications for power amplifiers in the frontend of transmitters, and oscillators in transmitting and receiving circuits as well [56], [59], [60] \(^2\).

In the work of [56], the actual output spectrum of the ideal oscillator is determined. It is shown that the power spectral density of the output noise is frequency dependent, and falls off as the inverse-square of the offset frequency $1/f^2$. This $1/f^2$ behavior simply reflects the fact that the frequency response of the voltage of a Resistor-Inductor-Capacitor (RLC) tank circuit rolls off as $1/f$ to either side of the center frequency, and the corresponding power is proportional to the square of voltage. It is shown in [56] that the normalized single-sideband noise spectral density

\(^2\)Due to the non-linear interaction between the low-frequency noise and the large high frequency signal, the low frequency noise is upconverted to the RF frequency of the oscillator resulting in lower and upper side bands around the RF signal.
denoted as $P_{SB}^1(\Delta w)$ is given by

$$P_{SB}^1(\Delta w) = 10\log \left( \frac{2FKT}{P_{sig}} \left\{ 1 + \left( \frac{\omega_0}{2Q\Delta \omega} \right)^2 \right\} \left\{ 1 + \left( \frac{\Delta \omega_{1/f}}{|\Delta \omega|} \right) \right\} \right) \quad (3.9)$$

where $\Delta \omega_{1/f}$ is the corner frequency as shown in Figure-3.8, $Q$ is a measure of the quality of the resonator, $\omega_0$ is the center frequency of the oscillator, $\Delta \omega$ is the offset frequency from the center frequency, $F$ is the Noise Factor of the system, $K$ is Boltzman’s constant ($8.617e^{-5}\text{eVolts/Kelvin}$), and $T$ is absolute temperature in $\text{Kelvin}$. The units of phase noise are commonly expressed as $\text{decibels}$ below the carrier per Hertz, or $\text{dBc/Hz}$, specified at a particular offset frequency from the carrier frequency. The phase noise performance specifications of a typical PLL synthesizer is shown in Table-3.1. This PLL, by MicroSource Inc., is designed to operate between $2.1\text{GHz}$ and $5\text{GHz}$.

<table>
<thead>
<tr>
<th>Phase Noise</th>
<th>Offset Frequency</th>
<th>Units</th>
<th>Typical</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSB Phase Noise</td>
<td>100Hz</td>
<td>dBC/Hz</td>
<td>-82</td>
<td>-77</td>
</tr>
<tr>
<td></td>
<td>1KHz</td>
<td>dBC/Hz</td>
<td>-80</td>
<td>-75</td>
</tr>
<tr>
<td></td>
<td>10KHz</td>
<td>dBC/Hz</td>
<td>-100</td>
<td>-96</td>
</tr>
<tr>
<td></td>
<td>100KHz</td>
<td>dBC/Hz</td>
<td>-125</td>
<td>-121</td>
</tr>
<tr>
<td></td>
<td>1MHz</td>
<td>dBC/Hz</td>
<td>-146</td>
<td>-142</td>
</tr>
</tbody>
</table>

Equation (3.9) reveals that phase noise at a given offset frequency, improves as the carrier power increases, since increasing the signal power improves the ratio of the carrier-to-noise power, simply because the thermal noise is fixed. The phase noise spectrum based on this model is shown in Figure-3.8.
A predictive model of the phase noise $\theta(t)$ with improved accuracy is given by [56]

$$\theta(t) \approx \frac{A_m \sin(\Delta \omega t)}{2q_{\text{max}} \Delta \omega} \quad (3.10)$$

where the coefficient $A_m$ and the term $q_{\text{max}}$ are known constants. Although (3.10) provides the spectrum of $\theta(t)$, the spectrum of the output voltage of the oscillator is desired. The two quantities are linked through the actual output waveform, given by

$$v_{\text{out}}(t) = \cos[\omega_0 t + \theta(t)] \quad (3.11)$$

where $\omega_0$ is the oscillator frequency. Applying the phase-to-voltage conversion in (3.11), and assuming negligible amplitude distortion, a single-tone signal results in two equal-power sidebands $P_{SB}^2(\Delta \omega)$ symmetrically located around the carrier frequency and given by

$$P_{SB}^2(\Delta \omega) = 10 \log \left( \frac{A_m}{4q_{\text{max}} \Delta \omega} \right)^2 \quad (3.12)$$

Equation-(3.12) predicts the $1/f$ behavior, $1/f^2$ behavior, and $1/f^3$ of the phase noise from close-in offset frequencies to large offsets as well.
The oscillator power spectrum in the close-in \((1/f^2)\) region is given by [56]

\[
P_{\frac{1}{f^2}}(\Delta \omega) = 10 \log \left( \frac{i_n^2 \Gamma_{\text{rms}}^2}{2 q_{\text{max}} \Delta w^2} \right)
\] (3.13)

where \(i_n\) denotes the current noise, \(\Gamma\) is a deterministic quantity, and \(\Gamma_{\text{rms}}\) is the Root Mean Square (RMS) of \(\Gamma\). All other factors held equal, it can be seen that reducing \(\Gamma_{\text{rms}}\) will reduce the phase noise across all frequencies. Among other attributes, equation (3.13) facilitates a quantitative study of the translation of \(1/f\) noise into close-in \((1/f^2)\) phase noise. The model described by (3.13) has been shown to be quite effective in predicting the phase noise in the close-in region, and as a result, has particular significance for applications with minimal tolerance of the \(1/f\) noise.

Phase noise near the carrier frequency\(^3\) is particularly important in communication systems with narrow channel spacings (or equivalently, narrow subcarriers spacing in multicarrier systems such as in OFDM). In fact, the allowable channel spacings are frequently constrained by the achievable phase noise performance of the oscillator.

In wireless communications, the statistical properties of random noise sources in the circuits within the system may change with time [57], [56], [65]. There are other noise sources whose statistical properties do not depend on time. These time-independent noise sources are referred to as stationary noise. Thermal noise of a resistor is an example of a stationary noise source. The noise sources in many oscillators are time-varying, however, and cannot be well described as stationary. They vary periodically with signal waveforms. The phase noise model described above can accommodate time-varying noise sources.

In the next section, the most commonly used oscillator models used to characterize the oscillator phase noise are discussed. Depending on the implementation of the oscillator, the appropriate model can be used to determine the effect of the phase noise contribution on system performance.

\(^3\)Phase noise near the carrier frequency is referred to as close-in phase noise.
3.4 Characterization: Phase Noise Models

The phase noise models frequently described in the literature follow one of two practical realizations of the local oscillator (LO). In the first case, the LO is realized as a free running oscillator [10], [61]. In the latter, it is realized as a Phase-Locked Loop (PLL) [62], [66], [67]. In the next sections, we describe these realizations.

3.4.1 Free-Running Oscillator

A block diagram of the realization of a free running LO in a wireless receiver circuit is shown in Fig-(3.9). The output signal of a noisy LO is described by

\[ x(t + \alpha(t)) \]  \hspace{1cm} (3.14)

where \( \alpha(t) \) is a random time shift. The phase is related to the time shift by

\[ \theta(t) = 2\pi f_c \alpha(t) \]  \hspace{1cm} (3.15)

where \( f_c \) denotes the carrier frequency. The time shift \( \alpha(t) \) is asymptotically a Wiener or Brownian motion process [18], [62], and is given by

\[ \alpha(t) = \sqrt{c} B(t) \]  \hspace{1cm} (3.16)
where \( c \) is a parameter describing the oscillator quality. \( B(t) \) represents the Wiener process, which is an accumulated Gaussian random variable with zero mean and unit variance denoted as \( N(0,1) \). The strength of the phase noise directly determines the quality of the oscillator output signal. A parameter widely used in the literature [15], [68], [69] in characterizing the phase noise strength in OFDM systems is the relative phase noise bandwidth \( \Delta f_{3dB}/\Delta f_{carr} \), where \( \Delta f_{3dB} \) is the absolute 3dB bandwidth of the phase noise, \( \Delta f_{carr} \) is the subcarrier spacing. For a given system bandwidth \( W \) and total number of subcarriers \( N \), the subcarrier spacing in the OFDM systems is given by \( \Delta f_{carr} = W/N \).

The variance of the Wiener process increases linearly with time [10], [61], [62] because \( B(t_2) - B(t_1) \propto (\sqrt{|t_2 - t_1|}) N(0,1) \), where \( N(0,1) \) describes a zero-mean and unit variance Gaussian random process. The variance is defined as

\[
\sigma^2_\alpha = c t 
\]

and the autocorrelation function of the process \( \alpha(t) \) is given by

\[
E[\alpha(t)\alpha(t+\tau)] = c \min(t,t+\tau) 
\]

In practice, the constant \( c \) is not readily available, but can be obtained through the phase noise 3dB bandwidth [55], [62], [70] denoted by \( \Delta f_{3dB} \). The relationship between \( c \) and \( \Delta f_{3dB} \) is given by

\[
c = \frac{\Delta f_{3dB}}{\pi f_c^2} 
\]

where \( f_c \) is the carrier frequency. Phase noise \( \theta(t) \) is also sometimes described as a zero mean Brownian motion process with variance \( 2\pi\beta t \), where \( \beta = 2\Delta f_{3dB} \). The relationship between \( c \) and \( \beta \) is obtained through the relation

\[
\sigma^2_\theta = 2\pi\beta t \\
= 4\pi^2 f_c^2 \sigma^2_\alpha \\
= 4\pi^2 f_c^2 c t \\
4\pi \Delta f_{3dB} = 4\pi^2 f_c^2 c \\
\underbrace{2\Delta f_{3dB}}_\beta = 2\pi f_c^2 c \\
\beta = 2\pi f_c^2 c 
\]
3.4.2 Phase-Locked Loop (PLL) Oscillator

The PLL realization of the local oscillator is the most popular implementation in practical wireless and electronics systems, and the model has been utilized in the works of [71], [70], [69], [18], [62], [65], [63], [72], [64]. In its simplest form, a PLL is basically a closed loop control system, and its functioning is based on phase detection of the phase difference between the input and output signals of the controlled oscillator. A block diagram of a typical PLL implementation is shown in Fig-(3.10). For the PLL synthesizer, a negative feedback loop is closed around a voltage controlled oscillator (VCO). The frequency of the VCO is controlled through the phase detector (PD) and the low-pass filter (LPF) by the reference signal. Although the reference signal is generated from a very stable oscillator, the signal in real world applications always exhibits a time deviation, denoted as $\alpha_{in}(t)$, compared with the desired signal. Since $\alpha_{in}(t)$ is at the output of the free-running oscillator, it is characterized as a Wiener process [63], [18], [62]. The time deviation at the output of the PLL is denoted as $\alpha_{VCO}(t)$. The time deviation at the input of the low-pass loop filter of the PLL $\beta(t)$ is defined as

$$\beta(t) = \alpha_{VCO}(t) - \alpha_{in}(t)$$

(3.21)
Figure 3.11  An illustration of the good approximation of the Tikhonov PDF by the Gaussian PDF, for small variances of the phase noise ($\sigma_\theta^2$)

In this case, the relation of the phase noise $\theta(t)$ to the random time shift $\alpha_{in}(t)$ is given by

$$\theta(t) = 2\pi f_c\left(\alpha_{VCO}(t)\right)$$
$$= 2\pi f_c\left(\beta(t) + \alpha_{in}(t)\right)$$

(3.22)

The phase noise for the PLL realization of the oscillator is generally modeled by a Tikhonov distribution [73], [66], [67]

$$f_\theta\left(\theta(t)\right) = \frac{\cos\left(\theta(t)\right)}{2\pi I_0\left(\frac{1}{\sigma_\theta^2}\right)} ; \quad |\theta(t)| < \pi$$

(3.23)

where $t$ denotes time, $I_0(x)$ is the $0^{th}$ order modified Bessel function of the first kind, and $\sigma_\theta^2$ is the variance of the phase noise.

The variance of the phase noise is inversely proportional to the loop SNR of the PLL, i.e., $\sigma_\theta^2 \propto \frac{1}{SNR}$. Depending on the instantaneous loop SNR inside the PLL, the PDF of the phase noise ranges from a Uniform distribution for low SNRs, to approaching an impulse function for infinite SNR [66], [67] as shown in Figure-3.11. It is also observed in Figure-3.11 that, for small values of the phase noise variance, the Tikhonov PDF can be well approximated by the Gaussian PDF.
Due to design limitations on the improvement of the quality of the oscillator, the PLL realization is more commonly utilized because they employ feedback loops, which help in containing frequency drifts common in free running oscillators. As a result, the Tikhonov model will be assumed in this thesis.
3.5 Phase Noise in Communication Systems

Oscillators, including free-running LOs and PLLs, are essential blocks in all wireless communication systems. However, the performance of the oscillator is limited by the impact of phase noise, reference spurs, and PLL lock time.

To understand the significance of phase noise of oscillators, consider a generic transmitter-receiver shown in Figure-(3.9), where a local oscillator (LO) provides the carrier signal for both the receive and transmit paths. If the LO output contains phase noise, both downconverted and upconverted signals are corrupted.

Spurs are noise concentrated at discrete offset frequencies from the carrier frequency. These offset frequencies are typically multiples of the channel spacing, exactly at frequencies of an adjacent user of the system [63], [72]. Spurs can combine with the signal of a desired user, and together be down-converted to baseband frequency, creating undesired noise products that appear in the desired frequency.

Oscillating circuits typically generate phase noise. At a lower offset from the carrier (in the close-in region), the phase noise has a tendency to increase the bit error rate (BER) and degrade the signal to noise ratio (SNR) of the system [72]. At farther offset frequencies from the carrier, phase noise can combine with other signals from other users in the system and create undesired noise products. Because it is a continuous function of the offset frequency, phase noise effects can also lead to the production of jamming signals.

Lock Time is defined as the time required by the PLL to change frequencies, and is dependent on the frequency spacing. Since data transmission is not permissible when a PLL is switching frequencies, a PLL must lock fast enough not to affect the data rate and maintain acceptable system performance. A shorter lock time also minimizes power consumption as the PLL can be in sleep mode for a longer time when there is no activity in the system.

In PLL frequency synthesis, reference sidebands and spurious outputs are an issue in system design. There are several types of spurious outputs with different causes. The most common is the reference spurs. Reference spurs are spurious output signals that occur at multiples of the reference frequency.

The impact of phase noise is well known and knowledge to combat such effects
is very valuable in wireless communication systems.
3.6 Phase Noise in OFDM

3.6.1 Effects of Phase Noise in OFDM

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation technique that is chosen as a basis for several wireless communication systems. It is implemented in several of the recent wireless systems due to its ability to combat impulsive noise and multipath effects and make better use of the system available bandwidth [74], [62]. Furthermore, to protect the detected symbol against time delayed versions of the previous symbol, which can cause inter-symbol interference (ISI), OFDM systems apply a guard interval, which is a cyclic extension of the OFDM symbols [70], [61].

A block diagram of an OFDM system is shown in Fig.3.12. The transmitted bits are fed into the channel encoder. The encoder provides channel error-correction capabilities by adding redundant bits to the transmitted data. The interleaver randomizes the encoded data, and is a simple yet powerful technique that is used to enable error correcting codes (such as the Reed-Solomon code) to perform burst error correction. The encoded and randomized bits are mapped into data symbols such as M-QAM, M-QPSK. Pilot symbols are inserted at specific positions for channel estimation purpose at the receiver. A cyclic prefix is added to the guard interval. The guard interval protects against intersymbol interference (ISI). The digital-to-analog converter (D/A) converts the OFDM signal to analog signal. The analog baseband OFDM signal at the output of the D/A is upconverted to RF by the oscillator for transmission through the wireless channel. At the receiver, the received RF signal is downconverted to baseband by the oscillator, followed by analog-to-digital conversion (A/D). The cyclic prefix is removed from the signal, and serial-to-parallel converted, followed by FFT by the FFT block. Channel estimation is performed on a subcarrier basis, making use of pilot symbols inserted at specific locations at the transmitter. The symbols are demapped to the transmitted bits by the M-QAM symbol demapper. The de-interleaver reverses the interleaving process performed at the transmitter, followed by decoding by the decoder.

The ubiquitous utilization of OFDM for its high data rate capability and
resistance to multipath effects, calls for a careful consideration and thorough understanding of the effects of hardware impairments and synchronization issues in system design. The high data rate in OFDM is achieved by employing a large number of subcarriers for a fixed bandwidth $W$. The large number of subcarriers is accomplished by reducing the subcarrier spacing. Unfortunately, phase noise limits the subcarrier spacing of the system [75]. This can be explained from the fact that higher phase noise implies larger relative phase noise bandwidth $\Delta f_{\text{dB}}/\Delta f_{\text{carrier}}$ for a given system bandwidth $W$, and a system with a larger relative phase noise bandwidth is more susceptible to synchronization errors. An accurate prediction of the tolerable phase noise is therefore very important, and will allow system designers to carefully relax design specifications of the OFDM system. Quantifying the phase noise is even more important in systems that are operating at much higher frequencies, as the effects of phase noise also increase with an increase in the carrier frequency [34], [72], [64], [63].

Striving for higher data rates in combination with high spectral efficiency, it is seen that solutions for the phase noise problem become even more crucial, due to the fact that a system that is almost free of phase noise permits the adequate use of higher order modulation. In addition, a system that is design against phase noise permits the use of a large number of subcarriers in system implementation as well. With recent advances in device technology, the use of higher frequency bands around 20-60 GHz, where substantial amount of unused bandwidth is available, becomes an option for next generation WLANs and other wireless applications such as cellular communication [62], [55], [57]. However, because the effect of a noisy oscillator can grow quadratically with the carrier frequency, the phase noise problem becomes even more critical at these high frequencies [57], [55]. Thus, the knowledge of the effects of phase noise, and keeping such effects within tolerable levels will play a crucial role in future next generation wireless communication systems.

The high spectral efficiency of OFDM is achieved by applying partly overlapping but orthogonal spectra of the different subcarriers. An illustration of overlapping subcarrier spectra to achieve the high spectral efficiency is shown in Fig-(3.13). When the OFDM system is perfectly synchronized, the subcarriers are orthogonal. However, due to imperfect local oscillators (LO) at either transmitter (TX) or receiver (RX) of the communication system, phase noise (PN) causes a spread in the spectra
of the subcarriers. The spread leads to spectral leakage of adjacent subcarriers into a desired subcarrier and destroys the orthogonality between subcarriers. As a result of the non-orthogonality, inter-carrier interference (ICI) occurs. These imperfections in the LOs will increasingly continue to present unwanted limitations on the performance of OFDM systems as next generation systems venture into the high GHz frequency range.

Low-cost implementations of oscillators or systems operating at high carrier frequencies are more susceptible to the effects of phase noise, since it is harder to design sufficiently stable oscillators in these scenarios. Therefore, it is very important to understand the influence of imperfect oscillators, i.e. phase noise as well as carrier frequency offset on the system performance.

The influence of phase noise on the performance of an OFDM system has been investigated and analyzed in several publications [14], [15], [10], [16], [19], [12], [76], [13], [77], [69], [62], [54], [78], [58], [59], [60], [21], [18], [70], [61], [68].

These works generally state that the influence of phase noise can be split into a multiplicative part, which is common to all subcarriers. It is therefore often referred to as common phase error (CPE). The second part is additive, and is often referred to as intercarrier interference (ICI). The CPE is identified in some works as the main performance limiting factor for coherent detection based receivers, and many
adequate correction approaches for the CPE term have been proposed [79], [11], [16]. Some approaches for correction of the ICI have also been proposed in [15], [17], [68], [61], [13], [80], [62]. In these works however, a high signal-to-noise ratio (SNR) is required to achieve reasonable performance gains with typically high implementation complexity.

In all practical systems, data detection must be accomplished in the presence of such effects as phase noise and unknown channel conditions. Some authors including [76] have attempted to describe in similar terms the effects of phase noise and unknown channel conditions. However, the channel impulse response (CIR) changes slowly with respect to the OFDM symbol rate, and, hence, channel estimates obtained at some time in the past remain valid in the present and can be used for data detection\(^4\). On the other hand, phase noise varies rapidly and estimates obtained in one OFDM symbol are not strongly correlated with the phase noise process in another symbol interval. Hence, phase noise cannot be easily mitigated by using a training symbols approach.

The effect of phase noise on the system performance has been further investigated [74], [16], [17], [10], [61], [18], [81], [82], [62], [70], [19] and is found that OFDM is much more sensitive to phase noise than a single carrier system [19], [76]. Tomba in [19] provides a more detailed treatment on the error probability of an OFDM sys-

\(^4\)This statement assumes frequency flat fading, which is true for OFDM per subcarrier.
tem in the presence of phase noise for different modulation schemes. Various methods to estimate and mitigate the effect of phase noise have also been investigated and presented in [62], [61], [18]. The CPE is shown to depend on the average value of the phase noise over one OFDM symbol and has the same effect on all subcarriers. The second component which is the intercarrier interference (ICI), is induced by random phase noise.

The properties of the ICI are studied by several authors, including [69], [10], [17]. These works, implicitly or explicitly, assume the ICI term is complex Gaussian, due to the Central-Limit-Theorem (CLT). This is, however, an approximation, as noted by the authors of [77], [10] and is only valid for some combinations of phase noise and subcarrier spacings.

Methods for the correction of the ICI were proposed in [17], [62], [68], [18], although the ones proposed in [17], [82] require high Signal-to-Noise ratio (SNR) in order to achieve reasonable performance. The work in [83] presented the SNR expressions for small angle phase noise. In [13], the authors presented a closed-form expression for the Signal-to-Interference-plus-Noise ratio (SINR) of the OFDM system affected by phase noise, together with a general phase-noise suppression algorithm and a good overview of the phase noise problem as well.

From the above summary, it is observed that since any practical coherent OFDM receiver is assumed to implement some kind of CPE correction, the main apparent performance limiting factor in phase noise impaired systems is the ICI. For this reason, we focus on the effects of the ICI contribution on the degradation of the system performance.

### 3.7 Conclusion

In this Chapter, a description of the theory of phase noise is presented. The oscillator, which is the primary source of phase noise in wireless communication systems, is described. Technical characterizations of the phase noise in oscillators is also discussed, including a presentation of the different distributions frequently used to describe the phase noise models.
4

Performance Analysis of SISO-OFDM with Phase Noise and Imperfect Channel Estimation

4.1 Introduction

In Chapter-3, we presented an overview of phase noise. We further characterize the phase noise and describe its effects on OFDM systems.

In this Chapter, we analyze the performance of OFDM systems that employ single antenna at both the transmitter and receiver. We derive the bit error probability for the OFDM system that is impaired by phase noise, and subject to channel estimation errors. We further investigate the effect of the distribution of the phase noise on the system performance.

We first assume the receiver has perfect knowledge of the channel, and derive the probability of bit error. In practical applications however, this assumption may not hold, even in slowly time-varying channels. We next extend the analysis further, to the case when the receiver does not have perfect channel knowledge.

Due to its remarkable data rate capability, superior spectral efficiency, and
robustness to frequency selective fading channels, OFDM is a promising candidate for next generation wireless communication, as these emerging wireless services require higher bandwidths to support not only voice and data, but multimedia formats as well. An attractive property of OFDM is that modulation and demodulation is efficiently implemented by the Discrete Fourier Transform (DFT). Though OFDM is robust to ISI compared to its counterparts, the performance and ease of implementation critically depends on the orthogonality between subcarriers.

It is known that the highly desirable orthogonality property in OFDM systems is destroyed by hardware imperfections such as local oscillator phase noise [83], [61], [16], [81]. Furthermore, due to its longer symbol duration compared to single-carrier systems, the orthogonality between subcarriers is also destroyed by Doppler effects due to the nature of fast fading channels [84], [24], [85]. In general, the loss of orthogonality causes dispersion of signal power from a particular subcarrier unto adjacent subcarriers, and results in intercarrier interference (ICI). As a consequence, the system bit error rate (BER) is severely degraded. An added performance degradation also results from imperfect CSI [86], [87], [88], [89], [90], [91].

The effects of phase noise in OFDM have been investigated and is available in the literature [16], [83], [61], [81]. In all these studies, perfect CSI is assumed. On the other hand, the study in [92] considered the effects of power amplifier and a jamming signal but with no phase noise or channel time-variations effects.

In addition, it is also known that for any robust communication system design, very accurate channel estimates are highly desirable as this has significant consequences on the performance of the system. The works of [84], [24], [85] investigated the performance of OFDM in time-varying fading channels. However, the effects of phase noise or of a jamming signal is not considered in these studies.

With the effects of phase noise significantly decreasing system performance through CPE and ICI contributions, different approaches have been proposed to eliminate, or at least considerably minimize, phase noise using in-band pilots [16], [10], [61], [18], [76], [15], [14], [93], [80]. These approaches can be categorized into a time-domain [59], [58], and a frequency-domain approach [12], [80].

The time-domain approach mitigates phase noise at the receiver before the Fast Fourier transform (FFT). The time-domain approach presented in [59], estimates
phase noise using a pilot tone surrounded by the guard band. This approach requires a special pilot pattern which, however, restricts its applicability. A more general approach has been proposed such as in [58] which interprets the time domain phase noise by orthogonal transforms, and turns phase noise mitigation into the recovery of orthogonal waveforms. This approach requires a large number of pilots to guarantee a satisfactory bit error rate (BER) performance.

The frequency-domain approach involves the correction of CPE and ICI. The method introduced in [12], [79], [16], compensates for the CPE. Such a method may not always be effective as it neglects ICI, an important contribution of phase noise. The authors in [17], [80] investigated the correction/compensation of the ICI term, in which the ICI is approximated as random Gaussian noise and obtained estimates of the ICI power. The work in [62] further compared the performance of free-running oscillator and PLL-based oscillator using the phase noise correction scheme. A system that employs the PLL-based oscillator shows improved performance with phase noise correction due to the fact that the PLL filters low-frequency components of the phase noise spectrum [71], [57], [56], [55]. This makes CPE correction easier and facilitates a improved ICI correction. By simultaneously mitigating both CPE and ICI, the above methods provides performance gain over the CPE-only correction.

In all the above work, it is assumed that perfect channel estimation is achieved at the receiver, which is not the case in any practical receiver in a wireless communication system. For correct signal detection, channel estimation is very important in wireless communication system. A good channel estimation scheme should provide reliable information on the state of the channel at the receiver. This enables the receiver to effectively compensate for the various effects of the channel and accurately estimates the transmitted symbols. The performance of OFDM systems in the presence of channel estimation error has received considerable attention from researchers. The authors in [94] analyzed the performance of an un-coded OFDM system in quasi-static channels, and employed optimum training for the acquisition of the channel state information. Applying a linear minimum mean square error (LMMSE) channel estimator, they considered the performance of both MPSK and MQAM modulations. The authors in [95] examined the performance degradation due to channel estimation error in OFDM based IEEE 802.11 WLAN systems. The average effective SNR
and average bit error probability are derived for Rayleigh fading channels. In [96], a systematic approach is presented to derive the bit error probability of OFDM systems in the presence of channel estimation errors, and closed form expressions for the bit error probability are derived for MPSK modulations, employing a linear pilot assisted channel estimation scheme. The authors in [97] used a similar approach to that in [98], and derived bit error probability expressions for OFDM systems in the presence of channel estimation error.

As robust designs are achieved when comprehensive investigation of practical system impairments are considered, it is observed from above that such a study that accounts for these effects will be very valuable. As such, this chapter investigates the system performance, by analysis and simulations, under a permutation of these impairments.

In this chapter, the performance of OFDM systems in the presence of phase noise and channel estimation errors is investigated, taking into account the scenarios of different distributions of the phase noise. We derive the analytical expression for the bit error probability of the OFDM system in the presence of channel estimation errors. Finally, the analysis is verified by employing system parameters of the IEEE 802.16 standard and comparing the simulation results with analysis.

### 4.2 Performance Analysis

A block diagram of the Single Input, Single Output (SISO) OFDM receiver is shown in Fig-4.1.

The transmitted bits are fed into the channel encoder. The encoder provides channel error-correction capabilities by adding redundant bits to the transmitted data. The interleaver randomizes the encoded data, and is a simple yet powerful technique that is used to enable error correcting codes (such as the Reed-Solomon code) to perform burst error correction. The encoded and randomized bits are mapped into data symbols such as M-QAM, M-QPSK. In the analysis, M-QAM data symbol mapping is assumed. Pilot symbols are inserted at specific positions for channel estimation purpose at the receiver.
4.2.1 Transmitted OFDM Signal

A block of $N$ complex-valued data symbols $\{X(k)\}_{k=0}^{N-1}$ are grouped and converted into a parallel set to form the input to the OFDM modulator, where $k$ is the subcarrier index and $N$ is the number of subcarriers. The modulator consists of an Inverse Discrete Fourier transform (IFFT) block. The output of the IFFT is the complex baseband modulated OFDM symbol in discrete time domain and is given by

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} ; \quad 0 \leq n \leq N - 1 \quad (4.1)$$

A cyclic prefix is added to the guard interval. The guard interval protects against intersymbol interference (ISI). The digital-to-analog converter (D/A) converts the OFDM signal to analog signal.

4.2.2 Phase Noise Model

The analog baseband OFDM signal at the output of the D/A is upconverted to RF by the oscillator for transmission through the wireless channel. Free-running oscillators, which can produce a carrier reference signals for RF signal transmission, are prone to frequency drifts. Making use of this type of oscillator in practical system design can lead to implementation issues such as synchronization between transmitter
and receiver. Furthermore, a drifting carrier signal can cause interference to neighboring frequency bands, which is undesirable in any communication system.

Due to these limitations on the quality of the oscillator, the PLL realization is more commonly utilized because it is more robust against the effects of drifts that is common in free running oscillators. The phase noise associated with the PLL realization of the oscillator is modeled by a Tikhonov PDF. As a result, the Tikhonov model is assumed in this thesis.

The phase noise \( \theta(n) \) is modeled by a Tikhonov distribution \([73], [66], [67]\), and is given by

\[
f_\theta(\theta(n)) = \frac{\cos(\theta(n))}{2\pi I_0(\frac{\sigma_\theta^2}{2\pi})}; \quad |\theta(n)| < \pi
\]

(4.2)

where \( n \) denotes time index, \( I_0(\alpha) \) is the 0\(^{th}\) order modified Bessel function of the first kind, and \( \sigma_\theta^2 \) is the variance of the phase noise.

### 4.2.3 Channel Model

The modulated RF signal is transmitted through the wireless channel. The channel is modeled by a tapped delay line with channel coefficients that are assumed to be slowly varying such that they are considered almost constant over the OFDM block. The channel frequency response for the \( k^{th} \) subcarrier is

\[
H(k) = \sum_{p=0}^{L-1} h(p)e^{j\frac{2\pi pk}{N}}
\]

(4.3)

where \( h(p) \) is the complex channel gain of the \( p^{th} \) multipath component.

### 4.2.4 Received SISO-OFDM Signal

At the receiver, the received RF signal is downconverted to baseband by the oscillator, followed by analog-to-digital conversion (A/D). The cyclic prefix is removed from the signal, and serial-to-parallel converted, followed by FFT by the FFT block. Channel estimation is performed on a subcarrier basis, making use of pilot symbols inserted at specific locations at the transmitter. The symbols are demapped to
the transmitted bits by the M-QAM symbol demapper. The de-interleaver reverses the interleaving process performed at the transmitter, followed by decoding by the decoder.

The received signal at the input to the FFT block in the receiver, is given by

\[ y(n) = \left[ \sum_{p=0}^{L-1} h(p)x(n-p) + w(n) \right] e^{j\theta(n)} \]

\[ = \left[ \sum_{p=0}^{L-1} h(p) \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi(n-p)k}{N}} + w(n) \right] e^{j\theta(n)} \] (4.4)

where \( \theta(n) \) is the phase noise.

The complex Gaussian random variable \( w(n) \) represents the Additive White Gaussian Noise term with \( E[|w(n)|^2] = \sigma_w^2 \). It is assumed that the channel on each subcarrier is flat and slowly time-varying, such that it is considered almost constant over a symbol duration. Using DFT, the received signal \( y(n) \) is demodulated by the OFDM demodulator, and the demodulated OFDM signal at the \( l \)th subcarrier is given by,

\[ Y(l) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n)e^{-\frac{j2\pi ln}{N}} \]

\[ = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{p=0}^{L-1} \sum_{k=0}^{N-1} h(p)X(k)e^{\frac{j2\pi(n-p)k}{N}} e^{-j2\pi pk/N} e^{j\theta(n)} \]

\[ + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w(n)e^{j\theta(n)} e^{-\frac{j2\pi ln}{N}} \]

\[ = (\frac{1}{N}) \sum_{n=0}^{N-1} H(l)X(l)e^{j\theta(n)} \]

\[ + (\frac{1}{N}) \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} H(k)X(k)e^{\frac{j2\pi(k-l)n}{N}} e^{j\theta(n)} + W(l) \]

\[ = H(l)X(l)V_A + \sum_{k=0}^{N-1} H(k)X(k)V_B(k-l) + W(l) \]

\[ = D_{siso}(l) + \beta_{siso}(l) + W_{siso}(l) \] (4.5)
The first term \( D_{\text{siso}}(l) \) includes the desired signal term on the \( l \)th subcarrier, \( \beta_{\text{siso}}(l) \) is the ICI term on the \( l \)th subcarrier, and \( W_{\text{siso}}(l) \) is the noise term, respectively. And we define

\[
V_A = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\theta(n)} \tag{4.6}
\]

\[
V_B(k - l) = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j2\pi(k-l)n/N} e^{j\theta(n)} \tag{4.7}
\]

The term \( V_A \) which is common to all subcarriers, is frequently referred to as the common phase error (CPE). It can be corrected using pilots or other special techniques [10], [61]. The term \( V_B(l - k) \) is the contribution of the RF impairments destroying the orthogonality of the subcarriers and leading to intercarrier interference (ICI).

### 4.3 Perfect CPE Correction

We derive the BER of the phase noise impaired system starting with the perfect CSI case, and extending to that with imperfect CSI. It is assumed that the phase noise of the local oscillators at the transmitter and the receiver are independent and identically distributed. In order to evaluate the performance of the system, we need to characterize the statistics of the received signal. We consider only the effect of phase noise at the receiver and assume that the transmitted data \([X(0) \ldots X(N-1)]\) on all sub-carriers are independent.

#### 4.3.1 SISO Receiver with Perfect CSI

The detected symbol \( \tilde{X} \) on the \( l \)th subcarrier is given by

\[
\tilde{X}_{\text{siso}}(l) = Y(l)H^*(l) = H(l)X(l)V_AH^*(l) + W(l)H^*(l) + \sum_{k=0,k\neq{l}}^{N-1} H(k)X(k)V_B(k - l)H^*(l)
\]

Assuming that the CPE term \( V_A \) is effectively estimated and fully compensated
by using pilots, the detected symbol $\tilde{X}$ on the $l^{th}$ subcarrier is given by

$$
\tilde{X}_{siso}(l) = H(l)X(l)H^*(l) + W(l)H^*(l)
$$

$$
+ \sum_{k=0,k\neq l}^{N-1} H(k)X(k)V_B(k-l)H^*(l)
$$

$$
= |H(l)|^2 X(l)
$$

$$
\tilde{D}_{siso}(l)
$$

$$
+ \sum_{k=0,k\neq l}^{N-1} H(k)X(k)V_B(k-l)H^*(l)
$$

$$
\tilde{\beta}_{siso}(l)
$$

$$
+ W(l)H^*(l)
$$

$$
\tilde{W}_{siso}(l)
$$

(4.8)

where $H(l)^*$ implies complex conjugate of $H(l)$. In (4.8), $\tilde{D}_{siso}(l)$ is the desired signal term. The second term $\tilde{\beta}_{siso}(l)$ is the ICI term which is approximated as a Gaussian random variable. The proof of the Gaussianity of the ICI is presented in the Appendix. The last term is the additive Gaussian noise term. The above equation can be written compactly as

$$
\tilde{X}_{siso}(l) = \tilde{D}_{siso}(l) + \tilde{\beta}_{siso}(l) + \tilde{W}_{siso}(l)
$$

(4.9)

Conditioned on $\theta(n)$ and the channel $H(l)$ on the $l^{th}$ subcarrier, it is observed that $\tilde{X}_{siso}(l)$ is a zero-mean complex Gaussian random variable with variance ($\sigma_{\tilde{\beta}_{siso}}^2 + \sigma_{\tilde{W}_{siso}}^2$).

### 4.3.2 Mean and Variance of ICI and Noise

The variance of the noise term $\tilde{W}_{siso}(l)$ is given by

$$
\sigma_{\tilde{W}_{siso}}^2 = E[\tilde{W}_{siso}(l)\tilde{W}_{siso}^*(l)]
$$

$$
= \sigma_W^2 |H(l)|^2
$$

(4.10)

where $\sigma_W^2$ is the variance of the additive Gaussian noise.
For the ICI term, since $H(k)$, $X(k)$ are zero mean and independent of random variables, it can be seen that the ICI term is zero mean.

The variance $\sigma_\text{siso}^2$ of the ICI term is the second moment of $\tilde{\beta}_\text{siso}(l)$, and conditioned on $\theta(n)$ and the channel $H(l)$, it is derived using a similar approach in [10] as follows:

$$\sigma_\text{siso}^2 = E[|\tilde{\beta}_\text{siso}(l)|^2]$$
$$= |H(l)|^2 E\left[\sum_{k=0 \atop k \neq l}^{N-1} X(k)H(k)V_B(k-l)|^2\right]$$
$$= |H(l)|^2 E_s \sum_{k=0 \atop k \neq l}^{N-1} |V_B(k-l)|^2$$

(4.11)

where we have assumed that the $E[|H(k)|^2] = 1$, and $E_s$ is the transmitted symbol energy. We will assume (without loss of generality), that the desired subcarrier is the 0th subcarrier ($l = 0$). Then $\sigma_\text{siso}^2$ is given by

$$\sigma_\text{siso}^2 = |H(0)|^2 E_s \sum_{k=1}^{N-1} |V_B(k)|^2 - |V_B(0)|^2$$

(4.12)

Since $V_B(k)$ is complex, the sum $\sum_{k=0}^{N-1} |V_B(k)|^2$ can be written as,

$$\sum_{k=0}^{N-1} |V_B(k)V_B^*(k)| = \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}} \sum_{k=0}^{N-1} e^{j2\pi \frac{k(n-m)}{N}}$$

(4.13)

where $\Delta \theta_{nm} = [\theta(n) - \theta(m)]$ is the phase noise increment at different time instances. Then, $|V_B(0)|^2$ can be written as

$$|V_B(0)|^2 = \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}}$$

(4.14)

From the orthogonality identity of complex exponentials,

$$\frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi \frac{(n-m)}{N}} = \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases}$$

(4.15)
and since $e^{j\Delta \theta_{nm}} = 1$ for $n = m$, the summation is given by

$$\sum_{k=0}^{N-1} [V_B(k)V^*_B(k)] = 1.$$ Therefore

$$\sigma_{\beta_{\text{siso}}}^2 = E_s |H(0)|^2 \left[ 1 - \left[ |V_B(0)|^2 \right] \right]$$

$$= E_s |H(0)|^2 \left[ 1 - \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}} \right]$$

$$= E_s |H(0)|^2 \Phi_{\Delta \theta}$$

$$= E_s \sigma_{\beta_{\text{siso}}}^2$$  \hspace{1cm} (4.16)$$

where we define $\sigma_{\beta_{\text{siso}}}^2 = |H(0)|^2 \Phi_{\Delta \theta}$ and

$$\Phi_{\Delta \theta} = \left[ 1 - \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}} \right]$$

$$\hspace{1cm} (4.17)$$

**4.3.3 Probability of Error**

We present the bit error rate (BER) analysis for the case of 16QAM modulation using Gray code mapping for $(b_1b_2b_3b_4)$, as shown in Figure-4.2.
Although only 16-QAM is considered in this presentation, note that the following analysis is valid for all square QAM constellations. Conditioned on $\theta(n)$, $H(l)$, the conditional bit error probability (BEP) for bit $b_1$ on the $l^{th}$ subcarrier is given by

$$P_{e(siso)}(b_1|H(l), \theta(n)) = \frac{1}{2} \left[ P_r(\tilde{X}_I < 0 | X_I = d, H(l), \theta(n)) + P_r(\tilde{X}_I < 0 | X_I = 3d, H(l), \theta(n)) \right]$$

(4.18)

where $X_I, \tilde{X}_I$ are the real parts of $X, \tilde{X}$ respectively, and $d^2$ is proportional to the average symbol energy $E_s$ such that $E_s = 10d^2$ (note that $2d$ denotes the distance between two neighboring points on the signal constellation). With four bits per symbol, the energy per bit $E_b$ is given by $E_b = 5d^2/2$. The conditional bit error probability for bit $b_3$ is

$$P_{e(siso)}(b_3|H(l), \theta(n)) = \frac{1}{2} \left[ P_r(|\tilde{X}_I| > 2d | X_I = d, H(l), \theta(n)) + P_r(|\tilde{X}_I| < 2d | X_I = 3d, H(l), \theta(n)) \right]$$

(4.19)

Rewriting (4.18) and (4.19) as a sum of Q-functions, and substituting for $d$ as a function of $E_b$, the conditional BEP for bit $b_1$ can be written as

$$P_{e(siso)}(b_1|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left(\sqrt{\frac{d^2}{\sigma_{\beta siso}^2 + \sigma_{W siso}^2}}\right) + Q\left(\sqrt{\frac{9d^2}{\sigma_{\beta siso}^2 + \sigma_{W siso}^2}}\right) \right]$$

$$= \frac{1}{2} \left[ Q\left(\sqrt{\frac{(2E_b/5)|H|^2}{(4E_b)\sigma_{\beta siso}^2 + \sigma_{W siso}^2}}\right) + Q\left(\sqrt{\frac{9(2E_b/5)|H|^2}{(4E_b)\sigma_{\beta siso}^2 + \sigma_{W siso}^2}}\right) \right]$$

$$= \frac{1}{2} \left[ Q\left(\sqrt{\frac{(2E_b/5)|H|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma_{W}^2}}\right) + Q\left(\sqrt{\frac{9(2E_b/5)|H|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma_{W}^2}}\right) \right]$$

(4.20)
where $\Phi_{\Delta \theta}$ is defined in (4.17). The term $\sigma^2_\beta = \Phi_{\Delta \theta}|H|^2$ and $\sigma^2_\beta = \sigma^2_W|H|^2$.

Similarly, after substitution and simplification, the conditional BEP for bit $b_3$ is given by

$$
P_{e(siso)}(b_3|H(l), \theta(n)) = \frac{1}{2} \left[ Q(\sqrt{\frac{9d^2}{\frac{\sigma^2_\beta}{\beta_{siso}} + \frac{\sigma^2_\beta}{W_{siso}}}}) ight.
$$
$$
+ Q(\sqrt{\frac{d^2}{\frac{\sigma^2_\beta}{\beta_{siso}} + \frac{\sigma^2_\beta}{W_{siso}}}})
$$
$$
+ Q(\sqrt{\frac{d^2}{\frac{\sigma^2_\beta}{\beta_{siso}} + \frac{\sigma^2_\beta}{W_{siso}}}})
$$
$$
- Q(\sqrt{\frac{25d^2}{\frac{\sigma^2_\beta}{\beta_{siso}} + \frac{\sigma^2_\beta}{W_{siso}}}}) \right] \quad (4.21)$$

$$
P_{e(siso)}(b_3|H(l), \theta(n)) = \frac{1}{2} \left[ Q(\sqrt{\frac{9(2E_b/5)|H|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2_W}}) ight.
$$
$$
+ Q(\sqrt{\frac{(2E_b/5)|H|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2_W}})
$$
$$
+ Q(\sqrt{\frac{(2E_b/5)|H|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2_W}})
$$
$$
- Q(\sqrt{\frac{25(2E_b/5)|H|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2_W}}) \right] \quad (4.22)$$

Note that (4.20) and (4.22) depend on $|H|$, which follows a Rayleigh distribution with PDF

$$
f_{|H|}(\alpha) = \frac{\alpha}{\sigma^2_\alpha} e^{-\frac{\alpha^2}{2\sigma^2_\alpha}}, \quad \alpha \geq 0 \quad (4.23)$$

and $\sigma^2_\alpha$ is the variance of $|H|$.

Due to the symmetry of square M-QAM constellations, the BEP for the in-phase and quadrature bits are equal such that $P_e(b1) = P_e(b2)$ and $P_e(b3) = P_e(b4)$. Therefore the average BER is obtained by averaging the conditional BEP of $b_1$ in (4.20) and $b_3$ in (4.22) over the PDF of $\theta(n)$ and $|H|$. The average BER is thus given by
\[ P_{e(siso)} = \frac{1}{2} \int_{|H|} f(|H|) d|H| \int_{\theta} f(\theta) d\theta \times \left[ P_{e(siso)}(b_1|\theta, |H|) + P_{e(siso)}(b_3|\theta, |H|) \right] \]

\[ = \frac{1}{2} \int_{0}^{\infty} f(|H|) d|H| \int_{-\pi}^{\pi} f(\theta) d\theta \times \left[ P_{e(siso)}(b_1|\theta, |H|) + P_{e(siso)}(b_3|\theta, |H|) \right] \quad (4.24) \]

4.3.4 SISO Receiver with Imperfect CSI

In the presence of imperfect channel estimation, we assume a channel estimation model such that the channel estimate \( \hat{H} \) of the true channel \( H \) is given by

\[ \hat{H} = H + \varepsilon \quad (4.25) \]

where \( \varepsilon \) is the error in the channel estimate, and is modeled as zero-mean complex Gaussian random variable with variance \( 2\sigma_\varepsilon^2 \), and is independent of \( H \).

We assume that the common phase error term \( V_A \) is effectively estimated and fully compensated for using pilots. Using similar steps as in the previous section, the detected symbol \( \tilde{X} \) on the \( l^{th} \) subcarrier is given by
\[
\tilde{X}_{sioso}^\varepsilon(l) = Y(l)\tilde{H}^\ast(l) \\
= Y(l) \left[ H^\ast(l) + \varepsilon^\ast(l) \right] \\
= H(l)X(l) \left[ H^\ast(l) + \varepsilon^\ast(l) \right] \\
+ \left[ H^\ast(l) + \varepsilon^\ast(l) \right] \sum_{k=0,k\neq l}^{N-1} H(k)X(k)V_B(k-l) \\
+ W(l) \left[ H^\ast(l) + \varepsilon^\ast(l) \right] \\
= \underbrace{|H(l)|^2X(l)}_{\tilde{D}_{sioso}(l)} + \underbrace{H(l)X(l)\varepsilon^\ast(l)}_{\tilde{\Psi}_{sioso}(l)} \\
+ \left[ H^\ast(l) + \varepsilon^\ast(l) \right] \sum_{k=0,k\neq l}^{N-1} H(k)X(k)V_B(k-l) \\
+ W(l) \left[ H^\ast(l) + \varepsilon^\ast(l) \right] \\
\quad + \underbrace{W^\ast(l)\varepsilon(l)}_{\tilde{W}^\varepsilon_{sioso}(l)} \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quad
where $\sigma_N^2$ is the variance of the Gaussian noise, and $\sigma_e^2$ the variance of the error in the channel estimate.

The variance of $\tilde{\Psi}_{siso}(l)$ on the $l^{th}$ subcarrier is given by

$$
\sigma_{\tilde{\Psi}_{siso}}^2 = E[\tilde{\Psi}_{siso}(l)\tilde{\Psi}^*_{siso}(l)]
= E_s |H|^2 \sigma_e^2
= E_s \sigma_{\tilde{\Psi}_{siso}}^2
$$

(4.29)

where $\sigma_{\tilde{\Psi}_{siso}}^2 = |H|^2 \sigma_e^2$ and $\sigma_{\tilde{\Psi}_{siso}} = E_s \sigma_{\tilde{\Psi}_{siso}}^2$.

Similarly, the variance of the ICI term $\tilde{\beta}_{\epsilon_{siso}}^{(l)}$ is given by

$$
\sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2 = E[\tilde{\beta}_{\epsilon_{siso}}^{(l)}\tilde{\beta}_{\epsilon_{siso}}^*(l)]
= E_s \Phi_\Delta\theta \left(|H|^2 + \sigma_e^2\right)
= E_s \sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2
$$

(4.30)

where $\sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2 = E_s \sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2$ and $\sigma_{\tilde{\beta}_{\epsilon_{siso}}} = \Phi_\Delta\theta \left(|H|^2 + \sigma_e^2\right)$ with $\Phi_\Delta\theta$ given by (4.17).

### 4.3.6 Probability of Error

The conditional BEP for bit $b_1$ can be written as

$$
P_{e_{siso}}(b_1|H(l),\theta(n)) = \frac{1}{2} \left[ Q \left( \sqrt{\frac{d^2}{\sigma_{\tilde{\Psi}_{siso}}^2 + \sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2 + \sigma_{W_{siso}}^2}} \right) 
+ Q \left( \sqrt{\frac{9d^2}{\sigma_{\tilde{\Psi}_{siso}}^2 + \sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2 + \sigma_{W_{siso}}^2}} \right) \right]$$

$$
= \frac{1}{2} \left[ Q \left( \sqrt{\frac{(2E_b/5)\left(|H|^2\right)^2}{\sigma_{\tilde{\Psi}_{siso}}^2 + (4E_b)\sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2 + \sigma_{W_{siso}}^2}} \right) 
+ Q \left( \sqrt{\frac{9(2E_b/5)\left(|H|^2\right)^2}{\sigma_{\tilde{\Psi}_{siso}}^2 + (4E_b)\sigma_{\tilde{\beta}_{\epsilon_{siso}}}^2 + \sigma_{W_{siso}}^2}} \right) \right]
$$

(4.31)
The conditional BEP for bit $b_3$ can be written as

$$P_{e(siso)}(b_3|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9d^2}{\sigma_{\Psi_{siso}}^2 + \sigma_{\beta_{siso}}^2 + \sigma_{W_{siso}}^2}} \right) + Q\left( \sqrt{\frac{d^2}{\sigma_{\Psi_{siso}}^2 + \sigma_{\beta_{siso}}^2 + \sigma_{W_{siso}}^2}} \right) \right]$$

$$+ Q\left( \sqrt{\frac{d^2}{\sigma_{\Psi_{siso}}^2 + \sigma_{\beta_{siso}}^2 + \sigma_{W_{siso}}^2}} \right)$$

$$- Q\left( \sqrt{\frac{25d^2}{\sigma_{\Psi_{siso}}^2 + \sigma_{\beta_{siso}}^2 + \sigma_{W_{siso}}^2}} \right)$$

As in the case with perfect channel estimation, due to the symmetry of square M-QAM constellations, the average BER is obtained by averaging the conditional BEP of $b_1$ in (4.31) and $b_3$ in (4.32) over the PDF of $\theta(n)$ and $|H|$. The average BER is thus given by
\[
\mathcal{P}_{e(siso)} = \frac{1}{2} \int_{|H|} f(|H|) d|H| \int_{\theta} f(\theta) d\theta \\
\times \left[ P_{e(siso)}^e \left( b_1 \theta, |H| \right) + P_{e(siso)}^e \left( b_3 \theta, |H| \right) \right] \\
= \frac{1}{2} \int_{0}^{\infty} f(|H|) d|H| \int_{-\pi}^{\pi} f(\theta) d\theta \\
\times \left[ P_{e(siso)}^e \left( b_1 \theta, |H| \right) + P_{e(siso)}^e \left( b_3 \theta, |H| \right) \right] 
\] (4.33)
4.4 No CPE Correction

In this section, the effect of the CPE is taken into account in order to observe the degradation without prior CPE correction.

4.4.1 SISO Receiver with Perfect CSI

The detected symbol \( \tilde{X} \) on the \( l \)th subcarrier is given by

\[
\tilde{X}_{\text{siso}}(l) = Y(l)H^*(l)
= H(l)X(l)V_A H^*(l) + W(l)H^*(l)
+ \sum_{k=0\text{,} k\neq l}^{N-1} H(k)X(k)V_B (k - l) H^*(l)
\]

where \( H(l)^* \) implies complex conjugate of \( H(l) \).

The above equation can be written compactly as

\[
\tilde{X}_{\text{siso}}(l) = \tilde{D}_{\text{siso}}(l) + \tilde{\beta}_{\text{siso}}(l) + \tilde{W}_{\text{siso}}(l)
= \left( \tilde{D}_{\text{siso}}^I(l) + \tilde{\beta}_{\text{siso}}^I(l) + \tilde{W}_{\text{siso}}^I(l) \right)
+ j \left( \tilde{D}_{\text{siso}}^Q(l) + \tilde{\beta}_{\text{siso}}^Q(l) + \tilde{W}_{\text{siso}}^Q(l) \right)
= \tilde{X}_{\text{siso}}^I(l) + j \tilde{X}_{\text{siso}}^Q(l)
\] (4.34)

where \( \tilde{X}_{\text{siso}}^I(l) \) represents the in-phase component of \( \tilde{X}_{\text{siso}}(l) \), and \( \tilde{X}_{\text{siso}}^Q(l) \) represents the quadrature component of \( \tilde{X}_{\text{siso}}(l) \).

Conditioned on \( \theta(n) \) and the channel \( H(l) \) on the \( l \)th subcarrier, it is observed that \( \tilde{X}_{\text{siso}}(l) \) is a zero-mean complex Gaussian random variable with variance \( \sigma_{\tilde{D}_{\text{siso}}}^2 + \sigma_{\tilde{W}_{\text{siso}}}^2 \). Furthermore, \( \sigma_{\tilde{\beta}_{\text{siso}}}^2 \) is equal to \( \sigma_{\tilde{\beta}_{\text{siso}}}^2 \) and is given by \( \sigma_{\beta_{\text{siso}}}^2 / 2 \). Similarly, \( \sigma_{\tilde{W}_{\text{siso}}}^2 \) is equal to \( \sigma_{\tilde{W}_{\text{siso}}}^2 \) and is given by \( \sigma_{\tilde{W}_{\text{siso}}}^2 / 2 \).

4.4.2 Probability of Error

We present the bit error rate analysis for 16QAM modulation using Gray code mapping with the four bits per symbol \( (b_1 b_2 b_3 b_4) \) representing the in-phase and quadrature components \( (i_1 i_2 q_1 q_2) \).
Conditioned on \( \theta(n), H(l) \), the conditional bit error probability (BEP) for bit \( b_1 \) on the \( l \)th subcarrier is given by

\[
P_{e(siso)}(b_1|H(l), \theta(n)) = \frac{1}{2} \left[ Pr\left(\tilde{X}_I < 0|X_I = d, H(l), \theta(n)\right) + Pr\left(\tilde{X}_I < 0|X_I = 3d, H(l), \theta(n)\right) \right]
\]

(4.35)

where \( X_I, \tilde{X}_I \) are the real parts of \( X, \tilde{X} \) respectively, and \( d^2 \) is proportional to the average symbol energy \( E_s \) such that \( E_s = 10d^2 \). With four bits per symbol, the energy per bit \( E_b \) is given by \( E_b = 5d^2/2 \).

The conditional bit error probability for bit \( b_3 \) is

\[
P_{e(siso)}(b_3|H(l), \theta(n)) = \frac{1}{2} \left[ Pr\left(|\tilde{X}_I| > 2d|X_I = d, H(l), \theta(n)\right) + Pr\left(|\tilde{X}_I| < 2d|X_I = 3d, H(l), \theta(n)\right) \right]
\]

(4.36)

Similarly, the conditional bit error probability for bit \( b_2 \) is

\[
P_{e(siso)}(b_2|H(l), \theta(n)) = \frac{1}{2} \left[ Pr\left(|\tilde{X}_Q| < 0|X_Q = d, H(l), \theta(n)\right) + Pr\left(|\tilde{X}_Q| < 0|X_Q = 3d, H(l), \theta(n)\right) \right]
\]

(4.37)

and that of \( b_4 \) is

\[
P_{e(siso)}(b_4|H(l), \theta(n)) = \frac{1}{2} \left[ Pr\left(|\tilde{X}_Q| > 2d|X_Q = d, H(l), \theta(n)\right) + Pr\left(|\tilde{X}_Q| < 2d|X_Q = 3d, H(l), \theta(n)\right) \right]
\]

(4.38)

Rewriting (4.35), (4.36), (4.37) and (4.38) as a sum of Q-functions, and substituting for \( d \) as a function of \( E_b \), the conditional BEP for bit \( b_1 \) can be written as

\[
P_{e(siso)}(b_1|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left(\sqrt{\frac{(2E_b/5)(V_A^I)^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}}\right) + Q\left(\sqrt{\frac{9(2E_b/5)(V_A^I)^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}}\right) \right]
\]

(4.39)

where \( V_A^I \) represents the real part of \( V_A \) and \( \Phi_{\Delta\theta} \) is defined in (4.17).
Similarly, after substitution and simplification, the conditional BEP for bit $b_3$ is given by

$$ P_{e(siso)}(b_3|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) ight. $$

$$ + Q\left( \sqrt{\frac{(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) $$

$$ + Q\left( \sqrt{\frac{(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) $$

$$ - Q\left( \sqrt{\frac{25(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) \right] \quad (4.40) $$

The conditional BEP for bit $b_2$ can be written as

$$ P_{e(siso)}(b_2|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) ight. $$

$$ + Q\left( \sqrt{\frac{9(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) \right] \quad (4.41) $$

where $V_A^Q$ represents the imaginary part of $V_A$.

Similarly, the conditional BEP for bit $b_4$ is given by

$$ P_{e(siso)}(b_4|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) ight. $$

$$ + Q\left( \sqrt{\frac{(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) $$

$$ + Q\left( \sqrt{\frac{(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) $$

$$ - Q\left( \sqrt{\frac{25(2E_b/5)(V_A')^2|H|^2}{(4E_b)\Phi_{\Delta\theta}/2 + \sigma_W^2/2}} \right) \right] \quad (4.42) $$

Note that (4.39), (4.40), (4.41) and (4.42) depend on $|H|$, which follows a Rayleigh distribution.
Due to the non-symmetry introduced by virtue of the CPE, the average BER is obtained by averaging the conditional BEP of $b_1$ in (4.39), $b_2$ in (4.41), $b_3$ in (4.40) and $b_4$ in (4.42) over the PDF of $\theta(n)$ and $|H|$. The average BEP is thus given by

$$
\overline{P}_{e(siso)} = \frac{1}{4} \int_{|H|} f(|H|) d|H| \int_{\theta} f(\theta) d\theta \sum_{i=1}^{4} P_{e(siso)}(b_i|\theta, |H|) \\
= \frac{1}{4} \int_{0}^{\infty} f(|H|) d|H| \int_{-\pi}^{\pi} f(\theta) d\theta \sum_{i=1}^{4} P_{e(siso)}(b_i|\theta, |H|)
$$

(4.43)

4.4.3 SISO Receiver with Imperfect CSI

In the presence of imperfect channel estimation, we assume a channel estimation model such that the channel estimate $\hat{H}$ of the true channel $H$ is given by

$$
\hat{H} = H + \varepsilon
$$

(4.44)

where $\varepsilon$ is the error in the channel estimate, and is modeled as zero-mean complex Gaussian random variable with variance $2\sigma^2$, and is independent of $H$.

Using similar steps as in the previous section, the detected symbol $\tilde{X}$ on the $l^{th}$ subcarrier is given by

$$
\tilde{X}_{siso}^e(l) = Y(l)\hat{H}^*(l)
= \frac{|H(l)|^2 X(l)V_A}{\tilde{D}_{siso}(l)} + \frac{H(l)X(l)V_A \varepsilon^*(l)}{\tilde{\Psi}_{siso}(l)}
+ \left[ H^*(l) + \varepsilon^*(l) \right] \sum_{k=0, k \neq l}^{N-1} H(k)X(k)V_B(k - l)
+ W(l) \left[ H^*(l) + \varepsilon^*(l) \right]
$$

(4.45)

In (4.45), $\tilde{D}_{siso}(l)$ is the desired signal term. The second term $\tilde{\Psi}_{siso}(l)$ is a noise term due to imperfect channel estimation and conditioned on $X$ and the channel $H$. This term is a Gaussian random variable since $\varepsilon$ is modeled as Gaussian. The third
term $\tilde{\beta}^e_{\text{siso}}(l)$ is the ICI term with channel estimation errors and is approximated as Gaussian. The last term is the additive Gaussian noise term. The above equation can be written compactly as

$$\tilde{X}^e_{\text{siso}}(l) = \tilde{D}_{\text{siso}}(l) + \tilde{\Psi}_{\text{siso}}(l) + \tilde{\beta}^e_{\text{siso}}(l) + \tilde{W}^e_{\text{siso}}(l)$$

$$= \left( \tilde{X}^e_{\text{siso}}(l) \right)^I + j * \left( \tilde{X}^e_{\text{siso}}(l) \right)^Q$$ (4.46)

Conditioned on $\theta(n)$ and the channel $H(l)$, it is observed that $\tilde{X}^e_{\text{siso}}(l)$ is a zero-mean complex Gaussian random variable with variance $(\sigma^2_{\Psi_{\text{siso}}} + \sigma^2_{\beta^e_{\text{siso}}} + \sigma^2_{W^e_{\text{siso}}})$.

### 4.4.4 Variance of ICI and Noise

The variance of the noise term $\tilde{W}^e_{\text{siso}}(l)$ on the $l^{th}$ subcarrier is given by

$$\sigma^2_{\tilde{W}_{\text{siso}}} = E[\tilde{W}^e_{\text{siso}}(l) \tilde{W}^{e*}_{\text{siso}}(l)]$$

$$= \sigma^2_{W} \left( |H|^2 + \sigma^2_{e} \right)$$ (4.47)

where $\sigma^2_{W}$ is the variance of the Gaussian noise, and $\sigma^2_{e}$ the variance of the error in the channel estimate.

The variance of $\tilde{\Psi}_{\text{siso}}(l)$ on the $l^{th}$ subcarrier is given by

$$\sigma^2_{\tilde{\Psi}_{\text{siso}}} = E[\tilde{\Psi}_{\text{siso}}(l) \tilde{\Psi}^{*}_{\text{siso}}(l)]$$

$$= E_{s} |H|^2 \sigma^2_{e}$$

$$= E_{s} \sigma^2_{\Psi_{\text{siso}}}$$ (4.48)

where $\sigma^2_{\Psi_{\text{siso}}} = |H|^2 \sigma^2_{s}$ and $\sigma^2_{\tilde{\Psi}_{\text{siso}}} = E_{s} \sigma^2_{\Psi_{\text{siso}}}$.

Similarly, the variance of the ICI term $\tilde{\beta}^e_{\text{siso}}(l)$ is given by

$$\sigma^2_{\tilde{\beta}^e_{\text{siso}}} = E[\tilde{\beta}^e_{\text{siso}}(l) \tilde{\beta}^{e*}_{\text{siso}}(l)]$$

$$= E_{s} \Phi_{\Delta \theta} \left( |H|^2 + \sigma^2_{e} \right)$$

$$= E_{s} \sigma^2_{\beta^e_{\text{siso}}}$$ (4.49)

where $\sigma^2_{\beta^e_{\text{siso}}} = E_{s} \sigma^2_{\beta^e_{\text{siso}}}$ and $\sigma^2_{\beta^e_{\text{siso}}} = \Phi_{\Delta \theta} \left( |H|^2 + \sigma^2_{e} \right)$ with $\Phi_{\Delta \theta}$ given by (4.17).
4.4.5 Probability of Error

The conditional BEP for bit $b_1$ can be written as

$$P_{e(siso)}(b_1|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5)(|H|^2)^2(V_A^l)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) + Q\left( \sqrt{\frac{9(2E_b/5)(|H|^2)^2(V_A^l)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) \right] \quad (4.50)$$

Similarly, the conditional BEP for bit $b_3$ can be written as

$$P_{e(siso)}(b_3|H(l), \theta(n)) =$$

$$\frac{1}{2} \left[ Q\left( \sqrt{\frac{9(2E_b/5)(|H|^2)^2(V_A^l)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) + Q\left( \sqrt{\frac{(2E_b/5)(|H|^2)^2(V_A^l)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) \right] + Q\left( \sqrt{\frac{(2E_b/5)(|H|^2)^2(V_A^l)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) \right] \right] \quad (4.51)$$

The conditional BEP for bit $b_2$ can be written as

$$P_{e(siso)}(b_2|H(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5)(|H|^2)^2(V_A^Q)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) + Q\left( \sqrt{\frac{9(2E_b/5)(|H|^2)^2(V_A^Q)^2}{\sigma_{\tilde{\psi}_{siso}}/2 + (4E_b)\sigma^2_{\beta_{siso}}/2 + \sigma^2_{W_{siso}}/2}} \right) \right] \quad (4.52)$$

Similarly, the conditional BEP for bit $b_4$ can be written as
Due to the non-symmetry introduced by virtue of the CPE, the average BER is obtained by averaging the conditional BEP of $b_1$ in (4.50), $b_2$ in (4.52), $b_3$ in (4.51) and $b_4$ in (4.53) over the PDF of $\theta(n)$ and $|H|$. The average BEP is thus given by

$$
\overline{P_{e(siso)}} = \frac{1}{4} \int_{|H|} f(|H|) |H| \int_{\theta} f(\theta) d\theta \sum_{i=1}^{4} P_{e(siso)}^{e}(b_i|\theta, |H|) = \frac{1}{4} \int_{0}^{\infty} f(|H|) |H| \int_{-\pi}^{\pi} f(\theta) d\theta \sum_{i=1}^{4} P_{e(siso)}^{e}(b_i|\theta, |H|) \quad (4.54)
$$
4.5 Results

In this section, the numerical results of the analysis is presented and compared with simulations using system parameters that are compliant to the IEEE 802.16 Standard.

4.5.1 Simulation Parameters

The main simulation parameters used in this section are summarized in Table 4.1. The simulation platform is implemented in MATLAB. An uncoded system is considered. As such, the blocks that involve convolutional encoding/decoding in the IEEE 802.16 standard are omitted. Using 64, 126, 256 subcarriers and 16-QAM as the modulation of choice, the transmitted signal is processed by a 64, 128, or 256-point FFT. With a Rayleigh fading channel, a normalized channel power is assumed. Hence a zero-mean and unity variance for the fading channel. The numerical results are obtained for the different cases of phase noise variance and channel estimation errors combination as summarized in Table 4.1.

Table 4.1 Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\theta^2$</td>
<td>0.0, 0.01, 0.03, 0.06, 0.09</td>
</tr>
<tr>
<td>$\sigma_\epsilon^2$</td>
<td>0.0, 0.02, 0.04, 0.06</td>
</tr>
<tr>
<td>Modulation</td>
<td>16QAM</td>
</tr>
<tr>
<td>Subcarriers (N)</td>
<td>64, 128, 256</td>
</tr>
</tbody>
</table>

4.5.2 Discussion

In Figures 4.3 - 4.5, the average bit error probability with prior CPE correction is shown. Extension to the case of no prior CPE correction is shown in 4.6 - 4.8.

Figure-4.3 illustrates the system performance subject to different phase noise contributions ($\sigma_\theta^2 = 0.01, 0.03, 0.06$), for $N = 256$ subcarriers, and with perfect channel estimation. Furthermore, it is assumed that the CPE term is perfectly compensated and corrected. It is observed that for different variances of the phase noise, the system performance degrades as the variance increases, and reaches an error floor
even at high SNRs. This is in agreement with the fact that $\sigma^2_\theta$ is proportional to the 3dB bandwidth of the oscillator power spectral density (PSD). This is expected since an increase in SNR implies an increase in the signal energy, and from (4.16) and (4.49), the ICI variance $\sigma^2_\beta$ is directly proportional to the signal energy.

In Figure-4.4, the combine effect of phase noise and imperfect channel estimation is illustrated, with the added assumption of perfect prior CPE correction. The system performance is further degraded by the additional effects of channel estimation errors due to imperfect channel estimation.

Figure-4.5 illustrates the degradation in system performance due to ICI contribution as the number of subcarriers $N$ increases from 64 to 256, with perfect channel estimates and the phase noise contribution due to $\sigma^2_\theta = 0.03$. It is observed that the higher the number of subcarriers, the more interference contribution on the desired subcarrier due to all adjacent subcarriers. This can be understood due to the fact that, for a fixed system bandwidth and increase in number of subcarriers, the subcarrier spacing is reduced correspondingly. For a given phase noise bandwidth, this
Figure 4.4 BER of 16QAM-OFDM in Rayleigh Fading channels with Phase Noise & Imperfect Channel Estimation, with perfect prior CPE correction assumed: $\sigma_\theta^2 = 0.03$, and Channel Estimation Error with $\sigma_\varepsilon^2 = 0.02, 0.04, 0.06$. 
Figure 4.5  BER of 16QAM-OFDM in Rayleigh Fading channels with Phase Noise & perfect Channel Estimation: Number of subcarriers (N) = 64, 128, 256 and $\sigma^2_\theta = 0.03$.

Reduction in subcarrier spacing leads to an increase in ICI contribution from adjacent subcarriers. Hence, an increase in system performance degradation.
Next, we consider the case with no CPE correction. Figure-4.6 compares the performance of two systems. The plot on the left assumes perfect CPE correction, while the one on the right assumes no CPE correction. Both systems are subject to phase noise and channel estimation errors. From the results, it is observed that a system with robust CPE correction schemes is highly desired.

In Figure-4.7, we compare the results obtained by analysis and simulations. For a given channel estimation error, the phase noise variance is increased from \( \sigma_\theta^2 = 0.01 \) to \( \sigma_\theta^2 = 0.09 \). The comparison is done between simulations and the analysis with no assumption of CPE correction. The results indicate good match between analysis and results obtained by simulations.

The comparison is also performed for a given phase noise variance of \( \sigma_\theta^2 = 0.01 \), and increasing the variance of the channel estimation error from \( \sigma_\epsilon^2 = 0.02 \) to \( \sigma_\epsilon^2 = 0.06 \). This comparison also indicate a good match between the two approaches.
Figure 4.7 Comparison of Analysis & Simulations: BER of 16QAM-OFDM in Rayleigh Fading channels with Phase Noise & Imperfect Channel Estimation, with no prior CPE correction: Number of subcarriers (N) = 256 and Channel Estimation Error with $\sigma^2 = 0.02$. 

16QAM OFDM with PN & ICE: $\sigma^2 = 0.02$ (Analysis & Sims)

SNR in dB

Average BER

0 5 10 15 20 25 30 35 40

$\sigma^2 = 0.01$

$\sigma^2 = 0.03$

$\sigma^2 = 0.06$

$\sigma^2 = 0.09$

Theory
Figure 4.8 Comparison of Analysis & Simulations: BER of 16QAM-OFDM in Rayleigh Fading channels with Phase Noise & Imperfect Channel Estimation, with no prior CPE correction: Number of subcarriers (N) = 256 and Phase Noise with $\sigma_\theta^2 = 0.01$. 
4.6 Conclusion

The performance analysis of OFDM systems with the joint effects of phase noise and channel estimation errors in Rayleigh fading channels is presented. It is shown that the phase noise impairments can cause severe system performance degradation even at high SNRs. The added effect of channel estimation errors further degrades system performance.

By initially assuming perfect CPE correction, we separately considered the effect of phase noise contribution to the ICI term only, which leads to performance degradation. We further extended this analysis to include the CPE contribution to the system performance. It is shown that, unless a very adequate CPE correction scheme is implemented, the phase noise impaired system is heavily affected.

Numerical evaluation shows that the analytical results can be applied to describe the performance of a practical system. As such, system parameters that are compliant to the IEEE 802.16 Standard are used for simulations and comparison with the results obtained by analysis.

The text in Chapter 4 is a revision of the material presented in:


The dissertation authors were the primary researchers and authors, and the co-authors listed in these publications directed and supervised the research which forms the basis for this chapter.
Performance Analysis of SIMO-OFDM with Phase Noise and Imperfect Channel Estimation

5.1 Introduction

The latest wireless communication techniques for wireless broadband applications, such as high speed wireless internet connectivity, demand higher data rates and better quality of service. However, transmission reliability is still degraded by harsh propagation channels. Multiple-input multiple-output (MIMO) communication systems can increase the system capacity and improve transmission reliability [4], [5], [37], [46], [47], [99], [45], with increase in diversity of the system.

Transmission diversity has been widely and successfully employed to enhance the performance of communication systems in wireless MIMO channels [54], [100], [101], and is an effective method to adequately combat channel impairments such as multipath fading and channel interference. The authors in [102], [103], [104] proposed algorithms to improve MIMO system performance, while [54], [78], [103], [105] investigated implementation issues and limitations in MIMO systems. Diversity systems improve performance by transmitting and/or receiving multiple copies of transmitted...
data redundantly over multiple independent channels, and intelligently combining them to obtain an effective increase in system SNR [31], [106], [35], [37] and hence an improvement in the overall system performance.

Depending on the domain in which the transmission redundancy is provided, diversity techniques are divided into Time diversity, Frequency diversity, and Space diversity, [37], [106], [31]. Furthermore, depending on whether it is applied to the transmitter or the receiver, transmission diversity can be classified into Transmit diversity and Receive diversity.

In this chapter, we focus on receive antenna diversity.

This chapter extends the work in [107] to a multi-antenna system by considering the performance of an M-QAM MRC-OFDM system with imperfect channel estimation, and subject to RF impairments. The analysis is presented for both perfect and imperfect channel estimation.

Specifically, we examine the performance of the impaired system employing Maximal Ratio Combining (MRC) receiver architecture in slowly varying Rayleigh fading channels. We derive the BER expressions for M-QAM OFDM with the effects of the RF impairments and employing R-branch MRC receiver structure with and without perfect channel estimation. Simulation results are also presented to illustrate the extent of the effects on system performance degradation.

5.2 Diversity Techniques

5.2.1 Time Diversity

In Time diversity, identical signals are transmitted during different time slots. These time slots are uncorrelated, i.e., the temporal separation between the slots is greater than the coherence time of the wireless [31], [106]. Examples of current implementations of Time diversity are interleavers, Forward Error Correction (FEC) in error correction codes, and RAKE receiver in CDMA systems.

A major drawback of the Time diversity technique is that the redundancy is provided in the time domain with a penalty of a loss in bandwidth efficiency [31], [106].
5.2.2 Frequency Diversity

In Frequency diversity, several frequencies, sometimes referred to as carriers, are used to transmit the same signal. The frequency separation between these carriers is much larger than the coherence bandwidth of the channel [106], and consequently, the carriers are assumed uncorrelated and do not experience the same channel fades.

Similar to the case of Time diversity, the redundancy in Frequency diversity is provided in the frequency domain with the penalty of a loss in spectral efficiency. This loss in efficiency is due to the guard bands existing between the carrier frequencies, and additionally, depending on the implementation, the structure of the receiver is complicated as it must be able to typically work with a large number of frequencies.

5.2.3 Space Diversity

Space diversity, also referred to as Antenna Diversity, has been frequently implemented in practice. This diversity technique can be further classified into various schemes such as Polarization diversity, Beamforming diversity, Antenna switching [22], [53], [108]. Depending on whether diversity is applied to the transmitter or the receiver, it can be classified into Transmit diversity and Receive diversity.

Furthermore, depending on how the replicas of the transmitted signals are combined at the receiver, space diversity techniques are classified into Selection Combining technique, Switched Combining technique, Equal-Gain Combining and Maximal-Ratio Combining techniques [108].

The concept of Space diversity uses multiple transmit and/or receive antennas to transmit and/or receive signals. For effective implementation, the antennas are spatially separated from one another by multiples of half wavelength [22], [108] in order to achieve independence across antennas. Unlike Time and Frequency diversity, the redundancy in Space diversity is provided for the receiver in the spatial domain, with no loss in spectral efficiency.

In practical wireless communication, a combined diversity technique of the aforementioned techniques is employed to provide multidimensional diversity. For instance, in wireless systems, a combination of space diversity (multi-antenna Base-stations and IEEE 802.11n WLANs multi-antenna Access-points) and time diver-
sity (interleaving as well as error correction/control coding) is utilized to provide 2-dimensional diversity for the mobile units or remote stations.
5.3 Receiver Diversity

Receive diversity is an effective method to effectively combat channel impairments such as multipath fading and channel interference. Receive diversity systems improve performance by receiving multiple copies of transmitted data redundantly over the multiple independent channels, and intelligently combining them to obtain an effective increase in system SNR, and hence the overall system performance.

In Receive antenna diversity systems, the decision variable is obtained by combining the signals received at the multiple antennas according to some desired criterion. Among the numerous combining schemes in receive diversity systems, Maximal Ratio Combining (MRC) is one of the most common linear combining technique.

5.3.1 Selection Combining

In this technique, the receiver comprises \( M \) receive antennas associated with \( M \) independent demodulators to provide \( M \) branches of which the gains are weighted to provide the same average SNR for each branch. The receiver selects the incoming signal with the highest instantaneous SNR to demodulate [106], [109]. In practice, the instantaneous power of the received signal including noise \( (S + N_0) \) is frequently used in place of \( S \) in obtaining the instantaneous SNR \( S/N_0 \), as the measurement of the instantaneous SNR is typically difficult [109]. This is the simplest spatial diversity combining method which requires only an SNR monitoring and an antenna switch at the receiver [109].

5.3.2 Scanning Combining (SC)

In this technique, the receiver scans all branches by observing a certain order and selects a particular branch which has an \( SNR \) above a predetermined \( SNR \) threshold. The signal of this branch is selected as the output until its power drops below the threshold. The receiver then starts searching again.

The advantage of this technique over the selection combining method is that the receiver does not need to monitor continuous and instantaneous \( SNR \) of all branches at all times. However, it is inferior to the selection combining method
because the best incoming signal is not always selected, and consequently, the average 
\( SNR \) of the output is smaller than that of selection combining.

### 5.3.3 Maximal Ratio Combining (MRC)

Maximum ratio combining (MRC) is a special form of diversity where multiple replicas of the same signal, received over different diversity branches, are combined in order to maximize the instantaneous \( SNR \) at the combiner output \[110\].

In this technique, the signal from \( M \) branches are weighted by \textit{weighting} factors of the branches and combined/summed together. In general, these weighting factors are proportional to the ratios of the \( SNR \) of those branches themselves \[106\]. Before summing, the signals must be co-phased to achieve constructive/coherence addition \[41\], \[50\]. The average \( SNR \) of the output signal is simply the sum of the individual \( SNRs \) of all the branches.

Although the structure and cost of MRC are higher than those of other combining methods, it is clearly a technique that can provide acceptable output signal with expected \( SNR \), even when the signal strengths of the incoming signals are significantly lower.

It is well known that MRC is the optimum diversity scheme in communication systems in the sense of maximizing the signal-to-noise ratio (SNR) at the output of the combiner \[111\], \[106\]. However, good channel estimates are required at the receiver to exploit the full potential of MRC \[111\].

The weighting factor in MRC corresponds to the complex conjugated channel coefficient of each receive branch. The signal received on each antenna is given by

\[
\begin{align*}
\ y_r &= h_r s + n_r, & r = 1, 2, \ldots, N_R,
\end{align*}
\]

where \( h_r \) and \( n_r \) are the channel coefficients and the noise experienced by the \( r^{th} \) antenna branch respectively, \( s \) is the transmitted signal, and \( N_R \) is the number of receive antenna branches. Moreover, it is considered that the antennas are sufficiently spaced apart such that the channel coefficients, affected by fading, can be assumed to be independent. The weighted combination for the input antennas to be taken into account is expressed as follows:
where \( \varrho_r \) are the combiner weighting coefficients that are chosen such that the SNR is maximized at the combiner output. From this combination, the SNR of the channel is given by

\[
\text{SNR}(h_1, h_2, \ldots, h_{NR}) = \text{SNR} \left( \sum_{r=1}^{NR} |\varrho_r h_r|^2 \right) = \sum_{r=1}^{NR} |\varrho_r|^2
\]

(5.3)

Applying the Cauchy-Schwartz inequality, it can be verified that \( \varrho_r = h_r^* \) maximizes the SNR. Replacing this value in Equation 5.3, the maximum SNR yields

\[
\text{SNR}(h_1, h_2, \ldots, h_{NR}) = \text{SNR} \sum_{r=1}^{NR} |h_r|^2
\]

(5.4)

The process described above is shown in Figure-5.1, where an example of a receiver with multi-antenna diversity is depicted. The signal is sent over a channel with transmission coefficients \( h_r \), and reaches all receive antennas with some added noise. Then, the process consists of multiplying the signal in each receive branch by the corresponding conjugated channel coefficient \( h_r^* \), and at the end, the signals from all branches are summed. It can be appreciated that the received signal is very similar to the one obtained with the Alamouti scheme in [50] for the two antenna scenario, as the same gain in the signal is achieved, as well as some modified noise. However, better performance is obtained with this scheme as only one symbol is transmitted in one time interval and the unity average transmit power is already achieved in each interval. Therefore, the resultant signal is not multiplied by the factor \( 1/\sqrt{2} \) as in the Alamouti scheme, and consequently, a gain of 3dB in power is obtained.

### 5.3.4 Equal Gain Combining (EGC)

This diversity combining technique is similar to the MRC method, except that all weighting factors are set to unity [112]. It is more frequently employed in systems
that utilize equal-energy constellations such as MPSK, where signal detection does not depend on the amplitude of the decision variable. The performance is marginally inferior compared to that of MRC.

With all these available and appealing performance enhancement combining schemes, the appropriate choice clearly depends on the application and system requirements. That choice may range from the pursuit of a combination of higher data rates, spectral efficiency, enhanced link reliability, interference minimization, etc. These schemes can be successfully applied to both single carrier as well as multicarrier communication systems, such as OFDM.
5.4 Maximal Ratio Combining in OFDM

As the demand for high-speed mobile wireless communications over band-limited channels continue to grow, so is the interest in research and development to accommodate these needs. Orthogonal Frequency Division Multiplexing (OFDM) is a proven technology and a key technique for achieving needed spectral efficiency requirements and higher data rates for wireless communication systems of the present and near future, and is being used in many wireless applications such as digital video broadcasting (DVB), wireless local area networks (WLAN) [81], [61], [82], [113] and possible next-generation wireless networks such as WiMax. OFDM is known to be very resistant to multipath fading compared to single carrier systems and efficiently utilizes the scarce frequency spectrum.

The performance of OFDM is known to be severely affected by hardware imperfections such as carrier frequency offset [82], [113] and oscillator phase noise [81], [18], as well as imperfect channel estimation [114], [115]. The channel estimates requirements for higher order modulation coupled with MRC make it highly desirable to obtain good channel estimates for a robust communication system design.

The effects of RF impairments in MRC-OFDM have been documented in the literature, including, but not limited to [81], [61], [82], [113]. These works consider a non-MRC system with perfect channel estimation. The effect of imperfect channel estimation in MRC is considered in [114], [115], but for a MRC non-OFDM system. Furthermore, no RF impairments were taken into account. While [116] investigated MRC with OFDM, the study only considered cochannel interference with only simulations and no analysis for the BER. The authors in [117] investigated the performance of MRC-OFDM in a time-varying fading environment, with no RF impairments.

As robust designs are achieved when comprehensive investigation of practical system impairments are considered, it is observed from above that such a study that accounts for these effects will be very valuable. As such, this Chapter investigates the performance of an M-QAM MRC-OFDM system with imperfect channel estimation, subject to RF impairments. The analysis is compared with simulation results, under a permutation of these impairments, making use of system parameters from the IEEE 802.16 standard.
5.5 MRC System Model

A block diagram of the MRC-OFDM receiver is shown in Fig-5.1. The OFDM-MRC receiver can be thought of as a parallel arrangement of single-antenna OFDM receivers\(^1\). The received signal at each antenna is assumed to experience independent channel fade, with the likelihood that at least one antenna branch receives a strong signal. The output of each of the single-antenna receivers is then fed into a combiner that is designed to maximize the SNR at the combiner output by employing appropriate weighting coefficients.

5.5.1 Transmitted OFDM Signal

A block of \(N\) complex-valued data symbols \(\{X(k)\}_{k=0}^{N-1}\) are grouped and converted into a parallel set to form the input to the OFDM modulator, where \(k\) is the subcarrier index and \(N\) is the number of subcarriers. The modulator consists of an Inverse Fast Fourier transform (IFFT) block. The output of the IFFT is the complex

\(^1\)The single-antenna OFDM receiver is described in Chapter 4.
baseband modulated OFDM symbol in discrete time domain and is given by

\[ x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{i2\pi kn/N}; \quad 0 \leq n \leq N - 1 \quad (5.5) \]

### 5.5.2 Channel Model

The channel is modeled by a tapped delay line with channel coefficients that are assumed to be slowly varying such that they are almost constant over at least one OFDM symbol duration. The channel frequency response for the \( k^{th} \) subcarrier at the \( r^{th} \) antenna is

\[ H_r(k) = \sum_{p=0}^{P-1} h_r(p)e^{i2\pi pk/N} \quad (5.6) \]

where \( h_r(p) \) is the complex channel gain of the \( p^{th} \) multipath component on the \( r^{th} \) antenna.

### 5.5.3 Phase Noise Model

The phase noise \( \theta(n) \) is modeled by a Tikhonov distribution \([73], [66], [67]\), and is given by

\[ f_\theta(\theta(n)) = \frac{\cos(\theta(n))}{\sigma_\theta^2 I_0(\alpha/\sigma_\theta^2)}; \quad |\theta(n)| < \pi \quad (5.7) \]

where \( n \) denotes time index, \( I_0(\alpha) \) is the 0th order modified Bessel function of the first kind, and \( \sigma_\theta^2 \) is the variance of the phase noise.

### 5.5.4 Received OFDM Signal

The received signal on the \( r^{th} \) antenna branch is given by

\[ y_r(n) = \left( \sum_{p=0}^{L-1} h_r(p)x(n-p) + w_r(n) \right)e^{i\theta(n)} \quad (5.8) \]

where \( \theta(n) \) is the phase noise process. The complex Gaussian random variable \( w_r(n) \) represents the noise term on the \( r^{th} \) antenna, with zero mean and variance \( \sigma_{w_r}^2 = \)
The demodulated OFDM signal of the \( r \)-th antenna branch on the \( l \)-th subcarrier is given by

\[
Y_r(l) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_r(n) \times e^{-j2\pi kn/N} = H_r(l)X(l)V_A + \sum_{k=0 \atop k \neq l}^{N-1} H_r(k)X(k)V_B(k-l) + W_r(l)
\]  

(5.9)

where the first, second and third terms are the desired signal component, the ICI, and noise term on the \( k \)-th subcarrier respectively. The complex valued fading channel \( H_r(k) \) on the \( r \)-th branch and the data symbols \( X(k) \) on each subcarrier are assumed uncorrelated.

### 5.5.5 MRC-OFDM Receiver with perfect CSI

We consider the performance of an OFDM receiver employing MRC. The demodulated OFDM signals are combined on a subcarrier basis. This optimizes the SNR per subcarrier [106], [111]. It is assumed that the channel on each subcarrier is flat and time-invariant, with perfect channel estimation at the receiver.

In the following analysis, it is assumed that the channel is spatially independent and identically distributed (i.i.d.) across antennas. Furthermore, the noise variances are i.i.d. across antennas, and the \( V_B(l,k) \)'s are not a function of the individual antennas. The decision variable at the output of the R-branch MRC is

\[
\tilde{X}_{mrc}(l) = \sum_{r=1}^{R} Y_r(l)H_r(l)^* \]  

(5.10)

where \( H_r^*(k) \) implies complex conjugate of \( H_r(k) \) and \( Y_r(l) \) is given by (5.9).

The decision variable at the output of the R-branch MRC is
\[
\tilde{X}_{\text{mrc}}(l) = \sum_{r=1}^{R} Y_r(l) H_r(l)^* \\
= \sum_{r=1}^{R} H_r(l) X(l) H_r^*(l) V_A + \sum_{r=1}^{R} W_r(l) H_r^*(l) \\
+ \sum_{r=1}^{R} \sum_{k=0, k \neq l}^{N-1} H_r(k) X(k) V_B(k - l) H_r^*(l) \\
= X(l) \sum_{r=1}^{R} |H_r(l)|^2 V_A + \sum_{r=1}^{R} W_r(l) H_r^*(l) \\
\underbrace{\tilde{D}_{\text{mrc}}(l)}_{\tilde{D}_{\text{mrc}}(l)} + \underbrace{\tilde{W}_{\text{mrc}}(l)}_{\tilde{W}_{\text{mrc}}(l)} \\
+ \sum_{r=1}^{R} \sum_{k=0, k \neq l}^{N-1} H_r(k) X(k) V_B(k - l) H_r^*(l) \\
\underbrace{\tilde{\beta}_{\text{mrc}}(l)}_{\tilde{\beta}_{\text{mrc}}(l)} \\
\tag{5.11}
\]

where \( H_r(l)^* \) implies complex conjugate of \( H_r(l) \). The term \( V_A \) is frequently referred to as the Common Phase Error (CPE) term. With prior CPE correction assumed, the decision variable at the output of the R-branch MRC is

\[
\tilde{X}_{\text{mrc}}(l) = X(l) \sum_{r=1}^{R} |H_r(l)|^2 V_A + \sum_{r=1}^{R} W_r(l) H_r^*(l) \\
\underbrace{\tilde{D}_{\text{mrc}}(l)}_{\tilde{D}_{\text{mrc}}(l)} + \underbrace{\tilde{W}_{\text{mrc}}(l)}_{\tilde{W}_{\text{mrc}}(l)} \\
+ \sum_{r=1}^{R} \sum_{k=0, k \neq l}^{N-1} H_r(k) X(k) V_B(k - l) H_r^*(l) \\
\underbrace{\tilde{\beta}_{\text{mrc}}(l)}_{\tilde{\beta}_{\text{mrc}}(l)} \\
\tag{5.12}
\]

In (5.12), \( \tilde{D}_{\text{mrc}}(l) \) is the desired signal term. The term \( \tilde{\beta}_{\text{mrc}}(l) \) is the ICI term due to phase noise and is approximated as Gaussian random variable. The proof of the Gaussianity of the ICI is presented in the Appendix. The term \( \tilde{W}_{\text{mrc}}(l) \) is the additive Gaussian noise term.
5.5.6 Mean and Variance of ICI and Noise

Conditioned on $H_r(l)$, the variance of the noise term $\tilde{W}_{mrc}(l)$ on the $l^{th}$ subcarrier is given by

$$
\sigma^2_{\tilde{W}_{mrc}} = E\left[ \tilde{W}_{mrc}(l) \tilde{W}_{mrc}^*(l) \right] \\
= E\left[ \left( \sum_{r=1}^{R} W_r(l) H_r^*(l) \right) \left( \sum_{\rho=1}^{R} W_{\rho}(l) H_{\rho}^*(l) \right)^* \right] \\
= E\left[\sum_{r=1}^{R} \sum_{\rho=1}^{R} W_r(l) H_r^*(l) W_{\rho}^*(l) H_{\rho}(l) \right] \\
= \sum_{r=1}^{R} \sum_{\rho=1}^{R} H_{\rho}(l) H_r^*(l) E\left[ W_r(l) W_{\rho}^*(l) \right] \\
= \sum_{r=1}^{R} \sum_{\rho=1}^{R} H_{\rho}(l) H_r^*(l) \sigma^2_{W_r} \delta_{r\rho}
$$

(5.13)

where $\delta_{r\rho}$ is defined as

$$
\delta_{r\rho} = \begin{cases} 
1, & r = \rho \\
0, & r \neq \rho 
\end{cases}
$$

(5.14)

Therefore, the variance of the noise term $\tilde{W}_{mrc}(l)$ on the $l^{th}$ subcarrier is given by

$$
\sigma^2_{\tilde{W}_{mrc}} = E\left[ \tilde{W}_{mrc}(l) \tilde{W}_{mrc}^*(l) \right] \\
= \sum_{r=1}^{R} \sigma^2_{W_r} |H_r(l)|^2 
$$

(5.15)

where $\sigma^2_{W_r}$ is the variance of the additive Gaussian noise on the $R^{th}$ antenna branch. Assuming the noise terms are i.i.d., $\sigma^2_{W_r} = \sigma^2_{W}$, we obtain

$$
\sigma^2_{\tilde{W}_{mrc}} = \sigma^2_{W} \sum_{r=1}^{R} |H_r(l)|^2 
$$

(5.16)
The variance \( \sigma^2_{\tilde{\beta}_{mrc}} \) of the ICI term is the second moment of \( \tilde{\beta}_{mrc}(l) \), and conditioned on \( \theta(n) \) and the channel \( H_r(l) \), it is derived using a similar approach in the previous Chapter as follows

\[
\sigma^2_{\tilde{\beta}_{mrc}} = E\left[\tilde{\beta}_{mrc}(l)\tilde{\beta}^*_{{mrc}}(l)\right] = E\left[\sum_{r=1}^{R} \sum_{k=0, k\neq l}^{N-1} H_r(k)X(k)V_B(k,l)H^*_r(l) \times \sum_{\rho=1}^{R} \sum_{m=0, m\neq l}^{N-1} H_\rho(m)X(m)V_B(m,l)H^*_\rho(l)\right]
\]

(5.17)

Due to the assumed independence of the channel fades \( H_r(k) \)'s across subcarriers and across antenna branches, we have

\[
E[H_r(k)H^*_r(m)] = E[|H_r(k)|] \delta_{km}
\]

\[
E[H_\rho(k)H^*_\rho(m)] = E[|H_r(k)|] \delta_{\rho}\rho
\]

(5.18)

where the functions \( \delta_{km} \) and \( \delta_{\rho}\rho \) are defined as in (5.14).

Making use of (5.18), the variance \( \sigma^2_{\tilde{\beta}_{mrc}} \) of the ICI term can be written as

\[
\sigma^2_{\tilde{\beta}_{mrc}} = \sum_{r=1}^{R} \sum_{k=0}^{N-1} |H_r(l)|^2 E\left[|X(k)H_r(k)V_B(k-l)|^2\right]
\]

\[
= E_s \sum_{r=1}^{R} \sum_{k=0}^{N-1} |H_r(l)|^2 |V_B(k-l)|^2
\]

\[
= E_s \sum_{r=1}^{R} \sum_{k=1}^{N-1} |H_r(0)|^2 |V_B(k)|^2
\]

\[
= E_s \sum_{r=1}^{R} \sum_{k=0}^{N-1} |H_r(0)|^2 \left(|V_B(k)|^2 - |V_B(0)|^2\right)
\]

(5.19)

where we have assumed that the \( E[|H_r(k)|^2] = 1 \), and \( E_s \) is the transmitted symbol energy. Following steps in the previous Chapter, we further simplify \( \sigma^2_{\tilde{\beta}_{mrc}} \) to
\[ \sigma^2_{\beta_{mrc}} = E_s \sum_{r=1}^{R} \sum_{k=0}^{N-1} |H_r(0)|^2 \left( |V_B(k)|^2 - |V_B(0)|^2 \right) \]

\[ = E_s \sum_{r=1}^{R} |H_r(0)|^2 \left[ 1 - \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}} \right] \]

\[ = E_s \Phi_{\Delta \theta} \sum_{r=1}^{R} |H_r(0)|^2 \]

\[ = E_s \sigma^2_{\beta_{mrc}} \quad (5.20) \]

where \( \sigma^2_{\beta_{mrc}} = \Phi_{\Delta \theta} \sum_{r=1}^{R} |H_r(0)|^2 \) and \( \Phi_{\Delta \theta} \) is given by

\[ \Phi_{\Delta \theta} = \left[ 1 - \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}} \right] \quad (5.21) \]

### 5.5.7 Probability of Error

Making use of Gray code mapping as shown in Figure-4.2, and conditioned on \( \theta(n) \) and the channel gains \( H_r(l) \) on the \( R \) antenna branches, the conditional bit error probability (BEP) for the bit \( b_1 \) on the \( l^{th} \) subcarrier is given by

\[ P_{e\text{(mrc)}}(b_1|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Pr\left( \tilde{X}_I < 0|X_I = d, H_1(l), ..., H_R(l), \theta(n) \right) \right. \]

\[ + \left. Pr\left( \tilde{X}_I < 0|X_I = 3d, H_1(l), ..., H_R(l), \theta(n) \right) \right] \]

\[ (5.22) \]

where \( X_I, \tilde{X}_I \) are the real parts of \( X, \tilde{X} \) respectively, and \( d^2 \) is proportional to the average symbol energy \( E_s \) such that \( E_s = 10d^2 \). With four bits per symbol, the energy per bit \( E_b \) is given by \( E_b = 5d^2/2 \). Using a similar approach, the conditional
bit error probability for the bit $b_3$ is

$$P_{e(mrc)}(b_3|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Pr\left(|\tilde{X}_I| > 2d|X_I = d, H_1(l), ..., H_R(l), \theta(n)\right) + Pr\left(|\tilde{X}_I| < 2d|X_I = 3d, H_1(l), ..., H_R(l), \theta(n)\right) \right]$$

(5.23)

Rewriting (5.22) and (5.23) as a sum of Q-functions, and substituting for $d$ as a function of $E_b$, the conditional BEP for the bit $b_1$ can be written as

$$P_{e(mrc)}(b_1|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Q\left(\sqrt{\frac{d^2}{\sigma_{\beta_{mrc}}^2 + \sigma_{W_{mrc}}^2}}\right) + Q\left(\sqrt{\frac{9d^2}{\sigma_{\beta_{mrc}}^2 + \sigma_{W_{mrc}}^2}}\right) + Q\left(\sqrt{9\left(\frac{2E_b}{5}\right)\left(\sum_{r=1}^{R}|H_r|^2\right)^2}\right) + Q\left(\sqrt{9\left(\frac{2E_b}{5}\right)\sum_{r=1}^{R}|H_r|^2}\right) \right]$$

(5.24)

where $\sigma_{\beta_{mrc}}^2 = \Phi_{\Delta \theta} \sum_{r=1}^{R}|H_r|^2$. The term $\sigma_{W_{mrc}}^2 = \sigma_{W}^2 \sum_{r=1}^{R}|H_r|^2$.

Similarly, the conditional BEP for the bit $b_3$ is given by

$$P_{e(mrc)}(b_3|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Q\left(\sqrt{\frac{9d^2}{\sigma_{\beta_{mrc}}^2 + \sigma_{W_{mrc}}^2}}\right) + Q\left(\sqrt{\frac{d^2}{\sigma_{\beta_{mrc}}^2 + \sigma_{W_{mrc}}^2}}\right) + Q\left(\sqrt{\frac{d^2}{\sigma_{\beta_{mrc}}^2 + \sigma_{W_{mrc}}^2}}\right) - Q\left(\sqrt{\frac{25d^2}{\sigma_{\beta_{mrc}}^2 + \sigma_{W_{mrc}}^2}}\right) \right]$$

(5.25)
which simplifies to

\[
P_{e(mrc)}(b_3|H_1(l), \ldots, H_R(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9(2E_b/5)\sum_{r=1}^{R}|H_r|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) ight. \\
+ Q\left( \sqrt{\frac{(2E_b/5)\sum_{r=1}^{R}|H_r|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
+ Q\left( \sqrt{\frac{(2E_b/5)\sum_{r=1}^{R}|H_r|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
- Q\left( \sqrt{\frac{25(2E_b/5)\sum_{r=1}^{R}|H_r|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] 
\] (5.26)

In (5.24) and (5.26) above, the conditional bit error probability (BEP) is now expressed in the SNR form of \( \gamma_{\text{snr}}(\theta) \) that is given by

\[
\gamma_{\text{snr}(mrc)}(\theta) = \frac{(2E_b/5)\sum_{r=1}^{R}|H_r|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2} 
\] (5.27)

where \( \gamma_{\text{snr}(mrc)}(\theta) \) is the SNR. Note that both (5.24) and (5.26) depend on \( \gamma_{\text{snr}(mrc)}(\theta) \), which is the sum of squares of independent, and identically distributed Rayleigh random variables \(|H_r|\) with variance \( \sigma^2_\alpha \). It is observed that \( \gamma_{\text{snr}(mrc)}(\theta) \) follows a Chi-square distribution [106] with \( \nu \) degrees of freedom, and probability density function (PDF) given by

\[
f\left( \gamma_{\text{snr}}(\theta(n)) \right) = \frac{\sqrt{\frac{\gamma^{\nu-1}}{2^{\nu/2}\sigma^\nu\Gamma(\nu/2)}}e^{-\frac{\gamma}{2\sigma} \sigma^\nu \Gamma(\nu/2)}}{2^{\nu/2}\sigma^\nu\Gamma(\nu/2)} \] (5.28)

where \( \sigma^2_\gamma = 2\nu(\sigma^2_\alpha)^2 \) and \( \sigma^2_\alpha \) is the variance of the Rayleigh distributed random variable \(|H_r|\). Furthermore, \( \nu = 2R \) since the channel \( H \) is a complex Gaussian random variable with independent in-phase and quadrature components.

Due to the symmetry of square M-QAM constellations, the BEP for the in-phase and quadrature bits are equal such that \( P_e(b_1) = P_e(b_2) \) and \( P_e(b_3) = P_e(b_4) \). Therefore the average BER is obtained by only averaging the conditional BEP of \( b_1 \) in (5.24) and \( b_3 \) in (5.26) over the PDF of \( \theta(n) \) and \( \gamma_{\text{snr}(mrc)}(\theta) \). The average BER is thus given by
\[
\overline{P}_{e(mrc)} = \frac{1}{2} \int_{-\gamma}^{\gamma} f(\gamma|\theta) d\gamma \int_{\theta} f(\theta) d\theta \\
\times \left[ P_{e(mrc)}(b_1|H_1, \ldots, H_R, \theta(n)) + P_{e(mrc)}(b_3|H_1, \ldots, H_R, \theta(n)) \right] \\
= \frac{1}{2} \int_{-\infty}^{\infty} f(\gamma|\theta) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta \\
\times \left[ P_{e(mrc)}(b_1|\gamma_{\text{snr}(mrc)}(\theta)) + P_{e(mrc)}(b_3|\gamma_{\text{snr}(mrc)}(\theta)) \right] 
\] (5.29)

### 5.5.8 MRC-OFDM Receiver with Imperfect CSI

We next consider the case of a system with imperfect channel estimation. Let \( \hat{H}_r(l) \) be the channel estimate on the \( l \)th subcarrier of the \( r \)th MRC branch. A linear channel estimation model is assumed as in the SISO case. In the presence of imperfect channel estimation, we assume a channel estimation model such that the channel estimate \( \hat{H} \) of the true channel \( H \) is given by

\[
\hat{H} = H + \varepsilon 
\] (5.30)

where \( \varepsilon \) is the error in the channel estimate, and is modeled as zero-mean complex Gaussian random variable with variance \( 2\sigma^2 \varepsilon \), and independent of \( H \).

The decision variable on the \( l \)th subcarrier after employing MRC with imperfect CSI is

\[
\hat{X}_{mrc}^\varepsilon(l) = \sum_{r=1}^{R} Y_r(l)\hat{H}_r(l)^* 
\] (5.31)

where \( H^*(l) \) implies complex conjugate of \( H(l) \) and \( Y_r(l) \) is given by (5.9).

With prior CPE (\( V_4 \)) correction assumed, the decision variable at the output of the R-branch MRC is
\[
\hat{\mathbf{X}}_{\text{mrc}}^\epsilon(l) = \sum_{r=1}^{R} Y_r(l) \tilde{H}_r(l)^* \\
= \sum_{r=1}^{R} H_r(l) X(l) \tilde{H}_r^*(l) + \sum_{r=1}^{R} W_r(l) \tilde{H}_r^*(l) \\
+ \sum_{r=1}^{R} \sum_{k=0, k\neq l}^{N-1} H_r(k) X(k) V_B(k, l) \tilde{H}_r^*(l) \\
= \sum_{r=1}^{R} H_r(l) X(l) \left[ H_r^*(l) + \tilde{\epsilon}_r^*(l) \right] \\
+ \sum_{r=1}^{R} \sum_{k=0, k\neq l}^{N-1} H_r(k) X(k) V_B(k, l) \left[ H_r^*(l) + \tilde{\epsilon}_r^*(l) \right] \\
+ \sum_{r=1}^{R} W_r(l) \left[ H_r^*(l) + \tilde{\epsilon}_r^*(l) \right] \\
\tag{5.32}
\]

\[
\tilde{\mathbf{X}}_{\text{mrc}}^\epsilon(l) = \sum_{r=1}^{R} H_r(l) X(l) H_r^*(l) + \sum_{r=1}^{R} H_r(l) X(l) \tilde{\epsilon}_r^*(l) \\
\overbrace{\tilde{D}_{\text{mrc}}(l)} + \overbrace{\tilde{\Psi}_{\text{mrc}}(l)} \\
+ \sum_{r=1}^{R} \sum_{k=0, k\neq l}^{N-1} H_r(k) X(k) V_B(k, l) \left[ H_r^*(l) + \tilde{\epsilon}_r^*(l) \right] \\
\overbrace{\tilde{\beta}_{\text{mrc}}(l)} \\
+ \sum_{r=1}^{R} W_r(l) \left[ H_r^*(l) + \tilde{\epsilon}_r^*(l) \right] \\
\overbrace{\tilde{W}_{\text{mrc}}(l)} \\
\tag{5.33}
\]

In (5.33), \( \tilde{D}_{\text{mrc}}(l) \) is the desired signal term. The term \( \tilde{\Psi}_{\text{mrc}}(l) \) is a noise term due to channel estimation error. The term \( \tilde{\beta}_{\text{mrc}}(l) \) is the ICI due to phase noise and is approximated as a Gaussian random variable. The proof of the Gaussianity of the ICI is presented in the Appendix. The last term is the additive Gaussian noise term.
5.5.9 Mean and Variance of ICI and Noise

The variance of the noise term $\tilde{W}_{mrc}^e(l)$ is given by

$$\sigma_{\tilde{W}_{mrc}}^2 = E\left[ \tilde{W}_{mrc}^e(l) \tilde{W}_{mrc}^{*e}(l) \right]$$
$$= E\left[ \left( \sum_{r=1}^{R} W_r(l) \tilde{H}_r^*(l) \right) \left( \sum_{\rho=1}^{R} W_{\rho}(l) \tilde{H}_{\rho}^*(l) \right)^* \right]$$
$$= E\left[ \sum_{r=1}^{R} \sum_{\rho=1}^{R} W_r(l) \tilde{H}_r^*(l) W_{\rho}^*(l) \tilde{H}_{\rho}(l) \right]$$
$$= \sum_{r=1}^{R} \sum_{\rho=1}^{R} E\left[ \tilde{H}_r(l) \tilde{H}_r^*(l) \right] E\left[ W_r(l) W_{\rho}^*(l) \right]$$
$$= \sum_{r=1}^{R} \sum_{\rho=1}^{R} E\left[ \tilde{H}_r(l) \tilde{H}_r^*(l) \right] \sigma_{W_r}^2 \delta_{r\rho} \quad (5.34)$$

where $\delta_{r\rho}$ is defined in (5.14).

The variance of the noise term $\tilde{W}_{mrc}^e(l)$ simplifies to

$$\sigma_{\tilde{W}_{mrc}}^2 = \sum_{r=1}^{R} E\left[ \left| H_r^*(l) + \varepsilon_r^*(l) \right|^2 \right] \sigma_{W_r}^2$$
$$= \sum_{r=1}^{R} \sigma_{W_r}^2 \left( |H_r(l)|^2 + \sigma_{\varepsilon_r}^2 \right)$$
$$= \sigma_{W_r}^2 \sum_{r=1}^{R} \left( |H_r(l)|^2 + \sigma_{\varepsilon_r}^2 \right) \quad (5.35)$$

where $\sigma_{W_r}^2$ is the variance of the Gaussian noise on the $R^{th}$ antenna branch. With the i.i.d. assumption, $\sigma_{W_r}^2 = \sigma_{W_r}^2$.

The variance of $\tilde{\Psi}_{mrc}(l)$ on the $l^{th}$ subcarrier is given by

$$\sigma_{\tilde{\Psi}_{mrc}}^2 = E\left[ \tilde{\Psi}_{mrc}(l) \tilde{\Psi}_{mrc}^*(l) \right]$$
$$= E_s \sum_{r=1}^{R} |H_r(l)|^2 \sigma_{\varepsilon_r}^2$$
$$= E_s \sigma_{\tilde{\Psi}_{mrc}}^2 \quad (5.36)$$
where \( \sigma_{\Psi_{mrc}}^2 = \sum_{r=1}^{R} |H_r|^2 \sigma_{\varepsilon_r}^2 \) and \( \sigma_{\Phi_{mrc}} = E \sigma_{\Psi_{mrc}}^2 \).

Conditioned on \( \theta(n) \) and the channel \( H_r(l) \), the variance \( \sigma_{\tilde{\beta}_{mrc}}^2 \) of the ICI term \( \tilde{\beta}_{mrc}(l) \) is derived using a similar approach as in the previous section.

\[
\sigma_{\tilde{\beta}_{mrc}}^2 = E \left[ \tilde{\beta}_{mrc}(l) \tilde{\beta}_{mrc}^*(l) \right] = E \left[ \sum_{r=1}^{R} \sum_{k=0,k \neq l}^{N-1} \tilde{H}_r(k) X(k) V_B(k,l) \tilde{H}_r^*(l) \sum_{\rho=1}^{R} \sum_{m=0,m \neq l}^{N-1} \tilde{H}_\rho(m) X(m) V_B(m,l) \tilde{H}_\rho^*(l) \right] \tag{5.37}
\]

Following similar steps as in the previous section, the variance of the ICI is given by

\[
\sigma_{\tilde{\beta}_{mrc}}^2 = \sum_{r=1}^{R} \sum_{k=0}^{N-1} E[|\tilde{H}_r(l)|^2] \cdot E[|X(k)H_r(k)V_B(k-l)|^2]
\]

\[
= E_s \sum_{r=1}^{R} \sum_{k=0,k \neq l}^{N-1} E[|H_r^*(l) + \varepsilon_r^*(l)|^2] |V_B(k-l)|^2
\]

\[
= E_s \sum_{r=1}^{R} \sum_{k=1}^{N-1} \left( |H_r^*(0)|^2 + \sigma_{\varepsilon_r}^2 \right) |V_B(k)|^2
\]

\[
= E_s \sum_{r=1}^{R} \sum_{k=0}^{N-1} \left( |H_r^*(0)|^2 + \sigma_{\varepsilon_r}^2 \right) \left( |V_B(k)|^2 - |V_B(0)|^2 \right) \tag{5.38}
\]

where we have assumed that the \( E[|H_r(k)|^2] = 1 \), and \( E_s \) is the transmitted symbol energy. Then, \( \sigma_{\tilde{\beta}_{mrc}}^2 \) simplifies to
\[ \sigma_{\beta_{mrc}}^2 = E_s \sum_{r=1}^{R} \left( |H_r^*(0)|^2 + \sigma_{\varepsilon}^2 \right) \left[ 1 - \frac{1}{N^2} \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} e^{j\Delta \theta_{nm}} \right] \]

\[ = E_s \Phi_{\Delta \theta} \sum_{r=1}^{R} \left( |H_r^*(0)|^2 + \sigma_{\varepsilon}^2 \right) \]

\[ = E_s \sum_{r=1}^{R} \left( |H_r^*(0)|^2 + \sigma_{\varepsilon}^2 \right) \]

(5.39)

where \( \sigma_{\beta_{mrc}}^2 = \Phi_{\Delta \theta} \sum_{r=1}^{R} \left( |H_r^*(0)|^2 + \sigma_{\varepsilon}^2 \right) \).

### 5.5.10 Probability of Error

Conditioned on \( \theta(n) \) and the \( H_r(l) \)'s on the \( R \) branches, the conditional bit error probability (BEP) for bit \( b_1 \) on the \( l^{th} \) subcarrier is given by

\[ P_{e(b_1)}(b_1|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Pr\left( \tilde{X}_I < 0 | X_I = d, H_1(l), ..., H_R(l), \theta(n) \right) \right. \]

\[ + Pr\left( \tilde{X}_I < 0 | X_I = 3d, H_1(l), ..., H_R(l), \theta(n) \right) \]

(5.40)

where \( X_I, \tilde{X}_I \) are the real parts of \( X, \tilde{X} \) respectively, and \( d^2 \) is proportional to the average symbol energy \( E_s \) such that \( E_s = 10d^2 \).

Using a similar approach, the conditional bit error probability for bit \( b_3 \) is

\[ P_{e(b_3)}(b_3|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Pr\left( |\tilde{X}_I| > 2d | X_I = d, H_1(l), ..., H_R(l), \theta(n) \right) \right. \]

\[ + Pr\left( |\tilde{X}_I| < 2d | X_I = 3d, H_1(l), ..., H_R(l), \theta(n) \right) \]

(5.41)

Rewriting (5.40) and (5.41) as a sum of Q-functions, and substituting for \( d \) as
a function of $E_b$, the conditional BEP for bit $b_1$ can be written as

$$P_{e_{\text{mrc}}}^e(b_1|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{d^2}{\sigma^2_{\Psi_{\text{mrc}}} + \sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}}} \right) + Q\left( \frac{9d^2}{\sigma^2_{\Psi_{\text{mrc}}} + \sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}} \right) \right]$$

$$= \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5)\left( \sum_{r=1}^{R} |H_r|^2 \right)^2}{\sigma^2_{\Psi_{\text{mrc}}} + (4E_b)\sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}}} \right) + Q\left( \frac{9(2E_b/5)\left( \sum_{r=1}^{R} |H_r|^2 \right)^2}{\sigma^2_{\Psi_{\text{mrc}}} + (4E_b)\sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}} \right) \right]$$

where $\sigma^2_{W_{\text{mrc}}} = \sigma^2_W \sum_{r=1}^{R} \left( |H_r^*(l)|^2 + \sigma^2_{\varepsilon} \right)$. The term $\sigma^2_{\beta_{\text{mrc}}} = \Phi \Delta \theta \sum_{r=1}^{R} \left( |H_r^*(l)|^2 + \sigma^2_{\varepsilon} \right)$.

Similarly, the conditional BEP for bit $b_3$ is given by

$$P_{e_{\text{mrc}}}^e(b_3|H_1(l), ..., H_R(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{d^2}{\sigma^2_{\Psi_{\text{mrc}}} + \sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}}} \right) + Q\left( \frac{9d^2}{\sigma^2_{\Psi_{\text{mrc}}} + \sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}} \right) + Q\left( \frac{d^2}{\sigma^2_{\Psi_{\text{mrc}}} + \sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}} \right) + Q\left( \frac{25d^2}{\sigma^2_{\Psi_{\text{mrc}}} + \sigma^2_{\beta_{\text{mrc}}} + \sigma^2_{W_{\text{mrc}}}} \right) \right]$$

(5.43)
The average BER is obtained by averaging the conditional BEP of $b_1$ in (5.42) and $b_3$ in (5.44) over the PDF of $\theta(n)$ and $\gamma_{\text{snr}(mrc)}(\theta)$. The average BER is thus given by

$$
\bar{P}_{e(mrc)} = \frac{1}{2} \int_{-\infty}^{\infty} f(\gamma|\theta) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta
\times \left[ P_{e(mrc)}(b_1|H_1, \ldots, H_R, \theta(n)) + P_{e(mrc)}(b_3|H_1, \ldots, H_R, \theta(n)) \right]
$$

$$
= \frac{1}{2} \int_{0}^{\infty} f(\gamma|\theta) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta
\times \left[ P_{e(mrc)}(b_1|\gamma_{\text{snr}(mrc)}(\theta)) + P_{e(mrc)}(b_3|\gamma_{\text{snr}(mrc)}(\theta)) \right]
$$

(5.45)
5.6 Results

In this section, the numerical and simulation results are presented, together with a discussion of the results. We simulate the performance for a 16-QAM OFDM system employing MRC and considering the effects of phase noise and channel estimation error.

5.6.1 Simulation Parameters

The parameters used in the simulations are summarized in Table 5.1, and are compliant to the IEEE802.16 standard. The simulation platform is implemented in MATLAB. An uncoded system is implemented. As such, the blocks that involve convolutional encoding/decoding in the IEEE 802.16 standard are omitted. Using 256 subcarriers and 16-QAM as the modulation of choice, the transmitted signal is processed by a 256-point FFT. For each of the independent receive antenna branches, a normalized channel power is assumed. Hence a zero-mean and unity variance for the fading channel. The number of receive antenna branches considered include $R = 2, 3, 4$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_\theta$</td>
<td>0.0, 0.01, 0.03, 0.06, 0.09</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.0, 0.02, 0.04, 0.06</td>
</tr>
<tr>
<td>Modulation</td>
<td>16QAM</td>
</tr>
<tr>
<td>Subcarriers (N)</td>
<td>256</td>
</tr>
<tr>
<td>Transmit Antennas</td>
<td>1</td>
</tr>
<tr>
<td>MRC branches (R)</td>
<td>2, 3, 4</td>
</tr>
</tbody>
</table>

5.6.2 Discussion

We present the numerical results of the average BER performance as a function of the average SNR for a 16-QAM MRC-OFDM system. Different permutations of the combined effect of phase noise and imperfect channel estimation, and for each of the Tx, Rx antenna configurations are considered. Since prior CPE correction is assumed in the analysis, the ICI is the only influence of the phase noise.
Figure-5.2 shows the results of the system performance with phase noise and perfect channel estimation, and further assuming perfect prior CPE correction, leaving the ICI as the only influence of the phase noise. For comparison, the results obtained for an ideal system with no phase noise and with perfect channel estimation, is also displayed on the same axis. It is observed that, with all the diversity benefits that an MRC system provides, the effect of phase noise is very significant in determining system performance.

Figure-5.3 illustrates the results of the system performance with phase noise \( \sigma_\theta^2 = 0.03 \) and imperfect channel estimation, with channel estimation error for \( \sigma_\epsilon^2 = 0.02 \). The added degradation due to the channel estimation error is also observed even for high number of receive antenna branches.

The effect of the phase noise contribution in Figure-5.4 is increased from \( \sigma_\theta^2 = 0.03 \) to \( \sigma_\theta^2 = 0.06 \), and with channel estimation error of \( \sigma_\epsilon^2 = 0.02 \). The degradation due to the increase in phase noise contribution is observed to be quite significant even with large number of antennas. This could be due to the fact that, although there is an increase in performance due to diversity, there is also increase interference from all antennas and from all subcarriers per antenna. Hence, more noise is introduced to the system, which consequently leads to further performance degradation.

Finally, the numerical results are compared with simulations in Figure-5.5, by employing parameters from Table-5.1. From the plots, it is observed that the analysis agrees with simulations.
Figure 5.2 Performance of 16QAM MRC-OFDM in Rayleigh fading channels with Phase Noise ($\sigma^2_{\theta} = 0.03$). Number of Receive Antenna branches = 1, 2, 3, 4. Number of subcarriers = 256.
Figure 5.3 Performance of 16QAM MRC-OFDM in Rayleigh fading channels with Phase Noise ($\sigma_\theta^2 = 0.03$) & Imperfect Channel Estimation ($\sigma_\epsilon^2 = 0.02$). Number of Receive Antenna branches = 1, 2, 3, 4. Number of subcarriers = 256.
Figure 5.4 Performance of 16QAM MRC-OFDM in Rayleigh fading channels with Phase Noise ($\sigma_{\theta}^2 = 0.06$) & Imperfect Channel Estimation ($\sigma_{\epsilon}^2 = 0.02$). Number of Receive Antenna branches = 1, 2, 3, 4. Number of subcarriers = 256.
Figure 5.5  Analysis and Simulation Results: Performance of 16QAM MRC-OFDM in Rayleigh fading channels with perfect Channel Estimation. Number of Receive Antenna branches = 1, 2, 3, 4. Number of subcarriers = 256.

5.7 Conclusion

The performance of MRC-OFDM systems with phase noise and channel estimation errors is analyzed. Analytic expression for the bit error probability of the MRC-OFDM system is derived. In most of the previous studies, the average BER requires an integral over Rayleigh fading for each of the receive antenna branches. It is quite evident that as the number of receive antenna branches \( R \) increases, the complexity of the \( R \)-fold numerical integral increases dramatically. In this analysis however, we exploit the fact that the SNR expression in the conditional BER expressions can be recognized as a random variable with a known PDF. This approach considerably simplifies the rest of the analysis, by reducing the \( R \)-fold integral of the \( R \) receive branches to a single integral.

Finally, numerical evaluation compared with simulation results show good match. Furthermore, the approach can be applied to any practical system.
The text in Chapter 5 is a revision of the material presented in:


The dissertation author was the primary researcher and author, and the co-author listed in these publication directed and supervised the research which forms the basis for this chapter.
6

Performance Analysis of MISO-OFDM with Phase Noise and Imperfect Channel Estimation

6.1 Introduction

In wireless channels, the signal power may fluctuate significantly during transmission. When the power drops considerably, the channel is said to be in deep fade.

The use of multiple antennas in wireless links with appropriate modulation and demodulation is rapidly becoming the new arena in wireless communications. This is even more so in wireless systems that employ multiple antennas at the transmitter. The use of multiple antennas at the transmitter in wireless communication links adds the new dimension of Space to the already exploited Time and Frequency dimensions. This new dimension, if leveraged correctly, can lead to substantial performance improvements.

Over the recent years, the field of multi-antenna communications has matured substantially, both in theory and practice. The advances in theory include a thorough understanding of the capacity and other performance limits of Space-Time wireless links, channel models and receiver design. A growing awareness of the huge perfor-
formance gains that are possible with Space-Time coding techniques has spurred efforts to integrate this technology into practical systems.

Space-Time block coding is a transmit diversity technique that relies on coding across transmit antennas, in order to extract diversity in the absence of channel knowledge at the transmitter. Diversity is used in wireless channels to combat channel fading, and is characterized by the number of independently fading branches, also known as diversity order. Diversity provides the receiver with multiple independent replicas of the transmitted signal. Each of the independent replicas constitutes a diversity branch. With an increase in the number of independent diversity branches, the likelihood that all branches are simultaneously in a deep fade is reduced considerably, leading to an increase in the wireless link reliability.

A typical Space Time wireless system is equipped with \( M_T \) transmit antennas and \( M_R \) receive antennas. The system shown in Fig-6.1 employs a receiver structure with \( M_R = 1 \). The input data bits are fed to the Space Time coding block for Space-Time encoding. The bits are interleaved to minimize burst errors, before they are mapped to data symbols and generate \( M_T \) symbols. The symbols are then upconverted to the high frequency passband for transmission by the \( M_T \) transmit antennas. The signals pass through the wireless channel, where they are attenuated and undergo fading before they arrive at the \( M_R \) receive antennas. Additive noise in the \( M_R \) receive chains corrupts the receive signal, which is then used for detection of the transmitted signals. These signal streams are de-interleaved and decoded to reproduce the transmitted data.

### 6.2 Transmit Diversity Techniques

Transmit diversity is applicable to MISO channels, and has become an active area of research. The use of diversity techniques in MIMO channels requires a combination of transmit and receive diversity. Transmit diversity techniques can be categorized into two groups [118], [119], [120]

- Transmit diversity with feedback
- Transmit diversity without feedback
Figure 6.1 STBC-OFDM System Block Diagram.
Within these two groups, various transmit diversity schemes have been proposed, such as \textit{Delay diversity} [121], [122], \textit{beamforming} [53], \textit{Antenna switching} [123]. In order to further improve the performance of the system, transmit diversity is combined with modulation and/or coding schemes with the consequence of improving both Diversity gain and Coding gain [109]. The combination of transmit diversity and Space-Time Codes (STC) that provide coding gain, is one of such combining techniques\(^1\).

Although Space-Time Block Codes (STBCs) do not provide coding gain, they possess a simple decoding algorithm, due to the orthogonality of the codes. As a result, STBCs provide full diversity in both Space and Time domains [33], [53], [124]. The simple decoding process facilitates a shorter time of implementation by reducing complexity.

When STBCs are employed, it is possible to employ Maximal Ratio Combining technique at the receiver. In general, the performance of systems can be further improved by combining transmit diversity and receive diversity.

\section{6.3 Space-Time Block Codes}

STBCs are among various practical and advanced coding techniques designed for the use of multiple antenna transmission, with the potential to approach the high capacity of MIMO systems [37], [44], [49], [109], [125]. Some of the most notable properties that make STBC so valuable and widely embraced in practical systems can be summarized as presented below. The STBC

\begin{itemize}
  \item decreases the sensitivity to multipath fading, and facilitates the utilization of higher order modulation schemes, which result in an increase in data rate.
  \item completely takes advantage of the channel bandwidth.
  \item requires the relatively simple Maximum Likelihood decoding technique due to the orthogonality of the codes [49], [50].
\end{itemize}

\(^1\)Space-Time Trellis Codes (STTC) are examples of codes having coding gain [49]. STTC is beyond the scope of this work.
• consequently improves the error performance and capacity of wireless communication channels.

From the above, it can be seen that STBC is a relative simple and cost-effective scheme to meet the stringent requirements on quality and spectral efficiency for next generation wireless systems without demanding much changes to the structure of existing systems.


6.4 Space-Time Block Codes in OFDM

High performance transceiver designs have been proposed for Orthogonal Frequency Division Multiplexing (OFDM) systems to develop high data rate wireless communication systems due to the compelling advantages over competing technologies. OFDM is a multi-carrier modulation technique in which the available bandwidth is divided among closely spaced but orthogonal frequency-flat sub-channels also referred to as subcarriers. This makes OFDM more robust in frequency-selective fading channels [17], [126], [127], [128]. The technology is proven to be a key technique for achieving the needed spectral efficiency and higher data rates requirements for current and future wireless communication systems, and is being used in many wireless applications such as digital video broadcasting (DVB), wireless local area networks (WLAN) and possible next-generation wireless networks such as fixed and mobile WiMax.

As the current trend of communication systems demands highly power-efficient and bandwidth-efficient schemes, techniques that provide such desirable properties are considered very valuable in next generation wireless systems. Making use of multiple antennas increases the capacity of the system [99] with the associated higher data rates than single antenna systems [48], [128], [129], [130], [131]. Space-Time coding is a power-efficient and bandwidth-efficient method of communication over fading channels by exploiting the benefits of multiple transmit antennas systems [33], [48], [50], [129], [132], [133]. The relative ease of implementation and the commercial viability of multi-antenna nodes or wireless access points (base stations in a cellular network, or access points in a WLAN for instance), coupled with the accompanied performance gains qualifies STBC with transmit diversity as a very attractive choice.

With the continuous increasing demand for higher transmission rates in wireless systems, the investigation of STBC that achieves maximum data rates only seems logical and inevitable. The design of maximum-rates STBC codes have been investigated and some are presented in the works of [48], [53], [129], [132] with special interest in orthogonal STBC (OSTBC) due to the simplicity of the linear processing involved at the receiver. Some of these higher-rate codes, however, are known to exhibit transmit power imbalance issues [48]. This becomes especially important in
multicarrier systems (with OFDM as a prime example) that are already plagued by Peak-to-Average-Power Ratio (PAPR) due to signal processing at the transmitter. Consequently, the choice of such codes becomes very relevant. A concise summary of complex orthogonal STBC and the maximum achievable rates of such codes are presented in [109].

With all the blessings of these celebrated practical advantages notwithstanding, the performance of OFDM is also known to be severely affected by hardware imperfections and RF impairments such as carrier frequency offset (CFO) and oscillator phase noise [17], [107] with further degradation due to imperfect channel estimation [24], [92], [107], [129].

The effects of phase noise in OFDM have been documented in the literature, including, but not limited to [17], [107]. These works investigated a non-STBC OFDM system with imperfect channel estimation considered in [107]. The performance of STBC-OFDM is available in the literature. While [130] and [131] investigated STBC-OFDM in a mobile environment, [130] presented only simulation results with no analysis for the BER. The author in [134] assumed perfect knowledge of the channel while [128] and [135] investigated performance gains and trade-offs in STBC-OFDM for higher number of transmit antennas ($N_t \geq 4$). The work of [127] on the other hand focuses on channel estimation with no phase noise impairment. In all these works, the joint effects of phase noise and imperfect channel estimation is not considered.

This Chapter extends the work in Chapter-4 to a Multi Input Single Output (MISO) communication system by considering STBC-OFDM employing multiple transmit antennas with applications to IEEE 802.16 standard. We investigate the performance of the system subject to phase noise and imperfect channel estimation, and derive the analytical BER expressions of the system.

6.5 System Model

We consider transmit diversity OFDM systems employing Space-Time Block Codes. A block diagram of the STBC system is shown in Fig-6.1. The information bits at the transmitter are grouped and mapped into the complex M-QAM symbols, and next fed to the STBC encoder. The STBC encoded bits are fed to the paral-
parallel arrangement of transmitter chains\(^2\) at the output of the STBC encoder. In all the analysis that follow, boldface letters represent vector quantities and underlined boldface letters represent matrix quantities.

For each transmit antenna, a block of \(N\) complex-valued data symbols \(\{X(k)\}_{k=0}^{N-1}\) are grouped and converted into a parallel set to form the input to the OFDM modulator, where \(k\) is the subcarrier index and \(N\) is the number of subcarriers. For each transmit-receive antenna combination, the total system bandwidth is divided into \(N\) equally spaced and orthogonal sub-carriers and used to transmit the complex symbols. The modulator consists of an Inverse Fast Fourier transform (IFFT) block. The output of the IFFT at each transmitter is the complex baseband modulated OFDM symbol in discrete time domain and is given by

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi kn}{N}}; \quad 0 \leq n \leq N - 1
\]  

(6.1)

where \(N\) is the number of subcarriers.

### 6.5.1 Channel

The channel is modeled by a tapped delay line with channel coefficients that are assumed to be slowly varying such that they are almost constant over the OFDM block. The channel frequency response for the \(k^{th}\) subcarrier is

\[
H(k) = \sum_{p=0}^{L-1} h(p)e^{j\frac{2\pi pk}{N}}
\]  

(6.2)

where \(h(p)\) is the complex channel gain of the \(p^{th}\) multipath component.

### 6.5.2 Phase Noise

The phase noise \(\theta(n)\) is modeled by a Tikhonov distribution [73], [66], [67], and is given by

\[
f_\theta(\theta(n)) = \frac{\cos(\theta(n))}{2\pi I_0\left(\frac{1}{\sigma_\theta^2}\right)}; \quad |\theta(n)| < \pi
\]  

(6.3)

\(^2\)The transmitter block is described in Chapter 4.
where \( n \) denotes time index, \( I_0(\alpha) \) is the \( 0^{th} \) order modified Bessel function of the first kind, and \( \sigma_\theta^2 \) is the variance of the phase noise.

### 6.6 2-Transmit Antenna STBC

We investigate an STBC-OFDM system employing the Alamouti code with two transmit antennas and one receive antenna. This code is used to encode the transmitted symbols at the two transmit antennas and can be described as follows. During the first time instant, the two symbols \([X_0 \ X_1]\) are transmitted from the two antennas simultaneously, with \(X_0\) and \(X_1\) transmitted from first and second antennas respectively. In the second time instant, the symbols \([-X_1^* \ X_0^*]\) are transmitted simultaneously from the two transmit antennas. This encoding of the transmitted symbol sequence from the transmit antennas is given by the encoding matrix [50], [48]

\[
g_2 = \begin{bmatrix} X_0 & X_1 \\ -X_1^* & X_0^* \end{bmatrix}
\]  

(6.4)

where subscript of 2 in \( g_2 \) represents number of transmit antennas.

#### 6.6.1 Received Signal

In all the analysis that follow, boldface letters represent vector quantities and underlined boldface letters represent matrix quantities. The time-domain received signals at the first and second transmission instances at the input to the FFT block are respectively given by

\[
y(2p - 1) = \left( h_0 \otimes x_0 + h_1 \otimes x_1 + w \right) e^{j\theta(n)}
\]

\[
y(2p) = \left( -h_0 \otimes x_1^* + h_1 \otimes x_0^* + w \right) e^{j\theta(n)}
\]

(6.5)

where \( p \) is the STBC time index, \( \otimes \) represents linear convolution, and subscripts indicate antenna index. The complex Gaussian random variable \( w(n) \) represents a zero-mean additive white Gaussian Noise with \( \sigma_w^2 = E[|w(n)|^2] \), and \( \theta(n) \) is the phase noise.
The frequency-domain received signal can be written as the sum of the desired signal component, the inter-carrier interference (ICI), and the additive noise term. We consider the \( l \)th subcarrier. The received signal on the \( l \)th subcarrier over the duration of the STBC which consists two consecutive OFDM symbols, is given by

\[
\begin{bmatrix}
Y_0(l) \\
Y_1^*(l)
\end{bmatrix} = \underbrace{\begin{bmatrix}
H_0(l)V_A & H_1(l)V_A \\
H_1^*(l)V_A & -H_0^*(l)V_A
\end{bmatrix}}_{\mathbf{D}_{2\chi_1}(l)} \begin{bmatrix}
X_0(l) \\
X_1(l)
\end{bmatrix} + \underbrace{\begin{bmatrix}
A + B \\
C + D
\end{bmatrix}}_{\beta_{2\chi_1}(l)} + \begin{bmatrix}
W_0(l) \\
W_1^*(l)
\end{bmatrix}
\] (6.6)

where the subscript in \( \mathbf{D}_{2\chi_1}, \beta_{2\chi_1}, \mathbf{W}_{2\chi_1} \) above describes the 2-Transmit, 1-Receive antenna configuration. The elements in \( \beta_{2\chi_1}(l) \) are defined as

\[
A = \sum_{k=0, k \neq l}^{N-1} H_0(k)X_0(k)V_B(k - l)
\]
\[
B = \sum_{k=0, k \neq l}^{N-1} H_1(k)X_1(k)V_B(k - l)
\]
\[
C = \sum_{k=0, k \neq l}^{N-1} H_1^*(k)X_0(k)V_B(k - l)
\]
\[
D = -\sum_{k=0, k \neq l}^{N-1} H_0^*(k)X_1(k)V_B(k - l)
\] (6.7)

where \( k \) is the index of interfering subcarriers, and \( N \) is the number of subcarriers. The received signal can be written compactly as

\[
\mathbf{Y}_{2\chi_1}(l) = \mathbf{D}_{2\chi_1}(l) + \beta_{2\chi_1}(l) + \mathbf{W}_{2\chi_1}(l)
\] (6.8)

The terms \( V_A \) and \( V_B(k - l) \) are defined as

\[
V_A = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\theta(n)}
\] (6.9)
\[ V_B(k - l) = \frac{1}{N} \sum_{n=0}^{N-1} e^{-\frac{j2\pi(k-l)n}{N}} e^{j\theta(n)} \]  

(6.10)

The term \( V_A \) is common to all subcarriers, and is frequently referred to in the literature as the common phase error (CPE). It can be corrected using pilots or other special techniques \[17\], \[107\].

### 6.6.2 Detection with Perfect Channel Estimation

We assume perfect channel estimation at the receiver. Furthermore, it is assumed that the CPE term \( V_A \) is effectively estimated and fully compensated using pilots that are continuously inserted within the OFDM symbols \[17\]. Assuming detection is performed per subcarrier, the detected symbols \( \tilde{X} \) on the \( l^{th} \) subcarrier are obtained as follows

\[
\tilde{X}_{2\chi_1} = H^H Y_{2\chi_1} 
\]  

(6.11)

where \( H^H \) implies Hermitian of \( H \).

\[
\begin{bmatrix}
    \tilde{X}_0(l) \\
    \tilde{X}_1(l)
\end{bmatrix} = \begin{bmatrix} H_0^*(l) & H_1(l) \\
                           H_1^*(l) & -H_0(l)
\end{bmatrix} \begin{bmatrix} H_0(l) & H_1(l) \\
                           H_1^*(l) & -H_0^*(l)
\end{bmatrix} \begin{bmatrix} X_0(l) \\
                           X_1(l)
\end{bmatrix} + \begin{bmatrix} H_0^*(l) & H_1(l) \\
                           H_1^*(l) & -H_0(l)
\end{bmatrix} \begin{bmatrix} A + B \\
                           C + D
\end{bmatrix} + \begin{bmatrix} H_0^*(l) & H_1(l) \\
                           H_1^*(l) & -H_0(l)
\end{bmatrix} \begin{bmatrix} W_0(l) \\
                           W_1^*(l)
\end{bmatrix}
\]  

(6.12)
Simplifying the above equation, we obtained

\[
\begin{bmatrix}
\tilde{X}_0(l) \\
\tilde{X}_1(l)
\end{bmatrix}
= \begin{bmatrix}
|H_0|^2 + |H_1|^2 & 0 \\
0 & |H_0|^2 + |H_1|^2
\end{bmatrix}
\begin{bmatrix}
X_0(l) \\
X_1(l)
\end{bmatrix}
+ \begin{bmatrix}
H_0^*(l) & H_1(l) \\
H_1^*(l) & -H_0(l)
\end{bmatrix}
\begin{bmatrix}
A + B \\
C + D
\end{bmatrix}
+ \begin{bmatrix}
H_0^*(l)W_0(l) + H_1(l)W_1^*(l) \\
H_1^*(l)W_0(l) - H_0(l)W_1^*(l)
\end{bmatrix}
\]

This equation can be written as

\[
\tilde{X}_{2\chi_1}(l) = \tilde{D}_{2\chi_1}(l) + \tilde{\beta}_{2\chi_1}(l) + \tilde{W}_{2\chi_1}(l)
\] (6.14)

In (6.14), the term $\tilde{D}_{2\chi_1}(l)$ is the desired signal component, the second term, $\tilde{\beta}_{2\chi_1}(l)$, is the ICI as a consequence of the phase noise contribution, and is approximated as a Gaussian random variable by invoking the Central Limit Theorem. The proof of the Gaussian approximation of the ICI term is presented in the Appendix. The last term, $\tilde{W}_{2\chi_1}(l)$, is the Gaussian noise.

We make the following assumptions in the analysis that follow. The transmitted data symbols $X_0(k), X_1(k)$ on antennas are independent of each other. The transmitted data symbols $X(k)'s$ on subcarriers are also independent. The channel is assumed to be slowly fading such that it is considered approximately constant over two consecutive OFDM symbols. It is further assumed that the transmit antennas are spaced sufficiently far apart that the channel gains can be assumed independent. The channel gains are complex Gaussian random variables with Rayleigh fading amplitudes and uniform phase. For each subcarrier, it is assumed that $X(k), H(k)$ and $V_B(k - l)$ are independent of each other. The term $V_B(k - l)$ is defined in (8).

It is also assumed that detection is accomplished on a per subcarrier basis. Due to the assumed independence of the channel gains and data on each subcarrier, the ICI term is the sum of conditionally independent (conditioned on the phase noise) random
variables from all interfering subcarriers. Assuming a sufficiently large number of subcarriers $N$ (for instance $N = 256$), and by invoking the Central Limit Theorem, the ICI term is approximated by a Gaussian random variable. The proof of the approximation is presented in the Appendix.

### 6.6.3 Mean and Variance of ICI and Noise

Conditioned on $\theta(n)$, the covariance matrix of the noise $\tilde{W}_{2\chi 1}(l)$ is given by

$$
\Omega_{\tilde{W}_{2\chi 1}} = E[\tilde{W}_{2\chi 1}(l) \tilde{W}_{2\chi 1}(l)^H]
$$

$$
= \begin{bmatrix} 
\sigma^2_{\tilde{W}_{2\chi 1}} & 0 \\
0 & \sigma^2_{\tilde{W}_{2\chi 1}} 
\end{bmatrix}
$$

$$
= \sigma^2_W \begin{bmatrix} 
\sum_{i=0}^{1} |H_i|^2 & 0 \\
0 & \sum_{i=0}^{1} |H_i|^2 
\end{bmatrix}
$$

(6.15)

As one notices above, $\sigma^2_{\tilde{W}_{2\chi 1}}$ is independent of $\theta(n)$, and is given by $\sigma^2_W \sum_{i=0}^{1} |H_i|^2$. The independence of $\theta(n)$ is presented in the Appendix of this Chapter. The term $\sigma^2_W$ is the variance of the Gaussian noise, and $H_i$ is the channel gain along the $i^{th}$ antenna.

Due to the effect of the phase noise, the ICI contribution $\tilde{\beta}_{2\chi 1}(l)$ manifests itself in the form of noise, which is approximated as a Gaussian random variable by invoking the Central Limit Theorem. The covariance matrix of the ICI term is given by
\[ \Omega_{\beta_{2\chi_1}} = E[\tilde{\beta}_{2\chi_1}(l) \tilde{\beta}_{2\chi_1}(l)^H] \]
\[ = \begin{bmatrix} E_s \Phi_{\Delta \theta} \sum_{i=0}^{1} |H_i|^2 & 0 \\ 0 & E_s \Phi_{\Delta \theta} \sum_{i=0}^{1} |H_i|^2 \end{bmatrix} \]
\[ = \begin{bmatrix} \sigma_{\beta_{2\chi_1}}^2 & 0 \\ 0 & \sigma_{\beta_{2\chi_1}}^2 \end{bmatrix} \]
\[ = E_s \Omega_{\beta} \quad (6.16) \]

where \( \sigma_{\beta_{2\chi_1}}^2 = E_s \sigma_{\beta_{2\chi_1}}^2 \). The transmitted symbol energy is \( E_s \) and \( \sigma_{\beta_{2\chi_1}}^2 = \Phi_{\Delta \theta} \sum_{i=0}^{1} |H_i|^2 \). The term \( \Phi_{\Delta \theta} \) is given by

\[ \Phi_{\Delta \theta} = \left( 1 - \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} e^{j\Delta \theta_{nm}} \right) \quad (6.17) \]

where \( \Delta \theta_{nm} = \left( \theta(n) - \theta(m) \right) \) is the phase noise increment.

### 6.6.4 Probability of Error

We present the bit error rate (BER) analysis for the case of 16-QAM modulation using Gray code mapping for \( (b_1 b_2 b_3 b_4) \) as shown in Figure-4.2. Although 16-QAM is only considered in this presentation, the analysis is valid for all square QAM constellations. Conditioned on \( \theta(n), H_0(l), H_1(l) \), the conditional bit error probability for the bit \( b_1 \) on the \( l \)th subcarrier is given by

\[ P_{e_{2\chi_1}}(b_1|H_0(l), H_1(l), \theta(n)) = \]
\[ = \frac{1}{2} \left[ Pr\left( \tilde{X}_I < 0 | X_I = d, H_0(l), H_1(l), \theta(n) \right) \right. \]
\[ + \left. Pr\left( \tilde{X}_I < 0 | X_I = 3d, H_0(l), H_1(l), \theta(n) \right) \right] \quad (6.18) \]
where $X_I, \tilde{X}_I$ are the real parts of $X, \tilde{X}$ respectively, and $d^2$ is proportional to the average symbol energy $E_s$ such that $E_s = 10d^2$ (note that $2d$ denotes the distance between two neighboring points on the signal constellation). With four bits per symbol, the energy per bit $E_b$ as a function of the energy per symbol $E_s$ is given by $E_s = 4E_b$. The energy per bit $E_b$ in terms of $d$ is given by $E_b = 5d^2/2$. Using a similar approach, the conditional bit error probability for the bit $b_3$ is

$$
\begin{align*}
P_{e_{2x1}}(b_3|H_0(l), H_1(l), \theta(n)) &= \\
&= \frac{1}{2} \left[ Pr\left(|\tilde{X}_I| > 2d|X_I = d, H_0(l), H_1(l), \theta(n)\right) \\
&+ Pr\left(|\tilde{X}_I| < 2d|X_I = 3d, H_0(l), H_1(l), \theta(n)\right) \right] \\
&= 1/2 \left[ Q\left( \sqrt{\frac{d^2}{\sigma_{\beta_{2x1}}^2 + \sigma_{W_{2x1}}^2}} \right) \\
&+ Q\left( \sqrt{\frac{9d^2}{\sigma_{\beta_{2x1}}^2 + \sigma_{W_{2x1}}^2}} \right) \right] \\
&= 1/2 \left[ Q\left( \sqrt{\frac{d^2}{\sigma_{\beta_{2x1}}^2 + \sigma_{W_{2x1}}^2}} \right) \\
&+ Q\left( \sqrt{\frac{9d^2}{\sigma_{\beta_{2x1}}^2 + \sigma_{W_{2x1}}^2}} \right) \right] \\
&= 1/2 \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
&+ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] \\
&= 1/2 \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
&+ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] \\
&= 1/2 \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
&+ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] \right)
\end{align*}

(6.19)$$

Rewriting (6.18) and (6.19) as a sum of Q-functions, and substituting for $d$ as a function of $E_b$, the conditional bit error probability for the bit $b_1$ can be written as

$$
\begin{align*}
P_{e_{2x1}}(b_1|H_0(l), H_1(l), \theta(n)) &= \frac{1}{2} \left[ Q\left( \sqrt{\frac{d^2}{\sigma_{\beta_{2x1}}^2 + \sigma_{W_{2x1}}^2}} \right) \\
&+ Q\left( \sqrt{\frac{9d^2}{\sigma_{\beta_{2x1}}^2 + \sigma_{W_{2x1}}^2}} \right) \right] \\
&= \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
&+ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] \\
&= \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
&+ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] \\
&= \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \\
&+ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right] \right)
\end{align*}

(6.20)$$

where the term $\sigma_{\beta_{2x1}}^2 = \Phi_{\Delta\theta} \sum_{i=0}^{1} |H_i|^2$, and $\sigma_W^2$ is the variance of the Gaussian noise. The term $\sigma_{W_{2x1}}^2 = \sigma_W^2 \sum_{i=0}^{1} |H_i|^2$, and $\Phi_{\Delta\theta}$ is given by (6.17).
Similarly, the conditional BEP for the bit $b_3$ is given by

$$P_e(3|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9d^2}{\sigma_{\beta_{2\chi_1}}^2}} \right) + Q\left( \sqrt{\frac{d^2}{\sigma_{\beta_{2\chi_1}}^2}} \right) + Q\left( \sqrt{\frac{d^2}{\sigma_{\beta_{2\chi_1}}^2}} \right) - Q\left( \sqrt{\frac{25d^2}{\sigma_{\beta_{2\chi_1}}^2}} \right) \right]$$

(6.20)

Denote $\gamma_{\text{snr}}$ as the SNR. Then, in (6.20) and (6.21) above, the conditional bit error probability is now expressed in a form $\gamma_{\text{snr}}(\theta)$ given by

$$P_e(3|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma_W^2}} \right) + Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma_W^2}} \right) + Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma_W^2}} \right) - Q\left( \sqrt{\frac{25(2E_b/5) \sum_{i=0}^{1} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma_W^2}} \right) \right]$$

(6.21)

Denote $\gamma_{\text{snr}}$ as the SNR. Then, in (6.20) and (6.21) above, the conditional bit error probability is now expressed in a form $\gamma_{\text{snr}}(\theta)$ given by

$$\gamma_{\text{snr}}(\theta) = \frac{2E_b}{5} \left( \sum_{i=0}^{1} |H_i|^2 \right) \left( (4E_b)\Phi_{\Delta \theta} + \sigma_W^2 \right)$$

(6.22)

where $\gamma_{\text{snr}}(\theta)$ is the SNR, and $\Phi_{\Delta \theta}$ is defined in (6.17). Note that $H_i$ is a zero mean complex Gaussian random variable with variance $\sigma^2$. The magnitude of $H_i$ follows a Rayleigh distribution, and the phase is uniformly distributed. Furthermore, it is observed that (6.20) and (6.21) depend on $\gamma_{\text{snr}}(\theta)$, which is the sum of squares of independent, and identically distributed Rayleigh random variables $|H_i|^2$ with variance $\sigma^2$. It is then observed that $\gamma_{\text{snr}}(\theta)$ generally follows a Chi-square distribution [106] with $\nu$ degrees of freedom, and the probability density function (PDF)
is given by

\[ f(\gamma_{\text{snr}}|\theta(n)) = \frac{\gamma^{\frac{\nu}{2} - 1}}{2^{\frac{\nu}{2}} \sigma^2} \Gamma \left( \frac{\nu}{2} \right) e^{-\frac{\gamma}{2\sigma^2}} \]  

(6.23)

where \( \Gamma \) is the Gamma function, and is defined as \( \Gamma(z) = (z - 1)! \), and \( \sigma^2 \) is the variance of \( H_i \). The variance of \( \gamma_{\text{snr}} \) as a function of the variance of \( H_i \) is given by \( \sigma^2_{\gamma} = 2\nu\sigma^4 \). In our case, the \( \gamma_{\text{snr}2\chi_1}(\theta) \) is the sum of squares of two complex, independent, and identically distributed Rayleigh random variables. The number of degrees of freedom is equal to the number of independent fading channels. Based on this, the PDF in this case is given by

\[ f(\gamma_{\text{snr}2\chi_1}|\theta(n)) = \frac{\gamma^{\frac{\nu}{2} - 1}}{2^{\frac{\nu}{2}} \sigma^2} \Gamma \left( \frac{\nu}{2} \right) e^{-\frac{\gamma}{2\sigma^2}} \]  

(6.24)

Due to the symmetry of square M-QAM constellations, the bit error probability for the in-phase and quadrature bits are equal such that \( P_{e2\chi_1}(b_1) = P_{e2\chi_1}(b_2) \) and \( P_{e2\chi_1}(b_3) = P_{e2\chi_1}(b_4) \). Therefore the average bit error probability is obtained by only averaging the conditional BEP of \( b_1 \) in (6.20) and \( b_3 \) in (6.21) over the PDF of \( \theta(n) \) and \( \gamma_{\text{snr}}(\theta) \). The average BER is

\[ P_{e2\chi_1} = \frac{1}{2} \int_0^\infty f(\gamma|\theta) d\gamma \int_{\theta} f(\theta) d\theta 
\times \left[ P_{e2\chi_1}(b_1|H_0, H_1, \theta(n)) + P_{e2\chi_1}(b_3|H_0, H_1, \theta(n)) \right] 
= \frac{1}{2} \int_{-\pi}^{\pi} f(\gamma|\theta) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta 
\times \left[ P_{e2\chi_1}(b_1|\gamma(\theta)) + P_{e2\chi_1}(b_3|\gamma(\theta)) \right] \]  

(6.25)

### 6.6.5 Detection with Imperfect Channel Estimation

In the presence of imperfect channel estimation, we assume a channel estimation model such that the channel estimate \( \hat{H}_i \) of the true channel \( H_i \) is given by

\[
\begin{bmatrix}
\hat{H}_0 & \hat{H}_1 \\
\hat{H}_1^* & -\hat{H}_0^*
\end{bmatrix} = \begin{bmatrix}
H_0 + \varepsilon_0 & H_1 + \varepsilon_1 \\
H_1^* + \varepsilon_1^* & -H_0^* - \varepsilon_0^*
\end{bmatrix}
\]  

(6.26)
where $\varepsilon_0$ and $\varepsilon_1$ are the errors in the channel estimates of $H_0$ and $H_1$ respectively, and are modeled as independent zero-mean complex Gaussian random variables with variances $2\sigma^2_{\varepsilon_0}$ and $2\sigma^2_{\varepsilon_1}$ respectively.

We assume that the common phase error, $V_A$, is effectively estimated and fully compensated for using pilots. For simplicity in the analysis that follow, the explicit use of the subcarrier index is suppressed. Using similar steps outlined in the analysis in the previous section, the detected symbols $\tilde{X}_{2\chi_1}$ on the $l$th subcarrier, in the presence of channel estimation errors are given by

\[
\begin{aligned}
\tilde{X}_{2\chi_1} &= \tilde{H}^H Y_{2\chi_1} \\
\begin{bmatrix}
\tilde{X}_0 \\
\tilde{X}_1
\end{bmatrix} &= 
\begin{bmatrix}
H_0^* + \varepsilon_0^* & H_1 + \varepsilon_1 \\
H_1^* + \varepsilon_1^* & -H_0 - \varepsilon_0
\end{bmatrix}
\begin{bmatrix}
H_0 & H_1 \\
H_1^* & -H_0^*
\end{bmatrix}
\begin{bmatrix}
X_0(l) \\
X_1(l)
\end{bmatrix}
\end{aligned}
\]

\[
+ 
\begin{bmatrix}
H_0^* + \varepsilon_0^* & H_1 + \varepsilon_1 \\
H_1^* + \varepsilon_1^* & -H_0 - \varepsilon_0
\end{bmatrix}
\begin{bmatrix}
A + B \\
C + D
\end{bmatrix}
\begin{bmatrix}
X_0(l) \\
X_1(l)
\end{bmatrix}
\]

\[
+ 
\begin{bmatrix}
H_0^* + \varepsilon_0^* & H_1 + \varepsilon_1 \\
H_1^* + \varepsilon_1^* & -H_0 - \varepsilon_0
\end{bmatrix}
\begin{bmatrix}
W_0(l) \\
W_1^*(l)
\end{bmatrix}
\]

(6.27)

where the elements $A, B, C, D$ are defined in (6.7). Expanding the above equation, we obtain
\[
\begin{bmatrix}
\tilde{X}_0 \\
\tilde{X}_1
\end{bmatrix} = 
\begin{bmatrix}
|H_0|^2 + |H_1|^2 & 0 \\
0 & |H_0|^2 + |H_1|^2
\end{bmatrix}
\begin{bmatrix}
X_0(l) \\
X_1(l)
\end{bmatrix}
\]

\[
+ 
\begin{bmatrix}
H_0\varepsilon_0^* + H_1^*\varepsilon_1 & H_1^*\varepsilon_0^* - H_0^*\varepsilon_1 \\
H_0\varepsilon_1^* - H_1^*\varepsilon_0 & H_1^*\varepsilon_1^* + H_0^*\varepsilon_0
\end{bmatrix}
\begin{bmatrix}
X_0(l) \\
X_1(l)
\end{bmatrix}
\]

\[
+ 
\begin{bmatrix}
H_0^* + \varepsilon_0^* & H_1 + \varepsilon_1 \\
H_1^* + \varepsilon_1^* & -H_0 - \varepsilon_0
\end{bmatrix}
\begin{bmatrix}
A + B \\
C + D
\end{bmatrix}
\]

\[
+ 
\begin{bmatrix}
(H_0^* + \varepsilon_0^*)W_0 + (H_1 + \varepsilon_1)W_1^* \\
(H_1^* + \varepsilon_1^*)W_0 - (H_0 + \varepsilon_0)W_1^*
\end{bmatrix}
\] (6.28)

In (6.28), the term $\tilde{D}_{2\chi_1}(l)$ is the desired signal component. The second term $\tilde{\Psi}_{2\chi_1}(l)$ is a noise term due to imperfect channel estimation. Conditioned on $\theta(n)$ and the channel gains $H_0(l), H_1(l)$, the term $\tilde{\Psi}_{2\chi_1}(l)$ is a Gaussian random variable since $\varepsilon_0$ and $\varepsilon_1$ are modeled as Gaussian. The term $\tilde{\beta}_{2\chi_1}(l)$ is the ICI which is approximated as Gaussian, and the last term is the additive Gaussian noise term. The above equation can be written compactly as

\[
\tilde{X}_{2\chi_1}^\epsilon(l) = \tilde{D}_{2\chi_1}(l) + \tilde{\Psi}_{2\chi_1}(l) + \tilde{\beta}_{2\chi_1}(l) + \tilde{W}_{2\chi_1}^\epsilon(l) 
\] (6.29)

Conditioned on $\theta(n)$ and the channel gains $H_0(l), H_1(l)$, then $\tilde{X}_{2\chi_1}^\epsilon(l)$ is a zero-mean complex Gaussian random variable with variance $(\sigma^2_{\tilde{\Psi}_{2\chi_1}} + \sigma^2_{\tilde{\beta}_{2\chi_1}} + \sigma^2_{\tilde{W}_{2\chi_1}})$. 
6.6.6 Mean and Variance of ICI and Noise

The covariance matrix of the Gaussian noise $\tilde{W}_{2\chi_1}(l)$ is given by

$$
\Omega_{\tilde{W}_{2\chi_1}} = E[\tilde{W}_{2\chi_1}(l) \tilde{W}_{2\chi_1}(l)^H]
$$

$$
= \sigma_W^2 \begin{bmatrix}
\sum_{i=0}^1 (|H_i|^2 + \sigma_{\tilde{\xi}_i}^2) & 0 \\
0 & \sum_{i=0}^1 (|H_i|^2 + \sigma_{\tilde{\xi}_i}^2)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\sigma_{\tilde{W}_{2\chi_1}}^2 & 0 \\
0 & \sigma_{\tilde{W}_{2\chi_1}}^2
\end{bmatrix}
$$

(6.30)

where $\sigma_{\tilde{W}_{2\chi_1}}^2 = \sigma_W^2 \sum_{i=0}^1 (|H_i|^2 + \sigma_{\tilde{\xi}_i}^2)$ and $\sigma_W^2$ is the variance of the Gaussian noise, with $\sigma_{\tilde{\xi}_i}^2$ the variance of the error in the channel estimate of $H_i$.

The covariance matrix of $\tilde{\Psi}_{2\chi_1}(l)$ is given by

$$
\Omega_{\tilde{\Psi}_{2\chi_1}} = E[\tilde{\Psi}_{2\chi_1}(l) \tilde{\Psi}_{2\chi_1}(l)^H]
$$

$$
= E_s \begin{bmatrix}
\left( \sum_{i=0}^1 |H_i|^2 \right) \left( \sum_{i=0}^1 \sigma_{\tilde{\xi}_i}^2 \right) & 0 \\
0 & \left( \sum_{i=0}^1 |H_i|^2 \right) \left( \sum_{i=0}^1 \sigma_{\tilde{\xi}_i}^2 \right)
\end{bmatrix}
$$

$$
= E_s \begin{bmatrix}
\sigma_{\tilde{\Psi}_{2\chi_1}}^2 & 0 \\
0 & \sigma_{\tilde{\Psi}_{2\chi_1}}^2
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\sigma_{\tilde{\Psi}_{2\chi_1}}^2 & 0 \\
0 & \sigma_{\tilde{\Psi}_{2\chi_1}}^2
\end{bmatrix}
$$

(6.31)

where $\sigma_{\tilde{\Psi}_{2\chi_1}}^2 = \left( \sum_{i=0}^1 |H_i|^2 \right) \left( \sum_{i=0}^1 \sigma_{\tilde{\xi}_i}^2 \right)$ and $\sigma_{\tilde{\Psi}_{2\chi_1}}^2 = E_s \sigma_{\tilde{\Psi}_{2\chi_1}}^2$.

Similarly, the covariance matrix of the ICI term $\tilde{\beta}_{2\chi_1}(l)$ is given by
\[ \Omega_{\tilde{g}_{2x1}} = E[\tilde{\beta}_{2x1}(l) \tilde{\beta}_{2x1}(l)^H] \]

\[ = E_s \Phi_{\Delta \theta} \begin{bmatrix} \sum_{i=0}^{1}(|H_i|^2 + \sigma_i^2) & 0 \\ 0 & \sum_{i=0}^{1}(|H_i|^2 + \sigma_i^2) \end{bmatrix} \]

\[ = E_s \begin{bmatrix} \frac{\sigma_{\tilde{g}_{2x1}}^2}{2} & 0 \\ 0 & \frac{\sigma_{\tilde{g}_{2x1}}^2}{2} \end{bmatrix} \]

\[ = \begin{bmatrix} \frac{\sigma_{\tilde{g}_{2x1}}^2}{2} & 0 \\ 0 & \frac{\sigma_{\tilde{g}_{2x1}}^2}{2} \end{bmatrix} \] (6.32)

where \( \sigma_{\tilde{g}_{2x1}}^2 = E_s \sigma_{\tilde{g}_{2x1}}^2 \), and \( \sigma_{\tilde{g}_{2x1}}^2 = \Phi_{\Delta \theta} \sum_{i=0}^{1}(|H_i|^2 + \sigma_i^2) \). In addition, the term \( \Phi_{\Delta \theta} \) is defined in (6.17).

### 6.6.7 Probability of Error

We present the BER analysis for the case of 1-6QAM only. Conditioned on \( \theta(n) \) and the channel gains, \( H_0(l) \text{and} H_1(l) \), the conditional BER for the bit \( b_1 \) is given by

\[ P_{e_{2x1}}^c(b_1|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Pr\left( |\tilde{X}_I| < 0 | X_I = d, H_0(l), H_1(l), \theta(n) \right) \right. \]

\[ + \left. Pr\left( |\tilde{X}_I| < 0 | X_I = 3d, H_0(l), H_1(l), \theta(n) \right) \right] \] (6.33)

where \( X_I, \tilde{X}_I \) are the real parts of \( X, \tilde{X} \) respectively. Using a similar approach as in the previous section, the conditional bit error probability for the bit \( b_3 \) is

\[ P_{e_{2x1}}^c(b_3|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Pr\left( |\tilde{X}_I| > 2d | X_I = d, H_0(l), H_1(l), \theta(n) \right) \right. \]

\[ + \left. Pr\left( |\tilde{X}_I| < 2d | X_I = 3d, H_0(l), H_1(l), \theta(n) \right) \right] \] (6.34)

Rewriting (6.33) and (6.34) as a sum of Q-functions, and substituting for \( d \) as a function \( E_b \), the conditional BER for the bit \( b_1 \) can be written as
\[
P_{e_{2x1}}(b_1|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \frac{d^2}{\sigma_{\Psi_{2x1}}^2 + \sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \right. \\
+ Q\left( \frac{9d^2}{\sigma_{\Psi_{2x1}}^2 + \sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \right] \\
= \frac{1}{2} \left[ Q\left( \frac{(2E_b/5)\left( \sum_{i=0}^1 |H_i|^2 \right)^2}{(4E_b)\sigma_{\Psi_{2x1}}^2 + (4E_b)\sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \right. \\
+ Q\left( \frac{9(2E_b/5)\left( \sum_{i=0}^1 |H_i|^2 \right)^2}{(4E_b)\sigma_{\Psi_{2x1}}^2 + (4E_b)\sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \right] \quad (6.35)
\]

and for the bit \( b_3 \) is given by

\[
P_{e_{2x1}}(b_3|H_0(l), H_1(l), \theta(n)) = \\
\frac{1}{2} \left[ Q\left( \frac{9d^2}{\sigma_{\Psi_{2x1}}^2 + \sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \right. \\
+ Q\left( \frac{d^2}{\sigma_{\Psi_{2x1}}^2 + \sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \\
+ Q\left( \frac{d^2}{\sigma_{\Psi_{2x1}}^2 + \sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \\
- Q\left( \frac{25d^2}{\sigma_{\Psi_{2x1}}^2 + \sigma_{\beta_{2x1}}^2 + \frac{\sigma_{\Psi_{2x1}}^2}{\sigma_{W_{2x1}}^2}} \right) \right] \quad (6.36)
\]

This can be further simplified to
\[ P_{e_{2x1}}^e (b_3|H_0(l), H_1(l), \theta(n)) = \]
\[
\frac{1}{2} \left[ Q \left( \sqrt{\frac{9(2E_b/5) \left( \sum_{i=0}^{1} |H_i|^2 \right)^2}{(4E_b)\sigma_{\Psi_{2x1}}^2 + (4E_b)\sigma_{\beta_{2x1}}^2 + \sigma_{f_{W_{2x1}}}^2}} \right) + Q \left( \sqrt{\frac{(2E_b/5) \left( \sum_{i=0}^{1} |H_i|^2 \right)^2}{(4E_b)\sigma_{\Psi_{2x1}}^2 + (4E_b)\sigma_{\beta_{2x1}}^2 + \sigma_{f_{W_{2x1}}}^2}} \right) + Q \left( \sqrt{\frac{(2E_b/5) \left( \sum_{i=0}^{1} |H_i|^2 \right)^2}{(4E_b)\sigma_{\Psi_{2x1}}^2 + (4E_b)\sigma_{\beta_{2x1}}^2 + \sigma_{f_{W_{2x1}}}^2}} \right) - Q \left( \sqrt{\frac{25(2E_b/5) \left( \sum_{i=0}^{1} |H_i|^2 \right)^2}{(4E_b)\sigma_{\Psi_{2x1}}^2 + (4E_b)\sigma_{\beta_{2x1}}^2 + \sigma_{f_{W_{2x1}}}^2}} \right) \right] \quad (6.37) \]

As mentioned earlier, due to the symmetry of square M-QAM constellations, we only need to evaluate the conditional bit error probability for \( b_1 \) and \( b_3 \) and average over the distributions of \( \theta(n) \) and \( \gamma_{snr_{2x1}}(\theta) \) which is given by

\[
\overline{P}_{e_{2x1}}^e = \frac{1}{2} \int f(\gamma) d\gamma \int f(\theta) d\theta 
\times \left[ P_{e_{2x1}}^e (b_1|H_0, H_1, \theta(n)) + P_{e_{2x1}}^e (b_3|H_0, H_1, \theta(n)) \right] \\
= \frac{1}{2} \int_0^\infty f(\gamma) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta 
\times \left[ P_{e_{2x1}}^e (b_1|\gamma(\theta)) + P_{e_{2x1}}^e (b_3|\gamma(\theta)) \right] \quad (6.38) \]

### 6.7 4-Transmit Antenna STBC

We next consider an OFDM system that employs STBC for a 4-Transmit, 1-Receive antenna configuration. The encoding of the transmission sequence from the transmit antennas is given by the code matrix as outlined in equation (38) of [48], and is shown below.
where the subscript $4$ in $g_4$ denotes the number of transmit antennas. Each column of the matrix corresponds to the transmission sequence per antenna over the total transmission time. This is a $1/2$ rate code, with four symbols transmitted across four transmit antennas over eight time instants.

### 6.7.1 Received Signal

In this section, we investigate the effect of phase noise on the performance of a 4-Transmit, 1-Receive antenna configuration. We consider the $l^{th}$ subcarrier and for simplicity, the subcarrier indices in $H(k)$, $X(k)$, and $V_B(k - l)$ are suppressed. The use of $\sum$ in the ICI term $\beta_{4x1}(l)$ below is for the purpose of notational compactness to represent $\sum_{k=0, k \neq l}^{N-1}$. The demodulated signal on the $l^{th}$ subcarrier can be written as the sum of the desired signal, ICI, and noise terms components, and is given by

$$g_4 = \begin{bmatrix}
X_1 & X_2 & X_3 & X_4 \\
-X_2 & X_1 & -X_4 & X_3 \\
-X_3 & X_4 & X_1 & -X_2 \\
-X_4 & -X_3 & X_2 & X_1 \\
X_1^* & X_2^* & X_3^* & X_4^* \\
-X_2^* & X_1^* & -X_4^* & X_3^* \\
-X_3^* & X_4^* & X_1^* & -X_2^* \\
-X_4^* & -X_3^* & X_2^* & X_1^*
\end{bmatrix}$$

(6.39)
The received signal where \( V \) depends on the subcarrier indices \( k \). The term \( V_A \) is defined in (6.9) and \( V_B \) depends on the subcarrier indices \( k, l \) as defined in (6.10).

The received signal \( Y_{4\times1}(l) \) can be written compactly as

\[
Y_{4\times1}(l) = D_{4\times1}(l) + \beta_{4\times1}(l) + W_{4\times1}(l) 
\]

(6.41)

### 6.7.2 Detection with Perfect CSI

The detected symbols on the \( i^{th} \) subcarrier are given by

\[
\tilde{X}_{4\times1}(l) = H^H Y_{4\times1}(l) 
\]

(6.42)
where $\mathbf{H}^H$ implies Hermitian of the channel matrix $\mathbf{H}$. With common phase error (CPE) correction assumed, this can be written as

$$
\begin{bmatrix}
\tilde{X}_1 \\
\tilde{X}_2 \\
\tilde{X}_3 \\
\tilde{X}_4
\end{bmatrix} = 2\left(\sum_{i=1}^{4} |H_i|^2\right)\mathbf{I}_{4\times4} + \tilde{\mathbf{D}}_{4\times1}(l) + \mathbf{H}^H \tilde{\mathbf{\beta}}_{4\times1}(l) + \mathbf{H}^H
$$

(6.43)

$$
\tilde{X}_{4\times1}(l) = \tilde{\mathbf{D}}_{4\times1}(l) + \tilde{\mathbf{\beta}}_{4\times1}(l) + \tilde{\mathbf{W}}_{4\times1}(l)
$$

(6.44)

where $\mathbf{I}_{4\times4}$ is the 4 by 4 identity matrix.

The detected signal is now written as a sum of the desired signal component $\tilde{\mathbf{D}}_{4\times1}(l)$, the ICI $\tilde{\mathbf{\beta}}_{4\times1}(l)$, which is approximated by a Gaussian random variable, and the Gaussian noise $\tilde{\mathbf{W}}_{4\times1}(l)$.

### 6.7.3 Mean and Variance of ICI and Noise

The covariance matrix $\Omega_{\tilde{\mathbf{\beta}}_{4\times1}}$ of the ICI term $\tilde{\mathbf{\beta}}_{4\times1}(l)$ is given by

$$
\Omega_{\tilde{\mathbf{\beta}}_{4\times1}} = 4E_s \Phi_{\Delta\theta} \left(\sum_{i=1}^{4} |H_i|^2\right)\mathbf{I}_{4\times4}
$$

$$
= 4E_s \sigma_{\tilde{\mathbf{\beta}}_{4\times1}}^2 \mathbf{I}_{4\times4}
$$

$$
= \sigma_{\tilde{\mathbf{\beta}}_{4\times1}}^2 \mathbf{I}_{4\times4}
$$

(6.45)

where $\sigma_{\tilde{\mathbf{\beta}}_{4\times1}}^2 = 4E_s \sigma_{\tilde{\mathbf{\beta}}_{4\times1}}^2$ and $\sigma_{\mathbf{\beta}}^2 = \Phi_{\Delta\theta} \left(\sum_{i=1}^{4} |H_i|^2\right)$. The average symbol energy is denoted as $E_s$, and $\Phi_{\Delta\theta}$ is given by (6.17).
The covariance matrix $\Omega_{\tilde{W}_{4x1}}$ of the additive Gaussian noise $\tilde{W}_{4x1}(l)$ is given by

$$\Omega_{\tilde{W}_{4x1}} = E[H^H W_{4x1} W_{4x1}^H H]$$
$$= H^H R_{W_{4x1}} H$$
$$= 4 \sigma_W^2 \left( \sum_{i=1}^{4} |H_i|^2 \right) I_{4x4}$$
$$= \sigma_{W_{4x1}}^2 I_{4x4}$$  \hspace{1cm} (6.46)$$

where $R_{W_{4x1}}$ is a 4 by 4 diagonal matrix with diagonal elements $\sigma_W^2$, and $\sigma_{W_{4x1}}^2 = 4 \sigma_W^2 \left( \sum_{i=1}^{4} |H_i|^2 \right)$.

### 6.7.4 Probability of Error

Using a similar approach described in the previous section, with the assumption that the CPE, $V_A$ is adequately corrected, the conditional bit error probability for the bit $b_1$ is given as

$$P_{e_{4x1}}(b_1|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5) \left( \sum_{i=1}^{4} |H_i|^2 \right)^2}{(4E_b)4 \sigma_{b_{4x1}}^2 + \sigma_{W_{4x1}}^2}} \right) + Q\left( \sqrt{\frac{9(2E_b/5) \left( \sum_{i=1}^{4} |H_i|^2 \right)^2}{(4E_b)4 \sigma_{b_{4x1}}^2 + \sigma_{W_{4x1}}^2}} \right) \right]$$
$$= \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_b/5) \sum_{i=1}^{4} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) + Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=1}^{4} |H_i|^2}{(4E_b)\Phi_{\Delta\theta} + \sigma_W^2}} \right) \right]$$  \hspace{1cm} (6.47)
The conditional bit error probability for the bit $b_3$ is also given by

$$P_{e_{4x1}}(b_3|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{9(2E_b/5) \sum_{i=1}^{4} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2 W}} \right) ight.$$  

$$+ Q\left( \sqrt{\frac{2E_b/5 \sum_{i=1}^{4} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2 W}} \right) 

+ Q\left( \sqrt{\frac{2E_b/5 \sum_{i=1}^{4} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2 W}} \right) 

- Q\left( \sqrt{\frac{25(2E_b/5) \sum_{i=1}^{4} |H_i|^2}{(4E_b)\Phi_{\Delta \theta} + \sigma^2 W}} \right) \right] \quad (6.48)$$

The conditional bit error probability (BEP) is expressed in the form of $\gamma_{snr_{4x1}}(\theta)$ given by

$$\gamma_{snr_{4x1}}(\theta) = \frac{2E_b}{5} \left( \sum_{i=1}^{4} |H_i|^2 \right) \quad (6.49)$$

where $\gamma_{snr_{4x1}}(\theta)$ is the SNR, and $\Phi_{\Delta \theta}$ is defined in (6.17).

The PDF of $\gamma_{snr_{4x1}}(\theta)$ is obtained from (6.23) and simplified for the case of 4-Transmit antennas, given by

$$f\left(\gamma_{snr_{4x1}}|\theta(n)\right) = \frac{\gamma^3}{2^4 \sigma^8 \Gamma^{4\gamma}(\frac{n}{2})} e^{-\frac{\gamma}{2\sigma^2}} \quad (6.50)$$

where $\sigma^2$ is the variance of $H_i$.

The average BER is then obtained by averaging over the PDFs of $\theta(n)$ and $\gamma_{snr_{4x1}}(\theta)$, and is given by

$$\bar{P}_{e_{4x1}} = \frac{1}{2} \int_{\gamma} f(\gamma|\theta) d\gamma \int_{\theta} f(\theta) d\theta \times \left[ P_{e_{4x1}}(b_1|H_0, H_1, \theta(n)) + P_{e_{4x1}}(b_3|H_0, H_1, \theta(n)) \right]$$

$$= \frac{1}{2} \int_{0}^{\infty} f(\gamma|\theta) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta \times \left[ P_{e_{4x1}}(b_1|\gamma(\theta)) + P_{e_{4x1}}(b_3|\gamma(\theta)) \right] \quad (6.51)$$
6.7.5 Detection with Imperfect Channel Estimate

In this section, we investigate and analyze the system performance of STBC-OFDM employing a 4-Transmit, 1-Receive antenna configuration with channel estimation errors. In [129], it was noted that STBC remains effective in the presence of imperfect channel estimation, with standard channel estimation techniques applied, as long as the number of transmit antennas, \( N_t \), remains small. In the presence of imperfect channel estimation, we assume a channel estimation model such that the channel estimate \( \hat{H}_i \) of the true channel \( H_i \) is given by

\[
\hat{H} = \begin{bmatrix}
H_1 + \varepsilon_1 & H_2 + \varepsilon_2 & H_3 + \varepsilon_3 & H_4 + \varepsilon_4 \\
H_2 + \varepsilon_2 & -H_1 - \varepsilon_1 & H_4 + \varepsilon_4 & -H_3 - \varepsilon_3 \\
H_3 + \varepsilon_3 & -H_4 - \varepsilon_4 & -H_1 - \varepsilon_1 & H_2 + \varepsilon_2 \\
H_4 + \varepsilon_4 & H_3 + \varepsilon_3 & -H_2 - \varepsilon_2 & -H_1 - \varepsilon_1 \\
H_1^* + \varepsilon_1^* & H_2^* + \varepsilon_2^* & H_3^* + \varepsilon_3^* & H_4^* + \varepsilon_4^* \\
H_2^* + \varepsilon_2^* & -H_1^* - \varepsilon_1^* & H_4^* + \varepsilon_4^* & -H_3^* - \varepsilon_3^* \\
H_3^* + \varepsilon_3^* & -H_4^* - \varepsilon_4^* & -H_1^* + \varepsilon_1^* & H_2^* + \varepsilon_2^* \\
H_4^* + \varepsilon_4^* & H_3^* + \varepsilon_3^* & -H_2^* - \varepsilon_2^* & -H_1^* - \varepsilon_1^*
\end{bmatrix}
\]  

(6.52)

Following similar steps as described in the previous section, the received signal on \( l^{th} \) subcarrier impaired by phase noise can be written compactly as

\[
Y_{4\times 1}(l) = D_{4\times 1}(l) + \beta_{4\times 1}(l) + W_{4\times 1}(l)
\]  

(6.53)

where \( \beta_{4\times 1}(l) \) is the ICI contribution on the \( l^{th} \) subcarrier due to the effect of phase noise.

The detected signal on the \( l^{th} \) subcarrier using the channel matrix in (6.52) is given by
\[ \tilde{X}_{4x1}^e = \hat{H}^H Y_{4x1} \]
\[ = 2 \left( \sum_{i=1}^{4} |H_i|^2 \right) I_{4x4} + \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} \Psi_{1,1} & \Psi_{1,2} & \Psi_{1,3} & \Psi_{1,4} \\ \Psi_{2,1} & \Psi_{2,2} & \Psi_{2,3} & \Psi_{2,4} \\ \Psi_{3,1} & \Psi_{3,2} & \Psi_{3,3} & \Psi_{3,4} \\ \Psi_{4,1} & \Psi_{4,2} & \Psi_{4,3} & \Psi_{4,4} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5^s \\ W_6^s \\ W_7^s \\ W_8^s \end{bmatrix} \]
\[ + \begin{bmatrix} \tilde{D}_{4x1}(l) \\ \tilde{\Psi}_{4x1}(l) \end{bmatrix} \]

(6.54)

\[ \tilde{X}_{4x1}(l) = \tilde{D}_{4x1}(l) + \tilde{\Psi}_{4x1}(l) + \tilde{\beta}_{4x1}^e(l) + \tilde{W}_{4x1}(l) \]

(6.55)

where \( \tilde{\Psi}_{4x1} \) is defined in the Appendix of this Chapter.

In (6.55), the term \( \tilde{D}_{4x1}(l) \) is the desired signal component. The second term is a noise term due to imperfect channel estimation and, conditioned on the phase noise and the channel gains, is a Gaussian random variable since the channel estimation errors are modeled as Gaussian. The third term is the ICI which is approximated as a Gaussian random variable and \( \tilde{W}_{4x1}^e(l) \) is the additive Gaussian noise.
6.7.6 Mean and Variance of ICI and Noise

The covariance matrix of $\tilde{\Psi}_{4\chi_1}(l)$ in (6.55) is given by

$$\Omega_{\tilde{\Psi}_{4\chi_1}} = E[\tilde{\Psi}_{4\chi_1}\tilde{\Psi}_{4\chi_1}^H]$$

$$= 4E_s \left( \sum_{i=1}^{4} |H_i|^2 \right) \left( \sum_{i=1}^{4} \sigma_{\varepsilon_i}^2 \right) I_{4x4}$$

$$= 4E_s \sigma_{\tilde{\Psi}_{4\chi_1}}^2 I_{4x4}$$

$$= \sigma_{\tilde{\Psi}_{4\chi_1}}^2 I_{4x4}$$  \hspace{1cm} (6.56)$$

where $\sigma_{\tilde{\Psi}_{4\chi_1}}^2 = \left( \sum_{i=1}^{4} |H_i|^2 \right) \left( \sum_{i=1}^{4} \sigma_{\varepsilon_i}^2 \right)$ and the term $\sigma_{\tilde{\Psi}_{4\chi_1}}^2 = 4E_s \sigma_{\Psi_{4\chi_1}}^2$.

Similarly, the covariance matrix of the ICI term $\tilde{\beta}_{4\chi_1}(l)$ in (6.55) is given by

$$\Omega_{\tilde{\beta}_{4\chi_1}} = E[\tilde{\beta}_{4\chi_1}\tilde{\beta}_{4\chi_1}^H]$$

$$= 4E_s \Phi_{\Delta \theta} \sum_{i=1}^{4} (|H_i|^2 + \sigma_{\varepsilon_i}^2) I_{4x4}$$

$$= 4E_s \sigma_{\tilde{\beta}_{4\chi_1}}^2 I_{4x4}$$

$$= \sigma_{\tilde{\beta}_{4\chi_1}}^2 I_{4x4}$$  \hspace{1cm} (6.57)$$

where $\sigma_{\tilde{\beta}_{4\chi_1}}^2 = 4E_s \sigma_{\beta_{4\chi_1}}^2$, and $\sigma_{\tilde{\beta}_{4\chi_1}}^2 = \Phi_{\Delta \theta} \sum_{i=1}^{4} (|H_i|^2 + \sigma_{\varepsilon_i}^2)$.

The covariance matrix of the additive Gaussian noise term $\tilde{W}_{4\chi_1}(l)$ in (6.55) is given by

$$\Omega_{\tilde{W}_{4\chi_1}} = E[\tilde{W}_{4\chi_1}\tilde{W}_{4\chi_1}^H]$$

$$= \tilde{W}_{4\chi_1}^H R_{W_{4\chi_1}} \tilde{W}_{4\chi_1}$$

$$= 4 \sigma_W^2 \sum_{i=1}^{4} (|H_i|^2 + \sigma_{\varepsilon_i}^2) I_{4x4}$$

$$= \sigma_{\tilde{W}_{4\chi_1}}^2 I_{4x4}$$  \hspace{1cm} (6.58)$$

where $\sigma_{\tilde{W}_{4\chi_1}}^2 = 4\sigma_W^2 \sum_{i=1}^{4} (|H_i|^2 + \sigma_{\varepsilon_i}^2)$.
6.7.7 Probability of Error

The conditional bit error probability for the bit $b_1$ can be written as

$$P_{e^{x_1}}^e(b_1|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left(\frac{2E_b/5\left(2 \sum_{i=1}^{4} |H_i|^2\right)^2}{(4E_b)4\sigma_{\psi}^2 + (4E_b)4\sigma_{\beta}^2 + \sigma_W^2}\right) + Q\left(\frac{9(2E_b/5\left(2 \sum_{i=1}^{4} |H_i|^2\right)^2}{(4E_b)4\sigma_{\psi}^2 + (4E_b)4\sigma_{\beta}^2 + \sigma_W^2}\right) \right]$$

and the probability of bit error for the bit $b_3$ is given by

$$P_{e^{x_1}}^e(b_3|H_0(l), H_1(l), \theta(n)) = \frac{1}{2} \left[ Q\left(\frac{9(2E_b/5\left(2 \sum_{i=1}^{4} |H_i|^2\right)^2}{(4E_b)4\sigma_{\psi}^2 + (4E_b)4\sigma_{\beta}^2 + \sigma_W^2}\right) + Q\left(\frac{25(2E_b/5\left(2 \sum_{i=1}^{4} |H_i|^2\right)^2}{(4E_b)4\sigma_{\psi}^2 + (4E_b)4\sigma_{\beta}^2 + \sigma_W^2}\right) - Q\left(\frac{25(2E_b/5\left(2 \sum_{i=1}^{4} |H_i|^2\right)^2}{(4E_b)4\sigma_{\psi}^2 + (4E_b)4\sigma_{\beta}^2 + \sigma_W^2}\right) \right]$$

The average BER is then given by

$$\overline{P}_{e^{x_1}} = \frac{1}{2} \int_\gamma f(\gamma|\theta) d\gamma \int_\theta f(\theta) d\theta$$

$$\times \left[ P_{e^{x_1}}^e(b_1|H_0, H_1, \theta(n)) + P_{e^{x_1}}^e(b_3|H_0, H_1, \theta(n)) \right]$$

$$= \frac{1}{2} \int_0^\infty f(\gamma|\theta) d\gamma \int_{-\pi}^{\pi} f(\theta) d\theta$$

$$\times \left[ P_{e^{x_1}}^e(b_1|\gamma(\theta)) + P_{e^{x_1}}^e(b_3|\gamma(\theta)) \right]$$
6.8 Results

The results of the analysis are presented next, and compared with simulations for validation.

6.8.1 Simulation Parameters

The parameters used for the simulation and analytical results are summarized in Table 6.1, making use of parameters and specifications that are IEEE802.16 compliant. The simulation platform is implemented in MATLAB. As in the previous Chapters, an uncoded system is implemented. As such, the blocks that involve convolutional encoding/decoding in the IEEE 802.16 standard are omitted. Using 256 subcarriers and 16-QAM as the modulation of choice, the transmitted signal is processed by a 256-point FFT. For each transmit signal path per antenna, a normalized channel power is assumed. Hence a zero-mean and unity variance for the fading channel. The number of transmit antennas for the STBC-OFDM system considered include $N_T = 2, 4$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_g^2$</td>
<td>0.0, 0.01, 0.03</td>
</tr>
<tr>
<td>$\sigma_x^2$</td>
<td>0.0, 0.02, 0.04</td>
</tr>
<tr>
<td>Modulation</td>
<td>16-QAM</td>
</tr>
<tr>
<td>Subcarriers (N)</td>
<td>256</td>
</tr>
<tr>
<td>Transmit Antennas ($N_T$)</td>
<td>2, 4</td>
</tr>
<tr>
<td>Receive Antennas</td>
<td>1</td>
</tr>
</tbody>
</table>

6.8.2 Discussion of Results

The results obtained from the analysis of M-QAM STBC-OFDM in the presence of phase noise is presented for both perfect and imperfect channel estimation.

Figure-6.2 illustrates the performance on 16QAM STBC-OFDM system in the presence of phase noise and perfect channel estimation, employing two transmit antennas and one receive antenna, making use of the Alamouti code.
Figure-6.3 illustrates the performance on 16QAM STBC-OFDM system impaired by phase noise with $\sigma_\theta^2 = 0.03$ and with imperfect channel estimation. The system employs two transmit antennas and one receive antenna, and makes use of the Alamouti code.

Figure-6.4 also illustrates the performance of 16QAM 2Tx, 1Rx STBC-OFDM system employing the Alamouti code, with the joint effect of phase noise ($\sigma_\theta^2 = 0.06$) and imperfect channel estimation.

From Figure-6.3 and Figure-6.4, it is observed that with improved channel estimates, the phase noise is the determining factor of system performance. However, as the channel estimates degrades, the channel estimation becomes the dominant factor. This observation signals that good channel estimation is also very crucial in obtaining good performance.

Figure-6.5 illustrates the performance on 16QAM STBC-OFDM system in the presence of phase noise with $\sigma_\theta^2 = 0.03$ and perfect channel estimation, employing four transmit antennas and one receive antenna.

Figure-6.6 also illustrates the performance on 16QAM STBC-OFDM system in the presence of phase noise with $\sigma_\theta^2 = 0.06$ and perfect channel estimation, employing four transmit antennas and one receive antenna.

A similar trend is observed between Figure-6.5 and Figure-6.6 as in the 2Tx, 1Rx case of Figure-6.3 and Figure-6.4. The two results indicate that robust channel estimation schemes are essential for good system performance.

Finally, the analytical results are compared with simulations in Figure-6.7, by employing parameters from Table-6.1. From the plots, it is observed that the analysis agrees with simulations.
6.9 Conclusion

The performance of MISO-OFDM systems employing STBC, with phase noise and channel estimation errors is analyzed. The analytical expression for the bit error probability of the STBC system is derived. Numerical evaluation shows that the results accurately describes the performance of the system, when compared to the simulation results of a practical system.

Although there is considerable performance gains, ranging from higher data rates to increase in diversity order of the system (with trade-offs between transmission rate and diversity), the transmit diversity scheme is shown to be severely impaired by the phase noise and channel estimation errors regardless. The results show that the system impairments cause significant performance loss as a consequence of these negative effects in the form of the CPE and ICI contributions.

The text in Chapter 6 is a revision of the material presented in:
Figure 6.3 Performance of 16QAM STBC-OFDM in Rayleigh Fading channels with Phase Noise ($\sigma_\theta^2 = 0.03$) and Imperfect Channel estimation, employing 2 Transmit Antennas and 1 Receive Antenna. The Number of subcarriers ($N$) = 256.
Figure 6.4 Performance of 16QAM STBC-OFDM in Rayleigh Fading channels with Phase Noise ($\sigma^2_\theta = 0.06$) and Imperfect Channel estimation, employing 2 Transmit Antennas and 1 Receive Antenna. The Number of subcarriers ($N$) = 256.
Figure 6.5 Performance of 16QAM STBC-OFDM in Rayleigh Fading channels with Phase Noise ($\sigma_\theta^2 = 0.03$) and Imperfect Channel estimation, employing 4 Transmit Antennas and 1 Receive Antenna. The Number of subcarriers ($N$) = 256.

Figure 6.6 Performance of 16QAM STBC-OFDM in Rayleigh Fading channels with Phase Noise ($\sigma_\theta^2 = 0.06$) and Imperfect Channel estimation, employing 4 Transmit Antennas and 1 Receive Antenna. The Number of subcarriers ($N$) = 256.
16-QAM STBC–OFDM for 2, 3, 4 Tx Antennas: Analysis & Simulations

Figure 6.7 Analysis and Simulation Results: Performance of 16QAM STBC-OFDM in Rayleigh Fading channels with Perfect Channel estimation. Number of Transmit Antennas ($N_T$) = 1, 2, 3, 4. The Number of subcarriers ($N$) = 256.

The dissertation author was the primary researcher and author, and the co-author listed in these publication directed and supervised the research which forms the basis for this chapter.
Conclusions and Contributions

In this dissertation, a detailed discussion of the performance degradation of OFDM systems subject to system non-idealities associated with RF impairments such as phase noise is discussed. In addition, the effects of channel estimation errors on the system performance, due to imperfect channel estimation in Rayleigh fading channels, is also investigated. We also presented a detailed discussion of the IEEE 802.16 standard, and for the purpose of investigating the effects of the system impairments on a practical system, the parameters of the IEEE 802.16 standard are applied in simulations and compared with results obtained from analysis. This comparison provides valuable insights to system performance limitations due to these non-idealities.

In Chapter 4, the performance analysis of SISO-OFDM systems with the joint effects of phase noise and channel estimation errors in Rayleigh fading channels is presented. The phase noise destroys the orthogonality between overlapping but orthogonal subcarriers, leading to the introduction of CPE and ICI. The detailed analysis of the phase noise impairment is first presented in the presence of perfect channel estimation, as a basis for the case for imperfect channel estimation. Two cases are considered in the above analysis: the first considers the case of perfect CPE correction. Since the problem of phase error is understood in communication systems and with adequate CPE correction schemes, we first proceed to investigate the effect of only ICI on system performance due to phase noise. The analysis is then extended to include the combined effect of CPE and ICI.

Numerical evaluation shows that the analytical results can be applied to de-
scribe the performance of a practical system. As such, system parameters of the IEEE 802.16 Standards are applied for simulations and comparison with the results obtained by analysis. In addition to the effect of channel estimation errors, it is shown that the effects of phase noise impairments limits system performance significantly.

Next, the performance analysis of SISO-OFDM is extended to OFDM systems employing multi-antennas at the receiver or transmitter. Although the implementation of multi-antenna architecture at either the transmitter or receiver leads to improved system performance, there are nevertheless physical implementation constraints such as antenna spacing and decoding complexity at the receiver as well. Therefore, we consider it very important to investigate each system configuration separately. Such insights will provide a valuable reference in determining which architecture is appropriate based on a combination of performance limitations and implementation complexity.

As such, the performance of SIMO-OFDM, employing Maximal Ratio Combining at the receiver, in the presence of phase noise and channel estimation errors is analyzed in Chapter 5. The analytical expression for the bit error probability of the MRC-OFDM system, in the presence of phase noise and imperfect channel estimation errors, is derived. Similar to Chapter 4, the detailed analysis of the phase noise impairment is first presented in the presence of perfect channel estimation. As in the SISO case, parameters of the IEEE 802.16 Standards are applied in simulations and compared with analysis.

In Chapter 6, an OFDM system configuration with multiple antenna at the transmitter and a single receive antenna is considered. Employing STBC in a MISO-OFDM, the performance of the system impaired by phase noise and channel estimation errors is analyzed. The analytical expression for the BER of the STBC-OFDM system is also derived. Numerical evaluations of the analytical results reveal that with all the performance gains that multi-antenna systems offer, system performance is still considerably limited by these non-idealities. As in the previous Chapters, parameters of the IEEE 802.16 Standards are applied for simulations and comparison with the results obtained by analysis.

Regardless of the transceiver architecture employed in the above investigations, it is observed that the combined effects of phase noise impairments and imperfect
channel estimation significantly limits the true system performance. Although there is considerable performance gains, ranging from higher data rates to increase in diversity order of the system, both transmit and receive diversity schemes pay a severe penalty for inadequately accounting for the effects of phase noise and the lack of improved channel estimates.

From the conclusions above, the main contributions in this dissertation can be summarized as follows:

- BER performance analysis of SISO-OFDM systems
  - Effects of phase noise on SISO-OFDM system performance
  - Effects of imperfect channel estimation on SISO-OFDM system performance
  - Combined effects of phase noise and imperfect channel estimation
  - Comparison of simulations and analytical results

- BER analysis of SIMO-OFDM systems employing MRC scheme at the receiver
  - Effects of phase noise on SIMO-OFDM system performance
  - Effects of imperfect channel estimation on SIMO-OFDM system performance
  - Combined effects of phase noise and imperfect channel estimation
  - Comparison of simulations and analytical results

- BER analysis of MISO-OFDM systems employing STBC
  - Effects of phase noise on MISO-OFDM system performance
  - Effects of imperfect channel estimation on MISO-OFDM system performance
  - Combined effects of phase noise and imperfect channel estimation
  - Comparison of simulations and analytical results
8

Appendix

8.1 Gaussianity of ICI

Following the proof in [136], and as cited in [24], we show that the intercarrier interference (ICI) term is conditionally asymptotically Gaussian. Re-stated here for convenience, the Lyapounov Theorem in [136] states that if $\Lambda_1, \ldots, \Lambda_{n-1}$ are independent random variables each with mean $\mu_k$, variance $\sigma_k^2$ and finite absolute third moment of $\varepsilon_k^3$, and if

$$\lim_{N \to \infty} \frac{\varepsilon}{\sigma} = 0 \quad (8.1)$$

where

$$\varepsilon = \left( \sum_{k=0}^{N-1} \varepsilon_k^3 \right)^{1/3} \quad (8.2)$$

$$\sigma = \left( \sum_{k=0}^{N-1} \sigma_k^2 \right)^{1/2} \quad (8.3)$$

then

$$\gamma = \sum_{k=0}^{N-1} \Lambda_k \quad (8.4)$$

is asymptotically Gaussian.
**Proof of Gaussianity of ICI**

To prove that the ICI term can be approximated by a Gaussian random variable, we proceed as follows. The ICI on the $l^{th}$ subcarrier due to phase noise is given by

$$
\beta(l) = \left( \frac{1}{N} \right)^{N-1} \sum_{n=0}^{N-1} \sum_{k=0, k \neq l}^{N-1} H(k) X(k) e^{j2\pi(k-l)n/N} e^{j\theta(n)}
$$

(8.5)

Re-writing the ICI term, we obtain

$$
\beta(l) = \sum_{n=0}^{N-1} \gamma_n
$$

(8.6)

where

$$
\gamma_n = \left( \frac{1}{N} \right)^{N-1} \sum_{k=0, k \neq l}^{N-1} H(k) X(k) e^{j2\pi(k-l)n/N} e^{j\theta(n)}
$$

$$
= \sum_{k=0, k \neq l}^{N-1} \Lambda_k
$$

(8.7)

where $\Lambda_k = \left( \frac{1}{N} \right) H(k) X(k) e^{j2\pi(k-l)n/N} e^{j\theta(n)}$. Conditioned on $\theta(n)$, it can be seen that $\Lambda_k$’s are conditionally independent since the transmitted data and the channel gains are independent. Making use of Lyapounov’s Theorem in [136], we derive expressions for the mean $\mu$, variance $\sigma^2$ and third absolute central moment $\varepsilon^3$ of $\Lambda_k$ and show that $\gamma_n = \sum_{k=0, k \neq l}^{N-1} \Lambda_k$ is asymptotically Gaussian. Then $\beta(l) = \sum_{n=0}^{N-1} \gamma_n$ is a sum of jointly Gaussian random variables, and can be approximated as a Gaussian random variable.

**Gaussianity of $\gamma$**

Conditioned on $\theta(n)$, the $\Lambda_k$’s are conditionally independent random variables. Since the transmitted symbols $X(k)$ are independent of the channel gains $H(k)$, and
the channel is a zero-mean complex Gaussian random variable, we obtain

\[ \mu_k = E[\Lambda_k] \]
\[ = \left( \frac{1}{N} \right) E[H(k)X(k)e^{j2\pi(k-l)n/N}e^{j\theta(n)}] \]
\[ = \left( \frac{1}{N} \right) E[H(k)]E[X(k)][e^{j2\pi(k-l)n/N}e^{j\theta(n)}] \]
\[ = 0 \] \hspace{1cm} (8.8)

\[ \sigma_k^2 = E[\Lambda_k^2] \]
\[ = \left( \frac{1}{N^2} \right) E[|H(k)|^2]E[|X(k)|^2]|e^{j2\pi(k-l)n/N}e^{j\theta(n)}|^2 \]
\[ = \left( \frac{1}{N^2} \right) R_H E_s \] \hspace{1cm} (8.9)

where \( R_H \) is the defined as the channel power \( E[|H(k)|^2] \), \( E[|X(k)|^2] \) is the transmitted signal energy \( E_s \), and the term \(|e^{j2\pi(k-l)n/N}e^{j\theta(n)}|^2 = 1 \). The third central moment \( \varepsilon^3 \) is given by

\[ \varepsilon_k^3 = E[|\Lambda_k|^3] \]
\[ = \left( \frac{1}{N^3} \right) E[|H(k)|^3]E[|X(k)|^3] \cdot |e^{j2\pi(k-l)n/N}e^{j\theta(n)}|^3 \]
\[ = \left( \frac{1}{N^3} \right) (R_H)^{3/2}\Gamma(5/2)\varepsilon_X^3 \cdot |e^{j2\pi(k-l)n/N}e^{j\theta(n)}|^3 \]
\[ = \left( \frac{1}{N^3} \right) (R_H)^{3/2}\Gamma(5/2)\varepsilon_X^3 \] \hspace{1cm} (8.10)

where \( \varepsilon_X^3 \) is the third absolute moment of \( X(k) \), \( (R_H)^{3/2}\Gamma(5/2) \) is the third absolute moment of \( H(k) \), \( \Gamma \) is the Gamma function as defined in [106] given by \( \Gamma(z) = (z-1)! \). The term \(|e^{j2\pi(l-k)n/N}e^{j\theta(n)}|^3 = 1 \). From the above expressions, and substituting in the limiting fraction of \( \varepsilon/\sigma \), we have

\[ \frac{\varepsilon}{\sigma} = \left( \frac{(R_H)^{3/2}\Gamma(5/2)\varepsilon_X^3}{(N-1)^{1/6}R_H E_s^{1/2}} \right)^{1/3} \] \hspace{1cm} (8.11)

It is observed that as \( N \) becomes large, the above fraction vanishes. Hence \( \gamma \) can be approximated as a Gaussian random variable.
Next we show that these $\gamma'_n$s in the ICI term $\sum_{n=0}^{N-1} \gamma_n$ are jointly Gaussian.

$$\gamma_n = \left( \frac{1}{N} \right) \sum_{\substack{k=0 \\k \neq l}}^{N-1} H(k) X(k) e^{j2\pi(k-l)n/N} e^{j\theta(n)}$$ \hspace{1cm} (8.12)

Rewrite the $\gamma_n$’s as

$$\gamma_1 = \sum_k C_{k,1} H_k X_k$$

$$\gamma_2 = \sum_k C_{k,2} H_k X_k$$

$$\vdots$$

$$\gamma_n = \sum_k C_{k,n} H_k X_k$$

such that

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{bmatrix} = \begin{bmatrix} C_{1,1} X_1 & C_{1,2} X_2 & \cdots & C_{1,k} X_k \\ C_{2,1} X_1 & C_{2,2} X_2 & \cdots & C_{2,k} X_k \\ \vdots & \vdots & \ddots & \vdots \\ C_{n,1} X_1 & C_{n,2} X_2 & \cdots & C_{n,k} X_k \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_k \end{bmatrix}$$ \hspace{1cm} (8.13)

$$\gamma = \mathbf{A} \mathbf{H}$$ \hspace{1cm} (8.14)

It is observed that $[\gamma_1 \gamma_2 \cdots \gamma_n]^T$ in (8.13) is a linear transformation of $H_k$’s as noted in [106]. Furthermore, the $H'_k$s are independent Gaussian random variables, and are jointly Gaussian. It is known that a linear transformation of a set of jointly Gaussian random variables results in another set of jointly Gaussian random variables [106]. Since $\gamma$ is conditionally asymptotically Gaussian and the $\gamma'_n$s are jointly Gaussian, then $\beta(l)$ is a sum of jointly Gaussian random variables. Hence, $\beta(l)$ is asymptotically conditionally Gaussian.
8.2 Conditional Variance of AWGN

Proof: Variance of \( \tilde{W} \) independent of \( \theta(n) \) We show that the conditional variance of the additive Gaussian noise term in (6.14), (6.29), (6.43), (6.55) is independent of the phase noise. We define the random variable \( \tilde{W}(t) = W(t) e^{j\theta(t)} \), where \( W(t) \) and \( \theta(t) \) are independent and \( W(t) \) is zero-mean complex Gaussian random variable with variance \( \sigma^2_W \). Conditioned on \( \theta(t) \), the PDF of \( \tilde{W}(t) \) is

\[
f_{\tilde{W}(t)\mid \theta(t)}(\nu\mid \theta(t)) = \frac{1}{2\pi\sigma^2_W} e^{-|\nu|^2/2\sigma^2_W}
\]

\[
f_{\tilde{W}(t)}(\nu) = E\left[ f_{\tilde{W}(t)\mid \theta(t)}(\nu\mid \theta(t)) \right]
= \int_{\theta} f_{\theta(t)}(\theta) d\theta \cdot f_{\tilde{W}(t)\mid \theta(t)}(\nu\mid \theta(t))
= \frac{1}{2\pi\sigma^2_W} e^{-|\nu|^2/2\sigma^2_W}
= f_W(t)(\nu)
\]

(8.15)

Hence the PDF of \( \tilde{W}(t) \) is Gaussian. Therefore

\[
\text{Var}\left[ W e^{j\theta} \mid \theta \right] = E\left[ |W e^{j\theta}|^2 \mid \theta \right]
= \left| e^{j\theta} \right|^2 E\left[ |W|^2 \mid \theta \right]
= \sigma^2_W
\]

(8.16)

Hence the variance of the Gaussian noise \( \sigma^2_W \) and independent of \( \theta(n) \).

8.3 The \( \tilde{\Psi}_{4\chi_1}(l) \) Term

The elements of \( \tilde{\Psi}_{4\chi_1}(l) \) in (6.55) are given by
\[ \Psi_{1,1} = H_1 e_1^* + H_2 e_2^* + H_3 e_3^* + H_4 e_4^* + H_1' e_1 + H_2' e_2 + H_3' e_3 + H_4' e_4 \]
\[ \Psi_{1,2} = H_2 e_1^* - H_1 e_2^* - H_4 e_3^* + H_3 e_4^* + H_1' e_1 + H_2' e_2 - H_3' e_3 + H_4' e_4 \]
\[ \Psi_{1,3} = H_3 e_1^* + H_4 e_2^* - H_1 e_3^* - H_2 e_4^* + H_1' e_1 + H_2' e_2 + H_3' e_3 - H_4' e_4 \]
\[ \Psi_{1,4} = H_2 e_3^* - H_1 e_4^* + H_4 e_1^* - H_2 e_3^* - H_1' e_1 - H_2' e_2 \]
\[ \Psi_{2,1} = H_1 e_2^* - H_2 e_3^* - H_3 e_4^* + H_4 e_1^* + H_1' e_2 - H_2' e_3 + H_3' e_4 + H_4' e_1 \]
\[ \Psi_{2,2} = H_1 e_1^* + H_2 e_2^* + H_3 e_3^* + H_4 e_4^* + H_1' e_1 + H_2' e_2 + H_3' e_3 + H_4' e_4 \]
\[ \Psi_{2,3} = H_1 e_4^* - H_2 e_3^* + H_3 e_2^* - H_4 e_1^* + H_1' e_4 - H_2' e_3 + H_3' e_2 - H_4' e_1 \]
\[ \Psi_{2,4} = H_3 e_1^* + H_4 e_2^* - H_1 e_3^* - H_2 e_4^* + H_3 e_1 + H_4 e_2 - H_1' e_3 - H_2' e_4 \]
\[ \Psi_{3,1} = H_1 e_3^* + H_2 e_4^* - H_3 e_1^* - H_4 e_2^* + H_1' e_3 + H_2' e_4 - H_3' e_1 - H_4' e_2 \]
\[ \Psi_{3,2} = H_2 e_3^* - H_1 e_4^* + H_4 e_1^* - H_3 e_2^* + H_2' e_3 - H_1' e_4 + H_2' e_3 - H_1' e_2 \]
\[ \Psi_{3,3} = H_1 e_4^* + H_2 e_3^* - H_3 e_2^* + H_4 e_1^* + H_1' e_4 + H_2' e_3 + H_3' e_2 + H_1' e_4 \]
\[ \Psi_{3,4} = H_1 e_2^* - H_2 e_1^* - H_3 e_4^* + H_4 e_3^* + H_1' e_2 - H_2' e_1 + H_3' e_4 + H_4' e_3 \]
\[ \Psi_{4,1} = H_1 e_4^* - H_2 e_3^* - H_3 e_2^* + H_4 e_1^* + H_1' e_4 - H_2' e_3 + H_3' e_2 + H_1' e_1 \]
\[ \Psi_{4,2} = H_1 e_3^* + H_2 e_4^* - H_3 e_1^* - H_4 e_2^* + H_1' e_3 + H_2' e_4 - H_3' e_1 - H_4' e_2 \]
\[ \Psi_{4,3} = H_2 e_1^* - H_1 e_2^* - H_4 e_3^* + H_3 e_4^* + H_2' e_3 - H_1' e_2 - H_3' e_4 + H_4' e_3 \]
\[ \Psi_{4,4} = H_1 e_1^* + H_2 e_2^* + H_3 e_3^* + H_4 e_4^* + H_1' e_1 + H_2' e_2 + H_3' e_3 + H_4' e_4 \] 
(8.17)
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