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Essays on Credit Risk and Bank Lending Standards in Loan Markets

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Author
Aproberts-Warren, Margaret

Publication Date
2017

Peer reviewed|Thesis/dissertation
ESSAYS ON CREDIT RISK AND BANK LENDING STANDARDS
IN LOAN MARKETS

A dissertation submitted in partial satisfaction
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

Margaret apRoberts-Warren

September 2017

The Dissertation of Margaret apRoberts-Warren is approved:

_________________________________________
Professor Carl Walsh, Chair

_________________________________________
Professor Johanna Francis

_________________________________________
Professor Kenneth Kletzer

_________________________________________
Tyrus Miller
Vice Provost and Dean of Graduate Studies
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Abstract

Essays on Credit Risk and Bank Lending Standards in Loan Markets

by

Margaret apRoberts-Warren

In this dissertation I explore how credit risk and bank lending standards affect financial markets and the greater economy. Credit risk refers to the probability that a borrower will be unwilling or unable to repay debt liabilities. Lending standards take on a broader interpretation: they may refer to the types of borrowers who access debt markets and other non-price loan terms such as loan covenants and other screening or monitoring activities. In addition, credit risk and lending standards are intrinsically linked: to the extent they affect asset quality, lending standards may have significant effects on the risk of default for both borrowers and banks alike. This point is well illustrated by the rise of, and subsequent crisis in, the subprime mortgage market during the mid–to late–2000s.

In chapter one, I construct a theoretical model to investigate the impact of credit default risk on the terms and provision of unsecured interbank credit. Bank-specific interest rates are derived via a Nash bargaining solution. However, interbank lending transactions are prone to counterparty default risk where the risk of default depends on both aggregate and bank-specific factors. Given the result of the bargaining problem, a cut-off borrower default risk is derived which determines participation in interbank lending
markets. The average interbank interest rate is also derived.

After a deterioration of the aggregate factor affecting default risk, participation in interbank lending markets declines and both individual and the average interbank loan rates rise. Monetary policies are also examined: individual and the average interbank loan rates rise when the cost of borrowing from the central bank’s lending facility or the return on holding reserves rise. However, an increase in the cost of funds borrowed from central bank increases participation in interbank markets while an increase in the return on reserves reduces participation. The results are discussed in the context of the distress and policy actions taken in the federal funds market during the financial crisis of 2008.

In chapter two, I explore the relationship between monetary policy, commercial bank lending standards, and the probability of bank default. In a partial equilibrium model of the commercial loan market, borrowers with heterogeneous abilities receive loans from financial intermediaries if they satisfy bank lending standards. These lending standards take the form of a minimum ability requirement: only borrowers with abilities that are at least as large as the minimum threshold receive a loan; the rest are rationed. After loans are made, a negative aggregate shock results in unexpected loan losses for the bank which may push them into default.

In the baseline model, the probability of bank default and the policy rate are negatively correlated: after a decrease in the policy rate, lending standards loosen and the probability of bank default rises. However, both the sign and magnitude of this correlation depends on the extent of bank capitalization: the risk of default is positively correlated
when banks have sufficiently low capital ratios, while strongly capitalized banks’ risk of
default falls with the policy rate. Additionally, this correlation grows stronger as bank
capital ratios increase relative to the baseline level. These results imply that the na-
ture and size of the trade-off between financial stability and traditional monetary policy
objectives depends crucially on bank capital ratios. This also suggests that optimal
monetary policy that incorporates financial stability objectives is linked to bank capital
structure and, by extension, bank capital regulation.

In chapter three, I investigate the impact of costly bank monitoring on the spread
between the loan rate and the risk-free rate and the response of output after a negative
bank liquidity shock. A simple RBC model of an economy with a commercial loan
market is constructed where financial transactions are characterized by moral hazard
on the part of borrowers that ultimately restricts borrower leverage. However, lenders
can alleviate the severity of the moral hazard problem by monitoring borrowers’ projects
after a loan has been made. The intensity of monitoring is optimally chosen by lenders:
while more intense monitoring lessens the borrower’s moral hazard problem, it is costly.

Compared to an economy without costly endogenous monitoring, costly monitoring
results in a less severe drop in capital and output after a surprise increase in the cost of
bank deposits. This is driven by the loan rate spread: this spread falls and leads to a
larger rise in the excess return to capital over the cost of bank loans which contributes
to a faster recovery in borrower net worth, capital, and output.
Acknowledgments

I would like to thank my committee members for their continuous support and feedback.

I would also like to thank my husband Zac. ‘Nuff said.
Chapter 1

Counterparty Risk and Unsecured Interbank Credit Markets

While each component of financial markets plays a distinct role in the provision of credit and liquidity, interbank lending remains an integral part of modern financial systems both in regards to their role in facilitating the flow of credit throughout the economy and in implementing monetary policy. However, like other segments of financial systems, interbank credit markets are not immune to turbulence. The turmoil is often minor, but, on occasion, it can be quite severe.

The financial crisis of 2007-2009 was a period of unprecedented turmoil in interbank credit markets and witnessed significant spikes in the cost and price volatility of in-
terbank credit. Figure 1 plots the TED spread, which gives the difference between the average unsecured interbank loan rate and treasury bonds and is a key indicator of stress in financial markets. During the crisis the cost of unsecured interbank loans rose substantially: in the six and half years prior to the crisis the TED spread averaged 27.72 basis points; during the crisis it average 165.61 and, at one point, reached 576 basis points.\footnote{We define the crisis period to be from August, 2007 through July, 2009. The pre-crisis period is from the beginning of 2001 through July, 2007.} The volatility of the TED spread more than quadrupled between the two periods. In the federal funds market, the volatility of the spread between the effective fed funds rate to the target rate nearly doubled, and spreads spiked anywhere from 64 to nearly 100 basis points on key dates. Similar patterns emerged in collateralized repo markets, where Gorton and Metrick (2012) document the substantial rise in both spreads and repo haircuts.

Empirical evidence suggests that counterparty risk was at the heart of increasing spreads. In unsecured interbank lending markets, lending banks face counterparty risk due to the fact that interbank loans may not be repaid if the borrowing bank defaults. During the crisis, as markets and bank balance sheets deteriorated and uncertainty and the risk of default rose, interest rates on interbank credit rose in order to compensate lenders for the higher degree of default risk. Afonso et al. (2011) find evidence that counterparty risk drove spreads in the federal funds market after the demise of Lehman Brothers. Using Fedwire data and additional information on individual bank size and financial health, they find that worse performing large banks saw sharp increases in rate spreads relative to smaller banks and banks with better finances. They also find that low qual-
ity banks reduced the amount borrowed in the federal funds market and were more likely to access the Federal Reserve’s discount window.\textsuperscript{2} The deterioration in interbank markets also prompted a response from policy makers. The Federal Reserve took the unusual action of lowering the discount rate, the cost of borrowing at its emergency lending facility, in August 2007\textsuperscript{3} and implemented a new policy in October 2008 of paying interest on bank reserves.\textsuperscript{4}

Given the role that counterparty risk played in interbank lending markets, and the response from the central bank, my motivation for this paper is to construct a model where the effects of counterparty risk and monetary policy actions on interbank lending markets can be analyzed. To this end, I construct a model of an unsecured interbank lending market where banks are capable of default on interbank liabilities. The probability that a bank defaults depend on two factors: an aggregate quality component that affects the risk of default at all banks and a second idiosyncratic quality component that is bank-specific. This set-up allows me to not only analyze the cross-section of loan rates but also the effect of aggregate shocks on individual loan rates. Banks with liquidity surpluses match with banks with liquidity shortages and bilaterally bargain over the interbank loan rate. The resulting match-specific interbank loan rate is modeled as the solution to a Nash bargaining problem and is affected by counterparty default risk.

\textsuperscript{2}Taylor and Williams \textit{Williams and Taylor (2009)} find evidence that counterparty risk played a key role in determining spreads in other unsecured interbank loan markets. In regressions, they find that higher credit risk is correlated with higher spreads and are significant in most specifications. Gorton and Metrick \textit{Gorton and Metrick (2012)} examine repo loan markets during the crisis and also find evidence of counterparty risk concerns. In regressions they find that the LIBOR-OIS spread, their variable to represent counterparty risk was positively and significantly correlated with repo loan spreads.


Although all banks may engage in the interbank loan market, not all banks will participate. Due to the heterogeneity of banks introduced by the idiosyncratic quality component, for some matches the borrowing bank will find it more attractive to borrow from the central bank’s lending facility and the lending banks will find it more attractive to continue to hold excess liquidity as reserves. Given the result of the interest rate bargaining problem, an endogenous threshold level of the idiosyncratic bank quality measure emerges. Only banks with quality of at least the threshold will participate in interbank markets; those with quality below the threshold do not participate.

Lastly, given the match-specific interest rates and participation threshold, I derive the average interbank loan rate. In the federal funds market, this is analogous to the effective federal funds rate and is one of the most closely watched measures of financial market conditions. Unlike other models of the interbank credit market, in this model the average interbank loan rate is affected by the match-specific loan rates and which banks participate in the market.

I then use the model to analyze events and policies that transpired during the financial crisis. Specifically, I analyze the effects of changes in the aggregate bank quality component, the cost of borrowing at the central bank’s lending facility, and the return on bank reserves. The first shock – a fall in the aggregate quality component – represents the financial distress and perceived increase in counterparty default risk that occurred during the crisis. The second and third factors represent actions taken by the Federal Reserve in response to interbank market stress: lowering the discount rate and beginning the policy of paying interest on reserves.
After a deterioration of the aggregate quality component, participation in interbank lending markets declines and individual loan rates rise. The effect on the average interbank loan rate is more nuanced and can be decomposed into two counteracting effects. First, the incumbent interest rate effect: when the aggregate quality component falls, default risk for borrowing banks already participating in the interbank market rises. As a result, risk-premia for all borrowers rise putting upward pressure on the average rate. However, there is second counteracting effect – the borrower threshold effect. A deterioration in aggregate conditions causes the lowest quality borrowing banks to drop out of the interbank market. Since these borrowers paid the highest risk premia, their exit from the interbank market causes the average premium to fall and puts downward pressure on the average interbank loan rate. However, under the assumptions of the model, the incumbent interest rate effect outweighs the borrower threshold effect, and the average loan rate rises when aggregate conditions deteriorate.

In regards to monetary policy, both individual and the average interbank loan rates rise when the cost of borrowing from the central bank’s lending facility or the return on holding reserves rise. However, an increase in the cost of funds borrowed from the central bank increases participation in interbank markets while an increase in the return on reserves reduces participation. Taken together these results can shed light on events that occurred in the federal funds market documented by Afonso et al. (2011).

This paper is related to a growing literature on counterparty risk in interbank credit markets. Several models (Heider et al. (2010), Heider and Hoerova (2009), and Freixas and Jorge (2008)) use a setting with informational asymmetries to analyze how interbank
credit markets function – and when they they break down. In these models, loan terms are assumed to result from a no-arbitrage condition, abstracting away from the reality of bilateral bargaining that occurs in interbank markets. In contrast, my model focuses on the impact of credit risk on interbank rates when banks bargain over the terms of the loan. I also focus on a full information setting where the probability of default for the lender and borrower is known by both parties. Participation is determined not by rationing, as in the aforementioned models, but by a participation constraint for both the borrowing and lending bank.

Several models do use a Nash bargaining process to model interbank market transactions. Bech and Klee (2011) and Ennis and Weinberg (2009) use a Nash bargaining process to help explain anomalies concerning the fed funds rate relative to, respectively, the discount window rate and the interest rate paid on reserves. Afonso and Lagos (2012) employ a continuous time search and bargaining model to investigate the determinants of the fed funds rate, and Acharya et al. (2012) model interbank transactions as a two-stage bargaining process with the threat of breakdown. However, these models do not focus on the effect of default risk on the interbank loan rate.

My model is most closely related to Vollmer and Wiese (2016) who construct an interbank lending market with counterparty risk where the interbank loan rate is determined by bilateral bargaining. In this setting, the authors examine the impact of interest on reserves and the central bank’s lending facility rate on the likelihood an interbank transaction occurs. There is heterogeneity in default probabilities between borrowing and lending banks, but no heterogeneity amongst borrowing banks nor amongst lend-
ing banks. In contrast, my model introduces heterogeneity across the set of borrowing banks and the set of lending banks in regards to default risk. This allows me to derive a distribution of loan rates and pin down the extent of participation in the interbank loan market.\(^5\) It also allows me to examine the impact of monetary policies on the average interbank loan rate, as well as the impact of an aggregate deterioration in bank quality on the cost of and participation in interbank markets. Factoring in the effect of participation on the average loan rate is important because the set of participating banks affects the resulting average rate, and it is the average rate that is most often referenced in regards to interbank market conditions.

The remainder of the paper is organized as follows: Section 2 outlines how the cost of and access to interbank credit is determined. Section 3 derives and analyzes the average interbank interest rate and Section 4 concludes.

1.1 The Model

1.1.1 Model Overview

The model consists of three periods, \(t = 0,1,2\), and is populated by a continuum of financial intermediaries (henceforth referred to as banks) of mass 1. The basic timeline of the model is illustrated in figure 2. In \(t = 0\) banks are endowed with 1 unit of funds from risk-neutral depositors. A fraction \(1 - \alpha, \alpha \in (0,1)\), of these deposits are held

\(^5\)Bech and Monnet (2016) also derive a distribution of loans rates in an interbank market with bilateral bargaining over loan terms. However, in their model the underlying heterogeneity causing rate dispersion stems from differences in reserves, not difference in default risk.
in a long-term illiquid asset, and the remaining $\alpha$ are held in a short-term liquid asset (henceforth referred to as reserves). The probability that bank $i$’s long-term assets are successful and return $R^L_i$ is equal to $P\theta_i$. $P \in [0, 1]$ is an aggregate quality component that affects all bank’s probability of success; $\theta_i \in [0, 1]$ is an idiosyncratic, bank-specific, quality component affecting bank $i$’s probability of success. If a bank’s long-term assets fail they earn a return of zero, and the bank fails and is shut down by regulators. I assume that $\theta_i$ is distributed uniformly on the interval $[0, 1]$. I also assume that both $P$ and $\theta_i$ are realized in period $t = 0$ and are public information.

To satisfy the liquidity needs of depositors, claims $d_1 > 0$ and $d_2 > 0$ are offered by banks at $t = 0$ for withdraw at dates 1 and 2.\footnote{I assume that the values of $d_1$ and $d_2$ are chosen by regulators to ensure that banks with successful long-term assets are solvent.} At the beginning of $t = 1$, banks are hit with a liquidity shock. While the aggregate demand for liquidity at each date is certain, banks face uncertain liquidity demand at the individual level. While a fraction $\lambda$ of depositors withdraw their claims at $t = 1$ (and $1 - \lambda$ withdraw at $t = 2$), half of banks receive a below average amount of depositor withdrawals $\lambda_l d_1$, where $\lambda_l < \lambda$. These banks (dubbed “lending banks”) are left with an excess supply of liquidity that they can either hold as reserves or lend to another bank on the interbank credit market. The other half of banks (dubbed “borrowing banks”) receive an above average amount of depositor withdrawals $\lambda_h d_1$, where $\lambda_h > \lambda_l$, and $\frac{1}{2}\lambda_h + \frac{1}{2}\lambda_l = \lambda$. Since it is assumed that banks cannot liquidate any of their illiquid assets, banks with high withdrawals must attempt to borrow funds on the interbank credit market or borrow from the central bank’s standing lending facility.
I interpret the interbank credit market as being analogous to the federal funds market where banks lend and borrow excess reserves via unsecured loans. Due to the decentralized nature of the fed funds market, the terms of interbank credit transactions are modeled as the result of a Nash bargaining solution. I assume that borrowing and lending banks are randomly matched and may only match with one counterparty. The interbank interest rate that results is bank specific in that the terms of the loan depend on the borrowing bank’s probability of success.

However, not all banks in need of liquidity will access the interbank market. Given the results of the Nash bargaining solution, only borrowing banks with an idiosyncratic quality value above a certain threshold will find it advantageous to borrow on the interbank market. Borrowing banks that fall below the threshold will find it more attractive to borrow from the central bank’s lending facility. At the same time, lending banks matched with these low-quality borrowing banks will find it more attractive to hold their excess liquidity as reserves, given the resulting expected return to lending. The threshold that determines participation in interbank markets depends on aggregate conditions and monetary policy. In $t = 2$ liabilities are repaid, less any defaults, and bank profits are realized.

In the remainder of this section, I describe in detail how the cost and use of interbank credit is determined in the model. I begin with interest rate determination and then derive the necessary conditions for a borrowing bank to participate in interbank lending markets. I then derive the average interbank lending rate. Comparative statics are conducted throughout. Of special interest is how the cost of and participation in interbank
lending markets changes with the aggregate success probability $P$ as well as monetary policy. I interpret the events that precipitated the financial crisis as a sharp increase in the probability of default for all banks, which is modeled as a decrease in $P$.

### 1.1.2 Interest Rate Determination

After withdrawal shocks are realized, lending bank $l$ and borrowing bank $b$ are randomly matched and negotiate over the interbank interest rate $R_{l,b}^{IB}$. The resulting interest rate is the solution to the following Nash bargaining problem:

$$\max_{R_{l,b}^{IB}} \left[ E(\pi_l) - E(d_l) \right]^\eta \left[ E(\pi_b) - E(d_b) \right]^{1-\eta} \tag{1.1}$$

where $\eta \in (0,1)$ is the lending bank’s relative bargaining power, $E(\pi_i)$ is the expected payoff for bank $i = \{l, b\}$ when a loan is made and $E(d_i)$ is the expected disagreement point for bank $i = \{l, b\}$ (that is, $E(d_i)$ is what bank $i$ expects to earn if no loan is made).

Banks in need of liquidity face date $t = 1$ deposit withdrawals of $\lambda_h d_1$ but only have $\alpha < \lambda_h d_1$ of reserves. If it cannot raise the appropriate funds by the end of $t = 1$ the bank is shut down by regulators. If the borrowing bank $b$ receives a loan from $l$, its expected profits $E(\pi_b)$ at the end of $t = 2$ are as follows:
\[ E(\pi_b) = \theta_b P[(1 - \alpha)R^L + \alpha R^R - R_{l,b}^{LB} L_b^{LB} - (1 - \lambda_h)d_2] \] (1.2)

s.t. \[ \lambda_h d_1 \leq \alpha + L_b^{LB} \] (1.3)

With probability \( \theta_b P \) bank \( b \) is successful and earns a return of \( R^L \) on its \( 1 - \alpha \) illiquid assets and \( R^R \) on its \( \alpha \) of reserves. \( R^R \) represents the interest paid on reserves held at the central bank and is equal to \( z_R R < R \) where \( R \) is the policy rate set by the central bank. When successful, bank \( b \) also repays its interbank liabilities and remaining depositor claims in \( t = 2 \). I assume that borrowing banks always repay their interbank liabilities, given they do not default, regardless of whether or not the lending bank survives. If the lending bank fails, bank \( b \)'s payment is used to repay the lending bank’s creditors. Lastly, equation 1.3 imposes a flow of funds constraint: bank \( b \)'s reserves and funds borrowed on the interbank market must be large enough to meet total depositor withdrawals in \( t = 1 \).

If \( b \) does not receive a loan on the interbank credit market it must borrow from the central bank’s lending facility and receives the following expected profits \( E(d_b) \):

\[ E(d_b) = \theta_b P[(1 - \alpha)R^L + \alpha R^R - R_{l,b}^{LF} L_b^{LF} - (1 - \lambda_h)d_2] \] (1.4)

s.t. \[ \lambda_h d_1 \leq \alpha + L_b^{LF} \] (1.5)

where \( R_{LF} = z_{LF} R > R \) is the cost of funds borrowed from the central bank and is equal to the policy rate times penalty \( z_{LF} \).\(^7\)

\(^7\)For simplicity, I assume that discount window loans require no collateral.
Banks that receive a positive liquidity shock have $t = 1$ depositor withdrawals of $\lambda_1d_1 < \alpha$. If the lender $l$ makes an interbank loan to bank $b$ its expected profits are as follows:

$$E(\pi_l) = \theta_lP[(1 - \alpha)R^L + \alpha R^R + \theta_b PR^B_{l,b} L^B_l - (1 - \lambda_l)d_2]$$  \hspace{1cm} (1.6)$$

s.t. \hspace{1cm} \lambda_1d_1 + L^B_l \leq \alpha \hspace{1cm} (1.7)$$

The lending bank’s expected profit from participation in the interbank lending market is interpreted analogously to the borrowing bank’s with two notable differences. First, repayment of the interbank loan made to bank $b$ is contingent on bank $b$’s success; thus, bank $b$’s probability of success affects lending bank $l$’s expected profits. Second, the flow of funds constraint for the lending bank states that bank $l$’s reserves must be large enough to cover funds offered on the interbank loan market plus date 1 deposit withdrawals.

If $l$ does not make the loan to $b$, the expected payout (and $l$’s disagreement point $E(d_l)$) is given by the following:

$$E(d_l) = \theta_lP[(1 - \alpha)R^L + (\alpha - \lambda_1d_1)R^R - (1 - \lambda_l)d_2]$$  \hspace{1cm} (1.8)$$

When $l$ does not make a loan to $b$, all of $l$’s reserves that remaining after paying date 1 depositor withdrawals $\alpha - \lambda_1d_1$ continue to be held as reserves.

Before solving the bargaining problem, a few simplifications can be made in regards to the loan quantities. First, since the cost of loans from the central bank’s lending
facility and interbank loans are greater than the opportunity cost of using reserves to meet liquidity needs, borrowing banks will use all of their reserves to meet date 1 deposit withdrawals and then borrows the shortfall (either from another bank or from the lending facility). This implies that

\[ L_b^{IB} = \lambda_t d_1 - \alpha \] (1.9)

The most that lending bank \( l \) can lend is \( L_l^{IB} = \alpha - \lambda_l d_1 \). To ensure that bank \( b \)'s liquidity needs can be satisfied by bank \( l \), I assume that \( 2\alpha \geq d_1 \). This implies that \( L_l^{IB} \geq L_b^{IB} \), although in equilibrium the actual loan amount will be given by \( L_b^{IB} \) in equation 1.9.

Given equations 1.2, 1.4, 1.6, 1.8, and 1.9, the Nash bargaining problem solves the following maximization problem:

\[
\max_{R_{l,b}^{IB}} \left[ \theta_l P(\theta_b P R_{l,b}^{IB} - z_R R)(\lambda_h d_1 - \alpha) \right]^\eta \left[ \theta_b P(z_{LF} R - R_{l,b}^{IB})(\lambda_h d_1 - \alpha) \right]^{1-\eta} \tag{1.10}
\]

Solving the maximization problem 1.10 yields the following equilibrium interbank interest rate \( R_{l,b}^{IB} \):

\[
R_{l,b}^{IB} = R \left( \eta z_{LF} + \frac{(1 - \eta) z_r}{\theta_b P} \right) \tag{1.11}
\]

Result 1. \( R_{l,b}^{IB} \) satisfies the following properties:
1. $R_{I,l,b}^{IB}$ is decreasing in the borrowing bank’s idiosyncratic success probability $\theta_b$.

2. $R_{I,l,b}^{IB}$ is decreasing in the aggregate success probability $P$.

3. $R_{I,l,b}^{IB}$ is increasing in the central bank lending facility premium $z_{LF}$.

4. $R_{I,l,b}^{IB}$ is increasing in the interest on bank reserves premium $z_R$.

5. $R_{I,l,b}^{IB}$ is increasing in the risk-free rate $R$.

Proof. Each part of result 1 can be shown by partially differentiating equation 1.11 with respect to relevant right-hand side variable.

Intuitively, if the borrowing bank has a lower probability of success due to either a lower value of $\theta_b$ or a lower value of $P$, the lending bank can negotiate a higher interbank interest rate to compensate for the higher level of counterparty risk. This result is consistent with empirical evidence from Afonso et al. (2011) that worse-performing banks saw larger increases in the spread between their interbank loan rate and the risk-free rate.

An increase in the risk-free rate $R$ leads to an increase in $R_{I,l,b}^{IB}$ for two reasons. First, an increase in $R$ increases the opportunity cost of funds to the lending bank; thus, $l$ must be compensated with a higher interbank rate. Second, an increase in $R$ increases the cost of funds borrowed from the central bank and increases the interbank rate that they are willing to pay. By similar intuition, when the premium charged on central bank-borrowed funds, $z_{LF}$, rises, borrowing banks are willing to pay a higher interbank loan rate and the equilibrium rate rises. Additionally, if the premium paid on bank
reserves, $z_R$, rises, the opportunity cost of funds rises for the lending bank and they demand a higher interbank loan rate.

1.1.3 Determining Access to Credit

The previous section describes how the interbank interest rate is determined for an individual lender-borrower match. However, not all matches will result in an interbank loan transaction. Participation constraints for both the borrowing and lending bank must be satisfied. If not, bargaining breaks down; the borrowing bank must turn to the central bank’s lending facility and the lending bank holds any excess liquidity as reserves.

For the lender to participate in the interbank loan, the expected return to lending to borrowing bank $b$ must at least as large as the interest paid on reserves:

$$\theta_b P R_{l,b}^B \geq R^R \quad (1.12)$$

$$\Rightarrow (\theta_b P \eta z_{LF} + (1 - \eta) z_R) R \geq z_R R \quad (1.13)$$

For the borrower to participate in the interbank loan, the expected cost of the interbank loan must be no larger than the expected cost of borrowing funds from the central bank’s lending facility:
Both borrower and lender’s participation constraints pin down a single threshold value of \( \theta_b \) which determines participation in the interbank lending market.

**Result 2.** There exists a threshold level of the borrowing bank’s idiosyncratic success probability \( \tilde{\theta}_b \) such that for any interbank loan matches where \( \theta_b < \tilde{\theta}_b \), negotiations break down and the transaction does not take place. Interbank lending transactions do occur, at the interest rate in 1.11, for matches with \( \theta_b \geq \tilde{\theta}_b \). The threshold is given by the following condition:

\[
\tilde{\theta}_b = \frac{z_R}{z_{LF}P} \tag{1.16}
\]

**Proof.** Result 2 can be shown using equations 1.13 and 1.15. Since the left-hand side of equation 1.13 is increasing in \( \theta_b \), the lowest level of \( \theta_b \) that satisfies the lender’s participation constraint occurs when 1.13 holds with equality:

\[
(\theta_b P \eta z_{LF} + (1 - \eta) z_R) R = z_R R \tag{1.17}
\]

Solving 1.17 for \( \tilde{\theta}_b \) yields the value of of threshold in equation 1.16.
At the same time, the borrowing bank’s participation constraint implies the same cutoff for $\theta_b$. Since the left-hand side of equation 1.15 is decreasing in $\theta_b$, the lowest level of $\theta_b$ that satisfies the borrower’s participation constraint occurs when 1.15 holds with equality:

$$
\left(\eta z_{LF} + \frac{(1-\eta)z_R}{\theta_b P}\right)R = z_{LF}R
$$

Solving 1.18 for $\tilde{\theta}_b$ yields the same value of of threshold in equation 1.16 that is also implied by the lender’s participation constraint.

$\Box$

The remainder of this section is concerned with how this idiosyncratic cutoff $\tilde{\theta}_b$ changes with respect to aggregate conditions. These results are formalized in the following proposition:

**Result 3.** The cutoff idiosyncratic success probability component $\tilde{\theta}_b$ satisfies the following properties:

1. $\tilde{\theta}_b$ is decreasing in the aggregate success probability component, $P$.

2. $\tilde{\theta}_b$ is decreasing in the premium on loans borrowed from the central bank’s lending facility, $z_{LF}$.

3. $\tilde{\theta}_b$ is increasing in the premium on interest paid on bank reserves, $z_R$.

**Proof.** Each component of Proposition 3 can shown to be true by differentiating equation
1.16 with respect to the respective variable of interest.

Intuitively, how each of the variables considered in result 3 affects cutoff $\tilde{\theta}_b$ depends on its effect on the lending and borrowing bank’s participation constraints. A higher level of $P$ simultaneous increases the expected return to lending and decreases the expected cost of borrowing for all levels of $\theta_b$. This loosening of both the lender and borrower participation constraints allows $\tilde{\theta}_b$ to fall and increases participation in the interbank market.

When the premium on loans from the central bank’s lending facility rises the expected return to lending rises, loosening the lender’s participation constraint and permitting a higher level of idiosyncratic risk. For the borrower, an increase in $z_{LF}$ increases the net cost of borrowing at the lending facility. This implies the borrower is willing to pay a higher risk premium and still borrow on the interbank market which results in a decrease in $\tilde{\theta}_b$ and increased participation in the interbank market.

Similarly, when the return on reserves $z_R$ rises, the net opportunity cost of lending on the interbank market rises. Lending banks require a higher expected return and $\tilde{\theta}_b$ rises. In addition, the cost of borrowing on the interbank market rises, tightening the borrower’s lending constraint. To counteract this, the risk premium must fall which necessitates an increase in $\tilde{\theta}_b$ and reduced participation in the interbank market.
1.2 Aggregation Results

The preceding section describes how the cost of and participation in unsecured interbank credit markets are determined at the individual bank level. This section derives and analyzes the average interbank rate for the entire interbank credit market.\(^8\)

The average, or effective, interbank interest rate, \(R_{IB}^{eff}\), is the value-weighted average of all individual interbank rates that occur given the cutoff level of \(\theta_b\). Using equation 1.11, 1.16, and the assumption that \(\theta\) is uniformly distributed on the interval \([0, 1]\), \(R_{IB}^{eff}\) can be expressed as follows:

\[
R_{IB}^{eff} = R\left(\eta z_{LF} + \frac{(1 - \eta)z_R}{P(1 - \bar{\theta}_b)} \int_{\bar{\theta}_b}^{\theta_b} \frac{1}{\theta_b} d\theta\right)
\]  
(1.19)

The first term parentheses in equation 1.19 can be thought of as a “base rate” that is common to all borrowing banks. The second term is the average risk premium paid by all borrowing banks that participate in the interbank market. Equation 1.19 shows that the effective interbank interest rate can be expressed as the base rate plus the average risk premium.

Of key interest in this paper is how the effective interbank loan rate responds to changes in periods of financial turmoil. In the model, financial distress in the interbank lending market is modeled as an unexpected drop in the aggregate success probability component \(P\). The effect of \(P\) on the effective interbank loan rate can be found by differentiating

\(^8\)In relation to the fed funds market, the average interbank interest rate is analogous to the effective federal funds rate.
1.19 with respect to \( P \), while internalizing the effect of \( P \) on the borrower threshold \( \tilde{\theta}_b \):

\[
\frac{\partial R^{IB}_{eff}}{\partial P} = \frac{(1 - \eta)z_R R}{(1 - \tilde{\theta}_b)} \left[ -\frac{1}{P^2} \int_{\tilde{\theta}_b}^{1} \frac{1}{\theta_b} d\theta - \left( \frac{1}{P \tilde{\theta}_b} - \frac{1}{P(1 - \tilde{\theta}_b)} \int_{\tilde{\theta}_b}^{1} \frac{1}{\theta_b} d\theta \right) \frac{\partial \tilde{\theta}_b}{\partial P} \right]
\]  (1.20)

A fall in \( P \) has two effects on the effective interbank interest rate. The first effect of fall in \( P \), which I dub the incumbant interest rate effect, is captured by the first term within the brackets in equation 1.20. This measures the effect of \( P \) on the risk premium paid by borrowing banks who were already participating interbank markets. Per result 1, when \( P \) falls the interbank interest rate rises for all borrowers who access funds. Thus, ceteris paribus, the incumbant interest rate effects causes \( R^{IB}_{eff} \) to rise after a fall in \( P \), and vice versa.

The second effect of \( P \) on the effective interbank rate is the borrower threshold effect. This is the effect on \( R^{IB}_{eff} \) when the borrower threshold \( \tilde{\theta}_b \) changes in response to a fall in \( P \), and is captured by the term in parentheses (within the brackets) in equation 1.20 that is multiplied by \( \frac{\partial \tilde{\theta}_b}{\partial P} \).

Looker closer at this second effect, the two components that constitute the borrower threshold effect move \( R^{IB}_{eff} \) in opposite directions. Per result 3, when \( P \) falls the threshold \( \tilde{\theta}_b \) rises implying that \( \frac{\partial \tilde{\theta}_b}{\partial P} < 0 \). This also implies that the number of borrowing banks falls, and, all else equal, averaging interbank rates over fewer borrowers causes \( R^{IB}_{eff} \) to rise after a fall in \( P \). This effect is captured by the first term inside the parentheses in equation 1.20. However, when \( \tilde{\theta}_b \) rises some banks who were not rationed prior to the fall in \( P \) are now rationed out of the market. Since these borrowers were the riskiest
borrowers and paid the highest risk premiums, the average interbank rate falls after a rise in $P$ due to these risky borrowers dropping out of the interbank market. This effect is captured by the second term inside the parentheses in equation 1.20.

Despite these counteracting effects, the net effect of the borrower threshold effect is known with certainty and is summarized in the following result.

**Result 4.** The net borrower threshold effect from a change in $P$ is positive. That is, the borrower threshold effect causes $R_{\text{eff}}^{IB}$ to fall after a fall in the aggregate success component, and vice versa.

**Proof.** Using equation 1.20, the necessary condition for the borrower threshold effect to be positive is

$$B(\tilde{\theta}_b) \equiv \frac{1}{\tilde{\theta}_b} + \ln(\tilde{\theta}_b) > 1 \quad (1.21)$$

To show that equation 1.21 is true for all $\tilde{\theta}_b \in (0, 1)$, first observe that

$$\lim_{\tilde{\theta}_b \to 1} B(\tilde{\theta}_b) = 1 \quad (1.22)$$

$$\lim_{\tilde{\theta}_b \to 0} B(\tilde{\theta}_b) = \infty \quad (1.23)$$

Second, $B(\tilde{\theta}_b)$ is strictly decreasing in $\tilde{\theta}_b$ for $\tilde{\theta}_b \in (0, 1)$:
\[ \frac{\partial B(\tilde{\theta}_b)}{\partial \tilde{\theta}_b} = \frac{1}{\tilde{\theta}_b} \left( 1 - \frac{1}{\tilde{\theta}_b} \right) < 0 \] (1.24)

Taken together, equations 1.22, 1.23, and 1.24 show that equation 1.21 holds for all \( \tilde{\theta}_b \in (0, 1) \).

Intuitively, the first term in the borrower threshold effect, \( \frac{1}{P\tilde{\theta}_b} \) represents the risk premium paid by the threshold borrower, and the second term, \( \frac{1}{P(1-\tilde{\theta}_b)} \int_{\tilde{\theta}_b}^{1} \frac{1}{y} dy \), represents the average premium paid by all borrowing banks. Since the threshold borrowing bank is the riskiest borrower, they pay a higher premium that the average which implies the borrower threshold effect must be positive.

Figure 1.2 illustrates the borrower threshold effect (in panel a) and the incumbent interest rate effect (in panel b). Given the two counteracting effects of \( P \) on \( R^{IB}_{eff} \), which effect will dominate? The answer is formalized in result 5.

**Result 5.** The effective interbank loan rate, \( R^{IB}_{eff} \), rises when the aggregate success probability component, \( P \), falls.

**Proof.** For result 4 to be true, the derivative of \( R^{IB}_{eff} \) with respect to \( P \) must be negative. Differentiating 1.19 with respect to \( P \), while internalizing the effect of \( P \) on the borrower threshold \( \tilde{\theta}_b \) yields the following expression

Equation 1.20 is strictly negative if the following condition holds
\[ A(\tilde{\theta}_b) \equiv \tilde{\theta}_b - \ln(\tilde{\theta}_b) > 1 \] (1.25)

To show that equation 1.25 holds for all \( \tilde{\theta}_b \in (0, 1) \), first note that

\[
\lim_{\tilde{\theta}_b \to 1} A(\tilde{\theta}_b) = 1 \quad (1.26)
\]

\[
\lim_{\tilde{\theta}_b \to 0} A(\tilde{\theta}_b) = \infty \quad (1.27)
\]

Second, \( A(\tilde{\theta}_b) \) is strictly decreasing in \( \tilde{\theta}_b \) for \( \tilde{\theta}_b \in (0, 1) \):

\[
\frac{\partial A(\tilde{\theta}_b)}{\partial \tilde{\theta}_b} = 1 - \frac{1}{\tilde{\theta}_b} < 0 \quad (1.28)
\]

Taken together, equations 1.26, 1.27, and 1.28 show that equation 1.25 holds for all \( \tilde{\theta}_b \in (0, 1) \).

It is also of interest to see how the effective interbank loan rate changes with monetary policy, namely the premium paid on bank reserves, \( z_R \), and the premium charged on funds borrowed from the central bank’s lending facility, \( z_{LF} \). This relationship is formalized in the following result.

**Result 6.** The effective interbank loan rate is increasing in \( z_{LF} \) and increasing in \( z_R \).
Proof. Differentiating 1.19 with respect to $z_{LF}$ yields the following condition:

$$\frac{\partial R_{eff}^{IB}}{\partial z_{LF}} = \eta R + \frac{(1-\eta)\tilde{\theta}_b R}{1-\bar{\theta}} \left( \frac{1}{P\theta_b} - \frac{1}{P(1-\theta_b)} \int_{\theta_b}^{1} \frac{1}{\theta} d\theta \right) > 0$$  \hspace{1cm} (1.29)

which is strictly positive given result 4.

The first term on the right-hand side of equation 1.29 represents the direct effect of an increase in $z_{LF}$ on $R_{eff}^{IB}$. The second term represents the indirect effect on $R_{eff}^{IB}$ through changes in the threshold level of $\tilde{\theta}_b$. This is equal to the increase in the average risk premium brought about by a rise in $z_{LF}$. Since an increase in the premium on funds borrowed from the central bank’s lending facility causes $\tilde{\theta}_b$ to fall, and thus higher risk borrowing banks to participate in the interbank loan market, the average risk premium rises and contributes to the increase in $R_{eff}^{IB}$.

Differentiating 1.19 with respect to $z_R$ yields the following condition:

$$\frac{\partial R_{eff}^{IB}}{\partial z_R} = \frac{(1-\eta)R}{1-\bar{\theta}} \left[ \frac{1}{P} \int_{\theta_b}^{1} \frac{1}{\theta} d\theta - \tilde{\theta}_b \left[ \frac{1}{P\theta_b} - \frac{1}{P(1-\theta_b)} \int_{\theta_b}^{1} \frac{1}{\theta} d\theta \right] \right] > 0$$  \hspace{1cm} (1.30)

Unlike $z_{LF}$, an increase in $z_R$ has two counteracting effects of $R_{eff}^{IB}$. The first term in the parentheses on the right-hand side of equation 1.30 represents the direct effect of a rise in $z_R$ on the effective interbank loan rate. The second term (in square brackets) represents the change in $R_{eff}^{IB}$ caused by a change in the average risk premium. Since
an increase in $z_R$ causes the threshold $\tilde{\theta}_b$ to rise and the marginally riskiest borrowing banks to drop out of the interbank market, an increase in $z_R$ causes the average risk premium to fall which exerts downward pressure on $R_{\text{eff}}^{IB}$. However, equation 1.30 holds if the following condition is met:

$$\tilde{\theta}_b - \ln(\tilde{\theta}_b) > 1$$  

(1.31)

which was shown to be true in result 5.

At different points during the financial crisis, the premium paid on the Federal Reserve’s discount rate was lowered and the Federal Reserve began to pay interest on reserves. In the model, the policy of paying interest on reserves would be shown as an increase in $z_R$. This would result in less participation in interbank loan markets and an increase the spread between the effective interbank loan rate and policy rate. While the policy of paying interest on reserves was designed to provide a floor on the interbank rate in the face of large liquidity injections by the Fed’s other liquidity facilities, the model suggests it raises the average cost of borrowing in the interbank loan market.

The Federal Reserve also lowered the discount rate during the financial crisis. In the model, this would be shown as a decrease in $z_{LF}$ which would decrease participation in the interbank loan market and decrease the spread between the effective interbank rate and the policy rate. While there is evidence that low quality borrowers were more likely to access the discount window, this does not fit with the rising spreads witnessed.

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9See Afonso et al. (2011).
However, it is possible that the premium on borrowing from the discount window includes costs other than the discount rate. Namely, borrowing from the discount window may carry an extra cost associated with the “stigma” of seeking an emergency loan from the Fed.\footnote{See Ennis and Weinberg (2009).} If the perceived cost of of this stigma rose during the crisis, when sending a signal of poor financial health to the markets may be particularly harmful, it is possible that the perceived $z_{LF}$ rose during the crisis despite the Federal Reserve’s policy actions.

\section*{1.3 Conclusion}

This paper presents a theoretical framework to analyze the effects of counterparty risk and monetary policy on the cost of and participation in unsecured interbank credit markets. I am able to derive a bank–specific interbank loan rate that is the result of a Nash bargaining problem. This individual loan rate is increasing in the borrower’s probability of default and rises after a decrease in the aggregate component of success probabilities.

Given the individual interbank loan rate, an endogenous threshold for the bank-specific success component emerges which determines participation in the interbank market. After a deterioration in aggregate conditions, this threshold tightens, fewer banks access interbank credit, and both individual and the average interbank loan rates rise. In this respect, my model is capable of capturing the key effects the financial crisis had on interbank markets both from the perspective of individual borrowing banks and from a
market-wide perspective.

Central bank policies also affect access and loan rates: lowering the cost of borrowing from the central bank’s lending facility and an increase in the return on bank reserves reduces participation. However, these policy actions have opposite effects on the average interbank loan rate. Lowering the lending facility rate lowers the average interbank rate while raising the return on reserves increases the average interbank rate.

While I am encouraged by the results my framework is capable of producing, there is still much room for further exploration and improvement moving forward. Ultimately, I would like to completely endogenize the probability of default. In the current framework, the cost of interbank credit has no effect of the probability of default. By endogenizing default probabilities I hope to create a richer setting to consider the effects that aggregate conditions and loan rates have on counterparty risk and the functioning of interbank credit markets.

Another potential extension of my model I am interested in pursuing is to analyze the effect that interbank markets have on the real economy. As the model currently stands, the cost of interbank credit has no effect on the quantity or price of commercial loans, and, more generally, my framework abstracts away from the linkages between real output and financial markets. Incorporating such linkages would move me in the direction of modeling and analyzing the effects of monetary policy, including both conventional and unconventional policy tools.
Figure 1.1: TED spread.
Table 1.1: Model timeline.

| $t = 0$          | Banks are endowed with 1 unit of deposits.
|                  | $1 - \alpha$ are held in an illiquid long-term asset.
|                  | $\alpha$ are held as liquid reserves.
|                  | The aggregate quality component and idiosyncratic quality components are realized.
| $t = 1$          | Liquidity shocks hit.
|                  | Half of banks have low withdrawals of $\lambda_l d_1$ and have excess liquidity.
|                  | Half of banks have high withdrawals of $\lambda_h d_1$ and need extra liquidity.
|                  | Each bank with excess liquidity is matched with a bank in need of liquidity and negotiate over the interbank loan rate.
|                  | If negotiations break down, the bank in need of liquidity borrows from the central bank’s lending facility and the bank with excess liquidity holds their funds as reserves.
| $t = 2$          | Loans, interbank liabilities, and remaining deposits are repaid, less defaults. |
Figure 1.2: The effects of a fall in $P$ on the effective interbank interest rate. After a fall in $P$ the threshold $\tilde{\theta}_b$ rises causing the average interbank loan rate to fall (panel a). Simultaneously, a fall in $P$ causes risk premia for existing borrowers to rise which causes the average interbank loan rate to rise (panel b).
Chapter 2

Lending Standards, Bank Risk-Taking and Monetary Policy

In the wake of the financial crisis of 2007-2009 the potential role that monetary policy played in stoking the crisis has emerged as a topic of considerable interest and debate.\(^1\)

The general narrative dictates that the low interest rate policy of the early-2000s fueled the spectacular boom in real estate by flooding financial markets with liquidity. This boom, further exacerbated by the deterioration of mortgage lending standards and the rapid expansion of subprime lending, ultimately left financial institutions more vulnerable to the risk of insolvency and set the stage for the damaging events of the crisis.

Multiple factors likely contributed to the loosening of lending standards in the run up to the crisis.\textsuperscript{2} This paper investigates the influence of conventional monetary policy on bank lending standards and, by extension, the risk of bank failure. I construct a partial equilibrium model of the commercial loan market where both borrowers and banks are capable of default. Monetary policy affects bank lending standards – which types of borrowers banks will, and will not, lend to – which influences the risk of bank default.

For the baseline calibration of the model, expansionary monetary policy, modeled as a lower policy rate, loosens lending standards and increases the probability of bank default – that is, the risk of bank default and the policy rate are negatively correlated. However, sensitivity analysis shows the both the sign and magnitude of the correlation between the probability of bank default and the policy rate is sensitive to the extent of bank capitalization (i.e., bank capital ratios). The results suggest that the nature and size of the trade-off between financial stability and the traditional policy objectives of full employment and moderate inflation depends crucially on bank capital ratios and, by extension, minimum capital requirements.

These results emerge from a partial equilibrium model of the financial immedialtion sector populated by entrepreneurs, financial intermediaries (“banks” from henceforth) and depositors, where both the deposit market and loan market are competitive. Entrepreneurs have access to a non-divisible investment project that yields uncertain returns. Since it is assumed that entrepreneurs have no existing net worth, they must borrow funds from a bank to invest. Additionally, since the project return is subject to

\textsuperscript{2}See Lo (2012) and Thakor (2015) for an overview of the causal factors of the crisis.
idiosyncratic shocks, entrepreneurs may default on their loan, in which case the bank seizes the project return.\footnote{One aspect of the model is that banks are willing to lend to borrowers that hold no collateral. While borrowers have no existing assets to pledge as collateral ex-ante, the realized project returns serve as a form of collateral ex-post: if the borrower defaults on their loan the bank seizes the project returns, similar to the loan contract in Bernanke et al. (1999).}

While it is assumed that all entrepreneurs seek to borrow the same amount of funds, they do vary in regards to their publically known “abilities” which affect their project returns. This assumption is similar to how heterogeneous productivity affects firm output in Melitz (2003) or worker productivity, as in Jovanovic (1979) and Strand (2000). All else equal, higher ability entrepreneurs have higher expected project returns and yield a higher expected return to a bank compared to low ability borrowers. This result plays a key role in the determination of lending standards in the model. In the context of this paper, lending standards refer to the internal guidelines used by banks to determine whether a given borrower is creditworthy or whether the borrower’s loan application should be denied. In the model, if the maximum expected return to lending to a given ability level is less than the bank’s cost of funds, entrepreneurs with that ability level will be denied a loan and rationed from the market.

Thus, lending standards, as interpreted in the framework of the model, refers to a minimum ability level where the expected return to lending to the minimum (“cutoff”) ability level is just equal to the bank’s cost of funds. Entrepreneurs with ability levels equal to or above the cutoff receive a loan, and those with abilities below the cutoff are denied loans. These rationed entrepreneurs are willing to pay a higher loan rate; however, no loan rate exists that guarantees that the bank will cover the cost of its
funds. Standards loosen when the cutoff ability level falls and tighten when the cutoff ability level rises.\(^4\)

The return on a bank’s loan portfolio is also subject to fluctuations induced by an aggregate shock that affects all individual entrepreneurial project returns. In the event of a negative shock, the realized return to lending to low ability borrowers is less than expected. If the shock is large enough, the realized return on these loans may fail to cover the cost of deposits and results in a net loss for the bank. If these loan losses are high enough, the bank will be unable to repay its deposit liabilities and will default.

The risk of bank default is intrinsically linked to lending standards: when standards are lax, loan losses in the event of a negative shock are larger due to lower bank asset quality. The increase in loan losses implies that the bank cannot withstand as large of a negative aggregate shock without defaulting, and the probability of bank default increases when standards loosen.

While looser standards unequivocally increase the risk of bank default, conventional monetary policy, modeled as a change in the policy rate, has two counteracting effects on the probability of bank default. The “deposits effect” refers to the impact of the cost of deposits on the risk of bank default: as the policy rate falls, the cost of a given amount of deposits also falls which, on its own, results in a lower risk of bank default. In contrast, the ”standards effect” causes the risk of bank default to move in the opposite direction. When the policy rate falls, lending standards loosen. This has the effect

\(^4\)This ability cut-off is similar to the firm productivity cut-off in Melitz (2003); the key difference is that in Melitz (2003) firms self-select out of production and/or exporting. In this model, borrowers do not self-select out of the loan market – they are rationed by banks.
of depressing bank profits in the event of a negative aggregate shock and, in isolation, increases the probability of bank default.

To determine which effect dominates, and the ultimate relationship between the policy rate and the probability of bank default, I calibrate and numerically solve the model. Under the baseline calibration, the probability of bank default increases when the policy rate falls. Even though the deposits effect pushes the bank further from default, this positive effect is outweighed by the negative effect of looser lending standards. Thus the net effect of a lower policy rate is a higher probability of bank default.

However, the relationship between the policy rate and the equilibrium probability of bank default is sensitive to equilibrium bank capital ratios. When model parameters are set to produce relatively high capital ratios the probability of default and the policy rate are negatively correlated. When parameters are set such that equilibrium capital ratios are low this relationship is reversed, and default risk and the policy rate are positively correlated.

This sensitivity is largely driven by the sensitivity of the deposits effect to bank capital ratios. In the model, the amount of deposits raised by a bank – and therefore the deposits effect – is highest when capital ratios are low. Low capital ratios cause the deposits effect to be large relative to the standards effect, and default risk and the policy rate are positively correlated. As bank capital ratios rise, deposits and the deposits effect falls rapidly relative to the standards effect. At sufficiently high capital ratios, the standards effect dominates the deposits effect, and default risk and the policy rate are negatively correlated.

The model is calibrated to match empirical observations on borrower credit scores, the average probability of bank default, and average bank capital ratios.
correlated.

In addition to determining the sign of the correlation between default risk and the policy rate, equilibrium capital ratios also affect the magnitude of this correlation. Relative to the baseline calibration, as capital ratios rise the correlation grows larger in absolute terms. This implies that monetary policy has a larger impact on the risk of bank default when bank capital ratios are relatively high. While this paper is certainly not the first to note that the relationship between risk-taking and monetary policy depends on bank capital structure, the results of this paper contribute to the growing debate on the impact of capital structure and regulation on risk-taking.\(^6\)

This paper contributes to the growing literature on the risk-taking channel of monetary policy. The risk-taking channel, first formalized by Borio and Zhu (2012), centers on the idea that expansionary monetary policy may increase risk-taking. This transmission mechanism focuses on the way in which changes in the policy rate affect risk perceptions and appetites, and how these further affect the cost and terms of funding, and thus real economic activity.

This paper adds to this literature by examining the relationship between the policy rate, lending standards, and the risk of bank default in a setting where both asset quality and leverage play a role in determining the risk of bank insolvency. In contrast with the existing literature, most theoretical models exploring the risk-taking channel do not

\(^6\)Dell’Ariccia et al. (2013) find empirical evidence that the effect of monetary policy on risk-taking is more pronounced for more capitalized banks. DellAriccia et al. (2011) construct a partial equilibrium model where the relationship between risk-taking and monetary policy depends on the bank’s (fixed) capital structure: looser monetary policy increases risk-taking for highly capitalized banks but reduces risk-taking for banks with low capitalization.
incorporate these three factors simultaneously. For example, Angeloni et al. (2013) and Valencia (2014) explore the relationship between bank default risk and monetary policy where bank leverage is the key determinant of the probability of bank default. In both models, a fall in the policy rate reduces the cost of deposits and incentivizes banks to increase leverage, thereby increasing the risk of bank failure. This model has a similar mechanism in regards to the cost of deposits and bank leverage; however, this model also incorporates a role for asset quality via bank lending standards. In contrast, Agur and Demertzis (2012) and Cociuba et al. (2012) investigate the impact of monetary policy on asset quality; however, unlike my model, there is no explicit discussion of the impact of changes in asset quality on bank default risk.

This paper also relates to the literature on lending standards by investigating the impact of monetary policy on lending standards in a full information setting. In the literature, lending standards often emerge due to some sort of managerial incentives or irrationalities. Examples include Rajan (1994), who examines how managerial incentives may result in lax lending standards, and Berger and Udell (2004), who focus the role short-sighted loan officer memory may play in the procyclicality of lending. Alternatively, lending standards may emerge as the result of informational asymmetries about the type or quality of borrowers, as in Dell’Ariccia and Marquez (2006) and Ruckes (2004).

In comparison, in my model lending standards emerge as a minimum ability requirement, which results from heterogeneous entrepreneurial abilities. Additionally, the model does so in a full information setting, nor does it rely on private managerial incentives or short-
comings to explain fluctuating standards.\footnote{The main mechanisms of the model still function in an asymmetrical information setting if the bank can get some sort of signal (via credit scores, income statements, etc.) about borrower ability.}

\section{The Model}

\subsection{Model Set-Up}

I assume there is a continuum of risk-neutral entrepreneurs and a continuum of risk-neutral banks, both of of mass one, that operate in the economy. Each entrepreneur has the opportunity to pursue a non-divisible investment project that requires $1 of initial funds. It is assumed that entrepreneurs have no existing net worth and cannot issue equity; therefore, they must obtain external financing from a bank.\footnote{Throughout the model I use the terms “entrepreneur” and “borrower” interchangably.}

Entrepreneurs are heterogenous in regards to their exogeneously given abilities $a$, where abilities are distributed across entrepreneurs according to the cdf, and associated pdf, $H(a)$ and $h(a)$ on a non-negative support. For simplicity, it is assumed that an entrepreneur’s ability is public information known by both the entrepreneur and bank; however, it is straightforward to extend the model such that entrepreneurial ability is private information and banks receive only a noisy signal of an entrepreneur’s ability level.

Entrepreneurial ability is of importance since it affects the project return $a\omega R^K$. $\omega$ is a random idiosyncratic component of the project return and is assumed to be drawn from the common distribution $F(\omega)$ on a non-negative support. $R^K$ is the aggregate return.
to capital and follows the process below:

\[ R^K = \epsilon R^K, \quad E(\epsilon) = 1 \]  

(2.1)

where \( R^K > 1 \) is the long-run gross return to capital and \( \epsilon \) is a mean one aggregate shock drawn from a non-negative support.

Due to the uncertainty of the project return, an entrepreneur may default on their debt if the project return is too low. I define a threshold \( \tilde{\omega}_a \) such that an entrepreneur with ability \( a \) and idiosyncratic return \( \tilde{\omega}_a \) is just able to make their debt payment:

\[ \tilde{\omega}_a = \frac{D_a}{aR^K} \]  

(2.2)

where \( D_a \) is the contracted loan repayment for a borrower with ability \( a \).\(^9\)

With probability \( F(\tilde{\omega}_a) \) the entrepreneur’s idiosyncratic return is too low and the entrepreneur defaults. Upon default, the project and its returns are seized by the bank while the entrepreneur receives nothing. With probability \( 1 - F(\tilde{\omega}_a) \) the borrower does not default, the bank receives the debt payment \( D_a \), and the entrepreneur keeps the project return net of the debt payment. The existence of heterogenous abilities implies that the the maximum return to lending is increasing in entrepreneurial ability – this result is key for determining lending standards in the model.

\(^9\)Since each loan is $1, \( D_a \) can also be interpreted as one plus the loan interest rate.
In relation to commercial banking, lending standards refer to the internal guidelines used by banks to determine the creditworthiness of potential borrowers. In the context of the model, lending standards take the form of a minimum ability requirement, much akin to minimum credit score requirements lenders use in practice. Specifically, lending standards refer to the lowest ability an entrepreneur can have and still receive a loan. Given the equilibrium market return to lending, $R^L$, only borrowers that can credibly commit to generating the bank an expected return of at least $R^L$ receive a loan. However, for lower ability borrowers the maximum expected return to lending may fall below $R^L$; these borrowers are denied loans and effectively rationed from the loan market.

Entrepreneurs are not the only agents capable of default – banks may also default. While banks can diversify away the risk embodied in $\omega$, they cannot diversify away all of the risk embodied in the aggregate return to capital. As in Bernanke et al. (1999), I assume that, when possible, the loan payment adjusts after the shock to $R^K$ so that the ex-post return to lending is equal to the equilibrium market return to lending, $R^L$. However, in the case of a negative shock this adjustment will not be possible for all borrowers. For lower ability entrepreneurs, the shock to $R^K$ may be “bad” enough that the maximum ex-post return to lending falls below $R^L$. For these loans, the loan payment is adjusted to maximize the ex-post return to lending; however, since this return is less than expected, banks incur unexpected loan losses. If the realized return to lending is low enough, and unexpected loan losses high enough, banks may become insolvent and default on their deposit liabilities. For simplicity, and to ensure that the probability of bank default may be studied analytically, I assume that bank deposits
are fully ensured, and the cost of deposits to the bank is equal to the policy rate chosen by monetary authorities.\textsuperscript{10}

The equilibrium market return to lending and lending standards are determined in a competitive loan market. I assume that banks are identical; thus, the market supply curve can be derived from the representative bank’s problem to choose lending standards to maximize expected profits.\textsuperscript{11} The assumption of identical banks is problematic in a general equilibrium setting in that, in equilibrium, either all banks default or none default. As this model is a partial equilibrium model I abstract away from such issues.

The question of what demand concept to use in determining the market equilibrium is a more nuanced issue. Since entrepreneurs have no existing wealth and enjoy limited liability, the willingness to pay for a loan is infinite. Thus if willingness to pay is used to determine equilibrium, as is standard in most supply and demand frameworks, the equilibrium return to lending is bid up to the point that all entrepreneurs receive loans. However, since the maximum expected return to lending to any given ability group is finite, not all entrepreneurs can credibly commit ex-ante to generating a return that is greater than or equal to the equilibrium market return. Given this, the relevant demand concept for determining the loan market equilibrium is not willingness to pay, but ability to pay. In the model, the maximum expected ability to pay is measured by the maximum expected return to lending to a given ability level.

\textsuperscript{10}If the cost of deposits incorporates a risk premium that depends on the risk of bank default, comparative statics involving the probability of bank default cannot be examined analytically but must be solved numerically.

\textsuperscript{11}Since loan size is fixed, total lending is determined solely by lending standards. Thus the banks problem is the same as choosing total lending to maximize expected bank profits.
This maximum expected ability to pay is upward sloping in $R^L$-ability space. When $R^L$ is relatively high, few borrowers have high enough abilities to generate an expected return of $R^L$. In this case, lending standards are relatively tight and total lending low. As $R^L$ falls, more borrowers are able to generate the necessary return to lending, standards loosens and total lending increases. In equilibrium, $R^L$ is determined by intersection of this maximum expected ability to pay curve and the bank’s loan supply curve.

The timing of events is as follows. After the announcement of the risk-free rate, but before the realization of idiosyncratic returns $\omega$ and aggregate return $R^K$, banks raise deposits, make loans to entrepreneurs, and state contingent loan repayments are stipulated. After entrepreneurs receive loans, all markets close and $\omega$ and $R^K$ are realized. Loan payments and bank liabilities, less defaults, are repaid. To aid the reader, the relevant notation used throughout the model is summarized in table 2.1.

### 2.1.2 Maximum Return to Lending and Borrower Ability to Pay

Access to a bank loan ultimately depends on a given borrower’s ability to generate a sufficiently high return to the bank, which depends on both the loan payment and the borrower’s ability level. The return to lending to an entrepreneur with ability $a$ is

$$ (r_1|a) = a\epsilon R^K \Gamma(\tilde{\omega}_a) \quad (2.3) $$

where $\Gamma(\tilde{\omega}_a) \in [0,1]$ is the share of gross project returns that accrues to the bank:
\[
\Gamma(\tilde{\omega}_a) = [1 - F(\tilde{\omega}_a)] \tilde{\omega}_a + \int_{0}^{\tilde{\omega}_a} \omega dF(\omega)
\] (2.4)

The choice of \( \tilde{\omega}_a \) determines how the gross project returns are split between the lender and borrower. The first term on the right-hand side of equation 2.4 represents the bank’s payoff when the borrower does not default and is instead able to repay their contracted payment. The second term represents the payoff when the borrower defaults and the project returns are seized by the bank.

The value of \( \tilde{\omega}_a \) that maximizes the return to lending to ability level \( a \) solves the following:

\[
1 - F(\tilde{\omega}_a) = 0
\] (2.5)

The solution to equation 2.5 implies that the maximizing value of \( \tilde{\omega}_a \) is equal to infinity for all ability levels and the maximizing value of \( \Gamma(\tilde{\omega}_a) \) is equal to one. Combining this with equation 2.3 implies that the maximum realized return to lending and maximum expected return to lending to ability type \( a \) are equal to, respectively,

\[
(r^*_l|a|\epsilon) = a\epsilon R^K
\] (2.6)

\[
E(r^*_l|a) = aR^K
\] (2.7)
The realized return to lending is maximized when the bank receives 100% of the gross project returns $a\epsilon K^r$; the maximum expected return to lending, equation 2.7, follows since the expected value of $\epsilon$ is equal to one.\footnote{That the bank can seize 100% of the project returns is a significant departure from the costly state verification literature originating with Townsend (1979) where only a fraction of a defaulting project can be successfully collected by the bank. Introducing verification costs to the model imply that the lender’s maximum share of project returns, net of verification costs, is strictly less than one; however, this does not change the key results of the model.}

Two results follow immediately from equations 2.6 and 2.7:

**Result 7.** The expected and realized return to lending is increasing in entrepreneurial ability.

**Result 8.** For a given ability level, the realized return to lending is increasing and linear in the aggregate shock.

*Proof.* Result 1 follows directly from the partial derivative of equations 2.6 and 2.7 with respect to $a$. Result 2 follows directly from the first and second partial derivates of equation 2.6 with respect to $\epsilon$.

In equilibrium, only ability levels that generate the bank an expected return of at least the equilibrium market return to lending, $R^L$, receive loans. Given Result 1, low ability entrepreneurs will be unable to generate a sufficiently high return for the bank and are rationed from the loan market. However, high ability entrepreneurs generate high enough returns and receive loans. In this sense lending standards take the form of a minimum ability requirement: given $R^L$ there exists an ability level cutoff $\bar{a}$ such that borrowers with abilities $a \geq \bar{a}$ receive a loan, and borrowers with abilities $a < \bar{a}$ are denied loans and rationed from the market. The threshold $\bar{a}$ is determined by the
The lowest ability group that is not rationed from the loan market, $\bar{a}$, can provide the bank with a maximum expected return just equal to $R^L$. However, since the maximum expected return to lending is decreasing in ability, all $a < \bar{a}$ cannot return at least $R^L$ in expectation, even when all of the project returns go to the lender.

Equation 2.8 is illustrated in figure 2.1. This function shows the maximum expected return to lending as a function of borrower ability. When taken together with the equilibrium return to lending, $R^L$, this curve shows the lowest ability level that can still commit to generate an expected return of at least $R^L$. Entrepreneurs with ability levels above or equal to the threshold can credibly commit to generating an expected return to lending that is at least as large as $R^L$. These borrowers receive loans. Conversely, entrepreneurs with ability levels below the threshold cannot commit to generating a return of $R^L$ and are rationed from the market. As $R^L$ rises fewer entrepreneurs are able to generate a large enough expected return and the threshold $\bar{a}$ rises.

This description of how lending standards are determined is closely related to how a good in a perfectly competitive market is distributed among consumers with varying willingness to pay. Given the market price, only consumers with a willingness to pay that is greater than or equal to the price of the good end up purchasing it. Those with a willingness to pay that is less than the price do not purchase the good. Similarly, in
the model only borrowers who are able to generate an expected return of at least the equilibrium market return receive a loan. The key difference here is that, unlike the analogy above, borrowers who do not receive a loan do not self-select out of the market but are instead rationed because of their inability to generate a high enough expected return. Thus, the relevant demand concept to use in the determination of equilibrium in the lending market is the maximum ability to pay, as measured by the maximum expected return to lending, equation 2.7.

While all non-rationed borrowers have an expected maximum return to lending of at least \( R_L \), an ex-post return of at least \( R_L \) is not guaranteed. Given result 2, that the realized maximum return of \( a\epsilon \bar{R}^K \) is linear and increasing in the shock \( \epsilon \), all borrowers are able to generate a return of \( R_L \) if \( \epsilon \) is at least 1 (its expected value). However, in the event of a negative aggregate shock not all borrowers will be able to generate a return of \( R_L \). For example, consider the return to lending to the threshold ability \( \tilde{a} \). Ex-ante, the maximum expected return to lending to the threshold ability is exactly equal to the equilibrium market return to lending. If the aggregate shock is less than expected, then the ex-post maximum return for the threshold ability group is less than expected and, therefore, falls below \( R_L \). The relationship between the shock and the ex-post maximum return to lending is summarized in table 2.2.

If the aggregate shock is negative, at the very least, lending to the threshold ability \( \tilde{a} \) fails to generate a realized return of \( R_L \). As the negative shock becomes worse and \( \epsilon \) falls, the number of ability levels that are unable to generate an ex-post return of \( R_L \) increases. I assume that in a such a situation \( \tilde{\omega}_a \) is set to maximize the ex-post return
to lending, yielding a return of \( a\epsilon R^K < R^L \). For borrowers with an ex-post maximum return of at least \( R^L \), \( \tilde{\omega}_a \) is set such that the realized return to the bank is equal to \( R^L \):

\[
a\epsilon R^K \Gamma(\tilde{\omega}_a) = R^L.
\]

Given a shock \( \epsilon \in (0,1) \), which borrowers will be able to generate an ex-post return of at least \( R^L \)? I define an ability level \( a^X \) such that, given a negative aggregate shock, the maximum ex-post return to lending to this group is exactly equal to \( R^L \); that is, \( a^X \epsilon R^K = R^L \) which, with equation 2.8, implies that \( a^X = \tilde{a}/\epsilon \). Since the maximum return to lending is increasing in entrepreneurial ability, borrowers with ability greater than or equal to \( a^X \) are able to generate an ex-post return of at least \( R^L \); however, groups with abilities \( a \in [\tilde{a}, a^X) \) are unable to do so. Loans to these ability levels yield \( a\epsilon R^K < R^L \). Seeing as \( a^X \) clearly increases as \( \epsilon \) falls, the number of loans that fail to return at least \( R^L \) rises as the aggregate shock gets worse.

Figure 2.2 illustrates how \( a^X \) is determined. Before the shock, borrowers with abilities \( a \geq \tilde{a} \) receive loans since the expected return to lending to these groups is at least as large as the market return to lending. However, a negative shock causes the ex-post maximum return schedule to shift down below the expected maximum return schedule. After the adverse shock, the maximum return to lending for all borrower is lower. Despite the negative shock, borrowers with \( a \geq a^X \) are still able to generate a realized return of at least \( R^L \). However, groups with abilities \( a \in [\tilde{a}, a^X) \) are unable to generate a return of at least \( R^L \). Thus, while the expected return to lending was at least \( R^L \) for all non-rationed borrowers, the realized return to lending is given by the bold, black curve in figure 2.2. Note that in the event of a positive shock, the ex-post maximum return
schedule will shift up above the expected maximum return schedule, and all borrowers are able to generate an ex-post return of at least $R^L$.

2.1.3 Bank Default and the Bank’s Problem

Bank loans are financed with a combination of deposits and bank capital. I assume that bank deposits are completely insured so that the relevant cost of deposits is equal to the exogenous policy rate $R$. The bank is also endowed with a fixed amount of capital $K$. Bank capital and deposits are used to make loans to entrepreneurs. The bank’s choice of which entrepreneurs to lend to – reflected by their chosen ability cutoff $\bar{a}$ – determines both the extent of bank leverage and the riskiness of the bank’s assets. I assume that after standards are chosen, the deposit market closes and no additional bank capital can be raised. This implies that banks cannot raise further funds when faced with insolvency.

Given that the distribution of entrepreneurs over ability levels is given by the cdf $H(a)$ with pdf $h(a)$, realized bank profits when the aggregate shock is greater than or equal to one is given by the following:

$$
(\pi_b|\epsilon \geq 1) = R^L[1 - H(\bar{a})] - R[1 - H(\bar{a}) - K]
$$

When the aggregate shock is positive (or equal to its expected value of one), each of the bank’s $1 - H(\bar{a})$ loans is capable of generating a return to the bank of $R^L$. This loan revenue less the cost of the bank’s $1 - H(\bar{a}) - K$ deposits gives the bank’s total profits.
When the aggregate shock is negative (less than one), realized bank profits are equal to

\[
(\pi_b|\epsilon < 1) = R^L \left[ 1 - H\left(\frac{\tilde{a}}{\epsilon}\right) \right] + \int_{\tilde{a}}^{\tilde{a}/\epsilon} a eR^K dH(a) - R \left[ 1 - H(\tilde{a}) - K \right]
\] (2.10)

The first term on the right-hand side of equation 2.10 represents loan revenue from borrowers with ability levels of at least \(a^X = \tilde{a}/\epsilon\); that is, it shows the return from borrowers that are able to generate an ex-post return of \(R^L\). The second term represents loan revenue from ability levels that, due to the shock, cannot generate an ex-post return of \(R^L\), but instead yield the bank \(aeR^K < R^L\). The third term represents the cost of deposits.

In the case of a negative shock, the realization of \(\epsilon\) has a positive impact on realized profits since the partial derivative of equation 2.10 is positive:

\[
\frac{\partial(\pi_b|\epsilon < 1)}{\partial \epsilon} = \left(\tilde{a}R^K - R^L\right) h\left(\frac{\tilde{a}}{\epsilon}\right) \left( - \frac{\tilde{a}}{\epsilon^2} \right) + \int_{\tilde{a}}^{\tilde{a}/\epsilon} a eR^K dH(a) = \int_{\tilde{a}}^{\tilde{a}/\epsilon} a eR^K dH(a) > 0
\] (2.11)

where the second equality follows from the definition of \(\tilde{a}\), equation 2.8.

Given that bad state profits are increasing in \(\epsilon\), realized bank profits may be positive, negative or zero depending on the severity of the negative aggregate shock. I define a threshold shock level \(\hat{\epsilon}\) such that realized profits when the aggregate shock is \(\hat{\epsilon}\) are exactly equal to zero. Given this, the shock threshold is implicitly defined by the following condition:

\[13\] Since I am not examining the real effects of bank default in a general equilibrium setting, I have
\[(\pi_b|\epsilon = \hat{\epsilon}) = R^L \left[1 - H\left(\frac{\tilde{a}}{\hat{\epsilon}}\right)\right] + \int_{\tilde{a}}^{a/\hat{\epsilon}} a\hat{\epsilon}R^K dH(a) - R[1 - H(\tilde{a}) - K] = 0 \quad (2.12)\]

The \(\hat{\epsilon}\) implied by equation 2.12 pins down the aggregate default threshold for the bank. Since bad state profits are increasing in \(\epsilon\) and the bank breaks even when \(\epsilon = \hat{\epsilon}\), if the shock falls below \(\hat{\epsilon}\) the return from the bank’s loan portfolio is not large enough to repay their deposit liabilities, and the bank defaults. For \(\epsilon > \hat{\epsilon}\) the bank is able to repay their deposits and remains solvent.

Realized bank profits, as a function of the aggregate shock, are illustrated in figure 2.3. When the shock is positive, bank profits are given by equation 2.9 and are independent of the size of the shock. Conversely, if the shock falls below the default threshold \(\hat{\epsilon}\) the bank defaults and, due to limited liability, earns zero profits. When the shock falls in the interval \((\hat{\epsilon}, 1)\) bank profits are positive and increasing in the shock.

Of key interest in this paper is how the probability of bank default changes with lending standards. This ultimately depends on the effect of \(\tilde{a}\) on the default threshold \(\hat{\epsilon}\): if \(\hat{\epsilon}\) rises when \(\tilde{a}\) falls then, all else equal, the probability of bank default increases when lending standards loosen, and vice versa. However, if \(\hat{\epsilon}\) is increasing in \(\tilde{a}\) then the probability of bank default falls when standards loosen.

**Result 9.** Looser lending standards result in a higher probability of bank default. Tighter lending standards lower the risk of bank default.

assumed that banks are identical which implies that either all banks default or all banks survive. Given this is a partial equilibrium model and I am not interested in the impact of bank default (only the probability) I abstract away from such issues.
Proof. To eke out the relationship between default and standards, I take the derivative of 2.12 with respect to $\tilde{a}$, keeping in mind that $\hat{\epsilon}$ is an implicit function of $\tilde{a}$:

$$0 = \left( R^L - \tilde{a}\tilde{R}^K \right) h\left( \frac{\tilde{a}}{\hat{\epsilon}} \right) \left( \frac{1}{\hat{\epsilon}} - \frac{\tilde{a}}{\hat{\epsilon}^2} \frac{\partial \hat{\epsilon}}{\partial \tilde{a}} \right) + (R - \tilde{a}\hat{\epsilon}\tilde{R}^K) h(\tilde{a}) + \int_{\tilde{a}}^{\tilde{a}/\hat{\epsilon}} \tilde{a} \hat{\epsilon} \left( a\hat{R}^K \frac{\partial \hat{\epsilon}}{\partial \tilde{a}} \right) dH(a)$$

which, using equation 2.8, becomes

$$0 = \left( R - \tilde{a}\hat{\epsilon}\tilde{R}^K \right) h(\tilde{a}) + \int_{\tilde{a}}^{\tilde{a}/\hat{\epsilon}} \tilde{a} \hat{\epsilon} \left( a\hat{R}^K \frac{\partial \hat{\epsilon}}{\partial \tilde{a}} \right) dH(a) \quad (2.13)$$

The first bracketed term on the right-hand side of equation 2.13 represents the net loss on loans to the lowest ability borrowers, conditional upon realized bank profits being equal to zero. Since the return to lending to these borrowers is always the lowest return amongst the different (non-rationed) ability levels and this return must be less than the cost of deposits if losses are large enough to push profits to zero, the first-term in 2.13 must be positive. Given this, $\frac{\partial \hat{\epsilon}}{\partial \tilde{a}}$ is necessarily negative: when lending standards are eased ($\tilde{a}$ falls) the probability of bank insolvency rises ($\hat{\epsilon}$ rises).

Intuitively, given $\hat{\epsilon}$, loan losses to the threshold ability group increase as standards loosen and minimum ability $\tilde{a}$ falls. This implies that the bank must earn a higher return on loans that do not yield the full return $R^L$ and, therefore, cannot withstand as large of a negative shock. Thus, the probability of bank default rises when lending standards loosen and falls when standards tighten.
Although bank capital $K$ is fixed, it is also useful to see how the probability of bank default is affected by the level of bank capital.

**Result 10.** *The probability of bank default falls when $K$ rises. The probability of bank default rises when $K$ falls.*

*Proof.* Differentiating equation 2.12 with respect to $K$, keeping in mind that $\hat{\epsilon}$ is an implicit function of $K$, yields the following

$$0 = \left( R^L - aR^K \right) h\left( \frac{\tilde{a}}{\bar{e}} \right) \left( \frac{1}{\bar{e}} \frac{\partial \tilde{a}}{\partial K} - \frac{\tilde{a}}{\bar{e}^2} \frac{\partial \hat{\epsilon}}{\partial K} \right) + R + \int_{\tilde{a}}^{\bar{a}} aR^K \frac{\partial \hat{\epsilon}}{\partial K} dH(a)$$

Using equation 2.8, this becomes

$$0 = R + \int_{\tilde{a}}^{\bar{a}} aR^K \frac{\partial \hat{\epsilon}}{\partial K} dH(a) \quad (2.14)$$

which clearly implies that $\frac{\partial \hat{\epsilon}}{\partial K} < 0$. All else equal, higher levels of bank capital lower the default threshold $\hat{\epsilon}$ and lower the probability of bank default.

Intuitively, when a bank has more capital it requires fewer deposits to finance a given amount of loans. The cost-savings associated with this are captured in the first term on the right-hand side of equation 2.14. These cost-savings imply that the bank can earn lower total loan revenues and still break even which results in a lower default threshold $\hat{\epsilon}$. 

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The Bank’s Problem

The bank chooses lending standards \( \bar{a} \) (and by extension, total lending) to maximize expected profits, which take the following form:

\[
E(\pi_b) = \int_1^\infty \left[ R^L [1 - H(\bar{a})] - R [1 - H(\bar{a}) - K] \right] d\Phi(\epsilon) + \cdots \\
\cdots + \int_1^{\hat{\epsilon}} \left[ R^L \left[ 1 - H \left( \frac{\bar{a}}{\epsilon} \right) \right] + \int_{\bar{a}}^{\bar{a}/\epsilon} aR^K dH(a) - R [1 - H(\bar{a}) - K] \right] d\Phi(\epsilon)
\]  

(2.15)

where \( \Phi(\epsilon) \) is the cdf for \( \epsilon \) with associated pdf \( \phi(\epsilon) \). The first line in equation 2.15 represents expected bank profits conditional on a positive aggregate shock. The second line represents expected bank profits conditional on a negative, but not default inducing, aggregate shock. The bank’s choice of \( \bar{a} \) affects expected profits both through the size of the bank’s balance sheet as well as the default threshold \( \hat{\epsilon} \).

The bank’s problem is to choose \( \bar{a} \) to maximize 2.15 which yields the following first order condition:

\[
0 = -\int_1^\infty (R^L - R) h(\bar{a}) d\Phi(\epsilon) + \cdots \\
\cdots + \int_1^{\hat{\epsilon}} \left[ (\bar{a}R^K - R^L) h \left( \frac{\bar{a}}{\epsilon} \right) \frac{1}{\epsilon} - (\bar{a}R^K - R) h(\bar{a}) \right] d\Phi(\epsilon) - \cdots \\
\cdots - \left[ R^L \left[ 1 - H \left( \frac{\bar{a}}{\epsilon} \right) \right] + \int_{\bar{a}}^{\bar{a}/\epsilon} aR^K dH(a) - R [1 - H(\bar{a}) - K] \right] \phi(\hat{\epsilon}) \frac{\partial \hat{\epsilon}}{\partial \bar{a}}
\]  

(2.16)

Notice that the large bracketed term in the last line of equation 2.16 is equal to bank profits conditional on the shock being equal to \( \hat{\epsilon} \) and, by equation 2.12, is equal to zero.
Using this, and equation 2.8, the first order condition simplifies to

$$0 = -[1 - \Phi(1)]R^L - \int_\epsilon^1 \tilde{a}\epsilon R^K d\Phi(\epsilon) + [1 - \Phi(\tilde{\epsilon})]R$$ (2.17)

The first two terms on the right-hand side of equation 2.17 represents the marginal cost of tighter standards. Namely, higher standards reduce total loan revenue both in the event of a positive aggregate shock (first term) and a negative, but not default-inducing, shock (second term). The third term in 2.17 represents the marginal benefit of tighter standards: by tightening standards, banks need fewer deposits which increases expected profits. If $R^L < R$ the bank makes no loans; however, if $R^L > R$ optimal lending standards are given by equation 2.17, where the marginal cost and marginal benefit of tighter standards are equal.

The second order condition necessary for equation 2.17 to define a maximum is

$$-R\phi(\tilde{\epsilon}) \frac{\partial \tilde{\epsilon}}{\partial a} + \tilde{a}\epsilon R^K \phi(\tilde{\epsilon}) \frac{\partial \tilde{\epsilon}}{\partial a} - \int_\epsilon^1 \epsilon R^K d\Phi(\epsilon) < 0$$ (2.18)

Given that $\frac{\partial \tilde{\epsilon}}{\partial a} < 0$, the second and third terms in 2.18 are negative and reflect the fact that the marginal cost of tighter standards is increasing in the choice of standards. On the one hand, a higher $\tilde{a}$ directly increases expected bad shock state loan revenue that the bank foregoes when standards are higher (third term). Additionally, tighter standards indirectly increase foregone expected loan revenue earned in the bad shock state by virtue of lowering the bank default threshold (second term).
However, the marginal benefit of a higher $\tilde{a}$ is increasing in $\tilde{a}$ and therefore positive. This is embodied in the first term in equation 2.18. Because the probability of default falls when standards rise, the expected deposit savings from tighter standards is increasing in the level of standards. Given that both the marginal cost and marginal benefit of higher standards are increasing in $\tilde{a}$, is not readily apparent that the second order condition holds in general. However, I assume that 2.18 is satisfied and verify in the numerical simulations that it does indeed hold, as well as examine the sensitivity of the result to other parameters of the model.

Before proceeding to the analysis of equilibrium in the loan market, it is useful to first examine how the bank’s optimal choice of $\tilde{a}$ varies with other key determinants.

**Result 11.** Bank lending standards loosen when $R^L$ rises and tighten when $R^L$ falls.

**Proof.** Differentiating the bank’s first order condition with respect to $R^L$ yields the following condition:

$$0 = -[1 - \Phi(1)] + \frac{\partial \tilde{a}}{\partial R^L} \left[ - (R - \tilde{a}\epsilon R^K) \phi(\epsilon) \frac{\partial \epsilon}{\partial \tilde{a}} - \int \epsilon R^K d\Phi(\epsilon) \right]$$

(2.19)

Notice that the bracketed term on the right-hand side of equation 2.19 is the second order condition for the bank’s maximization problem and is negative by assumption. Since the first term on the right-hand side of 2.19 is also negative, it must be that $\frac{\partial \tilde{a}}{\partial R^L} < 0$. This implies that lending standards loosen when the equilibrium market lending rate increases: when $R^L$ rises the marginal cost of tighter standards also rises.
as the bank is foregoing a higher amount of loan revenue in the good shock state. As a result, bank’s find it optimal to loosen standards and lower $\tilde{a}$ when the equilibrium market return to lending rises. Conversely, lending standards tighten when $R^L$ falls.

Optimal lending standards are also affected by the cost of deposits, which is assumed to be equal to the policy rate $R$.

**Result 12.** Bank lending standards loosen when $R$ falls and tighten when $R$ rises.

**Proof.** Differentiating 2.17 with respect to $R$ yields

$$0 = [1 - \Phi(\hat{\epsilon})] + \frac{\partial \tilde{a}}{\partial R} \left[ - (R - \tilde{a}\hat{\epsilon}R^K)\phi(\hat{\epsilon}) \frac{\partial \hat{\epsilon}}{\partial \tilde{a}} - \int_{\hat{\epsilon}}^{1} \hat{\epsilon}R^K d\Phi(\epsilon_K) \right]$$  \hspace{1cm} (2.20)

Given that the bracketed term on the right-hand side of equation 2.23 is negative by the second order condition and the first term is positive, it follows that $\frac{\partial \tilde{a}}{\partial R}$ is positive: when the policy rate – and, therefore, the cost of deposits – falls, the marginal benefit of tighter standards also falls as the bank now expects to save less by keeping $\tilde{a}$ relatively high. The fall in the marginal benefit of standards prompts to bank to loosen standards in response and lower their optimal $\tilde{a}$.

**Result 13.** Lending standards loosen when bank capital falls and tighten when bank capital rises.

**Proof.** Differentiating the bank’s first order condition with respect to $K$ yields the following condition:
\[ 0 = - (R - \hat{a}\epsilon R^K)\phi(\hat{\epsilon}) \frac{\partial \hat{\epsilon}}{\partial K} + \frac{\partial \hat{a}}{\partial K} \left[ - (R - \hat{a}\epsilon R^K)\phi(\hat{\epsilon}) \frac{\partial \hat{\epsilon}}{\partial a} - \int_{\hat{\epsilon}}^{1} \epsilon R^K d\Phi(\epsilon_K) \right] \quad (2.21) \]

Given equations 2.14 and 2.18, the bracketed term on the right-hand side of equation 2.21 is negative and the first term positive. This implies that \( \frac{\partial \hat{a}}{\partial K} \) is positive: lending standards tighten when banks have more capital, and loosen when bank capital is lower.

Intuitively, a higher level of bank capital, all else equal, reduces the default shock threshold and increases the marginal benefit of tighter standards at any given level of \( \hat{a} \) by increasing the probability that deposits will be repaid. This increase in the benefit of standards prompts the bank to tightening lending standards and increase \( \hat{a} \).

### 2.1.4 Equilibrium and the Effect of Monetary Policy

Equilibrium in the loan market is ultimately determined by the bank’s optimal lending standards, equation 2.17, and the ability cut-off level, equation 2.8. Combining these two equations pins down equilibrium lending standards \( \hat{a}^* \), which is given by the following expression:

\[ [1 - \Phi(1)]\hat{a}^*R^K - \int_{\hat{\epsilon}}^{1} \hat{a}^*\epsilon R^K d\Phi(\epsilon) = (1 - \Phi(\hat{\epsilon}^*))R \quad (2.22) \]

Equation 2.22 is essentially the bank’s optimality condition for \( \hat{a} \) with the exception that the market return to lending \( R^L \) has been replaced using the ability cut-off threshold 2.8. Figure 2.4 illustrates the determination of equilibrium. Since the bank’s optimal \( \hat{a} \)
is decreasing in $R^L$ and the ability cut-off threshold is increasing in $R^L$, $\tilde{a}^*$ is the unique equilibrium in the loan market.

Of key interest in this paper is the effect that monetary policy has on lending standards. While a reduction in $R$ has no effect on the borrower maximum expected ability to pay, it affect the bank’s optimal choice of $\tilde{a}$: when $R$ falls banks ease their lending standards which, in turn, lower equilibrium standards. This can be seen by differentiating equation 2.22 with respect to $R$:

$$\frac{\partial \tilde{a}^*}{\partial R} \left[ R^K - \left\{ - (R - \tilde{a}^* \tilde{\epsilon}^* R^K) \phi(\tilde{\epsilon}^*) \frac{\partial \tilde{\epsilon}^*}{\partial \tilde{a}^*} - \int_{\tilde{\epsilon}^*}^{1} \tilde{\epsilon} R^K d\Phi(\tilde{\epsilon} R) \right\} \right] = 1 - \Phi(\tilde{\epsilon}^*) \quad (2.23)$$

Notice that the term in curly brackets on the left-hand side of equation 2.23 is the second order condition for the bank’s maximization problem and therefore negative which implies that the entire term in the square brackets is positive. Since $1 - \Phi(\tilde{\epsilon} R)$ must be positive, $\frac{\partial \tilde{a}^*}{\partial R}$ must also be positive: when the policy rate falls, equilibrium lending standards loosen; when $R$ rises equilibrium standards tighten.

The effect of a fall in $R$ is illustrated in figure 2.5. A reduction in $R$ lowers the cost of deposits. This causes the bank to lower their optimal lending standards at all levels of $R^L$ leading to the leftward shift of the bank’s optimal $\tilde{a}$ curve and a decrease in the equilibrium $R^L$ and $\tilde{a}^*$.

The last key item of interest is the relationship between the probability of bank default and the policy rate. In equilibrium, the default shock threshold $\tilde{\epsilon}^*$ is given by the following equation:
\[
\tilde{a}^* R^K \left[ 1 - H\left(\frac{\tilde{a}^*}{\hat{\epsilon}^*}\right) \right] + \int_{\tilde{a}^*}^{\hat{\epsilon}^*/\hat{\epsilon}^*} a\tilde{a}^* R^K dH(a) - R[1 - H(\tilde{a}^*) - K] = 0 \quad (2.24)
\]

Equation 2.24 are bank profits in the event of a negative shock (equation 2.12) where the \(R^L\) has been replaced by the equilibrium market return to lending (the maximum expected return to lending to the equilibrium threshold ability group \(\tilde{a}^*\)). Setting this equal to zero yields the equilibrium bank default threshold \(\hat{\epsilon}^*\).

Ultimately, the relationship between the policy rate and risk of bank default depends on the net effect of a change in \(R\) on equilibrium bank profits in the face of a negative aggregate shock. For a given shock value, if a higher \(R\) results in higher bank profits the bank is less likely to default; however, if bad state bank profits fall when \(R\) rises, the bank is more likely to default. Taking the total derivative of equation 2.24 with respect to \(R\) which yields, after some rearranging, the following expression:

\[
\int_{\tilde{a}^*}^{\hat{\epsilon}^*/\hat{\epsilon}^*} a\tilde{a}^* R^K \frac{d\hat{\epsilon}^*}{dR} dH(a) = \cdots
\]

\[
\cdots = [1 - H(\tilde{a}^*) - K] - \frac{d\tilde{a}^*}{dR} \left[ [1 - H(\tilde{a}^* / \hat{\epsilon}^*)]R^K + (R - \tilde{a}^* \hat{\epsilon}^* R^K) h(\tilde{a}^*) \right] \quad (2.25)
\]

The sign of \(\hat{\epsilon}^*\) is the same as the sign of the right-hand side (second line) of equation 2.25. If this partial is positive, then the probability of bank default increases when the policy rate increases, and vice versa. Conversely, if this partial is negative, the probability of bank default decreases when the policy rate increases, and rises when the policy rate falls.
The right-hand side of equation 2.25 gives a measure of the net effect on bad state bank profits from a change in the policy rate. Specifically, it is the negative of the marginal effect of $R$ on bad state profits: a positive value implies profits fall when $R$ rises, while a negative value implies profits rise when $R$ rises. Taking a closer look at the second line of equation 2.25, a change in $R$ has two separate effects on the bank’s bad state profits and, in effect, on the probability of bank default. The relationship between the policy rate and the probability of bank default can be decomposed into a “deposits” and “standards” effect. The first term in the second line of 2.25 is the deposits effect and captures the negative effect on bank’s profits of a higher $R$. Since the cost of deposits is equal to the policy rate, as $R$ rises the bank’s total cost for a given amount of deposits also rises and bad state profits fall. Thus to avoid default, the bank must earn more revenue from loans, implying that the aggregate shock threshold $\hat{\epsilon}^*$ must rise and, all else equal, the probability of bank default increases.

The second term in the second line of equation 2.25 represents standards effect of a change in $R$ on the probability of bank default. When the policy rate increases, equilibrium lending standards tighten which increases bank profits in the event of a bad aggregate shock. The second bracketed term on the right-hand side of 2.25 measures the change in bank profits induced by a change in lending standards, given a bad aggregate shock. Two factors drive the relationship between bad shock profits and lending standards. First, as standards tighten the equilibrium market return to lending increases. This increases revenue from borrowers who are able to fully repay $R^L$ ex-post. Second, tighter standards reduce loan losses in the event of a bad shock by virtue of
increasing $\hat{a}^{*}\hat{c}^{*}R^{K}$, and thereby also contribute to higher profits.

This standards effect, in isolation, will drive $\hat{c}^{*}$ and the risk of bank default down as $R$ increases. Since tighter standards, induced by a higher policy rate, increase bank profits in the event of a bad shock, the bank can withstand a worse aggregate shock before profits become negative.

The sign of equation 2.25 depends on which effect dominates. If the deposits effect is large relative to the standards effect, the probability of bank default rises with $R$, and vice versa. Unfortunately, it cannot be determined analytically which effect will dominate. However, the model can be solved numerically and the sign of this key partial derivative determined.

### 2.2 Calibration

Table 2.3 summarizes the model parameters that require calibration. In all there are two distributions (plus associated parameters) and two remaining parameters to select.

The distributions in the model include the distribution for the shock to the aggregate return to capital, $\Phi(\epsilon_{K})$ and the distribution of abilities across entrepreneurs, $H(a)$.

The distribution of abilities across entrepreneurs is assumed to be log-normally distributed on interval $(0, \infty)$ with an expected value of one. The choice of variance is slightly complicated by the fact that entrepreneurial ability is not directly observable in
reality; however, I look to the distribution of FICO scores, a widely used credit scoring system, to inform the choice of $\sigma_a^2$. Credit scores such as the FICO are commonly used by lenders to assess the creditworthiness of potential borrowers. Since entrepreneurial ability determines creditworthiness in the model, FICO scores seem like a fitting proxy for calibrating the distribution of abilities. The average FICO score for US consumer has been reported by Fair Isaac on a bi-annual basis since October 2005; the average FICO score (data ending with April 2015) is 689.5 with a standard deviation of 100 points. Using these empirical moments, I calculate $\sigma_a^2$ by squaring the reported standard deviation and normalizing the resulting number by the mean score. This results in a variance of 14.5 for the distribution of entrepreneurial abilities.

I assume that $\epsilon$ is lognormally distributed on interval $(0, \infty)$ with an expected value of one. The shock’s variance, $\sigma_\epsilon^2$, is jointly chosen with capital, $K$, to match two empirical observations from financial market data: the average bank failure rate and the average bank capital ratio. For the model’s bank default measure, I calculate the ratio of the average probability of bank default to the average policy rate consider in simulations. Similarly, for the second measure I calculate the ratio of the average capital ratio to the average policy rate for all policy rates considered.

The empirical counterparts of these two measures are drawn from two sources. Data on bank capital ratios and the policy rate are drawn from FRED for 1993-2007. Specifically, I use the annual average effective federal funds rate as a proxy for the policy rate and the annual average aggregate bank equity to assets ratio for bank capital ratios. Data on the rate of bank failure is drawn from the FDIC from their historical data on bank
failures and assistance for 1993-2007. I restrict the sample to commercial banks failures, at the exclusion of saving and loan institutions and instances of assistance.

The choice of time frame is an important decision since the empirical average default rate is very sensitive to the time frame used. In the past 30 years the U.S. has experienced two episodes of unusual of financial market distress: the savings and loans crisis of the late 80’s and the financial crisis of 2008. Unsurprisingly, including these periods in our time frame greatly increases the average failure rate. Since these two crises are extreme events and not indicative of “normal” business conditions, it would be inappropriate to include the crises in our sample. During a crisis general equilibrium effects can have significant effects on outcome as the fall in aggregate returns associated with crisis periods have significant multiplier effects via aggregate demand. These forces, however, are not well captured by the partial equilibrium framework of the model. Guided by this reasoning, I restrict my time period from to 1993-2007 – essentially, the period after the end of the S&L crisis but before the onset of the recent financial crisis. As a robustness check, I recalibrate the model using the average capital ratio, fed funds rate, and bank failure rate from 1993-2014. Doing so increases $\sigma_e^2$ to 0.1024 and increases $K$ to 0.0186; however, this alternative calibration does not change the base line results.

Given the data sources and time frame, the average annual failure rate among US commercial banks was 0.089% and the average capital ratio 8.96%. Further, given the average annual effective federal funds rate of 4.04%, the empirical counterparts I attempt to match via my choice of $K$ and $\sigma_K^2$ are an average default rate of 0.0898% and an average capital ratio of 9.04%. Given the other parameters of the model, this
corresponds to $K = 0.0176$ and $\sigma_K^2 = 0.0614$. I also have verified that, given the two empirical targets I am trying to match, this choice of $K$ and $\sigma_K^2$ are unique in that there is no other combination of these two parameters that produce the desired average failure rate and average capital ratio.

Lastly, I set the long-run aggregate return to capital equal to the average annual return on the S&P 500 from 1950-2013, which corresponds to a $\bar{R}^K$ of 1.10, and I let $R$ vary from 1.00 to 1.10 while investigating the relationship between the policy rate and the model’s equilibrium probability of bank default. After solving the model numerically, I verify that the second order conditions hold and are reasonably robust to model parameters. The solution approach is detailed in Appendix A.

### 2.3 Numerical Results

The relationship between the model implied probability of default and policy rate under the baseline calibration is depicted in figure 2.6. As the figure clearly shows, a decrease in the policy rate results in a higher probability of bank default under the baseline calibration. As $R$ falls the negative effect of looser lending standards outweighs the positive effect associated with cheaper deposits, and the probability of bank default rises when the policy rate falls.

The baseline result – expansionary monetary policy increases the risk of bank default – is in line with the empirical literature on the risk-taking channel of monetary policy. Broadly speaking, this literature supports the idea that 1) looser monetary policy leads
to a higher probability of bank default, and 2) looser monetary policy results in looser lending standards.\textsuperscript{14} The baseline results mirror these empirical findings.

While the baseline results indicate that the probability of bank default falls when the policy rate rises, this relationship is sensitive to some of the model parameters, in particular, the amount of bank capital $K$ and the variance of the distribution of entrepreneurial abilities $\sigma_a^2$. Relative to the baseline level of bank capital, at low levels of $K$ the probability of bank default is increasing in $R$, and at intermediate levels of $K$ the probability of bank default increases initially but then begins to fall as $R$ rises.\textsuperscript{15}

This sensitivity is illustrated in figure 2.7.

Similarly, when $\sigma_a^2$ is low relative to the baseline level the probability of bank default is increasing in $R$, and at intermediate levels the probability of bank default is concave with respect to $R$. This sensitivity is illustrate in figure 2.8.

Lowering $K$ and $\sigma_a^2$ also has the effect of increasing the average probability of bank default and lowering the average bank capital ratio. For the alternative levels of bank capital considered in figure 2.7, if the remaining parameters are adjusted such that the resulting average probability of bank default and average bank capital ratio are equal to the targets discussed in the calibration section, the baseline result persists: the probability of bank default strictly falls when the policy rate rises.\textsuperscript{16} A similar pattern holds for $\sigma_a^2$: when the other parameters are adjusted to achieve the target average default probability and average capital ratio, the probability of bank default is strictly

\textsuperscript{14}See Maddaloni and Peydró (2011) and Altunbas et al. (2014) for details.

\textsuperscript{15}Increasing $K$ does not alter the baseline result. Of course, if $K$ is high enough, the probability of bank default is zero and independent of the policy rate.

\textsuperscript{16}The target average probability of bank default is 0.0089821\% and average capital ratio of 9.04\%.
decreasing with respect to $R$.

This suggests that the sensitivity displayed in figures 2.7 and 2.8 is driven by differences in equilibrium capital ratios. When banks are weakly capitalized and have low capital ratios, a lower policy rate tends to decrease the probability of default. However, if banks have high capital ratios, lower policy rates tend to increase the risk of default.

The relationship between the probability of bank default and the policy rate ultimately depends on the effect of $R$ on bank profits when there is a negative shock; this, in turn, depends on the relative sizes of the deposits and standards effects. The deposit effect captures the negative effect of the policy rate on bad state profits: a higher $R$ increases the cost of deposits which lowers bank profits and makes the bank more likely to default.

However, the standards effect, which represents the influence on bad state profits that results from tighter lending standards, works in the opposite direction: a higher $R$ results in tighter equilibrium lending standards which increases bad state profits and pushes the risk of default down.

\[
\text{Deposits effect: } D_X = [1 - H(\tilde{a}^*) - K] \quad (2.26)
\]

\[
\text{Standards effect: } S_X = \frac{\partial \tilde{a}^*}{\partial R} \left[ [1 - H(\tilde{a}^*/\tilde{\epsilon}^*')] \tilde{R}^K + (R - \tilde{a}^*\tilde{\epsilon}^*\tilde{R}^K)h(\tilde{a}^*) \right] \quad (2.27)
\]

\[
\frac{d\tilde{\epsilon}^*}{dR} \propto D_X - S_X \quad (2.28)
\]

The deposits effect is equal to the amount of bank deposits. When $R$ rises, each dollar of deposits is now more expensive and bad state bank profits fall. The standards effect shows the increase in profits that result from higher standards: bad state profits rise
due to both a higher equilibrium market return to lending (first term inside brackets in 2.27) and lower loan losses from the threshold ability group (second term inside brackets in 2.27). The relationship between the risk of bank default depends on the relative size of these two effects: if the standards effect dominates the deposits effect, the default threshold $\hat{\epsilon}^*$ falls, as does the probability of bank default, when the policy rate rises. However, if the deposits effect is larger than the standards effect, $\hat{\epsilon}^*$ and the probability of bank default increase with the policy rate.

In the baseline results, the standards effect dominates the deposits effect, and the risk of bank default falls as $R$ rises ($\frac{d\hat{\epsilon}^*}{dR} < 0$). However, as the results in figure 2.7 suggest, when capital or the variance of entrepreneurial abilities are sufficiently low, the deposits effect dominates the standards effect, and the probability of bank default increases when $R$ rises ($\frac{d\hat{\epsilon}^*}{dR} > 0$).

In regards to $K$, lowering the amount of bank capital increases the equilibrium quantity of deposits and magnifies the deposit effect. Since deposits are equal to total lending less capital, a lower $K$ directly increases deposits, given the amount of total lending. Additionally, since a lower amount of bank capital results in looser standards and higher lending, banks with less capital need even more deposits. Thus, as $K$ falls, bank leverage (loans to capital) increases resulting in a higher level of deposits and a larger direct effect.\(^{17}\)

Since bank capital affects the equilibrium level of standards, changing $K$ also influences

---

\(^{17}\)In reality, banks do not use 100% of their capital to finance risky loans; some capital may be used for riskless assets, like government bonds or reserves, or is used to finance specific loss buffers like loan loss provisions. Despite this, to the extent that capital is used to finance risky loans, a change in capital directly affects deposits.
the standards effect. Looking at equation 2.27, the standards effect has three components: the sensitivity of equilibrium lending standards to the policy rate \( \frac{\partial \tilde{a}_*}{\partial R} \), the number of loans that can repay their loan in full (first term inside brackets), and loan losses from the threshold group \( \tilde{a}_* \) (second term inside brackets). While less capital makes standards less sensitive to \( R \) and lowers loan losses, it does result in a higher amount of loans that repay their loan in full. Taken together, the net effect of lower \( K \) is to increase the standards effect: less bank capital increases the impact of tighter standards on bank profits in the event of a negative shock. Put another way, bank profits are more sensitive to changes in lending standards when capital is lower.

Intuitively, having less bank capital implies that to remain solvent the bank must have a larger number of loans that are able to generate a return to the bank that is at least as large as the equilibrium market return. Therefore when the equilibrium market return to lending increases as a result of tighter lending standards, the impact on bank profits is larger since the bank has more loans that must return this rate compared to a bank with higher capital levels.

While both the deposits and standards effects increase as capital falls, the deposit effect increases much more relative to the standards effect. This can been seen in figure 2.9 which illustrates the average standards effect and deposits effect (averaged across policy rates) for varying levels of bank capital. At low levels of bank capital, the deposits effect is large relative to the standards effect and the probability of bank default is increasing with the policy rate. As the amount of bank capital increases, both effects fall; however, the deposits effect falls much faster than standards effect, and at larger capital levels
the probability of bank default becomes decreasing with the policy rate.

As capital falls, the deposits effect increases faster than the standards effect because deposits are very sensitive to changes in bank capital relative to the impact of standards on bank profits. For example, if bank capital falls by $1, deposits rise by more than $1. On the one hand, a $1 reduction in capital means that deposits must rise by $1 to continue financing a given amount of loans. Additionally, since less capital implies more lending, deposits must increase even further to finance these additional loans. Thus, as capital falls the deposits effect of \( R \) on the risk of bank default increases relatively quickly.

Conversely, as \( K \) falls the standards effect increases relatively slowly. On the one hand, lower capital implies a larger number of loans must repay the full equilibrium market return to lending for the bank to still break-even; thus, when the market return to lending rises due to higher standards, the effect on profits is amplified which causes the standards effect to rise. However, less bank capital reduces the amount of loan losses from the threshold ability group \( \tilde{a}^* \): banks with lower levels of capital cannot withstand as large of loans losses from this threshold group and still break-even compared to a bank with more capital. The effect of loan losses, in isolation, causes the standards effect to fall. Figure ?? illustrates the contribution of loan losses and the number of fulling repaying loans to the average standards effect for varying levels of bank capital. While the net effect of less capital is to increase the standards effect, the effect of loan losses dampens the rise in the standards effect.
In regards to $\sigma_a^2$, changing the variance of entrepreneurial abilities also affects equilibrium lending standards and the relative magnitudes of the deposits and standards effects. On the one hand, a lower variance increases loans and therefore deposits which increases the magnitude of the deposits effect. However, by virtue of lowering standards, a lower variance also increases the standards effect. Changing standards have a larger effect on bad state profits when $\sigma_a^2$ is lower by virtue of increasing the number of loans that repay the full equilibrium market return to lending and larger loan losses from the threshold group. However, the deposits effect increases much faster than the standards effect as the variance of entrepreneurial abilities falls. At low values of $\sigma_a^2$ the deposits effect is large relative to the standards effect, and the probability of bank default is increasing in the policy rate. As $\sigma_a^2$ increases, the deposits effect falls quickly relative to the standards effect, and at higher variances the standards effect dominates the deposits effect and the probability of default falls when $R$ rises.

Since the primary effect of changing $K$ and $\sigma_a^2$ is to change equilibrium lending standards and, by extention, equilibrium bank capital ratios and the magnitude of the deposits effect, the previous results suggest that the extent of bank capitalization plays an integral role in the relationship between the policy rate and the probability of bank default. When $K$ and $\sigma_a^2$ fall, standards loosen, capital ratios fall, and the deposits effect is relatively high. As standards tighten in response to higher $K$ and $\sigma_a^2$, capital ratios rise, and the quantity of deposits and the deposits effect falls rapidly. However, the impact of standards on bad state bad profits (the standards effect) is relatively insensitive to changes in standards and capital ratios induced by different model parameters. While
the standards effect does fall in magnitude as standards tighten, it falls slowly relative to the change in the deposits effect. This result is displayed in figure 2.11: it shows the standards effect and deposits effect for varying levels of standards $\tilde{a}$. The deposits effect dominates when standards are very loose and capital ratios low; in this scenario, lower policy rates lead to lower probabilities of bank default. When standards are tighter and capital ratios high the standards effect dominates, and lower policy rates lead to higher bank default risk.\footnote{This sensitivity is similar to the result from DellAriccia et al. (2011) when capital structure is fixed. In their paper, a reduction in the policy rate increases risk-taking for highly capitalized banks but reduces risk-taking in banks that are less capitalized.}

The overall relationship between the risk of bank default and the policy rate as a function of bank capital is shown in figure 2.12 which plots the correlation between the probability of bank default and $R$ for various levels of bank capital. When $K$ and bank capital ratios are low, correlation is positive: a higher policy rate is associated with higher default risk. However, when $K$ and bank capital ratios are high, correlation is negative, and a higher $R$ leads to lower default risk. Figure 2.13 shows the similar relationship with respect to varying levels for the variance of entrepreneurial abilities.

The sign of the correlation measure tells us about the overall relationship between bank default and the policy rate – that is, if the policy rate has a positive or negative effect on the probability of bank default. However, the correlation between $R$ and the risk of default also informs us of the magnitude of this relationship: how large of an impact do changes in $R$ have on the probability of default? When bank capital or the variance of entrepreneurial abilities rise, equilibrium capital ratios rise leading to a lower
probability of bank default at all policy rates. However, as $K$ and $\sigma^2_a$ are raised above their baseline levels, the probability of bank default and the policy rate become more strongly correlated implying that changes in $R$ have a larger impact on the probability of bank default. \(^{19}\)

The numerical results from the model have important implications for central bankers. First, if central banks incorporate financial stability into their policy objectives, as many contemporary commentators have called for in the wake of the 2007-2009 financial crisis, policy makers will face a trade-off between full employment, stable inflation and financial stability. The results of this paper suggest that the exact nature of the trade-off depends on the relationship between the risk of bank default and the policy rate, which depends in large part on bank capital ratios. For weakly capitalized banks, bank default and $R$ are positively correlated. The trade-off of achieving lower inflation (i.e., the cost of a higher $R$) is driven by a widening output gap and higher probabilities of bank default. However, as the baseline results suggest, when the policy rate and risk of bank default are negatively correlated, higher bank default risk is a cost of reducing the output gap.

Second, the correlation results suggest that the magnitude of the trade-off between employment, inflation and financial stability also depends on the extent of bank capitalization: as the model parameters increase relative to the baseline level, equilibrium capital ratios rise and changes in the policy rate have larger effects on the risk of bank default. \(^{19}\)This result is in line with empirical evidence presented by Dell’Ariccia et al. (2013) where the authors find that the effect of monetary policy on bank-risk taking in less pronounced for banks with lower capital ratios.
default. This may introduce an interesting interplay between minimum capital requirements and optimal monetary policy. If bank capital ratios rise in response to stricter requirements, as they have done in the wake of the financial crisis, changes in the policy rate have a larger impact on the risk of bank default and, by extension, the stability of the financial system as a whole.\textsuperscript{20} This increases the trade-off policy makers face between achieving full employment and financial stability. The result that higher capital ratios strengthen the effect of the policy rate on the risk of bank default suggests that when financial stability is an objective, optimal monetary policy is affected by capital ratios and, by extension, minimum capital requirements.

Additionally, the asymmetric response of bank default should be taken into account. This asymmetry may be of heightened importance in determining the appropriate policy response to limiting systemic risk. Large banks which tend to be the most important in terms of systemic risk also tend to have lower capital ratios;\textsuperscript{21} managing risk at these banks may entail different policy responses compared to smaller, better capitalized banks.

2.4 Conclusion

This paper constructs a partial equilibrium model of the commercial loan market to investigate the impact of monetary policy on bank lending standards and the risk of bank default. In the model, a reduction in the policy rate results in a easing of lend-

\textsuperscript{20}The average capital ratio from 2007-2015 was 10.9\% compared to an average of 8.9\% from 1993-2006.
\textsuperscript{21}See Laeven et al. (2014).
ing standards and, in the baseline calibration, an increase in the risk of bank default. However, the relationship between the policy rate and the probability of bank default depends critically on the extent of bank capitalization. While the relatively high capital ratios in the baseline calibration produces a negative relationship between the policy rate and the probability of bank default, when parameters are changed to produce low equilibrium capital ratios, the risk of bank default is increasing in the policy rate. Additionally, as equilibrium capital ratios increase relative to baseline levels, changes in the policy rate have a larger impact on the risk of bank default.

While this paper does provide important insights into the role monetary policy and lending standards play in shaping the risk of bank default, several avenues for future research and extensions remain. First, incorporating entrepreneurial net worth and variable loan size would give a more complete picture of the commercial loan market. As is, changes in total lending (and therefore bank leverage) are driven completely by the “extensive” margin: after a fall in the policy rate, lending standards loosen thus increasing the number of borrowers and total lending. Introducing entrepreneurial net worth and variable loan size would incorporate the “intensive” margin as well: after a fall in the policy rate, existing borrowers would seek to borrow more, thus increasing total lending and bank leverage even further.

Second, seeing as the model results were sensitive to bank capital structure, it would be interesting to endogenize the amount of capital banks wish to hold. This would allow a more complete analysis of the bank’s problem, as well as an avenue to introduce and examine the direct effect of minimum capital requirements on bank behavior and
risk-taking.

Last, embedding the model into a general equilibrium setting could provide additional insights into how changes in lending standards and the quality of bank assets affect the overall economy, as well as the impact of asset prices on lending standards and risk-taking. Additionally, a general equilibrium model that incorporates bank default will be valuable for central bankers. On the one hand, bank default, and the risk thereof, may alter the effect of monetary policy on the real economy and inflation. In addition, if policy makers aim to achieve financial stability objectives, understanding the trade-off between bank risk, inflation and output will be vital.
Table 2.1: Notation guide.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^K$</td>
<td>Realized aggregate return to capital. Subject to shocks: $R^K = \epsilon_K R^K$.</td>
</tr>
<tr>
<td>$\mathbb{E}^K$</td>
<td>Expected aggregate return to capital.</td>
</tr>
<tr>
<td>$E(r^*</td>
<td>a) = a R^K$</td>
</tr>
<tr>
<td>$(r^*_i</td>
<td>a) = a \epsilon R^K$</td>
</tr>
<tr>
<td>$R^L$</td>
<td>Market return to lending.</td>
</tr>
<tr>
<td>$\epsilon \sim \Phi(\epsilon_K)$</td>
<td>Shock to the aggregate return to capital.</td>
</tr>
<tr>
<td>$\omega \sim F(\omega)$</td>
<td>Idiosyncratic component of the project return.</td>
</tr>
<tr>
<td>$\hat{\epsilon}$</td>
<td>Default shock threshold for bank.</td>
</tr>
<tr>
<td>$\tilde{\omega}_a = \frac{D_a}{a R^K}$</td>
<td>Default shock threshold for borrower with ability $a$. $D_a$ is the contracted loan repayment.</td>
</tr>
<tr>
<td>$H(a)$</td>
<td>Distribution of entrepreneurial abilities.</td>
</tr>
<tr>
<td>$K$</td>
<td>Bank capital.</td>
</tr>
<tr>
<td>$1 - H(\tilde{a})$</td>
<td>Total loans.</td>
</tr>
<tr>
<td>$1 - H(\tilde{a}) - K$</td>
<td>Total deposits.</td>
</tr>
<tr>
<td>$1 - H(a^X) = 1 - H(\tilde{a}/\epsilon)$</td>
<td>The total amount of loans that are able to generate a realized return of at least $R^L$ given the realized aggregate shock $\epsilon_K$.</td>
</tr>
<tr>
<td>$\tilde{a}$</td>
<td>Lending standards/minimum ability requirement to receive a loan. Satisfies $E(r^*_i</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Variable</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^X = \tilde{a}/\epsilon$</td>
<td>Lowest ability group that is able to repay $R^L$ after the realization of the aggregate shock. Satisfies $(r^*_l</td>
</tr>
<tr>
<td>$\tilde{a}^*$</td>
<td>Equilibrium lending standards.</td>
</tr>
<tr>
<td>$\hat{\epsilon}^*$</td>
<td>Equilibrium bank default shock threshold.</td>
</tr>
</tbody>
</table>
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Table 2.2: The relationship between the aggregate shock (relative to its expected value) and the ex-post maximum return to lending (relative to its expected value).

<table>
<thead>
<tr>
<th>Aggregate Shock</th>
<th>Maximum Return to Lending</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon &gt; 1$</td>
<td>$(r^*_t</td>
</tr>
<tr>
<td>$\epsilon = 1$</td>
<td>$(r^*_t</td>
</tr>
<tr>
<td>$\epsilon &lt; 1$</td>
<td>$(r^*_t</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(a); h(a)$</td>
<td>Lognormal on $(0, \infty)$; $E(a) = 1$</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma_a^2$</td>
<td>14.5</td>
<td>Chosen to match the standard deviation of US FICO scores, normalized by the average FICO score.</td>
</tr>
<tr>
<td>$\Phi(\epsilon); \phi(\epsilon)$</td>
<td>Lognormal on $(0, \infty)$; $E(\epsilon_K) = 1$</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma_\epsilon^2$</td>
<td>0.0614</td>
<td>Chosen (with $K$) to match empirical average bank capital ratios and default probabilities.</td>
</tr>
<tr>
<td>$K$</td>
<td>0.0176</td>
<td>Chosen (with $K$) to match empirical average bank capital ratios and default probabilities.</td>
</tr>
<tr>
<td>$\overline{R}^K$</td>
<td>1.10</td>
<td>Average return to the S&amp;P 500 from 1950-2013</td>
</tr>
<tr>
<td>$R$</td>
<td>range 1.00 – 1.10</td>
<td>–</td>
</tr>
</tbody>
</table>
Figure 2.6: The relationship between the policy rate and the probability of bank default under the baseline calibration. As the policy rate rises, the risk of bank default falls. At lower policy rates, the risk of bank default is higher.
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(a) Baseline calibration, $\sigma_a^2 = 14.5$.

(b) $\sigma_a^2 = 2$

(c) $\sigma_a^2 = 0.5$

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Chapter 3

Endogenous Bank Monitoring, Loan Spreads, and the Amplification of Liquidity Shocks

A considerable body of research has emerged exploring the impact of financial frictions on the business cycle and propagation of shocks. At the heart of these frictions lies some fundamental issue of asymmetric information which, through their effect on resulting financial contracts, act as financial accelerators in the business cycle and amplifies the effect of shocks when compared to a frictionless economy.

The propagation mechanism in these models underpins the broad credit channel: the deterioration in borrower net worth after a negative shock results in a larger contraction in capital and a larger downturn compared to a frictionless model. The key driver
of economic fluctuations are variations in borrower net worth, and these net worth dynamics are strongly affected by the excess return to capital over the cost of external (aka, bank) financing.

In reality, the cost of bank financing is influenced, among other factors, by bank behavior. In this paper I focus on one particular aspect of bank behavior: the costly monitoring of borrowers. Indeed, information acquisition lies at the heart of financial intermediation: banks exist, in part, due to their superior ability to overcome the informational asymmetries that characterize financial transactions.

While the source of this comparative advantage is multifaceted, information acquisition on the part of lenders by monitoring borrowers is a key component. Additionally, bank lending standards and the resources expended on monitoring activities, vary with underlying economic conditions.¹ This implies that the cost of external financing, and the dynamics of borrower net worth and output, are potentially impacted by the inclusion of costly monitoring.

However, existing models fail to incorporate a channel for costly information acquisition. In many models² the cost of external funds is simply equal to the risk-free rate. Banks act simply as a veil and play no meaningful role in determining the cost and availability of credit. Other models³ introduce a spread between the risk-free rate and loan rate, but the spread is imposed in a somewhat ad-hoc fashion without true microfoundations.

This paper fills this gap in the existing literature by incorporating costly endogenous

¹See Asea and Blomberg (1998).
²E.g., Bernanke et al. (1999) and Gertler and Kiyotaki (2010).
³E.g., Curdia and Woodford (2015), Goodfriend and McCallum (2007).
bank monitoring into a simple real business cycle model with a commercial loan market. In the model, entrepreneurs combine their existing net worth with external financing from financial intermediaries ("banks") to fund investment projects. These borrowers are subject to moral hazard issues akin to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) in that entrepreneurs are capable of absconding with the project funds. However, unlike previous models, the probability that borrowers are successful in their theft is neither fixed nor exogenous; rather, it depends on the bank’s monitoring activities. Bank monitoring affects the probability that entrepreneur theft will be detected by the bank: when banks choose to monitor projects more intensely, theft is more likely to be detected which effectively ameliorates the severity of the underlying moral hazard issue. On the other hand, monitoring projects is costly and more intense monitoring increases monitoring costs for banks.

The inclusion of costly endogenous bank monitoring affects the dynamics of borrower net worth in two ways. The first works through borrower leverage. Since borrower moral hazard results in an endogenous leverage constraint, changes in bank monitoring directly affect the severity of the moral hazard issues and the level of borrower leverage. All else equal, more intense monitoring permits an increase in borrower leverage and less intense monitoring requires leverage to fall. Second, by affecting bank intermediation costs, costly monitoring affects the cost of bank financing and the excess return to capital. This, in turn, plays a key role in determining borrower net worth and output dynamics.

4Unlike Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), it is entrepreneurs, not banks, that are subject to this moral hazard issue.
To investigate the impact of costly monitoring, I calibrate and numerically solve the model both with costly endogenous monitoring and costless fixed monitoring. I find that after a negative bank liquidity shock (modeled as an increase in cost of deposits), the negative response of capital and output is moderated in the costly endogenous monitoring compared to the costless exogenous monitoring model. This result is driven by the costly monitoring model’s procyclical spread between loan rate and risk-free rate. In the costless model, the loan rate spread is simply equal is zero. However, with costly endogenous monitoring the spread falls after a negative liquidity shock which results in a larger rise in the excess return to capital and results in a faster recovery in borrower net worth, capital, and output. I also find that this result is primarily driven by monitoring and not by changes in borrower leverage.

This paper is related and contributes to the existing literature on bank monitoring. In these models, costly bank monitoring is introduced as a response to borrower moral hazard. By effectively reducing the benefit to firms from “misbehaving”, monitoring ameliorates moral hazard pressures. In many monitoring models, including Holmstrom and Tirole (1997), Diamond (1984), and Besanko and Kanatas (1993), bank monitoring is presented as a binary choice: the bank either monitors borrowers or doesn’t. Other models, such as Hauswald and Marquez (2006) and Carletti (2004), allow for a continuous range of bank monitoring intensities, but do not examine the impact of said monitoring on aggregate real outcomes. This paper contributes to the literature on bank monitoring by incorporating a continuous range of bank monitoring intensities.

5 The costless fixed monitoring case boils down to the model in Gertler and Karadi (2011) with the exception that entrepreneurs, not banks, are subject to moral hazard.
into a simple RBC model of the macroeconomy. This permits monitoring intensities and costs to vary with economic conditions and also allows me to investigate the impact of monitoring on loan market outcomes as well as its aggregate implications for capital and output.

This paper also contributes to a growing literature of general equilibrium models that incorporate spreads between the cost of external financing and the risk-free rate. Both Curdia and Woodford (2015) (henceforth WC) and Goodfriend and McCallum (2007) (henceforth GM) incorporate a time-varying loan rate spread in a monetary DSGE framework. In both aforementioned models, the loan rate spread arises because loan production is assumed to use real resources. However, the underlying reasons why intermediation uses real resources are left unexplored. Ultimately much of these intermediation costs – and loan rate spreads – stem from default risk and measures taken by banks to manage asset risk. WC include borrower default, but it is assumed to be exogenous and its underlying source is not modeled. In addition, GM include “monitoring” in their loan production function, but this is really just a relabeled labor input; the underlying need for bank monitoring is unmodeled.

This paper contributes to this literature by constructing a DSGE model with loan rate spreads where the loan rate spread is caused by information costs whose source is explicitly modeled. Like WC and GM, a loan rate spread emerges because intermediation is costly. Unlike WC and GM, the source of the loan rate spread in this model is not exogenously imposed. Instead, banks monitor borrowers to reduce asymmetric information and ameliorate borrower moral hazard – a function that lies at the heart of
banking! Given the evidence that banking lending standards (monitoring included) vary
with respect to economic conditions, the inclusion of costly monitoring may potentially
impact loan rate spreads and aggregate dynamics.

The remainder of the paper is structured as follows: section 2 lays out the baseline
model, section 3 discusses calibration of the model, section 4 presents and discusses
numeric results, and section 5 concludes.

3.1 The Model

3.1.1 Households

There is a continuum of identical households of mass one. Each household consumes
final output goods, supplies labor, and saves by lending funds to financial intermediaries
(“banks”).

Each household consists of two types of members, workers and entrepreneurs, and within
each household there is perfect consumption insurance. Workers inelastically supply la-
bror and remit their wages back to the household. Each entrepreneur manages a firm
that is ultimately owned by the household. At any point in time, a fraction $f$ of
household members take the role of workers and the remaining $1 - f$ take the role
of entrepreneurs. The probability that an entrepreneur changes roles and becomes a
worker at the end of the period (or that a worker becomes an entrepreneur) is $1 - \sigma$;
the probability that a worker or entrepreneur remains in their current role is $\sigma$. This
probability is constant over time ensuring that the proportion of the household in each role is also constant. Additionally, incorporating exit ensures that no entrepreneur amasses enough wealth to independently finance their investment projects without external financing from lenders. Exiting entrepreneurs transfer their retained earnings to their household. New entrepreneurs receive start-up funds from their household upon entering the entrepreneurial sector, as will be outlined in proceeding sections.

Let $C_t$ be consumption. Household preferences are given by

$$\max E_t \sum_{i=0}^{\infty} \beta^i \epsilon_{t+i} \ln(C_{t+i})$$

(3.1)

where $\beta \in (0, 1)$ and $\epsilon_t$ represents a household preference shock and is included as a simple way to shock the risk-free interest rate.

In period $t$ the household receives the risk-free rate of return $R_t$ on deposits saved in the previous period $D_{t-1}$. Workers receive the real wage $W_t$ for each unit of labor supplied $L_t$. The household also receives net payout $\Pi_t$ from firms; this includes any profits earned by financial firms as well as non-financial firms the household owns (net of start-up funds paid out to new entrepreneurs). The household budget constraint takes the following form:

$$C_t = W_t L_t + \Pi_t + R_t D_{t-1} - D_t$$

(3.2)

6Note that the supply of labor is perfectly inelastic, $L_t = 1 \forall t$. 

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The household chooses consumption and deposits to solve 3.1 subject to the budget constraint 3.2. The problem results in conventional household euler equation:

$$E_t \beta \Lambda_{t,t+1} R_{t+1} = 1$$ (3.3)

where $\Lambda_{t,t+1}$ is the household’s stochastic discount factor:

$$E_t \Lambda_{t,t+1} = E_t \frac{C_t}{C_{t+1}^{\epsilon_{t+1}}}$$ (3.4)

By affecting the household’ stochastic discount factor, the preference shock $\epsilon_t$ provides a simple way to generate a liquidity shock for banks. A positive realization of $\epsilon_t$ increases the marginal utility of consumption in period $t$; this increase in the household’s preference for current consumption causes the risk-free rate $R_{t+1}$ to rise. Since the risk-free rate represents the cost of deposits for banks, an increase in $\epsilon_t$ results in higher financing costs for banks.

3.1.2 Entrepreneurs

Entrepreneurs\(^7\) manage the firms that produce final output goods and make decisions about how much physical capital to acquire for production purposes. At the end of period $t$ entrepreneur $j$ purchases $K_{jt}$ units of capital worth $Q_t$. Capital purchases are

\(^7\)I use the term entrepreneur and borrower interchangeably throughout the remainder of the paper.
funded with the entrepreneur’s existing net worth \( N_{jt} \) and bank loans \( B_{jt} \). This leads to the following flow of funds constraint

\[
Q_t K_{jt} = N_{jt} + B_{jt}
\]  

(3.5)

Entrepreneur \( j \) carries their capital acquired at the end of period \( t \) into period \( t + 1 \) where it is combined with labor to produce the final output good. Each dollar of capital earns the return \( R_{kt+1} \). After production occurs, bank loans are repaid at a cost of \( R_{jbt+1} \) per dollar borrowed.\(^8\) Net worth evolves as the entrepreneur earns capital returns above the total cost of borrowing:

\[
N_{jt+1} = R_{kt+1}Q_t K_{jt} - R_{jbt+1}B_{jt}
\]  

(3.6)

\[
= (R_{kt+1} - R_{jbt+1})Q_t K_{jt} + R_{jbt+1}N_{jt}
\]  

(3.7)

In the absence of costly bank monitoring, competition drives the loan interest rate down to the cost of deposits, implying \( R_{jbt+1} = R_{t+1} \). However, when bank monitoring is costly \( R_{jbt+1} > R_{t+1} \). The magnitude of this spread depends on the choice of bank monitoring intensity and plays a crucial role in the propagation of shocks.

Additionally, in the absence of any financial frictions entrepreneurs will continue to borrow and increase capital holdings until \( R_{kt+1} = R_{jbt+1} \). However, borrower moral

\(^8\)As I will show in the following section, the loan rate, as well as the level of monitoring intensity, does not depend on entrepreneur characteristics and is, therefore, identical for all borrowers. However, for now I will index both variables by entrepreneur.
hazard limits the ability to borrow and results in a positive spread between the project return and loan rate. Specifically, I assume that an entrepreneur is able to divert and abscond with the project $Q_t K_{jt}$. Banks can discourage bad behavior by monitoring borrowers after loans are made. Monitoring reduces the probability an entrepreneur will be successful in diverting funds to $1 - m_{jt}$ where $m_{jt}$ is the lender’s optimally chosen monitoring intensity.

With probability $m_{jt}$ the bank detects the borrower’s undesirable behavior, and the project is seized by the bank and liquidated. If the borrower is successful in diverting funds (with probability $1 - m_{jt}$), they default on the loan and their project is shut down. In the case of default, the lending bank is unable to recover the project funds and incurs a loss. I assume that the entrepreneur’s decision to abscond must be made before the realization of aggregate uncertainty. This ensures that incentive compatibility is not violated by unexpected movements in the project return and reflects the fact that stealing funds takes foresight and time.

Because the bank is aware of the borrower’s incentive to steal funds, it will limit the amount they lend to ensure that borrowers do not misbehave. Specifically, contracts must satisfy the following incentive compatibility constraint:

$$V(K_{jt}, B_{jt}) \geq (1 - m_{jt})Q_t K_{jt}$$  \hspace{1cm} (3.8)

In principle, a less extreme version of the moral hazard problem could be imposed where borrowers can abscond with a fraction $\lambda$ of the project and lenders can recoup the remaining $1 - \lambda$ in the event of borrower default (as in Karadi and Gertler (20–) and Gertler and Kiyotaki (20–)). However, since monitoring intensity $m_{jt}$ affects the expected return to asconding in an identical fashion as $\lambda$, I assume that $\lambda = 1$ to avoid the need to calibrate an additional parameter.
where $V(K_{jt}, B_{jt})$ is the maximized value of the entrepreneur’s objective function $V_{jt}$ at the end of period $t$ given their choice of capital $K_{jt}$ and bank loan $B_{jt}$:

\[
V_{jt} = E_t \sum_{i=0}^{\infty} (1 - \sigma)\sigma^i \Lambda_{t,t+1+i} N_{jt+1+i} \tag{3.9}
\]

Additionally, the entrepreneur’s value function at the end of period $t - 1$ satisfies the following Bellman equation:

\[
V(K_{jt-1}, B_{jt-1}) = E_{t-1} \Lambda_{t-1,t} \left[ (1 - \sigma)N_{jt} + \sigma \max_{K_{jt}, B_{jt}} V(K_{jt}, B_{jt}) \right] \tag{3.10}
\]

Solving the entrepreneur’s problem is trivial. As long as $R_{kt+1} > R_{jbt+1}$, the entrepreneur will continue to borrow until the incentive compatibility constraint is binding. However, in order to express the incentive compatibility constraint in a convenient form, it is useful to solve for the borrower’s value function. I posit that the value function takes the following form

\[
V(K_{jt}, B_{jt}) = \nu_{jt} K_{jt} - \mu_{jt} B_{jt} \tag{3.11}
\]

where $\nu_{jt}$ and $\mu_{jt}$ are timing varying parameters that measure, respectively, the marginal benefit and marginal cost of expanding the investment project by one unit.

Using 3.11, the incentive compatibility constraint 3.8 can be restated as follows:
\[ \nu_{jt} K_{jt} - \mu_{jt} B_{jt} \geq (1 - m_{jt}) Q_t K_{jt} \quad (3.12) \]

Assuming the incentive compatibility constraint holds, combining 3.12 and 3.5 yields entrepreneur j’s individual capital holdings:

\[ Q_t K_{jt} = \left( \frac{\mu_{jt}}{1 - m_{jt} - (\nu_{jt} Q_t - \mu_t)} \right) N_{jt} = \phi_{jt} N_{jt} \quad (3.13) \]

where \( \phi_{jt} \) is entrepreneur j’s leverage, and the parameters \( \nu_{jt} \) and \( \mu_{jt} \) are equal to

\[ \nu_{jt} = E_t \beta \Lambda_{t,t+1} \Omega_{jt+1} Q_t R_{kt+1} \quad (3.14) \]

\[ \mu_{jt} = E_t \beta \Lambda_{t,t+1} \Omega_{jt+1} R_{jbt+1} \quad (3.15) \]

and \( \Omega_{jt+1} \) is the marginal value of entrepreneurial net worth in period \( t + 1 \):

\[ \Omega_{jt+1} = (1 - \sigma) + \sigma \left[ \mu_{jt+1} + \phi_{jt+1} \left( \frac{\nu_{jt+1} Q_{jt+1} R_{jt+1}}{Q_{jt+1} - \mu_{jt+1}} \right) \right] \quad (3.16) \]

The marginal value of net worth \( \Omega_{jt+1} \) times the stochastic discount factor \( \Lambda_{t,t+1} \) can be thought of as an augmented discount factor; it measures the expected discounted value.
to the entrepreneur of an additional dollar of net worth in the future. With probability $1 - \sigma$ the entrepreneur becomes a worker and exits with her additional net worth. With probability $\sigma$ she continues her role as an entrepreneur and in the following period leverages her additional net worth by a factor $\phi_{jt+1}$ which earns the real excess return

$$\frac{\nu_{jt+1}}{Q_{t+1}} - \mu_{jt+1}.$$

The real marginal benefit to the entrepreneur of expanding capital in the current period, $\frac{\nu_{jt}}{Q_t}$, is equal to the expected return to capital $R_{kt+1}$ multiplied by the augmented discount factor. Each additional dollar of capital acquired at the end of period $t$ increases the entrepreneur’s net worth by $R_{kt+1}$. This multiplied by the augmented discount factor gives the real marginal benefit of additional capital. Note that borrower leverage is increasing in $\frac{\nu_{jt}}{Q_t}$: as the real marginal benefit of capital rises, entrepreneurs have more to lose if they are caught stealing funds and the incentive compatibility constraint loosens, allowing project size and leverage to rise.

The marginal cost to the entrepreneur of increasing their bank loan in the current period, $\mu_{jt}$, is similarly interpreted. An additional dollar of loans reduces future net worth by $R_{jbt+1}$ dollars which, when multiplied by the augmented discount factor, gives the marginal cost of bank loans. Unlike $\nu_{jt}$, borrower leverage is decreasing in $\mu_{jt}$: as the marginal cost of external financing rises, entrepreneurs have less to lose if they are caught stealing funds. This causes the incentive compatibility constraint to tighten and project size and leverage to contract.

Additionally, the bank’s choice of monitoring intensity also directly affects the entrepreneur’s incentive compatibility constraint and resulting leverage. If banks more
closely monitor borrowers, and \( m_{jt} \) rises, the probability of successfully stealing project funds falls. This lowers the borrower’s expected value of attempting to abscond with the project and results in a looser incentive compatibility constraint and higher borrower leverage, all else equal. This choice of monitoring intensity is determined by the bank’s decision problem and is outlined in the following section.

### 3.1.3 Banks

The primary role of banks is to channel savings from households to entrepreneurs and to monitor entrepreneur projects. I assume that there is a continuum of mass one of identical, competitive banks. Banks raise deposits from households and use these funds to make loans to entrepreneurs. I assume that there are no reserve requirements, implying that deposits are equal to total lending. Banks will expand the amount they lend as long as it is profitable and the borrower’s incentive compatibility constraint is not violated. Assuming the bank at least breaks even on its loans, this implies that total lending to entrepreneur \( j \), \( B_{jt} \), is equal to

\[
B_{jt} = (\phi_{jt} - 1)N_{jt} \tag{3.17}
\]

Banks monitor borrowers as a way to alleviate entrepreneurial moral hazard associated with stealing project funds. Specifically, I assume that banks choose monitoring intensity \( m_{jt} \) which directly affects the probability \( 1 - m_{jt} \) an entrepreneur \( j \) will successfully steal project funds without detection. When a bank increases monitoring intensity \( m_{jt} \),
the likelihood of successful theft, and the borrower’s moral hazard pressures, falls. The main idea behind this assumption is that when banks more closely monitor a borrower’s project and accounts, they are more likely to notice unusual account activity and transactions that indicate funds are being misused.

By relaxing the borrower’s incentive compatibility constraint, higher monitoring intensity permits higher entrepreneur leverage and an increase in loan size and bank profits. However, monitoring comes at a cost: I assume that monitoring is associated with convex costs that are proportional to the entrepreneur’s project size: \( c(m_{jt})Q_tK_{jt} = c(m_{jt})\phi_{jt}N_{jt} \), with \( c’ > 0 \) and \( c” > 0 \). When more resources are used to increase monitoring, costs rise at an increasing rate. In addition, given the level of monitoring intensity, larger, more complex projects are more difficult, and hence more costly, to monitor.

A bank earns \( R_{jbt+1} \) on each dollar lent to entrepreneur \( j \) at the end of period \( t \) and pays \( R_{t+1} \) for each dollar of deposits it takes on. I assume that banks face an infinitely elastic supply of deposits at the risk-free rate \( R_{t+1} \). For each borrower, the bank chooses monitoring intensity to maximize loan profit, which is equal to net loan revenue less monitoring costs, while internalizing the effect monitoring has on total loan size:

\[
\max_{m_{jt}} E_t\left[ (R_{bt+1} - R_{t+1})(\phi_{jt} - 1)N_{jt} - c(m_{jt})\phi_{jt}N_{jt} \right] \tag{3.18}
\]

\[
s.t. \quad \phi_{jt} = \frac{\mu_{jt}}{1 - m_{jt} - (\nu_{jt}Q_t - \mu_{jt})} \tag{3.19}
\]

Taking \( R_{jbt+1}, \nu_{jt}, \) and \( \mu_{jt} \) as given during optimization, the optimal monitoring inten-
sity is given by the following first order condition:

\[
E_t \left[ \frac{R_{bt+1} - R_{t+1} - c(m_{jt})}{1 - m_{jt} - \left( \frac{\nu_j t}{Q_t} - \mu_{jt} \right)} \phi_{jt} - c'(m_{jt}) \phi_{jt} \right] = 0 \tag{3.20}
\]

The first term on the left-hand side of 3.20 is the expected marginal benefit of more intense monitoring: higher monitoring intensity eases the borrower’s incentive compatibility constraint and allows lending to increase by a factor of \( \frac{\phi_{jt}}{1 - m_{jt} - \left( \frac{\nu_j t}{Q_t} - \mu_{jt} \right)} \). Each additional unit lent earns the bank net marginal loan revenue of \( R_{jbt+1} - R_{t+1} - c(m_{jt}) \).

The second term gives the marginal cost of more intense monitoring – increased monitoring costs. The optimal level of monitoring intensity occurs when the increase in net loan revenue realized from more intense monitoring is exactly equal to the additional cost.

Since banks are assumed to be identical and competitive, the loan interest rate \( R_{jbt+1} \) is determined by the following zero profit condition:

\[
E_t(\pi_b) = 0 \Rightarrow E_t \left[ (R_{jbt+1} - R_{t+1})(\phi_{jt} - 1)N_{jt} - c(m_{jt})\phi_{jt}N_{jt} \right] = 0 \tag{3.21}
\]

Since bank profits are linear in borrower net worth, the loan rate can be expressed as the following:

\[
E_t R_{jbt+1} = E_t[R_{t+1} + sp_t] \tag{3.22}
\]
where $sp_t$ is the spread between the loan rate and the risk-free rate that results from costly bank monitoring:

$$E_t sp_t = \frac{E_t \phi_{jt}}{E_t (\phi_{jt} - 1)} c(m_{jt})$$  \hspace{1cm} (3.23)

In a world without costly monitoring $sp_t = 0$ and $R_{jbt+1} = R_{t+1}$. However, costly monitoring introduces a wedge between the loan rate and the risk-free rate that depends on borrower leverage and the bank’s choice of monitoring intensity. All other variables held constant, an increase in bank monitoring raises monitoring costs $c(m_{jt})$ and raises the loan rate. On the other hand, higher borrower leverage allows the bank to spread the costs of monitoring over a large loan volume and lowers the loan rate.

The effect of endogenous movements in bank monitoring on the loan rate and loan spread plays a key role in propogating shocks due to its effect on the evolution of entrepreneur net worth. This relationship is outlined in the proceeding section.

3.1.4 The Evolution of Entrepreneur Net Worth and the Propogation of Shocks

Thus far, loan rates, monitoring intensity, and leverage have all been index by entrepreneur. However, when the equations for the loan rate (3.22), optimal monitoring (3.20), and borrower leverage (3.19) are taken together, entrepreneur-specific variables (namely net worth $N_{jt}$) do not enter into any of the conditions. This implies that all
entrepreneurs are subject to the same level of monitoring and loan rate and have the same leverage.

Since leverage is identical across entrepreneurs, aggregate capital demand is easily derived by aggregating 3.13 over entrepreneurs:

\[ Q_t K_t = \phi_t N_t \]  

(3.24)

The primary driver of fluctuations in aggregate capital demand is variation in aggregate net worth. In turn, total entrepreneurial net worth is the sum of surviving entrepreneur’s net worth, \( N_{et} \), and new entrepreneur’s net worth, \( N_{nt} \).

\[ N_t = N_{et} + N_{nt} \]  

(3.25)

Given that a fraction \( \sigma \) of entrepreneurs survive until period \( t \), surviving entrepreneur’s net worth is equal to

\[ N_{et} = \sigma [(R_{kt} - R_{bt})\phi_{t-1} + R_{bt}] N_{t-1} \]  

(3.26)

Note that \( N_{et} \) grows larger as the net return to capital \( R_{kt} - R_{bt} \) rises, and the effect of excess returns on net worth increases as leverage \( \phi_{t-1} \) grows.

New entrepreneurs receive start-up funds from their household. I assume that households transfer a fraction of exiting entrepreneurs’ assets to entering entrepreneurs.
Specifically, new entrepreneurs receive a fraction $\frac{\omega}{1-\sigma}$ of the value of exiting entrepreneurs’ assets at time $t$, $(1-\sigma)Q_t K_{t-1}$.

\[ N_{nt} = \omega Q_t K_{t-1} \quad (3.27) \]

Combining 3.25, 3.26, and 3.27 yields the following law of motion for entrepreneurial net worth:

\[ N_t = \sigma \left[ (R_{kt} - R_{kt}) \phi_{t-1} + R_{kt} \right] N_{t-1} + \omega Q_t K_{t-1} \quad (3.28) \]

The key motivation of this paper is to investigate how the effect of liquidity shocks differs between an economy with costly endogenous monitoring (endogenous $m_t$, $c(m_t) > 0$) and costless fixed monitoring (fixed $m_t$, $c(m_t) = 0$). The major difference between the costly and costless monitoring models stems from the loan rate and its effect on borrower net worth.

In the costless monitoring model, the loan rate is equal to the risk-free rate and 3.28 becomes

\[ N_t = \sigma \left[ (R_{kt} - R_t) \phi_{t-1} + R_t \right] N_{t-1} + \omega Q_t K_{t-1} \quad (3.29) \]

In the costly monitoring model, a wedge is introduced between the loan rate and the risk-free rate and 3.28 becomes
\[ N_t = \sigma \left[ (R_{kt} - R_t - sp_t)\phi_{t-1} + R_t + sp_t \right] N_{t-1} + \omega Q_t K_{t-1} \]  

(3.30)

where 3.30 makes use of 3.22.

In both models, the propagation of shocks is primarily driven by variation in entrepreneurial net worth. After a negative shock, aggregate net worth falls resulting in lower investment, a smaller capital stock, and lower output. This results in a spike in the spread between the return on capital and the cost of borrowing which, over time, pushes the economy back to its steady state. Since the loan spread \( sp_t \) caused by costly monitoring influences the magnitude of the excess return on capital, bank monitoring activities play a key role in the evolution of net worth and the overall effect of shocks on capital and output.

Analytically, the effect of monitoring is ambiguous; however, additional insights can be uncovered with numerical methods. Before I turn to calibration and numerical results, the description of goods producers, capital producers, equilibrium conditions, and shock processes are detailed in the remaining model sections.

### 3.1.5 Goods Production

Entrepreneurs enter period \( t \) with \( K_{t-1} \) units of capital with which they combine with labor to produce final output according to the following production function.
\[ Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha} \] (3.31)

where \( Y_t \) is aggregate output, \( K_{t-1} \) is aggregate capital acquired in period \( t - 1 \), the \( L_t \) is labor input\(^{10} \), and \( \alpha \in (0, 1) \) is capital’s share of output.

After production occurs in period \( t \) capital depreciates by a a fraction \( \delta \). The remaining \( 1 - \delta \) of depreciated capital is resold to capital producers at price \( Q_t \). This, and the production technology 3.31, implies that the expected gross return to holding capital from period \( t \) into period \( t + 1 \) is given by

\[ E_t R_{kt+1} = E_t \left[ \frac{\alpha Y_{t+1}}{K_t} + (1 - \delta) Q_{t+1} \right] \] (3.32)

3.1.6 Capital Producers

In period \( t \), capital producers use the final output good to produce new capital goods which are sold to entrepreneurs at price \( Q_t \). This capital is then used in the following period \( t + 1 \) for the production of output goods. Since it is assumed that capital producers are identical and owned by households, capital producers choose investment \( I_t \) to solve the following problem:

\[ \max_{I_t} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left[ (Q_{t+i} - 1) I_{t+i} - f \left( \frac{I_{t+i}}{I_{t+i-1}} \right) I_{t+i} \right] \] (3.33)

\(^{10}\text{Household inelastically supply labor, implying that } L_t = 1 \ \forall t.\)
where \( f(\phi) \) represents investment adjust costs and are included to permit a variable price of capital, and \( f' > 0, f'' > 0, f(1) = f'(1) = 0. \)

The first order condition for investment gives the following “Q” relation:

\[
Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + f'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} - E_t \beta A_{t,t+1} f'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2
\]

Any profits earned by capital producers (which occurs only outside of the steady state) are transferred back to the household. Lastly, given investment \( I_t \) and existing capital \( K_{t-1} \), the following gives the law of motion for capital:

\[
K_t = I_t + (1 - \delta) K_{t-1}
\]

### 3.1.7 Closing the Model

The previous sections lay out the bulk of the model. For simplicity, I strip out other complications such as price and wage rigidity, government, and money. All that remains to close the model is to specify shock processes, exogenous cost structures, and market clearing conditions.

The only shock present in the model is the household preference shock \( \epsilon \) that is introduced to generate a liquidity shock for banks. I assume this follows the AR(1) process
\[ \epsilon_t = \rho \epsilon_{t-1} + z_t \quad (3.36) \]

I require that the capital market, labor market, deposit market, and output market all clear. Since there are no reserve requirements for banks, deposit market clearing implies that total deposits are equal to total lending.

\[ D_t = (\phi_t - 1)N_t \quad (3.37) \]

Capital market clearing occurs when the supply of capital, given by the law of motion 3.35, is equal to capital purchases by entrepreneurs, given by aggregate capital demand 3.24.

Output goods are used for consumption, investment, investment adjustment costs, and bank monitoring costs. Therefore, output market clearing requires that

\[ Y_t = C_t + \left[ 1 + f\left( \frac{I_t}{I_{t-1}} \right) \right] I_t + c(m_t)\phi_t N_t \quad (3.38) \]

I assume that investment adjustment costs and bank monitoring costs adhere to the following quadratic structure:

\[ f\left( \frac{I_t}{I_{t-1}} \right) = \frac{\eta_i}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \quad (3.39) \]
The bank’s monitoring cost function has two components. The first component \( \frac{\eta c m^2}{2} \) depends on the bank’s choice of monitoring intensity while the second component \( f \) is fixed with respect to monitoring intensity. This assumption is included primarily for technical purposes: including a fixed cost component to monitoring ensures the bank earns positive net loan revenue on each dollar lent and will choose a non-zero monitoring intensity in equilibrium. Without the fixed cost, the bank’s marginal net loan revenue, and marginal benefit of monitoring, would equal zero and the bank would have no incentive to monitor borrowers.

\[
c(m_t) = \frac{\eta c m^2}{2} + f
\]  

(3.40)

3.2 Calibration

Table 3.1 shows the model parameters. For the costly endogenous monitoring model, there are eight parameters to calibrate; four are conventional, four (\( c, f, \omega, \) and \( \sigma \)) are unique to the model. The household discount factor \( \beta \) takes the standard value of 0.99; the rate of capital depreciation \( \delta \) and capital’s share of output \( \alpha \) also take their standard parameters of 0.025 and 0.33 respectively. I follow Gertler and Karadi (2011) in choosing an inverse elasticity of investment to the price of capital \( \eta_i \) of 1.728.

The values for the variable and fixed monitoring cost parameters \( c \) and \( f \) and the fraction of exiting entrepreneur assets to be transferred to new entrepreneurs \( \omega \) are chosen to hit three empirical targets: 1) a steady state spread between the return to capital and the
risk-free rate $R_k - R$ of 758 basis points which is roughly equal to the average spread between equities and government bonds in Mehra and Prescott (2003); 2) a steady state spread between the bank loan rate and the risk-free rate $R_b - R$ of 286 basis points which is roughly equal to the average spread between the Moody’s BAA corporate bond yield and the 3-month Treasury bill rate; and 3) a steady state entrepreneurial leverage $\frac{K}{N}$ of 2.86 which is roughly equal to the data observed on nonfinancial corporate businesses in aggregate balance sheet data. In order to match the horizon used by Mehra and Prescott (2003), the time period used for calculating average loan rate spreads and leverage is 1951-2000.

The last parameter to calibrate is the entrepreneur survival rate $\sigma$. Data on small business lifespans from the BLS suggests that the value of $\sigma$ depends on the time horizon used. Looking at average survival rates of small business five years out from creation implies a survival rate of 0.955 and an expected entrepreneur lifespan of 5.56 years. When this time horizon is increased to ten years, the implied survival rate increases to 0.963 and expected lifespan increases to 6.76 years. Data from publically traded companies suggest even larger survival rates and longer lifespans: 0.975 (average lifespan of 10 years) based on work by Daepp et al. (2015) and 0.986 (average lifespan of 18 years) based on work by Foster (2012).

However, given the other calibration targets, the steady state of the model cannot support such high survival rates. Given the relatively high steady state spread between capital and loan rates, survival rates must be moderated to achieve the steady state

\[11\text{Leverage is constructed using the series on nonfinancial corporate equity and assets.}\]
leverage target and maintain a non-trivial role for monitoring. As such, I have chosen a survival rate of 0.94. This is, roughly, the highest σ that can be used while achieving the other steady state targets and and non-zero monitoring at the steady state.

The costless fixed monitoring model must also be parameterized. For this model, there are no monitoring costs parameters by design, but the value of the (fixed) monitoring intensity \( m \), along with \( \omega \), must be chosen. I choose these parameters to match the data on aggregate leverage and spreads. However, which spread to use is another question to be answered. In contrast to the costly monitoring model, when monitoring is costless, there is no spread between the loan rate and risk-free rate. The only spread that remains is between the return on capital and the risk-free rate, \( R_k - R \). Since this spread represents the difference between the return on capital and the cost of borrowing, \( m \) is set so that the spread \( R_k - R \) in the costless fixed monitoring model is equal to the spread between \( R_k - R_b \) in the costly endogenous monitoring model. All other parameters remain the same.

### 3.3 Results

Of primary interest to this paper is how liquidity shocks are propagated with and without costly endogenous monitoring. Figure 3.1 shows the impulse responses function for a negative liquidity shock (a rise in the risk-free rate) for both the costly endogenous monitoring model and costless exogenous monitoring model.

In the model with costly endogenous monitoring, a negative liquidity shock results in
smaller fluctuations in net worth, capital, and output compared to the costless fixed monitoring case. In both models, entrepreneur net worth plays the key role in the propagation of shocks. While net worth initially falls by roughly the same amount in the two models, it recovers more quickly in the endogenous monitoring mode due to the larger response of the spread between the return on capital and the cost of external financing. This difference in responses, in turn, is driven by the response of the cost of bank financing in the two models.

Specifically, the cost of bank financing increases relatively less in the costly monitoring model compared to the costless monitoring model. In the costless monitoring model the cost of bank financing is simply equal to the risk-free rate, and, after the liquidity shock, the cost of bank financing rises. However, in the costly monitoring model the spread between the loan rate and risk-free rate falls leading to a smaller rise in the cost of bank financing compared to the costless monitoring model. This is driven by two forces: leverage and bank monitoring. After a negative liquidity shock, borrower leverage rises which causes the spread between the loan rate to decrease. Additionally, this fall in the bank’s interest margin causes banks to reduce monitoring, and lower monitoring costs cause the spread to fall even further. The smaller increase in entrepreneur borrowing costs results in a larger excess return on capital in the costly monitoring model and contributes to the faster recovery of borrower net worth, capital, and output.

The response of total lending and leverage to a negative liquidity shock is shown in figure 3.2. Although the costly endogenous monitoring model produces a milder downturn in output and capital, the decrease in lending is amplified in the costly endogenous mon-
itoring model relative to the the costless exogenous monitoring model. This is directly due to the effect of bank monitoring on borrower leverage. Recall, total bank lending is increasing in both borrower net worth and borrower leverage. Although net worth recovers faster, the increase in borrower leverage is dampened in the costly monitoring model as a result of the reduction in bank monitoring. Taken together, the effect of lower bank monitoring dominates and causes a steeper contraction in total lending.

Since the effect of borrower leverage on the loan rate plays a key part in the results, it important to see how much of the above results actually stem from the effect of endogenous monitoring, and not from borrower leverage, on the loan spread. To this end, a third version of the model is used where monitoring is costly, but the monitoring intensity is fixed (what I dub the “Exogenous Costly Monitoring” model). I use the same calibration as for the costly endogenous monitoring model, but in the model equations I fix the value of bank monitoring at the steady state value. This ensures that any fluctuation in the loan rate to risk-free rate spread is driven solely by movements in borrower leverage and not by changes in bank monitoring intensity. Responses for all three models are shown in figure 3.3.

Although the spread between the cost of bank funds and the risk-free rate falls in the exogenous costly model, the response of output and capital is nearly indistinguishable from the costless monitoring model until roughly 20 quarters after the shock. This suggests that endogenous bank monitoring is driving the difference between the costly endogenous monitoring model and the costless monitoring model, not changes in leverage.
A brief discussion of the results and their implications are warranted. First, the result that the spread between the cost of bank funds and the risk-free rate falls after a negative liquidity shock is in line with empirical evidence from Lown and Morgan (2002), and in line with WC and GM.\(^{12}\) Additionally, result that the output response is attenuated in the model with costly monitoring is also in line with WC. However, in the WC model, the authors find that changes in output after an increase in the risk-free rate are nearly indistinguishable between the model with a positive spread \(R_b - R\) and with no such spread. In the context of this paper’s results, this is not surprising. In WC, endogenous changes in the loan rate spread are solely driven by loan volume. This is analogous to the exogenous costly monitoring version of my model where changes in the loan spread are driven solely by borrower leverage. Like the WC results, the dynamics of output for the exogenous costly monitoring model are very similar to the model with no spread (for about 5 years upon which they diverge).

However, when monitoring intensity is both costly and endogenous there is an appreciable difference in output responses. When the loan spread is affected by both borrower leverage and bank monitoring, the decline in capital and output is less severe and recovers more quickly compared to the model without a loan spread. This result suggests bank monitoring (and more broadly, bank lending standards) may play an important role in the business cycle.

\(^{12}\)Recall, WC refers to Curdia and Woodford (2015) and GM refers to Goodfriend and McCallum (2007).
3.4 Conclusion

This paper investigates the impact of costly endogenous bank monitoring on the response of the economy to a negative liquidity shock. I find that the response of capital and output are attenuated in the model with costly endogenous monitoring compared to the model with costless fixed monitoring. This result is driven by the effect of monitoring on the loan rate and excess return to capital over the loan rate. After a negative liquidity shock, reduced monitoring causes the spread between the loan rate and the risk-free rate to fall, implying that the cost of bank financing rises less in the costly endogenous monitoring model. This leads to a larger spike in the excess return to capital over the cost of bank financing which causes a less severe fall in borrower net worth, capital, and output.

Given the role that costly endogenous monitoring plays in loan rate spreads and aggregate outcomes, a logical extension of the model would include a role for monetary policy. Examining optimal monetary policy in a setting where loan rate spreads are driven by underlying micro frictions in financial markets could potentially shed new light on the role and effects of monetary policy actions.
Table 3.1: Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Household discount factor.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Capital depreciation rate.</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>1.728</td>
<td>Investment adjustment cost parameter.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Capital output share.</td>
</tr>
<tr>
<td>$\rho_\epsilon$</td>
<td>0.9</td>
<td>AR coefficient for liquidity shock.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.0455</td>
<td>Variable monitoring cost parameter.</td>
</tr>
<tr>
<td>$f$</td>
<td>0.0045</td>
<td>Fixed monitoring cost parameter.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.94</td>
<td>Probability of entrepreneur survival.</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.0047</td>
<td>Fraction of assets transferred to new entrepreneurs.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.256</td>
<td>Fixed monitoring intensity.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.94</td>
<td>Probability of entrepreneur survival.</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.0068</td>
<td>Fraction of assets transferred to new entrepreneurs.</td>
</tr>
</tbody>
</table>
Figure 3.1: Responses of output, capital, monitoring intensity, borrower net worth, excess return to capital over the cost of bank loans, and the excess cost of bank loans over the risk-free rate to a negative liquidity shock (an increase in the risk-free rate) for the costly endogenous monitoring model and costless exogenous monitoring model.
Figure 3.2: Responses of total lending and borrower leverage to a negative liquidity shock (an increase in the risk-free rate) for the costly endogenous monitoring model and costless exogenous monitoring model.
Figure 3.3: Responses of output, capital, monitoring intensity, borrower net worth, excess return to capital over the cost of bank loans, and the excess cost of bank loans over the risk-free rate to a negative liquidity shock (an increase in the risk-free rate) for the costly endogenous monitoring model, costless exogenous monitoring model, and costly exogenous monitoring model.
Appendix A

Solution Strategy For Chapter 2

The equilibrium for the model was solved using MATLAB. The solution process involved three primary steps: 1) code a function that returns the bank’s default threshold for any level of standards; 2) code the model equilibrium condition, equation 2.22, while simultaneously imposing the relationship between the shock threshold and standards from part 1; 3) solve for the equilibrium condition. This process was then repeated for each of the policy rates in consideration.

Step 1: first, I wrote a function for bad state profits (equation 2.10) as a function of standards and the aggregate shock. To find the bank’s default threshold \( \hat{\epsilon} \), for any level of standards, I constructed a function that finds the value of \( \epsilon \) that makes bad state profits zero. This is achieved by finding the root of the bad state profit function using \texttt{fzero} while accounting for corner solutions where \( \hat{\epsilon} = \{0, \infty\} \). The resulting function outputs the bank’s default threshold for any level of standards inputted.
Step 2: I next wrote a function for the equilibrium condition, equation 2.22, as a function of lending standards. This function used the \( \hat{\epsilon} \) function from step 1 to impose the dependency of bank default on lending standards. The resulting function outputs the equilibrium condition for any level of standards inputted. The equilibrium level of standards is found when this function returns a value of zero.

Step 3: I solve for equilibrium lending standards by finding the root of the equilibrium condition coded in step 2 using lsqnonlin.\(^1\) The resulting equilibrium level of standards \( \tilde{a}^* \) was input back into the function from step 1 to find the equilibrium default shock threshold \( \hat{\epsilon}^* \).

\(^1\)I utilized lsqnonlin, and not fzero, so that I could specify upper and lower bounds for ability levels the solver considered. While both fzero and lsqnonlin produced nearly identical results for the baseline calibration, the bounds provided by lsqnonlin helped avoid errors when more extreme parameter values were considered in the sensitivity analysis.
Bibliography


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