Title
Beltrami Energy Based Partitioning Model for Image Segmentation

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Author
An, Jing

Publication Date
2015-04-01

Undergraduate
Reflective Essay

When I talk with people about my research on image processing, they often assume that what I am doing is engineering-based, not real math. But image processing is closely related to math subjects I learned in class. In fact, the initial motivation drove me to this fun field is due to its broadness and practicability. From my research experience, it is aesthetically pleasing to realize that numerous mathematics tools in various branches, not limited to general analysis, but also geometry, topology, combinatorics and more have proved their elegant success in solving difficult imaging problems. Besides, the image processing has seen its many applications in industry, which makes it different from isolated pure mathematics; it provides better problem-solving strategies and benefits people’s life perceptibly.

In Fall 2014, after finishing the UCLA department of mathematics REU in summer, in which my first research project was about improving classical segmentation models to achieve better segmentation results of microscopy images, I continued my research with my mentor on a brand-new project. My mentor Dr. Zosso and his co-workers have introduced a creative graph-partitioning model based on Beltrami energy, and made noticeable results using this model. However, there was some space left for improvement.
and extension. Because analysis interests me most among all math fields, my mentor suggested me focus on the analytical side and try to find the connections between the proposed model and the classical segmentation models I studied in summer. For an undergraduate who just stepped into her third year, this problem is high-level and clueless at the first sight. But fortunately, the excellent resources provided by UCLA library make the impossible becomes possible.

Without a doubt, every researcher needs to build a concrete understanding about the problem’s background first. With this in mind, I listed important papers and books I needed to read and began to search them online. At this time, UC-eLinks did me great favor by supporting me in freely accessing to all academic articles. Moreover, because Beltrami energy is fully constructed using differential geometry, a good grasp of this field is required. Therefore, I found several recommended differential geometry books through Melvyl UC Catalog online, and then I went to the Science and Engineering Library to check them out. Although the problem’s background for me was complex and took much time to comb and digest, the invaluable benefits brought by the UCLA library collections helped me a lot. It not only significantly speeded up my acquaintance with the materials, but also functioned as a bridge between inspiration and me. As a result, I had more weeks to complete the algorithm by adding details of projections, and managed to give heuristic proof of the connections.

Thanks to the assist of the Library resources, I attained good results so that I could step further. In the following winter, my mentor assigned me a task to numerically
analyze the convergence of our primal-dual projected gradients algorithm, which his submitted paper was required to add according to the referee’s comments. Once again, through the powerful library database, I delightfully gained a dozen of related papers discussing general primal-dual hybrid gradient algorithms, and one of which written by Chambolle and Pock provided an essential theorem to prove the convergence of our algorithm. However, the proof they gave has too many gaps to understand. At this point, I decided to fill these gaps to make our proof easier to understand by more undergraduate peers. This work was as challenging as translating languages. But I finally made it with the aid of online numerical analysis textbooks, which the UC-eLinks has bought copyrights to let students read for free. I even used the Interlibrary Loan to request books I wanted from other campuses. To my surprise, all these books arrived at UCLA within two days, and such efficiency and convenience made my research proceed unexpectedly smoothly.

Along with the end of winter quarter, my research work has come to an end. This experience accompanied by efforts and sweat is a wonderful journey to me, not only because of the progress I made, but also due to the uncountable knowledge I gained from the UCLA library resources. I believe myself has been more mathematically mature, and been closer to become a real applied math researcher. During this change, the UCLA library has played an influential role and should be highly appreciated.
Abstract: The Beltrami framework has seen its successful applications in many fields of image processing, so as in the graph partitioning problem. In this report, we present a Beltrami energy based graph partitioning model, which is a composition of two minimization problems. To optimize the energy, we can apply the efficient primal-dual projected gradients method to the inner part; and then given the inner optimizer has been found, the rearrangement algorithm can strictly decrease the outer energy in finite steps to achieve the optimization. Specifically for the inner part, we look into finding appropriate primal and dual time steps to ensure the convergence of the primal-dual projected gradients algorithm. Indeed, if the product of the primal and dual time steps is bounded by the inverse of the squared induced norm of $\nabla_w$, then we can show the convergence by squeezing the partial primal dual gap to zero, thanks to Chambolle and Pock's theorem. Moreover, we are interested in exploring the model's similarities to the state of the art, and we believe this geometric graph partitioning model can seamlessly unify both edge and region-based segmentation methods in a single functional by selecting proper conditions. With the stepsizes' boundedness, and the model's region-based and edge-based constructions for comparison purposes, in the end we present experimental segmentation results to confirm that the Beltrami graph partitioning model with our proposed algorithm has good performance in practice.