Measurments of neutral $B$ decay branching fractions to $K^0 \pi^+ \pi^-$ final states and the charge asymmetry of $B^0 \to K^+ \pi^-$
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RAPID COMMUNICATIONS


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We analyze the decay $B^0 \to K^0_S \pi^+ \pi^-$ using a sample of $232 \times 10^6$ $\Upsilon(4S) \to B\bar{B}$ decays collected with the BABAR detector at the SLAC PEP-II asymmetric-energy $B$ factory. A maximum likelihood fit finds the following branching fractions: $\mathcal{B}(B^0 \to K^0_S \pi^+ \pi^-) = (43.0 \pm 2.3 \pm 2.3) \times 10^{-6}$ and $\mathcal{B}(B^0 \to f_0(\to \pi^+ \pi^-)K^0_S) = (5.5 \pm 0.7 \pm 0.5 \pm 0.3) \times 10^{-6}$ and $\mathcal{B}(B^0 \to K^+ \pi^-) = (11.0 \pm 1.5 \pm 0.5 \pm 0.5) \times 10^{-6}$. For these results, the first uncertainty is statistical, the second is systematic, and the third (if present) is due to the effect of interference from other resonances. We also measure the $CP$-violating charge asymmetry in the decay $B^0 \to K^+ \pi^-$, $\mathcal{A}_{K^+\pi} = -0.11 \pm 0.14 \pm 0.05$.

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Measurements of charmless three-body $B$ decays, which are dominated by their intermediate quasi-two-body decays, are important in furthering our understanding of quark couplings described by the Cabibbo-Kobayashi-Maskawa matrix [1]. $CP$ violation can be probed through the investigation of neutral $B$-meson decays to resonance channels with the final state $K_S^0 \pi^+ \pi^-$, such as $f_0 K_S^0$ [2], $\rho^0 K_S^0$ [3] and $K^+ \pi^-$ [4].

By measuring the charmless branching fraction of $B^0 \to K^0_S \pi^+ \pi^-$, along with those of its dominant resonant submodes, we can obtain information about the structure of the decay Dalitz plot. Such measurements have previously been performed by the CLEO [5], Belle [6] and BABAR [2-4] experiments.

QCD factorization models [7] have predicted branching fractions and asymmetries for charmless $B$ decays. Predictions have also been made using flavor SU(3) symmetry [8]. For $B^0 \to K^+ \pi^-$, predictions [9] have been made for the branching fractions and charge asymmetry,

$$ \mathcal{A}_{K^+\pi} = \frac{\Gamma_{B^0 \to K^+ \pi^-}}{\Gamma_{B^0 \to K^0_S \pi^+ \pi^-}} - \frac{\Gamma_{B^0 \to K^+ \pi^-}}{\Gamma_{B^0 \to K^0_S \pi^+ \pi^-}} - \frac{\Gamma_{B^0 \to K^+ \pi^-}}{\Gamma_{B^0 \to K^0_S \pi^+ \pi^-}}, $$

which is a $CP$-violating quantity since the decay channel is a flavor eigenstate. $CP$ violation in charge asymmetry has already been observed by BABAR and Belle in $B^0 \to K^+ \pi^-$ [10].

In this paper the branching fractions of $B^0 \to K^0_S \pi^+ \pi^-$, $B^0 \to K^+ \pi^-$ and $B^0 \to f_0(980)(\to \pi^+ \pi^-)K^0_S$ are presented, averaged over charge-conjugate states, along with a measurement of the charge asymmetry in $B^0 \to K^+ \pi^-$. The selection criteria require events with a reconstructed $K^0_S$ in the final state. Results are stated in terms of the $K^0_S$ final state, taking into account the probabilities for $\mathcal{B}(K^0 \to K^0_S)$ and $\mathcal{B}(K^0_S \to \pi^+ \pi^-)$ [11]. For the $B^0 \to K^0_S \pi^+ \pi^-$ branching fraction, the total charmless contribution to the Dalitz plot is measured (with charmed and charmonium resonances removed), including contributions from resonant charmless substructure.

The data used in this analysis were collected at the PEP-II asymmetric-energy $e^+e^-$ storage ring with the BABAR detector [12]. The BABAR detector consists of a double-sided five-layer silicon tracker, a 40-layer drift chamber, a Cherenkov detector, an electromagnetic calorimeter and a magnet with instrumented flux return. The data sample has an integrated luminosity of 210 fb$^{-1}$ collected at the $\Upsilon(4S)$ resonance, which corresponds to $(231.8 \pm 2.5) \times 10^6 B\bar{B}$ pairs. It is assumed that the $\Upsilon(4S)$ decays equally to neutral and charged $B$-meson pairs. In addition, 21.6 fb$^{-1}$ of data collected at 40 MeV below the $\Upsilon(4S)$ resonance were used for background studies.

The reconstruction of candidate $B$ mesons combines two charged tracks and a $K^0_S$ candidate, with the $K^0_S$ being reconstructed from two oppositely charged tracks consistent with $\pi^+ \pi^-$. The $B^0$ decay vertex is reconstructed from the two charged tracks that were not daughters of the $K^0_S$, with the requirements that the tracks originate from the beam-spot, have at least 12 hits in the drift chamber and have a transverse momentum greater than 100 MeV/c. $K^0_S$ candidates are required to have a reconstructed mass within 15 MeV/c$^2$ of the nominal $K^0_S$ mass [11], at least a 5 standard deviation separation between the $B^0$ decay vertex and its own decay vertex, and a cosine of the angle between the line joining the $B^0$ and $K^0_S$ decay vertices and the $K^0_S$ momentum vector greater than 0.999. To identify pions we use measurements of energy loss (dE/dx) in the tracking system, the number of photons detected by the Cherenkov detector and the corresponding Cherenkov angle. Candidate pions must fail the electron selection, which is based on dE/dx measurements, shower shape in the calorimeter, and the ratio of energy in the calorimeter to momentum in the drift chamber. Using simulated Monte Carlo (MC) events, we determine an approximate mean and width ($\sigma$) of the mass distribution for the resonances, and choose the resonance band to be $\pm 3\sigma$ from the mean. For the decay $B^0 \to K^+ \pi^-$ we require 0.776 < $m_{K^0_S \pi^+} < 1.010$ GeV/c$^2$ and for $B^0 \to f_0 K^0_S$ we require 0.879 < $m_{\pi^+ \pi^-} < 1.069$ GeV/c$^2$.

The dominant source of background is continuum quark production ($e^+e^- \to q\bar{q}$ where $q = u, d, s, c$). An event-shape variable, the cosine of the angle $\theta_T$ between the
thrust axis of the selected B candidate and the thrust axis of the rest of the event [12], is used to suppress this background. The distribution of $|\cos \theta_3|$ is strongly peaked towards unity for continuum background but is flat for signal events. The requirement $|\cos \theta_3| < 0.9$ reduces the relative amount of continuum background.

To separate signal events from the remaining background events, we use two kinematic variables and one event-shape variable. The first kinematic variable $\Delta E$, is the difference between the center-of-mass (CM) energy of the B candidate and $\sqrt{s}/2$, where $\sqrt{s}$ is the total CM energy of the $e^+e^-$ beams. The second is the beam-energy-substituted mass $m_{\text{ES}} = \sqrt{(s/2 + p_B \cdot p_B/E_1^\gamma - p_B^2)$, where $p_B$ is the B momentum and $(E_\gamma, p_\gamma)$ is the four-momentum of the $Y(4S)$ in the laboratory frame. We require these variables to be in the ranges $|\Delta E| < 0.1$ GeV and $5.22 < m_{\text{ES}} < 5.29$ GeV/c$^2$. We construct a Fisher discriminant ($F$) [13] using a linear combination of five event-shape variables: the cosine of the angle between the B-candidate momentum and the beam axis, the cosine of the angle between the B-candidate thrust axis and the beam axis, the zeroth and second angular moments of the energy flow about the thrust axis of the B [2], and the output of the B-flavor tagging algorithm, which uses the information from the other B [14]. This forms a more efficient Fisher discriminant than used in our previous measurement, Ref. [4].

Other B-meson decays can mimic a $K^0_S\pi^+\pi^-$ final state. MC events are used to identify the B decays that contribute background events to the data sample, and we use the available information on exclusive measurements [11,15] to find how many events from this background to expect in the data set. The largest B background is seen to come from quasi two-body decays including charmonium mesons such as $J/\psi K_S^0$, $\chi_{c0} K_S^0$ and $\psi(2S) K_S^0$. In these cases the charmonium meson decays to $\pi^+\pi^-$ or to $\mu^+\mu^-$ that are misidentified as pions. Most of these events are removed by vetoing the reconstructed $\pi^+\pi^-$ masses consistent with $3.04 < m_{\pi^+\pi^-} < 3.16$ GeV/c$^2$, $3.32 < m_{\pi^+\pi^-} < 3.51$ GeV/c$^2$ and $3.63 < m_{\pi^+\pi^-} < 3.74$ GeV/c$^2$, identify-}

TABLE I. The B-background modes for the channels $B^0 \rightarrow K^{\pm}\pi^\mp$ and $B^0 \rightarrow f_0 K_S^0$, $B^0 \rightarrow \rho^0 K_S^0$ is included at a level consistent with Ref. [3]. $K^{\ast\ast}$ refers to heavier $K$ resonances, e.g. $K_0^{\ast\ast}(1430)$.

<table>
<thead>
<tr>
<th>B-background Mode</th>
<th>Number Expected $(B^0 \rightarrow K^{\ast\ast}\pi^\mp)$</th>
<th>Number Expected $(B^0 \rightarrow \rho^0 K_S^0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow K^{\ast\ast}\pi^\mp$</td>
<td>$-$</td>
<td>$5 \pm 1$</td>
</tr>
<tr>
<td>$B^0 \rightarrow f_0 K_S^0$</td>
<td>$4 \pm 1$</td>
<td>$-$</td>
</tr>
<tr>
<td>$B^0 \rightarrow \rho^0 K_S^0$</td>
<td>$5 \pm 2$</td>
<td>$14 \pm 4$</td>
</tr>
<tr>
<td>$B^0 \rightarrow K^{\ast\ast}\pi^\mp$</td>
<td>$23 \pm 3$</td>
<td>$4 \pm 1$</td>
</tr>
<tr>
<td>Nonresonant</td>
<td>$7 \pm 1$</td>
<td>$5 \pm 1$</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{\ast}\pi^\mp$</td>
<td>$16 \pm 2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$B^0 \rightarrow \eta' K_S^0$</td>
<td>$1 \pm 1$</td>
<td>$19 \pm 7$</td>
</tr>
<tr>
<td>$B^0 \rightarrow J / \psi K_S^0$</td>
<td>$6 \pm 1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

In a signal MC study, selecting the candidate whose $|\cos \theta_3|$ value is closest to zero is found to select the true signal candidate in $69.2\%$ of such events. These requirements result in a final sample size of approximately 80,000 events.

After all requirements, the largest charless B background to the $B^0 \rightarrow K^0_S\pi^+\pi^-$ measurement is the decay $B^0 \rightarrow \eta' K_0^0$, $\eta' \rightarrow \rho^0(770)\gamma$, $\rho^0 \rightarrow \pi^+\pi^-$, which tends to peak in the signal region and which contributes $54 \pm 19$ events. Table I shows the B-background modes for the $B^0 \rightarrow K^{\ast\ast}\pi^\mp$ and $B^0 \rightarrow f_0 K_S^0$ channels. These events are effectively subtracted from the measured signal. To measure the nonresonant $B^0 \rightarrow K^0_S\pi^+\pi^-$, we select a region of the Dalitz plot believed to be free of resonances, $(3 < m_{\pi^+\pi^-} < 4$ GeV/c$^2$ and $m_{K^0_S\pi^-} > 1.91$ GeV/c$^2$). Backgrounds from other B decays and from continuum events are subtracted. Assuming a uniform nonresonant distribution in the Dalitz plane, we set an upper limit of $2.1 \times 10^{-6}$ at a 90% confidence level on the nonresonant $B^0 \rightarrow K^0_S\pi^\mp\pi^-$ branching fraction. All other branching fractions are taken from Refs. [11,15].

We use an extended maximum likelihood fit to extract the signal yield for each of the channels being investigated. The likelihood function for $N$ events is:

$$L = \exp \left( -\sum_j N_j \prod_{i=1}^N (N_i P_j(\hat{x}_i)) \right)$$

(2)

where $i$ and $j$ are integers, $M$ is the number of hypotheses (signal, continuum background and B background), $N_j$ is the number of events for the $j$th hypothesis determined by maximizing the likelihood function, and $P_j(\hat{x}_i)$ is a probability density function (PDF) evaluated using the vector $\hat{x}_i$, in this case $m_{\text{ES}}, \Delta E$, and $F$. Correlations between these variables are small for signal and continuum background hypotheses and the total PDF is a product $P_j(\hat{x}_i) = P_j(m_{\text{ES}}) \cdot P_j(\Delta E) \cdot P_j(F)$. However for B background, it is necessary to account for correlations observed between
m_{ES} and \( \Delta E \) by using a two-dimensional PDF for these variables.

The parameters of the signal and \( B \)-background PDFs are determined from MC simulation and fixed in the fit, along with the \( B \)-background normalization. The continuum background parameters are allowed to vary in the fit, to help reduce systematic effects from this dominant event type. Sideband data (which lie in the region \( 0.1 < \Delta E < 0.3 \text{ GeV} \) and \( 5.22 < m_{ES} < 5.29 \text{ GeV}/c^2 \)) are used to model the continuum background PDFs. For the \( m_{ES} \) PDFs, a Gaussian distribution is used for signal and a threshold function [16] for continuum. For the \( \Delta E \) PDFs, a sum of two Gaussian distributions with the same means is used for the signal and a first-order polynomial for the continuum background. Finally, for the \( F \) PDFs, a sum of two Gaussian distributions with distinct means and widths is used for signal and a sum of two Gaussian distributions with the same means is used to model the continuum background. The Fisher discriminant distribution of the \( B \) backgrounds is modeled by an asymmetric Gaussian distribution that has different widths above and below the modal value. We use \( B^0 \to D^- (\to K^0 \pi^-) \pi^+ \) as a calibration mode since it exhibits a one-to-one signal to continuum background ratio, allowing the signal parameters in a fit to be floated. A fit to these data is used in order to quantize any corrections and uncertainties due to MC. These corrections are applied to the fits to the charmless data sample.

To extract the branching fractions for the decay modes \( B^0 \to K^{*+} \pi^- \) and \( B^0 \to f_0 K^0 \) we use the relation

\[
\mathcal{B} = \frac{N_{\text{sig}}}{2N_{B^0\bar{B}^0} \varepsilon},
\]

where \( N_{\text{sig}} \) is the number of signal events fitted, \( \varepsilon \) is the signal efficiency obtained from MC and \( N_{B^0\bar{B}^0} \) is the total number of \( B^0\bar{B}^0 \) pairs.

For the charmless \( B^0 \to K^0 \pi^+ \pi^- \) branching fraction (and also for the nonresonant upper limit in the \( B \)-background studies above), it is necessary to account for the variation in efficiency, between approximately 5% and 40%, across the Dalitz plot and to know how the signal events are distributed across the Dalitz plot. To do this we assign to the \( j \)th event \( W_j = \sum_i V_{sig,i} \tilde{P}_i (\tilde{x}_j) / \sum_i \delta_i N_i P_i (\tilde{x}_j) \) where \( V_{sig,i} \) are the signal components of the covariance matrix obtained from the fit. This procedure projects out the signal distributions [17] shown in Figs. 1–4. The branching fraction is then calculated as \( \mathcal{B} = \sum_j W_j / (\varepsilon_j N_{B^0\bar{B}^0}) \), where \( \varepsilon_j \) is the efficiency, as a function of Dalitz plot position, simulated in small bins using high statistics MC.

Figure 1 shows the signal distributions for \( B^0 \to K^0 \pi^+ \pi^- \) candidates and the distributions of events for all hypotheses. Figure 2 shows the signal distributions for both the \( B^0 \to K^{*+} \pi^- \) and \( B^0 \to f_0 K^0 \) channels. The fitted signal yield and measured branching fraction are shown in Table II for all the modes under study. The average efficiency for \( B^0 \to K^0 \pi^+ \pi^- \) signal events is 16.8% and the continuum background yield is \( 79000 \pm 280 \) events. Figure 3 shows the signal mass projections of \( m_{K_0^0\pi} \) and \( m_{\pi^+ \pi^-} \) using \( B^0 \to K^0 \pi^+ \pi^- \) candidates. The \( m_{K_0^0\pi} \) distribution clearly shows a peak at \( 0.9 \text{ GeV}/c^2 \), corresponding to the \( K^+ \) (892) mass and there is a broad structure above \( 1 \text{ GeV}/c^2 \) that is the region where heavier kaon resonances can occur. The \( m_{\pi^+ \pi^-} \) distribution shows evidence for resonance structure around 1 GeV/c^2 that corresponds to the \( f_0 \) and a broader structure below this that may be attributed as the \( \rho^0(770) \). Figure 4 shows the efficiency corrected signal distribution of the cosine of the helicity angle, \( \theta_H \), for \( B^0 \to K^{*+} \pi^- \).

Table III shows the systematic uncertainties that are assigned to the branching fraction measurements. Control channels in data and MC are used to assign uncertainties due to pion tracking, particle identification, and \( K_0^0 \) reconstruction efficiency. To calculate uncertainties due to the fitting procedure, a large number of MC samples are generated from the fitted PDFs, containing the amounts of signal and continuum events that are measured in data and the number of \( B \)-background events that were antici-

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**FIG. 1 (color online).** Plots of the maximum likelihood fit to data for \( B^0 \to K^0 \pi^+ \pi^- \) candidates. Plots (a)–(c) show the distributions of all events that pass the selection criteria for (a) \( m_{ES} \) (b) \( \Delta E \) and (c) Fisher, with the solid (blue) line indicating the total model, the (red) dotted line indicating shape of the continuum background model and the (black) dashed line indicating the signal model. Plots (d)–(f) show the signal distributions for (d) \( m_{ES} \), (e) \( \Delta E \) and (f) Fisher, where the (black) circles are the signal distribution [17] and the solid (blue) curve is the signal PDF that was fitted in the maximum likelihood fit.
The efficiency of selecting and any third uncertainty is due to possible interference effects. This uncertainty is statistical and where, in the case of the branching fraction, takes into account that the branching fraction into account that the branching fraction into account that the branching fraction.

TABLE II. Signal yields and branching fractions for $B^0 \rightarrow K^0 \pi^+ \pi^-$, $B^0 \rightarrow K^+ \pi^-$, and $B^0 \rightarrow f_0 K^0$ where the first uncertainty is statistical and where, in the case of the branching fraction measurements, the second uncertainty is systematic and any third uncertainty is due to possible interference effects. The efficiency of selecting $B^0 \rightarrow K^+(-K^0_{2} \pi^+) \pi^-$ and $B^0 \rightarrow f_0(\pi^+ \pi^-) K^0$ events was found to be 24% and 27% respectively, while the continuum background yields were $7300 \pm 86$ events and $13000 \pm 110$ events, respectively. The $B^0 \rightarrow K^+ \pi^-$ branching fraction takes into account that $B(K^+ \rightarrow K^0 \pi^-) = 2/3$, assuming isospin symmetry.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Signal Events (×10)</th>
<th>Branching Fraction</th>
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<tbody>
<tr>
<td>$B^0 \rightarrow K^0 \pi^+ \pi^-$</td>
<td>$860 \pm 47$</td>
<td>$43.0 \pm 2.3 \pm 2.3$</td>
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<tr>
<td>$B^0 \rightarrow f_0(\pi^+ \pi^-) K^0$</td>
<td>$120 \pm 16$</td>
<td>$5.5 \pm 0.7 \pm 0.6 \pm 0.3$</td>
</tr>
<tr>
<td>$B^0 \rightarrow K^+ \pi^-$</td>
<td>$140 \pm 19$</td>
<td>$11.0 \pm 1.5 \pm 0.5 \pm 0.4$</td>
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</table>

FIG. 2 (color online). Maximum likelihood fits for signal distributions. For $B^0 \rightarrow K^+ \pi^-$ the plots show (a) $m_{ES}$, (b) $\Delta E$, and (c) the Fisher discriminant. The (black) circles are the signal distribution extracted from the data with the method of Ref. [17] and the solid curve is the signal PDF that resulted from the maximum likelihood fit. For $B^0 \rightarrow f_0 K^0$, plots show the distributions for (d) $m_{ES}$, (e) $\Delta E$, and (f) the Fisher discriminant, in an analogous fashion.

FIG. 3 (color online). (a) shows the $m_{K^0 \pi^+}$ signal distribution of $B^0 \rightarrow K^0 \pi^+ \pi^-$ candidates [17]. The one-dimensional distribution obtained by merging $m_{K^+ \pi^-}^2$ and $m_{K^0 \pi^-}^2$ into one ($m_{K \pi}^2$) by folding the Dalitz plane along the line corresponding to $m_{K \pi}^2 = m_{K \pi}^2$ in order to obtain the above $m_{K \pi}$ mass distribution. (b) shows the $m_{\pi^+ \pi^-}$ signal distribution of $B^0 \rightarrow K^0 \pi^+ \pi^-$ candidates [17]. The dashed lines indicate the expected mass of the labeled resonances.

FIG. 4. Distribution of the efficiency corrected cosine of the helicity angle, $\theta_H$, for $B^0 \rightarrow K^{\pm} \pi^\mp$ signal events.
The uncertainty due to simulated PDFs is obtained from the uncertainties in the branching fractions measurements of $B^0 \to K^0 \pi^+ \pi^-$, $B^0 \to K^{*+} \pi^-$ and $B^0 \to f_0 K^0$. The uncertainties are shown as a percentage of the measured branching fraction.

<table>
<thead>
<tr>
<th>Error source</th>
<th>$B^0 \to K^0 \pi^+ \pi^-$</th>
<th>$B^0 \to f_0 K^0$</th>
<th>$B^0 \to K^{*+} \pi^-$</th>
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<td>PDF params.</td>
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<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$B$ background</td>
<td>4.2</td>
<td>5.9</td>
<td>2.0</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.9</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>No. of $B\bar{B}$</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5.4</td>
<td>9.1</td>
<td>4.5</td>
</tr>
<tr>
<td>Interference</td>
<td>-</td>
<td>4.7</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The uncertainty of the $B$-background contribution to the fit is estimated by varying the measured branching fractions within their uncertainties. Each background is varied by $\pm 1\sigma$ [11] and the effect on the fitted signal yield is added as a contribution to the uncertainty. For $B^0 \to K^{*+} \pi^-$ there is an additional uncertainty in the $B$-background contributions due to the possible lineshapes of the $K_0^{\pm}(1430)$, which can alter the amount of $B$-background expected. In order to assign a systematic uncertainty, fits to data are performed using two parametrizations, a relativistic Breit–Wigner lineshape and the LASS parametrization [18]. The latter is a coherent sum of a relativistic Breit-Wigner and an effective range term, and is used in the analysis of $B^\pm \to K^\pm \pi^\mp \pi^\pm$ [19]. The uncertainty due to simulated PDFs is obtained from the channel $B^0 \to D^- (\to K^0_S \pi^0) \pi^\pm$ and by varying the PDFs according to the precision of the parameters obtained from MC. In order to take correlations between parameters into account, the full correlation matrix is used when varying parameters. All PDF parameters that are originally fixed in the fit are then varied in turn and each difference from the nominal fit is combined and taken as a systematic uncertainty. The uncertainty in the efficiency is due to limited MC statistics, where over 1 000 000 MC events are generated for the decay $B^0 \to K^0 \pi^+ \pi^-$ and over 150 000 MC events are generated for the decays $B^0 \to K^{*+} \pi^-$ and $B^0 \to f_0 K^0$. The same uncertainty in the number of $B\bar{B}$ events is used for all channels.

For the quasi two-body modes, possible interference effects between the final state modes were investigated by simulating the Dalitz plot using the measured branching fractions and random phases. The root-mean-squared of the distribution of the branching fraction is taken to be the uncertainty.

We measure the $CP$-violating charge asymmetry for the decay $B^0 \to K^{*+} \pi^-$ to be $\mathcal{A}_{K^{*+}\pi^-} = -0.11 \pm 0.14 \pm 0.05$, where the first uncertainty is statistical and the second uncertainty is systematic. The charge asymmetry in the background is expected to be zero, as is the charge asymmetry in signal and background of the self-tagging decay $B^0 \to D^- \pi^+$. As a cross-check, these are measured to be $-0.018 \pm 0.009, -0.013 \pm 0.029$ and $0.005 \pm 0.031$ respectively, where the uncertainties are statistical only.

The systematic uncertainty on $\mathcal{A}_{K^{*+}\pi^-}$ is calculated by considering contributions due to track finding, particle identification, fit biases and $B$-background asymmetry uncertainties. Biases due to track finding and particle identification were found to be negligible. The fit-bias contribution to the systematic uncertainty is calculated using a large number of MC samples. The contribution from $B$ background is calculated by varying the number of expected events within their uncertainties [11] and by assuming a conservative $CP$-violating asymmetry of $\pm 0.5$ as there are no available measurements for these decays. The resulting systematic uncertainty on the asymmetry is measured to be $\pm 0.05$.

In summary, the branching fractions for $B^0 \to K^0 \pi^+ \pi^-$, $B^0 \to K^{*+} \pi^-$, and $B^0 \to f_0 (\to \pi^+ \pi^-) K^0$ decaying to a $K^0_S \pi^- \pi^+$ state are measured and all agree with previous measurements [2,4–6]. We measure the direct $CP$-violating parameter $\mathcal{A}_{K^{*+}\pi^-}$ for the decay $B^0 \to K^{*+} \pi^-$, with no evidence of $CP$ violation with the statistics used. These results supersede the previous results of the BABAR Collaboration [2,4].

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