Title
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Overreaction, Delayed Reaction, and Contrarian Profits

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ABSTRACT

This paper presents a decomposition of short-horizon contrarian profits into various sources based on an analysis of stock price reactions to common factors and firm-specific information. In sharp contrast with the conclusions in the extant literature, we find that the lead-lag structure in stock prices contributes less than 5% of the observed contrarian profits with most of the profits being attributable to stock price overreaction.
A number of recent papers present evidence that stock returns are predictable, and more importantly, show that trading strategies based on these empirical regularities yield significant profits. The consistent profitability of short-horizon contrarian strategies is particularly striking. For example, Jegadeesh (1990) documents profits of about 2% per month from a contrarian strategy that buys and sells stocks based on their previous month's return and holds them one month. A similar strategy applied to weekly formation and holding periods examined by Lehmann (1990) earns about 2% per week and generates positive profits in every 6 month period in his sample.

These short-term contrarian profits were initially regarded as evidence that market prices tend to overreact to information. Viewed from this perspective, the evidence has important policy implications. Some argue that the overreaction is caused by speculative trading and recommend policy initiatives to discourage short-term speculation, (e.g. Stiglitz (1989) and Summers and Summers (1989)). Another possibility is that the overreaction indicates that the market lacks sufficient liquidity to offset the short-term price swings caused by unexpected buying and selling pressure, (e.g. Grossman and Miller (1988) and Jegadeesh and Titman (1992)).

A recent paper by Lo and MacKinlay (1990) suggests that these earlier interpretations may be premature and demonstrates that the profitability of contrarian strategies need not imply that market prices overreact to information. They identify a second potential source of contrarian profits that arises when some stocks react more quickly to information than do others, or equivalently, if the returns of some stocks lead the returns of others. For example, if stock A leads stock B, a contrarian strategy may profit from buying stock B subsequent to an increase in stock A and selling stock B subsequent to a decline in stock A even if neither overreacts to information.
To analyze the various sources of contrarian profits, Lo and MacKinlay examine the returns of a portfolio with weights inversely proportional to each stock's past returns less the return on the equally-weighted index. This portfolio has the property that its expected profits can be easily decomposed into three components: a component due to the dispersion of expected returns, a component due to the serial covariances of returns and a final component due to the cross-autocovariances of returns. To assess the importance of the lead-lag structure, and to measure its contribution to contrarian profits, they examine the component due to the cross-autocovariances.

The pattern of cross-autocovariances documented by Lo and MacKinlay implies a size dependent lead-lag structure.¹ They find large positive covariances between the returns of small stocks and the lagged returns of large stocks but virtually no correlation between the returns of large stocks and lagged small stock returns. Based on this evidence as well as other empirical tests Lo and MacKinlay conclude that "a systematic lead-lag relationship among returns of size-sorted portfolios is an important source of contrarian profits." They further argue that "less than 50 percent of the profit from a contrarian investment rule may be attributed to overreaction."

From an academic perspective, it is of interest to identify the source of contrarian profits since it poses a formidable challenge to the efficient market hypothesis. In addition, the Lo and Mackinlay conclusions could potentially have important implications for investment practice. For instance, if the lead-lag relation across size-sorted portfolios were in fact a major source of contrarian profits, then investors can trade profitably in size-related portfolios rather than in individual stocks at lower costs.

The evidence documented in this paper, however, indicates that contrarian

¹See also Conrad, Kaul and Nimalendran (1991).
strategies applied to size-sorted portfolios do not generate significant abnormal profits. Specifically, the Lo and MacKinlay contrarian strategy applied to 50 size-sorted portfolios actually generates small negative returns despite the fact that the cross-autocovariances between these portfolios are significantly positive. This finding indicates that the average cross-autocovariance may be a misleading measure of the contribution of the lead-lag structure to the profitability of contrarian strategies.

This paper provides a different measure of contrarian profits due to the lead-lag structure and market overreaction by considering stock price reactions to common factors and firm-specific information. We consider the effect of stock price reactions to common factors and firm specific information on contrarian profits separately for two reasons. First, by definition, the lead-lag structure in stock returns arises because of differences in the timeliness of stock price reactions to common factors and not because of their reactions to firm-specific information. Therefore, by measuring the contrarian profits due to delayed reactions to common factors we are able to assess the importance of the source of lead-lag effects for contrarian profits. Secondly, over or underreaction to common factors affect the contrarian profits differently from over or underreaction to firm specific information. For instance, as we show here, even if stock prices systematically overreact to common factors it is possible that the contrarian strategies will not be profitable. In contrast, systematic overreactions to firm specific information will always contribute to contrarian profits. Therefore, to address the question posed by Lo and MacKinlay, viz. to what extent does overreaction contribute to contrarian profits, we should separately examine the contribution of over or underreaction to the firm-specific information and the common factors to contrarian profits.

The estimates based on our factor model based decomposition indicate that most of the short horizon contrarian profits arise because of the tendency of stock prices
to overreact to firm-specific information. Less than 5% of the contrarian profit is attributable to the lead-lag structure.

The rest of the paper is organized as follows: The next section provides a brief description of the contrarian strategy we analyze. Section II presents a model that analyzes the various sources of the profits from this strategy. Empirical tests based on this model are described in Section III and Section IV concludes the paper.

I The Lo and MacKinlay Contrarian Strategy

This paper examines a contrarian strategy that buys and sells stocks based on their returns in week $t - 1$ and holds the stocks in week $t$. Because of its analytic tractability, we examine the strategy proposed by Lo and MacKinlay (1990). With this strategy, the portfolio weight ($w_{i,t}$), assigned to stock $i$ at time $t$ is:

$$w_{i,t} = -\frac{1}{N}(r_{i,t-1} - \bar{r}_{t-1}),$$

where $N$ is the number of stocks and $\bar{r}_{t-1}$ is the returns on the equally-weighted index at time $t - 1$. By construction, the total investment at any given time is zero. However, the dollar investments in the long and short sides of the portfolio vary over time depending on the return realizations at time $t - 1$.

The time $t$ profit of this contrarian strategy, denoted as $\pi_t$, is:

$$\pi_t = -\frac{1}{N} \sum_{i=1}^{N} (r_{i,t-1} - \bar{r}_{t-1}) r_{i,t}.$$  

Lo and MacKinlay show that the expected profits of this strategy can be decomposed as:

$$\mathbb{E}(\pi_t) = C - O - \sigma^2,$$

where:

$$C = \mathbb{E}(\bar{r}_t \bar{r}_{t-1}) - \bar{\mu}^2 - \frac{1}{N^2} \sum_{i=1}^{N} \mathbb{E}(r_{i,t} r_{i,t-1} - \mu_i^2)$$

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\[ O = \frac{N-1}{N^2} \sum_{i=1}^{N} \text{E}(r_{i,t}r_{i,t-1} - \mu_i^2), \] and

\[ \sigma_{\mu}^2 = \frac{1}{N} \sum_{i=1}^{N} (\mu_i - \bar{\mu})^2, \]

where \( \bar{\mu} \) is the expected return on the equally-weighted index. In words, \( C \) is the average cross-autocovariance, \( O \) is the average autocovariance of raw returns and \( \sigma_{\mu}^2 \) is the cross-sectional variance of expected returns.

Since equation (3) is a mathematical identity, the sum of the above three components must equal the profits of the contrarian strategy irrespective of the sources of the cross-autocovariances and autocovariances. However, as we show in the next section, since the lead-lag structure will in general affect autocovariances as well as cross-autocovariances, the Lo and MacKinlay decomposition does not generally describe the relative contributions of the lead-lag structure and overreaction to contrarian profits. Indeed, even when stock return predictability is entirely due to the lead-lag structure it is possible for \( C \) to be positive and for the contrarian profit to be negative.

II Sources of Contrarian Profits

This section presents a model of stock returns that is used to analyze the contribution of the various sources of contrarian profits. In addition, we present the conditions under which \( C \) and \( O \) equal the contrarian profits generated by the lead-lag structure and overreaction.

II.1 A Single Factor Model

Consider the following model where stock returns are generated by a single common factor along with firm-specific or idiosyncratic return components. This factor model, described in the following equation, allows stock prices to react instantan-
neously as well as with a one period lag to factor realizations.\textsuperscript{2}

\[ r_{i,t} = \mu_i + b_{0,i}^tf_t + b_{1,i}^tf_{t-1} + e_{i,t}, \]  

(6)

where \( \mu_i \) is the expected return of stock \( i \), \( f_t \) is the unexpected factor realization, \( e_{i,t} \) is the firm-specific component of return at time \( t \) and \( b_{0,i}^t \) and \( b_{1,i}^t \) are the sensitivities of stock \( i \) to the contemporaneous and lagged factor realizations at time \( t \). The factor sensitivities are indexed by \( t \) since the timeliness of a stock’s price reaction to a common factor need not be constant over time. However, we assume that the factor sensitivities are uncorrelated with factor realizations; \( \text{i.e., } E(b_{0,i}^t|f_t, f_{t-1}) = b_{0,i} \) and \( E(b_{1,i}^t|f_t, f_{t-1}) = b_{1,i} \).

In addition, since \( f_t \) is defined as the unexpected factor realization \( \text{cov}(f_t, f_{t-1}) = 0 \), and since the comovements in stock returns are entirely captured by the common factor,

\[ \text{cov}(e_{i,t}, e_{j,t-1}) = 0 \ \forall \ k \ \text{and} \ i \neq j. \]

Given this return generating model, the cross-autocovariance between the returns of \( i \) and \( j \) is:

\[ \text{cov}(r_{i,t}, r_{j,t-1}) = E(b_{1,i}^t b_{0,j}^t) \sigma_f^2, \]  

(7)

where \( \sigma_f^2 \) is the variance of \( f \). As can be seen from the above expression, equation (6) allows for the cross-autocovariances to be asymmetric. For instance, if \( j \) reacts instantaneously to \( f_t \) but \( i \) reacts partially with a delay, \( \text{i.e., } if b_{1,j}^t = 0 \) and \( b_{1,i}^t > 0 \), then \( \text{cov}(r_{i,t}, r_{j,t-1}) > 0 \) but \( \text{cov}(r_{j,t}, r_{i,t-1}) = 0 \). In this case, \( j \) leads \( i \) since \( j \)'s return predicts \( i \)'s return but the reverse is not true.

\textsuperscript{2}For expositional convenience, the return generating process considered here allows for only one common factor and a one period lead-lag relation. Extending the model to allow for multiple factors and lead-lag relations over longer periods is straightforward.
II.2 A Decomposition of Contrarian Profits

This subsection decomposes the profits from the contrarian strategy described in equation (1) into components attributable to stock price reactions to firm-specific information and to common factor realizations. The decomposition of contrarian profits, derived under the assumption that stock returns are generated by the process described by equation (6), is given below:

\[
E(\pi) = -E\left(\frac{1}{N} \sum_{i=1}^{N} (r_{i,t-1} - \bar{r}_{t-1})r_{i,t}\right) \\
= -\sigma_{\mu}^2 - \Omega - \delta \sigma_f^2
\]  

(8)

where:

\[
\Omega \equiv \frac{1}{N} \sum_{i=1}^{N} \text{cov}(e_{i,t}, e_{i,t-1})
\]  

(9)

\[
\delta_t \equiv \frac{1}{N} \sum_{i=1}^{N} (\bar{b}^*_0,i - \bar{b}^*_0)(\bar{b}^*_1,i - \bar{b}^*_1)
\]  

(10)

\[
\delta \equiv E(\delta_t)
\]

and \(\bar{b}_0^i\) and \(\bar{b}_1^i\) are the cross-sectional averages of \(b_0^i\) and \(b_1^i\).

Equation (8) decomposes expected contrarian profits into three components. The first component, \(-\sigma_{\mu}^2\), which also exists in the Lo and MacKinlay decomposition, represents the cross-sectional dispersion in expected returns. Stocks that have higher expected returns tend to experience higher than average returns during both portfolio formation and holding periods and thus reduce contrarian profits. The second component, \(-\Omega\), which is the negative of the average autocovariance of the idiosyncratic component of returns, is determined by stock price reactions to firm-specific information. If stock prices tend to overreact to firm-specific information and correct the overreaction in the following period then \(\Omega\) will be negative and will thereby contribute to contrarian profits. We will refer to this as the overreaction
component of contrarian profits. The last term in (8) is the component of contrarian profits attributable to differences in the timeliness of stock price reactions to common factors. This is the component that gives rise to a lead-lag structure in stock returns. When $\delta < 0$ stock price reactions to factor realizations contribute positively to contrarian profits while the reverse is true if $\delta > 0$. Interestingly, $\delta$ can be greater than zero even when all cross-autocovariances are positive.

To illustrate why cross-autocovariances can provide a misleading indication of the effect of the lead-lag structure on contrarian profits and why the lead-lag structure can reduce as well as contribute to contrarian profits we will consider two examples. The first example illustrates the intuition provided by Lo and MacKinlay and is similar to example 2.3 used in their paper. The second example illustrates why this intuition sometimes fails. In both examples we assume that the stocks are subject to factor risk but not firm-specific risk and that all stocks have the same expected returns so that in these examples the contrarian profits arise solely from the assumed lead-lag structure.

In the first example, stock A, the leading stock, reacts instantaneously to the common factor with assumed factor sensitivities of $b_{0,A}^t = 1$ and $b_{1,A}^t = 0$ for all $t$. For stock B, the lagging stock, the sensitivities to the contemporaneous and lagged factor realizations are specified as $b_{0,B}^t = 0$ and $b_{1,B}^t = .3$. These parameters imply that $\delta = -.15$, and the average contrarian profit equals $.15 \sigma_f^2$, which from expression (7) equals the average cross-autocovariance.

In the second example the factor sensitivities of A are the same as in the first example but the factor sensitivities of B are specified as $b_{0,B}^t = 1.2$ and $b_{1,B}^t = .3$. As in the last example, A leads B, and the average cross-autocovariance is $C = .15 \sigma_f^2 > 0$. However, in this example, since $\delta = .015 > 0$, the expected contrarian profit is negative, and equals $-.015 \sigma_f^2$. To understand this, note that when the
factor realization is high the return of stock B will be higher than the return of stock A implying that a contrarian strategy will sell B and buy A. Since part of stock B’s reaction to the positive factor realization is delayed, its return in the following period will on average be higher than the return of stock A.

In the second example, as in the first, there is only a single source of contrarian profits, the delayed reaction of stock B to the common factor. The Lo and MacKinlay decomposition, however, identifies two distinct sources of contrarian profits based on the autocovariances and cross-autocovariances but attributes only the latter to the lead-lag effect. Since the average autocovariance is positive in the second example, the average cross-autocovariance overestimates the contrarian profit due to the lead-lag effect. As we show in the next subsection, the average cross-autocovariance will in general equal the contribution of the lead-lag relation to contrarian profits only in the cases illustrated by the first example, i.e., when some stocks react instantaneously to the common factor while others do not react to common factors contemporaneously, but react completely with a lag.

II.3 Delayed Reactions, Cross-Autocovariances and Autocovariances

This subsection derives the average cross-autocovariances and average autocovariances given the return generating process described by equation (6). These derivations allow us to compare the Lo and MacKinlay decomposition with the decomposition given in expression (8) and thus provide conditions necessary for C and O to equal the lead-lag and overreaction components of contrarian profits.

We first examine the average cross-autocovariances using the return generating process described in equation (6). From expressions (4), (6) and (7),

\[ C = \frac{1}{N} \sum_{i=1}^{N} b_{it}^1 b_{i,t+1}^1 \sigma_i^2. \]
The contrarian profit due to the lead-lag structure in expression (8) is:

$$-\delta \sigma_f^2 = -\frac{1}{N} \mathbb{E}(\sum_{i=1}^{N} b_{0,i}^t b_{1,i}^t)\sigma_f^2 + \mathbb{E}(b_0^t b_1^t)\sigma_f^2. \quad (12)$$

A comparison of the above expressions indicates that the contribution of the lead-lag structure to contrarian profits equals the average cross-autocovariance only when the second term in expression (11) equals the first term in expression (12). This condition will generally be met only when either $b_{0,i}^t$ or $b_{1,i}^t$ equal zero for all stocks. In other words, in general, cross-autocovariances measure the contribution to contrarian profits only when some stocks react instantaneously to the common factor, (for these stocks $b_{0,i}^t \neq 0$ and $b_{1,i}^t = 0$) and other stocks exhibit no contemporaneously reaction (not even partially) to the common factor but react with a one-period lag (for these stocks $b_{0,i}^t = 0$ and $b_{1,i}^t \neq 0$).

We next examine the average autocovariance. From expression (5) and the return generating process given in equation (6) we get the following expression for the average autocovariance:

$$O = \frac{1}{N} \mathbb{E}(\sum_{i=1}^{N} b_{0,i}^t b_{1,i}^t)\sigma_f^2 + \frac{1}{N} \sum_{i=1}^{N} \text{cov}(e_{i,t}, e_{i,t-1}). \quad (13)$$

The overreaction component of contrarian profit $\Omega$ given in expression (9) equals $O$ only if for some stocks $b_{0,i}^t \neq 0$ and $b_{1,i}^t = 0$ and the others $b_{0,i}^t = 0$ and $b_{1,i}^t \neq 0$. When some stocks react to the common factor partly contemporaneously and partly with a delay so that $b_{0,i}^t > 0$ and $b_{1,i}^t > 0$, the delayed reaction induces a positive autocovariance in returns. Therefore, delayed reaction at least partly masks the overreaction component of contrarian profits when $O$ is used as a measure of market overreaction.\(^3\)

\(^3\)Lo and MacKinlay observe that estimates of the cross-autocovariances and autocovariances are negatively correlated and conjecture that this occurs "perhaps as a result of co-skewness or kurtosis." The analysis here suggests that this correlation arises because of the functional relation between $C$ and $O$. To see this, let the expected values of the cross-autocovariances and autocovariances at time $t$ conditional on the factor realisation at time $t-1$ be $C_t$ and $O_t$ respectively. Given the return
III Empirical Tests

This section presents empirical tests that examine the relative importance of the different sources of contrarian profits. The sample period is 1963 to 1990. All firms traded on the New York Stock Exchange and the American Stock Exchange that had at least 260 consecutive weeks of return data are included in the sample. The 260-week data availability requirement is imposed since we examine autocovariance estimates in some of the tests and it is well known that these estimates are biased downward in small samples. In addition, stocks with prices below $1 are also excluded since a large fraction of the price changes of these stocks are due to the bid-ask bounce. On average, there are 1987 firms in the sample each week.

Table 1 reports the average profits of the Lo and MacKinlay contrarian strategy described in Section I implemented on the full sample as well as on five size-sorted subsamples. To put these profits in perspective we also report the profits to a contrarian strategy that normalizes the investments in the long and short positions to $1, as in Lehmann (1990). With the latter strategy, the average contrarian profit is 1.37% per week per dollar long for the entire sample and are are monotonically related to size; the contrarian profits for the small and large firm subsamples are generating process (6), $C_t$ and $O_t$ equal:

$$C_t = b_0 b_{1,t} f_{t-1} - \frac{1}{N} \sum_{i=1}^{N} b_{0,i} b_{1,i} f_{t-1}^2 \text{ and}$$

$$O_t = \frac{1}{N} \sum_{i=1}^{N} (b_{0,i} b_{1,i} f_{t-1})^2 + \frac{1}{N} \sum_{i=1}^{N} \text{cov}(e_{i,t}, e_{i,t-1}).$$

The second component of $C_t$ becomes arbitrarily small in large samples. The changes in both $C_t$ and $O_t$ are driven by the common factor realisation at time $t-1$. If $b_{0,i} > 0$ and $b_{1,i} > 0$ then both $C_t$ and $O_t$ are positive functions of $f_{t-1}^2$. As a result, $C_t$ and $-O_t$ will be negatively correlated.

41963 is the first full calendar year covered by the CRSP daily stock return database and 1990 was the last year in this dataset at the time this study was initiated.

5Our conclusions are not sensitive to any of these exclusion criteria although the contrarian profits were larger when these conditions were not imposed.

6Stocks are assigned to size subsamples when they first enter the sample.
2.43% and .6% respectively.

Table 1 also reports the profits of the contrarian strategy implemented on 50 size-sorted portfolios. As we stated at the outset, if the size-related lead-lag structure is an important source of contrarian profits then we expect this contrarian strategy to be profitable. The average profit of the contrarian strategy implemented on these portfolios, however, is not different from zero (−.02%).

This observation implies that the lead-lag structure across size-sorted portfolios cannot be exploited using the contrarian strategy. As we show later, the contrarian strategy fails in this case because small firms tend to have higher than average betas both with respect to contemporaneous and lagged common factor realizations.

To examine the various sources of contrarian profits, we first estimate the sensitivities of weekly individual stock returns to contemporaneous and lagged factor returns. The CRSP value-weighted index (VWI) is used as the proxy for the common factor and the following time-series regression is fitted:

\[ r_{it} = a_i + b_{0,i} r_{VW I,t} + b_{1,i} r_{VW I,t-1} + e_{i,t}, \]  

(14)

where \( r_{i,t} \) and \( r_{VW I,t} \) are the time \( t \) returns of security \( i \) and the VWI respectively.

Table 2 presents the average estimates of the slope coefficients in regression (14) for the entire sample and for size-sorted quintiles. The average contemporaneous beta is 1.0594 and the average lagged beta is .1631. The lagged betas for the quintile of the smallest firms is .2350 while that for the quintile of largest firms is close to zero. These results indicate that the large firms react almost instantaneously to the common factor while the small firms react with a delay. As a result, the large firms lead the small firms but the reverse in not true.

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7 The average profit for the Lo and MacKinlay strategy is significantly below zero while the average profit per dollar long is not reliably less than zero. This result is due to the fact that the dollar investment in the long or short side of the Lo and MacKinlay contrarian portfolio is correlated with the rate of return per dollar long.
To examine whether the lead-lag structure in stock returns could potentially contribute to contrarian profits, we examine the cross-sectional covariance of contemporaneous and lagged betas, defined as:

$$\hat{c} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}\{(b_{0,i} - \bar{b}_0)(b_{1,i} - \bar{b}_1)\}. \quad (15)$$

$\hat{c}$ defined above provides an estimate of $c$ defined in equation (10) under the assumption that the contemporaneous and lagged betas do not vary over time. As reported in Table 2, $\hat{c}$ is negative for all size quintiles except the large firm quintile and it is also negative for the full sample, suggesting that the lead-lag structure could potentially contribute to contrarian profits. Our subsequent tests assess the magnitude of this contribution.

The contrarian profits due to overreaction to firm-specific information and delayed reaction to common factors are given by expressions (9) and (12) respectively. Assuming, for now, that the factor sensitivities are constant, the average autocovariance of the error terms from regression (14) provides an estimate of the overreaction component of contrarian profits. This estimate is $-0.2881 \times 10^{-3}$ which is larger in magnitude than the contrarian profit of $0.2619 \times 10^{-3}$. The contrarian profits due to delayed reaction, given by $-\hat{c}_0 \sigma_{\epsilon w}^2$, is $0.0015 \times 10^{-3}$ which is less than 1% of the total contrarian profits. The effect of the cross-sectional dispersion in average returns ($\sigma_{\mu}^2$) on contrarian profits is also small, consistent with the earlier findings of Jegadeesh (1990) and Lo and MacKinlay (1990).

These results suggest that most of the contrarian profits are attributable to market overreaction to firm-specific information. However, if factor sensitivities change over time, the contribution of the lead-lag structure estimated above is likely to be biased upwards and the contribution of overreaction to firm-specific information is likely to be biased downwards. To illustrate this, consider the example where stock A always reacts instantaneously to the common factor (i.e., $b_{0,A}^0 = 1$ and
\( b^t_{1,A} = 0 \forall t \) and stock B reacts to the common factor instantaneously half of the time but with a one period lag the other half of the time (i.e., \( b^t_{0,B} = 1 \) and \( b^t_{1,B} = 0 \) half the time and \( b^t_{0,B} = 0 \) and \( b^t_{1,B} = 1 \) the other half). In this case, the unconditional estimates from regression (14) will be \( b_{0,A} = 1 \) and \( b_{1,A} = 0 \); and \( b_{0,B} = .5 \) and \( b_{1,B} = .5 \). From the decomposition in (8), it follows that the contrarian profit due to the lead-lag effect is underestimated by the above procedure by \( \frac{3}{10} \sigma_f^2 \) and the profit due to overreaction is overestimated by \( \frac{1}{8} \sigma_f^2 \).

The next subsection provides estimates of the relative contribution of the different sources of contrarian profits, allowing for time varying factor sensitivities.

A. Contrarian Profits Conditional on Lagged Return Realizations

Let expression (6) describe the return generating process. If in addition we assume that the \( \epsilon_{it} \)'s are normally distributed and let \( \text{corr}(\epsilon_{i,t}, \epsilon_{i,t-1}) = \rho_i, \forall i \), the expected contrarian profit at time \( t \) conditional on \( f_{t-1} \) and each \( \epsilon_{i,t-1} \), can be shown to equal:

\[
E(\pi_t | f_{t-1}, \epsilon_{i,t-1}) = \sigma^2_f - \delta_i f^2_{t-1} - \rho \theta_{t-1}, \\
\text{where,} \quad \theta_t = \frac{1}{N_t} \sum_{i=1}^{N_t} \epsilon^2_{i,t}.
\]

(16)

Intuitively, this expression captures the fact that if the stock price reactions to factor realizations are important for the profitability of contrarian strategies, then large factor realizations should lead to large contrarian profits. Likewise, if the contrarian profits are related to overreaction to firm specific information then \( \pi_t \) will be larger following periods with large cross-sectional dispersion in the firm-specific components of returns. To measure the contribution of the different components of contrarian profits we estimate the following time-series regression:

\[
\pi_t = \alpha_0 + \alpha_1 (r_{VW,t-1} - \bar{r}_{VW})^2 + \gamma \theta_{t-1} + u_t,
\]

(17)
The estimates of contrarian profits due to delayed reactions to the common factor and to overreaction are given by \( \alpha_1 \sigma_{vW}^2 \) and \( \gamma \left( \frac{1}{T} \sum_{t=1}^{T} \theta_{t-1} \right) \) respectively. This decomposition does not require that \( b_{0,i} \) and \( b_{1,i} \) be constant through time. The estimates of \( e_{it} \)s used to compute \( \theta \) are estimated from regression (14). Sampling error and possible changes in factor sensitivities will induce measurement errors in estimated \( e_{it} \)s, and consequently \( \theta \) used in regression (17), will be measured with error. Therefore, the estimate of the contrarian profit due to overreaction obtained from this regression will be biased downwards.

Table 4 presents the estimates of regression (17). The slope coefficient \( \alpha_1 \) is significant for the small firms but not for the large firms. For instance, the estimate (t-statistic) of \( \alpha_1 \) for the small and large firm quintiles are .07 (8.55) and .002 (.68) respectively. The contrarian profit due to the lead-lag structure is statistically significant for the full sample but the magnitude is small; only about 3.89\% of the contrarian profits can be attributed to the lead-lag relation in stock prices and the point estimate of the contribution due to overreaction to firm specific component is 105\% of the contrarian profits. These results indicate that most of the contrarian profits are due to overreaction to the firm-specific component of returns.

B. Possible Association between Factor Realization and Factor Sensitivities

Our assumption that the factor sensitivities \( b_{0,i} \) and \( b_{1,i} \) are uncorrelated with factor realizations enabled us to obtain a linear relation between the conditional expectation of contrarian profits and squared lagged factor realizations. In a more general setting one may expect the timeliness of stock price reactions to depend on the magnitude of factor realizations. For instance, it is possible that delayed reactions of lagging stocks may be more pronounced following large factor realizations than following small factor realizations. In general, if the timeliness of stock price

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reactions is correlated with the magnitude of factor realizations then our estimate of the contribution of the lead-lag effect to contrarian profits will be biased.

To examine whether the contribution of the lead-lag effect to contrarian profits depends on the magnitude of lagged factor realizations we divided the observation into two subsamples based on the lagged values of \((r_{ \bar{VW} I} - \bar{r}_{ \bar{VW} I})^2\). Regression (17) was fitted separately within these two subsamples. The contribution of the lead-lag effect to contrarian profits for the low and high lagged factor realization subsamples were 4.6% and 4.5% respectively.\(^8\) These results reinforce our conclusion that the relative contribution of the lead-lag effect to contrarian profits is small.

C. Non-synchronous Trading and the Bid-Ask Spread

Non-synchronous trading and bid-ask bounce can bias the estimates of contrarian profits, as previous authors have noted. To examine the effect of non-synchronous trading and the bid-ask bounce on the estimated contributions of various sources, we examined a contrarian strategy where we skip a day between the portfolio formation date and the holding period.\(^9\) Specifically, the portfolio weights are assigned on the basis of Tuesday through Monday returns and the portfolio is held from the following Wednesday through Tuesday. The average contrarian profit of this strategy is also reliably different from zero at \(.2045 \times 10^{-8}\) (1.04% per dollar long.). The contributions of the lead-lag structure and overreactions to firm-specific information estimated based on regression (17) are 4% and 125%, respectively. These results are consistent with the results documented earlier that the lead-lag effect is not an important source of contrarian profits.

\(^8\)We also estimated the contribution of the lead-lag effect within 5 sub-samples formed based on lagged values of \((r_{ \bar{VW} I} - \bar{r}_{ \bar{VW} I})^2\). The estimates of the contribution of the lead-lag effect varied from \(-14.74\%\) to \(5.9\%\) within these subsamples. The standard errors of the subsample estimates of \(\alpha_1\) in regression (17) were, however, large relative to that for the full sample estimates because of the smaller dispersion of \((r_{ \bar{VW} I} - \bar{r}_{ \bar{VW} I})^2\) within each subsample.

\(^9\)This procedure to circumvent potential biases in estimated contrarian profits has been used previously by Jegadeesh (1990) and Lehmann (1990).
IV Conclusion

This paper presents a decomposition of short-horizon contrarian profits into various sources based on an analysis of stock price reactions to common factors and firm-specific information. We find that the lead-lag structure caused by delayed stock price reactions to the common factor contributes less than 5% of observed contrarian profits. Most of the observed contrarian profits is attributable to reversal of firm-specific component of stock returns. This result is robust with respect to estimation based on a variety of different assumptions. Consistent with the general practice in the current literature we refer to this return reversal as overreaction. It is however possible that it may have other interpretations.\footnote{For instance, Jegadeesh and Titman (1992) provide evidence suggesting that the negative serial covariance of returns may be related to the decay of the inventory component of bid-ask spreads.} Irrespective of the interpretation, our analysis indicates that any explanation for contrarian profits will come from understanding the source of the negative serial covariance of firm specific returns rather than from the lead-lag relation across stock returns.

The decomposition provided in this paper is based on a single factor model. While it is possible that the lead-lag relations may not be fully captured by a single factor model, it does not seem likely that delayed reactions to additional factors is an important source of contrarian profits for two reasons. First, if stocks react with a delay to other common factors then the serial covariance of the residuals based on the single factor model will be underestimated. We find, however, that the average serial covariance of the residuals estimated from the single factor model is marginally larger than the average contrarian profits. Secondly, the evidence in the literature indicates that most of the the comovements in stock returns are captured by a single factor (see Trzcinka (1986)).
References


Table 1. This table presents the estimates of profits to the Lo and MacKinlay contrarian strategy. \( \pi \) is the average contrarian profit and \( \psi \) is the average contrarian profit per dollar long. The \( t \)-statistics are reported in parentheses. The sample period is 1963 to 1991.

<table>
<thead>
<tr>
<th>Size</th>
<th>Subsamples</th>
<th>( \pi^a )</th>
<th>( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1</td>
<td>0.6150</td>
<td>0.0243</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(36.11)</td>
<td>(43.31)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3246</td>
<td>0.0150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25.76)</td>
<td>(31.04)</td>
</tr>
<tr>
<td>Medium</td>
<td>3</td>
<td>0.2261</td>
<td>0.0116</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(20.45)</td>
<td>(24.84)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.1475</td>
<td>0.0085</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(17.87)</td>
<td>(21.92)</td>
</tr>
<tr>
<td>Large</td>
<td>5</td>
<td>0.0839</td>
<td>0.0060</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(16.12)</td>
<td>(19.02)</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td>0.2619</td>
<td>0.0137</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(27.83)</td>
<td>(36.73)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>50 Size-sorted portfolios(^b)</th>
<th>-0.0036</th>
<th>-0.0002</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-2.04)</td>
<td>(-0.69)</td>
</tr>
</tbody>
</table>

\(^a\)Multiplied by 1000.
\(^b\)This row reports the profits to the contrarian strategy implemented with 50 size-sorted portfolios.
Table 2. This table presents the average estimates of the sensitivities of stock returns to current and lagged value-weighted index (VWI) returns based on the following time-series regression:

\[ r_{it} = a_i + b_{0,i}r_{VWI,t} + b_{1,i}r_{VWI,t-1} + e_{i,t}, \]

where \( r_{it} \) and \( r_{vwi,t} \) are the returns on stock \( i \) and the VWI respectively. \( \hat{\delta} \equiv \frac{1}{N} \sum_{i=1}^{N} (b_{0,i} - \bar{b}_0)(b_{1,i} - \bar{b}_1). \)

These estimates are presented for the full sample and also for size-based subsamples. The sample period is 1963 to 1990.

<table>
<thead>
<tr>
<th>Size</th>
<th>Subsamples</th>
<th>( \bar{b}_0 )</th>
<th>( \bar{b}_1 )</th>
<th>( \hat{\delta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
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<td>1.0952</td>
<td>0.2355</td>
<td>-0.0112</td>
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<tr>
<td></td>
<td>2</td>
<td>1.0899</td>
<td>0.2065</td>
<td>-0.0105</td>
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<td>0.1712</td>
<td>-0.0071</td>
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<td></td>
<td>4</td>
<td>1.0209</td>
<td>0.1139</td>
<td>-0.0013</td>
</tr>
<tr>
<td>Large</td>
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<td>0.0272</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>1.0595</td>
<td>0.1631</td>
<td>-0.0033</td>
</tr>
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</table>
Table 3. This table presents the estimates of various sources of profits to the Lo and MacKinlay contrarian strategy. $-\hat{\sigma}_{y_{W,I}}^2$, $-\Omega$ and $-\sigma_\mu^2$ are the estimates of contrarian profits due to the lead-lag structure, overreaction to the firm-specific component of returns and the cross-sectional dispersion of expected returns respectively. The numbers within brackets are the ratios of each of these components relative to the contrarian profit ($\pi$) reported in Table 1. These ratios do not add up to 1 due to estimation errors.

<table>
<thead>
<tr>
<th>Size</th>
<th>Subsamples</th>
<th>$-\hat{\sigma}<em>{y</em>{W,I}}^2$</th>
<th>$-\Omega$</th>
<th>$-\sigma_\mu^2$</th>
</tr>
</thead>
<tbody>
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<td>0.4814</td>
<td>-0.0061</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.008]</td>
<td>[0.783]</td>
<td>[-0.010]</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0048</td>
<td>0.3546</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>[0.015]</td>
<td>[1.092]</td>
<td>[-0.017]</td>
</tr>
<tr>
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<td>0.2606</td>
<td>-0.0036</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.015]</td>
<td>[1.153]</td>
<td>[-0.016]</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0006</td>
<td>0.1645</td>
<td>-0.0037</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.004]</td>
<td>[1.115]</td>
<td>[-0.025]</td>
</tr>
<tr>
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<td>-0.0014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.011]</td>
<td>[1.111]</td>
<td>[-0.017]</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td>0.0015</td>
<td>0.2881</td>
<td>-0.0044</td>
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<tr>
<td></td>
<td></td>
<td>[0.006]</td>
<td>[1.100]</td>
<td>[-0.017]</td>
</tr>
</tbody>
</table>

Note: The sample estimates multiplied by 1000 are reported here.
Table 4. This table presents a decomposition of the contrarian profits based on the following regression:

\[ \pi_t = \alpha_0 + \alpha_1 (r_{VW,t-1} - \bar{r}_{VW})^2 + \gamma \theta_{t-1} + u_t, \]

where, \[ \theta_t = \frac{1}{N_t} \sum_{i=1}^{N_t} e_{i,t}. \]

\( \pi_t \) is the contrarian profit and \( r_{VW,t} \) is the return on the value-weighted index in week \( t \). The estimates of the firm-specific component of returns \( (e_{i,t}) \) are obtained from the regression in Table 3.

The estimates of contrarian profits due to delayed reactions to the common factor and overreaction to firm-specific information are given by \( \alpha_1 \sigma_{VW}^2 \) and \( \gamma (\frac{1}{T} \sum_{t=1}^{T} \theta_{t-1}) \) respectively. The numbers in square brackets are the ratios of each of these components relative to the average contrarian profit \( (\pi) \) presented in Table 1. These ratios do not add up to 1 due to estimation errors.

<table>
<thead>
<tr>
<th>Size</th>
<th>Subsamples</th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \gamma )</th>
<th>( \alpha_1 \sigma_{VW}^2 )</th>
<th>( \gamma (\frac{1}{T} \sum_{t=1}^{T} \theta_{t-1}) )</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0.0785</td>
<td>70.4662</td>
<td>96.7322</td>
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<td></td>
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<td>(8.55)</td>
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<td>[1.156]</td>
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</tr>
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<tr>
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<td>(18.45)</td>
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<td>[1.539]</td>
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<tr>
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<td>(15.50)</td>
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<td>[1.593]</td>
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<td>(4.48)</td>
<td>(15.59)</td>
<td>[0.039]</td>
<td>[1.048]</td>
<td></td>
</tr>
</tbody>
</table>

Note: The sample estimates multiplied by 1000 are reported here.