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CORPORATE EARNINGS AND THE EQUITY PREMIUM

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ABSTRACT

Economic shocks affect corporate cash flows far more than they do aggregate consumption. We examine the asset-pricing implications of corporate sensitivity to shocks using a continuous-time representative agent framework in which earnings are a stochastic fraction of total consumption. We provide closed-form solutions for equity values when earnings and consumption follow exponential-affine jump-diffusion processes. Calibrating the model to historical data, we show that the extreme sensitivity of corporate cash flows to shocks dramatically increases the equity premium. The model implies realistic values for the equity premium given modest levels of risk aversion and generates levels of equity volatility consistent with those experienced by the stock market.
1. INTRODUCTION

In the standard Lucas (1978) and Mehra and Prescott (1985) representative-agent framework, equilibrium asset prices equal the expected product of a pricing kernel and the cash flows from those assets. Because of this, the ultimate ability of this framework to capture the properties of market equity values clearly hinges on being able to successfully identify an appropriate pricing kernel and accurately model corporate cash flows.


In contrast, less attention has been paid to the problem of modeling corporate cash flows within this framework. In fact, many papers in this literature sidestep this issue altogether by simply constraining aggregate corporate dividends to equal aggregate consumption. Important exceptions include Merton (1971) and Santos and Veronesi (2001) who model aggregate corporate cash flows as consumption minus a labor income component. Other important recent papers that allow cash flows to differ from aggregate consumption include Campbell (1986), Cecchetti, Lam, and Mark (1993), Campbell and Cochrane (1999), Abel (1999), Brennan and Xia (2001), Barberis and Huang (2001), and others.

Modeling corporate cash flows distinctly from aggregate consumption is crucial since corporate earnings and dividends have historically been far more sensitive to economic shocks than has aggregate consumption. For example, corporate earnings have been more than ten times as volatile as consumption growth during the post-war period. Similarly, when the severe onslaught of the Great Depression in the early 1930s caused nearly a 10% decline in aggregate consumption, the impact of this event sent shock waves throughout the corporate sector resulting in aggregate corporate profits being completely obliterated as they fell 104%. Intuitively, the reason for this high sensitivity is that stockholders are residual claimants to corporate cash flows. Thus, as shocks propagate through the economy, corporate earnings are more susceptible than more senior claims and components of consumption such as wages and labor income.
This paper extends this literature by modeling corporate cash flows in a way that allows them to be more sensitive to different types of economic shocks, and then examines the asset-pricing implications of this sensitivity. A novel feature of our approach is that we calibrate the model to aggregate corporate earnings rather than to dividends. A key rationale for focusing on earnings is the concern that corporate managers tend to artificially smooth dividends over time. Specifically, managers often retain earnings within the firm during good periods and pay dividends out of capital during bad periods. In fact, Fama and French (1999) show that firms that do not pay dividends now account for one-quarter of the value of the stock market. In addition, as argued by Liang and Sharpe (1999), Hall (2001), and others, reported dividends may not include important cash distributions such as share repurchases or corporate acquisitions. Thus, the statistical properties of reported dividends may be quite different from those of the actual economic cash flows that accrue to shareholders. Although corporate earnings measures doubtlessly also contain noise, their covariation over time with the pricing kernel may more closely reflect that of the underlying cash flows. Furthermore, the use of earnings is actually more consistent with the spirit of the standard Lucas (1978) and Mehra and Prescott (1985) economy in which the production plan is fixed and the consumption good is nondurable. Thus, earnings cannot be retained in this exchange economy and must be entirely paid out to shareholders as dividends.

In our model, corporate earnings represent a small but highly volatile component of aggregate consumption. This volatility directly impacts the covariance between the pricing kernel and the corporate cash flows and can significantly affect equilibrium stock values. To capture the sensitivity of corporate cash flows to both the usual “small” economic shocks as well as to rare catastrophic “large” economic shocks, we extend the traditional representative agent framework to allow aggregate earnings and consumption to follow distinct exponential-affine jump-diffusion processes. The ratio of aggregate earnings to consumption, which we designate the corporate fraction, plays a key role in the model as a mean-reverting state variable. From the first-order conditions of the representative agent, we derive an explicit closed-form expression for the equity value which can then be used to explore the model’s asset-pricing implications.

We calibrate the model to match the properties of real per capita consumption and earnings growth in the U.S. during the 1929 to 2001 period. Furthermore, to hold fixed the properties of the pricing kernel throughout our analysis, the results are all based on a modest level of five for the risk aversion coefficient of the representative agent’s power utility function. This enables us to assess more directly the extent to which equilibrium stock prices are affected by the behavior of corporate cash flows.

We find that the high sensitivity of corporate cash flows to economic shocks maps into equity premia that are many times larger than in the standard framework. Specifically, when aggregate earnings and consumption growth are constrained to be equally sensitive to shocks, the equity premium is only 0.54%. When the model is allowed to match the historical sensitivity of corporate cash flows, however, the average
equity premium becomes 3.22%. Thus, allowing for differences in the sensitivity of consumption and earnings to shocks results in nearly a sixfold increase in the size of the equity premium. We note that most of this increase comes from the sensitivity of cash flows to continuous shocks, rather than from the sensitivity to jump events.

Exploring further the effects of corporate sensitivity, we examine the implications for equity volatility. We find that matching historical corporate sensitivity results in equity volatility levels that closely parallel those observed in the markets. In particular, the average stock market volatility implied by the model is 19.26%. This corresponds closely to the 19.20% volatility of the CRSP value-weighted index during 1929 to 2000. In contrast, specifications that do not allow for differential corporate and consumption sensitivity produce equity volatilities that are only on the order of three percent.

The model also allows us to compute the effect of catastrophic economic shocks or jump events on equity values. Assuming that these jumps occur once per century on average, the model implies that the realization of a jump event results in a stock market crash on the order of 67%. This is consistent with the 69% decline in the U.S. stock market during the early stages of the Great Depression, and also with the declines experienced by other major countries such as Japan, Germany, Greece, and Portugal during the past century (see Jorion and Goetzmann (1999)). Catastrophic events have been considered before by Rietz (1988) as a potential resolution to the equity premium puzzle. Rietz calibrates his model parameters such that a 25% drop in consumption occurs on average once per century. Together with a coefficient of relative risk aversion of ten, Rietz is able to match the equity premium. However, Mehra and Prescott (1988) argue that aggregate consumption has not dropped by nearly this much during the last century. Our approach is immune to this criticism since we calibrate jumps in consumption and earnings to the historical experience during the Great Depression.

It is important to recognize, however, that while our results indicate that this approach of modeling cash flows is promising, the average equity premium implied by the model is only about half as large as historical estimates. Thus, the model clearly cannot provide a complete resolution of the equity premium puzzle without increasing the assumed level of risk aversion. Moreover, the Euler equation for the risk-free bond in our model is the standard one, which means that we inherit Weil’s (1989) riskfree rate puzzle. Our results do suggest, however, that combining our model of corporate sensitivity to economic shocks with other elements such as habit formation (as in Campbell and Cochrane (1999)) or investor heterogeneity in incomplete markets (see the discussion in Constantinides (2002)) could play an important role in the ultimate resolution of asset-pricing puzzles.

There is an extensive literature on the equity premium puzzle. Detailed references can be found in the excellent surveys by Kocherlakota (1996), Cochrane (1997), Campbell (1999), Constantinides (2002), and Mehra (2002). Breeden (1979) and Bakshi and Chen (1996, 1997) are continuous-time versions of the endowment economy in
Lucas (1978) based on diffusions. Naik and Lee (1990) extend the setup to allow for jumps in consumption growth. Their model is the continuous-time analogue of Rietz (1988).

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 describes the properties of the historical earnings data. Section 4 explains how the model is calibrated. Section 5 examines the asset-pricing implications of the model. Section 6 summarizes the results and makes concluding remarks.

2. THE MODEL

In this section, we extend the basic Lucas (1978) and Mehra and Prescott (1985) framework in a way that allows dividends to have a different degree of sensitivity to economic shocks than does aggregate consumption. This feature has a number of important asset-pricing implications that we will explore throughout the paper. Note that since dividends must equal earnings in this framework, the terms earnings and dividends are used interchangeably throughout this section.

We consider a standard exchange economy populated by a representative agent who maximizes an expected power utility function of the form

$$E_t \left[ \int_t^\infty e^{-\delta(s-t)} \frac{C_t^{1-\gamma}}{1-\gamma} ds \right],$$

where $C$ represents consumption, $\gamma$ is the coefficient of relative risk aversion, and $\delta$ is the subjective discount rate. The agent has two sources of a nondurable consumption good. First, the agent is initially endowed with one share of a stock which pays dividends $D_t$ in the form of the consumption good. Second, the agent also receives an exogenous endowment $I_t$ of the consumption good which constitutes his nonfinancial income. The stock is thus a claim to dividends instead of consumption, which in equilibrium is given by the sum of the dividends and nonfinancial income $C_t = D_t + I_t$.

Dividends and nonfinancial income are cointegrated in the data. We therefore specify the corporate fraction $F_t$ of dividends in consumption exogenously as

$$F_t = \frac{D_t}{C_t} = \exp(-X_t),$$

where $X_t$ follows the square-root jump-diffusion process

$$dX = (\mu - \kappa X) dt - \eta \sqrt{X} dZ + \xi dq.$$  

Here, $Z$ is a standard Brownian motion and $q$ is a Poisson process with constant
intensity $\lambda$. Innovations in both $Z$ and $q$ make this mean-reverting process deviate from its unconditional mean value of $(\mu + \lambda \xi) / \kappa$. Assuming that the jump parameter $\xi$ is positive and $\mu > \eta^2 / 2$, the value of $X$ remains strictly positive. In turn, this ensures that the corporate fraction $F_t$ is always between zero and one.\footnote{Menzli, Santos, and Veronesi (2002a, b) also present a model in which dividends for individual firms are represented as a fraction of aggregate consumption. In their model, however, aggregate dividends equal aggregate consumption. They note, however, that this feature of their model can be relaxed.}

We next assume that aggregate consumption follows the jump-di**ffusion process

$$\frac{dC}{C} = (\alpha + \beta X) \ dt + \sigma \sqrt{X} \ dZ - \psi \ dq,$$

where $0 \leq \psi < 1$. This specification allows both the conditional mean and variance of consumption growth to depend on the state variable $X$, which is consistent with empirical evidence to be presented later. We note, however, that the traditional Mehra and Prescott (1985) specification for consumption growth can be nested within ours by setting $\beta = 0$ and requiring that $X$ be constant by imposing the parameter restrictions $\mu = \kappa = \eta = \xi = 0$.

From Eq. (2), dividends are given by

$$D_t = C_t \ F_t = C_t \ \exp(-X_t).$$

An application of Ito’s Lemma implies the dynamics for the dividend process

$$\frac{dD}{D} = \left(\alpha - \mu + (\beta + \kappa + \sigma \eta + \eta^2 / 2)X\right) \ dt + \left(\sigma + \eta\right) \sqrt{X} \ dZ - (\psi + \zeta) \ dq,$$

where $\zeta = (1 - \psi)(1 - e^{-\xi})$. Comparing these dynamics with those in Eq. (4) shows that both consumption and dividends are influenced by continuous or diffusive shocks to the economy through $dZ$. The degree of sensitivity to these economic shocks, however, depends on the values of the coefficients of $dZ$ in Eqs. (4) and (6). Taking the ratio of these coefficients implies that dividend growth is $(1 + \eta / \sigma)$ times as sensitive to continuous shocks as consumption growth. Thus, dividend growth is more sensitive to continuous economic shocks whenever $\eta > 0$. This is consistent with the empirical evidence that corporate profits and dividends are much more volatile than consumption even during periods where there are no major shocks or jumps in the economy.

Similar results hold for the relative sensitivity of dividend and consumption growth to catastrophic events or jumps in the economy. In particular, the relative
sensitivity of dividend and consumption growth to jumps is given by the ratio of the coefficients of $dq$ in Eqs. (4) and (6), which is $(1 + \zeta/\psi)$. It is easily shown that dividend growth is more sensitive to catastrophic economic shocks when $\xi > 0$. This feature is particularly important since it allows the model to avoid the Mehra and Prescott (1988) critique of the Rietz (1988) model. Rietz argues that the historical equity premium can be explained by the risk of large downward jumps in consumption. Mehra and Prescott argue that the size of the downward jumps in consumption necessary to explain the equity premium is many times larger (possibly as large as 90%) than any ever experienced in U.S. history. By allowing us to specify the jump in dividends separately from the jump in consumption, the model is in a better position to capture the historical equity premium without assuming unrealistically large consumption crashes.

Taken together, these results imply that dividend growth is more sensitive to economic shocks whenever the corporate fraction is stochastic. Only in the case where the corporate fraction is deterministic are dividends and consumption growth equally sensitive to economic shocks. In this case, our model reduces to the standard one.

In equilibrium, the price of the stock satisfies the Euler equation

$$
S_t = E_t \left[ \int_t^\infty e^{-\delta(s-t)} \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma} D_{t+s} \, ds \right] 
= E_t \left[ \int_t^\infty e^{-\delta(s-t)} \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma} C_{t+s} \, F_{t+s} \, ds \right].
$$

Using the results in Duffie, Pan, and Singleton (2000), the stock price is given by the following closed-form expression,

$$
S_t = D_t \int_t^\infty e^{-\delta(s-t)} A(s) \, F_t^{-B(s)-1} \, ds,
$$

where

$$
A(s) = \exp \left( \int_t^s -\alpha(1 - \gamma) - \mu B(u) - \lambda((1 - \psi)^{1-\gamma} \exp(B(u)\xi) - 1) \, du \right),
$$

and $B(s)$ is given by
\[ B(s) = \frac{2}{\eta^2(s-t) - k_1} + \frac{\kappa + \sigma \eta(1-\gamma)}{\eta^2}, \]

\[ B(s) = \frac{\phi}{\eta^2} \tanh \left( \frac{\phi}{2}(s-t) + k_2 \right) + \frac{\kappa + \sigma \eta(1-\gamma)}{\eta^2}, \]

\[ B(s) = \frac{\theta}{\eta^2} \tan \left( \frac{-\theta}{2}(s-t) + k_3 \right) + \frac{\kappa + \sigma \eta(1-\gamma)}{\eta^2}, \]

in the cases where \( \phi = 0 \), \( \phi < 0 \), and \( \phi > 0 \), respectively, and where

\[ \varphi = \eta^2 \left( (2\beta - \sigma^2)(1-\gamma) + \sigma^2(1-\gamma)^2 \right) - (\kappa + \sigma \eta(1-\gamma))^2, \]

\[ k_1 = \frac{-2\eta^2}{\eta^2 + \kappa + \sigma \eta(1-\gamma)}, \]

\[ k_2 = \tanh^{-1} \left( \frac{-(\eta^2 + \kappa + \sigma \eta(1-\gamma))}{\phi} \right), \]

\[ k_3 = \tan^{-1} \left( \frac{-(\eta^2 + \kappa + \sigma \eta(1-\gamma))}{\theta} \right), \]

and \( \theta = \sqrt{\varphi}, \phi = \sqrt{-\varphi} \). From this closed-form solution, it is clear that the stock price is a complex function of consumption, the corporate fraction, and the parameters governing their dynamics. Despite this, however, it is easily seen that the price/dividend ratio does not depend on the current level of consumption, for a given level of \( F_t \). The price/dividend ratio, however, is time varying through its dependence on the corporate fraction \( F_t \).

A similar approach can be used to solve for the equilibrium instantaneous riskless rate. Evaluating the appropriate Euler condition and taking limits implies that the riskless rate \( r \) in this economy is given by

\(^2\)Other recent papers that provide solutions for equity prices in terms of their fundamental cash flows include Ohlson (1990, 1995), Bakshi and Chen (1996, 1997), Bekaert and Grenadier (1999), Ang and Liu (2001), Vuolteenaho (2002), and Mamaysky (2002).
Thus, the equilibrium riskless rate is also time varying through its dependence on $X$. This dependence is supported by the empirical evidence reported in the next section. The direction of the relation between $r$ and $X$ depends on the value of $\beta$. For positive values of $\beta$, a higher $X$ predicts higher future consumption growth. The agent would like to borrow in order to smooth consumption which drives up the riskless rate. A higher value of $\alpha$ increases consumption growth on average, which also drives up the riskless rate. Consumption volatility $\sigma$ induces a precautionary savings motive. To insure against future consumption fluctuations, the agent wants to save which lowers the riskless rate in equilibrium. Catastrophic events reinforce the precautionary savings motive. The higher the Poisson intensity $\lambda$, the more likely these events, the lower the riskless rate. When the subjective discount rate $\delta$ is high, the agent does not care much about the future, and therefore saves less, which increases the riskless rate in equilibrium. The coefficient of relative risk aversion $\gamma$ has two counteracting effects on the riskless rate. A high $\gamma$ lowers the elasticity of intertemporal substitution, and therefore induces the agent to save more in a growing economy, increasing the riskless rate. This explains the $\alpha \gamma$ term. A high $\gamma$ also strengthens the precautionary savings motive, which explains all other terms involving $\gamma$.

Finally, to provide some additional intuition about how our model differs from the standard representative agent setup, recall that in Mehra and Prescott (1985), dividends are a constant fraction of consumption (the fraction is actually one). In this case, $D_t = C_t F$, and the Euler equation in (7) can be reexpressed as

$$
\frac{S_t}{D_t} = E_t \left[ \int_t^\infty e^{-\delta(s-t)} \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma} \left( \frac{C_{t+s}F}{C_tF} \right) ds \right] = E_t \left[ \int_t^\infty e^{-\delta(s-t)} \left( \frac{C_{t+s}}{C_t} \right)^{1-\gamma} ds \right].
$$

(10)

In our model, however, the fraction $F_t$ is stochastic. Thus, Eq. (7) can be rewritten as

$$
\frac{S_t}{D_t} = E_t \left[ \int_t^\infty e^{-\delta(s-t)} \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma} \left( \frac{C_{t+s}F_{t+s}}{C_tF_t} \right) ds \right] = E_t \left[ \int_t^\infty e^{-\delta(s-t)} \left( \frac{C_{t+s}}{C_t} \right)^{1-\gamma} \left( \frac{F_{t+s}}{F_t} \right) ds \right].
$$

(11)

Comparing Eqs. (10) and (11) shows that in our model, the percentage change in
the corporate fraction appears as an additional source of randomness multiplying the convex pricing kernel under the integral. Of course, when the percentage change in the corporate fraction is uncorrelated with consumption growth, the expectation of the product in Eq. (11) becomes a product of expectations and the model would then reduce to the usual Mehra and Prescott (1985) setting. In our model, however, changes in consumption and the corporate fraction are correlated because of their sensitivity to common economic shocks. Thus, the expectation in Eq. (11) may be influenced by both the volatility and persistence of changes in the corporate fraction. Furthermore, the dependence of the dynamics of consumption on the state variable $X$ in Eq. (4) induces an additional degree of long-term covariability between the pricing kernel and changes in the corporate fraction, again affecting the expectation in Eq. (11). As we demonstrate later, these correlation effects can create significant differences between the equity premia implied by our model and those implied by the standard framework.

3. PROPERTIES OF EARNINGS

In exploring the asset-pricing implications of the model, our first step will be to calibrate the model to be consistent with the basic properties of the historical data. In this section, we review the properties of the data on aggregate earnings. This data is then used in the next section in calibrating the parameters of the model.

In calibrating the model, we make the identifying assumption that the process $D_t$ in the model is equivalent to aggregate after-tax corporate earnings in the economy rather than to aggregate dividends. There are a number of justifications for adopting this interpretation. For example, by focusing on earnings rather than dividends, we avoid controversy about how dividends should be measured (e.g. whether share repurchases should be included in aggregate dividends as in Hall (2001); also see Liang and Sharpe (1999)). Further, in the simple Lucas (1978) and Mehra and Prescott (1985) exchange economy framework, firms (like trees) produce a single consumption good. Since the consumption good is nondurable, however, the firm’s entire net output or “earnings” is paid out to shareholders as “dividends”; earnings cannot be retained within the firm since the consumption good is not storable and the investment plan

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3It is important to note that the case where percentage changes in the corporate fraction are uncorrelated with consumption growth is not the same as the case where percentage changes in dividends are uncorrelated with consumption growth. In the first case, the Mehra and Prescott (1985) model holds and the equity premium may be nonzero. In the second case, however, the equity premium is zero. The reason for the distinction, of course, is that we define dividends as the product of consumption and the corporate fraction. Thus, dividends may be correlated with consumption even if changes in the corporate fraction are not.
is fixed. Because of this, identifying $D_t$ with corporate profits is more consistent with the spirit of the model. Finally, we focus on after-tax rather than pre-tax earnings since corporate taxes are not paid out to shareholders (see McGrattan and Prescott (2001) for a discussion of the effects on taxes on the equity premium).

To provide historical perspective, we first present summary statistics for real per capita consumption and earnings growth rates as well as for the corporate fraction for the period from 1929 to 2001. The consumption, earnings, and price data are from the National Income and Product Accounts reported by the Bureau of Economic Analysis. Year-end population estimates are from the Census Bureau. Consumption is defined as the sum of aggregate nondurable and services consumption. As the earnings measure, we use total after-tax corporate profits with inventory valuation and capital consumption adjustments. It is important to note that this measure of earnings does not simply equal total consumption minus labor income. The reason for this is that the national income accounts also include components such as proprietor’s income, rental income, and net interest. We exclude these components since our focus is on valuing equity claims and these components do not accrue to corporate shareholders.

In computing real growth rates, our estimates of realized inflation rates are based on the price series corresponding to our definition of consumption (not the CPI). Because corporate earnings are negative during the years of 1931 and 1932, growth rates for earnings and the corporate fraction cannot be computed for those years. Thus, summary statistics are reported separately for the full 1929-2001 sample period and for the 1929-2001 period where the years 1931-1932 are omitted. Table 1 presents the summary statistics.

Table 1 shows that real per capita consumption growth averaged 1.9% per year over the full sample period since 1929. Furthermore, consumption growth displayed relatively little volatility; the standard deviation of consumption growth is 2.4% during the full sample period. The largest shock in consumption occurred in 1932 when it declined 8.9%. During the three-year period from 1929 to 1932, consumption declined a total of 16%. While shocks of this magnitude clearly represent traumatic events for the economy, the results in Rietz (1988) and Mehra and Prescott (1988) indicate that infrequent consumption shocks on the order of 10% to 15% are probably not alone sufficient to account for the size of the historical equity premium.

Table 1 also shows that real per capita earnings growth is much more variable than consumption growth. During the 1929 to 2001 period (excluding the two years where aggregate earnings were negative), the standard deviation of per capita real earnings growth is nearly 27%. The largest shock occurred in 1931 when earnings dropped by 103.3%. Comparing these statistics with those for consumption growth suggests that earnings growth is at least ten times as volatile as consumption growth, both during a major shock to the economy such as the 1931-1932 period, as well as during less volatile periods.
The corporate fraction $F$ plays a central role in our model. To provide some insight into the historical behavior of the corporate fraction, Fig. 1 plots the corporate fraction over the 1929 to 2001 period. The corporate fraction is highly variable, ranging from a low of $-4.2\%$ in 1932 to a maximum of $15.6\%$ in 1950. The average value of the corporate fraction is $9.8\%$. The standard deviation of the corporate fraction is $3.3\%$ and its first-order serial correlation coefficient is $0.76$.

Similar results hold for percentage changes in the fraction of corporate earnings relative to aggregate consumption. The standard deviation of these percentage changes is $25.2\%$. The largest decline of $103.4\%$ also occurred during 1931. The growth rates in both real per capita earnings and the corporate fraction are highly correlated with the growth rate in real per capita consumption. From Table 1, the correlations are $0.67$ and $0.62$ respectively. To illustrate this, Fig. 2 plots growth rates in real per capita earnings against the corresponding growth rates in real per capita consumption. As shown, there is a strong positive correlation between the two growth rates. Furthermore, this positive correlation is not just due to a few extreme observations.

To provide some empirical support for the dynamic assumptions underlying the model, Table 2 reports the results of regressions of consumption and earnings growth on the lagged values of $X_t$. These regressions are motivated by Eqs. (4) and (6) in which the expected changes in consumption and earnings are affine functions of $X_t$. As shown, both consumption and earnings growth are predictable on the basis of the information in the corporate fraction. In particular, the $R^2$s for the two regressions are $0.099$ and $0.158$ respectively. Furthermore, the coefficient for the state variable $X_t$ is significant in each of these regressions. These results provide some support for the dynamic specifications used in developing the model. Also, these results parallel those obtained by Lettau and Ludvigson (2002), Ang (2002), and others who find evidence of predictability in dividend growth.

As an additional diagnostic, recall from Eq. (9) that the riskless rate is also time varying through its dependence on the corporate fraction. To examine this empirical implication, Table 2 also reports the results of regressing the real rate $r_t$ on the value of the state variable $X_t$. As shown, there is a strong relation between the real rate and the corporate fraction. The $t$-statistic for $X$ is $4.31$, and the $R^2$ is $0.304$. Similar results are obtained when the regression is estimated in differences rather than levels.

4. MODEL CALIBRATION

Turning now to the calibration of the model, it is important to be aware that the possibility of dramatic but infrequent events such as the Great Depression makes it

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4Note that in estimating these regressions with the 1931-1932 period excluded, we avoid the necessity of fitting a regression specification that includes jump effects.
difficult to pin down precise estimates of central model parameters. Because of this, the spirit of our approach is to adopt parameter values that capture the basic properties of the data, but then also report the results using a wide range of alternative parameter values. With this in mind, we adopt the following procedures in estimating a set of benchmark jump, diffusion, and drift parameters for the model.

4.1 The Jump Parameters

First, to calibrate the jump parameters of the model, we use estimates that reflect the U.S. experience during the Great Depression. In particular, we use a value for $\lambda$ of 0.01, implying that a major economic crash similar to the Great Depression occurs once per century on average. This mean frequency may actually be overly conservative given the experience of other major economies such as Germany and Japan during the past century.

For the consumption jump size $\psi$, we assume a downward jump of 10%. This value is consistent with 8.9% downward jump in real per capita consumption during 1932, and the 16% decline from 1929 to 1932. While downward jumps in consumption of this magnitude are large, we note that there are many examples of economies that have experienced downward jumps in consumption of as much as 20% to 30% during the past 50 years.\(^5\)

Next, we assume that the realization of the Poisson event in the model results in a 90% decrease in corporate earnings. Again, this is on the conservative side given the 103.3% decline in real per capita corporate earnings during the Great Depression. Since the exponential square root process in our model only allows positive values for the corporate fraction, however, this parameterization allows the corporate fraction to remain positive while capturing the greater sensitivity of earnings. This parameterization implies that corporate earnings are nine times as sensitive as consumption to the realization of the Poisson event.

4.2 The Diffusion Parameters

To provide an estimate for the parameter $\sigma$ governing the sensitivity of consumption growth to continuous economic shocks, we note from Eq. (4) that the unconditional variance of consumption growth in the absence of jumps is simply $\sigma^2$ times the mean value of $X$. From Table 1, the standard deviation of consumption growth from 1929 to 2001 (omitting the years 1931-1932) is 0.020. Similarly, the standard deviation of consumption growth from 1890 to 1999 (again omitting the years 1931-1932) is 0.033 based on the data tabulated in Shiller (2000). Similar standard deviations for consumption growth are reported in Campbell (1999) for a variety of other countries. Based on this range of historical volatilities, we set $\sigma = 0.02$ in the calibration.

\(^5\)In particular, countries that have experienced a one-year decline in real GDP of more than 20% since 1950 include Algeria, Angola, Chad, Iran, Iraq, Namibia, Nicaragua, Niger, Nigeria, Sierra Leone, and Uganda (see Heston and Summers (1991)).
This value, combined with the mean value of $X$ during the sample period, implies that the unconditional volatility of consumption growth in the absence of jumps is approximately 0.03, which is consistent with the longer-term historical evidence.

From Table 1, the standard deviation of earnings growth is roughly thirteen times that of consumption growth throughout the sample period. Again, similar ratios are obtained for the 1890-1999 period using the data tabulated in Shiller (2000). Recall from the previous section that dividend growth is $(1 + \eta/\sigma)$ times as sensitive to continuous economic shocks. To be on the conservative side, we use a slightly lower ratio and set $\eta = 10 \sigma$. Thus, given this calibration, earnings can be viewed as being roughly nine to ten times as sensitive as consumption to both continuous and catastrophic economic shocks.

4.3 The Drift Parameters
As estimates of the parameters $\alpha$ and $\beta$ in the drift of the consumption process, we simply use the values given in Table 2 where consumption growth is regressed on $X$. The regression intercept and slope coefficients map directly into the values of $\alpha$ and $\beta$. The remaining parameters $\mu$ and $\kappa$ are chosen to match the unconditional mean and first-order serial correlation coefficients for the corporate fraction. These values, along with the values for all other parameters, are given in Table 3.

5. ASSET PRICING IMPLICATIONS
In this section, we explore the implications of the model for the properties of stock returns. We focus first on the equity premium. We then consider the implications for the volatility of stock returns. Finally, we examine how stock prices react to the realization of a catastrophic shock or jump event in the economy. Throughout this section, we use the calibration from the previous section as the benchmark or primary scenario. To provide additional intuition, however, we also provide results for alternative scenarios and parameter ranges centered around the benchmark. We use a value of five for the risk aversion coefficient throughout this section. This value reflects a modest level of risk aversion, but is still well within the range of plausibility.

5.1 The Equity Premium
Given the closed-form solution for stock prices, we can solve for the instantaneous expected return by a direct application of Ito’s Lemma. Subtracting the expression for the riskless rate in Eq. (9) from the expected return gives the instantaneous equity premium. Values of the equity premium for the benchmark scenario and a variety of other scenarios are reported in Table 4.

As shown, the equity premium is 3.22% in the benchmark scenario. While this value is consistent with recent surveys of market participants regarding the size of the current equity premium (see Welch (2000, 2001)), this value is lower than the usual
historical estimate of about seven percent. Thus, this model clearly does not provide a resolution of the equity premium puzzle. In fairness to the model, however, allowing corporate cash flows to be more sensitive to economic shocks does produce equity premia that are much larger than in the standard representative agent framework. Specifically, consider the scenario in which the corporate fraction is constant. This case is equivalent to the Mehra and Prescott (1985) framework since dividends have the same stochastic properties as aggregate consumption. Table 4 shows that in this scenario, the equity premium is only 0.47% when consumption jumps do not occur, and 0.54% when consumption jumps may occur. Thus, allowing for the increased sensitivity of dividends results in a sixfold increase in the equity premium. Because of this, the equity premium implied by the benchmark scenario is only about half of the historical value for the equity premium, rather than being orders of magnitude less.

The other scenarios shown in Table 4 provide some sense of how the key parameters of the model affect the implied equity premium. For example, increasing the average frequency of jumps from every 100 years to every 50 years increases the equity premium to 3.72%. Similarly, increasing the average frequency of jumps to every 20 years results in an equity premium of 5.24%.

To illustrate the relation between the equity premium and the sensitivity of corporate earnings to both continuous and jump shocks, the top panel in Fig. 3 plots the equity premium as a function of the parameter \( \eta \) governing the sensitivity of the corporate fraction to continuous shocks. The middle panel in Fig. 3 plots the equity premium as a function of the size of the jump in the corporate fraction, where the size of the jump is expressed as a fraction of the initial value of the corporate fraction. The bottom panel in Fig. 3 plots the equity premium as a function of the current value of the corporate fraction. As shown, the equity premium increases with the sensitivity to continuous shocks. There is a similar increasing relation between the equity premium and the sensitivity to catastrophic events or jumps. Note, however, that the effect of doubling the jump size is far less dramatic than the effect of doubling the sensitivity to continuous shocks. One reason for this is simply the fact that catastrophic events or jumps are very infrequent; as the risk of a jump event becomes larger, the effects of an increase in the jump size become more pronounced. Finally, an increase in the current value of the corporate fraction results in a decrease in the equity premium. Furthermore, this decrease is very nonlinear with the equity premium, initially dropping rapidly with an increase in the fraction, but then becoming much less sensitive as the fraction increases beyond ten percent.

### 5.2 Equity Return Volatility

Another application of Ito’s Lemma allows us to solve for the instantaneous volatility of stock returns. Table 4 also reports these volatilities for the various scenarios. As indicated, the model results in levels of volatility that are very consistent with the properties of historical stock returns. For example, the benchmark scenario implies stock return volatility of 19.26%. The annualized volatility of monthly stock returns
on the CRSP value-weighted index for the period from 1929 to 2000 is 19.20%.

It is important to observe that these realistic equity volatilities are a result of allowing earnings to be more sensitive to economic shocks. In particular, Table 4 shows that equity return volatility is only slightly greater than three percent in the cases where the corporate fraction is constrained to be constant. The instantaneous volatility of equity returns, however, is not just a simple function of the sensitivity of corporate earnings or dividends to continuous shocks. Equity return volatility in this model is a complex function of both the effects of the sensitivity of earnings to economic shocks as well as the mean reverting nature of the corporate fraction.

The values in Table 4 are computed under the assumption that the current corporate fraction is at its unconditional mean value of 9.8%. In actuality, the corporate fraction was 7.78% as of December 31, 2001. Substituting this value into the benchmark scenario implies an instantaneous stock return volatility of 19.84% as of that date. This corresponds closely with the December 31, 2001, value of the VIX index of implied volatility which was 22.02%. The widely-reported VIX index represents an average of the implied volatilities for near-the-money short-term call and put options on the S&P 100 index traded at the Chicago Board Options Exchange.

Finally, Fig. 4 graphs equity return volatility as a function the sensitivity of the corporate fraction to continuous and catastrophic economic shocks, and also as a function of the value of the current corporate fraction. As illustrated, the behavior of equity volatility parallels that of the equity premium shown in Fig. 3. In particular, equity volatility is an increasing function of sensitivity to continuous shocks. Equity volatility is also increasing in the jump size, but is less sensitive to variation in this parameter. The current value of the corporate fraction has a large effect on equity volatility, which decreases sharply as the corporate fraction increases.

5.3 Stock Market Crashes

Given the closed-form solution for equity values in the model, it is straightforward to solve for the effect of a catastrophic event or jump on the equity value. Table 4 reports the size of the jump in the stock price resulting from the realization of a Poisson event.

Under the benchmark scenario, equity values decline by about 67% when a jump event occurs. While this is clearly much larger than the associated 10% decline in consumption, it is significantly less than the 90% decline in dividends. Intuitively, equity values decline less than dividends since the mean reversion in the corporate fraction implies that the effects of a jump downwards are not permanent. The 67% value for the benchmark scenario is in good agreement with the historical evidence from the Great Depression. For example, the Dow Jones 30 stock index declined by 69% during the two-year 1930-1931 period. During 1931 alone, the Dow Jones 30 index fell by 53%.

While jump events result in large negative stock market returns in the model, it is important to note that these jumps are not the primary source of stock market
volatility in the model. Intuitively, this is because jump events are relatively rare events and have only a limited effect on the total volatility of returns. This can be seen clearly in Table 4 by comparing the benchmark case with the pure diffusion case in which jumps do not occur. Specifically, equity return volatility is 19.26% in the benchmark case, but is 17.90% in the pure diffusion case. Thus, jump events represent only a minor component of total stock market volatility in the model.

Although our model is calibrated to U.S. data, these results are also consistent with the experience of many other major countries which have undergone large economic shocks. For example, Jorion and Goetzmann (1999) report stock market returns for countries which experienced shocks severe enough to result in temporary stock market closures. Specifically, they report the return for the period from just prior to the closure to the date of reopening. Their Table IV reports that Greece experienced a 58% decline, Japan, a 95% decline, Germany, a 84% decline, and Portugal, a 86% decline during the periods of closures. Of course, there are countries such as China where the economic shocks were so severe that investors presumably lost 100% of the value of their stockholdings.

The other scenarios in Table 4 show that the effects of a jump on equity values are driven by the extent to which dividends are more sensitive to economic shocks. In particular, for the scenarios in which the corporate fraction is constant, the decline in equity values is of the same size as the decline in consumption. Since the largest decline in consumption during the Great Depression was only on the order of nine percent, models which do not allow for differential sensitivity between dividends and consumption cannot capture the size of stock market crashes observed historically.

As before, the relation between the size of the stock market crash and the sensitivity of the corporate fraction to continuous and catastrophic economic shocks, along with the relation between the size of the crash and the current corporate fraction are graphed in Fig. 5. Interestingly, the effect of an increase in the sensitivity to continuous shocks on the crash return is not monotonic. In contrast, an increase in either the jump size or the corporate fraction results in a larger stock market crash.

6. CONCLUSION

Earnings growth is more sensitive to economic shocks, including catastrophic shocks, than consumption growth. This paper documents that this distinction is important for stock pricing. When our model is calibrated to U.S. data, it improves upon the standard model in terms of matching average stock returns. Moreover, the model comes close in matching the return volatility observed in the data.
REFERENCES


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Table 1

**Summary Statistics.** This table reports summary statistics for real per capita consumption and after-tax corporate earnings growth. Also reported are summary statistics for the level of the corporate fraction and the growth rate in the corporate fraction, where the corporate fraction represents the ratio of after-tax corporate profits to aggregate consumption. The statistics are based on annual data for the indicated periods. Full sample denotes the period 1929 to 2001. Ex. 1931-32 denotes the full sample period with the exception of these two years. Since earnings are negative during 1931-32, summary statistics for the corporate earnings and fraction growth rates cannot be computed for the full sample period.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Period</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>Serial Correlation</th>
<th>Correlation with Consumption Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption Growth Rate</td>
<td>Full Sample</td>
<td>0.019</td>
<td>0.024</td>
<td>-0.089</td>
<td>0.021</td>
<td>0.082</td>
<td>0.44</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Ex. 1931-32</td>
<td>0.021</td>
<td>0.020</td>
<td>-0.045</td>
<td>0.023</td>
<td>0.082</td>
<td>0.41</td>
<td>1.00</td>
</tr>
<tr>
<td>Corporate Earnings Growth Rate</td>
<td>Ex. 1931-32</td>
<td>0.034</td>
<td>0.269</td>
<td>-1.033</td>
<td>0.037</td>
<td>0.846</td>
<td>0.24</td>
<td>0.67</td>
</tr>
<tr>
<td>Corporate Fraction</td>
<td>Full Sample</td>
<td>0.098</td>
<td>0.033</td>
<td>-0.042</td>
<td>0.104</td>
<td>0.156</td>
<td>0.76</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Ex. 1931-32</td>
<td>0.102</td>
<td>0.025</td>
<td>0.028</td>
<td>0.104</td>
<td>0.156</td>
<td>0.68</td>
<td>0.46</td>
</tr>
<tr>
<td>Corporate Fraction Growth Rate</td>
<td>Ex. 1931-32</td>
<td>0.009</td>
<td>0.252</td>
<td>-1.034</td>
<td>0.020</td>
<td>0.766</td>
<td>0.25</td>
<td>0.62</td>
</tr>
</tbody>
</table>
Table 2

Estimation Results for Consumption Growth, Earnings Growth, and Short-Term Riskless Rate Regressions. This table reports the results from regressions of consumption growth and earnings growth on the state variable $X_t$ which equals the negative logarithm of the corporate fraction. Also reported are regressions of the real rate $r_t$ on $X_t$. The results are based on annual data from 1929 to 2001 (the years 1931-1932 are excluded).

<table>
<thead>
<tr>
<th>Regression</th>
<th>$a$</th>
<th>$b$</th>
<th>$t_a$</th>
<th>$t_b$</th>
<th>$R^2$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{C_{t+1}-C_t}{C_t} = a + b X_t + \epsilon_{t+1}$</td>
<td>$-0.0269$</td>
<td>$0.0206$</td>
<td>$-1.43$</td>
<td>$2.56$</td>
<td>$0.099$</td>
<td>$70$</td>
</tr>
<tr>
<td>$\frac{D_{t+1}-D_t}{D_t} = a + b X_t + \epsilon_{t+1}$</td>
<td>$-0.8119$</td>
<td>$0.3646$</td>
<td>$-2.15$</td>
<td>$2.14$</td>
<td>$0.158$</td>
<td>$70$</td>
</tr>
<tr>
<td>$r_t = a + b X_t + \epsilon_{t+1}$</td>
<td>$-0.1329$</td>
<td>$0.0638$</td>
<td>$-3.97$</td>
<td>$4.31$</td>
<td>$0.304$</td>
<td>$70$</td>
</tr>
</tbody>
</table>
Table 3

**Benchmark Parameter Estimates.** This table reports the calibrated values for the indicated parameters. The parameter $\lambda$ represents the intensity for the Poisson process, $\gamma$ is the risk aversion coefficient, $\delta$ is the subjective rate of time preference.

\[
\frac{dC}{C} = (\alpha + \beta X) \, dt + \sigma \sqrt{X} \, dZ - \psi \, dq
\]

\[
dX = (\mu - \kappa X) \, dt - \eta \sqrt{X} \, dZ + \xi \, dq
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.0269</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0206</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0200</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.1000</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5829</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.2514</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.2000</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2.1972</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0100</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>5.0000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0010</td>
</tr>
</tbody>
</table>
Table 4
Equity Premia, Return Volatility, and Stock Price Jump Sizes for Different Scenarios. This table reports the indicated equity premia, return volatility, and stock price jump values for the various scenarios listed. The probability of a jump refers to the Poisson intensity parameter $\lambda$. Volatility of consumption refers to the parameter $\sigma$. Volatility of the corporate fraction refers to the sensitivity parameter $\eta$. The jumps in consumption and in earnings denote the percentage changes in these variables resulting from a Poisson event. The equity premium, return volatility, and jump in stock value are all reported in percentages.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>0.01</td>
<td>0.02</td>
<td>0.20</td>
<td>−10</td>
<td>−90</td>
<td>3.22</td>
<td>19.26</td>
<td>−67.18</td>
</tr>
<tr>
<td>Mehra-Prescott (Constant Fraction)</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>0.47</td>
<td>3.05</td>
<td>0.00</td>
</tr>
<tr>
<td>Reitz (Constant Fraction)</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>−10</td>
<td>−10</td>
<td>0.54</td>
<td>3.20</td>
<td>−10.00</td>
</tr>
<tr>
<td>50 Year Jump Frequency</td>
<td>0.02</td>
<td>0.02</td>
<td>0.20</td>
<td>−10</td>
<td>−90</td>
<td>3.72</td>
<td>20.54</td>
<td>−67.52</td>
</tr>
<tr>
<td>20 Year Jump Frequency</td>
<td>0.05</td>
<td>0.02</td>
<td>0.20</td>
<td>−10</td>
<td>−90</td>
<td>5.24</td>
<td>23.96</td>
<td>−68.52</td>
</tr>
<tr>
<td>Pure Diffusion Case</td>
<td>0.00</td>
<td>0.02</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
<td>2.73</td>
<td>17.90</td>
<td>0.00</td>
</tr>
<tr>
<td>Pure Jump Case</td>
<td>0.01</td>
<td>0.00</td>
<td>0.20</td>
<td>−10</td>
<td>−90</td>
<td>0.48</td>
<td>6.74</td>
<td>−67.76</td>
</tr>
<tr>
<td>Increased Consumption Volatility</td>
<td>0.01</td>
<td>0.04</td>
<td>0.20</td>
<td>−10</td>
<td>−90</td>
<td>6.55</td>
<td>21.01</td>
<td>−64.36</td>
</tr>
<tr>
<td>Increased Consumption Jump Size</td>
<td>0.01</td>
<td>0.02</td>
<td>0.20</td>
<td>−20</td>
<td>−90</td>
<td>4.18</td>
<td>19.22</td>
<td>−70.50</td>
</tr>
<tr>
<td>Both</td>
<td>0.01</td>
<td>0.04</td>
<td>0.20</td>
<td>−20</td>
<td>−90</td>
<td>7.43</td>
<td>20.90</td>
<td>−67.86</td>
</tr>
<tr>
<td>Low Sensitivity Case</td>
<td>0.01</td>
<td>0.02</td>
<td>0.10</td>
<td>−10</td>
<td>−50</td>
<td>1.81</td>
<td>10.88</td>
<td>−31.85</td>
</tr>
<tr>
<td>High Sensitivity Case</td>
<td>0.01</td>
<td>0.02</td>
<td>0.40</td>
<td>−10</td>
<td>−100</td>
<td>5.48</td>
<td>32.96</td>
<td>−100.00</td>
</tr>
</tbody>
</table>
Figure 1: Time series plot of the corporate fraction from 1929 to 2001. The corporate fraction is the ratio of aggregate after-tax corporate earnings to aggregate consumption.
Figure 2: Plot of real per capita earnings growth against real per capita consumption growth from 1929 to 2001 (the years 1931-1932 are excluded).
Figure 3: The top panel plots the equity premium as a function of the sensitivity of the corporate fraction to continuous shocks (η) for different values of the corporate fraction F. The middle panel plots the equity premium as a function of the proportional size of jumps in the corporate fraction for different values of the corporate fraction F. The bottom panel plots the equity premium as a function of F for different values of the sensitivity of the corporate fraction to continuous shocks (η).
Figure 4: The top panel plots the equity volatility as a function of the sensitivity of the corporate fraction to continuous shocks ($\eta$) for different values of the corporate fraction $F$. The middle panel plots the equity volatility as a function of the proportional size of jumps in the corporate fraction for different values of the corporate fraction $F$. The bottom panel plots the equity volatility as a function of $F$ for different values of the sensitivity of the corporate fraction to continuous shocks ($\eta$).
Figure 5: The top panel plots the percentage stock jump as a function of the sensitivity of the corporate fraction to continuous shocks ($\eta$) for different values of the corporate fraction $F$. The middle panel plots the percentage stock jump as a function of the proportional size of jumps in the corporate fraction for different values of the corporate fraction $F$. The bottom panel plots the percentage stock jump as a function of $F$ for different values of the sensitivity of the corporate fraction to continuous shocks ($\eta$).