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STOCHASTIC PROPERTIES OF THE CHIRIKOV-TAYLOR MAPPING*

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The "standard" mapping

\[ I_{n+1} = I_n + \frac{k}{2\pi} \sin 2\pi \theta_n \]  
\[ \theta_{n+1} = \theta_n + I_{n+1} \quad (\text{mod } 1) \]  

arises in many problems in plasma dynamics. We have studied the statistical properties of a single orbit, for various values of the parameter \( k > 4 \), i.e., well within the stochastic domain. For most tests, we code the orbit by coarse-graining \( I \)-space into 10 equal cells, and expressing the orbit as a "semi-infinite" sequence of \( (N > 10^4) \) digits, e.g. 8, 1, 4, 4, 7, 9, 7, 8, 7, 2, 2, 3, 5, 1, 5, 5, 3, 2,...

We then examine various joint and conditional probabilities, by counting the relative frequency of finite sequences. For example, \( P(3,5) \) is the relative frequency of the 2-digit sequence 3,5.

First we test for ergodicity by comparing \( P(a) \) for all \( a = 0, 1, \ldots 9 \); we find them all equal (to within expected fluctuations), demonstrating essential ergodicity, for \( k > 4 \). Next we test for statistical independence of \( I \)-values \( m \) iterations apart, and find it for \( m \geq 8 \) at \( k = 5 \), for \( m \geq 6 \) at \( k = 7.5 \), for \( m \geq 1 \) at \( k = 50 \). A related test is for the Markov amnesia time \( m_A \); for \( m \geq m_A \), the conditional probabilities obey the Markov assumption for the \( m \)th iterate of the mapping. We find that \( m_A = 5 \) for \( k = 7.5 \). Under Markov and non-Markov conditions, we evaluate the transition matrix \( P(b|a) \) and its associated entropy. We relate the latter to the Kolmogorov-Sinai entropy and to the Lyapunov exponent. Finally, without coarse-graining, we evaluate the autocorrelation function \( \langle \exp 2\pi i (I_{n+m} - I_n) \rangle \) as a function of discrete \( m \) and continuous \( k \).

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