Time-variable gravity observations of ice sheet mass balance:
Precision and limitations of the GRACE satellite data

I. Velicogna\textsuperscript{1,2} and J. Wahr\textsuperscript{3}

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[1] Time-variable gravity data from the Gravity Recovery and Climate Experiment (GRACE) mission have been available since 2002 to estimate the mass balance of the Greenland and Antarctic Ice Sheets. We analyze current progress and uncertainties in GRACE estimates of ice sheet mass balance. We discuss the impacts of errors associated with spherical harmonic truncation, spatial averaging, temporal sampling, and leakage from other time-dependent signals (e.g., glacial isostatic adjustment (GIA)). The largest sources of error for Antarctica are the GIA correction, the omission of $l=1$ terms, nontidal changes in ocean mass, and measurement errors. For Greenland, the errors come mostly from the uncertainty in the scaling factor. Using Release 5.0 (RL05) GRACE fields for January 2003 through November 2012, we find a mass change of $-258 \pm 41$ Gt/yr for Greenland, with an acceleration of $-31 \pm 6$ Gt/yr\textsuperscript{2}, and a loss that migrated clockwise around the ice sheet margin to progressively affect the entire periphery. For Antarctica, we report changes of $-83 \pm 49$ and $-147 \pm 80$ Gt/yr for two GIA models, with an acceleration of $-12 \pm 9$ Gt/yr\textsuperscript{2} and a dominance from the southeast pacific sector of West Antarctica and the Antarctic Peninsula. Citation: Velicogna, I. and J. Wahr (2013), Time-variable gravity observations of ice sheet mass balance: Precision and limitations of the GRACE satellite data, Geophys. Res. Lett., 40, 3055–3063, doi:10.1002/grl.50527.

1. Introduction

[2] Recent observations of Greenland and Antarctica have shown that the ice sheets are losing mass at rapid rates [Krabill et al., 2004; Rignot and Kanagaratnam, 2006; Velicogna and Wahr, 2006a, 2006b; Chen et al., 2006; Rignot et al., 2008a, 2008b; Howat et al., 2007; Velicogna, 2009; Rignot et al., 2011]. Since 2002, Gravity Recovery and Climate Experiment (GRACE) measurements of time variable gravity have provided novel and critical observations to detect, monitor, and understand ice sheet mass balance [e.g., Velicogna and Wahr, 2006a, 2006b; Chen et al., 2006; Luthcke et al., 2006; Velicogna, 2009; Chen et al., 2009]. Despite this advance, significant differences exist between published estimates of mass loss essentially derived from the same raw GRACE data. A significant share of these discrepancies is due to differences in time spans and the time variable nature of the signal. But differences in analysis methods have also had an impact.

[3] We present a detailed analysis of the factors that contribute to errors in GRACE-derived ice sheet mass balance estimates. We group these errors into four broad categories that correspond to the four main limitations of the GRACE data: (1) the lack of vertical resolution of the data when inverting for mass variability, (2) the coarse horizontal resolution of the data, (3) the limited temporal resolution of the data, and (4) measurement errors in the GRACE gravity solutions. We describe analysis methods commonly used to deal with these issues and discuss procedures to counteract problems that those analysis methods introduce. In addition, we present updated estimates of the mass balance of the Greenland and Antarctic Ice Sheets for January 2003 to November 2012 using the latest release GRACE fields and the most recent compilation of glacial isostatic adjustment (GIA) models.

[4] We focus on methods that employ gravity field solutions provided to users by various processing centers in the form of spherical harmonic (Stokes) coefficients. These gravity solutions typically have a temporal sampling of 1 month, though solutions at shorter time spans are also available. Unless otherwise stated, when we present results for specific computations, we use the averaging kernel approach outlined in Velicogna and Wahr [2006a, 2006b] and abbreviated VW.

[5] There have been studies that determine mass balance by dividing the ice sheet and its surrounding area into small regions (mascons) and fitting mass amplitudes for those regions directly to the level-one data without going through the intermediate step of constructing gravity fields [e.g., Luthcke et al., 2006; Ivins et al., 2011]. We do not specifically discuss the error budget of that technique in this paper, though we note that those studies are subject to the same categories of errors discussed here.

2. Errors Caused by the Lack of Vertical Resolution

[6] Using external gravity alone, it is not possible to determine the exact vertical location of a mass anomaly; i.e., whether it is at the surface, in the underlying solid Earth, or in the overlying atmosphere. To recover changes in ice mass from GRACE, it is therefore necessary to first remove the GIA signal, i.e., the signal associated with the Earth’s ongoing viscoelastic response to ice mass variability that occurred over the past tens of thousands of years, and...
the signal associated with changes in the atmospheric mass distribution.

2.1. GIA Corrections

[7] The only significant time-dependent gravity signal from the solid Earth beneath the ice sheets is from GIA, i.e., the Earth’s ongoing viscoelastic response to ice mass variability that occurred over the past tens of thousands of years. Removing the GIA signal has a significant impact for Antarctica, where it is a major part of the GRACE signal [Velicogna and Wahr, 2006a]. In Greenland, the GIA contribution is much less important, because there, the average GIA gravity change is 2–3 times smaller, and the ice sheet area is 7 times smaller. Over the time period of the mission, the GIA signal appears as a linear trend in $M(t)$, the ice sheet mass at time $t$; i.e., it is a constant in the mass change rate, $dM(t)/dt$, and has no impact on the mass acceleration, $d^2M/dt^2$.

[8] The GIA signal is removed using an a priori model. There are two important model error sources: the ice history and the Earth’s viscosity profile. Uncertainties associated with GIA models can be estimated using a suite of ice histories and viscosity profiles. VW convolved two ice history models (ICE5G [Peltier, 2004] and Fleming and Lambeck, [2004] for Greenland; ICE5G [Peltier, 2004] and IJ05 [Ivins and James, 2005] for Antarctica) with deformation Green’s functions constructed using various plausible combinations of two-layer mantle viscosity profiles. The best GIA estimate is set equal to the midpoint of possible GIA estimates, with an uncertainty given by the range in GIA estimates.

[9] New regional ice deglaciation models have recently been developed to match a variety of geologic, glaciological, and geodetic observations [e.g., Simpson et al., 2009; Whitehouse et al., 2012; Ivins et al., 2013]. In Antarctica, these new models exhibit a smaller East Antarctic ice loss since the Last Glacial Maximum and hence yield a smaller GIA correction to the Antarctic estimates than those predicted by ICE5G. The Antarctic mass change estimates employing regional Antarctic models with ICE5G outside of Antarctica are ~70 Gt/yr less negative than those that use ICE5G everywhere. Because of their reduced ice loss, these new Antarctic models violate far-field sea level observations when combined with conventional northern hemisphere deglaciation histories such as ICE5G; and this issue still needs to be resolved. However, these new models represent a significant advance in Antarctic GIA modeling. For Greenland, GRACE estimates of total ice sheet mass change are relatively insensitive to the choice of GIA model.

[10] For future reference, when we use VW’s analysis method for Antarctica, we find a GIA correction of $-140 \pm 72$ Gt/yr for global ICE5G, and $-71 \pm 34$ Gt/yr when the ICE5G Antarctic component is replaced by Ivins et al.’s [2013] regional model (IJ05_R2). For Greenland, we find a GIA correction of $-2 \pm 21$ Gt/yr using Simpson et al.’s [2009] Greenland model combined with ICE5G for the rest of the globe. In all these cases, the uncertainties come from considering a range of viscosity profiles.

2.2. Atmospheric Effects

[11] Atmospheric fields, most commonly from the European Centre for Medium Range Weather Forecasts, are used by the GRACE processing centers to remove the atmospheric signal from the level-one data before constructing gravity fields. Errors in this correction impact ice mass estimates. By adding back these corrections and using VW’s method of ice sheet analysis, we find that they affect the January 2003 to November 2012 GRACE trends averaged over an entire ice sheet by 2 Gt/yr for both Antarctica and Greenland. Since it is likely that the errors in the atmospheric corrections are even smaller than the corrections themselves, we conclude that atmospheric errors have a negligible impact on ice sheet wide mass balance estimates.

3. Errors Caused by Limited Horizontal Resolution

[12] Because the GRACE satellites are 450 km above the Earth’s surface, they are relatively insensitive to short-scale terms in the gravity field, which decay with altitude more quickly than large-scale terms. For this reason, GRACE errors are larger at short scales than at large scales, and short-scale terms are significantly downweighted in every processing scheme and omitted entirely at scales below a chosen cutoff. As a result, all GRACE mass solutions, regardless of their processing details, are effectively bandlimited, and consequently, every geophysical signal is smeared out and reduced. This reduction/truncation of short-scale terms is done both when the processing centers generate their Stokes coefficients (the processing stage) and when users transform those coefficients into mass estimates (the postprocessing stage).

[13] The processing centers remove short-scale terms by truncating their solutions to a finite set of low-degree harmonics, typically corresponding to scales of a few hundred kilometer and larger. For example, in their Release 5.0 (RL05) fields, CSR (the Center for Space Research at the University of Texas) and GFZ (GeoforschungsZentrum in Potsdam) truncate their fields to maximum degrees of 60 and 90, respectively, and many users further truncate the GFZ fields to a maximum degree closer to the CSR value of 60. In addition, some processing centers impose additional damping criteria on the short-scale terms; e.g., GRGS (Centre National d’Etudes Spatiales) constrains the time dependence of harmonics at degrees >30 to be small, with the strength of those constraints increasing as the degree increases, i.e., as the scale decreases. The maximum degree ($l_{\text{max}}$) of the solution determines the smallest spatial scale ($r_{\text{min}}$) that can be resolved. A rule-of-thumb is that $r_{\text{min}} \sim (20,000/l_{\text{max}})$ km. Thus, for $l_{\text{max}} = 60$, $r_{\text{min}} \sim 330$ km.

[14] This truncation not only removes small-scale features but also reduces the overall amplitude of the recovered signal and introduces ringing into the solution [e.g., Press et al., 1992]. If nothing is done to reduce these effects, then they will degrade estimates of mass variability. To illustrate these effects, we construct a simulated, realistic map of mass change rates across the Greenland ice sheet based on mass balance estimates obtained using the mass budget method [Rignot et al., 2011] (Figure 1a). The simulated signal corresponds to a total mass change of $\sim 264$ Gt/yr. We convert this mass field into a corresponding map of what GRACE would recover, by (a) expanding it into Stokes coefficients of the gravity field, (b) truncating those coefficients to degrees $l \leq 60$, and (c) transforming that truncated set of coefficients back to a representation of mass in the spatial domain (Figure 1b). Note, by comparing with Figure 1a, that the concentrated mass loss regions located around the
output glaciers in the input signal have reduced amplitudes and are spread out across larger regions, and that there are alternating positive/negative stripes (ringing) away from those regions. This ringing (which is not the same as the north/south-trending stripes seen in real GRACE data; see below) has conspired to produce a prominent, focused mass gain feature in Greenland’s interior that is not present in the input signal. This feature counsels caution when interpreting interior mass growth features that appear in GRACE solutions, before steps have been taken to reduce the effects of ringing. The truncation has reduced the total mass change integrated over the ice sheet; the recovered ice mass is 32% smaller than that of the original input signal (Table 1).

[15] The errors in real, unconstrained GRACE gravity field solutions increase rapidly with increasing degrees (i.e., decreasing spatial scales) and are correlated between degrees in such a way as to produce north/south-trending stripes when transformed into the spatial domain. Various postprocessing methods have been developed to reduce the impact of those errors on mass solutions. These include Gaussian smoothing [Wahr et al., 1998], destriping [Swenson and Wahr, 2006], convolving with an averaging function [Swenson and Wahr, 2002], fitting mascons to the Stokes coefficients [Tiwari et al., 2009], and using empirical orthogonal functions to reduce the errors in monthly solutions by, for example, identifying and removing monthly solutions that are adversely affected by orbit resonance errors [Wouters and Schrama, 2007]. In analyses that fit mascons directly to the level-one GRACE data [Luthcke et al., 2006; Ivins et al., 2011], short scales are damped by representing each mascon with a large-scale (i.e., low harmonic) expansion and by requiring each mascon value to be close to the value of nearby mascons.

[16] Every postprocessing method used to reduce noise modifies the contributions from real geophysical signals. To examine effects of Gaussian smoothing (the most common such postprocessing method, used to create the GRACE-gridded products available on the GRACE Tellus website http://grace.jpl.nasa.gov) on the Greenland mass estimate, we apply a Gaussian smoothing function with a 250 km radius to our simulated, truncated Stokes coefficients (from step (b), above) and transform those smoothed coefficients into mass estimates in the spatial domain (step (c)). The results, Figure 1c, show that this filter largely removes the ringing outside Greenland, but the mass loss regions have spread out even further. And although the spurious, focused mass gain in the central interior has been reduced, it is still present in the solution. The application of this smoothing reduces the apparent total mass loss of the ice sheet by an additional 11%, so that the recovered mass loss is now 43% smaller than that of the original input signal.

[17] We also evaluate the effect of another postprocessing technique, the decorrelation (or “destriping”) filter of Swenson and Wahr [2006] on the Greenland ice mass estimates. When we apply this filter to the truncated spherical harmonic coefficients of our simulated signal and apply a 250 km Gaussian-smoothing to the results, we obtain (not shown) a total mass loss that is 56% smaller than the input

Table 1. Truncation and Spatial Averaging Errors

<table>
<thead>
<tr>
<th>R (km)</th>
<th>Filtering</th>
<th>AIS (%)</th>
<th>GIS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No</td>
<td>−24</td>
<td>−32</td>
</tr>
<tr>
<td>250</td>
<td>No</td>
<td>−43</td>
<td>−43</td>
</tr>
<tr>
<td>0</td>
<td>Yes</td>
<td>−38</td>
<td>−52</td>
</tr>
<tr>
<td>250</td>
<td>Yes</td>
<td>−48</td>
<td>−56</td>
</tr>
<tr>
<td>VW kernel</td>
<td>No</td>
<td>−35</td>
<td>−44</td>
</tr>
</tbody>
</table>

*aShown is the error in GRACE total ice sheet mass balance estimates, as a percentage of the input mass loss, due to truncation to $l = 60$, Gaussian-smoothing with radius $R$ in kilometer, applying a destriping filter to the fields (yes or no), and the use of VW’s averaging kernel, for Antarctica (AIS) and Greenland (GIS). Scaling has to be applied to correct the magnitude of the recovered signal.
signal. If the destriped results are not Gaussian-smoothed, the recovered signal is 52% smaller.

[18] In VW, the truncated (but otherwise unsmoothed and unfiltered) Stokes coefficients are convolved with an averaging kernel specially designed to fully recover mass variability within a specified region. This method provides an estimate of the total mass change of the region but does not provide a map such as those in Figure 1. The impact on the total mass change of the real signal depends on the parameter values used to construct the averaging kernel. When VW’s Greenland averaging kernel is applied to the Figure 1a simulated signal, the total Greenland mass loss is reduced by 44%.

[19] Table 1 summarizes the signal reduction caused by these approaches for both ice sheets when applied to our simulated Greenland signal (and a similarly constructed synthetic Antarctic signal). Every method underestimates the mass loss. It is essential to correct for this underestimate by generating a set of plausible change simulations similar to that shown in Figure 1a, applying the postprocessing method to each simulation to determine the signal reduction ratio and scaling the mass loss estimate accordingly. Different simulations will yield different scaling factors, and the uncertainty of the scaling factor impacts the uncertainty of the final mass loss estimate. By considering a set of plausible simulations, a range of ratios can be determined, and this range can be used to determine the scaling factor uncertainty.

[20] Each of these methods, and any other method that uses a linear combination of Stokes coefficients (or of level-one data directly) to estimate the mass change $M$ in a region, delivers an estimate that is a weighted average of the true mass loss:

$$ M = \int_{\text{Earth}} \alpha(\theta, \phi) A(\theta, \phi) a^2 \sin \theta \, d\theta \, d\phi, $$

where $\theta$ and $\phi$ are the co-latitude and eastward longitude, $a$ is the Earth’s radius, and $\alpha(\theta, \phi)$ is the change in surface mass density at $(\theta, \phi)$. $A(\theta, \phi)$ is the sensitivity kernel of the estimate and depends on the analysis method [Jacob et al., 2012]. For an ideal estimate, $A(\theta, \phi)$ would be one inside the region and zero outside, and so there would be no signal reduction. But any filtering method, combined with the truncation of the Stokes fields, causes $A$ to be distorted from this ideal. It is therefore critical that the postprocessing analysis be fine-tuned, so that $A(\theta, \phi)$ is small over any external region likely to exhibit a large mass change and is nearly uniform over the region of interest.

3.1. Leakage From Outside the Ice Sheet and Degree-1 Contributions

[21] Because of the truncation and the postprocessing used to reduce short-scale noise, mass signals from outside an ice sheet contaminate the mass loss estimate (i.e., $A(\theta, \phi) \neq 0$ for points outside the ice sheet). The main sources of external leakage are the following: (1) changes in the storage of liquid water and snow on land outside the ice sheet, (2) ocean mass variability, and (3) ice loss from nearby ice caps.

[22] Leakage problems are complicated by the fact that GRACE does not deliver spherical harmonic coefficients at degree = 1. GRACE level-one data are routinely processed in the Earth’s center-of-mass frame, where the degree-1 coefficients vanish identically. But since we and the ice sheets live in a coordinate system attached to the Earth’s surface and that surface moves relative to the center-of-mass, degree-1 terms are nonzero in our frame, and their omission causes the sensitivity kernel, $A(\theta, \phi)$, to have a small but nonzero tail extending all around the globe. This frame translation, as represented by the inclusion of degree-1 terms, should be incorporated into every processing method, whether that method involves the use of Stokes coefficients, or fits mascons directly to the level-one data. We know of no a priori reason why its impact would be smaller for one method than for another.

[23] Monthly values of the degree-1 terms can be indirectly computed from the GRACE data, combined with GIA and ocean model output, as described by Swenson et al. [2008]. When those terms are added back to the monthly sets of Stokes coefficients, the sensitivity kernel becomes more focused, and the leakage decreases. The impact on ice sheet estimates is significant, particularly for Antarctica because of its size and its position relative to the polar axis. Using VW’s analysis method and degree-1 coefficients provided by Sean Swenson, we find that if those coefficients are not included, then the January 2003 to November 2012 mass loss rates are overestimated by 38 Gt/yr for Antarctica and underestimated by 9 Gt/yr for Greenland. These errors differ in magnitude and sometimes in sign for shorter time spans, and they range between $-20$ and $50$ Gt/yr for Antarctica and $-19$ and $9$ Gt/yr for Greenland (Figure S1 in the auxiliary material). In the remainder of this paper, we assume the degree-1 terms are included in the analysis.

[24] Leakage source 1, from water and snow on land outside the ice sheet, can be estimated using monthly global water storage fields from a hydrology model. We calculate spherical harmonic gravity coefficients from the Noah version of the Global Land Data Assimilation System (GLDAS/Noah) [Rodell et al., 2004], remove them from the GRACE Stokes coefficients, and apply VW’s analysis to the Stokes coefficient residuals. We find a leakage of only 1–2 Gt/yr for both ice sheets, because neither ice sheet is close to land that has appreciable water storage variability. Presumably, the errors in these corrections would be even smaller. The differences become larger if degree-1 terms are not included, as the long tail of the resulting sensitivity kernel picks up hydrology signals from around the globe.

[25] There are two types of contributions to the leakage from nontidal changes in ocean mass (source 2): (a) those due to changes in the general circulation of the ocean and (b) changes in water mass due to the exchange of water between the ocean and land (i.e., eustatic contributions). Contributions from (a) are reduced by the processing centers prior to computing gravity field solutions, by using output from an ocean general circulation model (OGCM). CSR and GFZ solutions, for example, use output from the Ocean Model for Circulation and Tides OGCM [Thomas, 2002]. By applying VW’s postprocessing method to the output of this ocean model, we find that the ocean corrections make the January 2003 to November 2012 GRACE trend more negative than it would otherwise be, by 20 Gt/yr for Antarctica and by less than 1 Gt/yr for Greenland. It is not clear how to assess the errors in these ocean model trends. If we assume those errors are no larger than the model results themselves, then we conclude that ocean circulation errors have a negligible impact on ice sheet mass balance estimate for Greenland but could be significant for Antarctica.
This model, like most OGCMs, conserves mass and does not include contributions from (b). Those can be added separately by computing the total water mass lost from (or added to) land every month, adding that amount of water to the ocean and computing the Stokes coefficients caused by that addition. The water lost from land can be estimated as the sum of the total global change of water and snow predicted by the hydrology model, and the mass lost from the polar ice sheets themselves. For ice sheets, we use an iterative procedure by which the mass change is first estimated without including its contribution to (b), its contribution to (b) is then calculated, the resulting ocean corrections are made, and the polar ice change is computed again.

It is usual when modeling the addition of mass to the ocean, to assume the water is uniformly distributed over the ocean. In reality, the distribution depends on where the water comes from [e.g., Tamisiea et al., 2001]. The removal of water from land causes uplift of the surrounding crust. If the water comes, for example, from thinning glaciers along the coast of the ice sheet, then the adjacent sea floor can rise by more than the thickness of the added ocean water, and there can be a net oceanic mass loss near the coast and an increased mass gain further away. Furthermore, the removal of ice reduces the gravitational force pulling ocean water toward the ice sheet, and so that water tends to flow away, further reducing adjacent sea level. Oceanic changes near the coast are particularly important, since they are optimally positioned to contaminate the ice sheet estimate.

We improve the calculation of this leakage by evaluating the contribution to (b) assuming a spatially variable distribution for the additional mass. We find that the spatial distribution is not critical when computing contributions from water originating in continental regions outside the ice sheets. The land water lost from those regions, as inferred from the GLDAS/Noah model, has less than a 1 Gt/yr impact on either ice sheet, whether it is distributed into the ocean as a uniform layer or in the self-consistent manner described by Tamisiea et al. [2001].

When evaluating the effect of the water added to the oceans from the ice sheets, however, it is important to distribute the water in a self-consistent manner. The exact numerical impact depends on the details of the analysis method, since that determines how far the sensitivity kernel extends out into the ocean. For VW’s analysis, we find that for Antarctica, the GRACE-derived mass loss is reduced by 4% if the Antarctic meltwater is distributed as a uniform layer and by 9% if it is distributed self-consistently. For Greenland, the corresponding percentages are smaller: 0.5% and 3%, respectively. For January 2003 to November 2012, the inclusion of the ice sheet meltwater in a self-consistent manner increases the mass loss rates by ~10 Gt/yr for Antarctica and ~8 Gt/yr for Greenland.

The contamination of the GRACE ice sheet mass balance estimates by ice loss from nearby glaciers and ice caps, GICs, (leakage source 3) is difficult to determine. For Greenland, the contamination from Baffin, Ellesmere, Axel Heiberg, and Devon Island should be estimated and removed. To do this, we compute the Stokes coefficients representing uniform mass changes of ~35 ± 4 Gt/yr from Baffin island and ~41 ± 4 Gt/yr from Ellesmere, Axel Heiberg, and Devon Islands combined [Jacob et al., 2012, updated to January 2003 to November 2012], apply VW’s analysis to those simulated Stokes coefficients, and find a leakage of 13 ± 2 Gt/yr. This value depends on the values of the sensitivity kernel over those islands, which depends on details of the analysis method. The peripheral glaciers in both Antarctica and Greenland are too close to those ice sheets to be resolvable with GRACE, and their mass changes are not well determined by independent data. The best approach is to leave their contributions as part of the GRACE ice sheet estimates.

4. Errors Caused by Temporal Aliasing

Most GRACE processing centers generate global gravity field solutions at monthly intervals. These solution are not true monthly averages but depend on the way the satellite tracks sample the Earth during a month. As a result, submonthly gravity fluctuations do not necessarily average out in the monthly solutions, but alias into apparent longer-period variability. Processing centers reduce this aliasing by using model output to remove short-period signals caused by the atmosphere, nontidal ocean circulation, and Earth and ocean tides before solving for gravity. Still, there are errors in those models, which lead to aliasing errors in the monthly solutions.

Aliasing errors caused by errors in the atmospheric and nontidal ocean models are likely to appear as random noise from 1 month to the next and mostly average out when solving for long-period trends. This is also true for errors in most of the tidal lines. However, there are a few diurnal and semidiurnal tidal lines that give aliasing errors that appear as longer-period terms in the monthly solutions. The only tides with aliased periods long enough to possibly impact the mass balance trends, are K1 and K2, where the aliased periods are 7.5 years and 3.7 years, respectively [e.g., Ray and Luthcke, 2006]. Both periods are long enough that they should not affect the solutions for the trend over the entire data span, but they are of potential concern when looking at interannual variability. Even then, it is only the errors in the tidal models that are important, not the total tidal signals themselves. When we use VW’s analysis method to compute an ice sheet time series and fit a trend (and seasonal terms) to the time series, the trend results for each ice sheet are affected by only 3–4 Gt/yr when we also include 7.5 year and 3.5 year terms in the fit.

The choice of 1 month for the time span of a global solution is somewhat arbitrary. The longer the time span, the better the spatial coverage during that time span, and the better the spatial resolution. Some processing centers opt for submonthly time span to increase the temporal resolution. To compensate for the reduced spatial coverage in that case, those centers tie each solution together with solutions for neighboring time spans, using a smoothing function. Thus, the temporal resolution in those cases is often not as short as the length of the individual time spans tends to suggest.

5. Measurement Errors in the GRACE Gravity Solutions

We estimate the effects of measurement errors in the individual GRACE monthly fields by convolving the averaging function with uncertainty estimates for the GRACE Stokes coefficients, as described in Wahr et al. [2006]. The uncertainties in the Stokes coefficients are computed as the RMS of the difference between the Stokes coefficients and a
temporally smoothed version of those coefficients. We find monthly measurement uncertainties of 27 Gt for Greenland and 99 Gt for Antarctica. The large Antarctic uncertainty is due to the large month-to-month scatter caused by errors in the atmospheric corrections over Antarctica.

Estimates of the uncertainties in the Stokes coefficients are also included in the GRACE data release in the form of calibrated errors in the Stokes coefficients: diagonal elements of the covariance matrix, rescaled by the GRACE processing centers to match certain characteristics of the fields.

6. Comparison of Results From Different Centers

Ice mass trends inferred from the Stokes coefficients provided by different processing centers (CSR, GFZ, and GRGS) are in reasonably good agreement, as long as each center’s $C_{20}$ coefficients are replaced with $C_{20}$ coefficients inferred from satellite laser ranging (SLR) [Cheng et al., 2013] (Figure S2). Using VW’s analysis method, the inferred ice mass trends for January 2003 to April 2011 agree to within 6% for Antarctica and 4% for Greenland. This indicates that the constraints on degrees > 30 that GRGS imposes on the fields during processing have little impact on the ice mass trends, at least when averaged over an entire ice sheet.

7. Ice Sheet Mass Balance Estimates

We use VW’s analysis method to calculate time series of Greenland and Antarctic mass using RL05 GRACE gravity field solutions from CSR for January 2003 to November 2012 (Figure 2). We make all the corrections described above: scaling; including degree-1 terms; replacing the GRACE $C_{20}$ coefficients with those obtained from SLR; removing a GIA contribution; removing the GLDAS/Noah hydrology signal, redistributing any excess water into the ocean; distributing Greenland and Antarctic meltwater into the ocean with a spatial pattern that is consistent with the sea level equation; removing the contributions of Canadian Arctic ice caps from the Greenland estimate using Jacob et al. [2012] updated estimates. We obtain trends of $-258 \pm 41$ Gt/yr for Greenland; and $-83 \pm 49$ Gt/yr (U05_R2) and $-147 \pm 80$ Gt/yr (ICE5G) for Antarctica. The uncertainties include contributions, added in quadrature, from all the possible error sources described above: the scaling factor, the GIA correction, the atmospheric and oceanic corrections, as well as from the statistical uncertainty of the fit (Table 2). The statistical uncertainty includes both (a) the effects of measurement errors in the individual GRACE monthly fields (section 5) and (b) the fact that the real ice loss signal is not perfectly represented by the terms we fit to the data. The contributions from (b) are computed by accounting for the autocorrelation of the signal [e.g., Cowpertwait and Metcalfe, 2009].

Because the RL05 fields have only recently become available, most previously published ice sheet solutions use the RL04 fields. The main difference between the RL04 and RL05 solutions that impacts ice sheet mass estimates is an improved Atmosphere and Ocean De-aliasing (AOD) product that provides a more accurate correction for sub-monthly mass variability from the atmosphere and the ocean [Bettadpur et al., 2012]. Near Antarctica, in particular, the new AOD improves the representation of nontidal ocean mass variability, particularly beginning in mid-2009.

We compare the RL05 and RL04 solutions during their common period, January 2003 to April 2012. In Greenland, the difference between the two releases is negligible and less than 1%. But in Antarctica, the January 2003 to November 2012 mass change becomes more negative by 40 Gt/yr when the RL05 fields are used. The difference between the RL05 and RL04 Antarctic trends is not significant for time spans that end near or before mid-2009, though it steadily increases as the end of the time span extends past that date. When RL04 fields are used and the RL04 AOD correction is replaced with the RL05 correction, the results compare well with the one from the RL05 fields [Shepherd et al., 2012], which shows that the main difference between RL05 and RL04 solutions in Antarctica is the improved AOD product.

The GRACE time series in Figure 2 exhibit a curvature that indicates an above-linear change in ice mass, i.e., an acceleration in mass loss with time. When we fit a quadratic regression model to the data, we calculate an acceleration in ice mass of $-31 \pm 6$ Gt/yr$^2$ for Greenland and $-12 \pm 9$ Gt/yr$^2$ Antarctica for January 2003 to November 2012. The value of the acceleration is not affected by uncertainties in the GIA
Table 2. Error Sources and Their Estimated Magnitudes

<table>
<thead>
<tr>
<th>Error Source</th>
<th>AIS (Gt/yr)</th>
<th>GIS (Gt/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere</td>
<td>±2</td>
<td>±2</td>
</tr>
<tr>
<td>Ocean circulation</td>
<td>±20</td>
<td>±0.05</td>
</tr>
<tr>
<td>Scaling</td>
<td>±3</td>
<td>±29</td>
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<tr>
<td>GIA IJ05_R2</td>
<td>71 ±39</td>
<td>-2 ±1</td>
</tr>
<tr>
<td>GIA ICE5G</td>
<td>141 ±72</td>
<td>9 ±19</td>
</tr>
<tr>
<td>Use of original C50 terms</td>
<td>40</td>
<td>8</td>
</tr>
<tr>
<td>GIC leakage</td>
<td>-</td>
<td>13 ±2</td>
</tr>
<tr>
<td>Hydrology leakage</td>
<td>±2</td>
<td>±1</td>
</tr>
<tr>
<td>Omission of eustatic ocean</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Measurement error</td>
<td>±28</td>
<td>±8</td>
</tr>
</tbody>
</table>

*Error sources and corresponding amplitudes for January 2003 to November 2012, for the GRACE ice sheet mass balance trends in Giga-ton per year (Gt/yr) are shown for VW’s approach for Antarctica (AIS) and Greenland (GIS). See text for definition of error sources. For glacial isostatic adjustment (GIA) and glaciers and ice caps (GIC), amplitude of the correction is included.

We calculate the adjusted $R^2$ ($R^2_{adj}$) of the linear and quadratic data fit. For both Greenland and Antarctica, we find that $R^2_{adj}$ is larger when we use a quadratic fit, i.e., the data are better modeled by a linear increase in mass loss than by a constant mass loss. The $F$-test [e.g., Berry and Feldman, 1985] shows that the improvement obtained with the quadratic fit is statistically significant at or above the 98% confidence level for each ice sheet.

To understand what regions of the ice sheets are contributing to these increasing trends, we calculate the trend for the entire period and for two partial spans: January 2003 to December 2006 and December 2006 to November 2012. To calculate the trend for the two partial spans, we use a piecewise linear regression model [e.g., Berry and Feldman, 1985], which allows for a change in slope but with the condition that the lines be continuous at the intersection point (December 2006). We transform the trend estimates into estimates of surface mass trends as a function of latitude and longitude and apply Gaussian smoothing functions with 250 and 300 km radii for Greenland and Antarctica, respectively.

The results, Figure 3, show how the Greenland and Antarctic mass trends change between these two periods. In Greenland, the results reveal a widespread pattern of mass loss, except in the interior. During 2006–2012, the mass loss increased at northern latitudes, especially along the northwest coast and the northeast. In the southeast, the mass loss decreased slightly with respect to 2002–2006. In the summer of 2012, the Greenland time series Figure 3a reveals a
large mass loss, of about 700 ± 100 Gt, between January and September, which reflects the major melt event that took place that summer and resulted in a large increase in runoff [Hall et al., 2013].

[44] In West Antarctica, the ice mass loss increased in 2006–2012 relative to 2002–2006, which is consistent with the steady acceleration of outlet glaciers in that region [Rignot et al., 2011]. In the Pine Island/Thwaites sector, the mass loss spread inland and increased. In 2006–2012, East Antarctica experienced a gain in mass loss in Queen Maud Land. This increase has been analyzed independently and attributed to a sudden increase in snowfall in 2008–2009 that added about 300 Gt of mass to this part of the ice sheet [Boening et al., 2011]. This event is largely responsible for an increasing trend in total East Antarctic mass during 2003–2012. As this is a single snowfall event, its impact on the long-term trend will be limited, and the trend should decrease as the data span lengths.

[45] In Wilkes Land of East Antarctica, there are increasing mass loss trends in the region surrounding Totten and Cook Glaciers. While these trends are much smaller than the decreasing trends in the Amundsen Sea sector, the results indicate that a significant mass change is taking place in this region. This is consistent with recent analyses of ice volume changes from laser [Pritchard et al., 2009] and radar [Flament and Remy, 2012] altimetry, which reveal ice thinning along the glacier troughs, and mass budget calculations [Rignot et al., 2011].

[46] The use of more recent regional GIA models reduces the estimate of total mass loss from Antarctica by ~70 Gt/yr, for every time span [see, also, Shepherd et al., 2012; King et al., 2012; Ivins et al., 2013]. Further progress in resolving residual uncertainties in GIA modeling is inevitable. This is of special importance in East Antarctica, where even the sign of the mass balance of the ice sheet is not yet fully resolved, and which is also significantly affected by temporal fluctuations in snowfall.

8. Conclusion

[47] Published GRACE ice sheet estimates have not always agreed, in part because they have considered different time spans, but sometimes also because of incomplete appreciation of errors and how to compensate for them. When systematic errors are accounted for, and a consistent set of corrections and the same time span are used, different postprocessing methods produce consistent ice mass balance estimates [e.g., Shepherd et al., 2012]. The technical discussion in this paper may serve as a guide to help reduce those problems in the future. We review sources of systematic errors that occur when GRACE data are used to estimate ice sheet mass variability; and we discuss methods for quantifying, analyzing, and resolving those errors. We show that the largest sources of error for Antarctica are the GIA correction, the effects of $C_{20}$ coefficient, the omission of $l = 1$ terms, nontidal changes in ocean mass, and measurement errors. For Greenland, all the above sources impact the final result, but the largest error source is the uncertainty in the scaling factor. Using Release 5.0 (RL05) GRACE fields for January 2003 through November 2012, we find a mass change of $-258 \pm 41$ Gt/yr for Greenland, with an acceleration of $-31 \pm 6$ Gt/yr$^2$, and a loss that migrated clockwise around the ice sheet margin to progressively affect the entire periphery. For Antarctica, we report changes of $-83 \pm 49$ and $-147 \pm 80$ Gt/yr for two GIA models, with an acceleration of $-12 \pm 9$ Gt/yr$^2$ and a dominance from the southeast pacific sector of West Antarctica and the Antarctic Peninsula.

[48] We note the good agreement between data from different processing centers, and the robustness of the Greenland results with respect to uncertainties in the GIA model and to various GRACE corrections. In Antarctica, recent developments in GIA modeling have a significant impact on the GRACE trend estimates. The recent RL05 release includes an improvement in the ocean correction near the East Antarctic coast starting in mid-2009, which leads to larger GRACE-inferred mass loss (relative to RL04) for time spans that extend past that date, if that ocean model improvement is not incorporated into the RL04 analysis. Our results indicate that during January 2003 to November 2012, the rate of mass change was $-258 \pm 41$ Gt/yr for Greenland, and $-83 \pm 49$ Gt/yr (using the IJ05_R2 GIA model) and $-147 \pm 80$ Gt/yr (using ICE5G) for Antarctica. During the analyzed time period, however, the mass loss is not constant but increases linearly with time at a statistically significant level. A continuation of the gravity time series is critical for a better understanding of this acceleration and to improve the observational record of ice sheet mass changes.

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[50] The Editor thanks two anonymous reviewers for their assistance in evaluating this paper.

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