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IS Ni$^{56}$ A GOOD CLOSED SHELL?

Larry Zamick

August 14, 1968
IS $^{56}\text{Ni}$ A GOOD CLOSED SHELL?†

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August 14, 1968

The beta decay of $^{56}\text{Ni}$ to the $J = 1^+$ state of $^{56}\text{Co}$ proceeds about ninety-five times slower than is predicted by the shell model. Perturbation theory does not change this very much. Hence $^{56}\text{Ni}$ is not a good closed shell.

There is some controversy over whether or not $^{56}\text{Ni}_{28}$ is a good closed shell nucleus. On the one hand, Hartree-Fock solutions yield deformed ground states for the nickel isotopes [1] but there is always the possibility that the inclusion of pairing will restore the spherical symmetry [2]. In favor of a spherical solution is the fact that the energy spectra of the Nickel isotopes come out well if one uses an effective interaction and assumes a closed $f_{7/2}$ shell.[3, 4]. But it is well known that the effective interaction can mask the effects of deformation.

Since evidence based on theoretical grounds is not very conclusive it is well to look for empirical evidence. We wish to show that the beta decay

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of $^{56}\text{Ni}$ provides very strong evidence that $^{56}\text{Ni}$ is not a good closed shell nucleus.

The allowed Gamow-Teller transition of the $J = 0^+$ ground state of $^{56}\text{Ni}$ to the $J = 1^+$ state of $^{56}\text{Co}$ proceeds with a rate given by $\log ft = 4.4$. This rate is very easy to calculate in the shell model picture because both the initial and final states are unique. $^{56}\text{Ni}$ closes the $1f_{7/2}$ shell whereas the $^{56}\text{Co}$ wave function is $[f_{7/2}^{-1} f_{5/2}^{-1}] I = 1$.

The $ft$ values are given by

$$\log ft = 3.64 - \log M_{\text{GT}}^2$$

$$M_{\text{GT}} = \Sigma_{M\Gamma\alpha} \left| \langle \psi_{M\Gamma}^{JF} \Sigma \alpha \tau_+ \alpha \psi_{M\Gamma}^{Jf} \rangle \right|^2 .$$

We find

$$M_{\text{GT}}^2 = \left[ 2 \sqrt{2j + 1} \sqrt{(2j + 1)/2l + 1} \right]^2$$

where $j = 5/2$ and $l = 3$

$$M_{\text{GT}}^2 = 96/7 \quad \log ft = 2.5.$$

It may at first be surprising that $M_{\text{GT}}^2$ is so large (it is only 3 for a free neutron). But note that the transition from $J = 0$ to $J = 1$ is three times faster than from $J = 1$ to $J = 0$. Also the fact that there are many $f_{7/2}$ protons each of which can undergo a beta decay. The shell model transition rate is ninety-five times faster than experiment.

We should check, however, to see whether the transition rate is sensitive to small perturbations. There exist examples where this is so.
In lowest order perturbation theory there is only one additional mechanism which will affect the decay—a two particle—two hole component is admixed into the $^{56}\text{Ni}$ ground state and the process proceeds via $f_{7/2}^2 f_{5/2} \rightarrow f_{7/2 N}^1 f_{5/2 N}$.

Let us write the $^{56}\text{Ni}$ wave function as

$$\Psi = |0\rangle + \sum_{I_A T_A} b(I_A) \left[ [f_{7/2}^{-1} f_{7/2}^{-1}]^{I_A T_A} [f_{5/2} f_{5/2}]^{I_A T_A} \right]^{00} + \text{other configurations.}$$

Note that if $I_A$ is even $T_A = 1$ and if $I_A$ is odd then $T_A = 0$. In lowest order perturbation theory we find

$$M_{GT} = \frac{96}{7} \left[ 1 + \sum_{I_A = 0} b(I_A) \sqrt{2 I_A + 1} W(1 7/2, 5/2, I_A, 5/2, 7/2) \right]$$

$$\left( \frac{1}{\sqrt{3}} \quad \text{if } I_A \text{ is even} \right.$$  
$$\left. -1 \quad \text{if } I_A \text{ is odd} \right) \right]^{2}$$

$$= \frac{96}{7} \left[ 1 + 0.083 b(0) - 0.240 b(1) + 0.163 b(2) - 0.286 b(3) + 0.145 b(4) - 0.174 b(5) \right]^{2}.$$ 

Now

$$b(I_A T_A) = -\sqrt{(2 I_A + 1)(2 T_A + 1)} \langle f_{5/2}^2 I_A \frac{\Delta E}{2} f_{7/2}^2 I_A \rangle$$

where $\Delta E$ is minus twice the $f_{5/2} - f_{7/2}$ single particle splitting.
$\Delta E \approx -12$ MeV. Using the two body matrix elements of Kuo and Brown [5] the following values of $b(I_A)$ are obtained

\[
\begin{align*}
  b(0) &= -0.402 \\
  b(3) &= 0.114 \\
  b(4) &= -0.172 \\
  b(5) &= 0.047 \\
  b(2) &= -0.206
\end{align*}
\]

The signs of the $b(I_A)$'s are such that the sum over $I_A$ is coherent, all terms in the sum are important, and the correction goes in the direction of experiment. We find however that the correction is much too small.

\[
M_{GT}^2 = 96/7 [1 - 0.17]^2
\]

\[
\log ft. = 2.68.
\]

The combined set of circumstances—that the initial and final wave functions are unique in the shell model, that the Gamow Teller operator is an extraordinarily simple operator, that the deviation from experiment is large using shell model wave functions, and that this deviation persists after the use of perturbation theory—indicates without question that $^{56}$Ni is not a good closed shell nucleus.
REFERENCES

2. Y. Bar-Touv, private communication.
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