Party Discipline with Electoral and Institutional Variation

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First Version: October 21, 2003
This Version: March 9, 2004

Abstract

We study a model of party formation in which party discipline and inter-party ideological heterogeneity are endogenously and jointly determined. Discipline benefits party members because it gives risk-averse voters more confidence in the ideological composition of the party, but this discipline is costly to members who win office. Equilibrium is determined by balancing these forces. We show that this model can account for both comparative difference between parliamentary and presidential systems, and changes over time in the U.S. Congress.

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*We received helpful comments from Randy Calvert, John Duggan, Becky Morton, Steve Smith and seminar participants at Cornell, NYU, Princeton, and the Conference on Formal and Empirical Models of Politics at the Center in Political Economy, Washington University.
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There is substantial variation in party discipline and legislative cohesion, both across nations and over time. U.S. Congressional party leaders have greater formal authority over legislation than they had prior to 1970 (Rohde 1991), and roll call voting is more cohesive within congressional party caucuses today than before 1970 (Aldrich 1995, Rohde 1991). Observers as far back as Woodrow Wilson (1885) have claimed that parties are more cohesive in Westminster systems, such as that used in the U.K., than in presidential systems like the U.S.’s.¹ (See Cain, Ferejohn & Fiorina (1987) for more recent evidence.) Explaining the causes of this variation is the central theme of this paper.

It is often suggested that cross-country differences are caused by institutional variation. That is, different institutional structures (e.g., presidentialism versus parliamentarism) provide different incentives for parties to impose discipline and maintain cohesion (Cox & McCubbins 1993, Cain et al. 1987). While the purely institutional models identify an important explanation of comparative variation, a more complete theoretical model would also explain within-constitution variation (such as the over-time trends in the U.S.). Doing so requires something beyond a purely institutional account (since institutions have not changed within the United States Congress). In order to address a broader range of empirical variation, we propose a formal model in which party discipline is chosen strategically by parties, and show that it can account for both the comparative and historical evidence by employing both institutional and non-institutional causal mechanisms.

In our model, politicians endogenously choose the level of discipline their party will impose. Discipline benefits politicians because a more disciplined a party makes voters more confident about the policy positions members of that party will pursue. Since voters are risk averse, an increase in a party’s discipline increases the probability voters will vote for that party. However, legislators also bear costs from discipline. In particular, politicians are less able to pursue their own policy agendas once in office if their party is highly disciplined. Politicians, then, face a trade-off: they are more likely to be elected, but benefit less from holding office, when discipline increases. The equilibrium level of discipline balances these two concerns. We

¹A Westminster system is a parliamentary form of government in which legislators are elected in plurality rule, single-member districts.
identify two causal mechanisms that affect this trade-off, and thereby alter the level of party discipline: institutional variation and the competitiveness of elections.

Institutional variation explains why parliamentary systems are more disciplined than presidential systems. In presidential systems, the legislature’s policy setting power is constrained by the independent executive—a constraint that does not exist in pure parliamentary systems. This difference in legislative power leads voters in presidential systems to care less about uncertainty over their representatives’ policy preferences than do voters in parliamentary systems. This is because the president serves as a hedge against legislative extremism. Since the choice of a level of discipline affects affiliation decisions and, consequently, the ideological variation in the party, parties will choose different levels of discipline under different institutional structures. The less the voters value certainty, the lower the level of discipline the party will choose. Consequently, parties in parliamentary systems are more disciplined than those in presidential systems.

Changes in the competitiveness of elections explain over-time variation in the United States Congress. In the early years of the century, most Congressional districts were relatively uncompetitive. The south was solidly Democratic, while the non-urban parts of the north and west were solidly Republican. This trend reversed in the mid-1960s, due in part to the debate over civil rights. In addition, Cox & Katz (2002) show that redistricting in the wake of Baker v. Carr (decided in 1962) led to the end of many safe Republican districts in the north. As a result, at the state level party competitiveness increased. Within the model, this increase in competitiveness means that the pivotal voter in key districts became more likely to be nearly indifferent between the parties. This increased likelihood of indifference implies that a small increase in discipline by one party is more likely to swing the election in favor of that more-disciplined party. This increases the incentives for discipline for both parties, resulting in stronger parties in equilibrium. Thus, historical changes in party strength in the U.S. Congress can be understood with the same model that explains comparative differences in party strength between Westminster and presidential systems; however doing so requires a model that allows for both institutional and non-institutional causal mechanisms.

Careful observers of American politics will note that, while state level competitiveness has
increased since the 1960s, many individual districts have become less competitive because gerrymandering has created an abundance of “safe” seats. Importantly, the concept of increased competitiveness that is needed for our model is consistent with this trend. As will become clear, our account only requires that the median member of each party run in a more competitive district. Because the median member of the party is likely to be from a relatively moderate district in the model, this will occur when state-level competitiveness increases even if there is a concurrent increase in safe seats. Thus, it is important to recall, throughout the paper, that our claim that there has been an increase in competitiveness is related to the idea that the south has become less solidly Democratic and the north and west less solidly Republican, overall. It does not exclude the possibility that many seats have, nonetheless, become safer.

Our formalization also helps to resolve a puzzle about the relationship between party discipline and constituency service activities. Traditional political science thinking links weak legislative parties with strong incentives for legislators to engage in behavior that benefits their local constituency. This link is based on the idea that differences in party discipline and cohesion are ultimately caused by the effect of institutions on the tradeoffs voters are willing to make between the party affiliation and individual reputations of candidates (Cain et al. 1987, Cox & McCubbins 1993). The more institutions encourage a focus on party label, the argument goes, the stronger the parties and the lower the level of particularistic behavior by legislators.

We will show that while institutional changes do have this kind of effect, changes in the competitiveness of elections have a different effect on incentives to provide constituency service. In a companion paper (Ashworth & Bueno de Mesquita 2003a), we show that—unlike institutional changes where strong parties are associated with low levels of particularistic behavior—changes in the competitiveness of elections can lead, simultaneously, to stronger parties and enhanced incentives for particularistic behavior. Thus our theory can explain why party discipline and constituency service look like substitute instruments in cross-sectional comparisons where the causal variation is institutional, and like complementary instruments in U.S. history (both have increased since the 1960s) where the causal variation is in the electorate’s preferences.

The intuitions are the same as before. Moving from a presidential to a parliamentary system changes the marginal voter’s tradeoff between uncertainty about party ideology and
constituency service. Voters put more weight on uncertainty in parliamentary systems because of the absence of an independent executive branch. Consequently, parties are relatively strong and constituency service relatively weak in parliamentary systems. Increased competitiveness, however, does not change the voters' tradeoff. Instead, more competitive elections mean that the pivotal voter is closer to indifferent between the two parties and thus more likely to be swayed by either discipline or constituency service. Consequently, an increase in competitiveness leads to an increase in party strength and constituency service. Thus while the comparative evidence (and our comparative model) suggest a tradeoff between cohesion and particularism, changes in district level competitiveness lead to more of both.

The paper is organized as follows. The next section discusses some relevant literature and locates our work within it. Section 2 describes the model, and section 3 describes our concept of a stable party system. Section 4 presents the comparative static results, which are related to some debates in American politics in section 5. Section 6 outlines several extensions to the basic model. Section 7 concludes by informally describing how our companion paper builds on the intuition of this paper to study constituency service.

1 The Extant Literature

1.1 Theoretical Models of Parties and Cohesion

Several other papers model elections and legislation with endogenous party structures. The most closely related to our work is Snyder & Ting (2002), which we discuss in the next section, along with other work on informative party labels. Here, we briefly review work based on other motivations for party formation.

A variety of scholars have employed complete information models to examine incentives for party formation. Morelli (2002) studies a model in which parties provide two services: They help voters coordinate in the election and they allow candidates to commit to policies. Morelli compares plurality and proportional representation systems and finds that, if districts are similar enough, then there will be more parties under proportional representation than under plurality rule. If the districts are sufficiently heterogeneous, then these Duvergerian prediction
can be overturned.

Levy (2002) also focuses on the role parties play in insuring the credibility of policy commitments. Levy assumes that citizen-candidates can form a party and commit to any point in their Pareto set. This ability to commit proves beneficial to candidates as long as the policy space is multidimensional. Osborne & Tourky (2003) focus on the role of increasing returns to party size. They show that a two-party system will arise endogenously in many kinds of legislative institutions.

Diermeier & Feddersen (1998) and Huber (1996) study the role of legislative institutions, particularly the vote of confidence procedure, on party cohesion in legislation. They show that the ability to attach a confidence motion to a bill gives the party leadership an effective threat which helps keep the rank-and-file in line. Calvert & Fox (2000) demonstrate that control over agenda setting power in a repeated game can also be used to discipline legislative behavior and create party cohesion. Our work is complementary to these arguments. Diermeier and Feddersen, Huber, and Calvert and Fox identify important mechanisms through which party leaders control their members. Our model addresses incentives to grant and use such powers, which allows us to find within-institution comparative statics. Furthermore, our work provides a needed addition to Diermeier’s and Feddersen’s and Huber’s comparative conclusions—it is plausible that contemporary Congressional party leaders would like to increase discipline but cannot because they lack institutional tools like the vote of confidence. But this cannot be a complete explanation. For much of U.S. history, cohesion in Congress was even lower, relative to parliamentary systems, than it is today. This historical difference cannot be explained by institutional limits to cohesion in the U.S., since cohesion subsequently increased. Our model addresses why the change in party cohesion in the United States took place when it did.

1.2 Informative Party Labels

Perhaps the most well-known answer to the question of what role parties play in elections is that party labels provide voters with information about the candidates. Knowing that a candidate is a Democrat tells an American voter that the candidate is more likely than average to favor redistribution, abortion rights, etc. This idea has a long history, going back at least to Downs
(1957), who pointed out that because voters have little incentive to acquire costly information about candidates, they will rely on low-cost informational shortcuts. If the two parties offer different ideologies on average, the informational use of party labels is a rational response on the part of voters.

There is substantial evidence that voters learn about the policy positions of candidates from party labels. This literature is reviewed extensively in Snyder & Ting (2002); we summarize the relevant parts here. Snyder & Ting (2002) show that party dummies explain a significant part of the variation in voter placements of candidates on a left-right scale, and that these placements vary by party in the intuitive way—Democrats are placed about 2 points (out of 7) to the left of Republicans. Furthermore, conditioning on an estimate of the true ideology (the Poole-Rosenthal scores) adds little to the explanatory power of the regression. This suggests that voters are aware of the differences in candidate ideologies at the gross level of party differences, but that they have little additional information. Alvarez (1997) and Bartels (1986) show that voters are risk averse over the policy locations of candidates, so enhancing the informational content of the party labels should give an electoral boost to candidates, all else equal.

Cox & McCubbins (1993) have highlighted the role played by the legislative organization of the party in sustaining the electoral value of party labels. To protect the party label, legislators give party leaders the power to impose discipline. The leaders use this authority to induce members to vote in ways that support the desired interpretation of the party label, even when they would otherwise vote against these policies.\(^2\) Parties, on this view, are a type of institutional hand-tying.

There is direct evidence that parties secure the informativeness of their label by imposing costs on members that do not conform to the policy preferences of the party’s membership, thereby discouraging those who differ from the party’s platform from joining. For instance, retirement rates are higher for members of Congress who have preferences that lead them to vote against their party’s position (Hibbing 1982, Kiewiet & Zeng 1993, Snyder & Ting 2002), suggesting that such members derive less benefit from holding office. Snyder and Ting estimate that the probability that a member of Congress retires is 25% higher if his or her NOMINATE

\(^2\)See Aldrich (1995) and Calvert & Fox (2000) for related models
score is more than one standard deviation from the party's mean member (in either direction) than if he or she has a NOMINATE score within one standard deviation of the party mean.

Snyder & Ting (2002) formalize these ideas about informative party labels. In their model, the voters have concave utility over a one-dimensional policy space. They care directly about the ideological preferences of their representative, rather than about policy. Since voters do not observe a candidate's true ideology, they need to find informational cues that will help them vote correctly. If a political party imposes ex-post discipline on its members, then only members whose ideal policies are close to the platform will affiliate with the party. This means the voter can infer something about a candidate's ideology from the party label. The more disciplined is the party, the more homogeneous are the candidates who affiliate, and thus the more valuable is the label.³

Snyder and Ting go on to ask how platform choices are affected by these informative party labels. They find that Downsian parties will converge when information is good, while parties will diverge if information is bad enough. This is because moving away from the other party enhances the informational content of the label. These results do not actually depend on pressure being applied to party members during the policy-making process—all that is needed is that the party have an effective "testing" procedure to screen potential members. The same is true of most of our results, as long as more precise testing procedures impose more costs on members of the party than do less precise procedures.

1.3 This Paper's Contribution

We build on Snyder and Ting to provide a formal model of party discipline, candidate affiliation, and elections that addresses the whole range of empirical findings discussed in the introduction. As in Snyder and Ting, voters use party labels to learn about candidate ideology. The affiliations, and thus the informational content of the labels, depend on the strategically chosen levels of discipline. Thus, party discipline, affiliation, and ideological homogeneity are all determined endogenously within a strategic electoral-legislative setting. Since discipline is

³Levin & Tadelis (2002) study a related model of partnerships in which the partners ration access to their technology because doing so raises the market's estimate of their average quality.
endogenous, the model can explain comparative and historical variation in discipline using the causal mechanisms discussed earlier.

Our model does not address an important issue that has been the focus of most formal models of party politics: the location of party platforms in policy space. We take the location of platforms to be exogenous. This is a reasonable simplifying assumption given the phenomena we wish to study. In particular, our goal is to explain comparative and over-time variation in the level of discipline and ideological homogeneity of parties. As such, we focus on endogenizing discipline and affiliation decisions, just as other scholars have focused on the location of party platform while taking discipline to be exogenous.\footnote{Indeed, most models implicitly assume discipline is perfect by modeling the parties’ positions in the legislature as points in policy space.}

Clearly, a more complete model would treat discipline, affiliation, and platform location endogenously. However, doing so presents significant technical difficulties. If platforms and discipline are both choice variables, then the parties have two-dimensional strategy spaces. In our current model of party decision making, we would run into the problem of generic nonexistence of a Condorcet winner. Of course, this problem is not unique to our work; rather it is a general property of multi-dimensional, majority rule decision making (McKelvey 1976, Schofield 1978). As such, we abstract away from the strategic choice of platform location, in order to focus on the issues of cohesion and homogeneity.

2 Basic Model

A polity, divided into \( n \) electoral districts, must elect a legislature to set a policy in the one-dimensional policy space, \{R}. A left-wing (\( L \)) and a right-wing (\( R \)) party exist, with different, fixed platforms \( \pi_L < 0 < \pi_R \). However, the members of a party determine the extent to which legislators from that party will adhere to the party’s platform. This will, in turn, determine which politicians are willing to join each party.

There are three dates. At date 0, the two parties simultaneously hold conventions. During the conventions, party affiliations and party discipline are determined. These affiliation deci-
sions, in turn, determine the ideological composition of the parties. At date 1, each legislative
district elects a legislator. At date 2, the legislature convenes and sets policy.

A party is characterized by the set of its members, $\mathcal{P}$, and its level of discipline, $\alpha \in [0, \alpha^\prime]$. Parties are majoritarian institutions, so if a majority of $\mathcal{P}$ would prefer some other $\alpha^\prime$, they can force a change. We focus on configurations $(\mathcal{P}, \alpha)$ that are immune to such changes. Informally, a cost $\alpha_P$ and set of members $\mathcal{P}$ is stable if no majority in $\mathcal{P}$ prefers a different $\alpha$, taking into account the subsequent changes in affiliations and electoral results. A formal definition is given later.

Our model of discipline follows Snyder & Ting (2002). Discipline, $\alpha$, affects affiliation decisions because a potential legislator whose ideal policy is far from the platform particularly dislikes discipline. A legislator who has ideal point $x$ and is affiliated with party $P$ receives payoff $V(\alpha_P) = B - \alpha_P(x - \pi_P)^2$ if she wins office, while a candidate who does not win office gets 0. $B$ represents the non-policy rewards associated with holding office. A legislator’s utility is decreasing in the divergence between her policy preferences and her party’s platform, and the rate of decrease is greater for members with ideal points further from the platform. One interpretation of this functional form is that, the greater is this distance, the more often she will feel compelled to break ranks with her party. Each time she breaks party discipline she bears a cost, $\alpha$. Thus, when the members of a party choose the level of discipline, they are doing so by threatening to impose costs $\alpha$ on representatives who do not vote with the party. In section 6 we consider the conceptually more satisfying, but analytically more cumbersome, assumption that politicians have preferences over office and the final policy outcome rather than over office and the vote they cast. We show that our results hold so long as legislators are sufficiently office motivated.

There is a density of potential politicians for each party, $f^p$, and these densities are symmetric about the platforms $\pi_P$.\textsuperscript{5} We keep the model symmetric by assuming that $\pi_L = -\pi_R$ and that $f^L(\pi_L - x) = f^R(\pi_R - x)$ for all $x$. After politicians choose whether or not to affiliate with the parties during these conventions, it is common knowledge that the left party consists of

\textsuperscript{5}Ansolabehere, Snyder & Stewart (2000) show that, in Congressional elections, there are separate pools of potential candidates for the Republican and Democratic parties with essentially no overlap, just as we assume.
politicians with ideal points \( \mathcal{P}_L \subset \mathbb{R} \) and the right party consists of politicians with ideal points \( \mathcal{P}_R \subset \mathbb{R} \). Given these affiliation decisions, the voters believe that the ideology of a candidate from party \( P \) is a random variable with mean

\[
\mu_P = \int_{\mathcal{P}_P} x dF_P(x)
\]

and variance

\[
\sigma^2_P = \int_{\mathcal{P}_P} (x - \mu_P)^2 dF_P(x).
\]

The representative voter in district \( d \) has preferences represented by \(-(x^*_d - x)^2\), where \( x \) is a policy and \( x^*_d \) is the voter’s ideal point. The candidates and other voters do not know \( x^*_d \); their common belief is that \( x^*_d = \gamma_d + \epsilon_d \), where \( \epsilon_d \) is a mean zero random variable with an absolutely continuous distribution \( F \). The density, \( f \), is continuous and log-concave. This is a relatively weak restriction—it is satisfied by most of the usual distributions (e.g., normal, uniform, extreme value, etc.).\(^6\) These ideal points are mutually independent across districts.

Finally, we assume that \( \sup_x f(x)(\pi_R - \pi_L) < 2 \). This assumption requires that there is a sufficiently large amount of uncertainty regarding the preferences of voters (\( f(\cdot) \) must be sufficiently dispersed). The reason this assumption is important is the following. In the model, the direct effect of increasing a party \( P \)'s discipline is to make a risk-averse voter \( i \) more likely to vote for party \( P \) because he is more certain of \( P \)'s ideology. This is the effect in which we are interested. However, there is an indirect effect as well. When \( P \) increases discipline, this makes all voters more likely to vote for \( P \). Since voters are, on average, centrist, the fact that all other districts are now more likely to vote for party \( P \) has the indirect effect of making voter \( i \) lean back toward \( P \)'s rival to balance the expected ideology of the legislature. Assuming that the distribution of the stochastic component of the district ideal points has sufficient variance insures that the direct effect is large relative to the indirect effect. This is because, when there is sufficient uncertainty about the preferences of others, voters focus primarily on their own election.

The fact that \( \gamma_d \neq 0 \) means that we allow districts to have ideological leanings toward one or the other party. However, we assume that there is no aggregate imbalance between the

\(^6\)See Bagnoli & Bengstrom (1989).
parties—the $\gamma$ are distributed symmetrically about 0. Denote the voter’s choice of a winning candidate by $w_d \in \{L, R\}$.

After the election, the legislature sets national policy, which is a point on the ideological dimension.

2.1 Legislative Policy

The large literature on party pressure and roll-call voting has not reached a consensus on the issue of party effects on policy, conditional on the membership of the legislature. Krehbiel (1991, 1999) argues that parties do not exert a “real” effect in the legislature. In these models, policy is determined in the legislature by a majority vote of the members. Parties appear powerful when there is ideological homogeneity because the majority party votes together (due to having the same preferences) and therefore the median voter in the legislature as a whole happens to look like the party. However, on this view, legislative bargaining should be viewed as a purely majoritarian process.

On the other side of the debate are those who argue that parties play a significant role in shaping the votes of their members (Aldrich 1995, Cox & McCubbins 1993, Rohde 1991). These scholars contend that, because votes are influenced by parties, the outcome of legislative bargaining is likely to reflect the interests of the majority party, rather than the interests of the majority of the legislature as a whole.

We do not speak directly to this debate, since we are concerned here with the role of parties in elections rather than in the legislature. Fortunately, we also do not have to take a strong position because our results are robust to the specification of the legislative stage. That is, our findings are consistent with legislative policy being determined by the entire legislature or by only the majority party. In particular, we will consider two models of legislative policy making: majority party control, in which the policy is the average of the ideal points of the majority party, and floor control, in which the policy is the average of all of the ideal points. In the body of the paper we will primarily present the model of majority party control, as it is less technically cumbersome. However, the intuitions are similar for floor control and sufficient conditions for the results to hold under floor control are presented in the appendix.
3 Stable Party Systems

The main goal of this section is to characterize stable party systems. In later sections we will use this characterization to derive comparative statics results. These comparative statics will show how party strength and ideological homogeneity differ in presidential and Westminster systems, as well how changes in ideological divisions in the U.S. electorate caused over-time changes in party strength in Congress.

We solve the game in several steps. First, we derive the equilibrium affiliation decisions for arbitrary levels of party discipline, and use these decisions to derive the voters’ beliefs at the election stage. Then we find the voters’ optimal voting rule in the election stage, given these beliefs. This voting rule is used to construct each potential candidate’s preferences over discipline and affiliations. Then we formally define our stability notion, and analyze the choice of party discipline.

3.1 Affiliations and Beliefs

Consider a potential member of a party who must decide whether or not to affiliate. We focus on the decision of a potential member of party $L$—party $R$’s decision problem is symmetric. At the convention, a potential member of party $L$ with ideal point $x$ has indirect utility over the level of party discipline ($\alpha_L$) given by

$$\Pr(w = L|\alpha_L)(B - \alpha_L(x - \pi_L)^2).$$

Write $V_L(\alpha_L, x) = B - \alpha_L(x - \pi_L)^2$. The probability of election depends on $\alpha_L$ because party discipline will affect the affiliation decisions, which in turn affect the voters’ beliefs about the candidates’ ideological positions. A potential member affiliates if and only if $V_L(\alpha_L, x) \geq 0$, since the outside option has payoff 0.

Writing $f^L$ for the density of potential $L$ member ideal points, we can determine the variance of the ideological positions of party $L$’s members as a function of the level of party discipline ($\alpha_L$):

$$\sigma^2_L(\alpha_L) = \int_{V_L(\alpha_L, x) \geq 0} (x - \mu_L)^2 f^L(x) \, dx.$$
It is clear that $\sigma^2_L(\alpha_L)$ is decreasing in $\alpha_L$. The more disciplined a party is, the less uncertainty there is over the policy preferences of its membership. Because of the symmetry of the distribution of potential members around the party’s platform, the average ideology in party $L$ is equal to party $L$’s platform position, $\mu_L = \pi_L$.

An example will be instructive. Assume that the distribution of potential ideal points for a party’s candidates is uniform (this is the main case considered by Snyder and Ting). If the discipline level is $\alpha_L$, then all candidates with ideal points between $\pi_L - \sqrt{B/\alpha_L}$ and $\pi_L + \sqrt{B/\alpha_L}$ will join the party. Thus the conditional density of the ideal points is uniform on that interval, and the variance is

$$\sigma^2(\alpha_L) = \int_{\pi_L - \sqrt{B/\alpha_L}}^{\pi_L + \sqrt{B/\alpha_L}} (x - \pi_L)^2 \frac{\sqrt{\alpha_L}}{2\sqrt{B}} \, dx$$

$$= \frac{B}{3\alpha_L}.$$

### 3.2 The Election

The voter chooses which candidate to select in round 2 by comparing the expected utility of each choice. The voter in district $d$ votes for $L$ if and only if

$$-\mathbb{E}((x_{leg} - x_d^*)^2 \mid L) \geq -\mathbb{E}((x_{leg} - x_d^*)^2 \mid R),$$

where $x_{leg}$ is the policy chosen by the legislature. Taking the expectations, this becomes

$$-(\mu_{leg|L} - x_d^*)^2 - \sigma^2_{leg|L} \geq -(\mu_{leg|R} - x_d^*)^2 - \sigma^2_{leg|R},$$

where $\mu_{leg|P}$ is the expected legislative policy (the mean of the distribution) if the voter votes for party $P$ and similarly for $\sigma^2_{leg|P}$. Rearrange this to get

$$x_d^* \leq \frac{1}{2}(\mu_{leg|R} + \mu_{leg|L}) + \frac{\sigma^2_{leg|R} - \sigma^2_{leg|L}}{2(\mu_{leg|R} - \mu_{leg|L})}.$$

To get a feel for what this condition implies, consider majority party control: policy is the mean of the majority party ideal points. In this case, the voter calculates his optimal choice by conditioning on being in the pivotal district: he assumes that if he votes $L$ the national policy has mean $\pi_L$ and variance $2\sigma^2_L/(n+1)$, while if he votes $R$ the national policy has mean $\pi_R$ and
variance $2\sigma^2_R/(n + 1)$. Further, since we assume that the two party platforms are themselves symmetric about 0, we can conclude that \( \frac{1}{2}(\mu_L + \mu_R) = 0 \). Thus, the voting rule simplifies to

\[
x_d^* \leq \frac{\sigma_R^2 - \sigma_L^2}{(n + 1)(\pi_R - \pi_L)}.
\]

To simplify notation, define the cut-point

\[
c(\alpha_L, \alpha_R) = \frac{\sigma_R^2(\alpha_R) - \sigma_L^2(\alpha_L)}{(n + 1)(\pi_R - \pi_L)}.
\]

The voter prefers \( L \) if \( x_d^* < c(\alpha_L, \alpha_R) \). The cut-point, \( c(\alpha_L, \alpha_R) \), is increasing in \( \alpha_L \) and is decreasing in \( \alpha_R \). The voter is more likely to vote for \( L \) the more certain he is of the policy preferences of the left-wing candidate (low \( \sigma_L \)) or the less certain he is of the policy preferences of the right-wing candidate (high \( \sigma_R \)). This is because both of these scenarios make electing the left-wing candidate relatively less risky, which benefits the risk-averse voter.

It turns out that the conclusions of this analysis also hold in the case of floor control. In both cases, each voter uses a cut-point voting rule: vote \( L \) if and only if \( x_d^* \leq c^d(\alpha_L, \alpha_R) \). Furthermore, these cut-points have the same monotonicity properties as the special case we examined above.

**Proposition 1** Under both majority party control and floor control, every voting subgame has a unique equilibrium. The equilibria are cut-point equilibria, and the cut-points are increasing in \( \alpha_L \) and decreasing in \( \alpha_R \) for each \( d \).

Proofs of all propositions are in the appendix.

### 3.3 Party Conventions

Given the beliefs and the voting strategies, we can solve for the election probabilities as a function of the affiliation decisions. The incentive effects of party discipline are the same for both wings of a given party (that is, potential members to the right and to the left of the party platform). Because the distribution of potential members is symmetric, when a change is made in the level of discipline symmetric groups from both wings of the party either affiliate or disaffiliate. Consequently, a change in party discipline does not affect the mean ideology of the party (\( \mu_P \)), but it does affect the variance (\( \sigma^2_P \)).
A candidate from party $L$ is elected if and only if:

$$x_d^* \leq c(\alpha_L, \alpha_R).$$

We can use the definition of $x_d^*$ to rewrite this condition as $\gamma_d + \epsilon_d \leq c(\alpha_L, \alpha_R)$, which can again be rewritten $\epsilon \leq c(\alpha_L, \alpha_R) - \gamma_d$. Since $\epsilon_d \sim F$, the probability that the voter votes for the candidate from party $L$ is $F(c(\alpha_L, \alpha_R) - \gamma_d)$. Given party $R$’s level of discipline, $\alpha_R$, a politician’s indirect utility from affiliating with party $L$ can be written:

$$F(c(\alpha_L, \alpha_R) - \gamma_d)) V_L(\alpha_L, x),$$

and a politician will affiliate if and only if:

$$F(c(\alpha_L, \alpha_R) - \gamma_d) V_L(\alpha_L, x) \geq 0$$

Memberships for each party can be derived from these affiliation decisions. Let $\mathcal{P}_L$ be the set of politicians who choose to affiliate with party $L$. This set is determined by the level of party discipline. We will make use of the following definition.

**Definition 1** A pair $(\alpha_L, \mathcal{P}_L)$ is stable against $(\alpha_R, \mathcal{P}_R)$ if (1) $\mathcal{P}_L$ is exactly the set of people who want to affiliate with party $L$ given $\alpha_L$ and $(\alpha_R, \mathcal{P}_R)$ and (2) there is no $\alpha'$ such that a majority in $\mathcal{P}_L$ prefer $\alpha'$ to $\alpha_L$ given $(\alpha_R, \mathcal{P}_R)$ and the new implied affiliation decisions.

Stability represents a natural equilibrium concept for a party that allows free entry and exit and is governed by a majoritarian principle.\textsuperscript{7} If there is free exit, then equilibrium requires that no candidate be affiliated if $V(\alpha, x) < 0$. Affiliation must be better, from each individual party member’s perspective, than non-affiliation. Similarly, if there is free entry, then no perfect equilibrium can have a non-affiliated candidate with $V(\alpha, x) > 0$. No politician will turn down the opportunity to join a party if doing so will make her better off than non-affiliation. Thus, free entry and exit, in conjunction with endogenously chosen discipline, suggest stability as the equilibrium concept.

Turning from the internal stability of a single party, we can now look for an equilibrium between the parties.

\textsuperscript{7}This is similar to the equilibrium concept commonly used to study local public finance with mobility, for example Epple & Romer (1991). Our politicians are sophisticated in Epple and Romer’s terminology.
Definition 2  A 4-tuple $(\alpha_L, \mathcal{P}_L, \alpha_R, \mathcal{P}_R)$ is a stable party system if each party is stable against the other. Further, a symmetric stable party system is a stable party system in which $\alpha_L = \alpha_R$.

Proposition 2  There exists at least one stable party system (SPS) for any values of the parameters, as long as each candidate has a unique favorite level of discipline for each configuration of the opposing party. Moreover, when there are multiple SPSs, there exists a SPS with the greatest level of party discipline and a SPS with the smallest amount of party discipline.

Several of the steps of the proof (which is in the appendix) are rather involved. However, it will be useful to go over the main points here, since understanding the major components of the proof is the best way to understand how the model works.

An SPS involves a pair of discipline levels $(\alpha_L, \alpha_R)$ that reproduce themselves in the following sense: Given the set of candidates who affiliate with party $P$ given $\alpha_P$ (called $\mathcal{P}(\alpha_P)$) and party $-P$’s level of discipline $\alpha_{-P}$, the Condorcet winning discipline level for $P$ is exactly $\alpha_P$. The proof uses this “self-generating” idea to build a map whose fixed points are SPSs.

Clearly, a crucial step is showing that Condorcet winners exist. We do this by proving a version of the median voter theorem within each party which allows us to characterize the majority preference over changes in discipline in terms of the median voter’s preferences. To see the intuition for how this is possible, consider the special case where $\gamma$ is 0 for all districts, that is, there is no expected ideological heterogeneity across districts.\(^8\) Party discipline is more costly for members with ideal points far from the platform. Consequently, potential members have preferences over discipline that are ordered by the distance of their ideal points from the party platform. Politicians whose policy preferences are similar to the party platform prefer more discipline, while those whose policy preferences differ significantly from the party platform prefer less discipline. This gives the parties’ decision problems regarding discipline a natural one-dimensional structure which is needed for the median voter theorem to apply.

The next step is based on a different monotonicity property of candidate preferences. A candidate’s benefit from increasing discipline is increasing in the other party’s level of discipline. Since the median voter in the party is decisive at the convention, this means that the Condorcet

\(^8\)Variation in the $\gamma$ is the cause of most of the complications in the proof of the entire proposition.
winning level of discipline for party $P$ is increasing in $\alpha_{-P}$. This implies that the “best-response” functions are increasing, which is enough to ensure an equilibrium. In addition, this property is important for the comparative statics—any change that leads one party to increase its discipline will lead to feedback effects, increasing discipline for both parties.

To derive the comparative statics results later in the paper, we need a sharper characterization of a party median voter’s optimal discipline level. Since the median voter in each party is decisive, her first-order condition must hold with equality at any interior stable party system. This first-order condition is the key to all of our results, so we will go carefully through the intuition. Recall that the median member of party $L$ has utility given by the probability of election (given the stable party system) times the expected benefit of serving in office (given her policy preferences and the level of discipline):

$$F \left(c(\alpha_L, \alpha_R) - \gamma_{\text{d}}\right) \left(B - \alpha_L(x_{\text{med}} - \pi_L)^2\right).$$

Taking logs, we can write the party $L$ median voter’s maximization problem as:

$$\max_{\alpha_L} \left\{ \log F \left(c(\alpha_L, \alpha_R) - \gamma_{\text{d}}\right) + \log \left(B - \alpha_L(x_{\text{med}} - \pi_L)^2\right) \right\}.$$ 

The first-order condition for this maximization problem is:

$$c_L \frac{f(c - \gamma_{\text{d}})}{F(c - \gamma_{\text{d}})} = \frac{(x_{\text{med}} - \pi_L)^2}{B - \alpha_L(x_{\text{med}} - \pi_L)^2}.$$

Similarly, the first-order condition for the party $R$ median voter is:

$$-c_R \frac{f(c - \gamma_{\text{d}})}{1 - F(c - \gamma_{\text{d}})} = \frac{(x_{\text{med}} - \pi_R)^2}{B - \alpha_R(x_{\text{med}} - \pi_R)^2},$$

where $c_L$ and $c_R$ are the partial derivatives of $c$ with respect to $\alpha_L$ and $\alpha_R$, respectively.

Like all first-order conditions, this says that if $\alpha$ is optimal, then the marginal benefit from a small increase in the amount of party discipline must be exactly balanced by the marginal cost of that change. What are the marginal benefits and costs of discipline for the median member of a party? Recall that $c$ is a cut-point ($c(\alpha_L, \alpha_R) = (\sigma_R^2(\alpha_R) - \sigma_L^2(\alpha_L))/(2(\mu_R - \mu_L))$ defining when the pivotal voter in a district will choose the candidate from party $L$. A small increase from $\alpha_L$ to $\alpha_L + d\alpha_L$ moves the cut-point to the right by approximately $d\alpha_Le_L$, and
this increases the probability of election by approximately \( d\alpha_L c_L f(c - \gamma_d) \). Winning gives the candidate a payoff of approximately \( B - \alpha_L (x_{med} - \pi_L)^2 \), so the marginal benefit of an increase in discipline is:

\[
d\alpha_L c_L f(c - \gamma_d) (B - \alpha_L (x_{med} - \pi_L)^2).
\]

That is, the marginal increase in probability of election multiplied by the benefits of holding office.

The cost of an increase in discipline arises from the fact that, once in office, candidates are more constrained in the policies they can pursue. The extra costs of party discipline associated with a small increase in \( \alpha_L \) are approximately \( d\alpha_L (x_{med} - \pi_L)^2 \). The candidate, of course, only bears these costs if elected, which occurs approximately with probability \( F(c - \gamma_d) \). Thus the marginal cost of an increase in discipline is

\[
d\alpha (x_{med} - \pi_L)^2 F(c - \gamma_d).
\]

The first order condition equates these two expressions.

The marginal benefit of an increase in discipline is due to the possibility that a small increase in discipline will swing the election results. Said differently, the benefit of a small increase in discipline comes entirely from its effect on voters who are close to indifference between the parties. Consequently, over-time or cross-institutional variations in the level of discipline must be driven by one of two factors: (1) changes in the responsiveness to discipline of nearly indifferent voters or (2) changes in the number (measure) of such voters. We will show how these two factors can explain variance both between presidential and parliamentary systems and over-time within the U.S. Congress.

Before turning to the comparative statics, we need one more piece of technical apparatus. Note that proposition 2 demonstrates the existence of a stable party system but does not guarantee uniqueness. Indeed, in general there will not be a unique equilibrium. Thus, in order to compare the level of discipline that emerges under various institutional settings we need a way to compare sets of equilibria. We will say that one set of equilibria is greater than a second set if the greatest and least equilibria in the first set are greater than the greatest and least equilibria, respectively, of the second set. Echenique (2002) shows that if equilibrium sets are
ordered this way, then a broad class of adaptive adjustment processes will converge to greater equilibria whenever a shock increases the equilibrium set.

4 Comparative Statics

4.1 Presidential vs. Westminster Systems

We can now compare party discipline in presidential and Westminster systems. To focus on the role of the executive, we will consider two systems that are identical except for selection of the executive. There are a number of single-member legislative districts, each of which is contested by the same two parties. In the Westminster system, the winners of these elections make up the parliament and determine policy. In the presidential system, the majority party in the legislature must bargain with the president to set policy. For simplicity, we abstract from heterogeneity among districts for this section, so all of the $\gamma$ are 0. Since this makes everything symmetric, it is natural to focus on equilibria that are symmetric in the sense that (i) all of the voters use the same cut-point in the elections and (ii) the parties choose the same levels of discipline.\footnote{Standard arguments can be used to modify propositions 1 and 2 to show that a symmetric equilibrium exists.}

For the most part, we will work with a reduced form description of bargaining between the legislature and the president, although we demonstrate that the results hold under veto bargaining in section 6.2. Following Alesina & Rosenthal (1995), we assume that policy in a presidential system is a weighted average of the legislature’s proposal and the president’s ideal point, with weight $\beta$ on the legislative proposal: $x = \beta x_{\text{leg}} + (1 - \beta)x_{\text{pres}}$.

Since our focus is on the legislative election, we make the following simplifying assumptions. First, when casting his legislative vote, a voter assumes that he is not pivotal in the presidential election. Second, we focus on equilibria that treat the parties symmetrically. Jointly, these imply that the lottery over presidential ideology has mean 0.

Again, we can build some intuition by first considering the case of majority control of the legislature. The Westminster system corresponds exactly to the system we studied in the
previous section. Thus, the voter’s cut-point is:
\[ c^{wst}(\alpha_L, \alpha_R) = \frac{\sigma^2_R(\alpha_R) - \sigma^2_L(\alpha_L)}{(n+1)(\mu_R - \mu_L)}. \]  
(2)

The results of this section will be derived by comparing this cut-point to the cut-point in the presidential system.

In order to determine the optimal voting rule in a presidential system, we mimic the previous analysis, taking into account bargaining between the legislature and the president. Without repeating the algebra, it is clear that the voter in a presidential system will vote for legislative candidate \( L \) if and only if:
\[ x^*_d \leq \frac{1}{2} \beta(\mu_R + \mu_L) + (1 - \beta)\mathbb{E}\text{Pres} + \frac{\beta(\sigma^2_R(\alpha_R) - \sigma^2_L(\alpha_L))}{(n+1)(\mu_R - \mu_L)}. \]

Since \( \mu_L = -\mu_R \) and \( \mathbb{E}\text{Pres} = 0 \) in equilibrium, this simplifies to:
\[ x^*_d \leq \frac{\beta(\sigma^2_R(\alpha_R) - \sigma^2_L(\alpha_L))}{(n+1)(\mu_R - \mu_L)}, \]
so the cut-point is
\[ c^{\text{pres}}(\alpha_L, \alpha_R) = \frac{\beta(\sigma^2_R(\alpha_R) - \sigma^2_L(\alpha_L))}{(n+1)(\mu_R - \mu_L)}. \]  
(3)

Comparing equations (2) and (3) shows that voters have different incentives under each system. In particular, voters put greater weight on their uncertainty over candidate ideology in the Westminster system. This is because the legislature has less impact on national policy in a presidential system, since the legislature’s proposal does not determine national policy alone, but rather is averaged with presidential preferences. The president acts as a hedge against legislative extremism. Voters are more concerned about the possibility of an extreme legislature in a Westminster system because the legislature is unconstrained by an independent executive branch. Formally, this means that the marginal benefit of party discipline is attenuated in presidential systems relative to Westminster systems. To see this, recall that the marginal benefit of discipline for party \( P \) is proportional to \( c \). Direct calculation shows that \( c^{\text{pres}}_L = \beta c^{wst}_L < c^{wst}_L \) and \( -c^{\text{pres}}_R = -\beta c^{wst}_R < -c^{wst}_R \). This intuition suggests the following result.

**Proposition 3** The level of discipline is greater in Westminster systems than in presidential systems.
Party discipline is weaker in the presidential system because the pivotal voter is less responsive to changes in the variance of ideal points within a party. This means that an increase in discipline has a relatively small marginal impact on the probability of winning in a presidential system, since the voters do not care as much about the national policy preferences of their representatives as do the voters in a Westminster system.

4.2 More Competitive Districts

Institutional changes are not the only ones that lead to more disciplined parties—changes in the distribution of voter preferences can do so as well. In the next section, we will see that such changes can account for historical changes in party cohesion in the U.S.\(^\text{10}\) To capture increased electoral competitiveness, we consider changes in the distribution of the partisan leanings of the districts. Specifically, we consider a change in which the expected district median voter ideal points \((\gamma)\) become closer to 0, and we ask what happens to the equilibrium levels of party discipline. In right-leaning districts the median voter became more willing to consider a candidate from party \(L\), and in left-leaning districts the median voter became more willing to consider a candidate from party \(R\). We will assume that the median voter in each party is from a district controlled by that party.

What is the effect of a decrease in dispersion of district median voter ideal points on our model of party organization? The intuition in this case is different than in the comparative case where the variation was institutional. Recall that the benefit of party discipline is to improve voter confidence in the policy agenda that a candidate will pursue once in power. Unlike the move from presidential to parliamentary systems, an increase in competitiveness does not change the weight the marginal voter (that is, a voter that is exactly indifferent between the candidates) places on uncertainty. Instead, what changes is the probability that the indifferent voter actually swings the election outcome by changing his vote—this probability is greater in closer elections. Since it is precisely the indifferent voter who is affected by a small increase in party discipline, the increased probability that this voter’s vote matters increases the expected

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\(^{10}\)In thinking about the idea that congressional elections have become more competitive, recall our discussion of the meaning of competitiveness in the introduction. We revisit this issue in considerable detail in section 5.0.1.
marginal benefit of imposing discipline. That is, increasing discipline increases the probability of capturing the vote of an indifferent voter and an increase in competitiveness increases the value of capturing this vote by increasing the probability that this vote swings the election. At the same time, the expected cost of discipline decreases with an increase in competitiveness because, in a competitive district, any given candidate wins with lower probability. Thus, the marginal benefit of discipline increases and the marginal benefit falls, leading to more discipline in equilibrium. This intuition gives rise to the following result.

**Proposition 4** Consider the case of majority party control of the legislature. If the competitiveness increases between the parties, in the sense that each district’s expected median moves closer to the midpoint between the parties, then the level of party discipline increases.

Does this result extend to the case of floor control? The complication is that an increase in discipline has both the direct effect discussed above and the same counter-acting indirect effect discussed earlier: as discipline increases, the probability of the more disciplined party winning increases in every district, and because voters want to moderate policy they have a counter-acting incentive to lean toward the other party. As before, the comparative static result will hold as long as the indirect effects are not large relative to the direct effects. Earlier we showed that this was true as long as the uncertainty over voter preferences was sufficiently large. Here, demonstrating this exact result introduces significant technical difficulties. The complication arises because the cut-points in the election stage will generally depend on the entire distribution of the $\gamma$. However, the intuition remains the same and can be seen rather easily by considering the special case where the stochastic element ($\epsilon$) of each district’s median voter’s ideal point is distributed uniformly. (Note that a highly dispersed normal distribution is close to uniform over much of its domain.) Because uniformity adds significantly to tractability, we only demonstrate that the results in proposition (4) hold under floor control with the added assumption of uniformity.

**Proposition 5** Consider the case of floor control assuming a uniform distribution of the stochastic component of each district’s median voter’s ideal point. If the competitiveness increases be-
between the parties, in the sense that each district’s expected median moves closer to the midpoint between the parties, then the level of party discipline increases.

5 The Return of the Congressional Party

The middle of the 20th century was the height of the “textbook Congress”—a term sometimes used to describe the Congressional institutions familiar from works such as Mayhew’s Congress: The Electoral Connection (1974). The textbook Congress was characterized by the unwillingness of its members to let their parties exert much discipline in voting. Committees, and their chairs, were the true nexus of power (Shepsle 1989).

In retrospect, Mayhew’s classic description came at the end of the institutional equilibrium he was describing. The aftermath of Watergate saw a variety of reforms—such as making committee chairs accountable to parties—that strengthened parties in Congress. These reforms have been followed by increases in the cohesiveness of parties in legislative votes.

There is a substantial body of empirical work demonstrating these trends. To take a recent example, Aldrich, Berger & Rohde (2002) examine both houses of Congress between 1877 and 1994. They show that there is a strong correlation between measures of voting homogeneity, such as (minus) the standard deviation of Poole-Rosenthal ideal point estimates or the $R^2$ of a regression of those estimates on party dummies, and qualitative measures of the power of party leaders. In particular, they show that both quantitative measures of voting homogeneity and qualitative measures of party strength have increased since the mid-1960s.

We argue that the model of increased competitiveness from the last section can account for these over-time changes in the Congress. For much of the 20th century, many Congressional districts were relatively uncompetitive. The south was solidly Democratic, while the non-urban parts of the north and west were solidly Republican. This trend reversed in the 1960s, due in part to the debate over civil rights. In addition, Cox & Katz (2002) show that redistricting in the wake of Baker v. Carr (decided in 1962) led to the end of many safe Republican districts in the north. As a result, party competitiveness increased: southern voters became more willing to consider electing a Republican while non-urban western and northern voters became more
willing to consider Democratic candidates.

Today the parties are more competitive in most states. Erikson, Wright & McIver (1993) report that public opinion is roughly balanced between the two parties in most states, and Ansolabehere & Snyder (2002) find that the portion of vote shares explained by the state’s partisan leanings (“the normal vote”) has declined dramatically since mid-century. They write:

The normal vote accounts for 53 percent of the variation in the vote in the 1940s. It’s importance drops substantially in the 1950s, to 40 percent of total variance in the vote. And it collapses in the 1960s, explaining only 20 percent of the variance in the vote in the 1960s and 1970s. The decline of the normal vote as an explanatory factor continues in the 1980s, falling to 10 percent in the 1980s and 1990s.

Our model predicts that this change in competitiveness will lead to stronger parties, as observed in the historical record. Consequently, the model is consistent with the empirical finding that U.S. Congressional parties became stronger beginning in the late 1960s. According to the model, this institutional change was a rational response to shifting electoral conditions brought about by the civil rights movement and redistricting.

An important subtlety is that the district-level heterogeneity that we model ($\gamma$) need not necessarily reflect ideology. It can simply be interpreted as the level of commitment to the party due to some valence term. Thus, when we describe the solid south as having $\gamma_s$ to the left of zero, this need not imply that the south was “liberal” relative to other parts of the country. Rather, it implies that the south was more reliably committed to the Democratic party. All of our results are consistent with this more nuanced interpretation.

5.0.1 Competitiveness and Redistricting

Careful observers of American politics might object to our claim that competitiveness has increased since the 1960s. In particular, one might argue that while southern states have become less solidly Democratic and the northern and western states less solidly Republican, individual districts have become less competitive because gerrymandering has created an abundance of “safe” seats.
Importantly, our results do not depend on the claim that all seats have become more competitive. Rather, what is important is that the median member of each party is now running for a more competitive seat. As is clear from the proof of Proposition (4), our results hold if there is an increase in safe seats as long as the decisive voter in each party is from a district that feels the effect of an increase in competitiveness.

Further, a slight extension of the model provides reasons to think that this condition will hold. Assume that a candidate must bear some cost $\delta > 0$ if she affiliates with a party. In this case, she will affiliate with party $L$ if and only if

$$\Pr(w = L|\alpha)(B - \alpha(x - \pi_L)^2) \geq \delta.$$  

The left-hand side of this inequality is decreasing in $\gamma_d$ and in $(x - \pi_L)^2$. This means that in more left-leaning districts, candidates who have ideal points far from the platform are more willing to affiliate, which implies that the party label has more informational content in districts that lean toward the other party. Consequently, an absolute majority of the members of a party will be from districts that lean toward that party, even though the potential members are spread uniformly across districts. Thus, the median voter of a party is expected to be from a district that leans slightly toward that party. That is, the median member will hold a seat in a district that, while somewhat partisan, is not likely to be one of the party’s “safe” seats. Consequently, if an increase in state-level competitiveness coincides with an increase in safe seats our results will still hold because the median member of the party will be from one of the relatively centrist seats that feels the effect of the increase in competitiveness.

5.1 Conditional Party Government and its Critics

In the lively debate over party effects in Congress, changes in the ideological heterogeneity within parties plays a key theoretical role. The leading explanation for the changes in party strength in the U.S. Congress is Aldrich’s (1995) and Rohde’s (1991) idea of conditional party government (CPG). They argue that parties delegate authority to leaders only when there is sufficient ideological homogeneity among the party members and polarization between the parties. Members of homogeneous parties have no reason to fear that powerful leaders will
force them to support legislation they oppose, since their interests and the party’s interests are aligned. Thus, Aldrich and Rohde argue that the observed increase in party cohesiveness in roll call votes in the U.S. was caused by an increase in the ideological homogeneity in Congressional parties. Furthermore, they posit the following causal chain: increased ideological homogeneity led party members to grant the party leadership more disciplinary power which, in turn, led to greater cohesiveness in roll call votes.

Cox & McCubbins (1993) link the CPG theory with informative party labels, suggesting that parties and their leaders serve as “cartels” that prevent free-rider problems that could degrade the party label. Building on Aldrich’s (1995) and Rohde’s (1991) argument, Cox and McCubbins contend that ideological homogeneity causes a convergence of interests that make cartel-like parties particularly attractive. Hence, consistent with the earlier CPG theorists, Cox and McCubbins predict that parties will be strong when they are ideologically homogeneous.

Although the empirical work documenting the relationship between ideological heterogeneity and party voting is impressive, the CPG interpretation has inspired several theoretical critiques. Krehbiel (1993, 1999) and Calvert & Dietz (1996) argue that the empirical evidence is consistent with the possibility that parties are not actually fulfilling any important role in the legislature. In particular, the CPG model predicts that parties will become powerful when there is ideological homogeneity. But strong parties are not needed to control legislation or to protect the party label if the membership is ideologically homogeneous. The members will not be tempted to deviate from a party platform with which they agree. Thus, it is precisely at those times when they are least needed that CPG predicts that parties will be strongest.

Furthermore, both Krehbiel and Calvert and Dietz make the point that one cannot conclude that ideological homogeneity causes strong parties simply by observing that legislative votes are more cohesive in parties that are ideologically homogeneous. When a party is ideologically homogeneous the membership agree with one another on which policies are desirable. Consequently, cohesive votes should be expected among ideologically homogeneous legislative delegations with or without strong parties.

Our model makes three contributions to the debate over party discipline and ideological heterogeneity. First, membership is endogenous, so the model itself explains why parties become
more or less ideologically homogeneous. In particular, we establish the role of electoral concerns in the party formation process. Both Aldrich-Rohde and Krehbiel ultimately trace changes in legislative cohesion back to changes in the composition of parties, in spite of their differences over the precise channel of this effect. Second, there are clear comparative statics results—more legislative control of policy and more competitive elections lead to both more homogeneous parties and more discipline imposed by leaders. These features allow for a better understanding of the joint determination of the variables studied in the empirical debate over party strength in Congress. Finally, our model shows that it is possible to understand both historical variance in the power of U.S. parties and comparative variance in the power of parties across different institutional structures within a unified theoretical framework.

Importantly, by construction our model does not address the simultaneous determination of homogeneity and polarization, thus missing an important part of the CPG thesis. As discussed earlier, we do not endogenize party platform locations because to do so would cause intractable technical difficulties while distracting focus from the issues of party strength and homogeneity on which we wish to focus.

6 Extensions

6.1 Candidates with Preferences over Policy Outcomes

A potential weakness of the model we have considered so far is that legislators were assumed to have preferences over holding office and how they personally voted on policy issues. A more satisfying assumption would be that politicians care about holding office and final policy outcomes. In this section, we demonstrate that our results are robust to the assumption that politicians are concerned with policy outcomes, as long as they also put sufficient weight on the benefits of holding office.

For this section, we assume that each legislator $j$ in party $i$ takes position $\alpha\pi_i + (1 - \alpha)x_j$ and the policy is the average of these positions. Thus discipline actually compels members to support the platform $\alpha$ percent of the time.

Consider a politician who is choosing whether to affiliate with party $L$. Her payoffs condi-
tional on her decision and the electoral outcome in her district are described by the following
table:

<table>
<thead>
<tr>
<th>affiliate</th>
<th>$L$ wins</th>
<th>$R$ wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>not</td>
<td>$u(L)$</td>
<td>$u(R)$</td>
</tr>
</tbody>
</table>

where $u(S)$ is her expected utility for the policy lottery if she runs and wins, $u(L)$ is her expected
utility over the policy lottery if another member of her party wins the seat in her district, and
$u(R)$ is her expected utility if party $R$ wins in her district. Once she is affiliated, only the top
row is relevant for her preferences over $\alpha$. When choosing an affiliation, only the first column
is relevant.

She will affiliate if and only if $U = B + u(S) - u(L) \geq 0$. $u(S)$ is decreasing in $\alpha$, since she will be able to give her favorite policy less support as party discipline increases. Further,$u(L)$ is increasing in $\alpha$, since increased discipline decreases the risk associated with allowing
another politician (with a different ideal point) to serve as the district’s representative. Thus
$U$ is decreasing in $\alpha$, as before.

Now consider her preferences over $\alpha_L$ if she affiliates. The payoff in the top left cell is

$$
\mathbb{E} \left( B - (p - x)^2 \right) = B - (\mathbb{E}p - x_j)^2 - \sigma^2_P
$$

$$
= B - \left( \frac{n - 1 + \alpha}{n} \pi_i + \frac{1 - \alpha}{n} x_j - x_j \right)^2 - \sigma^2_P
$$

$$
= B - \left( \frac{n - 1 + \alpha}{n} \right)^2 (\pi_i - x_j)^2 - \sigma^2_P.
$$

This is almost the same as before. The only difference is the term $\sigma^2_P$. It turns out that, because
of this similarity, as long as politicians care enough about holding office ($B$ sufficiently large),
or our results are robust to allowing legislators to care about final policy outcomes, rather than
the policy for which they individually vote.\footnote{Technically, our earlier results depended on the logsupermodularity of the objective function. As long as the addition of the term $\sigma^2_P$ does not upset this logsupermodularity the results go through. The numerator of the cross-partial of the log of the payoff is $-B + \sigma^2 - \alpha \sigma^2$. For the previous results, this must be negative. This will be true so long as $B$ is sufficiently large. Thus, as long as politicians care enough about holding office, our}
6.2 Veto Bargaining

Much of the literature on congressional-presidential relations considers a bargaining protocol that more closely conforms to the actual strategic relationship between the president and Congress: veto bargaining. Under veto bargaining, there is a status quo policy \( (x_S) \) and the Congress proposes an alternative \( (x_{\text{leg}}) \). The president then chooses her favorite alternative from the set \( \{x_S, x_{\text{leg}}\} \). The equilibrium policy is

\[
x = \arg\max_{\hat{x} \in \{x_S, x_{\text{leg}}\}} |\hat{x} - x_{\text{pres}}|
\]

Under veto bargaining, variation in \( x_{\text{leg}} \) leads to variation in the equilibrium policy only when the status quo falls in the range where the legislature’s policy proposal would be chosen by the president. This fact can be used to demonstrate that our qualitative results comparing presidential and parliamentary systems still hold.

Imagine that there is pre-electoral uncertainty over the status quo, perhaps because of uncertainty over which issues will be important in a given legislative session. When the status quo is realized, after the election, changes in the variability of legislature policy proposals will have a smaller impact on the variability of policy under veto bargaining than under a system where policy is fully determined by the legislature (as in a parliamentary system). This is because, under veto bargaining, there is only variance in policy when the status quo is such that the legislature’s proposal will not be vetoed. That is, veto bargaining (like our earlier model of presidential systems) gives the legislature less control over policy. Consequently, voters are less concerned about uncertainty over their legislators’ policy preferences under veto bargaining than in a parliamentary system. As a result, increases in discipline have less marginal impact on the probability of election under veto bargaining than in parliamentary systems and so less discipline is imposed. Thus, our qualitative results are insensitive to the exact nature of the bargaining between the legislature and the executive and, in particular, are consistent with veto bargaining.

results are robust to allowing legislators care about final policy outcomes, rather than the policy for which they individually vote.
7 Conclusions

We presented a model in which party memberships and party discipline are jointly determined in equilibrium. The model is able to explain differences in party strength, both across different political systems and over time in the United States.

While we believe that the results we have presented regarding party strength and ideological homogeneity are interesting in their own right, this paper is part of a larger project that attempts to explain the relationship between a variety of legislative and electoral outcomes. Although the project is too large to be presented in a single paper, it is useful to see how the causal logic developed above can be applied to other empirical findings in the study of comparative legislatures.

Among the most interesting empirical findings in comparative legislative research are those having to do with the relationship between party strength, the level of constituency service in which legislators engage, and the size of the personal incumbency advantage. Comparisons between presidential systems like the U.S.’s and parliamentary systems like the U.K.’s have revealed that presidential systems have less cohesive parties (Cain et al. 1987), more constituency service (Cain et al. 1987), and a stronger incumbency advantage (Katz & King 1999, Gelman & King 1990)). Indeed, as a result of such comparisons, institutional theorists have argued that cohesive parties preclude high levels of constituency service and strong personal incumbency advantages (Cain et al. 1987, Cox & McCubbins 1993). However, compared to 1950, today’s U.S. Congress has more cohesive parties (as discussed in this paper) and it has more constituency service (Fiorina 1977, Fenno 1978) and stronger incumbency advantages (Gelman & King 1990). This presents a puzzle for traditional thinking since the relationship between party discipline, on the one hand, and constituency service and the incumbency advantage, on the other hand, is exactly the opposite in the U.S. time-series as the relationship in the comparative cross-section that gave rise to the theory that strong parties diminish constituency service and the incumbency advantage.

In order to address these empirical regularities, in other work we extend our model to include the provision of constituency service (Ashworth 2001, Ashworth & Bueno de Mesquita
Politicians are endowed with different levels of skill in providing constituency service and part of a voters’ decision calculus depends on his assessment of his incumbent politician’s ability relative to a challenger. Legislative outcomes provide voters with information about an incumbent’s ability, and they only reelect incumbents whom they believe to be high ability. This has two effects. First it gives legislators an incentive to provide constituency service. Second, over time voters become more confident in their incumbents, giving rise to the incumbency advantage.

The causal mechanisms underlying our account of differences in party strength also explain patterns of constituency service and the incumbency advantage. First, consider the comparison between presidential and parliamentary systems where the key causal mechanism in this paper is how much voters are concerned with uncertainty over candidate ideologies. In a model with constituency service, the voter has to consider two factors when deciding whether to reelect an incumbent: the incumbent’s policy preferences (as in the current model) and the incumbent’s ability to provide constituency service. As in the model in the current paper, voters place relatively more weight on uncertainty over candidate policy preferences than candidate ability to provide constituency service in a parliamentary system because legislators have more control over policy in a parliamentary system. Consequently, the payoffs to constituency service are greater in presidential systems. Thus, consistent with empirical findings, this argument predicts that there is more constituency service in presidential systems. Similarly, the incumbency advantage exists in this model because of voter concern over ability to provide constituency service. As such, again consistent with empirical findings, the incumbency advantage is expected to be larger in presidential systems than parliamentary systems.

We argue that the same model that accounts for the comparison between presidential and parliamentary systems can also account for the seemingly contradictory over-time empirical trend in the U.S. Congress where greater party strength coincided with increases in constituency service and the incumbency advantage. The analysis in this paper showed that increases in competitiveness of district races can explain heightened party strength. As competitiveness increases, so does the probability that greater discipline will swing an election. Similarly, when elections become more competitive a marginal increase in constituency service is more likely
to swing an election because the decisive voter is more likely to be swayed by an increase in
his assessment of the incumbent’s ability. Hence, competitive elections imply an increases in
constituency service and, for similar reasons, the incumbency advantage.

A Proof of Proposition 1

We have already show this for majority control; now we establish the result for floor control.
All best responses are cut-points since the voters’ payoffs are supermodular. As before, the
voter in district $d$ uses the cut-point

$$c^d = \frac{1}{2}(\mu_{\text{leg}|R} + \mu_{\text{leg}|L}) + \frac{\sigma^2_{\text{leg}|R} - \sigma^2_{\text{leg}|L}}{2(\mu_{\text{leg}|R} - \mu_{\text{leg}|L})},$$

Since the district-level elections are independent, we have

$$\sigma^2_{\text{leg}|R} - \sigma^2_{\text{leg}|L} = \frac{1}{n^2}(\sigma^2_R - \sigma^2_L)$$

and

$$\mu_{\text{leg}|R} - \mu_{\text{leg}|L} = \frac{1}{n}(\mu_R - \mu_L).$$

These quantities do not depend on the cut-points used in other districts. These cut-points do
matter for the term $(1/2)(\mu_{\text{leg}|R} + \mu_{\text{leg}|L})$. This can be written as

$$(1/2)(\mu_{\text{leg}|R} + \mu_{\text{leg}|L}) = \sum_{k \neq d} F(c_k - \gamma_k)\pi_L + (1 - F(c_k - \gamma_k))\pi_R.$$

Let

$$C = \left[\frac{n - 1}{2n} \pi_L + \frac{-\sigma^2_R(\pi)}{2n(\pi_R - \pi_L)}, \frac{n - 1}{2n} \pi_R + \frac{\sigma^2_L(\pi)}{2n(\pi_R - \pi_L)}\right].$$

This set is convex and compact, and it contains every cut-point that a rational voter might use.
Let $\phi : C^n \to C^n$ be the map given by

$$\phi_d(c, \alpha_L, \alpha_R) = \sum_{k \neq d} [F(c_k - \gamma_k)\pi_L + (1 - F(c_k - \gamma_k))\pi_R] + \frac{\sigma^2_R(\alpha_R) - \sigma^2_L(\alpha_L)}{2n(\pi_R - \pi_L)}.$$

This defines a map $c \mapsto \phi(c)$ whose fixed points are equilibria of the voting stage under floor
control. $\phi$ is continuous, so a fixed point exists by Brouwer’s theorem.
Let $\alpha$ be the vector of party disciplines $(\alpha_L, \alpha_R)$. The equilibrium cut-points, $c^*(\alpha)$ satisfy the equation $\phi(c^*(\alpha), \alpha) = c^*(\alpha)$. By the implicit function theorem, we have

$$
(I - D_c\phi)D_\alpha c^* = D_\alpha \phi.
$$

(4)

The Jacobian $D_c\phi$ has a zero diagonal and negative off diagonal terms, and the matrix $I - D_c\phi$ has a positive dominant diagonal, since our assumptions imply that

$$
\frac{\partial \phi_d}{\partial \epsilon_k} < \frac{1}{n}
$$

for $d \neq k$. This has two implications. First, equation 4 has a unique solution given by

$$
D_\alpha c^* = (I - D_c\phi)^{-1} D_\alpha \phi.
$$

Second, $(I - D_c\phi)$ positive dominant diagonal implies that $(I - D_c\phi)e \gg 0$, where $e$ is the vector of all 1s. Thus $D_c\phi$ is a productive Leontief matrix, and the proof of Proposition 5.AA.1 of Mas-Colell, Whiston & Green (1995) implies that $(I - D_c\phi)^{-1}$ has all nonnegative elements. Since each $\phi_d$ is increasing in $\alpha_L$ and decreasing in $\alpha_R$, the proof is complete.

### B Proof of Proposition 2

We will use the following result from Milgrom & Roberts (1994).

**Theorem** Let each $\phi_i(x_i, x_{-i}, t) : [0, 1]^N \times T \to [0, 1]$ be continuous but for upward jumps in $x_i$ and nondecreasing in $x_{-i}$ and $t$, where $T$ is any partially ordered set. Then there exist greatest and least fixed points of $\phi$ for each $t \in T$, and these fixed points are nondecreasing in $t$.

The strategy of the proof is to construct a function whose fixed points are stable party systems, and then show that this function satisfies the conditions of the theorem. This will imply the result.

This function will be based on the optimal levels of discipline of the members, so we record some facts about those first. Fix an arbitrary $\alpha_R$. Let $\alpha^*(\Delta, \gamma)$ be the level of discipline most preferred by a candidate whose ideal policy’s squared distance from the party platform is $\Delta$ and who is running in a district where the median voter’s expected ideal point is $\gamma$. That is

$$
\alpha^*(\Delta, \gamma) = \arg \max_{\alpha_L} \{ F(c(\alpha_L, \alpha_R) - \gamma) (B - \alpha_L \Delta) \},
$$

where $F$ is the voter’s expected ideal point in the district.
where \( \Delta = (x-x_L)^2 \). Berge’s theorem of the maximum implies that \( \alpha^* \) is an upper-hemicontinuous correspondence. The objective function of the candidate is differentiable and strictly logsupermodular in \((\alpha, -\Delta, \gamma)\). (To see this, just compute the cross-partial derivatives.) Thus the strict monotonicity theorem of Edlin & Shannon (1998) implies that if \((\Delta', \gamma') > (\Delta, \gamma) \) then \( \alpha^*(\Delta', \gamma') > \alpha^*(\Delta, \gamma) \).

The next step is to show that, for any discipline level \( \alpha'_{-P} \) and any potential coalition \( \mathcal{P}' \), there is a Condorcet winning level of discipline for party \( P \). Notice that this is the only step of the proof for which singleton-valuedness of \( \alpha^* \) is needed.

Let \( \alpha_{\text{med}} \) be the median of the \( \{ \alpha^*(\Delta, \gamma) \} \). Now consider some \( \alpha < \alpha_{\text{med}} \). Let \( h = \{ (\Delta, \gamma) \mid \alpha^*(\Delta, \gamma) = \alpha_{\text{med}} \} \). Since \( \alpha^* \) is singleton-valued and upper hemicontinuous, it is continuous. This implies that every \((\Delta', \gamma')\) is ordered (in the usual vector order) with respect to some \((\Delta, \gamma) \in h \). It is greater if and only if \( \alpha^*(\Delta', \gamma') < \alpha_{\text{med}} \) and is less if and only if \( \alpha^*(\Delta', \gamma') > \alpha_{\text{med}} \). (This is logsupermodularity.) If it is less, then we have

\[
u(\alpha_{\text{med}}, \Delta', \gamma') - \nu(\alpha, \Delta', \gamma') \geq \nu(\alpha_{\text{med}}, \Delta, \gamma) - \nu(\alpha, \Delta, \gamma)
\]

so all candidates with \( \alpha^* > \alpha_{\text{med}} \) prefer \( \alpha_{\text{med}} \) to \( \alpha \), and \( \alpha_{\text{med}} \) is a Condorcet winner.

Now we construct our function. Define \( \mathcal{BR} : [0, \bar{\alpha}]^2 \rightarrow [0, \bar{\alpha}]^2 \) by

\[
\mathcal{BR}(\alpha'_L, \alpha'_R) = (\mathcal{BR}_L(\alpha'_L, \alpha'_R), \mathcal{BR}_R(\alpha'_L, \alpha'_R)),
\]

where

\[
\mathcal{BR}_P(\alpha'_L, \alpha'_R) = \text{med}\{ \alpha^*(\Delta, \gamma) \mid (\Delta, \gamma) \in \mathcal{P}(\alpha'_L, \alpha'_R) \}.
\]

By the previous result, this correspondence picks the Condorcet winning levels of \( \alpha \) for each party, given the affiliations implied by the \( \alpha' \). At a fixed point of this correspondence, the optimal choices recreate the status quo, which is stability. All that’s left is to argue that the correspondence has a fixed point.

Now we show that \( \mathcal{BR} \) is continuous but for upward jumps in \( \alpha_P \). It’s clear from Topkis’s theorem that \( \mathcal{BR}_P \) is increasing in \( \alpha'_{-P} \), so we can restrict attention to sublattices with fixed \( \alpha'_{-P} \). To derive a contradiction, assume that there is an interval \( A = (\alpha, \hat{\alpha}) \) such that
inf \mathcal{BR}_P(A) > \mathcal{BR}_P(\hat{\alpha}). This means there is a sequence \{\alpha^n\} such that \alpha^n \to \hat{\alpha} and \alpha^n \in A for all n and an \epsilon > 0 such that \mathcal{BR}_P(\alpha^n) - \mathcal{BR}_P(\hat{\alpha}) > \epsilon for all n. From the definition of \mathcal{BR}, this means that \alpha^n_{med} - \alpha_{med}(\hat{\alpha}) > \epsilon for all n. Since the ideal \alpha lie in the compact set [0, \overline{\alpha}], we can choose the sequence to be convergent. Finally, observe that \mathcal{P}(\hat{\alpha}) \subset \mathcal{P}(\alpha^n) for all n.

Define \mu to be the restriction of the measure induced by \mu^p to \mathcal{P}(\hat{\alpha}), and let \mu^n be the restriction of this measure to \mathcal{P}(\alpha^n) \setminus \mathcal{P}(\hat{\alpha}). Let \phi = \mu \circ (\alpha^n)^{-1} be the push-forward measure on ideal levels of discipline, and define \phi^n similarly. The median ideal discipline level at \alpha^n satisfies

\phi(\alpha^n_{med}) + \phi^n(\alpha_{med}) = \frac{1}{2} \phi(\overline{\alpha}) + \phi^n(\overline{\alpha}).

Let \eta^n = \frac{1}{2} \phi^n(\overline{\alpha}) - \phi^n(\alpha_{med}). Then

\lim_{n \to \infty} \phi(\alpha^n_{med}) = \lim_{n \to \infty} \frac{1}{2} \phi(\overline{\alpha}) + \eta^n = \frac{1}{2} \phi(\overline{\alpha}),

where the second equality follows from \phi^n(\alpha) \leq \phi^n(\overline{\alpha}) \to 0. We claim that \phi is continuous on [0, \overline{\alpha}], which contradicts \lim_{n \to \infty} \alpha^n_{med} \geq \alpha_{med}.

Now we prove the claim. To see that \phi is continuous on [0, \overline{\alpha}], note that each interior \alpha^* satisfies the Edlin-Shannon conditions for strict monotonicity. We claim that \alpha^*(\overline{\alpha}) has Lebesgue measure 0 for any interior \alpha. Given this claim, the result follows since there can be no interior atoms. To prove the claim, observe that if such a set had positive measure, then it would contain an open set, and that open set would contain two points that are strictly ordered, contradicting the strict comparative statics result.

Finally, the result is trivial if \alpha_{med} = \overline{\alpha}.

C Proof of Proposition 3

We begin by recording a fact about the cut-point functions.

Lemma 1 For all \alpha_L and \alpha_R, we have \ellst_L > \ellst_R and \ellst_L > \ellst_R.

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**Proof** First, consider the case of floor control. In a symmetric equilibrium, the Westminster cut-point solves the equation

$$c^{wst} = \frac{n-1}{2n} \left( F(c^{wst})\pi_L + (1 - F(c^{wst}))\pi_R \right) + \frac{\sigma_R^2 - \sigma_L^2}{2n(\mu_R - \mu_L)}$$

$$\equiv \phi(c^{wst}).$$

The cut-point for the presidential case can be written

$$c^{pres} = \beta \phi(c^{pres}) + (1 - \beta) x_{Pres},$$

where $\phi$ is the function determining the Westminster cut-point. Applying the implicit function theorem, we see that

$$c^{pres}_L = \frac{\beta \phi_L}{1 - \beta \phi_c}.$$

Since

$$\phi_c = \frac{n-1}{2n} f(c)(\pi_L - \pi_R) < 0$$

(all other cut-points move right, mine moves left), this implies that $c^{pres}_L < c^{wst}_L$. A similar argument works for $\alpha_R$. Finally, in the case of majority control, $\phi_c \equiv 0$, so the result is trivial.

\[\Box\]

Now consider a symmetric stable party system $(\alpha_L, \alpha_R)$. Associated with this pair is a pair of memberships, $\mathcal{P}_L(\alpha_L)$ and $\mathcal{P}_R(\alpha_R)$. Rational affiliation implies that

$$\mathcal{P}_p(\alpha_p) = \left\{ x \mid x \in \left[ \pi_p - \sqrt{\frac{B}{\alpha_p}}, \pi_p + \sqrt{\frac{B}{\alpha_p}} \right] \right\}.$$

Let $\Delta_p(\alpha_p)$ be the median of $(x - \pi_p)^2$ over $\mathcal{P}(\alpha_p)$.

In a SPS, each party’s median voter’s first-order condition for $\alpha$ must be satisfied. Thus a stable party system in a Westminster system is a zero of the system

$$c^{wst}_L \frac{f(c)}{F(c)} - \frac{\Delta_L(\alpha_L)}{B - \alpha_L \Delta_L(\alpha_L)} = 0$$

$$-c^{wst}_R \frac{f(c)}{1 - F(c)} - \frac{\Delta_R(\alpha_R)}{B - \alpha_R \Delta_R(\alpha_R)} = 0,$$

while a stable party system in a presidential system is a zero of the system

$$c^{pres}_L \frac{f(c)}{F(c)} - \frac{\Delta_L(\alpha_L)}{B - \alpha_L \Delta_L(\alpha_L)} = 0$$

$$-c^{pres}_R \frac{f(c)}{1 - F(c)} - \frac{\Delta_R(\alpha_R)}{B - \alpha_R \Delta_R(\alpha_R)} = 0.$$
The lemma implies that the LHS of the first system is point-wise greater (in the usual vector order) than the LHS of the second. Furthermore, these functions are continuous in \((\alpha_L, \alpha_R)\) since \(c, f, \) and \(\Delta\) are all continuous, and party \(P\)'s first-order condition is increasing in \(\alpha_{-P}\), since the objective functions are logsupermodular in \(\alpha_P\) and \(\alpha_{-P}\). Thus theorem 4 of Milgrom & Roberts (1994) implies that the set of solutions to the first system is larger than that of the second.

D Proof of Proposition 4

The party medians choose the levels of discipline. At an interior equilibrium, a median voters' optimal levels of discipline satisfy the first-order condition:

\[
\begin{align*}
\frac{c_L}{F(c(\alpha_L, \alpha_R) - \gamma)} & - \frac{(x_{med} - \pi_L)^2}{B - \alpha_{L}(x_{med} - \pi_L)^2} = 0 \\
\frac{c_R}{1 - F(c(\alpha_L, \alpha_R) - \gamma)} & - \frac{(x_{med} - \pi_R)^2}{B - \alpha_{R}(x_{med} - \pi_R)^2} = 0,
\end{align*}
\]

where \(c_L\) is the partial derivative of \(c\) with respect to \(\alpha_L\). A SPS is a zero of this system.

Consider first the median member of party \(L\)'s first-order condition. If the district expected median increases to be closer to the midpoint between the parties (from \(\gamma \) to \(\gamma'\) with \(\gamma < \gamma' < 0\)), then the argument of the hazard rate decreases. Since logconcavity is the same as a decreasing hazard rate, this means the left hand side of the first order condition increases.

We proceed similarly for party \(R\)'s median voter. The argument of the hazard rate is increasing in \(\gamma\). Thus as the expected district median decreases toward the parties' midpoint, the left hand side of the first order condition increases. Thus, the left hand side of the system with less competitive parties is point-wise greater than the left hand side of the system with more competitive parties in the usual vector order. Theorem 4 of Milgrom & Roberts (1994) therefore implies that the set of solutions to the first system is larger than that of the second.

E Proof of Proposition 5

Consider the case where \(F\) is the uniform distribution on \([-\bar{r}, \bar{r}]\). In this case, the equilibrium cut-points are additively separable in the \(\gamma\) and the \(\alpha\).
District $d$'s cut-point is

$$c^d = \frac{1}{2n} \sum_{k \neq d} \left( \frac{\pi_R + (\pi_L - \pi_R) c^k - \gamma_k}{2\tau} \right) + \frac{\sigma^2_R - \sigma^2_L}{2n(\pi_R - \pi_L)}.$$

Rewrite this as

$$c^d + \frac{(\pi_R - \pi_L)}{4n\tau} \sum_{k \neq d} c^k = -\frac{n - 1}{n} \pi_R + \frac{1}{4n\tau} \sum_{k \neq d} \gamma_k - \frac{\sigma^2_R - \sigma^2_L}{2n(\pi_R - \pi_L)}.$$

Stacking these equations gives us

$$Xc = g(\alpha_L, \alpha_R) + h(\gamma).$$

At a solution to this system, the cut-points satisfy

$$c = X^{-1}g(\alpha_L, \alpha_R) + X^{-1}h(\gamma).$$

Thus the derivatives of $c$ with respect to $\alpha_L$ and $\alpha_R$ do not depend on the $\gamma$ so the results follow from the same argument as in the proof of Proposition (4).
References


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