Stock Price Volatility, Learning, and the Equity Premium

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Abstract

The determination of stock prices and equilibrium expected rates of return in a general equilibrium setting is still imperfectly understood. In particular, as Grossman and Shiller (1981) and others have argued, stock returns appear to be too volatile given the smooth process for dividends and consumption growth. Mehra and Prescott (1985) claim that this smoothness in consumption and dividend growth gives rise to an “equity premium paradox” since it makes it impossible to explain the equity risk premium with a risk aversion parameter of less than an implausible 35.

This paper reconciles the apparent smoothness of aggregate dividends and the volatility of observed stock prices by developing a model of stock prices in a dynamic general equilibrium setting in which learning is important. Dividends, which are one component of the aggregate consumption endowment, are assumed to follow a stochastic process with a mean-reverting drift that is not directly observable by the representative agent but must be estimated from the realized growth rates of dividends and aggregate consumption. The stock price-dividend ratio is shown to depend on the current estimate of the dividend growth rate as well as on the level of uncertainty about the true growth rate. This non-observability of the growth rate of dividends introduces an element of learning into the stock valuation process which is shown to increase the volatility of the stock price and therefore reduce the level of risk aversion required to explain the equity premium. The model is calibrated to the observed joint dividend and consumption process for the US, and is shown to yield an interest rate and stock price process that conform closely to the stylized facts for US capital markets.
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Abstract

A model of stock prices in a dynamic general equilibrium setting in which learning is important is developed. Dividends follow a stochastic process with a mean-reverting drift that is not directly observable by the representative agent but must be estimated from the realized growth rates of dividends and aggregate consumption. This non-observability of the drift of the dividend process introduces an element of learning into the stock valuation process which is shown to increase the volatility of the stock price for realistic parameter values. The model is calibrated to the observed dividend process for the US and is shown for certain risk aversion parameters to yield an interest rate and stock price process that conform closely to the stylized facts for US capital markets.
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1. Introduction

The determination of stock prices and equilibrium expected rates of return in a general equilibrium setting is still imperfectly understood. In particular, as Grossman and Shiller (1981) and others have argued, stock returns appear to be too volatile given the smooth process for dividends and consumption growth\(^1\). Mehra and Prescott (1985) claim that this smoothness in consumption and dividend growth gives rise to an “equity premium paradox” since it makes it impossible to explain the equity risk premium with a risk aversion parameter of less than an implausible 35. Moreover, the risk free rate is too low to be explained by a representative agent model with values of the risk aversion parameters that are high enough to explain the equity premium and a positive rate of impatience. There is a fourth puzzle which is less often referred to. This is that, if economists believe there is an equity risk premium paradox, we should expect to find them holding very highly levered equity portfolios. Yet, although Siegel and Thaler (1997) claim that “most economists that we know have a very high proportion of their retirement wealth invested in equities (as we do)”, there is no evidence that even economists hold the highly levered equity portfolios that their theory, combined with “reasonable values” of the risk aversion parameter, demands.

\(^1\) Campbell (1996) claims that “the deeper puzzle is the high volatility of stock prices, which seems to be associated with predictable time-variation in excess stock returns.” This “excess volatility” is sometimes taken as evidence against market efficiency. Hagiwara and Herce (1997) state that “Empirical research carried out during the past 15 years has repeatedly found that news about future real dividends cannot account for the variability of stock prices observed in annual US data.”
The early work on the equity premium puzzle, starting from Mehra and Prescott’s classic paper, treated dividends on the equity security as either identical to aggregate consumption, or as perfectly correlated with aggregate consumption. For example, Benninga and Protopapadakis (1990) and Kandel and Stambaugh (1990) were able to match the first and second moments of consumption growth and riskless and equity returns by treating equity as a levered claim on aggregate consumption and choosing the degree of leverage to match the volatility of stock returns; however, as Cecchetti, Lam and Mark (1993) point out, these authors are implicitly allowing the ratio of dividends to consumption to vary in order to match the volatility of stock returns, and the assumed variation is much greater than the data on consumption and dividends reveal.

Other authors take the volatility of the equity return as exogenous and consider utility specifications that will yield the equity premium. Examples include Epstein and Zin (1991) who adopt a recursive utility specification, and Constantinides (1990) who develops a model with habit formation\(^1\). Abel (1990) considers a model in which the equity claim is a claim to aggregate consumption and introduces a utility function that depends on lagged values of individual and aggregate consumption; although he is able to produce a reasonable unconditional equity premium and riskless rate, he reports that “the conditional expected returns vary too much”. More fundamentally however, the volatility of equity returns is endogenous since it reflects the volatility of prices and not the volatility

\(^1\) Constantinides (1990) actually assumes a production economy and identifies the return on equity with the return to a levered equity claim on a constant returns to scale production process. Ferson and Constantinides (1991) find that habit formation cannot explain the mean equity premium.
of some exogenously determined production process, and therefore should be modelled explicitly.

Cecchetti et al. (1993) endogenize the price of the equity security and distinguish between the flows to equityholders which they represent by dividends, and aggregate consumption. Matching the moments of the joint dividend-consumption process by a bivariate lognormal process with a two-state Markov drift, they are able to match the risk free rate and the estimated equity premium when account is taken of the estimation error in the latter (although with a negative rate of impatience). However, they are not able to match both the first and the second moments of the equity return and risk free rate process, concluding that “the model is rejected because of its inability to match simultaneously the relatively low standard deviation of the risk free rate and the relatively high standard deviation of the equity premium.” Abel (1994) considers a production economy in which there is non-traded labor income as well as the equity security and technological uncertainty follows a Markov switching regime, but finds that the Markov switching regime exacerbates the equity premium puzzle. Thus, these authors who attempt to endogenize the volatility of equity prices, are unable to calibrate their models to the high volatility of stock returns relative to the low volatility of dividend changes.

A common feature of almost all of the prior analyses is that they have been conducted under the assumption of perfect information; investors are assumed to know the process generating dividends or returns. Yet it seems clear that investors are uncertain

\[\text{\footnotesize\textsuperscript{2}}\] Abel (1994) (as well as Kandel and Stambaugh (1989,1990)) makes the strong assumption that the innovation in the dividend is independent of the innovation in the dividend growth rate.
about the stochastic process generating stock returns and consumption. For example, there is considerable debate today as to whether or not we have entered a new regime in which corporate profits and payouts will grow at a faster rate than in the past. Indeed, even Siegel and Thaler (1997) who treat the historical level of the equity premium as a puzzle, admit to some uncertainty about the current size of the premium.

Brennan (1998) has shown that uncertainty about the size of the equity premium, and the possibility of learning about it, can have a drastic effect on investor’s allocation to equities, and Timmerman (1993, 1996) demonstrates that, in a simple setting in which the stock price is equal to the sum of expected future dividends discounted at an exogenous rate, changing expectations about future dividends arising from a simple learning heuristic can increase stock return volatility relative to a setting in which the stochastic process for dividends is known.

In this paper we analyze the volatility of stock prices and dividends in a stochastic dynamic general equilibrium setting in which rational learning is important. Instead of the

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3 A typical quote is: “Copious efforts have been made to explain the (high) valuation ratio away. It is suggested, for example, that economic growth - and so returns on capital - will be higher than previously, or that intangible assets are more important than before. Neither of these arguments is compelling”. Martin Wolf, Financial Times, October 7, 1997, p14.

4 “We must stress that our analysis has all been on historical data which suggest that the equity premium has been too large in the past...there is reason to believe that it is lower than it has been in the past.” Siegel and Thaler (1997).

5 Barsky and de Long (1993) also develop a heuristic stock valuation model with stochastic dividend growth in which the discount rate is exogenous and investors ignore the fact that their current estimate of the dividend growth rate will be revised in the future.

6 Kandel and Stambaugh (1996) analyze the implications of the predictability of stock returns for a risk-averse Bayesian investor who allocates his wealth between stocks and cash and uses available data to update his prior beliefs about returns. The one period investment horizon assumption eliminates the dynamic effects induced by learning. In a more closely related setup, Gennaioli (1986) models an economy in which mean returns are unobservable and must be estimated, and shows how this affects the asset allocation.
Markov switching regimes used by Cechetti, Lam and Mark (1990, 1993), Kandel and Stambaugh (1989, 1990) and Abel (1994), we model the dividend as a lognormal process with a time varying mean that follows an Ornstein-Uhlenbeck process; the mean is not observable by investors but must be inferred or learned from the history of the observed dividend process and aggregate endowment. In addition to allowing a continuous state-space and introducing learning by investors, this framework has the advantage over the above Markov switching models of allowing for correlation between innovations in the dividend process and innovations in the (investors’ estimate of the ) dividend growth rate. This induces a positive correlation between the dividend innovation and the innovation in the price-dividend ratio which increases the risk of the stock return, but is missing in the models of Cechetti, et al. (1990, 1993), Kandel and Stambaugh (1989, 1990) and Abel (1994), mentioned above. Further, by endogenizing the stock price we are able to show how the risk of stock returns depends on the level of the investor’s uncertainty about the current dividend growth rate. This allows us to reconcile the riskiness of stock returns with the apparent smoothness of dividends. For simplicity, we assume that the endowment process of the single consumption good follows a lognormal process with constant parameters. This implies that the riskless interest rate is constant. In order to obtain a reasonable value of the riskfree interest rate with a risk aversion parameter which is consistent with the historical premium, it is necessary to assume that the representative

\[ \text{Brennan (1997) also treats the mean return of stock as an unobservable state variable. Everything else the same, investors tend to hold a much smaller percentage of their wealth in the risky asset.}\]

\[ 7 \text{Campbell (1996) finds that it is easier to explain the equity risk premium when consumption growth rates are negatively autocorrelated, but concludes (p23) “It is troubling that the standard model (for which } C = D \text{) must rely so heavily on negative autocorrelation of consumption growth for which there is no strong evidence.”}\]
agent’s rate of impatience is negative. The need to assume a negative rate of impatience is a well-known limitation of the representative agent paradigm, which ignores lifecycle considerations and makes the counterfactual assumption that the representative agent has an infinite horizon.

The model is developed in Section 2. Section 3 calibrates the model to data on consumption, dividends, interest rates and stock prices. Section 4 concludes.

2. The Model

Consider an infinite horizon pure exchange economy in which the instantaneous flow of the aggregate endowment of a single, non-storable, consumption good, $Y$, evolves according to a lognormal diffusion with constant coefficients:

$$
\frac{d\ln Y}{dt} = \gamma dw_Y
$$

where $\gamma$ is a constant and $dw_Y$ is the increment to a standard Wiener process.

Without loss of generality, we assume that there is a single risky asset in the economy that pays a continuous dividend at the rate, $D$; the logarithm of the dividend rate, $D$, is assumed to follow an arithmetic Brownian motion with constant volatility but a stochastic mean, so that:

$$
\frac{d\ln D}{dt} = \mu dt + Dw_D
$$

\*Constantinides, Donaldson and Mehra (1997) model life-cycle considerations in an overlapping generations model. For a general discussion of the risk free rate “puzzle” see Kocherlakota (1996). Kocherlakota (1990) points out that a negative rate of impatience is consistent with the existence of equilibrium in an infinite horizon growth model.
and $\mu$ follows an Ornstein-Uhlenbeck process:

$$\frac{d\mu}{\mu} = (\mu - \bar{\mu})dt + \sigma d\tilde{w}_\mu$$  \hspace{1cm} (3)

$\mu$ and $D$ are constants and $d\tilde{w}_\mu$ and $d\tilde{w}_D$ are, possibly correlated, increments to standard Wiener processes. This structure for the dividend process, in which the instantaneous expected rate of growth of the dividend flow is stochastic and reverts to a fixed long run value, $\bar{\mu}$, allows for either positive or negative autocorrelation in the dividend growth rate.

The parameters of the joint Markov process (1)-(3) are assumed to be known. In addition to the risky security, there is an instantaneously riskless security whose rate of return at time $t$ is denoted $r(t)$. The state of the endowment and dividend process at time $t$ can be represented by the vector of state variables $(Y, D, \mu)$.

The economy is assumed to be populated by a continuum of identical agents. Each agent $i$ has a Neumann-Morgenstern utility function, so that the agent’s lifetime expected utility is of the form:

$$E_t \sum_{t=1}^{\infty} \frac{u_i(c_t)}{(1+r)^t}$$

subject to the budget constraint:

$$dW_i = (W_i x_i P) r dt + x_i (D dt + dP) (y_i c_t) dt$$  \hspace{1cm} (5)

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9 This simplifying assumption means that our model fails to capture significant aspects of the learning process.
In the budget constraint (6), $W_i$ is the agent’s wealth, $x_i$ is the number of shares held, $y_i$ is the agent’s share of the aggregate endowment, and $P$ is the price of the risky security.

The dividend rate, $D$, is a cash flow rate and is therefore observable. However, we assume that the instantaneous expected growth rate of dividends, $\mu$, is not observed directly by agents, but must be estimated by them from the past values of $D$ and $Y$. This estimation problem introduces learning and estimation risk into the agent’s decision problem. As Gennotte (1986) has shown in a similar setting, the investor’s decision problem may be decomposed into two separate problems: an inference problem in which the investor updates his estimate of the current value of the unobserved state variable, $\mu$; and an optimization problem in which he uses his current estimate of $\mu$ to choose an optimal portfolio. We consider first the inference problem:

**The Agent’s Inference Problem**

At time zero each agent is assumed to have a normal prior distribution over $\mu$, with mean $m_0$ and variance, $\sigma_0^2$. As time evolves, $\mu$ changes according to (3) and the agent updates his assessment using the information from $D$ and $Y$. Following Liptser and Shiryayev (1978), it is shown in the Appendix that the mean and variance of the conditional distribution of $\mu$, $m$, and $\sigma$, evolve according to:

\[
\begin{align*}
\frac{dm}{dt} &= (\bar{\mu} - m) + \begin{pmatrix} a_{s1}^T & q_{s0} \end{pmatrix} q_{s1}^{-1} d\tilde{w} \\
\frac{d\sigma}{dt} &= 2\mu \sigma + 2 \begin{pmatrix} a_{s1}^T & q_{s0} \end{pmatrix} q_{s1}^{-1} \begin{pmatrix} a_{s1}^T & q_{s0} \end{pmatrix}^T dt
\end{align*}
\]  

(6) 

(7)

where

\[
\begin{align*}
d\tilde{w} &= ds \begin{pmatrix} a_{s0} & a_{s1} m \end{pmatrix} dt
\end{align*}
\]  

(8)
ds is the (2x1) vector of the dividend and consumption signals (dlnD, dlnY), \( \mathbf{\hat{d}} \) is a (2x1) vector of the innovations in lnD and lnY, given investor i’s information set at time t, \( \mathbf{\hat{a}} = \{ D_s, Y_s, s < t \} \). \( \mathbf{a}_0 \) and \( \mathbf{a}_1 \) are (2x1) vectors, \( \mathbf{q}_{ss} \) is a (1x2) vector, and \( \mathbf{q}_{ss} \) is a (2x2) matrix, which are all defined in the Appendix. Equation (7) is the usual Riccati equation:

this shows that evolves deterministically as the agent learns. In general, the agent will never learn \( \mu \) perfectly, and there will be a steady state value of \( * > 0 \), such for \(* \), \( d \), represents the investor’s uncertainty about \( \mu \) in the long run after he has observed the joint evolution of D and Y. It is reasonable to posit that \( * > 0 \), and then declining towards its asymptotic value of \(* \).

The Agent’s Optimization Problem

Given the agent’s information set, \( \mathbf{\hat{a}} \), the state of the economy is completely characterized by his current assessment of the instantaneous dividend growth rate, as measured by its mean, \( m_\tau \), and variance, \( \tau \), the current rates of endowment and dividend flow, Y, D. Thus the investor’s lifetime expected utility under the optimal policy may be written as:

\[
J_i(W, m, Y, D, t) = \max_{\mathbf{c}_t} \mathbb{E}_t \left[ \mathbf{u}_i(\mathbf{c}_t) d \right]
\]

The price of the risky security, P, will depend on the same set of state variables and may be written as: \( P(W, m, Y, D, t) \). Ito’s Lemma then implies that its stochastic process is of the form:
where \( p \) are to be determined. Using (12), the agent’s budget constraint (6) may be written as:

\[
\frac{dp}{p} dt = p \, dw_p \tag{10}
\]

Then the condition for optimality in agent i’s constrained decision problem (9) is \(^10\):

\[
0 \, \max \left[ U(c,t) \right. \\
J_w(r_w \, rP \, xD \, xP \, c \, y) \\
J_m(\mu \, m) J_D(m \, \frac{1}{2} D^2) D J_Y(\frac{1}{2} Y^2) Y \\
J_i \left( \frac{1}{2} J_{ww} x^2 p(x(t))^2 \frac{1}{2} J_{mm} m(t)^2 \frac{1}{2} J_{dd} D^2 \right) \right] \\
\left. \frac{1}{2} J_{yY} Y^2 Y \frac{1}{2} J_{wm} xP_{mp}(t) J_{wp} xPD_{mp}(t) \right. \\
\left. J_{wy} xPY_{py}(t) J_{dm} D_{mp}(t) J_{ym} Y_{mp}(t) \right] \\
J_{yd} YD_{yd} 
\tag{12}
\]

where \( mp \) is the instantaneous covariance between the innovations in m and P etc. The first order conditions corresponding to the choice of optimal consumption and investment, \( c^*_i \) and \( x^*_i \), are:

\[
x \left( \frac{J_w}{J_{ww}} (P^2 \, p(t))^2 \right. \left( P \, D \right. \left. rP \right) \\
\left. \frac{J_{wm}}{J_{ww}} (P^2 \, p(t))^2 \right. \left. P_{mp}(t) \right) \frac{J_{wp}}{J_{ww}} (P^2 \, p(t))^2 \left. D_{mp}(t) \right) \\
\left. \frac{J_{wy}}{J_{ww}} (P^2 \, p(t))^2 \right. \left. PY_{py}(t) \right) 
\tag{14}
\]

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\(^{10}\) The i subscript is omitted from equations (12)-(14) for clarity.
Following Breeden (1979), these first order conditions imply that appreciation in the price of the risky security must satisfy the condition:

\[ T_i(c) \frac{P'(1)}{P} \frac{D}{P} \frac{rP}{P} \frac{\text{cov}_t(d_c, \frac{dP}{P})}{P} \]  

(15)

where \( T_i(c) \frac{c}{U_{cc}(c, z)} \), is the inverse of the investor’s absolute risk aversion.

Equation (15) states that, at the optimum, the covariance between the changes in the agent’s optimal consumption rate, \( c_t \), and the rate of the return on the risky asset is equal to the product of the agent’s risk tolerance and the expected rate of return on the risky asset.

**Market Equilibrium**

We assume that the aggregate endowment flow, \( Y \), includes the flow of dividends from the risky security, which is therefore in zero net supply. Then the market equilibrium conditions for the goods and securities markets respectively, are:

\[
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & C & Y & y_t \frac{dP}{P} \\
0 & x_t \frac{dP}{P} & 0 \\
0 & 0 & 0 \\
\end{bmatrix} = 0
\]  

(16)

(17)

In order to obtain simple pricing functions we assume that each agent has an instantaneous state-dependent utility function of the CRRA class that can be written as:
Under the utility function (18), the agent’s risk-tolerance is \( T(c) = c/ \)
(15) over \( i \), and imposing the market clearing condition (16), we have the equilibrium rate of return condition:

\[
T(C) = \frac{P(1)}{P} \frac{D}{P} \frac{r}{P} \text{ cov}_t (dC, \frac{dP}{P})
\]  
(19)

where \( T(C) \)
clearing conditions (16)-(17) imply that the riskless interest rate, \( r \), is given by:

\[
r = \frac{1}{2} \gamma^2
\]  
(20)

Note that, as mentioned above, the interest rate in this model is constant, because, growth rate of the endowment process is constant. Thus, equations (19) and (20) determine the two prices, the price of the risky asset, \( P \), and the interest rate, \( r \).

**The Equilibrium Price of the Risky Asset**

In order to obtain an explicit expression for the price of the risky asset and the equity premium, we conjecture that the price of the risky asset depends only on \( D \), \( m \), and \( t \), and that it is homogeneous of degree one in \( D \); then it can be written as:

\[
P(D,m,t) = p(m,t)D
\]  
(21)

Since Ito’s Lemma implies that:
and the stochastic process for the price-dividend ratio is:

$$\frac{dp}{p} = \left( \frac{P_m}{p} - \mu m \right) dt + \frac{1}{2} \left( \frac{P_m}{p}^2 \right) dw_m$$

(23)

the stochastic process for the price, P, may be written:

$$\frac{dP}{P} = \left\{ \frac{1}{2} \left( \frac{P_m}{p} \right)^2 + \frac{P_m}{p} \left[ (\mu m) - D \right] \right\} dt + \frac{P_m}{p} dw_m$$

$$dt + p dw_P$$

(24)

Then, using equations (1) and (24), the covariance between the change in aggregate consumption and the return on the risky security, \( \text{cov}(dC, dP/P) \), is \( C \left( \frac{mY m}{p} + \frac{D_Y}{p} \right) \).

Finally, substituting for \( T(C) \), condition (23), the pricing conjecture is verified, and the price depends only on \( D, m, \) and \( t \), as stated in the following theorem

**Theorem 1**

The price of the risky asset at time \( t \) is given by

$$P(D, m, t) = p(m, t)D$$

(25)

where the price-dividend ratio \( p(m, t) \) is the solution to the following partial differential equation:
It is shown in Appendix B that, in order for equation (26) to have a well-defined solution, it is necessary and sufficient that the expected long run growth rate in dividends satisfy the following growth condition, which requires that the expected asymptotic dividend growth rate under the equivalent martingale measure be less than the interest rate:

**Condition A**

\[
\frac{-\bar\mu - \mu_D - \mu_Y}{\frac{1}{2} \frac{\mu}{D^2}} < r
\]

The coefficients of equation (26), the differential equation for the price-dividend ratio, depend on the coefficients of the joint stochastic process for the dividend and endowment under the investor’s information set, equations (1), (2), and (6)-(8), as well as on the investor’s risk aversion coefficient, the stochastic process for \(\mu\), equation (3), only indirectly through equation (6), the process for \(m\), which is the estimate value of \(\mu\).
The pricing function, \( p(m, t) \), is time dependent because the precision of the agent’s assessment of the current value of \( \mu \), \( \mu^* \), increases over time as the agent learns. In the limit, as \( t \) becomes large, \( \mu \) approaches \( \mu^* \), and the coefficients of the partial differential equation (26) become time invariant; then \( p(m, t) \) approaches zero and the equation becomes an ordinary differential equation whose solution we denote by \( p(m) \). The boundary condition for equation (26) is then

\[
\lim_{t \to \infty} p(m, t) = p(m) \tag{27}
\]

where \( p(m) \) is given by the following theorem:

**Theorem 2**

(i) When \( \mu = \mu^* \), the price-dividend ratio, \( p(m) \), satisfies the ordinary differential equation:

\[
A \frac{d^2}{dm^2} B(m) \frac{d}{dm} C(m) p = 1 + 0 \tag{28}
\]

where \( A, B(m) \) and \( C(m) \) are as given in the corresponding expressions below equation (26), with \( \mu = \mu^* \).

(ii) Then, under Condition A, the price-dividend ratio is given by:

\[
p(m) = \int_0^\infty e^{i\mu^* s^2} \frac{m^2}{2} \left( \frac{s^2}{4s^2 + \frac{m^2}{2}} \right) ds \tag{29}
\]
where

It may be shown that expression (29) for the price-dividend ratio is equivalent to:

\[
p(m) = E \left\{ \frac{D(s)}{D(0)} e^{r^* s} ds \right\} \tag{30}
\]

where \( E^* \{ \} \) denotes that expectations are taken with respect to the equivalent martingale measure under which:

\[
d \ln D = (m - \delta_Y) dt + \sigma_D d \hat{W}_D \tag{31}
\]

\[
d m = (\mu - m) dt + \sigma_m d \hat{W}_m \tag{32}
\]

Having determined the price-dividend ratio, \( p(m, t) \), it is straightforward to obtain the equity risk-premium from the equilibrium return condition (19). The equity premium is expressed in terms of the price-dividend ratio and its derivatives in the following theorem:

**Theorem 3**

(i) In the short run, when there is learning so that \( \gamma > \gamma^* \), the equity risk premium, current growth rate in dividends, \( m \), as well as calendar time:
In the long run, when $\alpha = \gamma$, the equity risk premium, no longer time dependent, and may be written in terms of the steady state price-dividend ratio, $p(m)$, and its derivatives:

\[
\begin{align*}
(m, t) & \quad \frac{1}{p} \quad r \\
\frac{1}{2} \frac{p_{mm}}{p} m(t) & \quad \frac{p_m}{p} [ (\bar{\mu} - m) \quad m_D(t)] \\
(m & \quad \frac{1}{2} D^2) \quad \frac{1}{p} \quad \frac{p_t}{p} \quad r
\end{align*}
\]

(ii) In the long run, when $\alpha = \gamma$, the equity risk premium, no longer time dependent, and may be written in terms of the steady state price-dividend ratio, $p(m)$, and its derivatives:

\[
\begin{align*}
(m) & \quad \frac{1}{p} \quad r \\
\frac{1}{2} \frac{p_{mm}}{p} m & \quad \frac{p_m}{p} [ (\bar{\mu} - m) \quad m_D] \\
(m & \quad \frac{1}{2} D^2) \quad \frac{1}{p} \quad r
\end{align*}
\]

The third term in (34) is the expected growth rate in the dividend, while the fourth is the dividend yield. Thus, if $m$ were non-stochastic ($m = 0$) and equal to $\bar{\mu}$, then would be a constant equal to the sum of the expected growth rate in dividends, $\frac{1}{2} D^2$, and the dividend yield, $1/p$, less the riskless interest rate, $r$. Thus the first two terms arise from variation in $m$ which gives rise to changes in the price-dividend ratio, $p$. The changes in $m$ arise, as shown in equation (6), as the investor changes his assessment of the dividend growth rate $\mu$, due to the mean reversion in $\mu$ and, more importantly, due to the inference he draws about $\mu$ from the innovations in the two “signals”, the realized
instantaneous growth rates in dividends and the aggregate endowment. Thus, the consumption risk of a security arises not only from the covariance of the innovations in its current dividend with the innovation in consumption, but also from the covariance between innovations in the investor’s assessment of expected future cash flows (or, equivalently, in the growth rate) with the change in consumption. As the following lemma shows, \( m^2 \) and \( mD \) are increasing in \( mY \) is independent of

**Lemma 1**

(i) Since \( a_{11} > 0 \), equation (8) implies that \( m^2 \) is increasing in

(ii)

\[
\begin{align*}
\quad & mY \quad \mu Y \\
\end{align*}
\]

(iii)

\[
\begin{align*}
\quad & mD \quad \mu D \\
\end{align*}
\]

Note that the risk premium does not depend directly on \( mY \), or \( DY \), the covariances of innovations in the dividend and its estimated growth rate with the innovations in the endowment process: however, these covariances, which enter into the coefficients of the differential equation for the price-dividend ratio, do affect the risk premium indirectly through the \( p(m, t) \) function and its derivatives.

**Equilibrium Pricing in a Fully Observable Economy**

It will be useful to compare the equity prices in our economy with those in a fully observable economy in which the representative investor knows not only \( D \) and the parameters of the joint stochastic process for consumption and dividends (1) - (3), but also
the current dividend growth rate, $\mu$. In the fully observable economy the price of the risky security will be of the form $P(D, \mu)$. Condition (19), the price dividend ratio, $q(\mu)$, satisfies

$$Aq_{\mu \mu} + B(\mu)q_{\mu} + C(\mu)q = 1$$

(35)

where $A = \frac{1}{2} \mu^2$, $B(\mu) = (\mu - \mu_D)\mu_Y$, and $C(\mu) = \frac{1}{2} D^2 r$.

3. Model Calibration

In order to assess the ability of the model to generate an endogenous stock price process and interest rate with properties similar to those of the empirical data, the stochastic process for dividends and consumption, equations (1)-(3), was calibrated to the moments of the joint distribution of consumption and dividends reported in Table 1, which are based on the Shiller (1989) data. Expressions for the following six unconditional moments were calculated as shown in Appendix C: the mean and variance of per capita consumption growth; the mean, variance and first order autocorrelation of dividend growth; the covariance between innovations in consumption and dividends. The consumption process (1) was chosen so that the autocorrelation in consumption growth rates is zero. The values of $\mu_C$ follow directly from the first two moments of the consumption process, and $\mu$ was set equal to the mean change in the log dividend. Then the six parameters of the stochastic process for dividends, and the correlations between innovations in $\mu$, $\ln D$, and $\ln C$, were chosen to match the values of the three empirical moments: $\text{var}(\ln D_t - \ln D_{t-1})$, $\text{cov}(\ln D_t - \ln D_{t-1}, \ln C_t - \ln C_{t-1})$, and $\text{cov}(\ln D_t - \ln D_{t-1}, \ln D_{t-1} - \ln D_{t-2})$. Since this allows three degrees of freedom in the choice of parameters, different
values of \( \mu_D \) and \( \mu_C \) were prespecified and then \( \mu \), \( D \), and \( D_C \) were chosen to satisfy the three moment conditions.

were chosen to yield the historical risk premium and risk-free rate.

Table 2 reports statistics for selected combinations of the prespecified parameter values that satisfy the growth condition, \emph{Condition A}. The selected combinations, or models, are given in Columns (a) - (c) of the table. Columns (d) - (f) show for each model, the values of \( D \), \( \mu \), and \( D_Y \) that are consistent with the matched moments of the joint process for dividends and consumption. Column (g) reports the steady state value of the standard error of the estimate of \( \mu \). Columns (h)- (k) report the endogenously determined values of the dividend yield, equity premium, return volatility, and the correlation between the stock return and the innovation in consumption growth when the true dividend growth rate, \( \mu \), is not directly observed, the current estimate of the growth rate is \( m = 2\% \), and the learning steady state has been reached so that the pricing function \( p(m) \) is time-independent. In order to assess the extent to which the results are due to the fact that investors are continuously learning about the current value of \( \mu \) from their observations of the dividend process, the pricing function for each model was obtained for the corresponding fully observable economy, by solving numerically the ordinary differential equation (35): the results are shown in columns (l) - (o).

Observe first that it is possible to find a wide range of parameter values that are consistent with the joint process for dividends and consumption and which, with learning in the steady state, yield stock price volatility of the order of 16-20\%, an equity premium of the order of 5-6\%, a risk free rate of 2.5\%, and a dividend yield of 3-5\%, and all with a risk aversion parameter of 15. For example, we can generate an economy (line viii) in which, for
m = 2%, the stock price volatility is 17.2%, the equity premium is 6.06%, the dividend yield is 5.2%, and the risk free rate is 2.5%, which compare with the corresponding historical averages of 17.4%, 5.75% ,4.8%, and 1.74%\textsuperscript{11}. Thus, in this example the endogenous characteristics of the model stock and bond returns are very close to their empirical counterparts, while the moments of the dividend and consumption processes are equal to their empirical counterparts by construction. Moreover, the model correlations between stock returns and consumption, which range between 0.49 and 0.69, straddle the empirical value of 0.51. This example clearly demonstrates that the apparent high volatility of stock prices compared with the apparent smoothness of dividends is insufficient reason\textit{per se} for concluding that stock prices are “too volatile” to be supported by a rational model.

The reason for the high volatility of the stock price in the model is that small changes in the slowly mean-reverting assessment of the growth rate, m, have relatively large consequences for stock prices. Figure 1 plots, for Model viii, the price-dividend ratios as functions of the current assessment of the instantaneous dividend growth rate both when there is learning, p(m), and for the corresponding fully observable economy, q(µ). The price-dividend ratio is an increasing convex function of m, rising from 6.1 when m = -15% to 76 when m = 15%. Perhaps surprisingly, the price-dividend ratio is generally lower in the fully observable economy. The reason for this is that the aggregate social risk in the two economies is the same, being determined by \( Y \), while expected future dividends are a convex function of the current growth rate, \( µ \), so that uncertainty about that growth rate increases the value of the expectation.

\textsuperscript{11} However, the current yield on the newly issued US indexed bonds is around 3.5%.
*, the steady state value of the standard error of the estimate of the current dividend growth rate, ranges from 1.9% to 7.1% for the different models. By comparing columns (i) and (m) it can be seen that the effect of learning in the economy with a non-observable dividend growth rate is to increase the equity premium slightly. However, the effect of learning on the volatility of stock returns (columns (j) and (n)) is large in many cases; for example in Model viii the volatility of stock returns is only 12.5% when µ is observable, while in the corresponding economy with learning the volatility is 17.2%. Thus, it is plausible that the high level of stock volatility observed in the data is due to learning effects of the type that we have modelled here. In what follows we select Model viii for further analysis.

Figure 2 shows that for Model viii the equity premium is an approximately linear increasing function of the current assessment of the growth rate m. Since the dividend yield is inversely related to m, Figure 3 shows that in this model the equity premium (and since the interest rate is constant, the expected return on stocks) is decreasing in the dividend yield.

To gain further insight into the properties of the model, we take Model viii of Table 2 and use it to generate sample paths of dividends and stock prices for 1000 years. The representative investor is assumed to begin the 1000 years with a prior distribution on the dividend growth rate which is normally distributed with mean of 2% and standard deviation of 3.7% and then the dividend and consumption rates are assumed to change according to a (monthly) discrete approximation to the assumed stochastic process (1)-(3); then m, the estimated growth rate of dividends is updated each month using discrete approximations to equations (6) and (8). The mean and standard deviation of the simulated excess returns were
5.4% and 17.2% which correspond to the historical statistics of 5.75% and 18.7% respectively. The 1000 years was then broken down into ten 100 year subperiods. In each subperiod the dividend rate was normalized to start at $1 per year. The first ten panels of Figure 4 show the simulated end-of year prices and dividends over the ten 100 year periods on a log scale. Panel k shows the (normalized) historical values of real dividends and stock prices from Shiller (1989) for the period 1890-1985, and panel l shows the same historical series of dividends, but with the prices constructed as the product of the historical dividends and the price-dividend ratio yielded by Model viii with Bayesian updating of the estimated dividend growth rate from the historical path of dividends and consumption: we call these the “Model viii historical prices”. The charts are quite similar in appearance, though it is noticeable that there is only one simulated 100 year subperiod which appears to be as favorable as the historical period 1890-1985\(^{12}\).

Figure 5 plots the historical real annual dividend series along with the historical price series and the Model viii historical price series - the latter if of course based on the dividend series. The two price series track each other quite well except for a period from 1955 to 1975 when the actual prices exceed the model prices for an extended period.

Finally, Figure 6 plots the historical real annual equity returns against the Model viii historical returns. The historical returns have a mean and standard deviation of 10.4% and 18.8% while the corresponding figures for the Model viii historical returns are 10.9% and 18.1%. The correlation between the two series is 0.64. The less than perfect correlation between the return series is to be expected since we have assumed in constructing the Model

\(^{12}\) See Brown, Goetzmann, Ross (1995) who argue that this period in US stock market history may suffer from a positive survival bias.
returns that the only signals the investor receives about the current value of \( \mu \) are from the realized dividend and consumption growth rates, whereas in setting the historical prices which form the basis for the historical return series investors had access to a broad variety of information signals including earnings.

Since the *Model* historical prices reflect only the limited information set of the past dividend series, while the historical prices reflect (under the assumption of market efficiency) a possibly broader information set, we might expect the historical returns to forecast the *Model* historical returns. To test this, the *Model* historical returns were regressed on the contemporaneous and lagged historical market returns, and the results are reported in Panel A of Table 3. The coefficient of the contemporaneous market return is highly significant confirming the importance of the news in dividend innovations for stock returns, but the lagged market return is insignificant.

Panel B of Table 3 reports the results of regressions, following Hodrick (1992), of log real annual returns on the dividend yield at the start of the year. Line (i) reports the results for the whole 1000 year simulation period. Line (ii) reports results using the historical data from 1890 to 1985. Line (iii) reports the results using the *Model* historical returns and dividend yields. Consistent with the flat relation between the equity premium and dividend yield shown in Figure 2, there is no significant relation between the simulated real annual return and lagged dividend yield. The historical data yield an *negative* relation between returns and dividend yield while the *Model* historical data yield an insignificant relation

\[ \text{A significant positive relation between nominal returns and lagged yield is apparent in the historical data (not reported). This is consistent with Fama and French (1988a) and Hodrick (1992). Goetzmann and Jorion (1993), and Kothari and Shanken (1997) offer a more circumspect interpretation of the evidence.} \]
Panel C of Table 3 reports the results of regressions, following Fama and French (1988b), of real annual returns on their lagged values. Line (i) reports the results for the whole 1000 year simulation period. Line (ii) reports results using the historical data from 1890 to 1985. Line (iii) reports the results using the Model viii historical returns and dividend yields. The simulated data reveal no evidence of serial dependence. For the historical data there is a significant negative coefficient on the lag 2 market return. For the Model viii historical data there is a marginally significant positive coefficient on the lag 1 return and a marginally significant negative coefficient on the lag 2 return. In summary, the simulated model data and the historical data are qualitatively similar\textsuperscript{14}.

3. Conclusion

We have shown in this paper that, by distinguishing between the cash flows received by equity holders and aggregate consumption, and by allowing for learning and a stochastic but unobservable dividend growth rate, it is possible to generate a representative agent model with rational behavior which yields a stock price series whose first two moments closely match those of the historical price series and yet which is consistent with the relatively smooth behavior of the dividend and consumption series.

The high volatility of the empirical stock price series combined with the low volatility of the dividend series has suggested to some scholars that either prices are “too volatile” to be consistent with a rational model or that there are time varying components to expected returns. We have been able to generate the empirical stock price volatility from a model based on the actual dividend volatility within a rational model. That of course is not

\textsuperscript{14} It is important to recall that there are significant small sample biases when returns are regressed on their lagged values or on lagged dividend yields using ordinary least squares.
to say that there are not in fact time-varying components to expected returns, and extension of the current model to allow for time-varying interest rates and learning with respect to the consumption process seems worthwhile.

We have been able to generate a theoretical equity risk premium of around 6%, and model returns conditional on the realized path of dividends whose first and second moments are very close to those of the realized returns over the period 1890-1985. We have done this in a model with a real interest rate of 2.5% which is close to the historical average of 1.74%. Perhaps the most debatable element of our model is the treatment of tastes. We have assumed a coefficient of relative risk aversion for the representative agent of 15. Several authors, including Mehra and Prescott (1985) and Grossman and Shiller (1981), have argued that any figure for relative risk aversion above 10 is unreasonable. However, Kandel and Stambaugh (1991) point out that intuitions about reasonable levels of risk aversion are typically derived either from the Friend and Blume (1975) study or from thought experiments. The Friend and Blume study implicitly assumes that consumption and stock returns are perfectly correlated and equally volatile which is clearly counterfactual, while “inferences about introspective context of thought experiments. It seems possible in such experiments to choose the size of a gamble so that any value of for distrusting the results of thought experiments is that the willingness to bear (stock-market) risk can be reduced by the kind of “background risk” to which real world agents are routinely exposed. As Fama (1991) argues that “.. a large equity premium is not necessarily a puzzle; high risk aversion (or low intertemporal elasticity of substitution) may be a fact”.

---

Dividends (1871-1996)
Log real dividend growth rate:
Mean 1.55%
Standard deviation 12.9%
First order autocorrelation 0.07

Dividend Yield (1871-1996)
Mean 4.8%
Standard deviation 1.2%

Returns (1871-1996)
Log Real Returns
Mean 6.99%
Standard deviation 17.43%
Equity Premium
Mean 5.75%
Standard deviation 18.72%

Consumption (1889-1985)
Log real consumption per capita growth rate
Mean 1.69%
Standard deviation 3.44% (M-P 3.56%)

Consumption-Correlations (1889-1985)
Correlation between growth rates
of \( \ln D \) and \( \ln C \) 0.34
Correlation between growth rate
of \( \ln C \) and \( \ln \) (Stock Return) 0.51

Real Interest Rate (1871-1996)
Mean 1.74%
Standard Deviation 5.57%

Empirical Moments of Consumption, Dividends, and Interest Rates
Table 1
### Equity Premia, Stock Volatility, Dividend Yields and Interest Rates for different Exogenous Parameter Values

#### Table 2

The table shows values of the equity premia, stock volatility, dividend yield and interest rate for various parameter combinations for the joint process of dividends and consumption which are consistent with the empirical moments of dividend and consumption shown in Table 1. For all the examples shown, the risk aversion coefficient, $2.5\%$, the impatience parameter, $\gamma$, is $2\%$. $\hat{\mu}$: standard error of estimate of dividend growth rate, $\mu$, in steady state. D/P: dividend yield; $\mu_Y$: stock return volatility; $\rho_{RC}$: correlation between equity return and innovation in consumption.

<table>
<thead>
<tr>
<th>Selected Parameters</th>
<th>Calculated Parameters</th>
<th>Model Outcomes</th>
<th>With Learning</th>
<th>$\mu$ observable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (b) (c)</td>
<td>(d) (e) (f)</td>
<td>(g) (h) (i) (j) (k)</td>
<td>(l) (m) (n) (o)</td>
<td></td>
</tr>
<tr>
<td>$\mu_Y$</td>
<td>$\mu_D$</td>
<td>D/RY</td>
<td>D/P</td>
<td>$\rho_{RC}$</td>
</tr>
<tr>
<td>(i) 0.01 0.6 -0.5</td>
<td>12.4% 0.6% 0.34</td>
<td>2.7% 3.9% 5.12%</td>
<td>18.4% 0.54</td>
<td>4.5% 4.71% 11.0% 0.83</td>
</tr>
<tr>
<td>(ii) 0.01 0.6 -0.1</td>
<td>12.4 0.5 0.34</td>
<td>2.2 3.7 5.02</td>
<td>19.0 0.51</td>
<td>4.1 4.70 14.2 0.64</td>
</tr>
<tr>
<td>(iii) 0.02 0.4 -0.7</td>
<td>12.4 0.9 0.34</td>
<td>3.7 3.6 5.11</td>
<td>20.1 0.49</td>
<td>4.5 4.65 9.5 0.95</td>
</tr>
<tr>
<td>(iv) 0.02 0.7 -0.2</td>
<td>12.4 0.7 0.33</td>
<td>2.5 4.5 5.63</td>
<td>17.9 0.61</td>
<td>4.9 5.42 13.9 0.76</td>
</tr>
<tr>
<td>(v) 0.03 0.5 -0.5</td>
<td>12.4 1.1 0.33</td>
<td>3.5 4.3 5.48</td>
<td>18.8 0.56</td>
<td>4.8 5.22 12.2 0.83</td>
</tr>
<tr>
<td>(vi) 0.03 0.7 0.2</td>
<td>12.4 0.8 0.33</td>
<td>1.9 4.3 5.5</td>
<td>18.7 0.57</td>
<td>4.5 5.45 16.9 0.62</td>
</tr>
<tr>
<td>(vii) 0.04 0.7 -0.3</td>
<td>12.4 1.1 0.32</td>
<td>2.9 5.2 6.07</td>
<td>17.1 0.69</td>
<td>5.4 5.94 13.8 0.84</td>
</tr>
<tr>
<td>(viii) 0.05 0.6 -0.5</td>
<td>12.4 1.1 0.32</td>
<td>3.7 5.2 6.06</td>
<td>17.2 0.68</td>
<td>5.6 5.91 12.5 0.92</td>
</tr>
<tr>
<td>(ix) 0.05 0.8 0.0</td>
<td>12.4 1.1 0.32</td>
<td>2.1 5.2 6.09</td>
<td>17.1 0.69</td>
<td>5.3 6.04 15.8 0.74</td>
</tr>
<tr>
<td>(x) 0.13 0.4 -0.7</td>
<td>12.4 3.3 0.30</td>
<td>5.9 5.0 5.60</td>
<td>16.6 0.65</td>
<td>5.3 5.55 12.6 0.86</td>
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<tr>
<td>(xi) 0.16 0.3 -0.8</td>
<td>12.4 4.2 0.31</td>
<td>7.1 4.5 5.06</td>
<td>16.5 0.59</td>
<td>4.8 5.02 12.5 0.78</td>
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<tr>
<td>(xii) 0.18 0.8 0.2</td>
<td>12.4 1.8 0.30</td>
<td>1.8 4.7 5.18</td>
<td>16.2 0.62</td>
<td>4.7 5.18 16.0 0.63</td>
</tr>
<tr>
<td>(xiii) 0.20 0.6 -0.1</td>
<td>12.4 2.6 0.29</td>
<td>3.3 4.7 5.13</td>
<td>16.0 0.62</td>
<td>4.7 5.13 15.4 0.64</td>
</tr>
</tbody>
</table>
Panel A: Regression of log Model viii historical returns (R_m) on current and lagged log historical real market returns (R_M). (1890-1985)

\[ \ln R_m(t) = a_0 + a_1 \ln R_M(t) + a_2 \ln R_M(t-1) \]

<table>
<thead>
<tr>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(R^2)</th>
<th>Nobs</th>
<th>F-stat</th>
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</thead>
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<tr>
<td>0.028</td>
<td>0.667</td>
<td>0.085</td>
<td>0.49</td>
<td>95</td>
<td>46.33</td>
</tr>
<tr>
<td>(1.88)</td>
<td>(9.50)</td>
<td>(1.21)</td>
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</table>

Panel B: Regressions of log Real Annual Returns (R) on Lagged Dividend Yields (D/P)\(^{16}\)

\[ \ln R_t = a_0 + a_1 (D/P)_{t-1} \]

\(i\) 1000 year simulation

<table>
<thead>
<tr>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(R^2)</th>
<th>Nobs</th>
<th>F-Stat</th>
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<tr>
<td>0.002</td>
<td>0.059</td>
<td>-0.000</td>
<td>999</td>
<td>0.31</td>
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<td>(0.28)</td>
<td>(0.55)</td>
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\(ii\) Historical Data (1890-1985)

<table>
<thead>
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<th>(R^2)</th>
<th>Nobs</th>
<th>F-Stat</th>
</tr>
</thead>
<tbody>
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<td>-0.389</td>
<td>-6.401</td>
<td>0.24</td>
<td>95</td>
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<td>(6.75)</td>
<td>(5.52)</td>
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\(iii\) Model viii Historical Data (1890-1985)

<table>
<thead>
<tr>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(R^2)</th>
<th>Nobs</th>
<th>F-Stat</th>
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</thead>
<tbody>
<tr>
<td>-0.003</td>
<td>1.753</td>
<td>-0.002</td>
<td>95</td>
<td>0.79</td>
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</table>

Panel C: Regression of log Real Annual Returns on Lagged Values

\[ \ln R(t) = a_0 + a_1 \ln R(t-1) + a_2 \ln R(t-2) \]

\(i\) 1000 year simulation

<table>
<thead>
<tr>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(R^2)</th>
<th>Nobs</th>
<th>F Stat</th>
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<tbody>
<tr>
<td>0.005</td>
<td>0.048</td>
<td>0.002</td>
<td>0.0003</td>
<td>998</td>
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<tr>
<td>(3.10)</td>
<td>(1.53)</td>
<td>(0.06)</td>
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\(ii\) Historical Data (1890-1985)

<table>
<thead>
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<th>(a_2)</th>
<th>(R^2)</th>
<th>Nobs</th>
<th>F Stat</th>
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<tbody>
<tr>
<td>0.102</td>
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<td>(4.69)</td>
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<td>(2.37)</td>
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</table>

\(iii\) Model viii Historical Data (1890-1985)

<table>
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<th>(a_2)</th>
<th>(R^2)</th>
<th>Nobs</th>
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<tr>
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<td>(4.20)</td>
<td>(1.88)</td>
<td>(1.73)</td>
<td></td>
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</table>

Model and Historical Stock Return Regressions

Table 3

\(^{16}\) The dividend yield is defined as the dividend for the previous year divided by the price for January of the current year
**References**


**Legends**

**Figure 1: Price-Dividend Ratio and Current Assessed Dividend Growth Rate.**
The figure shows the relation between the price-dividend ratio, \( p(m) \), and \( m \), the estimated dividend growth rate for *Model viii*, and the price-dividend ratio, \( q(\mu) \), and \( \mu \), the dividend growth rate, for the corresponding fully observable economy.

**Figure 2: Equity Premium and Current Assessed Dividend Growth Rate**
The figure shows the relation between the steady state equity premium, estimated dividend growth rate calculated from equation (42) for *Model viii*.

**Figure 3: Equity Premium and Dividend Yield.**
The figure shows the relation between the steady state equity premium, \( \pi \), the instantaneous dividend yield for *Model viii*.

**Figure 4: Simulated and Historical Real Annual Stock Prices and Dividends.**
Figures 4a-4l show time series plots of 100 year samples of log real annual dividends (dotted) and stock prices (continuous) generated from a 1000 year sample from *Model viii* starting with \( D = 1 \) and prior on \( \mu \) normally distributed with mean 2% and variance (3.7%). For the plots the dividend is renormalized to \( D = 1 \) for each 100 year subsample. Figure 4m shows the historical log real annual dividends and stock prices for the period 1890-1995.

Figure 4n shows the *Model viii* historical log real annual dividends and stock prices for the period 1890-1995, starting in 1890 with a prior on \( \mu \) with mean 2% and variance (3.7%).

**Figure 5: Normalized Historical and *Model viii* Historical Real Annual Stock Prices and Dividends (1890-1985).**
The figure shows the real annual dividends (crosses), the historical real annual stock prices (dotted) and the *Model viii* (continuous) historical real annual stock prices starting in 1890 with a prior on \( \mu \) with mean xxxx and variance xxxx.

**Figure 6: Actual and *Model viii* Historical Real Annual Stock Returns (1890-1985).**
The figure plots the historical real annual returns (vertical axis) against the *Model viii* historical real annual returns (horizontal axis) which are constructed from the *Model viii* historical prices and the historical dividends.
As a preliminary to analyzing the relation between the dividend yield and the risk premium, and between the risk premium and $\mu^*$, the uncertainty about drift of the dividend process, we establish the following results:

**Lemma 2**

(i) $p(m) > 0$

(ii) $p(\mu) > 0$.

**Theorem 4:** At $m = \mu^*$,

(i) The risk premium is increasing in the assessed drift of the dividend process: $\frac{m}{\mu^*} > 0$.

(ii) The risk premium is increasing in the dividend yield $(1/p)$.

Theorem 4 is consistent with the extensive evidence that expected returns on stocks are increasing in the dividend yield.  

**Theorem 5:** At $m = \mu^*$,

(i) $\text{var}\left(\frac{dp}{p}\right)$ is increasing in $m$.

(ii) $\text{cov}\left(\frac{dp}{p}, dC\right)$ is increasing in $m$.

(iii) $\text{cov}\left(\frac{dp}{p}, dC\right) > 0$.

The theorem establishes that at least around the certainty equivalent long run growth rate, $\mu^*$, the volatility of the stock price and the equity risk premium is increasing in the level of estimation risk about the current dividend growth rate.

We consider next the consistency of the model developed above with the stylized facts concerning the interest rate, the variance of stock returns, the equity premium, the variance and autocorrelation in dividend growth rates, and the covariance between innovations in consumptions and dividends.

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