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Dollarization: An Irreversible Decision

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Abstract:

After faithfully maintaining a fixed exchange rate and fully convertible currency for almost a decade Argentina still pays higher interest rates on peso denominated debt than on US dollar denominated debt. The interest rate spread is the price of keeping the option to devalue alive. Argentina has sufficient foreign reserves to defend the currency against any attack, but they could choose to devalue if the pain of maintaining the currency peg got too great. Investors fear devaluation and they charge Argentina for the option to devalue. In 1999 President Menem recommended “dollarizing” the economy. Dollarization extinguishes the option to devalue.

If dollarization is a credible commitment to maintain a fixed exchange rate, then it is an irreversible decision. This is the first paper, as far as I know, to explicitly model the unique feature -- irreversibility -- of a dollarization policy. In addition, it is the first paper to recognize that if a dollarization is a potential exchange rate regime choice, then the equilibrium is a mixed strategy equilibrium. Argentina might dollarize.

I compute Nash equilibria and the transition probabilities that a country will move from a currency board regime to dollarizing, or floating, in one year, or two years, out to seven years. I model shocks to the exchange rate as a jump-diffusion process. The jumps represent large “asymmetric” shocks to the exchange rate — such as Brazil’s 1999 devaluation. The probability of dollarization is inversely related to the jump probability. When the jump probability is small, the probability of dollarization is large and the probability of floating is small. When the jump probability is large, the probability of floating is large and the probability of dollarization is small.

*I thank my friend and colleague Steve Goldman for many valuable suggestions and corrections, and I thank Richard Saouma for help with the data.
Introduction

If dollarization is a credible commitment to maintain a fixed exchange rate, then it is an irreversible decision. This is the first paper, as far as I know, to explicitly model the unique feature -- irreversibility -- of a dollarization policy. In addition, it is the first paper to recognize that if dollarization is a potential exchange rate regime choice, then the equilibrium is a mixed strategy equilibrium.

In 1991 Argentina passed the Convertibility Law as part of a reform package to attack the crippling hyperinflation that paralyzed the country. The Convertibility Law fixed the exchange rate between the peso and the US$ at one and backed the peso with a quasi-currency board. Fixing the exchange rate was a politically courageous, and transparent, method to impose monetary discipline on the government. It worked. Inflation fell from over 1000% in 1990 to –2% in late 1999.

Now after a decade of monetary discipline and faithfully maintaining the peg, Argentina still suffers the dual curses of a currency board regime: the curse of a fixed exchange rate and the curse that the fixed exchange rate regime is not perfectly credible.

In a fixed exchange regime nominal shocks have temporary real effects unless prices are perfectly flexible. For example, when Brazil, Argentina’s largest trading partner, floated in January of 1999, the Real depreciated by 50%. Argentine GDP growth fell to –4% and its official unemployment rate hit 18%. As long as Argentina maintains the peg it cannot offset nominal shocks with exchange rate or monetary policy.

Furthermore, since a currency board (even though it has sufficient reserves) is not a perfectly credible commitment to maintaining the fixed rate Argentina has to pay a currency premium. Argentina did not follow Brazil into a floating regime, but they could have. A currency board regime keeps the option to devalue alive. The currency premium, which shows up in the interest rate spread, is the price of keeping the option to devalue open. In the spring of 1999, the Argentine President, Carlos Menem, proposed dollarizing the economy. Many others, eg, Barro (1999), or Eichengreen and Hausman (1999), also favor dollarization. In a dollarized Argentina the US$ replaces the peso -- in transactions, in contracts, everywhere. The peso does not exist. The US$ exchange rate is Argentina’s exchange rate and the US monetary policy is Argentina’s monetary policy. Dollarization extinguishes the option to devalue.

The unique feature of a dollarization policy is that the decision is irreversible. If dollarization is a credible commitment then it must be too expensive to reintroduce the peso. This paper analyzes dollarization as an irreversible decision. The Central Bank and investors optimize. Under a currency board the Bank holds a perpetual option -- the option to devalue. The option to devalue is an insurance policy. Investors fear devaluation and they charge a spread that equalizes expected returns. The spread is the premium the country must pay to keep the option alive. Floating exercises the option. Dollarization extinguishes the option. The Bank continues with the currency board regime as long as the expected present value of continuing is greater than the expected present value of stopping. It extinguishes the option if the expected present value of dollarizing is greater than the value of continuing. And it exercises the option if the value of floating is greater than the expected present value of dollarizing.

I examine the equilibria and transition probabilities for three specifications of the nominal shocks that hit the exchange rate. In the fixed exchange regime nominal shocks have temporary real effects. I assume that the shocks follow a jump diffusion process. The jumps represent unusual and large shocks that hit the system. The option to devalue provides protection against low probability large losses. I increase the probability of a draw from the jump process from 1 ½% to 3%. A jump probability of 1 ½% means that on average every 17 years a shock ten times the normal magnitude hits the system. Doubling the jump probability roughly halves the frequency. When the probability of a draw from the jump process is 1 ½%, the probability of dollarization is very high -- over 90% in a year -- even though it is cheap to keep the option to devalue alive. The spread averages only 1.8%. When the probability of a draw from the jump distribution doubles to 3% the average cost of keeping the option open almost triples to 5.2%. But the option is too valuable to extinguish. The probability of dollarization is zero. The probability of floating within a year is over 30% and goes to almost 90% in seven years.
These results are a natural extension of the conventional Mundellian wisdom to a dynamic environment with dollarization as a potential exchange regime. Mundell argued that relatively larger real sector shocks favored a flexible exchange regime, and relatively larger nominal sector shocks favored a fixed exchange regime. In this setup nominal shocks have only temporary real effects. But, the choice to dollarize is extremely sensitive to the distribution of the shocks because the choice is irreversible. The expected present value of the losses matters. Dollarization is not a good choice when the potential shocks are large.

The paper is organized as follows. Section 1 presents data from Argentina. It shows that periods with large spreads are periods with larger negative potential output gaps. Section 2 presents a formal model. The model has no closed form solution. I solve it numerically. Section 3 contains the results and section 4 concludes.

1. Argentina

Argentina is frequently mentioned as a prime candidate for dollarization. In 1999 the Argentine President Menem proposed dollarizing the economy. The Chief Economist at the Inter-American Development Bank, Ricardo Hausmann (1999), recommends dollarization. And academics such as Robert Barro (1999) and Barry Eichengreen (1999) also favor dollarization for Argentina. Goldfajn and Olivares (2000) give an excellent summary of the advantages and disadvantages of dollarization. This section presents some Argentine data to frame the issues.

**Price of Keeping the Option to Devalue Alive**

The currency premium is the price of keeping the option to devalue (float) open. At times the price was very high for Argentina.

The spread between the interest rate on an Argentine bond denominated in Pesos and the interest rate on the same Argentine bond denominated in US Dollars is the currency premium. The spread isolates the currency premium from the default, or country risk, premium. Holders of both peso and dollar denominated bonds are subject to default risk. But, the holders of dollar denominated bonds are not exposed to peso devaluation risk.

The top panel in Figure 1.1 shows interest rates on peso and dollar denominated debt with a ninety-day maturity over the 1990s. The middle panel shows the interest rate spread — the currency premium. The data are daily and the interest rates are expressed at annual rates.

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1 The Argentine peso denominated interest rate can be decomposed,

\[ i(\text{Peso}) = i(\text{US}) + \text{differential default premium} + \text{Currency Premium}, \]

into the US interest rate plus a differential default premium plus the currency premium. The US dollar denominated Argentine bond has no currency premium—devaluation risk.
Figure 1.1: The Spread and Output Gap

Peso and Dollar Denominated Interest Rates

Spread

GDP GAP  SPREAD
The currency premium is large, it averages 2.7% over the decade, and it spikes in times of currency crisis. When the Mexican currency collapsed at the end of 1994 the premium jumped to over 15%. In the Asian and Russian crises in 97 and 98 it jumped to 5%. And when Brazil floated in 1999 investors got worried and stayed worried through the Argentine election campaign.

The currency premium is very costly. It raises the real cost of capital for all domestic investment. The bottom panel in Figure 1.1 shows the spread and the real GDP gap (log(GDP/trend)) for quarterly data from 1994-1999. A high spread depresses real investment and output. Footnote 2 shows least squares regression estimates that give more formal, but statistically weak, evidence of a negative relationship between the spread and GDP growth.²

**Benefit from Dollarization**

Dollarization extinguishes the option to devalue and eliminates the currency premium. Dollarization would significantly lower Argentina’s real cost of capital and immunize them against the contagious spread of currency crises from other emerging markets.

Dollarization also locks in the benefits of a fixed exchange regime. For countries like Argentina it would commit future political regimes to monetary discipline and might bring positive externalities such as better access to international capital markets and deeper domestic markets, see IMF Occasional Paper #171, or *Latin American Economic Policies* (1999).

**Cost of Extinguishing the Option to Devalue**

The most important cost of dollarization is the permanent loss of an independent monetary policy. Dollarization, if it is effective, is an irreversible decision. Dollarizing permanently makes the Argentine exchange rate the US exchange rate and US monetary policy Argentina’s monetary policy. It also eliminates the Central Bank’s ability to act as a lender of last resort.

Figure 1.2 shows the GDP gap for Argentina, Canada, and the US the 90s.

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² The least-squares regression

\[ dy_t = 0.02 - 0.79 y_{t-1} - 0.42 s_{t-1} \]

\[(0.16) \quad (0.001) \quad (0.24)\]

shows a negative relation between detrended GDP growth (dy) the GDP gap (y) and the spread (s). The depressing effect of spread is weak statistically, but there are only 23 observations. (The numbers in parentheses below the coefficients are p values.) Neumeyer and Perri show similar pictures documenting a negative relationship between the level of interest rates and the output gap for several Latin-American countries including Argentina. They present no statistical evidence.
Figure 1.2
Argentine, Canadian, and US GDP GAP
Argentina suffers now because it pegs the peso to the dollar. Argentine output is much more volatile than US output and the cycles are not in phase with the US cycle. The Argentine economy is not like the US economy. The correlation between the output gap for the US and Argentina is 25%. In contrast, the Canadian and US economies move together. The correlation between the output gap for the US and Canada is 69%. Many of the shocks that hit the Argentine economy are not correlated with the shocks that hit the US economy — so called asymmetric shocks. For example, when Argentina’s neighbor and largest trading partner, Brazil, devalued in 1999 it had almost no impact on the US but it pushed Argentina into a deeper recession. Dollarizing makes the loss from increased volatility permanent and closes the option to float.

The country also loses seigniorage. Hausman (1999) estimates that dollarization means a $750 million annual loss in revenue (0.3% of GDP) to the Argentine government. There may be other potential downside risks. Some worry that it will increase the country risk premium.

**Section 2: A Formal Model**

This section presents a formal model. The next section gives numerical results and the Appendix shows the solution algorithm.

The model is stylized, but rich enough to capture key features of the exchange regime choice problem. Investors and the Bank optimize. Investors set the currency premium. The Bank chooses the exchange regime. The Bank chooses to remain in the currency board, or to switch. Switching is an irreversible decision. In the currency board regime the country must pay the cost of keeping the option to float alive — the currency premium. Floating (exercising the option) or dollarizing (extinguishing the option) ends the process.

**Assumptions**

1. Perfect capital mobility
2. Risk neutral investors
3. Some domestic price stickiness

**Notation**

- $R$ = the exchange regime: currency board, $R^c$, dollarization, $R^d$, float, $R^f$.
- $Z$ = the official exchange rate (pesos per dollar)
- $z$ = percentage deviation of the official exchange rate from the peg, $ln(Z/Z_0)$, where $Z_0$ is the fixed rate
- $x$ = percentage deviation of the shadow exchange rate from the official exchange rate, $ln(X/Z)$
- $y$ = percentage deviation of actual output from potential output, $ln(Y/Y_0)$
- $s(\tau)$ = the spread between the interest rate on a bond of maturity $\tau$ denominated in local currency and the interest rate on the same bond denominated in US$, i-\bar{i}^*$

**Equations of Motion:**

**Exogenous Shocks**

Nominal exogenous shocks drive the system. I assume that the shocks, $u$, are drawn from a mixture of normals,

$$u_{t+1} = \sigma(qw_{t+1} + (1-q)k_{t+1})$$  \hspace{1cm} (2.1)

The first term, $w$, represents the standard small shocks that continually hit the system, and the second term, $k$, represents large jumps that occasionally hit the system — the dollarization literature refers to them as

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3 The models in the papers by Ozkan and Sutherland (1998, 1995) are the closest to the model presented here. In their papers the Bank chooses when to abandon a fixed exchange rate regime (to float.). The choice is irreversible. The continuous time version of this model nests their models.

4 Or expectations are calculated under a “Martingale equivalent probability measure.”
“asymmetric” shocks. \( q \) is the binomial mixing process, \( p = \text{prob}(q = 0) \) is the probability of a draw from the jump distribution.

The State Variable

The exchange rate is the state variable. This is a regime switch model and the state variable depends on the regime. In the floating regime the official exchange rate is the state variable. It absorbs the nominal shocks. In the fixed regime the shadow exchange rate is the state variable. Shocks cause the shadow exchange rate to deviate from the official peg. The shadow exchange rate measures the over, or under, valuation of the currency. It returns to the peg as the real economy adjusts.

\[
\begin{align*}
\{ z_{t+1} = z_t + u_{t+1} & \quad x_{t+1} = (1 - a)x_t + u_{t+1} \quad z_t \in R^c \\
\{ z_{t+1} = z_t & \quad x_{t+1} = (1 - a)x_t + u_{t+1} \quad z_t \in R^s \\
\{ z_{t+1} = z_t & \quad x_{t+1} = 0 \quad z_t \in R^f
\end{align*}
\]

The Economy:

Output Gap

When the exchange rate is fixed, domestic output and prices must adjust to absorb the exogenous shocks.\(^5\)

\[
y_t = -b_1x_t - b_2A(s(x_t \mid R))
\]

The output gap is not a causal relationship. All the variables are endogenous. It expresses a “Phillips’ curve” type tradeoff. The real output gap is negatively related to currency overvaluation and to the spread.

Adjustment

The output gap links the adjustment processes for real output and the shadow exchange rate,

\[
\begin{align*}
\Delta y_{t+1} &= -b_1\Delta x_{t+1} - b_2\Delta A(s(x_{t+1} \mid R)) \\
&= -ay_t - b_2(\Delta A(s(x_{t+1})) - aA(s(x_t))) - b_1u_{t+1} \\
\Delta x_{t+1} &= -ax_t + u_{t+1}
\end{align*}
\]

Nominal shocks have temporary real effects in the fixed exchange rate regime.

Behavior

\(^5\) Of course, real shocks also cause real output to deviate from potential output. This is a single shock model. I ignore the real shocks. The model focuses on the link between the exchange rate and GDP because the paper focuses on the choice of an exchange regime.
Risk neutral investors maximize expected returns. The investors’ problem is to calculate expected depreciation given their perception of Bank behavior, \( \hat{R} \). The interest rate spread on a peso denominated bond of maturity \( \tau \) equals the expected currency depreciation\(^6\):

\[
s(\tau, x_t) = E_t[z_{t+\tau} | \hat{R}]
\]  

(2.5)

**The Bank**

The Bank pegs the exchange rate with a currency board. It chooses whether to continue in the currency board regime, or stop. Exercising the option to float, or extinguishing the option by dollarizing stops the process. Switching regimes is an irreversible decision. The Bank takes the spread (investor behavior) as given when it makes a decision.

**Utility**

**Continuation Region**

The Bank’s flow utility in the currency board regime is,

\[
\pi(x_t) = B - \pi_1 D(y(x_t), A(s(x_t))) - \pi_2 A(s(x_t))
\]  

(2.6)

where \( \pi_1 \) and \( \pi_2 \) are weights. There are three components: the country receives a constant benefit\(^7\) from fixing the exchange rate, \( B \), and it suffers utility losses, \( D(y) \), because shocks to the exchange rate drive output away from potential output and because the currency premium, \( A(s) \), depresses capital accumulation and output.

\( D \) is the disutility from lost potential output,

\[
D(y) = e^{-D_{yT}} - 1
\]  

(2.7)

When output equals potential output (\( y = 0 \)), the disutility is zero. When output is less than potential output the disutility is positive, and when it is greater the disutility is negative (positive utility.) The exponential function weights output losses much more heavily than similar output gains.

\( A \) measures the utility loss from the currency premium,

\[
A(s(x_t)) = \sum_{\tau=t+1}^{\tau} s(\tau, x_t) / (\tau - t)
\]  

(2.8)

where \( s(\tau) \)\(^8\)is the spread on a bond of maturity \( \tau \). The currency premium increases the cost of capital on all assets. I use a linear utility weighting function that puts more weight on short-maturity assets.

---

\(^6\) The Appendix shows the formula for expected depreciation using a finite state Markov chain.

\(^7\) The benefit could be a function of the state variable, see Section 1. Many South Americans believe that the “benefit” of a fixed exchange rate is monetary discipline. They fear a return to the bad old days of hyperinflation under a floating regime. Here \( B \) represents the net benefit between the fixed and floating regimes.

\(^8\) The current spread \( s(\tau, x_t) \) is observable. The Bank conjectures a spread function, \( \hat{s}(\tau, x) \) that holds for all values of \( x \).
Stopping Region

Dollarization

Flow utility in the dollarized regime is,

\[ \omega(x) = \frac{B}{S} - \pi(x)D(y^*) \]  \tag{2.9}

the benefit of the fixed exchange rate, \( B \), discounted for the revenue loss from seigniorage, \( S > 1 \), minus the disutility of real output losses, \( D \). The flow utility in the dollarized regime is similar to the flow utility in the currency board regime, but the country no longer suffers from the direct burden of the currency premium, i.e., \( A(s(x)) = 0 \), or the indirect burden that depresses real output, \( y^*(x) = -b_1 x \geq y(x) \). It, however, loses seigniorage.

Floating

In the floating regime the flow utility is zero. The country ends the suffering from output losses due to nominal exchange rate shocks and it loses the benefit of belonging to a fixed exchange regime. The currency premium is zero since I assume that floating exchange rates follow a random walk.

Bank’s Problem

The Bank solves a stopping problem.

If the Bank dollarizes the terminal payoff is the expected present value of utility in the dollarization regime,

\[ \Omega(x) = \omega(x) + \beta E\Omega(x' | x) \]  \tag{2.10}

where \( x' \) denotes next period’s value, \( x_{t+1} \), and \( \beta \) is a time-discount factor. The Bank weighs the terminal value of dollarization against the value of the option to float,

\[ g(x) = \max \{ 0, \pi(x) + \beta E g(x' | x) \} \]  \tag{2.11}

Here \( \pi(x) + \beta E g(x' | x) \) is the expected present value of utility from continuing in the currency board regime from state \( x \). The option value is non-negative because the Bank can exercise the option and receive the payoff in the floating regime of zero.

Formally the Bank solves the optimal stopping problem,

\[
\begin{align*}
    f(x) &= \max_R [(1 - m(x))g(x) + m(x)\Omega(x)] \\
    &= \max_R [(1 - m(x))[0, \pi(x) + \beta E f(x' | x)] + m(x)\Omega(x)] \\
    R &:= \{ x^f, x_s, x^s, m(x) \} = \arg \max[(1 - m(x))g(x) + m(x)\Omega(x)]
\end{align*}
\]  \tag{2.12}

for the value function \( f(x) \) and switching thresholds, \( x^f \) (float), \( x_s \), \( x^s \) (lower and upper dollarization bounds)\(^9\), and \( m(x) \) the probability of dollarizing.

---

\(^9\) The thresholds that solve the Bank’s problem are unique. Define \( \phi(x) = g(x) - \Omega(x) \), then
Figure 2.1 plots the functions, $g(x)$ and $\Omega(x)$. The functions cross, or are tangent, at the thresholds.

**Figure 2.1 Bank’s Stopping Problem**

In the continuation region the shadow exchange rate is low (currency appreciation or not much depreciation) so the expected present value of continuing is higher,

$$x \leq x_5 \quad f(x) = g(x) > \Omega(x) \quad m(x) = 0,$$

than the value of stopping.

If there is a region where the value of continuing equals the value of dollarizing,

$$x_3 < x \leq x^5 \quad f(x) = g(x) = \Omega(x) \quad 0 < m(x) < 1,$$

then, the bank is indifferent between continuing and dollarizing.

For higher values of the shadow exchange rate the value of continuing is less than the value of dollarizing and the Bank dollarizes with probability one,

$$x^5 < x \leq x^f \quad f(x) = \Omega(x) > g(x) \quad \Omega(x) > 0, \ m(x) = 1.$$

Finally, for large values of the shadow exchange rate—the currency is badly overvalued—the value of continuing or dollarizing is negative so the Bank exercises the option to float,

$$x^f < x \quad f(x) = g(x) = 0, \quad \Omega(x) < 0, \ m(x) = 0.$$

\[\lim_{x \to -\infty} \varphi(x | x) = \frac{\beta}{1-\beta} - \frac{\beta / S}{1-\beta} > 0, \text{ and } \varphi'(x | x = 0) < 0, \text{ there is, at most, a single crossing. If it occurs where } \Omega(x) > 0, \text{ then a dollarization region exists and the solution looks like figure 2.1.}\]
The Bank also chooses the probability of dollarizing, \( m(x) \) in the pure strategy regions, \( g(x) \neq \Omega(x) \). A value of zero, don’t dollarize, or one, dollarize, maximizes the value function (2.12). In the mixed strategy region, \( g(x) = \Omega(x) \), the Bank is indifferent between continuing and dollarizing for any positive probability. In the mixed strategy region the equilibrium conditions determine the probability of dollarizing.

**A Nash Equilibrium**

Define, \( f(x, s(x, \hat{R})) \), \( R \), as the value function and decisions that satisfies the Bank’s maximization problem equation (2.12). And define \( s(x, \hat{R}) \), as the spread function that satisfies the investors’ problem. The Bank’s decisions, \( R \), depend on the spread set by investors. And the spread depends on investors’ perception of Bank behavior, \( \hat{R} \). In a Nash equilibrium investors’ perceptions are correct,

\[
\hat{R} = R \\
s(\tau, x, \hat{R}) = E(z_{\tau+t} \mid R)
\]

**Mixed Strategy Equilibrium**

If a dollarization region exists, then no pure strategy Nash equilibrium exists. The equilibrium is a mixed strategy equilibrium. To illustrate, consider a simple case where the state variable only takes on three values, \( x \in \{ x_c, x^\$, x^f \} \), continue, dollarize, or float. Assume a pure strategy equilibrium and suppose the realization is dollarization. In a rational expectations equilibrium investors charge no currency premium in the dollarization region because dollarization locks in the fixed rate forever so there is no possibility of a loss from devaluation. But if investors charge no premium, then the Bank will not dollarize. The value of continuing is greater than the termination value from dollarizing. But, if the Bank continues, then this region is not a dollarization region.\(^{10}\) A pure strategy equilibrium does not exist, but a mixed-strategy equilibrium exists. The Bank dollarizes with probably \( m \) and continues with probability \( 1-m \). Investors charge a premium based on the probability of continuing. The equilibrium condition requires that the premium is just enough so that the Bank is indifferent between continuing and dollarizing.

**Section 3: Numerical Solutions**

This section examines the choice of the exchange rate regime as a function of the distribution of the exogenous shocks. Mundell argued that a fixed exchange rate led to more stable income if nominal shocks were relatively larger than real shocks. This section can be thought of as the new millennium version of the traditional Mundell’s experiment. All the shocks are nominal, but they have temporary real effects. I calculate the regions and the transition probabilities for moving from the currency board regime to a dollarization or floating regime. Large shocks (high volatility) make it more likely that the country will end up in a floating regime and small shocks make it more likely the country will end up dollarizing.

The system has no closed form solution. I solve it numerically. The Appendix gives the algorithm.

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\(^{10}\) I discovered that no pure strategy equilibrium existed when my numerical algorithm would not converge. Steve Goldman finally convinced me that it wouldn’t converge because the equilibrium was a mixed-strategy equilibrium.
Parameterization

The Exogenous Shocks

I model the shocks as a mixture of normals,

\[ u_{t+1} = \sigma(qw_{t+1} + (1-q)k_{t+1}) \]

\[ \begin{bmatrix} w \\ k \end{bmatrix} \sim N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 10^2 \end{bmatrix} \right) \]  

(3.1)

p = prob(q = 0)

I set the volatility of the “jump” process, k, at ten times the volatility of the usual shocks, w. \( \sigma \) is a scale factor. The jump distribution represents large “asymmetric” shocks.

The key parameter is p, the probability of a draw from the jump distribution. I vary p from 1 ½% to 3%. Loosely speaking p =1 ½% means that on average the country suffers from a shock 10 times the normal magnitude once in 17 years. Doubling the probability to 3% means on average it suffers from a shock 10 times the normal magnitude once in 9 years.

Adjustment

\[ \Delta x_{t+1} = -.08x_t + u_{t+1} \]  

(3.2)

I chose the mean-reversion coefficient, 0.8, from the quarterly output adjustment equation in Section 2, footnote 2.

Output Gap

\[ y = -b_1x - b_2A(s(x)) = -0.1x - 0.05A(s(x)) \]  

(3.3)

Tastes

The key taste parameter weights the disutility from output losses 100 times the loss from the currency premium. Section 3.2 shows the flow utility in each regime as a function of the state variable.

<table>
<thead>
<tr>
<th>( \pi_1 )</th>
<th>( \pi_2 )</th>
<th>D_1</th>
<th>B</th>
<th>S</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>0.05</td>
<td>0.65</td>
<td>1.5</td>
<td>1.1</td>
<td>0.975</td>
</tr>
</tbody>
</table>

3.1 Equilibria

The three equilibria are presented in three sets of Figures. The top panel of the figures labeled a plot the functions, \( g(x) > 0 \), the value of the option to float, and, \( \Omega(x) > 0 \), the value of stopping. These figures are similar to Figure 2.1 that shows the Bank’s decisions. The lower panel shows the spread on a one-year bond. The

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11 Asymmetric is not precisely defined in the exchange rate literature. It seems to mean the shock is country specific and does not affect the country the currency is pegged to. This is exactly what I have in mind.
separate figures labeled \( b \) show the cumulative probability of moving from the currency board regime to a dollarized regime or a floating regime in one year, in two years, out to eight years.

### 3.1.1 Low Jump Probability \( (p = 0.015) \)

In the low volatility regime dollarization is almost inevitable. The dollarization region is large. The probability of dollarization given a realization in the dollarization region is large. And the probability of floating is small.

Figure 3.1.1a shows the low volatility equilibrium.

**Figure 3.1.1a**
The state space is divided into three regions: continue, dollarize, and float. A realization of the shadow exchange rate, \( x \), in a region triggers an action. For realizations where the currency appreciates by 20% or more, the Bank always continues. Realizations in the 15% depreciation to 20% appreciation — a huge interval — trigger a mixed strategy. In this region the Bank dollarizes with probability 85\%.\(^{12}\) In the mixed strategy dollarization region investors charge a currency premium — there is a 15% probability that the Bank will not dollarize if it lands in this region. If it does not dollarize, then the next realization could be in the floating region and investors will lose. For realizations that depreciate the currency by 15\% to 24\% the Bank dollarizes with probability one. The bottom panel shows that there is no spread for this region.

Investors know the Bank will dollarize — so there is no potential loss from devaluation. The region to the far right is the floating region — a realization that depreciates the shadow exchange rate by more than 24\% triggers floating. The top panel shows that the option, \( f(x) \), has no value in this region (it is exercised) and the bottom panel shows the spread is zero. There is no spread in the floating regime. The bottom panel shows the spread for the continuation region. Realizations of the shadow exchange rate near the floating region result in a relatively high spread — up to 8\%—because the conditional probability of landing in the floating region is higher. The average spread of 1.8\% is less than the spread in the Argentine data.

Figure 3.1.1b shows the probability of landing in a stopping region—dollarization, or floating—in one year, two years, …conditional on starting at central parity, \( Z_0 \).

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\(^{12}\) In fact there should be a state dependent probability that increases from \( m(x) = 0 \) at the lower threshold, \( x_s \), to one at the upper boundary. So far I have not been able to solve for a state dependent probability. I restrict the choice to a single probability and use the condition that the average value of continuing, \( g(x) \), equals the average value of dollarizing, \( \Omega(x) \), in this region. See the Appendix.
The probability of dollarization is very high. In a year the probability is over 95% and in seven years almost 98%. And the probability of floating is only about 2%.

### 3.1.2 Moderate Jump Probability (p = 0.02)

In the moderate volatility regime dollarization is likely, but not inevitable. The continuation and floating regions are large. After a year the probability that the country remains in the currency board regime is almost 50%.

Figure 3.1.2a shows a moderate volatility equilibrium. The layout is the same as in Figure 3.1.1a. Notice the dollarization region is much smaller and the spread much larger.

**Figure 3.1.2a**
A realization of the shadow exchange rate that depreciates the currency by 22% or more triggers floating. The floating region in this equilibrium is slightly larger—it starts at 22% versus 24% in the low volatility equilibrium. (But the probability of jumping into this region is considerably higher, see Figure 3.1.2b). The most striking result is that the dollarization region is much smaller. For realizations that depreciate the currency by 15% to 22% the Bank dollarizes with probability one. For realizations that depreciate the currency from 8% to 15% the Bank dollarizes with probability 0.85. For realizations that depreciate the currency by less than 8% the Bank continues in the currency board regime. The threshold for the continuation region dropped from 20% currency appreciation in the low jump probability equilibrium to 8% depreciation in moderate jump probability equilibrium. The option to float is much more valuable in the moderate jump probability regime.

Figure 3.1.2b shows the probability of dollarizing or floating.

**Figure 3.1.2b**

The spread in the bottom panel of 3.1.2a reflects the significant probability of floating. The spread averages 3.2% which is ½% higher than the average spread in the Argentine data.
3.1.3 High Jump Probability \((p = 0.03)\)

In the high volatility regime floating is likely. And dollarization is not a policy alternative. The option to float is too valuable to extinguish.

Figure 3.1.3a shows a high volatility equilibrium. There is no dollarization region.

![Figure 3.1.3a](image)

The floating region is larger. A realization of the shadow exchange rate that depreciates the currency by 14\% or more triggers floating. For realizations that depreciate the currency by less than 14\% the Bank continues to peg the exchange rate with a currency board.

Figure 3.1.3b shows the probability of floating.
Figure 3.1.3b

Now the probability of floating soon is high. After a year the probability is 30% and after seven years it is 85%.

The spread in the bottom panel of 3.1.3b reflects the high probability of floating soon. It peaks at almost 10% which less than the spread in Argentina during the Mexican crisis. It averages 5.2% which is almost twice the average Argentine spread.

3.2: Tastes

The thresholds obviously depend on the Bank’s (country’s) tastes. The basic tradeoff is between the pain of lost output and the cost of the currency premium.

In the continuation region the Bank suffers from lost potential output and from paying the cost of the currency premium,
\[ \pi(x_t) = B - \pi_1(e^{-D_{1y}} - 1) - \pi_2 A(s(x_t \mid \hat{x}^f, \hat{x}^S, \hat{m}(x))) \]

\[ y_t = -b_1 x_t - b_2 A(s(x_t)) \] (3.4)

If a shock pushes the Bank into the floating region, then it floats and receives no benefit and suffers no losses (from shocks to the exchange rate), \( \pi(x_t) = 0, x_t \geq x^f \).

If it dollarizes it saves the cost of the currency premium, but it loses seigniorage revenue and it still suffers output losses for all realizations of the state variable,

\[ \pi(x_t) = B / S - \pi_1(e^{-D_{3y}} - 1) \]

\[ y^d_t = -b_1 x_t \] (3.5)

Figure 3.3 shows the flow disutility if the bank continues, or flows, or if it dollarizes.

**Figure 3.3**

Tastes

The flow disutility in the continuation region is greater than the flow disutility in the dollarization regime up to the floating threshold. If the Bank keeps the option to float open (continuation), then a realization \( x \) in
the floating region triggers the floating regime and flow disutility drops to zero\textsuperscript{13}. On the other hand, if the Bank dollarizes it gives up the option to float and bad realizations of the shadow exchange rate lead to large output losses and lots of pain. The exponential function puts a very heavy weight on large output losses.

**Section 4: Conclusions**

Dollarization is an irreversible decision. Pegging the exchange rate with a currency board keeps the option to float alive. Dollarization extinguishes the option. The main contribution of this paper is to explicitly model the choice of exchange regimes as an irreversible decision.

The value of the option to float depends on the stochastic process for the driving state variable. I modeled the driving process as jump-diffusion. The larger the jump probability the less likely the country will choose dollarization as an exchange regime.

The paper is abstract, but it has a simple practical message. When examining dollarization as a potential exchange rate regime the process of shocks that hits the system is crucial. In some sense this is the same message that Mundell gave for the traditional choice between fixed and flexible exchange rates thirty years ago. But irreversibility makes it even more important. The European Currency Union set strict standards for admission so that countries that were economically similar and geographically close could tie their currencies to a common standard. These might be interpreted as preconditions to reduce the volatility of the shock process.

\textsuperscript{13} Figure 3.3 is a continuous plot of a discrete state space. The flow utility should drop immediately to zero at the floating threshold. I used a coarser grid for this plot.
References


Calvo, Guillermo (1991), The Perils of Sterilization, IMF Staff Papers, 38, 921-926.


Goldfajn, Ilan and Gino Olivares (2000), Is Adoption of Full Dollarization the Solution? Looking at the Evidence, Department of Economics, Pontificia Universidade Catolica do Rio de Janeiro.


Appendix A: Algorithm

This appendix gives the algorithm I used to solve for the equilibria. I approximate the continuous state-space with a finite state-space\(^{14}\) and I iterate between solving the Bank’s problem and solving the investors’ problem until I find a fixed point. All the programs are in MATLAB.

Environment:

Let, \(x\), denote the finite vector of states and \(P\) the transition matrix between states. Define,

\[
x := \{x_{-N}, x_{-N+1}, \ldots x_0, \ldots x_{N-1}, x_N\}
\]

\[
P := \{P_{ij} \equiv \text{prob}(x' = x_j | x = x_i)\}
\]

Tauchen (1986) shows how to approximate a continuous state-space autoregressive process with a finite state Markov process.

Initialization

Form the column vectors,

\[
y^8(x) := \{y(x_j) = -b_t x_j\}
\]

\[
\omega(x) = B / S - \pi D(y^8)
\]

Calculate the vector of expected present values from dollarization,

\[
\Omega(x) = [I - \beta P]^{-1} \omega(x)
\]

Bank’s Problem:

Solve the stopping problem,

\[
(1.1) \ f(x) = \max\{(1 - m(x)) g(x) + m(x) \Omega(x)\}
\]

Here the \(f(x)\) and \(g(x)\) are unknown vectors, and \(m(x)\) is the state dependent probability of dollarizing.\(^{15}\)

Initialization

Calculate the flow utility vector \(\pi(x)\).

The utility cost of the spread is,

\[
A(s(x)) = \sum_{\tau=1}^{T} s(\tau, x) / \tau
\]

Where, the Bank takes the spread,

---

\(^{14}\) Judd (1998) and Ljungqvist and Sargent (2000) have excellent discussions of finite state approximation techniques.

\(^{15}\) The multiplication \((1 - m(x_j)) \ast g(x_j)\) is element by element.
\[ s(\tau, x) = E[z_{\tau+t} | \tilde{R}] \]

set by investors as given.

Now form the state dependent output and flow utility vectors,

\[ y(x) \equiv \{ y(x_j) = -b_1 x_j - b_2 A(s(x_j)) \} \]  
\[ \pi(x) = B - \pi_1 D(y) - \pi_2 A(s(x)) \]

Find the function \( g(x) \)

To calculate the value of the option to float partitioned the state vector \( x \) and the transition matrix, \( P \), into a continuation region \( (c) \) and a floating region \( (f) \) and make the floating (stopping) region absorbing,

\[
X = \begin{bmatrix} x^c \\ x^f \end{bmatrix}
\]

\[
P_B \equiv \begin{bmatrix} P_{cc} & P_{cf} \\ 0 & I \end{bmatrix}
\]

And define a conformable flow utility vector, \( \nu \), that reflects the fact that flow utility is zero in the floating region,

\[
\nu = \begin{bmatrix} \pi_c \\ 0 \end{bmatrix}
\]

To find the function \( g \) I use the fact that the option value function is monotonically decreasing to search efficiently. I search down the state space by increasing the floating region, i.e., \( x^f = \{ x_N, \ldots, x_{N-1}, \ldots \} \), until the value of the option to float is positive in all the continuation states,

\[ (1.4) \quad g(x) = [I - \beta Q]^{-1} \nu \geq 0 \]

And zero in the stopping region.

Check for a dollarization region,

If \( g(x) > \Omega(x) \) then, there is no dollarization region and \( f(x) = g(x) \)

If \( g(x) < \Omega(x) \) then, there is no equilibrium

If \( g(x_i) > \Omega(x_k) \quad i < k \)

and \( g(x_j) < \Omega(x_j) \quad j > k \)

then, look for a dollarization region.
Choose lower and upper thresholds\(^\text{16}\)
\[x_{d-} < x^{k-d} \text{ and } x_{d+} > x^{k+d} \]
so that,
\[
g(x_j) = \begin{cases} \Omega(x_j) & j = k-d \\ \Omega(x_j) & j = k+d \end{cases}
\]
This gives a region of indifference. Find the mixed strategy dollarization region, \(x_{ms}^*: = (x_{k-d}, x_{k+d})\), that maximizes \(f(x)\).

Notice this approximation will not converge to the optimal policy in Section 2 as the grid for the finite state space gets large. The Bank should be indifferent in each state, \(g(x_j) = \Omega(x_j)\) in the region. I have not been able to solve this problem.\(^\text{17}\)

Choose a probability of dollarizing, \(m\), in the mixed strategy region.

**Investors’ Problem: Choose the Spread**

The investors problem is to set the spread,
\[s(\tau, x \mid \hat{R}) = E(z_{\tau} \mid \hat{R})\]
equal to expected depreciation given their conjecture about bank behavior.

Initialization

Let,
\[
\begin{align*}
x^c \\
x_{ms}^c \\
x^s \\
x^f \\
m
\end{align*}
\]
represent the state vector that comes out of the Bank’s maximization problem. Choose \([x, m]’\) as the investors’ conjecture.

Investors faced an expanded the probability space because the Bank’s mixed strategy adds endogenous uncertainty. Double the mixed strategy region,
\[
x := \{ x^c, x^{1-m} \}, \quad x_{ms} = x_{ms}^c \]

to account for the fact that a realization \(x_i \in x_{ms}^c\) results in dollarization with probability \(m\) and continuation with probability \(1-m\).

\(^{16}\) The region is not necessarily symmetric, \(d^- \neq d^+\).

\(^{17}\) The function \(g(x)\) depends on the spread, \(s(x|m(x),..)\), and the spread depends on the probability of dollarizing. I need to find the fixed point of the Bank’s and investors’ problems. Allowing a separate probability \(m(x)\) for each state in the dollarization region increases the dimension of the problem beyond what can handle.
Define the companion transition matrix,

\[
P_t = \begin{bmatrix}
P_{cc} & P_{c\rightarrow m} & P_{cm} & P_{c\rightarrow s} & P_{cf} \\
P_{1\rightarrow m&c} & P_{1\rightarrow m\&1\rightarrow m} & P_{1\rightarrow m\&m} & P_{1\rightarrow m\&s} & P_{1\rightarrow m\&f} \\
0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & I
\end{bmatrix}
\]

(1.7)

The transition probability for the first row show the probability of moving from the currency board region (c) to another region. For transitions to the mixed strategy region (ms) the transition probabilities equal the probability of moving to the mixed strategy region times the probability of continuing or dollarization,

\[
P_{c\rightarrow m} = (1 - m)P_{cm}
\]

\[
P_{cm} = mP_{ms}
\]

The second rows reflects the fact that if the Bank continues when there is a realization in the dollarization region, then transition to any other state occurs with some probability. The remaining rows are absorbing states.

Use the transition matrix to compute expected change in the official exchange rate (currency depreciation) next period,

\[
Ez^\tau = P_t z_0 =
\]

(1.8)

\[
\begin{bmatrix}
Ex^{cf} \\
Ex^{1\rightarrow m&f} \\
Ex^f
\end{bmatrix}
= \begin{bmatrix}
P_{cc} & P_{c\rightarrow m} & P_{cm} & P_{c\rightarrow s} & P_{cf} \\
P_{1\rightarrow m&c} & P_{1\rightarrow m\&1\rightarrow m} & P_{1\rightarrow m\&m} & P_{1\rightarrow m\&s} & P_{1\rightarrow m\&f} \\
0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
x^f
\end{bmatrix}
\]

For example starting from a state in the currency board regime (c) devaluation only occurs when the next realization is in the floating region. Expected depreciation is the probability of a realization in floating region times the realization, \(P_{cf}x^f\). Dollarization and floating are absorbing regions, if the country starts in an absorbing state it stays there.

Using the Markov chain property, the expected change in the official exchange rate \(\tau\) periods in the future is,

\[
Ez^\tau = (P^\tau I + P^\tau - I + ..P_I)z_0 - (\tau - 1)z_0
\]
I search for equilibria by iterating between the Bank’s and the investors’ problems until I find a fixed point. I do the search in two stages. In stage 1 I search for the floating region by iterating between equation 1.4 (the Bank’s problem) and equation 1.8 (the investors’ problem). When there is no dollarization region the state vector and transition matrix in the investors’ problem collapses to the state vector and transition matrix in the Bank’s problem, equation 1.3. The search is systematic and easy. In stage 2 I look for the dollarization region and mixed strategy equilibrium. First I choose a region and then iterate between the Bank’s problem and the investors problem varying the probability of dollarization on a grid from zero to one. If there is a fixed point, I save it. If not I change region. When I find a solution I save it and I reset the region. I repeat the process until I find the region, $x^{m}$, and probability, $m$, that maximizes $f(x)$. Then I iterate between stage 1 and stage 2 until I find a fixed point.

$$s(\tau, x) = \sum_{j=1}^{\tau} E z^{u(j)}$$

**Equilibrium**

I search for equilibria by iterating between the Bank’s and the investors’ problems until I find a fixed point. I do the search in two stages. In stage 1 I search for the floating region by iterating between equation 1.4 (the Bank’s problem) and equation 1.8 (the investors’ problem). When there is no dollarization region the state vector and transition matrix in the investors’ problem collapses to the state vector and transition matrix in the Bank’s problem, equation 1.3. The search is systematic and easy. In stage 2 I look for the dollarization region and mixed strategy equilibrium. First I choose a region and then iterate between the Bank’s problem and the investors problem varying the probability of dollarization on a grid from zero to one. If there is a fixed point, I save it. If not I change region. When I find a solution I save it and I reset the region. I repeat the process until I find the region, $x^{m}$, and probability, $m$, that maximizes $f(x)$. Then I iterate between stage 1 and stage 2 until I find a fixed point.