Lawrence Berkeley National Laboratory
Recent Work

Title
SPIN DETERMINATION FOR BOSON RESONANCES: A SUPPLEMENTARY NOTE

Permalink
https://escholarship.org/uc/item/40n574kw

Author
Chung, Suh Urk.

Publication Date
1965-07-13
SPIN DETERMINATION FOR BOSON RESONANCES:
A SUPPLEMENTARY NOTE

Berkeley, California
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
SPIN DETERMINATION FOR BOSON RESONANCES: A SUPPLEMENTARY NOTE

Suh Urk Chung

July 13, 1965
SPIN DETERMINATION FOR BOSON RESONANCES:
A SUPPLEMENTARY NOTE

Suh Urk Chung

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 13, 1965

I. INTRODUCTION

The purpose of this note is to clarify a few points on the author's paper "Spin Determination for Boson Resonances" and to write the spin formula in a way that is more convenient for practical applications. The simplest of the spin formulae is expressed directly in terms of experimental quantities, and details are given for obtaining these quantities.

II. DEFINITION OF EXPERIMENTAL QUANTITIES

Suppose a boson resonance of spin $S$ is produced and decays as follows:

$$\pi + N \rightarrow B(\text{spin } S) + N'$$

$$B \rightarrow V(\text{spin } 1) + P(\text{spin } 0),$$

with the $V$ subsequently decaying in either of two channels,

$$V \rightarrow P_1 + P_2 + P_3$$

or

$$V \rightarrow P_1 + P_2$$

Here $P_1$, $P_2$, and $P_3$ are spin-0 bosons. The above symbols are used also for the momenta of the respective particles.

In the $B$ rest frame (BRF), one may define a coordinate system where the $z$ axis is parallel to the beam direction (along $\pi$) and the $y$ axis parallel to the production normal $Q = N' \times \pi$. The momentum $V$ makes angles $(\theta, \phi)$ with respect to this coordinate system as shown in Fig. 1. It is convenient to introduce another vector $R = \pi \times V$, which lies in the $x$-$y$ plane. Note that with the chosen coordinate system, $\phi$ is just the Treiman-Yang angle.
Using $R$ as the axis of rotation, one rotates the above coordinate system through $\theta$ and then goes to the $V$ rest frame (VRF) by a Lorentz transformation. The decay of $V$ is characterized by a vector $\vec{n}$ which makes angles $(\theta^i, \phi^i)$ with respect to this transformed coordinate system as shown in Fig. 2.

Fig. 1. Rest frame of $B$.

Fig. 2. Rest frame of $V$. 
Thus, if $V$ is a vector meson, $n$ is proportional to $\mathbf{P}_1 \times \mathbf{P}_2$ for channel (2a), whereas for (2b) $n$ is proportional to $\mathbf{P}_1 - \mathbf{P}_2$.

Using Figs. 1 and 2, one can easily get

$$a = \sin \theta \cos \phi = \mathbf{Q} \times \pi \cdot \mathbf{V} \quad a^I = \sin \theta^I \cos (\phi^I - \phi) = \mathbf{R} \times \mathbf{V} \cdot \mathbf{n}$$

$$b = \sin \theta \sin \phi = \mathbf{Q} \cdot \mathbf{V} \quad b^I = \sin \theta^I \sin (\phi^I - \phi) = \mathbf{R} \cdot \mathbf{n}$$

$$c = \cos \theta = \mathbf{Q} \cdot \mathbf{V} \quad c^I = \cos \theta^I = \mathbf{V} \cdot \mathbf{n},$$

where $a$, $b$, $c$, $a^I$, $b^I$, and $c^I$ are the experimental quantities. Note that vectors $\pi$, $V$, $Q$, and $R$ are defined in the BRF, while $n$ is defined in the VRF. The six quantities defined above are those most easily evaluated from experiment; one simply begins with relevant vectors defined in any arbitrary frame, transforms them to the BRF by the Lorentz transformation and forms the vectors $\pi$, $V$, $Q$, and $R$. Then one applies the Lorentz transformation once more to vectors $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ to get to the VRF, where one forms the vector $n$.

### III. SPIN FORMULAE

It is advantageous to rearrange formula (23) of Ref. (1) as

$$\epsilon_0 \epsilon_1 \epsilon S(-)^{S+1} \frac{S(S+1)}{L(L+1)-2S(S+1)} = \frac{[3(L-1)(L+2)]^{1/2}}{2 L(L+1)} \frac{G(22 LM)}{\sqrt{2} G(00 LM) + G(20 LM)} \quad \text{(even } L), \quad (4)$$

where $S \geq 1$ and $L \geq 2$, and $\epsilon_0$, $\epsilon_1$, and $\epsilon_S$ are intrinsic parities of the spin-0, spin-1, and spin-$S$ bosons, respectively. Note that the notation $\epsilon$ as defined in Ref. (1) is related to the intrinsic parities by the formula $\epsilon = \epsilon_0 \epsilon_1 \epsilon S(-)^{S+1}$. As shown in (A.12) of Ref. (1), $G(lm LM)$ can be evaluated by using

$$G(00 LM) = \left( \frac{3 \pi \omega_s}{3} \right)^{1/2} \left\langle Y_L^M (\theta, \phi) \right\rangle$$

and

$$G(2m LM) = -\left( \frac{10 \pi}{3} \right)^{1/2} \cdot (2L+1)^{1/2} \left\langle Y_2^m (\theta^I, \phi^I) \mathcal{O}_{mm}^{(L)}* \mathcal{O}_{mm} \ (\phi, \theta, -\phi) \right\rangle.$$

The conventions used for $Y_2^m$ and $\mathcal{O}_{mm}^{(L)}$ are those of Rose. Angles $\theta, \phi, \theta^I,$ and $\phi^I$ are defined in Sec. II.

The simplest formula results if one specializes to $L = 2$ and $M = 0$ in (4). Using (5), one can easily obtain

$$\epsilon_0 \epsilon_1 \epsilon S(-)^S \frac{S(S+1)}{5 - S(S+1)} = \frac{5 \sin^2 \theta \sin^2 \theta^I \cos 2(\phi^I - \phi)}{(3 \cos^2 \theta - 1)(3 - 5 \cos^2 \theta^I)} \quad \text{(6)}.$$
In this form, the similarity of this formula to that of Berman and Jacob\textsuperscript{7} is apparent. In fact, relation (6) can be derived in a straightforward way by using their formalism.

For \( L = 2 \) and \( M = 2 \), one sets in a similar way

\[
\epsilon_0 \epsilon_1 \sin[(-)S] \frac{S(S+1)}{3-S(S+1)} = \frac{5 \langle (1+\cos \theta)^2 \sin^2 \theta \, e^{2i\phi'} \rangle}{3 \langle \sin^2 \theta \, e^{2i\phi(3 - 5 \cos^2 \theta')} \rangle},
\]

and for \( L = 2 \) and \( M \leq 1 \),

\[
\epsilon_0 \epsilon_1 \sin[(-)S] \frac{S(S+1)}{3-S(S+1)} = \frac{-5 \langle (1+\cos \theta) \sin \theta \, \sin^2 \theta \, e^{-i(\phi-2\phi')} \rangle}{3 \langle \cos \theta \, \sin \theta \, e^{i\phi(3 - 5 \cos^2 \theta')} \rangle}.
\]

Formulae (7) and (8) are true both for real and imaginary parts separately.

One can write similar relations using negative values of \( M \), but they are related to positive values of \( M \) through Eq. (20b) of Ref. (1). One may use them, however, as a consistency check. For higher values of \( L \) (i.e., 4, 6, 8, etc.), it is perhaps easier to employ (1) and (2) directly by using general formulae for \( Y^m_l \) and \( X^m_m \). Finally, it is to be emphasized that formulae (6), (7), and (8) are true only for \( S \geq 1 \). For \( S = 0 \), the averages appearing in both the numerator and the denominator of these relations should be equal to zero separately.

A more convenient way of writing (6), (7), and (8) would be to express them directly in terms of the experimental quantities defined in (3). From (6), one gets

\[
\epsilon_0 \epsilon_1 \sin[(-)S] \frac{S(S+1)}{3-S(S+1)} = \frac{5 \langle (1 - c^2) (a^1b^2 - b^1a^2) \rangle}{\langle (3c^2 - 1)(3 - 5c^1c^2) \rangle},
\]

while (7) gives

\[
\epsilon_0 \epsilon_1 \sin[(-)S] \frac{S(S+1)}{3 - S(S+1)} = \frac{5 \langle \left( \frac{1+c}{1-c} \right) \left[ (a^2-b^2) (a^1b^2 - b^1a^2) - 4ab a'^1 b' \right] \rangle}{3 \left\langle (a^2-b^2)(3 - 5c^1c^2) \right\rangle},
\]

and from (8),

\[
\epsilon_0 \epsilon_1 \sin[(-)S] \frac{S(S+1)}{3 - S(S+1)} = \frac{-5 \langle (1+c) \left[ a(b^2 - b'^2) + (a^2 - b^2) a' b' \right] \rangle}{3 \left\langle ab (3 - 5c^1c^2) \right\rangle},
\]

and

\[
\epsilon_0 \epsilon_1 \sin[(-)S] \frac{S(S+1)}{3 - S(S+1)} = \frac{-5 \langle (1+c) \left[ 2a a'^1 b' + b(a^1 - b'^2) \right] \rangle}{3 \left\langle bc (3 - 5c^1c^2) \right\rangle}.
\]
Note that formulae (10) and (11) should be treated with care, because the numerators can become indeterminate for \( C = 1 \). \(^8\)

Relations (9) through (13) can be used to test both the spin and parity of \( B \). \(^4\) However, for testing parity alone, additional tests can be made, namely, relations (24a) and (24b) of Ref. (1). Again, they are written explicitly for \( L = 0 \) and \( L = 2 \).

For \( \epsilon_S = \epsilon_0 \epsilon_1 (-)^S \), the following averages should equal zero. \(^9\) Using (24b) of Ref. (1), one gets for \( L = M = 0 \) (\( S \geq 1 \)),

\[
\langle 5 \cos^2 \theta^* - 1 \rangle = 0 .
\] (14)

Using the same formula for \( L = 2 \) (\( S \geq 1 \)), one has

\[
\langle (3 \cos^2 \theta - 1) (5 \cos^2 \theta^* - 1) \rangle = 0 \\
\langle \sin^2 \theta \ e^{2i\phi} (5 \cos^2 \theta^* - 1) \rangle = 0 \\
\langle \sin \theta \ \cos \theta \ e^{i\phi} (5 \cos^2 \theta^* - 1) \rangle = 0 .
\] (15)

One may also write (24a) of Ref. (1) for \( L = 2 \) (\( S \geq 1 \)) as

\[
\langle \sin \theta \ \cos \theta \ \sin \theta^* \ \cos \theta^* \ e^{i(\phi^* - \phi)} \rangle = 0 \\
\langle (1 + \cos \theta) \ \sin \theta \ \sin \theta^* \ \cos \theta^* \ e^{i(\phi^* + \phi)} \rangle = 0 \\
\langle (1 + \cos \theta) (2 \cos \theta - 1) \ \sin \theta^* \ \cos \theta^* \ e^{i\phi^*} \rangle = 0 .
\] (16)

Again, it is to be noted that some care needs to be taken for certain averages in (14), (15), and (16) because of the possibility of becoming indeterminate when they are expressed in terms of experimental quantities in (3).

Finally, the analysis presented here and in Ref. (1) must be modified when Bose Symmetrization is required. It can be applied without modification, however, to the decay \( B \to \omega + \pi \), since the interference due to Bose Symmetrization is not very important because of the narrow width of the \( \omega \).
ACKNOWLEDGMENTS

The author wishes to express his appreciation to Professor D. H. Miller, Dr. Janos Kirz, and R. I. Hess for their interest and help. He also wishes to thank Professor Luis W. Alvarez for his encouragement and support.

This work was done under the auspices of the U.S. Atomic Energy Commission.
FOOTNOTES AND REFERENCES


2. This corresponds to a rotation of the coordinate system by Euler angles \((a, \beta, \gamma) = (\phi, \theta, -\phi)\). See page 50 of reference (5).

3. The Lorentz transformation under discussion is given below. Let \(\beta\) be the velocity of transformation and \(\gamma = 1/(1 - |\beta|^2)^{1/2}\). Then, the 4-vector \(P = (P, E)\) transforms as follows:

\[
P' = P + \left[(\gamma - 1) \frac{P \cdot \beta}{|\beta|^2} - \gamma E\right] \beta
\]

\[
E' = \gamma(E - \frac{P \cdot \beta}{\gamma})
\]

where \(P' = (P', E')\) is the transformed 4-vector.

4. Note that in this particular form both the spin and parity can be determined at the same time except for special cases. For example, one cannot distinguish between \(S = 1\) and \(S = 2\) when using \(L = 2\). This ambiguity is resolved in the following ways. First, one may employ the parity tests given in (14), (15), and (16). A second way is to note that \(G(l m LM)\) is not zero in general for \(L = 4\) if the spin is two, whereas for \(S = 1\) it should be identically zero. In addition, one can use formula (4) for \(L = 4\) if the spin is two.


6. It is rather amusing to see that the relation (6) can be checked analytically for a simple case. Suppose a boson of \(S = 1\) decays into a vector and a pseudoscalar meson (i.e., \(\varepsilon_0 = \varepsilon_1 = \varepsilon_S = -1\)). Then, the left side of (6) is equal to +2. To calculate the right side, one needs to know the angular distribution, the simplest of which can be written

\[
I(\theta, \phi, \theta', \phi') \sin^2 \theta' \left[ \cos \phi \cos (\phi' - \phi) - \cos \theta \sin \phi \sin (\phi' - \phi) \right]^2.
\]

Using this, one can show after a simple integration that the right side of (6) is also +2. Q.E.D.

7. See formula (2.12) of S. M. Berman and M. Jacob in Spin and Parity Analysis in Two-Step Decay Processes, Stanford Linear Accelerator Report SLAC-43, 1965, Stanford University, Stanford, California. See also their appendix for the explicit form of \(G(l)\) for smaller values of \(l\).

8. For \(C \approx 1\), one has \(\theta \approx 0\). By examining (6), (7), and (8), one can see easily how to treat the particular event for which \(C \approx 1\). For instance, in case of (6), the event does not contribute to the numerator, while its contribution to the denominator is \(2(3 - 5C^2)\).
9. Relation (14) is the same as that derived in reference (7). For 
\[ \epsilon_S = \epsilon_0 \epsilon_4 (-)^{S+1} \]
the angular distribution for the decay, i.e., \( S - 1 \) and \( S + 1 \), and correspondingly two amplitudes \( a_- \) and \( a_+ \). In this case, the average in (14) is zero only if \( a_- \) and \( a_+ \) are relatively real and one has [See Eq. (11), Ref. (1)]
\[ \frac{a_+}{a_-} = \left( \frac{S}{S+1} \right)^{1/2}. \]

Presumably, the lower angular-momentum state dominates the decay, and the ratio \( \frac{a_+}{a_-} \) is a small number, while \( \left[ \frac{S}{S+1} \right]^{1/2} \) is greater than \( (2)^{-1/2} \). So, one may conclude that relation (14) is not likely to hold for the parity case \( \epsilon_S = \epsilon_0 \epsilon_4 (-)^{S+1} \).

The same remark applies to averages in (15) and (16), except that here averages depend on the production variable, which may cause them to become zero accidentally for the case \( \epsilon_S = \epsilon_0 \epsilon_4 (-)^{S+1} \). However, it is unlikely that all averages in (15) and (16) become zero simultaneously by accident.

Note added in proof:
Relation (14) results from the fact that the angular distribution in \( \theta' \) for \( \epsilon_S = \epsilon_0 \epsilon_4 (-)^{S} \) is just \( \sin^2 \theta' \) i.e.
\[ \int I(\theta, \phi, \theta', \phi') \, d\phi' \, d\cos \theta \, d\phi \sim \sin^2 \theta' \]

Relations (15) are not redundant; they tell us that \( \sin^2 \theta' \) occurs as an overall factor in the angular distribution \( I(\theta, \phi, \theta', \phi') \). In other words, one must have for \( \epsilon_S = \epsilon_0 \epsilon_4 (-)^{S} \)
\[ I(\theta, \phi, \theta', \phi) \sim \sin^2 \theta' f(\phi, \theta, \phi), \]
where \( f(\phi', \theta, \phi) \) is some function appropriate to a given spin and parity.
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.