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PION PRODUCTION WITH HEAVY IONS*

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INTRODUCTION

A major contemporary effort in nuclear physics involves the investigation of nuclear collisions of a wide variety of projectiles (e.g., p, d, α, C) and targets (p, d, ... U) combinations in the energy range from ~500 MeV to ~10 GeV per projectile nucleon. As might be expected, the final states in these reactions are very complicated and have been observed to contain a spectrum of particles, e.g., π, p, d, t, α, ... Though complicated, we now have some knowledge of the characteristic distributions of particles and momenta in these final states — but little information on the correlations of different particles.

Faced with this incredibly multi-dimensional problem which defies exact quantum mechanical treatment, is it reasonable to expect that the study of just one facet — the production of pions, say — might be any more illuminating than the study of any other particles? To appreciate the essence of the moderately affirmative answer to this question (which we outline in this comment) we should recall that the real theoretical solution to our problem is an impossibly hard admixture of many-body physics, relativistic field theory and nuclear physics. Even the much less complicated A(p,π)B* problem, extensively studied at the meson factories, contains many ambiguities and is without any solid theoretical framework. As a consequence of this difficulty, the interpretation of the inclusive spectra presently being produced with heavy-ion beams has utilized a collection of models, each of which presumably contains some of the same physics as occurs in the experiments. Our view is that within this phenomenological context the pions represent a sensitive test of what the essential physics may be. This testing is rather crucial
since the phenomena described by the simple models are background to the fascinating events which may occur during a collision when the nuclear density is two or more times its normal value.

In dealing with complex, multi-particle problems such as collisions of heavy nuclei, there is a hope that a statistical or thermodynamic approach may explain essential features. Indeed, the "fire" models (fireball and firestreak) and the associated coalescence model have enjoyed considerable qualitative success. Pions have a special place in these models since with their 140 MeV rest mass production requirement, they are more likely emitted from the hotter "participant" regions of the collision and not from colder spectator matter. The evaporated nucleons, on the other hand, come from nearly all regions.

There are also theoretical treatments of the heavy-ion reactions which employ a non-equilibrium framework and treat the interacting hadronic matter as only partially evolved toward a thermal limit before nuclear disassembly, or simply view the collision as multiple nucleon-nucleon collisions smeared by Fermi motion. The Monte Carlo cascade codes belong in this family. In this framework, pion formation is greatest in the early stages, before the nucleon kinetic energy is degraded by collisions. Thus again, pion production cross sections provide a complementary probe to nucleon and stable cluster production.

In the next sections we will describe the experimental features and phenomenological understanding of pion production in the regions of momentum space which have been probed. As we have indicated, pions play a particularly important role in thermal models and may provide the best signal of the freeze-out density. In the multiple collision and hard
scattering models\textsuperscript{12,13} the pions are also importance since one knows they were actually "produced" in the reaction and did not arise from simple knock-out reactions. Although there are no quasi-elastic peaks in pion production to signal the reaction mechanism, we can trace the dominance of the $P_{33}$ isobar in the production, absorption, and propagation mechanisms\textsuperscript{13} in nuclear matter. In this way there is a healthy synergism between heavy-ion physics and meson-nucleus physics. We conclude with a short discussion on the use of pion interferometry as a possible tool with which to deduce the size of the nuclear region which produces pions.

**EXPERIMENTAL OVERVIEW**

Let us briefly note the kinds of pion data from heavy-ion reactions that are now available in the energy range $\sim (0.4 - 2)$ GeV/nucleon. Track detection methods such as photographic emulsions\textsuperscript{15} and the streamer chamber\textsuperscript{16} distributions have given the general features: charged-pion multiplicities, angular and energy spectra. Indeed, the streamer chamber has already been used to provide information on $\pi^-\pi^-$ correlations.$^\text{17}$

Counter experiments give a more restricted view than track detectors, but they can give higher statistical accuracy and explore rarer processes.$^\text{4}$ Some experiments have concentrated on pion spectra at very forward angles$^\text{1}$ and some on backward angles.$^\text{6}$ Others with rotatable solid-state detectors, scintillation range telescopes or magnetic spectrometers, have given differential cross sections over a range of intermediate angles.$^\text{2,3}$ The experiments with variable angle detectors have usually employed some tag counter system to allow sorting between central and peripheral collisions. Some experiments are attempting to follow the cross sections
down to even lower bombarding energies, while others have concentrated on measuring the rapidly falling spectra for the highest energy pions produced.

MULTIPPLICITIES

If pions are useful as indicators of the processes which occur when two relativistic heavy ions collide, and in particular, if copious production indicates the onset of exotic phenomena, then one of the first questions to answer is how many pions are produced in a typical reaction? The experimental answer has been somewhat confused, in part due to the difficulty of measuring pion multiplicities with emulsions. The most recent streamer chamber results using an inelastic trigger do, however, indicate that few pions are produced and that the production is only weakly dependent upon the beam and target. For example, the average number of $\pi^-$'s produced per (inelastic) interaction for an $^{40}$Ar beam on (LiH,Pb) is $(0.04, 0.10)$ at $0.4$ GeV/n, $(0.31, 0.92)$ at $0.9$ GeV/n, and $(1, 3.3)$ at $1.8$ GeV/n. Although the average number per interaction is rather low, as we see from the data in Fig. 1.a, pion multiplicities up to $\sim 14$ have been observed (this is some 50 times less probable than $n_{\pi} = 2$). There does not, however, appear to be anything striking about the pion multiplicity; the $p_l$ distribution is independent of pion multiplicity, the momentum distribution is independent of it, and the beam and target dependences are weak. In fact, the pion multiplicity simply appears proportional to the multiplicity of all charged particles.

Although pion production theories are just beginning to be developed, at present it appears these general multiplicity distributions,
which effectively average over all impact parameters for the collision (Fig. 1.b), contain little more information than the single pion spectra.\textsuperscript{19}

For example, collective effects may increase the production rate, but may also decrease the thermalization time, leaving the average multiplicity constant. In fact, it has been shown that many models (see e.g., Fig. 1.a) of the pion production dynamics yield a sum over Poisson distributions as the average pion multiplicity distribution — unless there are still-to-be-discovered multi-pion correlations.\textsuperscript{19} Clearly, we need to probe the reaction mechanism with a technique which determines the impact parameter of a reaction so that a more specific multiplicity distribution (e.g., $b = 0$) is obtained. (Speaking of impact parameters makes sense since there are hundreds of $l$ values involved.) Figure 1.b shows the breakdown of calculated multiplicity distributions for specific impact parameters.

In the pioneering paper of Chapline et al.\textsuperscript{9} it was suggested that a "fireball" — very hot, dense, nucleon matter — is formed in relativistic heavy-ion collisions. In the ensuing five years many refinements have been made and the agreement of this model with the data, particularly for central collisions, has been impressive.\textsuperscript{4,9} The essential physics of this model is thermal and chemical equilibrium among all active hadrons within an isoenergetic expanding interaction volume, with all interactions ceasing as some "freezeout" density is reached. The proton spectra are fit best with a density $\rho_c \approx 0.12 \pm 0.02$ hadrons/fm$^3$.

Since the nucleon-nucleon total cross section in the 1 GeV region is dominated by $\pi$ and $2\pi$ production (50\% and 25\% respectively), within the fire models pions play a crucial role in determining the properties
of the fireball material: they raise its specific heat, they increase its compressibility (thus affecting the creation of shock waves), and are present in large numbers \( N_\pi \leq 0.8 N_N \) at 2 GeV/N. In fact, it appears that the pion spectrum may be a better indicator than protons of the critical (freezeout) density, with little uncertainty introduced by considering the pions as independent free particles or as bound up in \( \Delta \)'s.

SINGLE PARTICLE SPECTRA

A very recent calculation \(^{10}\) has examined pion spectra within a refined firestreak model including equilibrium with light nuclei as well as nucleons. The \( \pi^\pm \) spectra from 800 MeV/nucleon Ne on NaF and Pb data of Nagamiya et al. \(^{2}\) \( (\pi^-) \), and Nakai et al. \(^{3}\) \( (\pi^+) \), were studied.

In the nearly equal target and projectile case \( (\text{Ne} \approx \text{NaF}) \), the simple fireball model with completely thermalized pions would predict a cross section \( \frac{d^3\sigma}{dp^3} \), which falls exponentially with pion energy and is isotropic in the c.m. system. For pion energies above about 150 MeV this exponential "Boltzmann-like" behavior is observed, although the apparent temperature of pions is somewhat lower than that of the protons. Figure 2 shows a comparison of \( \pi^- \) data with the firestreak calculation. The curves are slices of the Lorentz-invariant cross section at constant \( p_\perp (p^\star \sin \theta) \) plotted against rapidity \( y (\tanh^{-1} p_\parallel) \). Although the calculation generally fits the \( y \) (momentum) dependence of the spectra for differing \( p_\perp \) (angle), it overestimates the overall pion production by nearly an order of magnitude. Furthermore, as remarked above, the apparent pion temperature is too low and the data fall off with \( p_\perp \) more rapidly than calculated.
A hydrodynamic blast model explaining the apparent different temperatures was recently proposed. In it, the fireball assembly involves some hydrodynamic flow in which the isotropic thermal motion of the fireball is partially converted to the energy of an ordered, outward blast. This model, treating only the proton and pion data at 90° c.m. in the Ne + NaF 800 MeV/n case, deduces that the available thermal energy of the fireball is about 20% converted to pion rest mass, 40% converted to ordered outward blast energy, and 40% remaining as random thermal motion. In addition, because of the Lorentz transformation of rather large blast velocities, the pions appear cooler than protons (50 vs. 75 MeV).

Pions produced with low energies have been studied via range telescope measurements. As seen in Fig. 3 for Ne + NaF + π⁺ + X with 800 MeV/nucleon Ne, the distribution has a rich structure when viewed on a "p perp vs. y" plot. Here we present slices of the invariant cross section in p -rapidity space. The curves are the contours of equal values of \( d^2\sigma/(d^3p/E) \) and the dots are the location of the experimental measurements.

As becomes evident from a careful study of Fig. 3, for lower pion energies, \( T_\pi \approx 100 \text{ MeV} (p_\perp \lesssim 0.9) \), the angular distribution develops a forward-backward peaking - resembling in fact the peaking found in elementary \( p+p \rightarrow \pi^++X \) data. This is shown by the peaks at \( y = (0.6, -0.3) \). At still lower \( \pi^+ \) energy a peak at 90° c.m. and about 30 MeV (\( p_\perp \approx 0.6 \)) appears. Similar features have also been seen in the \( ^{20}\text{Ne} + ^{27}\text{Al} \) and \( ^{40}\text{Ar} + ^{40}\text{Ca} \) systems at 1 GeV/nucleon.

The fact that the cross sections of Fig. 3 fall off at lowest energy
along 90° (c.m.) was at first puzzling as none of the ordinary models (fireball, firestreak and hydrodynamic blast) show the fall off. However, rough Coulomb correction to the hydrodynamic blast model calculation appears to explain the suppression of low energy pions — just as the Coulomb correction suppresses lowest energy positrons in a beta decay spectrum. In the same manner the Coulomb correction is expected to enhance \( \pi^- \) cross sections. We eagerly await data on \( \pi^+/\pi^- \) ratios at lowest pion energies. Indeed, we know that at low energies the elastic \( \pi^- \)-nucleus interaction contains very strong Coulomb-nuclear interference effects and that the \( \pi^+ \) and \( \pi^- \) cross sections are quite different.

Since the \( pp + \pi X \) system also shows a forward-backward peaking, it is not surprising that one can explain the gross features of the heavy ion spectra with a nucleon-nucleon collision model with the nucleon momenta smeared (Fermi motion). In this case the forward-backward peaking of the pion distribution is a consequence of the dominance of the \( P_{33}(1238) \) in the production processes \( (3\cos^2\theta + 1) \). In addition, there is some limited evidence that resonant absorption by the spectator nuclear matter may contribute to the valley in Fig. 3 at \( y \approx 0.2 \).

In contrast to the angle dependence, the 90° (c.m.) peak in Fig. 3 is not explained by either the single collision models or the firestreak ones. A possibility is that pions in this region arise mainly from the hydrodynamic blast mechanism.

CASCADE CALCULATIONS

Given the inclusive and incoherent nature of most of the pion production measurements, the most precise calculational reproduction of
them may, to date, arise from the convolution of classical and quantum physics known as the intranuclear cascade. Essentially this approach simulates in a step-by-step manner the many possible reaction paths of a nucleon after two nuclei collide. Since refined versions of this approach have shown moderate success in reproducing proton-nucleus \( \pi \) production, it is natural to believe that the heavy-ion beam versions (now in their infancy) should produce comparable agreement. Indeed, a very recent comparison of preliminary calculational and preliminary experimental results show at least qualitative agreement on shapes and angular dependences. Since these codes treat pion production and absorption only via an isobar mechanism, \( \text{NN} \leftrightarrow \text{NN}^* \leftrightarrow \text{NN} \), it seems likely that inclusion of an additional (S-wave) two-nucleon absorption mechanism would decrease and distort the calculated pion spectra and bring the absolute value of the cross sections into better agreement.

The cascade calculations contain many of the same uncertainties as single-step models, e.g. off-shell kinematics and dynamics, Pauli effects, energy-momentum conservation, Fermi motion, and degree of coherence. However, since they are computer simulations they do have the option of selectively including many effects and determining the importance of each — for every step in the reaction! There are, of course, limits to this technique; for example, we question their usefulness in studying the high momenta tails of the spectra. Since these rare events are down by 4 to 5 orders of magnitude they are quite outside of the kinematic limits of free NN reactions, and may require a more careful treatment of the overall energy-momentum constant, and correlation effects.
THE KINEMATIC LIMIT

A pion produced in a collision between two nuclei attains its maximum possible energy when the two final nuclei recoil together; this is a high degree of coherence and is quite similar to elastic scattering. If the pion's momentum is slightly less than this maximum, there must still be a high degree of coherence in the final state. In fact, by counting the number of (initially "frozen") nucleons which must recoil together, we obtain what Baldin\(^25\) has labeled the "degree of cumulativity" for a reaction. Whereas these statements may appear rather obvious, actually calculating the inclusive cross sections in terms of nucleon-nucleon reactions is more subtle since a free target nucleon at rest can only produce a relatively low energy pion (the NN limit). Thus if a one-step mechanism is visualized, the participant target nucleon must have a rather large internal ("Fermi") momentum — so that momentum can be conserved. In particular, the production of high energy \(\pi\)'s in the backward hemisphere requires very large internal momenta (0.5 - 2 GeV/c). The magnitude of the probability needed for these momenta is simply too large to arise from the independent particle model's bound state wavefunctions, and apparently must be generated by nucleon-nucleon correlations. This then brings us back to the cluster ideas introduced because of kinematics!

The modern Bevalac experiments which probed the kinematics limit in \(\pi\) production began with that of Papp et al.,\(^1\) who measured pions produced at 2.5° with 1 - 4 GeV/nucleon beams of \(p\)'s, \(d\)'s and \(\alpha\)'s. Some of their data are shown in Fig. 4. They discovered that for a given projectile nucleus, the invariant cross sections \((E/k^2), (d^2\sigma/d\Omega dk)\) approximately
"scale", i.e. are independent of beam energy if plotted as a function of the Feynman scaling variable, $x_R = \frac{k_{c.m.}^\pi}{(k_{c.m.}^\pi)_{\text{max}}}$. Although one would not expect scaling to hold at such low energies, Schmidt and Blankenbecler subsequently showed that application of the parton model to these reactions yielded counting rules which predicted — with amazing success — the power $S$ which describes the fall-off of these spectra $[d\sigma \propto (1 - x_R)^S]$. Essentially this theory is a hard-scattering, single collision, impulse approximation picture with neither initial nor final state interactions, but with relativistically valid, nuclear momentum distributions. These distribution functions are of the type never probed in conventional nuclear physics since they deal with such asymptotically large internal momentum transfers that all nucleons in the residual nucleus must share this momentum equally.

Since the Papp et al. data were in the projectile fragmentation region (high forward velocity, small transverse momentum transfer), they appeared to examine the (small A) projectile's nuclear wavefunction. However, the earlier experiments of Baldin et al., Cochran et al., and the latter experiment of Chessin et al., all examined π's produced with large momenta in the target fragmentation region (high backward velocity, very large momentum transfer). These latter experiments have beam energies of 0.73–7.5 GeV/N, show a very rapid (5–7 orders of magnitude) drop-off, but do not show any scaling.

The parton model (which assumes all target nucleons act coherently) cannot account for the lack of scaling (energy independence) in the $\theta_\pi = 180^\circ$ data. Instead, two varieties of more standard models have shown some success. The first represents the p-nucleus production
cross sections as an incoherent sum over p-correlated cluster rates with higher-energy pions arising from progressively heavier clusters. These clusters, or nuclear density fluctuations, contain N (from 1 to \( \sim 6 \)) nucleons, occur with probabilities given by a binomial distribution, and have a center of mass momentum distribution of Gaussian form with increasing width for heavier clusters, \( \rho(p) \propto \exp\left[-p^2/N\sigma_0^2\right] \). Similar models are currently popular in explaining the high-energy backward protons observed in nuclear collisions.\(^{28}\)

The second type of model which has provided some understanding of the shapes of the backward angle pion spectra are also single-collision models (PWIA), but with an empirical nucleon-cluster \( \pi \) production rate,\(^{13}\) and a cluster-independent exponential (\( \rho(p) \propto \exp(-p/\sigma_0) \)) or Gaussian momentum distribution. In this model the shapes of the spectra in both the projectile \((p,d,\alpha,C)\) and target fragmentation regions are accounted for by either nucleon-nucleon or nucleon-'deuteron' production with the exponential form best for the single nucleon case. For example, in Fig. 4 we see that either "d" or "p" clusters can explain the shapes and the scaling at forward angles. Furthermore, it appears that the projectile fragmentation region is rather insensitive to the dynamical details of the production mechanism — which helps explain why the counting rules appeared to work so well here. We see this by the relatively small differences in Fig. 4 produced by different momentum distributions.

In another variation of this "cluster" picture, the incident hadron is viewed as sweeping out a straight tube of nucleons from the nucleus, and then interacting coherently with the tube as if it were one particle. Whereas this model has been applied successfully at very high energies,\(^{29}\)
there still remains questions concerning its validity at energies of \( \leq 2 \text{ GeV/N} \), where there is evidence that a larger number of nucleons than those contained within a tube participate in 180° pion production.

Major questions concerning the validity of all models still remain. On the theoretical side, we do not understand 1) how the spectra are influenced by the dispersion and absorption introduced by final state interactions; 2) the foundation of exponential internal momentum distributions; 3) and 3) the off-energy/mass shell aspects of the problem. On the experimental side, if these processes do arise from hard scatterings we should see a good number of fast, correlated nucleons in the final state - the exact number depending on the reaction model and the kinematic regime. At present these less-inclusive measurements are not yet available, although there is experimental evidence \(^{24}\) for quasi-elastic processes in proton "production."

### PION INTERFEROMETRY IN RELATIVISTIC HEAVY-ION COLLISIONS

We conclude this comment by discussing the quantum optics of pions - an idea which may be fruitful in understanding the 2 and 3 particle inclusive spectra which represent the next generation of relativistic heavy ion experiments. A textbook example of photon quantum optics is the Hanbury Brown-Twiss effect in which the intensity interference of light is used to measure the spatial extent of a light source, e.g., a distant star. Likewise, since pions are also bosons it has been possible to measure a correlation function for two identical pions created in pp collisions \(^{31}\) and thus determine the geometric size of the pion source (\( \sim 1 \text{ fm} \)). Several people \(^{32}\) have recently suggested that a deduction of
the appropriate correlation function from $2\pi$ and $1\pi$ inclusive spectra might also be tried for the relativistic heavy ion case. Indeed the first measurements\textsuperscript{17} have indicated that for a 1.8 GeV/N Ar beam on Pb, the pion source radius is $\sim 3-4$ fm.

There are two critical pieces of information which may be learnt from pion interferometry and the correlation function. The first is the Fourier transform of the space-time evolution of that part of two interacting nuclei which acts as a source of the pion field; this comes from the width of the correlation function. By determining this function for variously sized nuclei at different energies, one can, for example, test the geometrical assumptions of the fireball models, estimate the time sequence of the reaction mechanism (direct or equilibrated), and deduce the amount of coherence. In other words, pion correlation measurements could provide an experimental test of the geometrical aspects of pion production models.

The second critical piece of information pion interferometry may ideally provide (via the value of correlation function at zero momentum transfer) is the actual degree of coherence of the pion source field. The present theories assume a chaotic source. Therefore, pion interferometry could provide dynamic as well as geometric information about this pion source; however, preliminary calculations have shown that it will be essential to account for the large distortion due to final state interactions. The successful application of pion interferometry therefore also appears to depend upon our ability to calculate the effects of final state distortions and absorptions.
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REFERENCES


FIGURE CAPTIONS

Fig. 1. (a) $\pi^-$ multiplicity distribution for Ar + Pb$_3$O$_4$ at 1.8 GeV/n: closed circles are data from Ref. 16; triangles, the multiple collision model of Ref. 18; curve 1 is the statistical model of Ref. 19 with no $b$ cutoff, $\rho_{cut} = 0.16 m_\pi^3$; curve 2, the same statistical model with $b_{max}(Pb,0) = (9.6, 5.0)$ fm; dashed curves give contributions to 2 from Pb and O targets.

(b) $\pi^-$ multiplicity distribution as a function of impact parameter $b$ for Ar+Pb at 1.8 GeV/nucleon.

Fig. 2. The Lorentz invariant doubly differential cross section for pion production in the Ne+NeF $\rightarrow \pi^- + X$ reaction at 800 MeV/nucleon, versus rapidity $y (\tanh^{-1} \beta_H)$. The data are from Ref. 2 and the curves result from firestreak calculations$^{10}$ for constant values of $p_\perp (p*\sin\theta)$.

Fig. 3. Contour plots of the invariant cross section for Ne+NaF $\rightarrow \pi^+ + X$ at 800 MeV/nucleon in $p_\perp$-rapidity space. The curves represent contours of constant cross sections as deduced from the data of Ref. 3 (dots).

Fig. 4. (a) The invariant cross section for d+C $\rightarrow \pi^- (2.5^\circ) + X$ and (b) $\alpha+C \rightarrow \pi^- (2.5^\circ) + X$. The data are from Ref. 1 and the curves are calculated with the single step, hard scattering model of Ref. 13. As indicated, the elementary subprocess is considered to be either p+C $\rightarrow \pi^- + X$ [(a) and (b)], or d+C $\rightarrow \pi^-$ [(b) only]. The dot-dashed curves are calculated with no nucleon internal momentum distribution, the solid curves with a Gaussian one, and the dashed curves with a (long tailed) exponential one.
Fig. 2
Fig. 3
Fig. 4 XBL 784-8256
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