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Structure-Mapping Theory and Lexico-Semantic Information

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Abstract

In modelling analogy the Structure Mapping Engine (Gentner, 1983, Falkenhainer, Forbus and Gentner, 1989) can only map successfully on representations in a canonical form because it only permits mappings between relations with lexically-identical functors. We examine whether co-occurrence statistics can remedy this by providing an appropriate basis for modelling lexicosemantic relations. Using a co-occurrence model we reimplement SME to allow it to map between relations with functors that are lexically-distinct. Computational experiments are then reported which show that the resulting model, M-SME, maps successfully on representations which faithfully encode lexical properties, indicating that semantic constraints should only play a minimal role in the mapping process.

The Structure-Mapping Theory

The structure-mapping theory was originally proposed as a set of constraints defining permissible mappings between a base and target domain in analogy (Gentner, 1983), and implemented in the Structure-Mapping Engine (Falkenhainer, Forbus and Gentner, 1989). Structure-mapping theory constructs analogical mappings between discrete domains (called ‘Dgroups’) of propositional statements, with the main focus being on mapping interconnected relational structure.

The Lexical-Identicality Constraint

In detecting shared relational structure the structure-mapping theory only permits mappings to be made between relations if, and only if, they have lexically-identical functors and the same number of arguments. Thus there are two constraints on the formation of an initial match hypothesis. We call the first constraint on match hypothesis formation the lexical-identicality constraint, and it is important to observe that it carries a commitment to a canonical theory of representation because it requires that mappable relations are represented with identical names. For example, structure-mapping theory would not permit an alignment between the following two relations, even though it might be appropriate in a wider context:

(ORBITS PLANET SUN)

(REVOLVES_AROUND ELECTRON ATOM)

The fact that ORBITS is not lexically-identical to REVOLVES_AROUND also means that the corresponding

analogue mappings between the arguments of the relations (PLANET with ELECTRON, and SUN with ATOM) are not made. Holyoak and Thagard (1995) have argued that this constitutes a significant weakness in structure-mapping theory: “with its emphasis on structure to the exclusion of all other constraints, SME does not simply discourage mappings between non-identical but semantically similar items; it does not even permit them.”

Both the ACME (Holyoak and Thagard, 1989) and LISA (Hummel and Holyoak, 1997) models of analogy avoid this objection by postulating semantic links that hold between the names of relations. These links are hand-coded into the propositional representations on which the analogical mappings are generated. If a sufficiently strong semantic link is coded between two relations then a mapping can be countenanced between them. Thus, in the example above, ACME’s or LISA’s representations could incorporate a sufficiently strong semantic link between ORBITS and REVOLVES_AROUND to enable a mapping to be generated from one relation to the other.

The Canonical Representation Theory

Holyoak and Thagard’s criticism of the structure-mapping theory is not entirely fair, however, as it ignores SME’s commitment to a canonical representation (CR) theory. The CR theory claims that relations that are sufficiently similar in ‘meaning’ to facilitate mappings (e.g. ‘orbits’ and ‘revolves around’) are coded with identical tokens (in this case both might be coded as ‘orbits’). This extra assumption of the structure-mapping theory would allow the intuitively correct mapping to be made in the above case. However, since the postulation of semantic links and the CR theory rely on human-based coding decisions – and neither subscribe to a worked out model of semantics – both are ultimately equivalent in terms of their explanatory power.

The CR commitment of structure-mapping theory allows a modular approach to be taken to the cognitive modelling of analogy. By mapping across canonical representations questions of semantics are left outwith the scope of structure-mapping theory - SME thus remains noncommittal with respect to a theory of lexical semantics. In the experiments that follow we exploit SME’s modular approach to modelling by using the information provided by a co-occurrence model of lexical semantics to see if this allows SME to map successfully on non-canonical representations, and avoid the underspecifica-
tion inherent in the CR theory.

**Experimental Materials**

The ‘Karla the Hawk’ stories. The Karla the Hawk materials were chosen as the test domain in this study (Gentner, Ratterman and Forbus, 1993). The materials consist of twenty sets of stories written in natural language. Each set consists of a base story, and four systematic variations of that story. Two factors are crossed over the four variant stories, as shown below.

<table>
<thead>
<tr>
<th>+ST</th>
<th>-ST</th>
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<tbody>
<tr>
<td>+SF</td>
<td>Literal Similarity</td>
</tr>
<tr>
<td>-SF</td>
<td>Analogical</td>
</tr>
</tbody>
</table>

Table 1: The commonalities each variant category shares with the corresponding base it is derived from.

The four story categories systematically vary the commonalities that are shared with the base-story from which they are derived. Each variant can either share or not share surface (+SF) and structural (+ST) commonalities with the corresponding base-story. Because analogy consists in two domains possessing a shared structure, this 2 x 2 materials design allows for the controlled examination of SME’s performance. If SME is performing appropriately then we would expect a better mapping performance when mapping the base representations on to the LS and AN category materials, as they share structural commonalities.

The **Faithful Dgroups**. The standard representations that SME operates on, the Original Dgroups, encode relation names in canonical form in accordance with the CR theory. In order to test the performance of SME on representations that do not embody a commitment to the CR theory we developed our own representations that faithfully encode the relation names as used in the original natural language Karla stories. We call this set of representations the Faithful Dgroups, and they were produced by transferring the lexemes used to express relations in the original natural language Karla materials directly into the propositional form required by SME.

**Experiment 1A**

This first experiment was conducted to test the performance of SME on the Original Dgroups, which are the original encodings of nine of the twenty Karla the Hawk story-sets (Forbus, Gentner and Law, 1994). This was in order to provide a base measure of SME’s performance.

**Method.** For each of the nine sets of Original Dgroups SME was used to map the base Dgroup onto its four variants. The Structural Evaluation Score (SES)\(^1\) and number of match hypotheses formed for each mapping were then recorded.

\(^1\)SES scores are automatically calculated by SME and provide a measure of the quantity of structure that has been mapped between two domains.

**Results.** The data for Experiment 1A can be seen in Table 2. The results of two-factor repeated-measure ANOVA testing are given below.

**SES scores**: the only significant effect was for ±ST \((F(1,8) = 5.43, p < 0.05)\), and interaction \((F(1,8) = 1.24, p > 0.05)\) factors produced nonsignificant effects.

**Match hypothesis formation**: the only significant effect was for ±SF \((F(1,8) = 51.44, p < 0.01)\), Both the ±ST \((F(1,8) = 1.12, p > 0.05)\) and interaction \((F(1,8) = 1.29, p > 0.05)\) factors produced nonsignificant effects.

<table>
<thead>
<tr>
<th></th>
<th>LS</th>
<th>SS</th>
<th>AN</th>
<th>FOR</th>
</tr>
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<tbody>
<tr>
<td>SES Category Mean</td>
<td>21.51</td>
<td>17.14</td>
<td>21.16</td>
<td>16.23</td>
</tr>
<tr>
<td>MH Category Mean</td>
<td>240.5</td>
<td>230.0</td>
<td>214.3</td>
<td>205.4</td>
</tr>
</tbody>
</table>

Table 2: The SES scores and number of match hypotheses formed with the SME model on the nine Original Dgroups.

**Discussion.** As expected, SME exhibits the required sensitivity to the structural commonalities of the Original Dgroups (witness the higher SES scores for the LS and AN mapping tasks). This is demonstrated by the fact that the only significant factor in the analysis of the SES scores was ±ST. Interestingly, the number of match hypotheses formed for each category of match is sensitive to ±SF. This reflects the fact that lexically-identical functors are more likely to occur in the Original Dgroups when there are shared surface features, and SME can only form match hypotheses between relations with lexically-identical functors.

**Experiment 1B**

**Method.** The format of this experiment is the same as the previous one, except that this time SME was required to map across the Faithful Dgroups that faithfully encode the lexical properties of the original Karla representations.

**Results.** The results for Experiment 1B can be seen in Table 3. The details of repeated-measure ANOVA testing for two factors are given below.

**SES scores**: All three factors produced nonsignificant effects: ±ST \((F(1,8) < 1)\); ±SF \((F(1,8) = 4.72, p > 0.05)\); and interaction effects \((F(1,8) < 1)\).

**Match hypothesis formation**: Again, all three factors produced nonsignificant effects: ±ST \((F(1,8) < 1)\); ±SF \((F(1,8) = 3.21, p > 0.05)\); and interaction effects \((F(1,8) < 1)\).

Testing on both the SES scores \((t = 11.37, df = 35, p < 0.01)\) and the number of match hypotheses \((t = 8.38, df = 35, p < 0.01)\) revealed that there was a significant decrease in the the means of both from mapping on the Original Dgroups.

**Discussion.** As expected, SME does not exhibit the required sensitivity to ±ST on the Faithful Dgroups, and the greatly reduced SES scores from its performance on the Original Dgroups show that it fails to map signifi-
Table 3: The SES scores and number of match hypotheses formed with the SME model on the Faithful Dgroups.

<table>
<thead>
<tr>
<th>SES Category Mean</th>
<th>SS</th>
<th>AN</th>
<th>FOR</th>
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<tbody>
<tr>
<td>LS</td>
<td>1.62</td>
<td>1.21</td>
<td>1.47</td>
</tr>
<tr>
<td>MH</td>
<td>92.1</td>
<td>84.7</td>
<td>93.1</td>
</tr>
</tbody>
</table>

Table 3: The SES scores and number of match hypotheses formed with the SME model on the Faithful Dgroups.

significant quantities of structure from one domain to another. Furthermore, the greatly reduced number of match hypotheses formed for each category of mapping (reduced from an overall mean of 224.8 in Experiment 1A to 87.13 in 1B) suggests a possible explanation of this failure: the constraints on the formation of match hypotheses are too strict to allow the appropriate local alignments to be made on the Faithful Dgroups (because there are an insufficient number of lexically-identical relations between different domains). This means that the raw material is not there for SME to combine to form the appropriate global mappings, and suggests that the process of match hypothesis formation needs to be altered if SME is to perform successfully on the Faithful Dgroups.

As noted above, the only point at which SME is committed to the CR theory is during the formation of match hypotheses. Therefore, if we are to remove SME’s commitment to the CR theory we need to do so by changing the constraints on the formation of match hypotheses to allow them to be formed between relations that are sufficiently similar instead of identical. This begs the question of what ‘sufficiently similar’ means.

Co-occurrence Statistics

There is a growing body of evidence that the frequency with which different lexemes co-occur with one another (that is, are used together within a particular context, such as a paragraph or moving-window) can provide useful information about the semantic properties of those lexemes. For example, Landauer and Dumais (1997) report that the LSA model can pass a multiple-choice TOEFL synonym test. Lund, Burgess and Atchley (1995) present evidence that co-occurrence data can act as a good predictor of priming effects. Burgess and Lund (1997) demonstrate that the HAL model can produce clustering in its high-dimensional space according to the grammatical category of different lexemes.

We therefore decided to investigate the possibility of using the Latent Semantic Analysis (LSA) model (Landauer and Dumais, 1997; Landauer, Foltz and Laham, 1998) to see if it could provide SME with the sort of lexico-semantic information required for it to map successfully on the Faithful Dgroups (Note that although we use the LSA model, this does not indicate a particular commitment to that model alone, but rather we use it as an exemplar of the more general approach).

Relaxing the Lexical-Identicality Constraint

Since the only commitment SME makes to the CR theory is during the formation of match hypotheses, where it requires that relations have lexically-identical functors and the same number of arguments if they are to support a match hypothesis, SME’s code was altered so that it enforced different constraints on the formation of match hypotheses. In the modified version of SME (M-SME) two relations still have to have the same number of arguments to warrant a match hypothesis, but the lexical-identicality constraint is relaxed. Instead of the two relations also having to have identical functors, the functors are compared with one another using the LSA model2. Only if they are assigned a score greater than a threshold value (called the reconciliation-threshold) is a match hypothesis formed. In this way, the relations with functors REVOLVES_AROUND and ORBITS might be combined in a match hypothesis because the LSA model assigns them a score of 0.48.

The possibility of assigning different values (between 0 and 1) to the reconciliation-threshold generalises the original constraints that SME places on match hypothesis formation. When the threshold is set to 1 the re-implented model performs just like the original SME because LSA only assigns lexically-identical functors a score of 1. When the threshold is set to 0 any two functors will be assigned an LSA score greater than or equal to the threshold, and so the only constraint on match hypothesis formation is that the relations in question have the same number of arguments3. It is clear that the reconciliation-threshold needs to be assigned a value that maximises the performance of M-SME.

Setting the Reconciliation-Threshold

In order to determine a value for the reconciliation-threshold it is necessary to establish some criterion by which the quality of mappings can be assessed. The following experiments investigate whether such a measure can be derived from the number of match hypotheses and the SES scores of M-SME on a variety of mapping tasks.

Experiment 2A

This experiment investigates the effect that varying the reconciliation-threshold has on the number of match hypotheses formed for each category of mapping (LS, SS, etc.). We predict that the number of match hypotheses formed for each match will decrease as the reconciliation-threshold increases because the semantic constraints on match hypothesis formation become stricter. This result will indicate that M-SME is functioning as expected. Furthermore, if the reconciliation-threshold can be used to reduce the number of match hypotheses formed then this could be used to limit the computational complexity of the mapping process.

Method. M-SME was used to map between the base domain and its four variants on the nine sets of Faith-

\(^2\)The LSA model assigns two functors a score between 0 and 1, depending on their location in the high-dimensional space defined by taking each lexeme sampled as a dimension.

\(^3\)Note that the introduction of a reconciliation-threshold only affects the formation of mappings: the evaluation of mappings remains unaffected; M-SME calculates SES scores in exactly the same way as SME.
ful Dgroups, as the reconciliation-threshold was adjusted between 0 and 1.

**Results.** The results of Experiment 2A can be seen in Figure 1. The reconciliation-threshold is plotted against the number of match hypotheses formed for each category of the mapping task. This shows that the number of match hypotheses formed for each category of the mapping task decreases in a regular nonlinear fashion as the reconciliation-threshold is increased from 0 to 1.

![Figure 1: A plot of the number of match hypotheses that M-SME produces in matching the base stories with their four variants as the reconciliation-threshold is adjusted.](image1)

**Discussion.** The regular decrease in the number of match hypotheses formed offers preliminary evidence that M-SME is performing as expected, and that the computational complexity of the mapping process can be limited by increasing the reconciliation-threshold. However, it is possible that in doing this the semantic constraints on mappings become too strict to allow the appropriate analogical mappings to be constructed. This clearly requires further investigation.

**Experiment 2B**

This experiment investigates the effect of the reconciliation threshold on the SES scores produced for each category in the standard mapping task on the Faithful Dgroups.

**Method.** M-SME was used to perform the same mapping task as in Experiment 2A, but this time the SES scores for each category were recorded as the reconciliation-threshold was adjusted from 0 to 1. We predicted that there would be a consistent separation in SES scores between those materials exhibiting $+S'T'$ and $-S'T'$ as the reconciliation-threshold was varied, indicating that M-SME is sensitive to the structural aspects of the Faithful Dgroups.

**Results.** The results of this experiment are shown in Figures 2-3. Figure 2 shows the SES category scores against the reconciliation-threshold. Figure 3 shows the same data, but this time with the mapping task with Faithful Dgroups against the reconciliation-threshold.

![Figure 2: A plot of M-SME's SES scores on the standard mapping task with Faithful Dgroups against the reconciliation-threshold.](image2)

![Figure 3: A plot of M-SME's SES scores on the standard mapping task with Faithful Dgroups against the reconciliation-threshold.](image3)

**Discussion.** Figure 3 offers preliminary evidence that M-SME is sensitive to the $\pm S'T'$ factor on the Faithful Dgroups, as predicted. This represents a large improvement over SME's performance on these materials. However, SES scores are a measure only of the *quantity* of structure that is mapped between two domains. Basing our evaluation of M-SME on SES scores alone is insufficient evidence of its success, because we need to ensure that it is sensitive to genuine analogies between domains and is not mapping inappropriate structure. So, a measure sensitive to the *quality* instead of just the *quantity*
of mapped structure is required.

Experiment 3

To gain a useful measure pertaining to the quality of mappings made by M-SME, each of the individual alignments made in the successful global mappings were examined and rated for correctness.

Method. Each of the individual alignments produced by M-SME on the standard mapping task on the Faithful Dgroups were inspected and assessed for correctness (i.e., whether or not they represented genuine analogical alignments). The LSA score that sanctioned each alignment was also recorded, to see if the reconciliation-threshold could be set so as to prevent incorrect alignments from being made whilst still permitting correct alignments to be made.

Alignments made between the base and the SS and FOR categories were rejected, because it was unclear what would constitute a correct or incorrect alignment in these cases, as the materials were designed to share little or no structure with the corresponding base representation. The matches were performed with the reconciliation-threshold set to 0 to make the alignments generated as inclusive as possible. This was in order to collect the largest possible set of match hypotheses to see what the LSA scores were for each alignment.

Note that not all of the match hypotheses formed for each match were inspected, but only the ones that were included in the highest scoring global mapping for each attempted match. Although it would have been informative to consider all these hypotheses, there would have been approximately 16,200 of them, which is too many to inspect by hand! This evaluation procedure imposes limitations on the information available. No conclusion can be drawn using this method about (i) the number of correct alignments that should have been, but are not, included within the best global mapping, and (ii) the number of incorrect alignments that are not included in the best global mapping.

Results. 85.99% of the alignments inspected were designated ‘correct’, whilst the remaining 14.01% were designated ‘incorrect’. The mean LSA score between the two functors featuring in correct alignments was 0.731; the same score for incorrect alignments was 0.294. Statistical analysis showed this difference to be significant ($t = 8.35, df = 255, p < 0.01$).

Discussion. The large proportion of alignments that are correct indicates that M-SME is mapping with great success on the Faithful Dgroups. The evidence of a significant separation between the LSA scores warranting the correct and incorrect alignments supports a naive hypothesis that all match hypotheses a fixed number of standard deviations from the mean LSA score of the correct alignments could be rejected on the grounds that they are unlikely to be correct alignments. We feel that this is a bad hypothesis for the following reason:

Many functors that appear in the Faithful Dgroups are lexically-identical because they represent higher-order or structural relations that are not explicitly mentioned in the original natural language stories. For example, causal sequences and relations of temporal succession are rarely flagged explicitly in narratives, but instead have to be inferred. However, such relations are essential to producing the structured representations that SME and M-SME operate on. Therefore, because their lexical form is not given explicitly in the original materials they have to be assigned a canonical form (in the case of the Original Dgroups CAUSE and FOLLOWS were used chiefly). The great frequency of such functors in the Dgroups, which were generally aligned correctly, increases the mean of the LSA scores supporting correct alignments because identical functors receive an LSA score of 1. This makes the actual separation between the scores of the correct and the incorrect alignments smaller than the mean statistic indicates.

A consequence of this is that there is no one optimal value for the reconciliation-threshold that will effectively separate the correct from the incorrect alignments (because of the lack of a sufficiently distinct boundary between the two populations). Furthermore, a closer inspection of the LSA scores sanctioning correct alignments revealed that they were subject to a fairly wide distribution. If LSA is taken as a reasonable model of lexico-semantic information then this offers evidence that the relations that should be analogically aligned need not be semantically similar in a fixed way.

In this light, the nature of the structure-mapping algorithm urges caution in enforcing a prohibitively high value to the reconciliation-threshold. The structure-mapping algorithm makes match hypotheses, and combines them in an appropriate fashion to form global mappings. However, if the reconciliation-threshold is set at too high a value certain match hypotheses will not be formed. This can, in turn, inhibit further structural alignments (because match hypotheses can sanction other alignments under the parallel-connectivity constraint), resulting in the poor mapping performance that SME exhibits in Experiment 1B. It is sensible, therefore, to take the line of caution when it comes to setting the value of the reconciliation-threshold, and aim for a lower value that is more permissive.

The results here suggest that a suitable value for the reconciliation-threshold would be in the range 0.0-0.3. This should reduce the number of match hypotheses formed considerably (there are around 900 on average when the threshold is 0, and about 450 on average when it is 0.3; cf. Figure 1), and thus decrease the computation required to combine the match hypotheses into global mappings, whilst preserving SES scores at a reasonable level and ensuring that a minimal number of correct alignments are prevented from being formed.

Experiment 4

This final experiment is designed to conclusively test the mapping performance of M-SME on the Faithful Dgroups, with a fixed reconciliation-threshold.
**Method.** M-SME was used to perform the standard inter-set mapping task of Experiments 1A-B, with its reconciliation-threshold fixed to 0. The SES scores and number of match hypotheses formed were recorded for each category of match.

**Results.** The results of Experiment 4 are shown in Table 4. The results of the two-factor repeated-measure ANOVA analysis are as below.

*SES scores:* The only factor that produced a significant effect was $\pm ST$ ($F(1, 8) = 19.00, p = 0.02$). Both $\pm SF$ ($F(1, 8) < 1$) and interaction ($F(1, 8) < 1$) effects were nonsignificant.

*Match hypothesis formation:* All three factors produced nonsignificant effects: $\pm ST$ ($F(1, 8) = 2.09, p > 0.05$); $\pm SF$ ($F(1, 8) < 1$); and interaction effects ($F(1, 8) = 1.40, p > 0.05$).

<table>
<thead>
<tr>
<th>SES Category Mean</th>
<th>LS</th>
<th>SS</th>
<th>AN</th>
<th>FOR</th>
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</thead>
<tbody>
<tr>
<td>MH Category Mean</td>
<td>903.3</td>
<td>874.6</td>
<td>931.8</td>
<td>853.2</td>
</tr>
</tbody>
</table>

Table 4: The SES scores and number of match hypotheses formed with M-SME mapping on the Faithful Dgroups. The reconciliation-threshold is set to 0.

**Discussion.** The SES scores demonstrate the appropriate sensitivity to the $\pm ST$ factor on the Faithful Dgroups, thus indicating that M-SME successfully generates analogical mappings on Dgroups that faithfully encode the lexical properties of the materials they are derived from. The number of match hypotheses is insensitive to $\pm SF$ indicating that surface features are irrelevant to the formation of match hypotheses; this is a marked difference from the performance of SME in Experiment 1A.

**Conclusion**

We have shown that SME’s commitment to the CR theory prevents it from generating analogical mappings on representations that faithfully encode lexical information (Experiments 1A-B). We then used the information provided by a co-occurrence model of semantics to produce an alternative model of analogical mapping, M-SME. Experiments 2A-B showed that M-SME functions as expected, but that there is no convenient measure of the quality of analogical mappings. In Experiment 3 the quality of alignments made by M-SME were inspected and rated for correctness. A detailed analysis of this data supported the idea that to maximise the quality of analogical mappings it is necessary to minimise the role that semantic constraints play during mapping. This result supports Gentner’s (1983) original insight that it is primarily structural constraints that determine analogical mappings (indeed, in Experiment 4 semantic constraints are effectively redundant in the mapping process). In the final experiment evidence was presented that M-SME is sensitive only to the structural properties of representations that faithfully encode lexical properties. Because a commitment to semantic links or the CR theory allows coding decisions to reduce the search space that analogical mappers face, it is significant that M-SME can still produce mappings when presented with problems of this greater complexity. Whilst M-SME is a more expensive mapper overall, we think that a similarly improved model of the retrieval of analogies may enable the use of contextual information to reduce the search space in the mapping phase.

**Acknowledgements**

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**References**


