Lawrence Berkeley National Laboratory
Recent Work

Title
Application of a Pulsed, RF-driven, Multicusp Source for Low Energy Plasma Immersion Ion Implantation

Permalink
https://escholarship.org/uc/item/42d7744t

Author
Wengrow, A.B.

Publication Date
1996-06-01
RESONANCES THAT OVERLAP

G. Smadja

October 1971

AEC Contract No. W-7405-eng-48
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
When a final state can be reached by the production of resonances in different subsystems, the usual approximation is to add them. We show that the "exact" treatment introduces correction terms which are quite small.
Conversely if we choose for $J$ any function of the momenta free from discontinuity in the three subenergies, this is enough to insure that $T$ obeys (4). To prove this one can use a consequence of (3):

$$\left(1 - \left(\frac{k_4}{k_5}\right)^* T(\cdot) - i \frac{M}{\alpha} T^{*}(\cdot)\right) = \left(1 - \frac{1}{2} i k_3\right)^* \left(\Phi_+ (\cdot) - \Phi_- (\cdot)\right)$$

where $\Phi_+ = \frac{1}{2} k_4 + T - k_5$. 

2. Making the kernel smooth

We shall take the driving term $J$ to be $J = \frac{1}{2} i k_4 + k_5$ with $\eta = \frac{1}{2} k_4^* J_\eta$. The $\Delta_t$ factors ($\Delta_t = \frac{q_t^\ell}{\beta^t}$) cancel the threshold behaviour of the phase space integrals. Equation (3) is, however, rather singular when $k_\eta$ are pole terms, i.e. when we deal with resonances. We shall recast it into an equivalent form with a smooth kernel.

It is natural to seek solutions of (3) of the same form as $J$: namely $T = \sum \frac{1}{\Delta_t} H_1 T_i$ so that

$$\frac{1}{\Delta_t} \left[ \frac{1}{2} H_1 \tau^{*} T_i + \frac{1}{2} \frac{H_2 \tau^{*} T_i}{\Delta_2} + \frac{1}{2} \frac{H_3 \tau^{*} T_i}{\Delta_3} + \frac{1}{2} \frac{H_1 \tau^{*} T_i}{\Delta_1} \right] = \left(1 - \frac{1}{2} i k_3 \right)^* \left(\Phi_+ (\cdot) - \Phi_- (\cdot)\right)$$

substituting (2) yields

$$\left(1 - \left(\frac{k_4}{k_5}\right)^* T_d + \frac{1}{2} \frac{k_3^* \left(\frac{1}{\Delta_2} H_2 \tau^{*} T_i + \frac{1}{\Delta_3} H_3 \tau^{*} T_i\right)}{\Delta} \right) + \frac{1}{2} \frac{k_3^* \left(\frac{1}{\Delta_1} H_1 \tau^{*} T_i + \frac{1}{\Delta_3} H_3 \tau^{*} T_i\right)}{\Delta_3}$$

When $J$ has the previous form $\frac{1}{2} k_4 T_\eta$ one solution is

$$T_d = \frac{1}{2} \frac{H_1 \tau^{*} T_i}{\Delta_1} + \frac{1}{2} \frac{H_2 \tau^{*} T_i}{\Delta_2} + \frac{1}{2} \frac{H_3 \tau^{*} T_i}{\Delta_3} + \frac{1}{2} \frac{H_1 \tau^{*} T_i}{\Delta_1} \left(1 - \frac{1}{2} i k_3 \right)^* \left(\Phi_+ (\cdot) - \Phi_- (\cdot)\right)$$

immediately suggested: $T_d = \frac{1}{2} \frac{H_1 \tau^{*} T_i}{\Delta_1} + \frac{1}{2} \frac{H_2 \tau^{*} T_i}{\Delta_2} + \frac{1}{2} \frac{H_3 \tau^{*} T_i}{\Delta_3} + \frac{1}{2} \frac{H_1 \tau^{*} T_i}{\Delta_1} \left(1 - \frac{1}{2} i k_3 \right)^* \left(\Phi_+ (\cdot) - \Phi_- (\cdot)\right)$

The new kernel is very convenient for computation purposes, and is reminiscent, formally, of the full solution to our problem in the non-relativistic case.*

III. CONNECTION WITH THE ISOBAR MODEL

We shall see later on that the right-hand side of (4) is usually small. At order zero: $T_k = J_k$, which is the isobar approximation if the partial waves of $J_k$ have just the minimum variation required by $\alpha \gamma^\ell \bar{\gamma}^\ell$ at thresholds (centrifugal barriers).

At first order: $T_d = J_d + \frac{1}{2} i A_d \sum_{k \neq d} \left(\frac{1}{2} \frac{H_1 \tau^{*} J_k}{\Delta_1} \right)$

1. Partial Waves

The effect of the correction terms are best seen in a partial wave analysis. As usual (Ref. [2]) we define

$$\Sigma_{\ell = 0}^{\infty} \sum_{CINH} \sum_{\ell = 0}^{\infty} \left(\frac{q^\ell}{q^\ell}\right) \left(\frac{Y_\ell(\alpha_t)}{l \eta}\right) \left(\frac{Y_\ell(\alpha_t)}{l \eta}\right)$$

which is the spherical harmonics expansion for subsystem $\ell$. $\ell = n$ is the incoming state, and other factors are explained in the text.*

* While this work was in progress, a considerable simplification of the non-relativistic case has been achieved in a pre-print of R.T. Cahill (Australia National University of Canberra)

* : The unexpected $\left(\frac{q^\ell}{q^\ell}\right)$ stems from the later multiplication by $\bar{\eta}$.
appendix. When \( \gamma_j \) is written in the same way with reduced amplitudes \( \tilde{\gamma}_{jl} \), the quantities \( \tau, \gamma \) are free from kinematical singularities and obey

\[
\tau_j \tilde{\gamma}_{jl}(s) = \frac{1}{2} \sum_{q_i, q_j} \left[ \frac{d}{d^4 q_i} \frac{d}{d^4 q_j} \right] I_j(s) J_j(s)
\]

where \( I_j(s) = \Gamma / \left( \sqrt{4s_j - 1} \right) \) in the resonance approximation. Within our conventions, recouplings should be as follows: \( \langle p, q_j \rangle J_j \rangle = \langle p, q_j \rangle J_j \rangle \). Only Clebsch Gordan coefficients and spherical harmonics appear in the last two scalar products, defined with the z axis normal to the Dalitz plot. As \( \tilde{T}_{jl} \) will be multiplied by \( \tilde{H}_{jl} \), only states with \( \tilde{N}_{jl} \neq 0 \) need to be considered.

2. Two Examples

a) The \( A_4(1270) \rightarrow J^+ \)

On Fig. 1a) we plot the amount of correction from the \( A_4 \rightarrow \pi \pi \) branching in system 2 upon the \( A_4 \rightarrow f \pi \) mode in system 1, assuming \( \gamma_f = \gamma_\pi = 1 \). Fig. 2a) shows the effect of this correction on contour lines of the Dalitz plot if we take for granted the results of D.V. Brockway [3] from a partial wave analysis (\( \gamma_f = 4, \gamma_\pi = 2 \)).

We observe a small modification in the overlap region.

* We stress that we only use these results for illustrative purposes, without worrying about the difficulties of the partial wave analysis.

b) \( \vec{p} \pi \rightarrow n^+ \pi^- \pi^- \quad \text{at} \quad 2.5 \text{ GeV} \quad \text{in the center of mass} \)

Approximate starting values were found from a first study by Ron Huesman (LBL), with production of \( \rho(750), \pi^0(1250) \), and \( \pi^0(1750) \). Here again, as can be seen from Figs. 1b), 2b), corrections lead to very small effects. Fig. 3 shows the experimental distribution (Ref. [4]) for comparison.

IV. EXTENSION TO HIGHER ENERGIES

It is an experimental fact that most events have at least one small associated mass, which justifies taking as ansatz \( J = \sum_{\lambda_1, \lambda_2, \ldots} \lambda \gamma_\lambda \left( 1 + \lambda \right) c^{\lambda \phi_\lambda} \left( \lambda \right) \frac{d}{d^4 s} \frac{d}{d^4 t} \frac{d}{d^4 u} \) in equation (4). The derivation is no more valid, since other discontinuities in \( S \) are now present, associated with inelastic channels, but the error thus made is likely to be even smaller than the overlap correction.
ACKNOWLEDGMENTS

Ron Huesman kindly provided us with his first fits. We also thank G.C. Fox for challenging us to solve this problem, Professor A.H. Rosenfeld for his constant interest, and Professor H. Stapp who straightened the errors, discussed the manuscript, and encouraged us on the way.

APPENDIX

We recall our notations: $Q_1$, $q_1$ are the lengths of the momentum $\mathbf{Q}_1$ of $(\mathbf{Jk})$ in the center of mass, and of its decay momentum $\mathbf{q}_1$, respectively. $\Omega_1, \omega_1$ are the directions of $\mathbf{Q}_1, \mathbf{q}_1$. \[ \langle \mathbf{p}, \mathbf{p}_1 | J \mathbf{h}, \mathbf{J} \ell, \mathbf{s}_1, \mathbf{s}_1' \rangle = \frac{1}{Q_1} \sum_{k=m} \frac{Y_{k}^{m}(\Omega_1)}{Y_{k}^{m}(\omega_1)} \cdot \delta_{33'} \delta_{NN'} \cdot \mathbf{H}_{\alpha_{1}1} \]

Furthermore, we use in the partial wave expansion factors $Q_1, q_1$ inside "centrifugal barriers". These terms were chosen so that in average, the kinematical factor would be of the order of unity.

REFERENCES


FIGURE CAPTIONS

Fig. 1: Ratio of the overlap corrections to the amplitudes assuming all amplitudes are equal to unity.
   a) Effect of $\sigma$ on $\rho$ amplitude b) effect of $\rho$ on $\ell^6$ amplitude

Fig. 2: Contour lines of the density in the Dalitz plot associated to $\Lambda_4 \rightarrow \pi^+\pi^-\pi^-$ at 1.07 GeV
   a) without correction. b) with the overlap correction.

Fig. 3: Contour lines of the density in the Dalitz plot for the reaction $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$ at 2.15 GeV a) without correction. b) with the overlap correction.

Fig. 4: Experimental distribution of the events in $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$ at 2.15 GeV.
Fig. 1

(a) \( J^P = 1^+ \)
\[ W = 1.07 \text{ GeV} \]
\[ \langle p | \sigma \rangle \]

(b) \( J^P = 2^+ \)
\[ W = 2.15 \text{ GeV} \]
\[ \langle f^0 | \rho \rangle \]

Fig. 2

Fig. 3
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.