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KAES: An Expert System for the Algebraic Analysis of Kinship Terminologies

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INTRODUCTION

In this paper we describe an algebraic approach to the analysis of kinship terminology structures and a computer software program called Kinship Algebra Expert System, or KAES (Read and Behrens, n.d.), which will guide the user in such an analysis. Though this statement is an accurate summary of what the paper is about, neither the term algebra nor the reference to a computer program that will model kinship terminology structures and a computer software program called Kinship Algebra Expert System, or KAES (Read and Behrens, n.d.), will bring up images of various application programs—wordprocessing, spreadsheets, and so on, or perhaps computer based symbolic notation schemes for kinship terms—and the reference to a computer program that will model the logic of, and to compare, kinship terminology structures.

Examples of models produced through the KAES program are discussed, along with their theoretical implications.

KEY WORDS: kinship, algebraic analysis, computer modeling, cognitive anthropology, artificial intelligence

ABSTRACT: In this paper we discuss a new algebraic approach for analyzing kin terminology structure and implementing the algebraic approach. A key aspect of our algebraic analysis is a shift away from a genealogical orientation to one of viewing a kinship terminology as a structured, culturally defined conceptual system. The basic idea is that a kinship terminology can be viewed as a structure consisting of a set of symbols (kin terms) interconnected through a binary product of kind terms subject to certain structure defining equations.

Because algebraic modeling uses a language unfamiliar to many anthropologists, we are developing a computer program, KAES, based on (1) the expertise marshaled by a mathematical anthropologist when deriving algebraic solutions, and (2) the knowledge used by a cultural anthropologist for relating kin terms as part of a logical system. The program KAES will provide the user with the capacity to creatively work with abstract algebras as a means both to model the logic of, and to compare, kinship terminology structures.
statistical procedures — not abstract symbol manipulation. But the logic of structures and the means for symbol manipulation are the two key concepts involved here.

That kinship terminologies have an internal logic and thus reflect a deeper structure than the surface level of kin terms and relatives to whom terms may be properly applied has been repeatedly noted from early on by kinship theorists (e.g., Morgan 1871; Tax 1937; Radcliffe-Brown 1950; Kroeber 1952), and it is towards elucidation of this logic that the algebraic modeling is aimed. The modeling is grounded in the concepts of modern algebra — concepts aimed at exposition of the properties of structures that result from rules defining how symbols may be manipulated and combined together. The goal of the algebraic analysis, then, is to capture and express the logic of terminology structures through constructing algebraic structures isomorphic to kinship terminology structures. The means is through symbolic representation and manipulation according to certain rules. Here is where software becomes important. The software serves as a means both to carry out abstract, symbolic reasoning and to present the structures defined thereby in the more familiar idiom of a graph — be it that of a kinship terminology as the target structure for the analysis, or the algebraic model produced by the analysis.

Modeling kinship terminology structures with abstract algebras is a powerful means for elucidating the properties giving kinship terminologies their structural form as we illustrate with analyses of the American, Shipibo, and Trobriand terminologies using KAES. The end goal, though, is not merely to produce algebraic representations of kinship terminology structures. The latter, we suggest, allows one to make the comparative study of kinship terminologies more effective and provides a basis for building a general theory of kin terminology structures as a kind of cognitive construct.

PART I. THE DOMAIN OF KINSHIP ANALYSIS

Previous analyses of kinship terminologies have used different analytical domains and, therefore, different definitions of what one treats as “data.” Standard approaches have sought structuring principles for kinship terminologies in a model based on procreation formed through examining how kin terms group kin types arranged in a genealogical grid, making kin types the basic data. Some authors, however, have viewed kin terms as part of a conceptual structure definable without necessary reference to other structures, and have found it advantageous to disentangle terminological data from genealogical data (Leaf 1971; Read 1976, 1984; Read and Behrens 1990) and to consider kin terms as constituting an internally structured system separate from reference to a genealogical space. While both the genealogical and the terminological approaches have spawned formal methods aimed at elucidating and expressing the structure of kinship terminologies, different concepts, and hence different properties to be expressed in a formal representation, are involved. Before discussing the KAES program and our algebraic modeling of a kinship terminology structure, we need to clarify what constitutes these differences.

GENEALOGICAL APPROACHES

For some anthropologists the study of kinship has been inseparable from genealogy, with information on the classes of kin types to which kin terms are applicable serving as the basis for analysis of terminological structure (e.g., Scheffler and Lounsbury 1971). Genealogies have also been seen as fundamental for eliciting kin terms by asking informants for the terms that would be applied to members of his/her genealogy (RAI 1971). And classifications of terminologies, such as Murdock’s (1949) system using cousin terms and Lü’s (1986) more recent system using an algebraic structure known as a quotient monoid, are based on the groupings imposed upon kin types by kin terms.

Correspondingly, most analytical methods aimed at elucidation of terminological structure, such as componential analysis (e.g., Goodeough 1956; Wallace and Atkins 1960; Romney and D’Andrade 1964), rewrite rules (e.g., Lounsbury 1964a, 1964b, 1965; Scheffler and Lounsbury 1971) and representation schemes (e.g., Atkins 1974), consider properties that emanate from the structure imposed on a genealogical space when kin terms are given kin type definitions such as Grandmother is mother’s mother or father’s mother. Most algebraic modeling (e.g., Boyd 1969, Boyd et al. 1972; Lehman and Witz 1976, 1979; Liu 1986; Lucich 1987) and computer based implementations (e.g., Kronenfeld 1976; Ottenheim 1989) have accepted the genealogical foundation as the appropriate basis for formal modeling, even if only to serve as a kind of etic grid. Scheffler and Lounsbury (1972: 69–70), however, have rejected the etic grid viewpoint and have made the stronger claim that genealogy is conceptually primary and analytically necessary. Others (Leaf 1971; Schneider 1972, 1984, 1986; Read 1984, 1986) have disputed the basis for this strong claim that kinship terms are essentially a system for classifying genealogical relationships by arguing, in effect, that merely because it is possible to map kin terms onto kin types does not make a terminology primarily a system for classification of kin types. These authors have, in varying ways, suggested that a kin terminology involves a conceptual framework more extensive than that entailed by genealogy and, even though genealogical relations may serve as a model, a framework that has properties transcending those of genealogy. Schneider (1972, 1984, 1986) has extended
this argument to a strong counterclaim in which it is disputed that "kinship as genealogy" even exists and if it exists, it must be demonstrated on a case by case basis.

Analytically, the strong version of the genealogical claim; i.e., that analysis must necessarily proceed using the language of kin types (Scheffler and Lounsbury 1971: 69), is deficient since none of the methodologies based on genealogical spaces can account for the existence of different structures when a single, common underlying structure, the genealogical space, is asserted to be the common framework for all terminologies. A single space cannot account for different structures, hence those differences must owe their origin to other, non-genealogical, concepts. When the methodologies aimed at explicating terminological structure in terms of genealogically defined properties are examined in detail it is evident that elucidation of structure is not fully achieved. Componential analysis, for example, only provides a parsimonious description of structure expressed using attributes based on a genealogical space, and rewrite rules presume a structure for the kernel meanings of kin terms without elucidating how that structure can be explicated. Elucidation of structure requires that the principals upon which structure is constructed be uncovered, and to do that we need to examine the way in which a kinship terminology forms a structure in its own right, not merely as a structure that can be mapped onto a genealogical space. Understanding the structure of a terminology in terms of its internal logic will, we suggest, lead to a more secure, and more profound, basis for comparison of different terminologies and thereby add to our understanding of the nature of culturally defined conceptual constructs.

The analytical separation proposed here does not imply that there is no connection between a genealogical space structured according to "universal regularities in the way people trace genealogical relationships and make genealogical claims" (Lehman and Witz 1974: 113, emphasis in the original) and kinship terminologies, for trivially kin terms are applied to persons for whom genealogical claims may be made. Rather, the distinction being made is between, on the one hand, viewing kin terminology structure as a system for categorization of kin types, with the terms serving as semantic labels for those categories — a viewpoint that justifies placing emphasis on rules specifying how a category can be recovered from one of its members via rewrite rules — or, on the other hand, viewing kin terminology structure as a structured system in its own right, hence placing emphasis on the generation and construction of structure — the topic of this paper.

If analysis of terminology structure is to be separated from a genealogical space as the reference point, then one must be able to represent kin terms analytically as a system of interrelated abstract symbols and then develop appropriate "machinery" for modeling the properties of that system. The former has been addressed with a kin term map and the latter with the machinery of abstract algebras, in particular, semigroups, in the terminological approach.

**TERMINOLOGICAL APPROACH**

An alternative to assuming the "universality" of a genealogical grid as a native conceptual framework is to disentangle the genealogical and terminological domains and to analytically consider a kinship terminology as a (cultural) construct in its own right (e.g., Read 1976, 1984, 1986) whose reference in use may, but need not necessarily, entail genealogical relationships. Here, structure is sought not in the way a genealogical space is structured by kin terms, but through the internal logic giving a terminology its form; i.e., the relationships among kin terms taken as linguistic objects and determined by deeper structuring principles.

One means for graphically representing the structure of a kinship terminology viewed as a cultural construct is with a kin term map (Leaf 1971, 1974). A kin term map lists each kin term only once as a node, and connections are made between nodes to indicate linkages among the kin terms. Figure 1 shows a kin term map for the American Kinship Terminology (AKT). In this map the kin terms Mother and Father, for example, each label a single node and each is linked to another node with the label "Etc."

![Fig. 1. Kin term map for the American Kinship Terminology.](image)
Grandfather. The connections in this map are conceptual linkages, not genealogical ones. This contrasts with a genealogical chart where one would label two genealogical positions, the kin types, mother's father and father's father, with the kin term Grandfather, and the connections between kin terms represent genealogical linkages.

An algebraic account for a terminology takes the kin terms as objects (nodes in the kin term map) and the kin term map as a structure to be generated. The means for generating a structure include a binary operation that maps pairs of symbols to other symbols and certain structural equations that specify the properties of the binary operation. The binary operation is based on a kin term product defined as follows (Read 1984; see also Kronenfeld 1984):

If ego (properly) calls alter, by the kin term K and alter, (properly) calls alter, by the kin term L, then the product of L and K (denoted \( L \times K \) and read 'L of a K') is a kin term M, if any, that ego (properly) uses for alter.

For example, if \( K \) is the kin term, Father, and \( L \) is the kin term, Mother, then \( L \times K = \) Mother of a Father = Grandmother. That is, if ego calls alter, by the kin term, Father, and alter, calls alter, by the kin term, Mother, then ego (properly) calls alter, by the kin term, Grandmother. Note that this is not an assertion about genealogical positions, but refers only to kin term usage.

With respect to the kin term product, some terms in a terminology, such as Father in the American kinship terminology (AKT), are "atoms" (indivisible) and others, such as Grandmother, or Mother of a Father, are "compound" (products of atoms). The kin term product may thus be viewed abstractly as a binary operation that acts on atoms to construct compounds. At the level of symbols, the binary operation combines atoms together (e.g., MOTHER, FATHER) to form compounds, or "words" (e.g., MOTHER \( \times \) FATHER) that, within the terminology, have linguistic labeling (e.g., Grandmother). These compounds constitute the symbol forms that can be generated from the set of atoms.

A particular structure is described through identifying kin term equations that can be used to produce the structure. These equations reduce certain words to other words or to atoms. The choice of equations is motivated by relations that are valid when the kin terms are given proper interpretation and applied to egos and alters. For example, if CHILD, PARENT, and SELF are atoms, and if, in ego's consanguineal space, ego (properly) refers to alter, as Child and alter, (properly) refers to alter, as Parent, then it follows that ego (properly) refers to alter, as Self since alter, is ego. This structural property can be expressed using the atoms and the binary operator, \( \times \), in the following equation:

\[
\text{PARENT} \times \text{CHILD} = \text{SELF},
\]
or restated using kin terms, Parent of a Child equals Self. In the structure formed through combination of symbols via the binary operation, \( \times \), whenever the product PARENT \( \times \) CHILD appears, it is replaced by the atom, SELF, thus producing a structure in which the above equation becomes a structural property. (The importance of this equation for the structure of the AKT will be shown below in Part 3.)

The algebraic approach is inductive, in the sense of determining what equations should be used, but leads to structures logically derivable from postulated atoms and structural equations; i.e., it utilizes the axiomatic method. The choice of atoms and equations is either confirmed or disconfirmed by considering the structure they define. If that structure diverges from the target structure (displayed as the kin term map), then different atoms and equations should be postulated and the derivation repeated until the algebraic structure converges on the target structure. The means for generating and comparing these structures is provided by the KAES program.

PART 2. THE KAES PROGRAM

While the analysis of kinship terminologies represented as abstract algebras is both elegant and exact, the algebraic approach often requires a degree of mathematical background that is not common to most anthropologists. To eliminate this technical "bottleneck," we developed an AI based computer program that recreates the mathematical anthropologist's derivation of algebraic structures and generates graphical representations of the algebraic structures, thereby making them visually comprehensible.

WHY AN EXPERT SYSTEM?

Within the last two decades, "expert" or "knowledge based" systems that use computers for heuristic modeling have developed from artificial intelligence research (Duda and Shortliffe 1983; Weiss and Kulikowski 1984; Charniak and McDermott 1985; Harmon and King 1985). Expert systems differ from conventional "number-crunching" programs in that they draw conclusions through logical or plausible inference, not through numerical calculation. Built into these software systems are the same kind of rules employed by human experts when they make decisions in their fields of expertise and so are designed to help one solve problems in a "commonsense" manner.

One problem domain where expert systems have been successfully applied is the inference of structure (Dietterich and Michalski 1983). For example, DENDRAL takes a set of spectrographic data and enumerates.
all the possible molecular structures that can account for these data (Buchanan and Feigenbaum 1978) and R1 (sometimes called XCON) configures VAX computers, another form of structure, from a list of components (McDermott 1982). The mathematical analysis of kinship terminology structures seems equally suitable for an "expert system" type treatment: The problem domain is well-specified and the solution space is small; reliable knowledge and data exist for many kinship terminologies; they are relatively time invariant; formal analysis requires only a single line of reasoning at a time; and the algebraic approach provides strong problem solving techniques together with a language which easily permits symbolic representation (cf. Stefik et al. 1982; Hayes-Roth et al. 1983).

SOFTWARE DESIGN CONSIDERATIONS

A number of important choices were made while designing KAES. Some of these addressed problems of representing abstract algebras with appropriate computer language structures. Other decisions concerned performance issues and the need for software portability.

Data Representation

The mathematical properties of kin term algebras and the inductive process whereby kin terminology structures are generated require a computer language that allows one to replicate this abstract process. Atoms in an algebra are symbols and these can be concatenated to build words which can be replaced with other words, and so on. Thus, this application requires a computer language that distinguishes symbols and lists of symbols as legitimate data types, not merely character strings that are manipulated in a character by character fashion.

Furthermore, because the correct algebraic structure for a terminology is not known in advance, one needs to search through potential solutions until a structure isomorphic with the kin term map is discovered. The search must be exhaustive and requires that one describe both the problem and a "sufficient" search, often recursively.

PROLOG was the computer language selected for writing the algebra constructor in KAES. PROLOG’s symbol and list processing capabilities are well developed and has built-in search algorithms. This language has already been applied by others for developing computer based mathematical reasoning (e.g. Bundy 1982).

One advantage of PROLOG for KAES is its manipulation of data types in a kinship algebra (such as symbols, words, word lists, and so on) without having to convert these to another form (such as character strings). For example, the following PROLOG representations were given to parts of an algebraic structure:

<table>
<thead>
<tr>
<th>Algebraic Form</th>
<th>PROLOG Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator Set:</td>
<td>generators ([FA], [BR])</td>
</tr>
<tr>
<td>Equation:</td>
<td>equation ([FA, BR], [FA]).</td>
</tr>
</tbody>
</table>

This example, whose content will be used when developing the structure of the Trobriand terminology in Part III, compares an algebraic representation to another form based on the idiom of PROLOG. Here, the algebraic facts (left column) have been represented in PROLOG as facts in predicate form (right column); i.e., as ordered n-tuples of objects included in the relations named generators and equation, respectively. The symbols FA and BR are shortened versions of FATHER and BROTHER, respectively, introduced for convenience and the square brackets delineate a list of symbols (see Clocksin and Mellish 1984; Bratko 1986; Sterling and Shapiro 1986 for extensive discussions of PROLOG and its syntax).

PROLOG also lends itself neatly to translation of rules and recursive definitions, both of which are common in algebraic systems and form a fundamental search condition in PROLOG. For example, the algebraic concept, word, is defined recursively: each atom is a word, and if w and w’ are already defined words, then w = w’ w” is also a word. A PROLOG program segment corresponding to this recursive definition of words, and in the context of the AKT, would be:

(1) atom([PARENT]). atom([CHILD]). atom([SELF]).
(2) word(X) IF atom([X]).
(3) word(W) IF word(U) AND word(V) AND product (U, V, W).

The first line declares that the symbols PARENT, CHILD, and SELF are atoms. The second line declares that a symbol, X, is a word if the symbol is an atom. The third line states that if U and V are words, then the word W, which is the product of U and V, is also considered to be a word. The predicate product has its own, separate PROLOG definition and declaration (not shown) that asserts W will be the list formed from the lists U and V through concatenation. By backtracking through a search tree, PROLOG would use the above rules to find all solutions satisfying this definition, namely: PARENT, CHILD, SELF, PARENT × PARENT, PARENT × CHILD, PARENT × SELF, CHILD × PARENT, and so forth.

User Interface

Perhaps the most important consideration in the design of KAES was to make the symbolic manipulations used in the algebraic analysis "trans-
parent" to users. To accomplish this, we used graphical representations of algebraic structures to make their comprehension and comparison easier.

This required a procedural language that offered fast, high-resolution bit-mapped graphics, windowing capabilities, and, preferably, mouse tracking. Turbo Pascal (Borland 1988) was selected for its modularity, execution speed, and widespread use. Moreover, a library of object-oriented graphical procedures and functions called "MetaWindows" (MetaGraphics 1986) is available that provides all of the above mentioned features as an extension of Pascal. MetaWindows also queries a computer's hardware configuration at run-time and loads all of the necessary device drivers required for color graphics output and mouse tracking.

Performance and Portability

At the time we began the programming, some of the then available symbolic languages were only interpreted, rather than compiled, and so executed relatively slowly. We wanted an implementation of PROLOG that offered a compiler and comparatively fast execution speeds as well as an easy to use editor. For these reasons, we decided to develop the KAES algebra constructor using Turbo PROLOG (Borland 1986).

An important motivation for developing KAES has been to make the expertise required for algebraic analyses of kinship terminologies available to a wider community of anthropologists. Therefore, another major concern during the development of KAES has been software portability. We decided to host our system on IBM DOS type machines because of MetaWindows's capability to query hardware configurations at run-time and by compiling KAES's Turbo PROLOG and Turbo Pascal modules into executable files, it is possible to distribute our system among the greatest number of users without the cost of royalties for run-time libraries and special device drivers.

SYSTEM ARCHITECTURE

Because of the two languages implemented in KAES, the program has a "split personality." This section details the design and specific tasks performed in each language environment, as shown in Figure 2.

Prolog Environment

PROLOG has been used to build the algebra constructor and to manage "facts" in a data base of kinship terminologies. Within this environment, the KAES program is separable into an inference engine assisted by an algebra constructor and a knowledge base designed in accordance with most expert system architectures.

Our algebra constructor exploits the built-in backtracking of PROLOG whereby a variable in a PROLOG clause is identified as a goal, then ancestral clauses (containing variables identified as subgoals) are evaluated to establish values that satisfy the goal. By using this inference strategy, KAES is able to examine a search tree in depth until it either finds all solutions that satisfy a goal or determines that no solution exists. The PROLOG instructions that manage all symbolic manipulation, e.g., word matching, word replacement, and construction of word lists, are built into KAES and augment the inference engine. Information input by the user is automatically implemented in the form of PROLOG clauses used to construct an algebra for the targeted kinship terminology, consistent with existing facts in its data base.

Two kinds of expertise are encoded in the knowledge base: (1) rules that control the order in which problem-solving tasks are conducted, such as when new structural equations are entered or isomorphic copies of an algebra (see below) are generated, and (2) rules that guide a user in the choice of atoms, equations and the like. The latter kind of rules sometimes queries the user for information they already know about a terminology, e.g., Does the terminology have a term, such as Self in the AKT, that acts
like an identity element? At other times KAES suggests equations needed to maintain certain properties in the algebra, e.g., Should the reciprocal position for an already entered kin term also be given symbolic representation? Once the user responds to these prompts, the information is passed on to the algebra constructor.

PROLOG is also used to perform bookkeeping tasks. As an algebra is derived for a kinship terminology, a formal account is written to a disk file. The algebraic data base for a kinship terminology is used to retrieve formal (mathematical) descriptions of terminologies, to graph terminologies, and to compare the graphs for different terminologies.

**Pascal Environment**

Pascal procedures are used for screen management and to supervise all user-KAES dialogue. We have built a user interface that manipulates objects such as pop-up menus, graphics windows, and text windows. Once an object is declared, it is possible to manipulate it as a program unit or to create new forms of the object merely by supplying the object with values for the arguments needed in its definition. For example, by defining a pop-up menu to be a distinct class of objects, it is possible to easily generate a new form of pop-up menu which differs only in the text drawn to it, its number of selections, or its location on the screen. These software tools have enabled us to develop generic graphics procedures that can be rapidly drawn with either local or virtual coordinate systems, making possible graphics rotation, rescaling, and zoom-in/zoom-out effects.

A user may interact with KAES by "clicking" a mouse to select from a menu, to accept one of KAES's suggestions, or to input text from the keyboard. Text is parsed to extract the information needed for drawing graphical representations of algebraic structures stored in the Turbo PROLOG database described above.

**File Interface**

For KAES to execute all of its tasks, it is necessary for the Turbo PROLOG and Turbo Pascal modules to communicate with one another. This is accomplished through a file interface. The information produced in Turbo Pascal is written to a virtual disk file (for speed of access) where it can be read as input to Turbo PROLOG modules, and vice-versa.

With these aspects of the KAES program in mind, we may turn to analysis of kinship terminology structure using KAES.

**PART 3 ALGEBRAIC ANALYSIS WITH KAES**

As we have noted, the target structure to be produced through the algebraic analysis is the kin term map for the terminology. The kin term map empirically illustrates the form of the structure linking kin terms together, but does not inform us about the internal organization of that structure. For this we turn to algebraic modeling.

To be answered with the algebraic analysis of a terminology are the questions: Can this target structure be generated from a more limited set of information?; and if so, What is this set of information? By answering these questions for a variety of terminologies a more comprehensive picture of the nature of kin terminology structures will be obtained.

Our discussion in this section will be organized around these steps and the results they produce as illustrated by the KAES program screen output. When clarification is needed, details of the algebraic manipulations performed by the computer will be given.

Three kinship terminology models will be constructed. The first model, for the American terminology (as described in Read 1984), was initially developed as an exercise in algebraic modeling and has sewed as a guide for the development of the KAES program. A second model, for the Trobriand terminology (as described in Lounsbury 1965), has been developed both as an exercise in algebraic modeling and through the KAES program. The third, for the Shipibo terminology (as described in Behrens 1984), has been developed primarily through the KAES program.

Underlying the operation of the KAES program is a theory of how kinship structures can be generated. The KAES programs signals the user about properties that need to be introduced for the construction to be consistent with that theory. As a consequence, the KAES program is more than a collection of tools that aid in algebraic modeling of kinship terminology structures. It is also an exposition and test of an underlying theory. Only part of that theory will be illustrated here, namely the role of reciprocity in the production of kinship terminology structures. As will be seen below, reciprocity—ego has a kin term for alter and alter has a reciprocal kin term for ego—is fundamental to constructing an algebraic model for what we call the core structure of a terminology. We now illustrate the modeling produced through the KAES program for the core structure of the American Kinship terminology.

**MODEL 1: AMERICAN KINSHIP TERMINOLOGY**

**Simplification and Core Structure**

The core structure is derived from the kin term map and is a simpler structure from which the complete, complex, terminological structure can be produced (see Read 1984 for an example that relates the core structure to the kin term map for the AKT). The AKT kin term map (see Figure 1)
was simplified by (1) separation of the consanguineal from the affinal structure and (2) replacement of kin term positions with sex markings to ones without sex marking (Read 1984). Both operations can be performed by KAES. The resulting structure — the core structure — is shown in Figure 3 and is the initial, target structure for the algebraic modeling of the AKT. That is to say, with the help of KAES we want to find an algebra whose graph is structurally isomorphic with Figure 3.

Identity Element and Generator
The analytical part of the KAES program begins by requesting specification of what kin term, if any, should be an identity element under the kin term product, what term(s) serves as a generator(s) and what, if any, will be the initial structural equations. The user need only identify the minimal set of information necessary for the construction to begin.

For the AKT, it suffices to take the term Self as an identity element and the term, Parent, as a generating element. No structural equations are introduced at this point, except those produced automatically by the program in order to define an element as an identity element. These equations are (stated in terms of PARENT and SELF):

1. \[ \text{PARENT} \times \text{SELF} = \text{SELF} \times \text{PARENT} = \text{PARENT} \]

We now introduce, for the purposes of this exposition, a notational convention and simplification for helping keep clear the fact that 'structure, from the algebraic viewpoint, is a consequence of logical relations amongst objects independent of labeling. Henceforth we will use single capital letters, rather than the capitalized form of the kin term, for labeling objects (i.e. terms) in an algebra. Within the KAES program, though, the user may introduce whatever symbols are convenient for representation of kin terms. With this notational convention, we can restate the generators and equations identified above by specifying

\[ G = \{P, I\} \]

to be the generating set, where I is an identity element, and the equations defining I as an identity element are the following equations:

1. \[ I \times P = P \times I = P \]

and

1. \[ I \times I = I. \]

(Henceforth, we will usually write XY in lieu of X \times Y and leave the operation “\times” implicit.) The algebra generated by G as produced by the KAES program is shown graphically in Figure 4.

While the structure captures the sense of backward extension expressed by the ancestral terms Parent, Grandparent, GreatGrandparent, etc., through the algebraic words P, PP, PPP, etc., a key part of what distinguishes kin terminologies as a structural form is missing. The missing part is the pairing that may be made with reciprocal terms: Parent with Child, “Sibling” with “Sibling” (self-receiprocal), and so on.

Reciprocal Terms
Though one could include a list of terms and their reciprocals as native knowledge to be incorporated in the construction process, we want to penetrate deeper into the structure and explore the role of properties such as reciprocity as a principle for the production of structure. To do so, a structural criterion is needed for knowing when for one term, call it Term,, another term, Term,, is the reciprocal of the first. We begin by specifying what is meant by reciprocal terms:

Reciprocal terms: If ego has a kin term, Term,, for alter and if alter has a kin term,
Fig. 4. Algebraic structure constructed from the generating set for the AKT. Here, and in the other figures, the numbers indicate the number of times the atomic element is "multiplied" together. Thus the node marked with a "1" corresponds to the atom \( P \), the node marked with a "2" corresponds to the compound \( P \times P \), and so on.

Term, for ego, hence making the following diagram valid, then Term, and Term, are reciprocal terms:

\[
\text{Term,} \quad \frac{\text{ego}}{\text{Term,}} \quad \frac{\text{Term,}}{\text{alter}}
\]

Now by definition of the kin term product, the product Term, \( \times \) Term, satisfies the diagram:

\[
\text{Term,} \quad \frac{\text{alter}_1}{\text{Term,}} \quad \frac{\text{Term,}}{\text{alter}_2} \quad \frac{\text{Term,} \times \text{Term,}}{\text{Term,}}
\]

In this diagram ego \( \neq \text{alter}_1 \), for if ego = alter, then Term, \( \times \) Term, would be the identity element, I. But it cannot be the identity element for if it were then the structure would be too restricted; i.e., the AKT structure would have the equation PARENT \( \times \) CHILD = SELF = CHILD \( \times \) PARENT and this would imply that the terminology structure consists only of lineal terms. Nonetheless, this gives us a clue as to the structural condition that Term, and Term, should satisfy if they are to behave in the algebra like reciprocal terms.

Were it the case that TERM, \( \times \) TERM, = TERM, \( \times \) TERM, = SELF, then TERM, would be an inverse element for TERM, under the product "\( \times \)". We can relax the criterion of an inverse slightly and use in place of an inverse element the notion of a semigroup inverse for defining reciprocals. In a semigroup \( S \), \( y \) is a semigroup inverse of \( x \) if, and only if,

\[
(5) \quad xyx = x
\]

and

\[
(6) \quad yxy = y.
\]

As shown in Read (1984), the semigroup inverse captures the sense of reciprocals for the AKT. However, the criterion is a bit too strong for other terminologies and a slightly weaker condition is needed for the Trobriand terminology (see below). The algebraic concept of an idempotent element seems to provide the needed, weaker, structural property.

By definition, an element \( x \) in a semigroup is an idempotent element if, and only if:

\[
(7) \quad xx = x.
\]

Observe that sibling terms satisfy the idempotent property under kin term products; e.g., BROTHER \( \times \) BROTHER = BROTHER.

If \( y \) is a semigroup inverse element for \( x \), note that the product \( xy \) is an idempotent element, for \( (xy) (xy) = x(yxy) = xy \). Hence if \( x \) has a semigroup inverse \( y \), then \( xy \) is an idempotent element, but the converse need not be true. A product \( xy \) may be an idempotent element but \( y \) not a semigroup inverse of \( x \). An example of such a case will be seen below with the Trobriand terminology.

We use the idea of an idempotent to algebraically define reciprocals:

**Definition:** If \( x, y \in G \), a minimal generating set for a semigroup, \( S \), then \( y \) (alternatively, \( x \)) will be called a reciprocal term for \( x \) (alternatively, \( y \)) if both \( xy \) and \( yx \) are idempotent elements.

Reciprocals Viewed as Forming a Reciprocal Structure

Next, we use reciprocity to define what will be called a reciprocal structure. If one considers the terms Parent and Child in the AKT, the same initial structural form would be produced regardless of whether one used PARENT as the single generating element, or used CHILD in this role. Further, Parent and Child are reciprocal kin terms. These two observations suggest that the core structure (see Figure 3) can be decomposed into a "product" of a structure and an isomorphic structure and
further, these isomorphic structures are linked through the respective generating elements also being reciprocals to each other in this larger structure. (A second situation where a similar construction of a larger structure from isomorphic structures arises will be seen below in the way sex marked terms are introduced into the model for the structure of the Trobriand and Shipibo terminologies). More precisely, a semigroup $S$ with isomorphic subsemigroups $G$ and $H$ (with, say, $i: G \to H$ the isomorphism) having minimal generating sets $G$ and $H$, respectively, satisfying:

1. $G$ and $H$ are not both subsemigroups of any proper subsemigroup of $S$ and (2) each generator $G$ in $G$ satisfies the reciprocal property in conjunction with the generator $H = i(G) \subseteq H$ and conversely, will be said to have a reciprocal structure. In other words, $S$ may be constructed from the sets $G$ and $H$, though possibly including additional equations (condition (1)), and every element in $G$ has a corresponding reciprocal in $H$ and vice-versa (condition (2)).

The method for introducing a reciprocal structure into the algebraic construction using the structural definition of reciprocals given above is as follows. First, given the set $A = \{A_i\}, i \in I$ (I a finite index set) of generators for the semigroup $A$, construct an isomorphic semigroup $B$, with generating set $B = \{B_i\}, i \in I$, where for at least one $i \in I, B_i \neq A_i$. Second, construct $P$, the free product (see Clifford and Preston 1967B: 140-41) of the semigroups $A$ and $B$. The semigroup free product $P$ will be denoted by $P = A \otimes B$ and consists of terms of the form $X_1X_2 \ldots X_n$, where $X_i \subseteq A$ or $X_i \subseteq B$. Third, the semigroup $P$ will be reduced by certain equations, with an equation introduced for each element $A_i$ as needed so as to define the element $B_i$ (the element corresponding to $A_i$ under the isomorphism) as the reciprocal of the element $A_i$. The equation will define $B_i$ as a semigroup inverse for $A_i$, if possible, otherwise it will define the products $A_iB_i$ and $B_iA_i$ as idempotent elements. The semigroup $B$ will be called the reciprocal semigroup for the semigroup $A$.

Now let us apply this procedure to the semigroup, $G$, produced by $G = \{P, I\}$ for the AKT. Let

$$H = \{C, I\}$$

and let $H$ be the semigroup generated by $H$. Now $H$ has the same structure as $G$, except for use of the label "C" in place of the label "P". We define an isomorphism $i: G \to H$ as follows:

$$i(I) = I$$

$$i(P) = C.$$  

Next form $P = G \otimes H$, the free product of $G$ and $H$. A typical term in $P$ is a product of the form $X_1X_2 \ldots X_n$, where $X_i \subseteq \{P, C\}$. Note that $I$ is an identity element for $P$.

In order that $P$ and $C$ should be reciprocals, introduce the equation

$$PC = I.$$  

With this equation, $C$ is a semigroup inverse of $P$ since $PCP = IP = P$ and $CPC = CI = C$. The equation can be rewritten in kin term form as Parent $\times$ Child $=$ Self, which has diagram:

![Diagram]

In the consanguineal space this diagram has the interpretation that alter, is ego, hence the equation is consistent with a genealogical interpretation. Note, however, that the genealogical interpretation only makes the equation plausible and is not validation for its use in the algebraic structure for the AKT. Validation comes through examining the structure produced when this equation is introduced into the algebra and comparing it with the core kin term structure shown in Figure 3.

The equation $PC = I$ is used to replace certain symbol strings in the free product $P = G \otimes H$ by simpler ones. For example, an expression of the form $PC \ldots$ is replaced by $C \ldots$ and $I$ is then eliminated since it is an identity element (with all of these operations carried out in the PROLOG part of the program through symbol matching and replacement).

Comparison of the Algebraic Structure with the Kin Term Map

Visual comparison of Figures 3 and 5 shows clearly that the two structures are isomorphic. Hence the core structure of the AKT can be algebraically characterized by:

$$\text{(12)} \quad \text{Generating set:} \{P, I\}, \text{I an identity element}$$

$$\text{(13)} \quad \text{Reciprocal set:} \{C, I\}$$

and

$$\text{(14)} \quad \text{Reciprocal structural equation:} PC = I.$$  

The corresponding algebraic structure (Figure 5 for the AKT) isomorphic to the core structure will be called the core algebra for the terminology.
MODEL 2: SHIPIBO TERMINOLOGY

For the Shipibo terminology we will demonstrate that its core algebra can be obtained from the core algebra for the AKT merely by introducing a few more structural equations. This contrasts with an initial impression that the Shipibo terminology differs radically from the AKT. The Shipibo terminology is usually classified as Hawaiian in ego’s generation and Sudanese in the 1st generation (see references in Behrens 1984) — not an entirely satisfactory classification. Nonetheless, it is clearly not linked to the AKT by the criteria used in Murdock’s classification scheme. However, as we now show, the Shipibo core structure can be algebraically modeled using the same primitives as for the AKT, hence it would appear that it ought to be linked with the Eskimo terminological type even though it does not exhibit the distinctive attributes of Murdock’s Eskimo type. The anomaly suggests that Murdock’s archetypical classification scheme may be defective in ways not previously mentioned in the literature.

Simplification and Core Structure

Simplification of the Shipibo terminology, whose kin term map is given in Figure 6A, is through (1) separation of kin terms into those used by male versus female speakers followed by (2) separation of affinal from consanguineal terms. The core structure so determined is given in Figure 6B.

Selection of Generators and Structural Equations

We begin the construction with the structure given in Figure 5. Although we will use the same symbols, \( P, C \) and \( I \), for the generating elements of the algebra, these will be given a different interpretation as kin terms. We will interpret \( P \) to be \( \text{Papa} \) ("Father"), \( C \) to be \( \text{Bake} \) ("Son") and \( I \) to be \( \text{Ea} \) ("Male Self"). In other words, we are keeping fixed the structural form, but changing the meaning of the symbols \( P \) and \( C \). Now let us introduce three equations:

\[
\begin{align*}
(15) & \quad \text{CCPP} = \text{CP} \\
(16) & \quad \text{PPPP} = \text{PP} \\
(17) & \quad \text{CPPP} = \text{PP}
\end{align*}
\]

In kin term form, the first equation asserts that "Son" \( \times \) "Son" \( \times \) "Father" \( \times \) "Father" = "Son" \( \times \) "Father", which may be roughly glossed as "Male Cousin" = "Brother" (Huetsa). The second equation may be glossed as "Great Great Grandparent = Great Grandparent" (Papaisi shoko) and the third equation, "Son of Great Grandparent" = "Grandparent" (Papaisi).

Reciprocal Structure

In order to preserve the reciprocal structure, the equation obtained by replacing the product or term on each side of an equation by its reciprocal must also be an equation. For the equation, \( PC = I \), both \( PC \) and \( I \) are self-reciprocal, hence the reciprocal equation is the same as this equation. Similarly, the product \( CCPP \) is self-reciprocal since \( (CCPP) \times (CCPP) = CCPP \), \( CPP \) is self-reciprocal. Hence the reciprocal equation for \( CCPP = CP \) is just the same equation. But for Equation (16), the reciprocal of \( PPPP = CCC \) and the reciprocal of \( PP = CC \). For Equation (17), the reciprocal of \( CPP = PCCC \) and the reciprocal of \( CCPP = PPP \). (These assertions may be verified by showing that \( PCCC \) is the semigroup inverse of \( CPP \), that \( CC, CCC, CCPP \) are the semigroup inverses of \( PP, PPP, PPPP \), respectively.) Hence we add the equations:

\[
\begin{align*}
(18) & \quad \text{CCCC} = \text{CCC} \\
(19) & \quad \text{PCCC} = \text{CC}
\end{align*}
\]
with kin term interpretation, "Great Great Grandchild" = "Great Grandchild" (Baba) and "Father" of "Great Grandchild" = "Grandchild" (Baba), respectively.

From the viewpoint of the KAES program, the user is prompted when an equation is introduced and its reciprocal equation is not yet part of the construction. The user has the option of accepting or rejecting the prompt. Thus when the user introduces equations (15)–(17) the KAES program automatically verifies if the reciprocal equations have yet been included in the construction and if not, equations (18) and (19) are displayed (using the symbols the user has introduced for kin terms) and the user is asked if these equations should be included.

**Comparison Between the Algebraic Structure and the Kin Term Map**

We add these equations to the algebraic specification and the KAES program determines the new algebra, \( M \), and its graphical form displayed in Figure 7. Visual comparison with the core structure given Figure 6B shows that the two structures are isomorphic. Hence the core structure for the Shipibo terminology has the following algebraic specification:

\[
\begin{align*}
&\text{(20)} & \text{Generating set: } \{P, I\} & \text{I an identity element} \\
&\text{(21)} & \text{Reciprocal set: } \{C, I\}
\end{align*}
\]

**Fig. 6a—b.** (A) Kin term map for the consanguineal terms of the Shipibo terminology from the viewpoint of a male speaker. (B) Core structure for the male terms of the Shipibo terminology derived from the kin term map given in Figure 6A.
Fig. 7. Algebraic structure (core algebra) formed from the structure given in Figure 5 by addition of equations (15)–(17), along with their reciprocal forms (equations (18) and (19)). Compare with the structure given in Figure 6B.

(22) Reciprocal structural equation: \[ PC = I \]

(23) Structural form equations: \[ PPPP = PP \]

(24) \[ CPPP = PP \]

(25) Reciprocal structural form equation: \[ CCCC = CC \]

We have given the algebraic terms interpretation as male marked kin terms. The female marked kin terms, and hence the distinction between terms used by male speaker versus terms used by female speaker in the Shipibo terminology, arises through a construction process whose details will only be outlined here.

Free Products Again — Sex Marking of Kin Terms

Essentially the same procedure is used to introduce sex marking as for the reciprocal structure. An isomorphic copy, \( F \), of the algebra \( M \), displayed in Figure 7 is created and interpreted as the female marked terms. Let

\[ \{ p, c, e \} \] be the generating set for the isomorphic algebra, \( F \), with isomorphism, \( i: M \rightarrow F \), given by: \( i(P) = p \), \( i(C) = c \), and \( i(I) = e \).

Because \( i: I \rightarrow e \) and \( I \neq e \), the structure, \( M \otimes F \), will have two positions where an alter can be "located," depending on the sex assigned to alter. Hence the distinction between male speaker and female speaker arises out of the logic for the generation of structure.

MODEL 3: TROBRIAND TERMINOLOGY

Simplification and Core Structure

A different simplification is used for the Trobriand terminology (kin term map given in Figure 8A). Here, (1) the circular nature of the terminology as indicated by the same kin term being used for alters two generations up or down from ego is "broken" by replacing the term "Tabu" by "Tabu," and "Tabu,". Then, (2) like the Shipibo, terms are separated into those used by male speakers for alters of the same sex, versus terms used by females for alters of the same sex. Lastly, (3) consanguineal and affinal relations are separated. The core structure is given in Figure 8B.

Selection of Generators and Structural Equations

With the Trobriand terminology we will demonstrate yet another way in which structural forms arise. This time a different generating set will be used. For the Trobriand terminology we take as atoms the kin terms "Father" (Tama) and "Older Brother" (Tuwa), and we will include two structural equations as part of the initial algebra.

Let \( G = \{ F, B \} \) be the generating set input to KAES to begin the algebraic construction. (Here we use B as a mnemonic for "Older Brother." Later we will use \( b \) as a mnemonic for "Younger Brother"). We introduce the following equations to establish the structural relationships between \( F \) and \( B \) so that the structural property of these terms (i.e., a "Father" of a "Older Brother" is a "Father" and an "Older Brother" of a n "Older Brother" is an "Older Brother") becomes part of the algebra:

\[ FB = F \]

\[ BB = B. \]

These may be glossed as "Father" of "Brother" = "Father" and "Older Brother" of "Older Brother" = "Older Brother." The KAES program builds the algebra, \( G \), with these generators and equations, and represents this algebra by the graph given in Figure 9.
Reciprocal Structure

The same construction procedure is used by KAES to produce the reciprocal structure. Let the reciprocal generating set, H, be given by:

\[ H = \{ S, b \} \]

with isomorphism \( i(F) = S \) and \( i(B) = b \). To complete the isomorphism, the following two equations must also be introduced:

\[ Sb = S \]

and

\[ bb = b. \]

The first equation has gloss, "Son" (Latu) of "Younger Brother" (Bwada) = "Son" and the second has gloss, "Younger Brother" of "Younger Brother" = "Younger Brother." Note the appearance in the structure, via equation (29), of the relationship that is usually attributed to the classificatory property: Brother of Father = Father. No "classificatory property" has been introduced in the structure as a separate concept, and in the construction the relationship \( Sb = S \) only involves the notion of atoms and reciprocity as applied to the Trobriand terminology. It is not included as instance of a classificatory property (merging of lineal and colineals).

Fig. 8a—b. (A) Kin term map for the terms of the Trobriand terminology from the viewpoint of a male speaker. (B) Core structure for the male consanguineal terms of the Trobriand terminology, derived from the kin term map given in Figure 8A.
This establishes that $Bb$ is an idempotent element, much like a "sibling" term, so can be glossed as "Brother." Note that the theorem proves $Bb$ is an idempotent element, but $b$ is not a semigroup inverse for $B$ since $BbB = BbB = Bb = B \neq 1$.

Next we introduce an equation to make $F$ and $S$ reciprocal, terms, namely

$$ (32) \quad FS = Bb = bB, $$

and its isomorphic version,

$$ (32') \quad SF = Bb. $$

Since $Bb$ is an idempotent element, $FS$ is also an idempotent element. At this stage in the construction $S$ is not a semigroup inverse since $FSF = BbF$ and the latter term has no further reduction. When additional equations are introduced below, $S$ will become a semigroup inverse for $F$, and conversely.

Before constructing the reduced algebra defined by introducing Equations (32) and (32'), another property, namely structural equivalence, is prompted by the program. (Two nodes in a structure are structurally equivalent if the two nodes are connected to other nodes in an equivalent manner; for example, structurally equivalent pairs of nodes are the nodes Mother and Father in Figure 1, or the nodes Bwada and Tuwa in Figure 8.) Under the interpretation of $B$ and $b$ as the kin terms "Older Brother" and "Younger Brother", it seems plausible that they should satisfy an analogy condition for structural equivalence:

$$ (33) \quad "Jump": "Older Brother" :: "Father": "Younger Brother." $$

The program suggests that the equation,

$$ (34) \quad Fb = F, $$

should be introduced and its reciprocal, the equation

$$ (35) \quad SB = S, $$

should also be entered.

There is still another set of equations that are required for the structure to be a reciprocal structure. Equation 26, $FB = F$, and Equation 31, $Fb = F$, have as reciprocal equations,

$$ (26') \quad bS = S $$

and

$$ (34') \quad BS = S, $$

respectively, which may be glossed as: "Younger Brother" of "Son" = "Son" and "Older Brother" of "Son" = "Son." While these properties are

---

When viewed from the perspective of a genealogical space), but is structurally necessary for the reciprocal structure to be part of the terminology structure. The classificatory property, since it derives from production of the reciprocal structure, implies that the classificatory property need not be attributed to factors reflecting social factors exogenous to kin terminology structure such as group marriage, as originally suggested by Morgan (1871), but can be accounted for simply by reference to the internal logic of the Trobriand terminology.

Let $H$ be the algebra isomorphic to the algebra $G$. We continue the construction using KAES by forming the free product $P = G \otimes H$ of the isomorphic algebras $G$ and $H$. In order to define the terms $B$ and $b$ as reciprocal terms in $P$, the program prompts the user with the following equation as a suggestion:

$$ (31) \quad Bb = Bb. $$

If this equation is accepted, the program informs the user of the property given in the following theorem:

Theorem: For the semigroup $P$, the equation $Bb = Bb$ implies the equation $(Bb)(Bb) = Bb$.

Proof: $(Bb)(Bb) = B(bB)b = B(Bb)b = (BB)(bb) = Bb$. 

---

Fig. 9. Algebraic structure constructed from the generating set and the initial structural equations for the Trobriand terminology.
consistent with a genealogical interpretation, we emphasize that these equations have not been introduced into the construction merely to satisfy a genealogical interpretation but in order to ensure that the structure is logically consistent with the reciprocal structural property. Validation of these equations depends on structural isomorphism between the algebra generated by them and the core structure given in Figure 8B, not their genealogical interpretation.

Yet one more pair of equations needs to be introduced for the same reason. Equation 29, $Sb = S$, and equation 34', $SB = S$, have as reciprocal equations,

\[(36) \quad BF = F\]

and

\[(37) \quad bF = F,\]

respectively. Hence these two equations, which give the Trobriand terminology its classificatory form in conjunction with Equations (29) and (35), are necessary for the reciprocal structure of reciprocity to be preserved. For these equations the genealogical interpretation is not true, yet they are necessary for the reciprocal structure property! In other words terminologies have an internally produced structural form whose features result from a particular choice of atoms, such as "Father" and "Older Brother" for the Trobriand terminology versus Parent and Child for the AKT, and general kinship properties such as reciprocity and the reciprocal structure.

We now determine the reduced algebra produced by these equations through the KAES program. The result is given in Figure 10.

**Comparison of the Algebraic Structure with the Kin Term Map**

Lastly, we introduce two equations used to limit the vertical extension of the Trobriand terminology. These equations are $FFF = FF$ and reciprocally, $SSS = SS$. The structure produced by these equations is determined by the KAES program to have the graph shown in Figure 11. The isomorphism with Figure 8B may be seen by inspection.

We may characterize the Trobriand terminology as follows:

\[(38) \quad \text{Generating set: } \{F, B\}\]

\[(39) \quad \text{Structural equations: } FB = F\]

\[(40) \quad BB = B\]

\[(38') \quad \text{Reciprocal set: } \{S, b\}\]

\[(39') \quad \text{Reciprocal structural equations: } Sb = S\]

\[(40') \quad bb = b;\]
from the above equations that $Bb$ is an identity element for this sub-
semigroup. We use this property to motivate the claim that the structural
term $Bb$ is almost an identity element in the algebra that has been
constructed:

$$Fb = F$$

While the generating set $\{F, B\}$ does not contain an identity element,
the term $Bb$ is almost an identity element in the algebra that has been
constructed:

$$BbF = F$$

From equations (32), (34'), and (36') it follows that $S$ is now a semi-
group inverse of $F$, since $FbF = F$ and $SFS = SS = S$.

The results given here, though based on only three studies, suggest quite
strongly that (1) kin terminologies are structured through the logic of kin
term products and general properties such as reciprocity and (2) kin term
structures form a distinct class of structures based on what has been called
the reciprocal structural property. The implications are numerous and
only a few will be highlighted here. First, and perhaps most important, it
follows that terminologies can be analytically taken as an abstract,
symbolic system of relationships which, though possibly motivated by
external phenomena, transcends these and becomes structured through its
own internal logic. Second, it opens the possibility for comparing kinship
terminology structures at a deeper level than has heretofore been possible;
that is, at the level of production of structure. This approach stands in
contraposition to previous ones that compare the relationship of kin
terms to kin types only after kin terms are mapped onto a genealogical grid.

We may illustrate the latter claim through a comparison of descriptive
versus classificatory kinship terminologies. Morgan's (1871) interest in
eliciting the underlying principles that distinguished his two major
classes of terminologies foundered on his failure to find a structural
property inherent to the logic of terminology structure from which the
distinction could be derived and fell back on an eventually discredited
explanation based on a presumed event such as group marriage. In
contrast, the algebraic modeling has uncovered a way in which the
classification principle may arise without reference to factors exogenous to
the logic of kin terminology structure. The construction suggests that the
distinction is structural in origin and owes its proximal genesis to factors such as (1) choice
of atoms, (2) equations that define the relations among these atoms and
(3) an architecture for the development of a reciprocal structure from a
simpler one.

Another implication relates to transformation of structures. The
assumption has often been made that transformations provide an underlying
commonality which transcends particular instances, hence a common-
ality which derives from deeper processes than the history of experiences
of a particular society. Despite the central role that has been given to such
transformations by many authors, it has generally remained more a dogma
guiding studies than an established tenet. The models developed here, however, make explicit the manner in which one structure can be a transformation of another; e.g., the Shipibo core algebra has been demonstrated to be a transformation of the core algebra for the AKT. However, the Trobriand case equally illustrates that we are not just dealing with transformations of one structure into another, but with a class of structures wherein commonality stems from the principals upon which they are based.

A more extensive cross-cultural comparison is needed to lay a more secure groundwork for a general theory of kin term structures. Part of such a theory has been given here through representation of reciprocity as an algebraic property and through production of an initial structure which satisfies a property based on reciprocity. Other structural features not discussed here become necessary as the construction process continues. For example, computation of the reciprocal of a kin term product becomes more complicated when terms with sex markings are part of the semigroup. Thus the reciprocal of Father, in the AKT might be Son or Daughter, depending on the sex of the speaker. This suggests the need for structurally specific definitions for computing the reciprocal of kin terms and their products.

There is also the relationship between terminology structure as an abstract system and implementation of terminologies as concrete systems. If one takes the terminology structure as an abstract, created system, it is also necessary to model the interconnection between a terminology and the structure of other systems, be they strictly genealogical or more extensive. From this vantage point it is not particularly useful to argue whether or not terminologies are an overlay on a more fundamental genealogical grid, for by framing the argument in that fashion one has already created a distortion. Rather, we suggest that it will be more informative to recognize that we are not dealing with one kinship structure, but with many kinship structures. From this perspective the problem is to delineate these different structures, together with their interconnection and interaction, as empirically resolvable problems.

Finally, we make a few comments about the implications of this work for the study of conceptual structures in general and their relationship to the learning of structures as systems of knowledge. Conceptual structures of this kind serve to create a universe within which external events are interpreted and through interpretation, become the basis for action (Read and Behrens 1989). By making structural forms explicit, it becomes possible to determine whether or not there are relatively few structural forms at the deep level of structure production. What may turn out to be critical is the means by which complex structures are built up out of a relatively limited range of atoms and equations. If so, it becomes clearer how there can be both a “superorganic,” yet have cultural information be individually learned and reside imperfectly in each individual. If we refer back to kin terminology structures, the existence of an internal logic which constrains their production also implies that the learning process need not be perfect. This is because “errors” made by one person will be “recognized” by others who “understand” that these errors violate the logic of the structure. To use an example from arithmetic, products are not a list of independent facts subject to possible modification, but facts resulting from the structural implications of the concept of multiplication whether or not each individual correctly learns the multiplication table. Errors are recognized and corrected without appeal to any authority other than the logic of the system. It would appear that kinship terminology structures, and by implication, conceptual structures in general, may have this same kind of built in correction mechanism. Perfect, equally shared knowledge about terminologies is not necessary. Errors which contradict the logic of the terminology are self correcting in the same manner that imperfect knowledge of the multiplication table is self correcting. Such self correction could give culture stability despite imperfect learning and, through stability, make culture a phenomenon which transcends individual experience.

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NOTES

1 It needs to be emphasized that two relatable systems are involved: (1) a genealogical space, G, produced and structured through a system of genealogical reckoning (see Lehman and Witz 1974; Liu 1986) and (2) a kin terminology structure, T, of kin terms viewed as a structured set of abstract symbols. These two systems are linkable through the fact that persons for whom genealogical claims are made are also persons for whom kin terms may sometimes be used. In ethnographic reports, the linkage has been simplified through providing “definitions” of kin terms as sets of kin types (though these become awkward with classificatory systems). Correspondingly, the more common analytical approach has been to recreate these “definitions” through a series of rewrite rules that operate on a reduced set of kin types. In other words, the approach has been to examine the structure of terminologies indirectly through the structure imposed on the genealogical space, G, when the symbols making up the terminology are given interpretation in the language of kin types.

Once it is recognized that a space, T, of kin terms exists separate from, and is not
structured by, mapping kin terms onto the space $G$, then analysis can focus on the structure of $T$ directly. This does not obviate interest in understanding the structure imposed on $G$ by mapping the terminology structure onto a genealogical space. Ultimately, the two kinds of analysis—analysis aimed at elucidating the structure of $T$ directly and analysis aimed at elucidating that structure through the structure imposed on $G$—will reach closure with each other as Lehman (1984, personal communication) has suggested.

2 This and all other figures except Figure 2 are screen dumps from the KAES program.

3 In Read (1984) the definition of a kin term product uses the phrase, "the kin term." However, at the surface level of kin terms viewed as labels to be applied to concrete egos and alters, the definition of a kin term product need not lead to a unique kin term. For example, if ego is a male, alter, is male parent of ego and alter, is ego, then ego calls alter, Father, alter, calls alter, Son and ego calls alter, Self, which would make Son of Father = Self, whereas if alter, is a male child of alter, other than ego then ego calls alter, Brother and now Son of Father = Brother. This indeterminism is considered here to result from the interpretation and application of a deeper structure to the more surface level of concrete egos and alters. In other words, we distinguish between the properties of a conceptual structure viewed at an abstract level (which is essentially static) versus the application of a conceptual structure to a genealogy (which is essentially dynamic) where additional properties may arise or be introduced in order to accommodate disjunctions that can occur when a static structure is mapped onto a dynamic system. Read's earlier definition has been corrected here by substituting the phrase "a kin term" instead of "the kin term" with its implication of uniqueness.

4 Three levels of notation are used here. First, an expression such as father without capitals refers to a genealogical position. Second, when capitalized as in Father, the reference is to a kin term used by native speakers. Finally, when written all in capitals, such as FATHER, the expression is being considered as an abstract symbol; for example, when a static structure is mapped onto a dynamic system. Read's earlier definition has been corrected here by substituting the phrase "a kin term" instead of "the kin term" with its implication of uniqueness.

5 Observe that kin term products are written in the order $L \times K$, where $K$ is the term used by ego for alter, and $L$ is the kin term used by alter, for alter, whereas for a kin type product, the product is written in the reverse order. This corresponds to the difference in the verbal gloss used here for kin term products as opposed to kin type products. For example, the kin term product Mother $\times$ Father is glossed as, "Mother of a Father," or Grandmother, whereas the kin type product $fm$ (for the kin types $f = \text{father}$ and $m = \text{mother}$) is glossed as "father's mother"; i.e., a kin type product for which the kin term Grandmother is applicable. The kin term product, Mother $\times$ Father, thus represents the kin term applicable to the kin type product, $fm$ (and to the kin type product, $mm$, as well).

6 The simplification of the kin term map is the most inductive part of the analysis and has been guided by the assumption that the kin term map is constructable from simpler structures. Simplification has been based on global properties such as separation of affinal from consanguineal terms, or grouping together kin terms in structurally equivalent positions, or an underlying symmetry based on male marked versus female marked terms, etc. The same general idea of collapsing a larger, more complex structure into a simpler one has also been used by Liu (1986: 40—42) in his descriptive analysis of terminologies based on genealogical spaces. The criterion for favoring one possible simplification over another has primarily been Occam's Razor; e.g., separating male marked from female marked terms in the AKT does not lead to a simpler structure that can be accounted for by a few, general properties in contrast to what occurs when the simplification given in Figure 3 is used.

7 Roughly, an algebra consists of a set of symbols and a set of operations that may be performed on the symbols to produce other symbol combinations. A set of symbols includes all the distinct symbol combinations that can be produced using the operation(s) of the algebra, taking into account any structural equations that identify which symbol combinations can be replaced by other symbols or simpler symbol combinations.

A familiar structure which has the form of an algebra is provided by the counting numbers, $0, 1, 2, 3, \ldots$ (the set of symbols) with the operation of addition (a binary operation) and equations such as $1 + 0 = 1 + 0$ (the additive identity element), $1 + 1 = 1 + 1$ (addition is commutative) and $1 + (1 + k) = (1 + 1) + k$ (addition is associative), where $1$ and $k$ are arbitrary counting numbers. The binary operation and these equations allow one, for example, to form the expression $(2 + 0) + 0$ (a symbol combination) and to replace it by the symbol, $2$.

8 The algebra $G$ generated from the set $G$ with binary operation $\times$ is constructed by the KAES program as follows (see Clifford and Preston 1981: 40—45 for a more formal discussion of the construction procedure upon which the KAES program is based). First, the free algebra (technically, a free semigroup) of all symbol combinations, or words, that can be formed using the symbols in $G$ and the binary operation $\times$ is constructed. Second, symbol combinations that are equivalent in the sense that any one combination can be reduced (or expanded) to another combination by use of the equations are identified and placed into a single symbol class. Lastly, each class is identified by the simplest symbol or symbol combination in the class. The algebra $G$ then has as its objects these symbol classes. For convenience, the objects of $G$ may be considered to be just the symbol used to identify the class to which that symbol belongs. Thus, for the set $G = \{P, C, I\}$, where $I$ is an identity element for the binary operation $\times$, the distinct symbol combinations $PC, IP, IP = (P \times P), PC, CP, CC, PPC, PPPCC, \ldots$ may be formed. Under the equation $PC = I$ the symbol combinations in $\{P, PC, PPPCC, PPPPPPCC, \ldots\}$ are all equivalent and this class may be identified by its simplest member, namely the symbol $I$. In the algebra $G$, one of the objects would be the class $\{PC, PPPCC, PPPPPPCC, \ldots\}$ but for convenience it may be replaced by its simplest representative, namely the symbol $I$. In the algebra $G$, the class corresponding to $PC$ and the class corresponding to $I$ are the same, so $PC = I$ is a valid equation in $G$.

9 A semigroup is the name given to a structure with a single binary operation, $\times$, which is associative. For example, the (binary) operation of addition over the nonzero counting numbers $1, 2, \ldots$ is associative, so the nonzero counting numbers along with the operation $+$ is an example of a semigroup which lacks the necessary notion of an identity element and an inverse for each element that characterizes a group.

10 Roughly, a reciprocal term "undoes" what a term does in that if a term maps one node to another, the reciprocal term should map the second node back to the first. With genealogical claims, a reciprocal claim can be formed by tracing a claim backwards. This can be done through two steps (at least for claims that do not include sex): (1) the terms in the genealogical claim are written in the reverse order and (2) each kin type in the genealogical claim is replaced by its reciprocal kin type. For example, the genealogical claim "ego's parent's parent's child has reciprocal claim formed by first reversing the claim to get "ego's child's parent's parent" and then replacing child by parent and parent by child to arrive at "ego's parent's child's child." The equivalent property for products of kin terms would be to reverse the order of the product and then replace the terms in the product by their reciprocal terms. The latter step is where the difficulty arises that is addressed by the algebraic definition of a reciprocal term, for what are the
reciprocals of the kin term atoms from which compound kin terms are constructed? While these reciprocals could be introduced as additional native knowledge, the aim in the formalism is to reduce the production of kin term structure to its minimal assumptions. Hence we need to determine if there is a structural property that is satisfied, in general, by reciprocals of kin terms; more specifically, by reciprocals of kin terms which are atoms so that they can be algebraically introduced.

What is needed, then, is an algebraic property of a structure that captures the sense of "undoing" what a term does. For structures such as groups (a semigroup with an identity element and an inverse for each element), the inverse "undoes" what an element does in the following sense. In a group, G, with identity element 1, if x is an element in G and x⁻¹ is its inverse (so that x • x⁻¹ = 1), and if y and z are elements with y = xz = z⁻¹x, then y⁻¹z⁻¹ = x⁻¹z⁻¹x = 1, so that if x "acts on" y to map it to z, then taking the product with the inverse of x maps z back to y. For a semigroup (which need not have an identity element) a semigroup inverse is the analog of an inverse in a group. Though Read (1984) showed that semigroup inverses express the structural property that a reciprocal term should satisfy for the AKT, analysis of the Trobriand terminology has led to the conclusion that a semigroup inverse is, in some cases, too strong a property.

The semigroup inverse has been weakened here, as expressed in the definition, via a property satisfied by a semigroup inverse, but where the reverse need not be true. When a product x • y of two kin terms, K and L, is an idempotent, then the term represented by the product K • L will be self-reciprocal, which is close to the idea of an identity for, example, Son of Mother = Brother and Brother is self-reciprocal, Brother of Son is Son, etc. Whether this definition (or any definition, for that matter) is satisfactory for all terminology structures will need empirical test.

11 A set G of elements in a semigroup S is said to be a generating set for S if no proper subsemigroup of S contains the set G. The generating set G for the semigroup S is a minimal generating set for S if no proper subset of G generates S.

12 That is, there is a mapping, call it i, from the set A = {A, B, C, D} of generators for the semigroup A onto the set of generators, B = {B, C} of generators for the semigroup B, say i: A → B, such that all equations which are valid for the semigroup A are also valid for the semigroup B under this mapping i that replaces symbols from the generating set for A by symbols from the generating set for B.

13 It should be noted that in this example the choice of the equation used to produce a homomorphic image (the result of reducing the free product via Equation (1 1)) of the free product of the kin terms Latu and Bwada is the kin term Latu, not what kin types can be replaced by other kin types. Second, unlike a merging rule which is introduced on an ad hoc basis in order to account for the pattern of kin types classified together when kin terms are mapped onto a genealogical space, Equation (29) is introduced via the same principle invoked for the production of a reciprocal structure for the AKT; i.e., the structure determined by the reciprocals of the generating kin terms should be isomorphic with the structure determined by the generating kin terms (the structure given in Figure 4 for the AKT and in Figure 9 for the Trobriand terminology). Equation (29) is the equation that results when the isomorphism is applied to Equation (26) and is necessary for this isomorphism of structures to be valid.

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