Lawrence Berkeley National Laboratory
Recent Work

Title
POLARIZATION OF SINGLE-PARTICLE STATES INVOLVED IN HEAVY ION TRANSFER REACTIONS

Permalink
https://escholarship.org/uc/item/4428d5tt

Author
Pruess, K.

Publication Date
1976-03-01
POLARIZATION OF SINGLE-PARTICLE STATES
INVOLVED IN HEAVY ION TRANSFER REACTIONS

K. Pruess, G. Delic, L. A. Charlton, and
N. K. Glendenning

March 1976

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
ABSTRACT

If two nuclei could be placed next to each other at a distance typical of grazing collisions, the single-particle states in each would adjust themselves to the presence of the other nucleus. They would develop an appreciable probability of being in the other nucleus, and the more weakly bound the state, the more strongly it would be polarized by the other nucleus. The solution to this situation has been realized in the two-center shell model. Whether such an adjustment can occur during a reaction depends on the ratio of the transit time to the nuclear periods of the more weakly bound nucleons. In typical low-energy experiments this ratio appears to allow for such a polarization. Clearly if it occurs, the form factor for a transfer reaction will be modified from those currently employed, which use the asymptotic single-particle states of the non-interacting separated nuclei. From the foregoing discussion we expect that 1) for stripping reactions to a sequence of excited states in the target nucleus, the polarization effects will increase with increasing excitation (i.e., decreasing binding), 2) since transfer can take place for larger impact parameters if polarization occurs, the effect on the angular distribution will be a shift to smaller angles compared to normal DWBA, 3) the polarization effect will ultimately vanish at sufficiently high bombarding energy. We have found from calculations so far that the single particle states of the heavier of the reaction partners suffer the greater polarization.

This work performed under the auspices of the USERDA.
POLARIZATION OF SINGLE-PARTICLE STATES INVOLVED
IN HEAVY ION TRANSFER REACTIONS*

K. Pruess, G. Delic, L. A. Charlton, and N. K. Glendenning
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Direct reactions between heavy ions usually involve relatively little change in center-of-mass energy, angular momentum, charge or mass of any of the reacting nuclei;

\[
\frac{\Delta E}{E}, \frac{\Delta L}{L}, \frac{\Delta Z}{Z}, \frac{\Delta A}{A} \ll 1
\]

The strong absorption of close collisions out of the direct reaction channels also implies that the collision is peripheral. Under all of these circumstances it follows that the optical potential which describes the elastic scattering also plays a dominant role for the direct reactions. Taking account also of the fact that the wave number is large, which implies that the density of angular momentum states per unit impact parameter is large, then classical conditions apply approximately. In this circumstance the deflection function is a very useful vehicle for a discussion of the angular distribution of nucleon transfer reactions. One need only supplement it with several additional relevant facts to understand the angular distribution of a quantal calculation or experiment. One of these facts is that absorption out of the simple channels occurs if the reaction partners come too close. A second fact is that nucleons cannot be transferred if the reaction partners are too distant. These two effects which reduce respectively the contribution of low and high partial waves, imply the dominance of a band of grazing partial waves. Since most of these scatter within a small range of angles at or below the grazing angle, \( \theta_g \), a one-step transfer cross section will have a peak in the neighborhood of the classical grazing angle of the real part of the optical potential that describes the elastic scattering.

There are many examples in the literature where the angular distribution has a grazing peak shape located near the angle defined by the elastic optical potential. This situation is consistent with, but not a proof of the interpretation that the reaction is a one-step reaction and that the transfer occurs between normal states of the non-interacting nuclei. There are also a growing number of observations of angular distributions which do not have this form, but are biased toward the forward angle, even though the grazing angle is well removed from the forward direction. There are also many cases where, although the reaction has a peaked cross section, the peak is substantially shifted from the grazing angle. When the \( Q \) of the reaction is small, so that \( \Delta E/E \ll 1 \), the grazing angle can be read from the elastic cross section. When the \( Q \) is large, the grazing angle can be substantially different in the entrance and exit channels. A DWBA calculation will locate the position of the grazing peak that is expected in the sense described above. In either of these two situations, where the cross section is either forward biased, or has a grazing peak substantially shifted from the expected location, we have a clear signal that the reaction cannot
be described as a dominantly one-step reaction between asymptotic states of the separated nuclei. It is interesting nonetheless that the DWBA can be forced to fit the anomalies rather consistently for a number of cross sections by the same change in optical model parameters.\(^2,^3\) However, once having made this observation, it does not make sense to continue to try to force these reactions into that framework by changing the optical model parameters so that they no longer describe the elastic scattering. This violates a sensible physical description and amounts to no more than a very elaborate (and costly) parameterization for which the parameters have no physical meaning or applicability beyond the isolated reaction.

When the angular distribution of a reaction is anomalous in either sense described above, we have the opportunity of learning something new about the reaction.

At the last Argonne Conference on heavy ions, theoretical calculations were presented which predicted the first kind of anomaly.\(^4\) A strongly biased forward cross section resulted from the interference of a one-step amplitude having a grazing peak, and a two-step amplitude, which was more forward biased. The destructive interference further resulted in a reduction of the grazing-angle cross section resulting in a forward-angle cross section with no evidence of a grazing peak. Experiments\(^5\) from Brookhaven were reported at the same conference which had this form of angular distribution and they have recently been interpreted successfully in the above way.\(^6\)

This paper is devoted to a possible explanation of the second kind of anomaly where the angular distribution although having a grazing peak, is shifted substantially from the position expected from classical considerations, or in the case of large \(Q\), from the position expected on the basis of a DWBA calculation.

The physical process that we consider in this paper is the following. As two nuclei approach each other, the more weakly bound of the shell model states may be polarized by the field of the other nucleus. In the static case of two nuclei which are held stationary the polarization can be computed in the two-center shell model.\(^7\) The result is that a nucleon which is originally in one nucleus may have an appreciable probability of being in the other when their surfaces are close. Whether during the time of a typical reaction, the shell model states undergo an adiabatic polarization depends on the ratio of transit time to nuclear period. This can be estimated as

\[
\frac{\text{transit time}}{\text{nuclear period}} \sim \left( \frac{E_{\text{Fermi}}}{E/A} \right)^{1/2}
\]

where \(E_{\text{Fermi}} \sim 30\) MeV and the collision energy per projectile nucleon, \(E/A\) is of the order of 10 MeV. If this ratio were very large compared to unity then we would expect the adiabatic polarization to occur, whereas if it were very small, we would not. For the conditions cited it is \(\sqrt{3}\) which is indecisive, but it appears worth investigating the magnitude and trends of the effect.
The trends that we expect are the following. 1) Since transfer can take place for larger impact parameters if polarization occurs, then the effect on the angular distribution will be a shift to smaller angles compared to normal DWBA. 2) The effect will increase for a sequence of states of decreasing binding in a nucleus. 3) The polarization effect will ultimately vanish at sufficiently high bombarding energy. We have also found from our calculations so far that 4) the states of the heavier of the reaction partners undergo the greater polarization for similar binding energies.

The cross section for the physical process described above can be computed approximately in the DWBA framework. We propose to compute a possibly upper limit on the effect in the following way. We will replace the asymptotic shell model states used in normal DWBA calculations by the polarized shell model states corresponding to the distance of closest approach of the grazing orbit. In some sense this is an upper limit on the magnitude of the effect, since the degree of polarization depends on the separation of the reacting nuclei. However since the transfer is still expected to occur in a narrow annulus around the nucleus (with radius somewhat increased over that expected for the asymptotic states), the use of a fixed polarization distance may be a good first approximation of the effect.

In order to use existing DWBA computer programs it is most convenient to express the polarized wave function on a spherical harmonic oscillator basis. Thus a single-particle state which, asymptotically is described by the quantum numbers \( n\ell j \Omega \) (with \( \Omega \) being the 3-projection of \( j \) on the axis joining the centers of the nuclei) at a finite separation will appear as

\[
\phi_{n\ell j \Omega}(R, \xi_B) = \sum_{n' \ell' j'} A_{n \ell j \Omega}^{n' \ell' j'}(R_B) \phi_{n' \ell' j'}(\xi_B)
\]

where \( R_B \) is the distance separating the mass centers in the system \( b + B \) and \( r_B \) is the displacement of the transferred nucleon from the mass center of \( B (= A+1) \). For two colliding spherical nuclei, the axes joining their centers is a symmetry axis. Choosing it to define the intrinsic 3-axis, the projection of \( j \) on it, which we call \( \Omega \), is conserved. For large separation, \( R_B \), the polarized state goes over to the normal shell model state; i.e.

\[
A_{n \ell j \Omega}^{n' \ell' j'}(\infty) = \delta_{nn'} \delta_{\ell \ell'} \delta_{jj'}
\]

In Fig. 1 we show a contour plot of these amplitudes for a proton \( f7/2 \) state in \(^{40}\text{Ca} \) which is polarized by a nearby \(^{16}\text{O} \) nucleus, and in Fig. 2 we show the approximate radial wave function along the line of centers of the two nuclei. The approximation involves taking only the components in the representation (1) which have a probability greater than about 0.1%. One sees here clearly how the probability is distributed in part in the neighboring oxygen nucleus. A proton in the oxygen nucleus can more easily be transferred to such a polarized state than to the asymptotic state, also shown which overlaps with oxygen only in its tail.
The wave function (1) is expressed on a basis which has the symmetry axis connecting the line of centers of the two nuclei as the Z-axis. We have to relate this correctly to the laboratory fixed axis, which we may take to be defined by $k_\alpha$ for the reaction

$$A(a,b)B, \ a = b+1, \ B = A+1$$

The usual DWBA amplitude we take to be

$$T_{j_1 m_1} \rightarrow j_2 m_2 \propto \int \psi(-)(k_\beta, R_\beta) \phi_n^*(r_2) V(r_a) \phi_{j_1} \psi(+) (k_\alpha, R_\alpha)$$

$$dr_a \ dR_\beta$$

where

$$r_a = R_1 a, \ R_\alpha = R_a A, \ r_2 = R_1 B, \ R_\beta = R_B$$

The connection between wave functions expressed in the intrinsic frame and the laboratory frame is

$$(\phi_n j j\Omega)^{\text{int.}} = \sum_m D^j_{m\Omega}(\omega) \phi_{m\Omega}$$

where $D$ is the rotation function that rotates $k_\alpha$ into the intrinsic axis (denote the Euler angles by $\omega$). Since

$$\sum_{\Omega_m} D^j_{m\Omega} D^{j*}_{m\Omega} = \delta_{j m}$$

then

$$\sum_{\mu \Omega} D^j_{\mu\Omega} D^{j*}_{\mu\Omega} \phi_{j \mu} = \phi_j \Omega$$

Hence according to (5) we replace

$$\sum_{\mu} D^j_{\mu\Omega} \phi_{j \mu} \ \text{by} \ \phi_j \Omega (R,r)$$

Thus the amplitude for the transfer between polarized states is

$$T^p_{j_1 m_1} \rightarrow j_2 m_2 = \sum_{\Omega_1 \Omega_2} \int \psi(-)^*(R_\beta) D^{j_2*}_{m_{2}\Omega_2} (\omega_\beta) \phi_{n_2 j_2 \Omega_2} (R_\beta, R_\beta)$$

$$V(r_a) D^1_{m_{1}\Omega_1} (\omega) \phi_{n_1 j_1 \Omega_1} (R_\alpha, R_\alpha) \psi^+(R_\alpha) \ dr_a \ d\Omega_\alpha \ dR_\beta$$
Introducing the expansion (1) for the polarized wave functions, but expressing the components through (5) in terms of the laboratory frame we have

\[
T_{j_1 l_1 m_1}^{P} + j_2 l_2 m_2 = \sum_{\Omega_1 \Omega_2} \sum_{n_1' \Omega_1' \Gamma_1} \sum_{n_2' \Omega_2' \Gamma_2} \int_{A_n' \Omega_2' \Gamma_2} \int_{n_2 \Omega_2 \Gamma_2} \left( R_{\beta} \right) \left( R_{\alpha} \right) \psi^{(-)}(r_a) \phi^{*}_{n_1' \Omega_1'}(r_a) \psi^{(+)}(R_{\alpha}) \phi^{*}_{n_2' \Omega_2'}(R_{\beta})
\]

The approximation that we will make in our initial exploration will be to take the amplitudes A out of the integration, and use their values at the grazing distance in each channel. These amplitudes for the f7/2 proton state in the heavy nucleus involved in

\[^{40}\text{Ca}(^{16}\text{O}, ^{15}\text{N})^{49}\text{Sc} \]

are shown in Fig. 1.

If the polarization of the single-particle states produces the expected trends, and an effect of the magnitude commensurate with the anomalies observed in experiment, then we would contemplate more refined calculations. There are a hierarchy of improvements. The potential model for the two-center shell model is based on modified oscillator potentials. One could improve the model by using, say Woods-Saxon potentials. Beyond this a dynamical evolution of the polarization could be formulated.

FOOTNOTES AND REFERENCES

* Work supported by U. S. Energy Research and Development Administration.
5. Auerbach, Baltz, Bond, Chasman, Garrett, Jones, Kahana, LeVine, Schnieder, Schwarzschild, and Thorn, Argonne Symposium on Heavy Ion Transfer

Fig. 1. Corresponding to a separation of 10 Fm between the centers of $^{48}$Ca and $^{16}$O a polarized proton wave function corresponding to the asymptotic 0 $f7/2$, $\Omega = 1/2$ state in Ca is represented by contour plots in the $n,\ell$ plane, the basis functions being harmonic oscillators with $\hbar \omega/h = 0.2718$. The $j = \ell+1/2$ and $j = \ell-1/2$ components have to be represented separately. The point at $n = 0$, $\ell = 3$ represents the residue of the original state which has been reduced to an amplitude of 80.5/100, the units on the plot being 1/100.
Fig. 2. The polarized wave function along the axis joining the centers of $^{48}$Ca and $^{16}$O is shown corresponding to the contour plot in Fig. 1. For comparison the unpolarized wave function is shown. The center of $^{48}$Ca is at the origin and the center of $^{16}$O is at 10 Fm.
LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.