Minimality Criteria in Spatial Belief Revision

Leandra Bucher (leandra.bucher@psychol.uni-giessen.de)
Justus Liebig University, Experimental Psychology and Cognitive Science, Otto-Behaghel-Straße 10F, 35396 Giessen, Germany

Paul D. Thorn (thorn@phil-fak.uni-duesseldorf.de)
Heinrich Heine University, Institute for Philosophy, Universitaetsstr. 1, 40204 Duesseldorf, Germany

Abstract

Agents typically revise their beliefs when confronted with evidence that contradicts those beliefs, selecting from a number of possible revisions sufficient to reestablish consistency. In cases where an individual's beliefs concern spatial relations, belief revision has been fruitfully treated as a decision about which features of an initially constructed spatial mental model to modify. A normative claim about belief revision maintains that agents should prefer minimal belief revisions. Yet recent studies have rebutted the preceding claim, where minimality is understood to consist in modifying the position of the fewest objects, showing instead that reasoners prefer revisions that modify the position of an object x while retaining the position of an object y, when the agent's new evidence is a relational statement of the form 'xRy'. We here present cases where the preceding effect is reduced, and show an effect of minimality as measured by the number of initial premises preserved.

Keywords: Relational reasoning; Spatial reasoning; Spatial cognition; Belief revision; Mental models

Belief Revision as Variation of Spatial Mental Models

Whenever one's beliefs are contradicted by compelling evidence, one is apt to revise one's beliefs in order to maintain the consistency. For example, suppose, you believe that Bob is a baker from Bavaria, and you then come across conclusive evidence that Bob is not from Germany. Presumably, you will adopt the belief that Bob is not from Germany, and retract the belief that Bob is from Bavaria. Although there are many possible revisions that would allow you to achieve consistency in this case, it is very likely that you will continue to believe that Bob is a baker. If you revise your beliefs in this way, then your revision will be accord with a normative principle that states that one should minimize the changes made to one's belief set when revising one's beliefs in the face of new (contradicting) evidence (Gärdenfors, 1992; Harman, 1986; Krumnack, Bucher, Nejasmic, & Knauff, 2011).

In the domain of spatial belief revision, and particularly in research concerning the revision of beliefs about spatial relations between objects, belief revision has been described as a process of modifying of spatial mental models (Bucher, Krumnack, Nejasmic, & Knauff, 2011; Bucher, Nejasmic, Bertleff, & Knauff, 2013; Krumnack, Bucher, Nejasmic, & Knauff, 2011; Bucher & Nejasmic, 2012; Knauff, Bucher, Krumnack, & Nejasmic, 2013; Mikheeva, Bucher, Nejasmic, & Knauff 2013; Nejasmic, Bucher, & Knauff, 2013). Such work follows the general approach of positing mental models as the basis for relational reasoning (Johnson-Laird & Byrne, 1991; Knauff, 2009).

In a prototypical task that is used to study spatial belief revision, participants are provided with two (or more) statements, S_i, about object arrangements. For example:

(S_1) "The apple is left of the mango."
(S_2) "The mango is left of the pear."

The example description results in the arrangement (or mental model M):

(M) Apple – Mango – Pear

Participants are then told that they cannot be entirely sure that the description is correct, but that a third statement (the "fact", F) – subsequently given (and partly contradicting) the initial description – is incontrovertibly true and has to be taken into account. For example:

(F) "The pear is left of the apple."

The task of the participants is to revise the initially constructed model (M) such that it coheres with the "fact" (F). Multiple revisions (R_i) are possible in order to reestablish consistency, and take the fact into account, e.g.:

(R_1) Pear – Apple – Mango
(R_2) Mango – Pear – Apple

Both revised arrangements (R_1 and R_2) preserve the initially constructed model (M) to the same extent, and each revision preserves one of the initial statements: R_1 preserves S_1, and R_2 preserves S_2. From the viewpoint of logic, R_1 and R_2 are equally acceptable. Nevertheless, multiple studies (discussed below) have found that reasoners have a clear preference for R_1.

Preferred Spatial Model Revision

The preference for R_1, in the preceding example, is based on linguistic cues that are provided in the expression of binary relations of the form xRy, in particular by the functional asymmetry between x and y (with the "pear" as x, and the "apple" as y, in the above example). The asymmetry is often

represented via a distinction between figure and ground, target and anchor, or (in the favored terminology of the present paper) "to-be-located object" (LO) and "reference object" (RO) (Miller & Johnson-Laird, 1976; Talmy, 1983; Landau & Jackendorff, 1993). The location of the LO is specified relative to the RO, and reasoners treat the LO as more flexible and relocatable than the RO (Logan, 1994; 1995).

R_1 results from the relocation of the *pear* (the LO of the fact F) within the initial model (M), while R_2 results from the relocation of the *apple* (the RO of the fact F). In tasks of the above type, reasoners relocate the LO in order to revise their initial arrangement, about 80-90% of the time. The preference has been termed the "LO principle" and the "LO preference" (Bucher, Krumnack, Nejasmic, & Knauff, 2011; Bucher & Nejasmic, 2012; Bucher, Nejasmic, Bertleff, & Knauff, 2013; Knauff, Bucher, Krumnack, & Nejasmic, 2013; Krumnack et al., 2011; Knauff, Bucher, Krumnack, & Nejasmic, 2013; Mikheeva, Bucher, Nejasmic, & Knauff, 2013). The LO principle is a strong effect, and there are only a few factors that are known to modulate (but do not override) the effect (e.g., Nejasmic, Bucher, & Knauff, 2013).

Challenging the LO Principle

In a recent study (Bucher & Nejasmic, 2012), participants were asked to revise arrangements of a similar kind to the one in the example above, but with a small modification: an additional object was 'attached' (above or below) to either the object in the leftmost or the rightmost position. Arrangements and subsequently presented counter-facts were counterbalanced across the experiments such that in half of the items the additional object was 'attached' to the object which corresponded to the LO of the counter-fact. Participants of a first experiment of this type were asked to draw initial and revised arrangements (Bucher & Nejasmic, 2012; experiment 1). Hence, participants were completely free to generate whatever solution they had in mind. The assumption was that participants were inclined to keep their revisions as minimal as possible, with minimality referring to the number of objects that are relocated in order to revise an initial arrangement. The surprising finding was that participants not only stuck to the LO principle (i.e., chose to relocate the LO in almost 90% of the cases, regardless of whether there was another object 'attached' to the LO or not), but in addition, they relocated the 'attached' object along with the LO. See the following example for illustration:

Initial arrangement: Hammer – Pliers – Saw
Drill

Counter-fact: "The Hammer is right of the Saw"
(with the "Hammer" as the LO of the fact)

Preferred revision: Pliers – Saw – Hammer
Drill

Please note that participants could have reached an equally correct solution, by choosing to relocate the LO (without the attached object), leaving the (formerly) attached object in its original position. However, among 768 drawings of revised arrangements (drawn by 24 participants, each of whom completed 32 revision tasks), there was not a single drawing that depicted an alternative solution of this sort. In a second experiment that was similar to the preceding, participants had to select an arrangement from several revised arrangements (instead of drawing the arrangements) (Bucher & Nejasmic, 2012; experiment 2). The result was analogous to the first experiment. Revisions were preferably based on the LO principle, indicated by the choice of respective arrangements by participants. Overall, the results suggest that revisions in these experiments did not follow a principle of minimality that is based on the number of objects relocated. If participants' revisions had been based on such a minimality principle, they would have relocated ROs (instead of LOs) in half of the items presented in these experiments. Subjects were, apparently, unmotivated by the (proposed) normative principle stating that they *should* prefer revisions that minimized the number of relocated objects.

Minimality Criteria in Spatial Belief Revision

The prescription to opt for minimal belief revisions is not unambiguous. So the question remains of whether human agents are to any extent influenced by a 'minimality norm'. The aim of the present study is to test the 'minimality norm' by applying an alternate minimality criterion. The study reported in the above section 'defined' minimality in terms of the number of relocated objects, and the results suggest that subjects did not strive to minimize such changes (or that the impetus was rather weak in comparison to the effect of the LO principle). In the present study, we look at minimality as measured by the number of initial premises preserved.

We present two experiments, where we administered tasks similar to the ones described above: two initial statements describe a spatial layout of three horizontally arranged objects. The arrangement must then be revised after a third statement (the "fact") is presented. In half of the items, the LO revision permits the subject to preserve one of the initial statements (LOpreserve condition), in the other half, the LO revision entails the rejection of both initial statements (LOreject condition). The question was whether participants try to preserve one of the initial statements. If so, the effect of the LO principle should be reduced in the LOreject condition.

The first experiment allowed participants to generate their arrangements freely in drawing tasks. In the second experiment, participants accomplished a verification task, selecting the respective arrangements among several arrangements simultaneously presented on a computer screen.

Experiment 1: Drawing Spatial Arrangements

Participants received two statements about the arrangement of three objects, presented on PowerPoint slides via video projection. The task was to draw the arrangements (in specially prepared booklets). Subsequently, participants received a "fact" inconsistent with the previous information, and were asked to draw a revised arrangement.

Method

Participants Twenty-three participants (11 male; age: M = 23.57; SD = 7.24), all undergraduate students (among them 8 students in psychology) from the University of Giessen, gave written informed consent to participate. Participants were tested in small groups (n = 6-10) and received course credit or were paid at a rate of 8 Euros per hour for participation.

Materials, Procedure, and Design Verbal descriptions of horizontal one-dimensional spatial arrangements of three small, equal-sized, disyllabic-termed objects (in German), belonging to one of two categories (fruits or tools) were presented. The descriptions consisted of two statements (premises), expressing a binary relation between x and y of the form xRy. Premises were presented in a sequential manner, displayed one at a time, with a display duration of 10s each. The premises contained the relational expressions "directly left of" and "directly right of". The occurrence and combination of the relational expressions were counter-balanced across the experimental items, such that all possible combinations of the two types of relational expression occurred equally often. There were no instructions dictating how to interpret 'directly left/right of' and 'directly below/above'. An example description (resulting in the arrangement Mango - Apple - Pear) is provided below:

1st premise: "Mango directly left of apple."
2nd premise: "Pear directly right of apple."

The description was followed by the prompt "Please sketch the arrangement of the objects." with display duration of 20s, allowing the participants to sketch the arrangement. The prompt "Please turn the page." with a duration of 3s, and a blank slide with the duration of 2s were shown before a third statement (fact) was presented for 10s. The instructions that the participants received at the beginning of the experiment included the statement that they cannot be sure whether the information about the object arrangement provided by the first two statements is correct, but that the fact (presented in red letters to contrast it from the initially presented premises which were black) should be taken as incontrovertible. The fact – again expressing a binary relation of the form xRy – provided information about the relation between the central object and an object at an outermost position (leftmost in half of the items; rightmost in the other half). The relations specified by the fact statements were (in half of the items) "directly below" and (in the other half of the items) "directly above". For example:

Contradicting fact: "The apple is directly above the mango."

In half of the items, the fact presented the central object of the initial arrangement as object x (the to-be-located object, LO, of the binary relation), in the other half as object y (the reference object, RO). Accordingly, in half of the items, one of the outermost positioned objects instantiated object y (RO), in the other half, object x (LO).

The fact was followed by the prompt "Please sketch the arrangement of the objects." with display duration of 20s, and subsequently, the prompt "Please turn the page." with a duration of 3s. There was a blank slide presented for 2s before the next item was presented. The task of the participants was to follow the prompts, i.e., to sketch the object arrangements and turn the page, when instructed by the presented slides. Participants used a pencil to sketch their arrangements in specially prepared booklets. Participants were instructed to draw the object arrangement according to the description of the first two premises, and then re-draw the arrangement, after the fact had been shown, thereby taking the fact into account (i.e., such that the drawing was consistent with the fact). In each trial, there were several ways to achieve consistency with the fact, among them the revision by relocation of the fact's LO:

LO revision: Apple
Mango – … – Pear

In half of the items, relocation of the LO entails that the information provided by exactly one of the two premises (ratio 50:50) was preserved in the resulting revised arrangement. These items are referred to as "LOpreserve" items. In the other half of the items, relocation of the LO entails that the information provided by neither of the initial premises was preserved in the resulting revised arrangement (as is the case in the above example).These items are referred to as "LOreject" items.

Sixty-four experimental items were presented, preceded by four practice items (not analyzed). Each group received the items in a different randomized order. Descriptions were provided using Microsoft PowerPoint (Version 2007) running in the windows XP environment on a standard personal computer. PowerPoint slides were presented on a large screen via video projector. Drawings for each trial were analyzed after the experimental session.

Percentage values for correctly drawn arrangements were calculated. The drawings produced after the fact statement had been presented were of special interest. We were interested in the solutions participants offered, and whether the solutions differed between LOpreserve and LOreject tasks.

Result

Mean percentage rate of correctly drawn initial arrangements was 91.56 % (*SD* = 6.47). Erroneous trials were excluded from further analyses. All drawings of the initial arrangements were horizontally aligned, with roughly equally sized small spaces between the objects. The drawings thus reflect that participants were disposed to interpret the expression "directly left/right of" as implying adjacency and horizontal alignment. Differences in the response patterns within LOpreserve and LOreject test conditions (see below) provides further evidence that participants tended to adopt this interpretation of such relational expressions. Mean percentage rate of correctly drawn revised arrangements was 96.63 % (SD = 3.62). Erroneous trials were excluded from further analyses.

For the revision phase an ANOVA with the factors Condition (LOpreserve, LOreject) × Revision Type (LO, Non-LO) was conducted, and revealed a main effect of Revision Type, $F(2,22) = 20.49$; $p < .001$ $\eta^2_{part} = .48$], and a significant interaction [$F(2,22) = 6.49$; $p = .019$; $\eta^2_{part} = .23$]. LO revisions were performed significantly more often than Non-LO revisions ($t(22) = 4.81$; $p < .001$). In the LOpreserve condition, there were significantly more LO revisions than in the LOreject condition ($t(22) = -2.54$; $p = .019$). The results are depicted in figure 1.

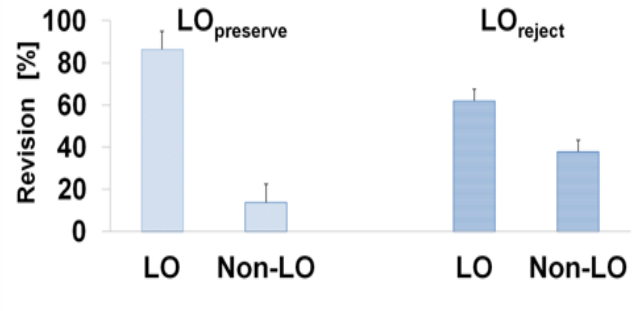

Figure 1. In the LOpreserve condition, participants generated more LO revisions than in the LOreject condition. Error bars showing standard deviations.

The LO effect was mitigated in the LOreject condition. Apparently, participants were more inclined to apply alternative strategies in tasks where the preferred strategy (to relocate the LO) led to the rejection of both initial premises. 10.54% of subject responses represented an (unexpected) alternative revision type (what we call a "hybrid" revision, HY) that represents a sort of compromise between a model resulting from an LO and an RO relocation. The remaining Non-LO revisions reflected (expected) RO relocations. Examples of the three types of response are depicted in figure 2. All correct drawings of revisions, participants provided, were easily and unambiguously identifiable as belonging to one of the three revision types.

Initial Model: Pflaume – Kiwi – Apfel
(Plum – Kiwi fruit – Apple)

LO revision: Kiwi
Pflaume – … – Apfel

RO revision: Pflaume – Kiwi
Apfel

HY revision:

Figure 2. Example of LO, RO, and HY revisions, in response to fact: Kiwi is above Apple. (HY revision is an actual subject drawing.)

In order to investigate the interest of participants in HY revisions, we conducted a second experiment which required participants to select (or verify) different solutions. In the 'revision phase' of the experiment, we presented subjects with all three revised model types (LO, RO, and HY).

Experiment 2: Selecting Spatial Arrangements

Experiment 2 resembled experiment 1. The crucial difference was that experiment 2 required verification of constructed and revised arrangements where in experiment 1 participants needed to generate the required arrangements.

Method

Participants Twenty participants (4 male; age: *M* = 25.60; *SD* = 4.54), all undergraduate students (among them 1 student of psychology) from the University of Giessen, gave written informed consent to participation. Participants were tested individually, and received course credit or were paid at a rate of 8 Euro per hour for participation.

Materials, Procedure, and Design The same verbal descriptions in experiment 1 were presented. Premises occurred on a computer screen, displayed one at a time, in a self-paced manner. Subsequent to premise presentation, the correct spatial arrangement and an incorrect arrangement (correct arrangement inverted), were presented. For example:

1st premise: "Mango directly left of apple."
2nd premise: "Pear directly right of apple."

Correct arrangement:	Incorrect arrangement:
Mango - Apple - Pear	Pear - Apple - Mango

Participants were instructed to select the correct object arrangement, and indicate their choice by pressing a left or right response button with the left or right hand, accordingly. Left and right locations for correct and incorrect arrangements were counter-balanced across the experiment. The number of correct decisions and corresponding decision times were recorded.

Subsequently to the construction phase, a third statement (fact) was presented. The facts resembled the facts presented in experiment 1. Participants were then presented with three object arrangements, among which they had to select a revised arrangement, indicating their choice by pressing the respective left, central, or right response button. In fact, all three arrangements were consistent with the presented "fact". However, the arrangements differed from the initial arrangement according to the position of the three objects, representing an LO, RO, or HY revision. The locations where LO, RO, and HY revisions were displayed were counter-balanced. The revised arrangements chosen, along with corresponding response times, were recorded.

Sixty-four experimental, preceded by 4 practice trials (not analyzed), were presented in a random order. All stimuli were generated and presented using Superlab 4.0 (Cedrus Corporation, San Pedro, CA, 1999) with an RB-530 response box running on a standard personal computer connected to a 19''-monitor.

Result

Mean percentage rate of correctly selected initial arrangements was 89.69 % (*SD* = 11.34). Erroneous trials were excluded from further analyses. Mean percentage rate of correctly selected revised arrangements was 99.89 % (SD = 3.42). Note that participants had to select among correct revisions, and errors were due to presses of "wrong" answer keys. Erroneous trials were excluded from further analyses.

For the revision phase, separate ANOVAs for revision rates and revision times were calculated, respectively with the factors Condition (LOpreserve, LOreject) × Revision Type (LO, RO, HY).

The ANOVA for selection rates revealed a significant interaction Condition × Revision Type [$F(2,18) = 4.94$; $p = .012$; $\eta^2_{part} = .21$], and a marginally significant main effect of Revision Type [$F(2,18) = 3.25$; $p = .070$; $\eta^2_{part} = .15$]. Follow-up tests revealed significantly more selections of HY than LO revisions ($t(19) = -5.80$; $p < .001$) and more RO than LO revisions ($t(19) = -2.54$; $p = .02$) in the LO reject condition (see figure 3). Choices of HY and RO revisions were comparable ($p > .40$) as were revision choices in the LOpreserve condition ($p > .50$).

LO revisions were significantly more frequent in the LOpreserve than in the LOreject condition ($t(19) = -3.22$; $p = .005$; There was also a marginally significant preference for RO revisions in the LOreject compared to the LOpreserve condition ($t(19) = 1.79$; $p = .09$). HY models were chosen comparably often in both conditions ($p > .15$).

The ANOVA for revision times revealed a main effect of Condition [$F(1,19) = 7.68$; $p = .012$; $\eta^2_{part} = .29$]. Revisions in the LOreject condition took significantly longer than in the LOpreserve condition (6.21s; *SD* = 2.52 vs. 5.29s; *SD* = 2.11; $t(19) = 2.77$; $p = .012$). LOreject condition tasks were apparently more difficult to solve.

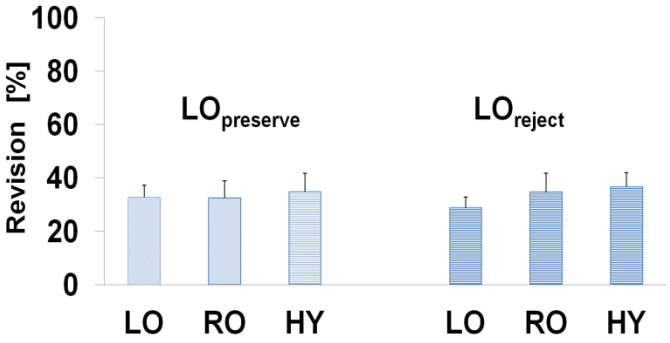

Figure 3. In the LOreject condition, HY and RO revisions were chosen more often than LO revisions. LO revisions were chosen more often in the LOpreserve compared to the LOreject condition. Error bars showing standard deviations.

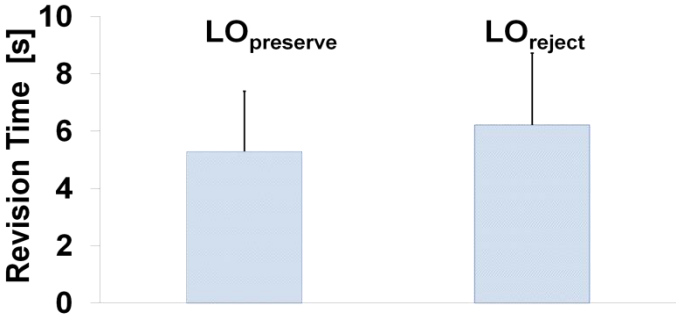

Figure 4. Mean revision times were lower in the LOpreserve than the LOreject condition. Error bars showing standard deviations.

In the LOreject condition, participants provided more Non-LO solutions than in the LOpreserve condition, although (unlike in experiment 1), participants were more likely to consider the HY solution. In fact, there was a slight preference for selecting HY models. However, LO revisions were still more frequent in the LOpreserve than in the LOreject condition, suggesting an effect of minimality, as measured by the number of initial premises preserved.

Discussion

In the case of (spatial) belief revision, it seems reasonable (from an epistemological perspective) to keep changes as small as possible, i.e., to make the revision 'minimal'. The preceding prescription is complicated by the fact that it is not always clear what counts as "minimal" in a given context. The aim of the current study was to assess whether naive subjects exhibit any preference for minimal spatial belief revisions, where minimality is understood in terms of preserving initial premises.

Results of previous studies suggest that reasoners have a strong preference for LO revisions in the context of spatial belief revision (e.g., Bucher et al., 2011; Knauff et al., 2013). Indeed, a recent study showed that the preference for LO revisions supersedes the influence of the putative norm that states that agents should prefer revisions that minimize

the number of objects that are relocated (Bucher & Nejasmic, 2012).

The present study investigates whether the number of initial premises preserved, could serve as an appropriate minimality criterion in spatial belief revision. To that end, we conducted two experiments where we varied the number of premises that are preserved when a spatial model is revised according to LO relocation versus RO relocation. The results show an effect of minimality. In a first (drawing) experiment, the LO effect was reduced in the condition where the LO revision entails the rejection of both premises of the initial description (as opposed to one premise). Participants make more Non-LO revisions in an LOreject condition than in an LOpreserve condition. Many of the Non-LO revisions were (as expected) RO revisions, but many of the Non-LO revisions were (unexpectedly) HY revisions (a compromise between an LO and an RO revision). In a second experiment, participants were required to select one of three possible revision types (LO, RO, or HY). While all three models were selected in almost equal measures in the LOpreserve condition, LO revisions were less often selected in the LOreject condition. This result also suggests an influence of a minimality norm as measured by number of initial premises preserved.

While the effect of the LO principle was strongly present in experiment 1, when participants had to generate the solutions "actively", the effect failed to be present in experiment 2, where possible solutions were given. In fact, in experiment 2, there was a slight preference for selecting HY revisions. Presumably, participants adapted their revision strategy to the conditions of the experiment, and learned to select a solution that is appropriate under both test conditions. Nevertheless, participants were still inclined to select the 'pure' LO revisions more frequently in the LOpreserve condition, and the 'pure' RO revisions more frequently in the LOreject condition. We note, moreover, that subjects (in experiment 2) found trials under the LOreject test condition more difficult than those under the LOpreserve test condition, as reflected by longer response times. This may reflect a tension between a 'basic' preference for LO revisions, and sensitivity to other considerations, such as minimality, in the course of spatial belief revision.

Finally, we want to point out that HY revisions reflect a tendency to preserve features of the initially constructed spatial arrangement (model), rather than the precise content of initial premises (which express, e.g., x *directly* left of y, and z *directly* right of y). When participants select a revised spatial arrangement, it appears that they proceed by modifying the initially constructed spatial model. They do not re-construct the revised model 'from scratch' by reflection on the verbatim content of the given premises. This finding is in accord with previous findings about spatial mental models (e.g., Mani & Johnson-Laird, 1982), and provides evidence for the claim that spatial belief revision can indeed be described as a modification of spatial mental models.

References

Bucher, L. & Nejasmic, J. (2012). Relocating multiple objects during belief revision. In C. Stachniss, K. Schill, & D. Uttal (Eds.), *Lecture Notes in Artificial Intelligence: Spatial Cognition*. Berlin, Germany: Springer.

Bucher, L., Krumnack, A., Nejasmic, J., & Knauff, M. (2011). Cognitive processes underlying spatial belief revision. In L. Carlson, C. Hölscher, & T. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Bucher, L., Nejasmic, J., Bertleff, S., & Knauff, M. (2013). Plausibility and visualizability in relational belief revision. In: M Knauff, M Pauen, N Sebanz, I Wachsmuth (eds) In: *Proceedings of the 35th Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Gärdenfors, P. (1992). Belief revision. Cambridge: Cambridge University Press.

Harman, G. (1986). *Change in view*. Cambridge, MA: MIT Press.

Johnson-Laird, P. N., & Byrne, R. M. J. (1991). *Deduction.* Hove: Erlbaum.

Knauff, M. (2009). A neuro-cognitive theory of deductive relational reasoning with mental models and visual images. *Spatial Cognition and Computation, 9*, 109-137.

Knauff, M., Bucher, L., Krumnack, A., & Nejasmic, J. (2013). Spatial belief revision. *Journal of Cognitive Psychology, 25*, 147-156.

Krumnack, A., Bucher, L., Nejasmic, J., & Knauff, M. (2011). Efficiency and minimal change in spatial belief revision. In L. Carlson, C. Hölscher, & T. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Landau, B., & Jackendoff, R. (1993). 'What' and 'where' in spatial language and spatial cognition. *Behavioral and Brain Sciences*, *16*, 217-265.

Mani, K. & Johnson-Laird, P. N. (1982). The mental representation of spatial descriptions. *Memory & Cognition, 10*, 181–187.

Mikheeva, M., Bucher, L., Nejasmic, J., & Knauff, M. (2013). Spatial reasoning in native speakers of Russian and German. In: M Knauff, M Pauen, N Sebanz, I Wachsmuth (eds) In: *Proceedings of the 35th Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Miller, G.A., & Johnson-Laird, P.N. (1976). Language and Perception. Cambridge: Havard University Press.

Nejasmic, J., Bucher, L., & Knauff, M. (2013). Grounded spatial belief revision. In: M. Knauff, M. Pauen, N. Sebanz, I.Wachsmuth (eds) In: *Proceedings of the 35th Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Talmy, L. (1983). How language structures space. In H. L. Pick & L. P. Acredolo (Eds.), *Spatial orientation: Theory, research and application*. London, UK: Plenum.