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UNSPLIT K$^*$'s

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ABSTRACT

It is shown that the $K_N(1420)$ and the $K^*(890)$ in our 12-GeV/c $K^+p$ experiment fit poorly to the double-pole structure reported for the $A_2$ meson. The probability that our data agree with the double pole hypothesis is less than 1% for the $K_N(1420)$ and completely negligible for the $K^*(890)$, although both resonances are fitted well by a Breit-Wigner shape. Our mass resolution is ±7 MeV at the $K_N(1420)$ and ±5 MeV at the $K^*(890)$.

The reported splitting$^1,2$ of the $A_2$ meson has stimulated a large amount of speculation about possible similar structure in other mesons. The source of the excitement seems to be the suggestion that, if the splitting exists, it may not arise from mundane interference between two resonances (this possibility fits the $A_2$ data), but rather it may be that the $A_2$ itself is a new kind of particle; a double pole.$^3$

If the $A_2$ is a double pole, it quite naturally becomes of extreme interest to examine the other members of the SU(3) multiplet to which the $A_2$ has been assigned; i.e., the $K_N(1420)$ and the $f^0$. 
We present here an analysis of the $K_N(1420)$ and the $K^*(890)$. We have studied the $K^*(890)$ because it seems useful to examine all possible mesons for fine structure and because it happens to be copiously produced in the same topology as the $K_N(1420)$.

The data for this experiment were obtained from a 600,000-picture exposure of the SLAC 82-inch hydrogen bubble chamber to an rf-separated 12-GeV/c $K^+$ beam. Copious production of both $K^*$'s, with subsequent decay into $K^+\pi^-$, is seen in the reaction

$$K^+p \rightarrow K^+\pi^-\pi^+p,$$

of which we have analyzed 27,000 events (see Fig. 1a). This sample represents more than 80% of the events of reaction (1) in our 30-event/μb experiment. The mass distribution in the region of the $K_N(1420)$, in 5-MeV bins, is shown in Fig. 1b, and the $K^*(890)$, in 2-MeV bins, is shown in Fig. 1c.

No statistically significant fine structure is seen for either $K^*$. The data have been divided into several regions of momentum transfer; no significant fine structure is seen in any momentum-transfer region. Also the decay angular distribution of the $K^+\pi^-$ system does not change significantly as a function of the $K^+\pi^-$ mass in the $K_N(1420)$ region.

In order to include the effects of finite mass resolution, and to reveal whether either of the resonance mass distributions is consistent with the double-pole structure considered in Ref. 1 for the $A_2$ meson, two hypotheses have been fitted—a Breit-Wigner and a double pole.

Since this experiment contains the largest single sample of both $K^*$'s published to date, it was felt that an effort should be made to present values of the masses and widths which are as accurate as possible considering the limitations of our experiment. Therefore spin effects have been included in
the amplitudes. However, details of the formulas used (such as spin terms) do not affect the general conclusion as far as the double-pole hypothesis is concerned.

For the Breit-Wigner hypothesis the data have been fitted to the expression

$$\frac{dn}{dm} = B \left( 1 + \alpha F_{BW}(\frac{mR}{p/m}) \right),$$

where the background is approximated in the resonance region by $B = a + bm$, $F_{BW}$ is an $\ell$-wave Breit-Wigner given by

$$F_{BW} = \left( \frac{\Gamma_R/2}{(mR - m)^2 + \left[ \frac{\Gamma(m) / 2}{\Gamma(m) / 2} \right]^2} \right),$$

$m$ is the $K^+\pi^-$ mass, $p$ is the momentum of the $K^+$ in the $K^+\pi^-$ center-of-mass system, $\Gamma(m)$ is the mass-dependent width of the $K^+\pi^-$ resonance, and $mR$, $\Gamma_R$, $pR$ are the mass, width, and decay momentum of the $K^+\pi^-$ system evaluated at the center of the resonance.

We represent $\Gamma(m)$ by

$$\Gamma(m) = \Gamma_R \left[ \frac{p^2 + X^2}{p^2 + (p^2 + X^2)^5} \right] \frac{\ell(p/m)}{\ell(pR/mR)}$$

where we set $X = 100$ MeV/c. Finally $a$, $b$, and $\alpha$ are treated as parameters independent of $m$. 6

For the double pole hypothesis we have used the expression

$$\frac{dn}{dm} = B \left( 1 + \alpha F_{DP}(\frac{mR}{p/m}) \right),$$

$$F_{DP} = F_{BW} \left( \frac{4 (mR - m)^2}{(mR - m)^2 + \left[ \frac{\Gamma(m) / 2}{\Gamma(m) / 2} \right]^2} \right),$$

where $\Gamma_R$ (implicit in the above expression) is no longer the full width at
half maximum (as it was for the Breit-Wigner hypothesis) but the approximate
distance between the two maxima of the double pole.

For each hypothesis, the five parameters $a, b, \alpha, m_R$, and $\Gamma_R$ have
been fitted to the data. The best-fit curves are shown in Fig. 1b and 1c, and
the parameters are presented in Table I.

In evaluating the $\chi^2$ for each fit we have used only the bins within one
full width of the central resonance mass. The confidence levels for the
Breit-Wigner hypothesis are quite acceptable; 47\% for the $K_N(1420)$ and 42\%
for the $K^*(890)$. On the other hand the confidence levels for the double pole
hypothesis are low; < 1\% for the $K_N(1420)$ and $< 10^{-10}$ for the $K^*(890)$. If
we compare the two hypotheses for the $K_N(1420)$ by the likelihood ratio method,
we find that the Breit-Wigner hypothesis is favored by a factor of 600 000.

In each fit the theory has been folded with an experimental mass-
resolution function which represents the effects of our measurement errors.
The mass-resolution function was obtained from the calculations of the kinematic fitting programs. If our mass resolution at the $K_N(1420)$ were worse
than the calculation by a factor of two, then our data could not display a
double-pole shape, even if one existed, because the narrow dip would have
been washed out. Therefore the question of whether our resolution function
is correct must be answered with great care.

Fortunately a method is available that provides a very firm answer
to the resolution question. More than 1000 events of the type

$$K^+ p \rightarrow K^0 \pi^+ p$$

are available in our data. Reaction (2) has many similarities to reaction (1).
It has a peripheral proton much of the time. The dominant effects in both
reactions are either (a) formation of a $\Delta^{++}$ by the $\pi^+p$ system, or (b) formation of a low-mass system recoiling peripherally against the proton.

If reaction (2) is treated without the assumption that a $K^0$ is formed—that is, as $K^+p \rightarrow \pi^+\pi^-\pi^+p$—and the $\pi^+\pi^-$ system is examined, then (a) the resolution function for the $\pi^+\pi^-$ system can be calculated, and (b) this calculated resolution function can be compared with the actual distribution of $\pi^+\pi^-$ mass, which would be essentially a delta function at the $K^0$ mass, if it were not for our measurement errors.

The result of this analysis is shown in Fig. 2a, where it is seen that our calculated resolution function fits the $K^0$ data extremely well (CL = 71%). Figure 2b shows the computed half width of the resolution function as a function of the $Q$ value of the $K^+\pi^-$ system in reaction (1), with the results of the $K^0$ analysis included to illustrate how well the $K^0$ reaction simulates reaction (1).

The above analysis shows that it is extremely unlikely that our mass resolution could be underestimated by as much as 20% (see Fig. 2a); but in that case, the confidence levels for the double pole would be 3% for the $K^0_N(1420)$ and $< 10^{-10}$ for the $K^*(890)$; and for the $K^0_N(1420)$, the likelihood ratio test favors the Breit-Wigner hypothesis by a factor of 15 000.

We conclude that the $K^0_N(1420)$ and the $K^*(890)$ do not exhibit the double pole structure reported for the $A_2$ meson, in fact, they exhibit no obvious fine structure of statistical significance at all.
Acknowledgments

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References


5. We have found that our fits are insensitive to the value of X for 0 ≤ X ≤ 200 MeV/c and slightly poorer for X > 200 MeV/c.

6. We have investigated the effect of mild mass dependences on these parameters, and they do not change our results in any significant way.

Table 1. Results of resonance shape fits.

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<th>$K^*(890)$</th>
<th>$K_N(1420)$</th>
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<td>Mass range for fit (MeV)</td>
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<td>Mass range for $\chi^2$ (MeV)</td>
<td>840-940</td>
<td>1320-1520</td>
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<td>$\Gamma/2$ of resolution function (MeV)</td>
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<td>Bin size (MeV)</td>
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<td>5</td>
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<td>Approximate number of events in resonance</td>
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<td>2 200</td>
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<td>Background/signal at peak</td>
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**Breit-Wigner fit**

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<tr>
<td>$\chi^2$/degrees of freedom</td>
<td>46/45</td>
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<td>Confidence level</td>
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<td>47%</td>
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<td>$M_R$ (MeV)</td>
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<td>1421.1±2.6$^a$</td>
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<tr>
<td>$\Gamma_R$ (MeV)</td>
<td>53.2±1.6</td>
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**Double pole fit**

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<td>$\chi^2$/degrees of freedom</td>
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<td>62/35</td>
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<td>Confidence level</td>
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<td>&lt;4%</td>
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<td>$M_R$ (MeV)</td>
<td>892.3±0.5$^a$</td>
<td>1421.7±1.3$^a$</td>
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<td>$\Gamma_R$</td>
<td>16.3±0.3</td>
<td>27.6±1.6</td>
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a. Statistical errors only. We estimate the systematic error of $M_R$ to be 2 MeV for the $K^*(890)$ and 3 MeV for the $K_N(1420)$. See Fig. 2a caption.
Figure Captions

Fig. 1. $K^+\pi^-$ mass distributions in the reaction $K^+_p \rightarrow K^+\pi^-\pi^+_p$ at 12 GeV/c. No cuts have been made. (a) All events (27,000 events). (b) $K_N(1420)$ region. The solid curve is the best fit to a Breit-Wigner ($CL = 47\%$), the dashed curve is the best fit to a double pole hypothesis ($CL < 1\%$). Both curves have the experimental mass resolution function ($\Gamma/2 = 6.5$ MeV) folded in. (c) $K^*(890)$ region. The Breit-Wigner fit (solid curve) has a confidence level of 42%, the dipole fit (dashed curve) has a confidence level of $< 10^{-10}$. The resolution function has $\Gamma/2 = 5.1$ MeV.

Fig. 2. (a) $\pi^+\pi^-$ Mass distribution of 1000 events of $K^+_p \rightarrow K^0\pi^+_p \rightarrow (\pi^+\pi^-)\pi^+_p$ where the constraints on the $K^0$ have been discarded. This simulates the reaction of Fig. 1 (see text). The calculated resolution function is shown as the solid curve. Note the poor fit that results when the width of the resolution function is increased by 20% (dotted line). The central mass of the $K^0$ from this sample is $496.7 \pm 0.1$ MeV, 1 MeV below the accepted value of $497.76 \pm 0.1$ MeV, indicating a systematic error in mass. Accordingly we have indicated a systematic error in our determination of both $K^*$ masses (see Table I).

(b) Mass resolution ($\Gamma/2$) for the $K^+\pi^-$ system in $K^+_p \rightarrow K^+\pi^-\pi^+_p$ as a function of $Q$ value of the $K^+\pi^-$ system. The points (closed circles) were derived from the errors assigned to the actual events. The closed diamond is the calculated resolution function ($\Gamma/2$) for the $\pi^+\pi^-$ system in $K^+_p \rightarrow (\pi^+\pi^-)\pi^+_p$, where the $\pi^+\pi^-$ comes from a $K^0$. The open diamond is the $\Gamma/2$ for the actual distribution of $\pi^+\pi^-$ mass in Fig. 2a.
Fig. 2b
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