LOCALITY AND REALITY

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ABSTRACT

Einstein's principle that no signal travels faster than light suggests that observations in one spacetime region should not depend on whether or not a radioactive decay is detected in a spacelike separated region. This locality property is incompatible with the predictions of quantum theory, and this incompatibility holds independently of the questions of realism, objective reality, and hidden variables. It holds both in the pragmatic quantum theory of Bohr and in realistic frameworks. It is shown here to hold in a completed realistic quantum theory that reconciles Einstein's demand for a description of reality itself with Bohr's contention that quantum theory is complete. This completed realistic quantum theory has no hidden variables, and no objective reality in which observable attributes can become definite independently of observers. The theory is described in some detail, with particular attention to those aspects related to the question of locality. This completed realistic quantum theory is in principle more comprehensive than Bohr's pragmatic quantum theory because it is not limited in principle by the requirement that the observed system be physically separated from the observing one. Applications are discussed.

1. INTRODUCTION

Einstein's principle that no signal travels faster than light suggests that observations in one spacetime region should not depend on whether or not a radioactive decay is detected in a spacelike separated region. An indication that this locality property might not hold in nature is contained in a 1964 theorem by J. S. Bell, which asserts that no deterministic local hidden-variable theory can give all the predictions of quantum theory. However, since the prevailing belief among physicists is that nature is both nondeterministic and without hidden-variables the theorem of Bell is usually interpreted merely as a confirmation of those prevailing beliefs, rather than an indication that nature is nonlocal.

Actually, however, the requirements of determinism and of hidden variables play no essential role in the proof of Bell's theorem, which has been reformulated as the assertion that no theory that predicts individual results that conform to the contingent predictions of quantum theory can be local. The discussion given there stresses that the issues are not "theory" and "predict" in the sense that what is excluded is a theory that would allow some human being to predict the results. The result is better expressed by the statement: "No process that selects observations that conform to the contingent predictions of quantum theory can be local."

The nature of this process is completely unrestricted. It can be a process of creation of the kind described by Whitehead, or the hand of the Almighty, or His dice. It need not be reducible to mathematical form, nor be expressible in terms of hidden-variables defined on a measure space.
Even the concept of process is inessential. The result can be express also in this way: "No selection of observations that conform to the contingent predictions of quantum theory can be local."

The word "contingent" stresses the fact that quantum theory makes predictions for alternative mutually exclusive experiments. The selected observations are required to conform to the predictions of quantum theory in four alternative experimental set-ups. These correspond to four alternative settings of a pair of devices. The setting of each device is controlled by whether or not a radioactive decay is detected within the device during a specified time interval. Thus one may speak of the dependence of the selected observations on whether or not a decay is detected under these specified conditions.

According to quantum theory the choice between whether or not the decay is detected is a stochastic variable: it is a variable with two possible values, and a weight, or probability, assigned to each one. Thus we may speak of the dependence of the selected observations on these stochastic variables.

Each device is confined, during the entire period associated with the specified conditions of detection, and the subsequent period during which the results of the experiment are observed, to a certain spacetime region, and these two regions are arranged to be spacelike-separated.

The selected observations represent observations that could occur under the specified conditions. All possibilities are initially considered.

According to quantum ideas the choice between whether or not the decay is detected is a matter of pure chance, with no significant deterministic roots in the past. But then the information about which choice is made in either region cannot travel the other region without moving either faster than light or backward in time. These possibilities are excluded by normal ideas. But if the information about which choice is made in one region cannot get to the other region then a change in the choice in the first region should leave unchanged the observations selected in the spacelike separated region. This argument suggests that the following locality condition should hold: A change in the stochastic variable that specifies the setting in either region leaves unchanged the observations selected in the spacelike separated region.

This locality condition imposes certain relationships among the observations selected in alternative experimental situations. However, there is no way to select observations that satisfy both these relationships and the contingent statistical predictions of quantum theory.

This claim that the locality condition is incompatible with the predictions of quantum theory can be compared to a weaker claim, made in recent articles on the interpretation of Bell's theorem, that the validity of these predictions entails the failure of either locality or realism, where "realism is a philosophical view according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone."
In the context of those papers realism means hidden variables. Given a choice between locality and hidden variables most physicists would follow Bohr and choose locality. However, this option is excluded by the stronger result stated above: Given the validity of the predictions of quantum theory the locality condition must fail regardless of the existence or nonexistence of hidden variables or "objective reality."

One aim of the present paper is to affirm this result, first by logical analysis, and then by exhibiting the nonlocality argument in two explicit models of reality neither of which has either any "objective reality" in which observable attributes can become definite independently of observers, or any variable that is "hidden" in the sense that it is not present in the pragmatic quantum theory of Bohr.

These two models are not just idle schemes, concocted to prove the point. Each is a completed realistic quantum theory that reconciles Einstein's demand that the complete physical theory be local, and describe physical reality itself (rather than merely the rules by which scientists can calculate correlations among their observations under certain idealized and neverfully realized conditions of separation) with Bohr's contention that quantum theory is essentially the complete theory of physical reality.

The structure of the paper is this. In §2 the proof of the nonlocality theorem is reviewed, with particular attention to a criticism of it made in Ref. 4. The basic ingredient is a purely mathematical property of quantum theory symbolized by the equation \( L \cap Q = \emptyset \). In §3 this mathematical property \( L \cap Q = \emptyset \) is shown to entail Bell's result that no deterministic-stochastic hidden-variable theory that gives the predictions of quantum theory can be local. In §4 this property \( L \cap Q = \emptyset \) is shown to entail also the result of Bell and Clauser and Horne that no probabilistic hidden-variable theory that gives the predictions of quantum theory can be local.

In §5 it is shown that the fact that the analysis of §2 deals with alternative mutually exclusive experiments does not bring it into conflict with Bohr's complementarity principle. This discussion is framed within a general description of the Copenhagen interpretation needed later.

Section 6 describes a completed realistic quantum theory that has no hidden variables at all. It is a version of the many-worlds view of reality, called here dual-reality quantum theory because it consists, explicitly and exclusively, of two types of reality. The first is the absolute reality demanded by Einstein, and represented by the wave function that develops always lawfully according to the Schroedinger equation. The second is the experienced reality consisting of the myriads of personal realities that were the focus of Bohr's epistemological considerations. These two distinct realities are interwoven in a way that is controlled by the Schroedinger equation. The nature of this interweaving is described in some detail, with particular attention to those aspects that are relevant to the question of locality. Particular attention is paid also to the delicate but important question of the interpretation of the quantum theoretical probabilities.

The idea that reality consists of two distinct but related parts, the absolute impersonal reality, and the experienced personal
realities, is hardly new: it is an innate part of adult thinking. However, the absolute reality of dual-reality quantum theory is not a resident of ordinary spacetime, and this creates certain divergences from naive ideas. The way in which the classical worlds of ordinary experience, which reside in ordinary spacetime, arise automatically from the workings of the Schroedinger equation, is described in enough detail to permit a discussion of the way in which the nonlocality property manifests itself in this model of reality. The question of faster-than-light signals in this dual-reality model is discussed in §7.

In the dual-reality model all possible observations become actual observations, and hence no absolute choice or selection is ever made between different possible observations. Two objections to choiceless models of the many-worlds type are raised in §8, and a second model of reality is proposed. This second model consists of the dual-reality quantum theory supplemented by both "choices" associated with the experienced personal realities, and also "selections" associated with the absolute reality. The "choices" are naturally localized, as are the personal realities with which they are associated.

This completed quantum theory, like the first one, has no objective reality in which observable attributes can become definite independently of observers. It has no hidden variables referring to entities outside the pragmatic quantum theoretical framework. It reconciles the demands of Bohr and Einstein, and it allows meaningful choices.

The nonlocality theorem of §2 applies directly to this completed quantum theory and entails that the "selections" have a nonlocal or global character. This is not unnatural, since the absolute reality with which they are associated is not a resident of ordinary spacetime.

This completed quantum theory and its possible applications are discussed in the concluding section. The main point is that this completed quantum theory, though containing no elements not recognized in Bohr's pragmatic quantum theory, is in principle more comprehensive than Bohr's version because it is not restricted in principle to idealized situations in which the observed system can be physically separated from the observing system.

2. THE NONLOCALITY THEOREM

Consider the following experimental set-up: a pair of low-energy spin-\( \frac{1}{2} \) particles is allowed to scatter in a small region that is surrounded by an array of detectors. These detectors are arranged to cover almost completely a sphere centered on the collision region. Only two small escape holes are left uncovered. These lie at polar extremities of the sphere. The two particles are detected before the scattering. Thus if they are not detecting Shortly afterward by the spherical array then they have escaped through the two holes and are traveling on trajectories that will lead one into a Stern-Gerlach device A and the other into a Stern-Gerlach device B. The directions of the deflecting fields in these two devices are both perpendicular to the common line of flight of the particles, and they are initially fixed to be parallel to each other. However, each device is attached to an apparatus that will rotate the direction of the deflecting field to a new setting if it is activated by a detection, during a
prescribed time interval, of the products of the decay of a radioactive nucleus placed within it. The prescribed time interval is programmed to be shortly before the programmed time of arrival of the particle.

This single pair of particles is part of a large set of \( n \) similar pairs that can be separately analyzed by fast electronics, but that are all bunched together so that each Stern-Gerlach device has one setting or the other for the entire set of \( n \) particles passing through it.

Since each Stern-Gerlach device has two possible settings there are four possible settings of the pair of devices. According to quantum theory the question of whether or not the decay will be detected is a matter of pure chance, and the two possible settings of the two devices can be represented by the two possible values of two independent stochastic variables \( x_A \) and \( x_B \). Each of these two variables has two possible values, say 1 and 2, and each possible value is assigned a certain weight.

Let the spacetime region corresponding to the entire laboratory \( L_A \), from the beginning of the specified time interval associated with the setting of device \( A \) until the time in which the results at \( A \) are observed, be called \( R_A \), and let \( R_B \) be defined similarly. Let the arrangements be such that \( R_A \) and \( R_B \) are spacelike separated. Then the locality condition described in §1 is this: Changing the value of \( x_A \) leaves unchanged the observations selected in \( R_B \), and changing the value of \( x_B \) leaves unchanged the observations selected in \( R_A \).

The observations referred to here are precisely the observables of quantum theory. They are explicit and integral parts of the quantum formalism, which makes contingent statistical predictions about the observations selected in alternative mutually exclusive experimental arrangements, and treats all of the alternative possibilities on an equal footing.

Consider now the collection of all conceivable observations of the results of the experiment for all four possible values of \((x_A, x_B)\). For each of the four possible settings there are \( 4^n \) different conceivable observations. Let the \( 4^n \) different conceivable observations in the case \((x_A, x_B) = (1,1)\) be labelled by the index \( j \), which runs from 1 to \( 4^n \). Let the \( 4^n \) different conceivable observations in the case \((x_A, x_B) = (2,2)\) be labelled by the index \( k \), which also runs from 1 to \( 4^n \). This gives two sets of \( 4^n \) conceivable observations. The \( j \)th conceivable observation is identified by a sequence of \( n \) pairs of numbers \((r_{ij}(A,1,1,j), r_{ij}(B,1,1,j); i = 1,2,\ldots,n)\).

The \( k \)th conceivable observation is identified by a sequence of \( n \) pairs of numbers \((r_{ij}(A,2,2,k), r_{ij}(B,2,2,k); i = 1,2,\ldots,n)\).

Here \( r_{ij} \) is plus one or minus one according to whether the individual observed result on the \( i \)th pair is a deflection up or down, relative to the direction of the deflecting field, and the argument \( A \) or \( B \) specifies the deflecting device.

If the locality condition is valid then changing \( x_A \) or \( x_B \) leaves unchanged the observation selected in the other region. But then one can generate, for each conceivable pair of selected
observations \((j,k)\) for the \((1,1)\) and \((2,2)\) cases, the observations selected in the \((1,2)\) and \((2,1)\) cases. In particular, the locality condition gives

\[
\begin{align*}
    r_i(A,1,2,j,k) & = r_i(A,1,1,j,k) \\
    & = r_i(A,1,1,j) \\
    r_i(B,1,2,j,k) & = r_i(B,2,2,j,k) \\
    & = r_i(B,2,2,k) \\
    r_i(A,2,1,j,k) & = r_i(A,2,2,j,k) \\
    & = r_i(A,2,2,k) \\
    r_i(B,2,1,j,k) & = r_i(B,1,1,j,k) \\
    & = r_i(B,1,1,j)
\end{align*}
\]

(2.1)

These equations are called the locality equations.

The locality condition thus generates a set of \(4^n \times 4^n\) possible quartets of selected observations. If these quartets are labelled by a variable \(q\), which runs from 1 to \(4^n\), then for each \(q\) the following locality condition is satisfied: for all \(i \in \{1, \ldots, n\}\)

\[
\begin{align*}
    r_i(A,1,2,q) & = r_i(A,1,1,q) \\
    r_i(B,1,2,q) & = r_i(B,2,2,q) \\
    r_i(A,2,1,q) & = r_i(A,2,2,q) \\
    r_i(B,2,1,q) & = r_i(B,1,1,q)
\end{align*}
\]

(2.2)

These equations are called the locality equations. The range of \(q\) is now extended to \(2^8 n\), and each different conceivable quartet is labelled by an index \(q\). Let \(S\) denote the set of all of the conceivable quartets and let \(L\) denote the subset of \(S\) upon which the locality equations (2.2) are satisfied. This set \(L\) consists of precisely the \(4^n \times 4^n\) quartets \((j,k)\) constructed above. If one had started with all conceivable pairs of selected observations in the \((1,2)\) and \((2,1)\) cases then the locality condition would have generated this same set \(L\) of quartets. Thus the locality condition ensures that each selected observation is a member of a possible quartet of observations, and each of these quartets belongs to the subset \(L\) of \(S\). The subset \(Q\) of \(S\) is defined by the condition that for each of the four observations comprising any quartet \(q\) in \(Q\) the observed value of the correlation parameter lies within 3\% of the large \(n\) limit predicted by quantum theory. The subset
\( \emptyset \) of \( S \) is the empty set.

An easily proved mathematical property of the quantum theoretical predictions for the correlation parameter is the following "mathematical nonlocality property of quantum theory":

\[ L \cap Q = \emptyset. \tag{2.4} \]

This equation says that there is among the \( 4^n \times 4^n \) quartets \( q \in L \) not even one that satisfies the condition that the observed correlation parameter be within 3% of the quantum-theoretical large \( n \) limit, for all four members of the quartet. By taking \( n \) sufficiently large the total probability that the correlation parameter will differ from its large \( n \) limit by more than 3% can be made arbitrarily small. Thus for every one of the quartets \( q \in L \) at least one of the four members is an observation that can be made to lie in a preassigned region of arbitrarily small quantum-theoretic probability. Thus no process that selects observations that conform to the contingent predictions of quantum theory can be local, in the sense that a change of \( x_A \) leaves unchanged the observation selected in \( R_B \), and a change in \( x_B \) leaves unchanged the observation selected in \( R_A \).

In this argument it was tacitly assumed that the order of making the changes of \( x_A \) and \( x_B \) was immaterial. Since the regions \( R_A \) and \( R_B \) are spacelike-separated this commutivity property is demanded by the same relativistic notions that were the basis of the locality condition: this commutativity condition should be considered to be an implicit part of the locality condition.

Clauser and Shimony have objected to this commutativity assumption in my proof. Strangely, they have not objected to this same assumption in their own proofs, or in Bell's proof. All these proofs collapse if the commutivity assumption is not made.

To construct a counter-example to Bell's theorem that no deterministic local hidden-variable theory can give all the predictions of quantum theory one can proceed as follows: Let the hidden variable be expressed as a pair of variables \((\lambda, \theta)\), where \( \lambda \) lies in some set \( A \), and \( \theta \) can be 0 or 1. Let the four possible settings be labelled by the index \( s \) according to the prescription

\[
\begin{align*}
  s &= 1 \sim (1,1) \\
  s &= 2 \sim (1,2) \\
  s &= 3 \sim (2,2) \\
  s &= 4 \sim (2,1).
\end{align*}
\tag{2.5}
\]

For each value of \( s \) and \( \lambda \) there are two possible values of \( \theta \), which correspond to two different physical situations. For fixed \( \lambda \) let the eight possibilities be labelled by the index

\[ t = s + 4 \theta, \tag{2.6} \]

which runs from 1 to 8. The locality condition may then be cast into the form

\[
\begin{align*}
  r_i(A,t,\lambda) &= r_i(A, t+1,\lambda) \quad (t \text{ odd}) \\
  r_i(B,t,\lambda) &= r_i(B, t+1,\lambda) \quad (t \text{ even}).
\end{align*}
\tag{2.7}
\]

This condition ensures that changing the setting of \( x_A \) (resp. \( x_B \)) leaves unchanged the result in \( R_B \) (resp. \( R_A \)). Both the results, and the change in the results, naturally depends on the value of the
hidden variables, and hence upon $\theta$.

It is now easy, and amusing, to construct deterministic models
that satisfy both locality and the predictions of quantum theory.
In these models the changes in $x_A$ and $x_B$ do not commute,
and one no longer has only the quartets constructed above. However,
these models are not really local due to this noncommutivity
of the changes of $x_A$ and $x_B$.

§ 3. BELL'S THEOREM

Bell's theorem is conventionally formulated as the assertion that
no deterministic local hidden-variable theory can give the predictions
of quantum theory. This formulation of Bell's result is misleading
on two counts. In the first place the word "deterministic" suggests
that the result applies only to deterministic theories, whereas the
hidden variables can equally well be stochastic. Hence Bell's result
covers the cases where some or all of the results are determined
partly or wholly by chance.

In the second place the word "hidden-variable" suggests some
dependence of the result on the assumption of objective realities
that lie outside the framework of conventional quantum theory.
Actually, Bell started from conventional quantum theory plus locality,
and what he actually derived is essentially the result stated in §2.
But the tie he made with the Einstein-Rosen-Podolsky paradox has
tended to confuse this simple result with the question of the
existence of an objective physical reality of the kind sought by
Einstein, and hence with such extraneous and murky issues as the
EPR criterion of physical reality.

The mathematical result $\mathcal{L} Q = 0$ is essentially the mathe-
matical core of Bell's proof, and his result follows directly from
it. A local deterministic hidden-variable theory is, in this
case, defined to be one in which the results arrange themselves
in quartets that satisfy the locality equation (2.2), but with the
label $q$, which in §2 runs over all conceivable quartets,
replaced by a variable $\lambda$, which is supposed to give a specification
of the state that is more complete than the one provided by quantum
theory, and which in the context of the EPR paper was the combined
momentum and coordinate of the particle. Since the momentum and
coordinate of a particle are never directly observed the hidden
variable $\lambda$ appears to represent a set of variables that would
describe an objective microscopic reality of the kind occurring
in classical physics.

This imagery is irrelevant. The important condition, which follows
from locality alone, is that the results be arranged in quartets that
satisfy the locality condition (2.2). For then the mathematical
result $\mathcal{L} Q = 0$ entails that there is no $\lambda$, and no weighted
sum over various $\lambda$'s, such that a theory of this kind can give the
contingent predictions of quantum theory.

§4. PROBABILISTIC LOCAL HIDDEN-VARIABLE THEORIES.

Bell$^6$ and Clauser and Horne$^7$ have considered probabilistic local
hidden-variable theories. These are characterized by the requirement
that the conditional probability of the pair of results $(r_A, r_B)$,
subject to the conditions that the two setting parameters have the
values $x_A$ and $x_B$, has the form
\( \{r_A, r_B | x_A, x_B \} \)

\[
\begin{align*}
= N 
\sum_{\lambda=1}^{\infty} \rho(\lambda) P_A(\lambda, r_A, x_A) P_B(\lambda, r_B, x_B),
\end{align*}
\]

whereas the conditional probabilities for \( r_A \) and \( r_B \) separately are

\[
\begin{align*}
\{r_A | x_A\} = N 
\sum_{\lambda=1}^{\infty} \rho(\lambda) P_A(\lambda, r_A, x_A),
\end{align*}
\]

and

\[
\begin{align*}
\{r_B | x_B\} = N 
\sum_{\lambda=1}^{\infty} \rho(\lambda) P_B(\lambda, r_B, x_B),
\end{align*}
\]

Here the weight functions satisfy

\[
\sum_{\lambda=1}^{\infty} \rho(\lambda) = 1
\]

\[
\sum_{\lambda=1}^{\infty} P_A(\lambda, r_A, x_A) = 1
\]

\[
\sum_{\lambda=1}^{\infty} P_B(\lambda, r_B, x_B) = 1
\]

and

\[
\rho = |\rho|, \quad P_A = |P_A|, \quad P_B = |P_B|.
\]

This probabilistic formulation of the locality condition is equivalent to the formulation (2.2). In particular, the following result can be proved.\(^{10}\)

**Equivalence Theorem** If a quartet of individual results satisfy the locality condition

\[
\begin{align*}
\{r_A, r_B | x_A, x_B \} = \{r_A, r_B | x_A, x_B \}
\end{align*}
\]

and

\[
\begin{align*}
\{r_A | x_A \} = \frac{1}{n} \sum_{i=1}^{n} \theta_i(A, r_A, x_A),
\end{align*}
\]

then the average values defined by

\[
\begin{align*}
\{r_A, r_B | x_A, x_B \} & = \frac{1}{n} \sum_{i=1}^{n} \theta_i(A, r_A, x_A),
\end{align*}
\]

\[
\begin{align*}
\{r_B | x_B \} & = \frac{1}{n} \sum_{i=1}^{n} \theta_i(B, r_B, x_B)
\end{align*}
\]

where

\[
\theta_i(A, r_A, x_A) = \begin{cases} 
1 & \text{if } r_i(A, x_A) = r_A \\
0 & \text{if } r_i(A, x_A) \neq r_A
\end{cases}
\]

and

\[
\theta_i(B, r_B, x_B) = \begin{cases} 
1 & \text{if } r_i(B, x_B) = r_B \\
0 & \text{if } r_i(B, x_B) \neq r_B
\end{cases}
\]

can be expressed in the form (4.1) (trivially, by identifying \( \lambda \) with \( i \)) Conversely, probabilities satisfying the locality conditions (4.1) can be reproduced—up to terms that vanish as \( n \) tends to
infinity—as averages (4.2) over individual results that satisfy the individual-result locality condition (2.2).

This result can be summarized as follows: Let

\[ P \equiv \{(r_A|A), (r_B|B), \{r_A, r_B|x_A, x_B\}; x_A \in \{1,2\}, x_B \in \{1,2\}\} \]

be any set of probabilities, defined for all four possible values of \((x_A, x_B)\). Let \( \mathcal{P} \) be the set of all \( P \)'s. Let \( L_P \) be the subset of \( \mathcal{P} \) consisting of those \( P \)'s that satisfy the probabilistic locality conditions (4.1). Let \( P(q) \) be the set of probabilities constructed from quartet \( q \) of §2 according to the equations (4.2). Then the following equivalence theorem holds: 10

**Theorem I (Equivalence)**

a) If \( q \in L \) then \( P(q) \in L_P \).

b) If \( P \in L_P \) then there is a \( q \in L \) such that \( P(q) \cong P \),

where \( \cong \) means that the difference can be made smaller than any preassigned number \( \epsilon \) by taking \( n \) sufficiently large.

The locality property \( L \cap Q = \emptyset \) of §2 can be restated as follows:

**Theorem II (Nonlocality)**

Let \( Q_P \) be the set of \( P \) such that the correlation parameter calculated from the statistical weights \( P \) agrees with the value predicted by quantum theory in the large \( n \) limit to within 3\%, for all four values of \((x_A, x_B)\). Then

\[ \{q; q \in L, P(q) \in Q\} = \emptyset . \]

That is, there is no \( q \) such that \( q \) is in \( L \) and \( P(q) \) is in \( Q_P \).

These two mathematical results entail the following result: 10

**Theorem III (Probabilistic Nonlocality)**

\[ L_P \cap Q_P = \emptyset . \]

That is, no set of probabilities \( P \) can satisfy the probabilistic locality condition and give correlation parameters that agree with the predictions of quantum theory to within 3\% (i.e., to < 3\%).

**Proof.** Suppose there were a \( P \) such that \( P \in L_P \) and \( P \in Q_P \). Since \( P \in L_P \) we conclude from theorem I(b) that there is an \( q \in L \) such that \( P(q) \cong P \). Since \( P \) is in \( Q_P \) the \( P(q) \) will be within \( Q_P \) if we chose \( \epsilon \) appropriately. Thus there must be a \( q \) such that \( q \in L \) and \( P(q) \in Q_P \). However, this possibility is excluded by theorem II. Therefore there is no \( P \) such that \( P \in L_P \) and \( P \in Q_P \):

\[ L_P \cap Q_P = \emptyset . \]

Q.E.D.

The property \( L \cap Q = \emptyset \) therefore entails that no deterministic-stochastic or probabilistic hidden-variable theory that gives the predictions of quantum theory can be local. But what does it say about non-hidden-variable theories, and is particular about quantum theory itself, both within the orthodox Copenhagen interpretation and outside that interpretation? This question is answered in the following sections.
§5. THE COPENHAGEN INTERPRETATION

The Copenhagen interpretation can be divided into two parts, characterized by the words "pragmatic" and "complementarity". The first part affirms that quantum theory is fundamentally a set of rules by which scientists can calculate the probabilities that their observations will conform to certain experimental specifications in circumstances that conform to other experimented specifications. In the words of Bohr "Strictly speaking, the mathematical formalism of quantum mechanics and electrodynamics merely offers rules of calculation for the deduction of expectations about observations obtained under well defined conditions specified by classical physical concepts." The basic format of the calculation is this: A set of experimental specifications $A$ on the preparation of the system under study is mapped into a density matrix $\rho_A$, and a set of experimental specifications $B$ on a subsequent observation of this system is mapped into a density matrix $\rho_B$, and the conditional probability of $B$ subject to $A$ is

$$\{B|A\} = \text{Tr}_{A} \rho_B \cdot (5.1)$$

The second part of the Copenhagen interpretation is an attempt to explain how a theory that is admittedly merely a set of rules for calculating the probabilities of specified observations under specified conditions can be considered a complete theory of physical reality (or of atomic phenomena). The basis of this attempt is the notion of "complementarity", which is the idea that a complete physical theory need not represent conjunctively information obtainable only from alternative mutually exclusive experiments.

In more detail the point is this. The quantum theoretical formula (5.1) for the conditional probability $\{B|A\}$ can be expressed in the equivalent form $^{13,14}$

$$\{B|A\} = \int dx dp \rho_A(x,p,t) \rho_B(x,p,t) . \quad (5.2)$$

This form is identical to that used in classical theory where $\rho_A(x,p,t)$ is the probability that a system prepared according to specifications $A$ will have coordinates $x = (x_1, \ldots, x_n)$ and momenta $p = (p_1, \ldots, p_n)$ at time $t$, and $\rho_B(x,p,t)$ is the probability that a system with coordinates $x$ and momenta $p$ at time $t$ will lead to an observation satisfying specifications $B$. The right-hand side of (5.2) is independent of $t$ by virtue of the equations of motion for the functions $\rho_A(x,p,t)$ and $\rho_B(x,p,t)$. For free particles these equations are the same for the classical and quantum cases, namely

$$\rho_C(x,p,t) = \rho_C(x - v(p), p, t)$$

where $v(p)$ is the set of velocities of the particles that have momenta $p$, and $C$ is $A$ or $B$.

One important difference between the classical and quantum cases is that according to classical theory there are, in principle, for any arbitrarily small neighborhood $\eta$ of any point $(p,x)$, and for any $t$, specifications $A$ and $B$ such that $\rho_A(x,p,t)$ and $\rho_B(x,p,t)$ are zero outside $\eta$, whereas in quantum theory there is a mathematical substructure that prevents the region in $(p,x)$ space on which the probability function is (essentially) nonzero from being smaller than $\hbar^2$, where $\hbar$ is Planck's constant. This limitation is a form of Heisenberg's uncertainty principle. Thus although one
may consider a set of physical specifications that correspond to concentrating the probability in a region very narrow in \( p \)-space and very broad in \( x \)-space, and also an alternative set of physical specifications that correspond to concentrating the probability in a region very narrow in \( x \)-space and very broad in \( p \)-space, the mathematical structure does not permit the quantum theoretical probabilities to be concentrated in the intersection of two such regions. The cornerstone of the Copenhagen interpretation is the assertion that the physical specifications in two such cases demand experimental conditions that are mutually exclusive. More generally, the assertion is that the uncertainty principle limitations inherent in the mathematical structure correspond to physical limitations in our ability to obtain knowledge about \( \rho_A(x,p,t) \) and \( \rho_B(x,p,t) \). This result would presumably be a tautology if quantum theory were a homogeneous structure, since if one assumes quantum theory then, by virtue of the inherent mathematical structure of the theory itself, no probability distributions that violate the uncertainty principle limitation could ever arise. However, according to the Copenhagen interpretation the experimental specifications are made in terms of classical concepts, based on classical physics, and they depend on a separation of the world into the system being studied, which is represented quantum theoretically, and the observing system, which is treated classically. Within this hybrid structure the uncertainty principle limitation appears tied to the loss of information due to the uncontrollable character of the interaction between the two differently described systems. The paradigm is Heisenberg's analysis of the information obtainable about the preparation or measurement of the position and momentum of a particle by examining it under a microscope.

The Copenhagen interpretation unconditionally rejects, therefore, the idea that a probability function \( \rho_A(x,p,t) \) (or \( \rho_B(x,p,t) \)) can represent the information about a system extracted from two alternative mutually exclusive experiments. Yet our analysis in §2 rests directly upon the use, in one theoretical analysis, of the conceivable results of mutually exclusive experiments. Thus the question arises whether our analysis violates the injunction of Bohr's complementarity principle.

The Copenhagen interpretation certainly issues no blanket injunction against combining into one theoretical structure various alternative possibilities. In fact, quantum theory itself does just that: the single initial density function \( \rho_A(x,p,t) \) contains the information on the probabilities associated with all possible alternative final observations \( B \). Thus there can be no blanket denial of the right of an analyst to consider, in one logical analysis, the information that could be obtained from alternative mutually exclusive experiments if he carefully treats this information exclusively as information that could be obtained from alternative mutually exclusive experiments. This is demanded by the laws of logic, which Bohr accepted.

The paradigm of the use of information that could be obtained from alternative mutually exclusive experiments as something else is the naive analysis of the double-slit experiment. The naive analysis concludes from the fact that one could have done an alternative experiment, and thereby determined that in this second case the particle passed through one slit or the other, that also in the
original case the particle passed through one slit or the other. This naive analysis ascribes to the particle itself, without explicit reference to the experimental arrangement, attributes that can become manifest only in conjunction with alternative mutually exclusive experimental arrangements.

The crucial distinction here is between the viewpoint of "realism", or "objective reality", in which the observed attribute is assumed to be present in the object itself, and the viewpoint of "contextualism" in which the observed attribute is assumed to arise only in the confluence of object and observer. From this contextual viewpoint it is improper to ascribe to the object itself attributes that can arise only in the context of an observation. And it is totally meaningless to assign to it attributes that can arise only in the context of mutually exclusive observations. However, that is precisely what is done in the naive analysis of the double-slit experiment.

However, in the analysis of §2 the various possible observations are carefully associated with the local experimental situations in which they arise. There is no suggestion that any observed attribute has a physical existence outside the observer who observes it some particular local experimental situation. The analysis would be --and is--perfectly legitimate in a model in which the observed attribute is explicitly a joint characteristic of object and observer together, having no meaning whatever except in the conjunction or confluence of these two parts. Thus the analysis is predicated not on the assumption of realism or objective reality, but on quantum theory and locality.

The only way in which the analysis goes beyond usual quantum theory is precisely in the locality condition under discussion, which effects a disjunction of the experiments and observers in $R_A$ from those in $R_B$. Since these two regions are disjoint it is logically possible to make this disjunction within a contextual framework in which the observed attributes arise only in the confluence of object and observer.

Thus the analysis of §2 conforms to the local contextualistic form of Bohr's complementarity principle. Moreover, it is expressed completely in terms of the observables of quantum theory. It therefore follows that the idea of locality embodied in the locality condition is incompatible with pragmatic quantum theory itself. The issues of the validity of the doctrine of realism, or of the notion of objective reality, along with the question of hidden variables, are all irrelevant to this conclusion, because the argument of §2, like pragmatic quantum theory itself, avoids these issues.

The following section gives an explicit example of the application of the nonlocality argument to a concrete contextual theory of reality in which there are no hidden variables at all, and the observed attributes arise only in the confluence of object and observer.

6. DUAL-REALITY QUANTUM THEORY

Anyone reading von Neumann's chapter on the measuring process, with its two differing modes of development of the wave function, one continuous and lawful according to the Schroedinger equation, the other abrupt and stochastic, associated with the process of measurement, is led to ask, after reflection upon fact that measurement processes are fundamentally no different from other processes, whether changes of the stochastic kind are needed at all. For it seems unnatural that the Schroedinger equation should work up to a point, then suddenly fail.
Of course, if the wave function (or probability function) describes only a system being studied, and not the devices that are used to measure its properties, then one naturally expects the lawful development to cease during the process of measurement, due to the disturbing influence of the measuring device. However, if the wave function represents the whole universe then there can be no disturbance from outside, and hence no cause for a disruption of the lawful development.

On the other hand, as noted in §5, the functions \( \rho_A(x,p,t) \) and \( \rho_B(x,.p,t) \) of quantum theory are used in exactly the same way as the corresponding probability functions of classical theory. And the classical and quantum functions share also many other common mathematical properties. Thus it is natural to regard the quantum mechanical functions as probability functions. But then one must then answer the question "Probability of what?"

The Copenhagen interpretation answers this question by asserting the functions, \( \rho_A(x,p,t) \) and \( \rho_B(x,p,t) \) are to be used to calculate the conditional probabilities \( (B|A) \) from formula (5.2), and are to be ascribed no further meaning or significance. However, this interpretation demands two separate probability functions, one for the initial specifications \( A \) and one for the find specifications \( B \). Thus it assigns no meaning at all to a single function \( \rho(x,p,t) \) for the whole universe.

The probability functions \( \rho_A(x,p,t) \) and \( \rho_B(x,p,t) \) of quantum theory have, however, "interference properties" that are completely unlike those of classical probability functions. These properties suggest that these functions represent physical reality itself.

If this realistic interpretation is accepted then it becomes sensible to introduce, as our representation of the universe itself, a single universal function \( \rho(x,p,t) \), and to believe that this function always develops lawfully according to the Schroedinger equation.

As already stressed, the quantum probability function has many of the mathematical properties of a probability function. One of these properties is of critical importance. Consider an observer who is watching a device whose setting will either shift to position 2 or remain at 1 according to whether or not a decay is detected. Suppose he is programmed to walk into room 1 if the setting remains at 1, and go to room 2 if the setting is shifted to 2. Suppose the quantum probability for the detection of the decay is 50%. Then the function \( \rho(x,p,t) \), as governed by the Schroedinger equation, will split into a sum of two parts. In part 1 the setting remains at 1 and the observer walks to room 1; in part 2 the setting shifts to position 2 and the observer walks to room 2. This behavior is perfectly natural for a probability function: the observer has a certain probability of being in room 1 and a certain probability of being in room 2, and hence the probability function should, be a sum of two terms, one representing each possibility. However, if we now interpret the function \( \rho(x,p,t) \) as a representation of reality itself then the observer himself has split into two branches: in one branch the observer is in room 1; in the other branch the observer is in room 2. This splitting of the observer into two branches will be accompanied by a splitting of his environment, and eventually of the whole world, into two parts.
The analysis of von Neumann shows that these two parts of the world will, for all practical purposes, develop completely independently of each other: there will be effectively no interaction between them. The two parts will develop as though they are in different universes, although each part is really merely a localized region of the multiparticle \((x,p)\) space where the magnitude of the single universal function \(\rho(x,p,t)\) becomes relatively large (i.e., has a bump). With the human observer now split in two parts we may ask: "what does the observer experience?" The localized bump in the function \(\rho(x,p,t)\) that represented the observer before the split has now separated into two bumps, each developing independently of the other, in the way appropriate for the development of two independent possibilities.

If we accept the naturalistic view that conscious experience is an epiphenomena tied to brain processes, and is, in particular, an aspect of the formation of a memory structure, then the experience of the observer will now consist of two separate parts, because the two memory structures form independently of each other. That is, the perceptions received and recorded in one branch cannot affect physical processes in the other, and hence the two processes of memory formation will proceed independently, with the memories recorded by each unavailable for recall in the memory system formed by the other. Thus we can understand, in a general way, directly for the Schroedinger equation, and without any assumption other than the natural association of conscious experience with memory formation, how, in the absolute physical reality represented by the single function \(\rho(x,p,t)\), human memory structures, and hence the associated experienced personal realities, are continually splitting in to essentially independent branches.

This model of reality accords with the usual idea that there are two kinds of reality, first the absolute impersonal reality that contains the unwatched clock, and second the experienced reality which contains the individual personal realities that we experience directly.

This model accords with Einstein's demand that a complete physical theory describe "any real (individual) situation (as it supposedly exists apart from the act of observation)". It accords also with Bohr's contention that quantum theory is the complete theory of physical reality. But this reality includes now the absolute impersonal reality along with the experienced reality that was the focus of Bohr's thinking.

To see more clearly the way this works consider a device that flashes, at a preassigned sequence of \(n\) times, either a red light or a green light, depending on whether or not a radioactive decay is detected within the device during an immediately preceding time interval. A computerized robot is programmed to sense the red and green flashes, and to store in its memory the sequence of results, red or green, and then to compute and print out the fraction \(f_R\) of reds. It also computes, from the quantum-theoretical relative probabilities of red and green, the predicted value of \(f_R\) in the large \(n\) limit, and the range about this value such that the chances that the observed \(f_R\) will lie in this range are \(\approx 99\%\). Then it prints out "yes" or "no" according to whether or not the
observed $f_R$ falls in this range.

In this situation the function $\phi(x,p,t)$ will develop $2^n$ bumps corresponding to the $2^n$ possible memory structures. Let these $2^n$ possible memory structures be labelled by $i$. Each will be associated with a print-out giving the associated average $f_R^i$.

For large $n$ almost all of the printed results $f_R^i$ will lie within a small interval of the value $f_R^i = \frac{1}{2}$. This clustering about the value $f_R^i = \frac{1}{2}$ occurs, of course, no matter what the relative quantum theoretical probabilities of red and green are. To bring expectations regarding the number of "yes" answers into accord with the quantum calculations one must assign weights $w_i$ to the $2^n$ different possible memory structures $i$ according to the quantum theoretical rules, and give to the weight $w_i$ the intuitive meaning "the probability of finding the memory structure to be the one labelled by $i$, subject to the conditions of the experiment."

If the robot is now replaced by a similarly programmed human observer-scientist, then the intuitive meaning of $w_i$, is, for him, changed to "the probability that my experience will correspond to a memory structure in my brain that includes a record of memory structure $i$ if it includes a record of the verification of the conditions of the experiment."

This formulation ties expectations about conscious experiences to the physical system that evidently corresponds closely to it, namely the developing memory structure in the brain of the individual having the experiences. This association of conscious experience with brain processes is, of course, altogether natural, and is the inevitable result of pushing von Neumann's boundary between the quantum and classical worlds, to the quantum limit, so that the whole physical world is treated quantum mechanically, and only the world of experience is left on the classical, or observing side of von Neumann's boundary.

This way of interpreting quantum theory is, in principle, much more comprehensive than the Copenhagen interpretation, which works in principle only in certain idealized situations in which one can make a physical separation of "the system under study" from the surrounding universe.

On the other hand, this dual-reality model is, at present, totally useless for practical calculations because we do not know the detailed connection between brain processes and conscious experience. Bohr's intent was to formulate quantum theory as a practical theory.

The question of the connection between brain-processes and conscious experience never enters into the normal applications of classical physics: we use classical physics without knowing this connection. Bohr formulated quantum theory in the analogous way, and thus circumvented both the mind-body problem, and the associated question of the reality of the branches of the wave function that are inevitably generated by the universally valid Schroedinger equation, but which are forever unrelated to one's own personal experience.

By focusing in this way on practical matters, and avoiding all involvement of the mind-body problem and question of the ontological status of the nonexperienced branches, Bohr kept physics on a productive course. This was achieved, however, only at the expense of introducing a logical fuzziness associated with the need for imposing an idealized, and never perfectly realized, separation of one part of the world from the rest. The whole scheme rested, therefore, on an approximation, and it was
impossible to say what it was that was being approximated: there was no clear conception of the unapproximated whole. The dual-reality quantum theory supplies the needed conception of the whole.

Let us examine now the experimental set-up described in §2 from the dual-reality standpoint. Consider first a single observer who has arranged to receive information on all aspects of the experiment under discussion. There are four possible choices of the settings. When the information about the choices reaches him his personal reality will branch into four parts, one corresponding to each of the four possible settings. Later each of these branches will separate into $4^n$ branches, one for each of the $4^n$ possible combinations of the $n$ pairs of deflections. Thus the personal reality of any observer of the full experiment will separate into $4 \times 4^n$ branches.

Quantum theory assigns a weight $w_i$ to each of these $4 \times 4^n$ branches.

If there were many such observers of the whole experiment then the personal realities of each would divide into $4 \times 4^n$ branches. And the full collection of all of the personal realities would divide into $4 \times 4^n$ lots, with all the personal realities in each one of the $4 \times 4^n$ lots forming a community of observers who can communicate with each other, and who all agree on what has happened. Such a community will be called a "classical world", in order to indicate a connection to Bohr's thought. Each of these $4 \times 4^n$ classical worlds $i$ will be assigned a weight $w_i$, which can be intuitively understood as the fraction of an imaginary ensemble of identical initial classical worlds that develops into the image of the classical world $i$.

In this ensemble-conception of the meaning of the probabilities $w_i$ each member of the initial ensemble becomes one particular member of the final ensemble, and this particular member is labelled by one of the $4 \times 4^n$ values of $i$. This conception is basically different from the picture of reality itself provided by the dual-reality picture. In the dual-reality picture one initial classical world develops into all of the $4 \times 4^n$ possible final classical worlds. In the first conception a "choice" is involved: each initial classical world develops into some particular final classical world. But how is this particular one picked out?

This brings one up against the principles of insufficient reason, and the identity of indistinguishables: how can different choices arise in truly identical circumstances. And why must imaginary ensembles of identical systems be introduced to describe what happens to the actual system, which contains all of the classical worlds. Dual-reality quantum theory solves these problems by invoking the two levels of reality, absolute reality and personal experienced reality.

At the level of absolute reality the actual initial classical world develops into all of the final classical worlds. No choice is required, since all the possibilities are generated. At the level of experienced personal reality each of the $4 \times 4^n$ final classical worlds is independently experienced as the unique successor to the initial classical world, and $w_i$ is the "intrinsic likelihood" for classical world $i$ to be experienced as the successor. No imaginary ensemble or impossible choice is required. And no stochastic variable is introduced it represent any choice. The only ingredients in the theory are the absolute reality represented by the wave function and the myriads of experienced personal
realities. But the wave function and the experienced realities are precisely the two integral parts of the quantum formalism: they are not hidden variables.

To make the dual-reality picture completely clear it is helpful to visualize the spacetime location of the classical worlds. The $4 \times 4^n$ final classical worlds were defined to be the community of communicating observers (i.e. personal realities) who have received information about all aspects of the experiment. These observers must lie in the intersection of the forward light cones drawn from regions $R_A$ and $R_B$. This is the darkened region II of Fig.1.

![Figure 1. Locations of classical worlds.](image)

At the first time, $t_1$, there is, we suppose, a community of communicating observers who have set up the experiment. It is arranged that they all will have received by the time $t_3$ all the results from both $R_A$ and $R_B$. At time $t_2$ some of these observers are in region I and have observed the setting and results that occurred in $R_A$. These observers are grouped into $2 \times 2^n$ lots. Each lot is a community of communicating observers who all agree on what happened in $R_A$. Thus each lot is a classical world. These $2 \times 2^n$ different classical worlds occupy the same spacetime region I, but they develop essentially independently of each other, due to the linearity property of quantum theory and the fact that they are composed of very large numbers of particles. In region II there are also $2 \times 2^n$ essentially independent classical worlds. Although the sets in I and II, each consisting of $2 \times 2^n$ classical worlds, are both present at time $t_2$ there is no linkage between them: as far as experienced personal realities are concerned there those that contain information about what has happened in $R_A$ and those that contain information about what has happened in $R_B$, but none that contain information about what has happened in both $R_A$ and $R_B$. However, at the later time $t_3$, which intersects region III, all the observers have received the information about what has happened in both $R_A$ and $R_B$. The experienced personal realities now fall into $(2 \times 2^n)^2$ lots each consisting of a community of communicating observers who agree about what happened in both $R_A$ and $R_B$. These are the $4 \times 4^n$ final classical worlds, and they lie in region III. (I locate the experienced personal reality at the site of the brain of the observer who is having the experience). These $4 \times 4^n$ different final classical worlds all lie in region III, but they develop essentially independently of each other. This dual-reality picture of the classical worlds arises directly from the mathematics of quantum theory.15
Let us now examine in the dual-reality framework our locality condition that a change in the setting in one region leave unchanged the results in the other region. At the level of absolute reality all the possible results are simultaneously present, and hence the idea of a "change" in the settings, or in the results, has no meaning. It is only at the level of the experienced realities that the results are definite and one can, by considering different personal realities, or classical worlds, give meaning to the notion of a "change" in the setting or results.

Applied at time \( t_2 \) the locality condition still has no immediate meaning. For at time \( t_2 \) there is one set of classical worlds in each of which the choices and results in \( R_A \) are definite, and another set of classical worlds in each of which the choices and results in \( R_B \) are definite. But there is no personal reality, or classical world, in which the results and choices from both regions are definite. Thus the locality condition cannot be implemented, within the set of realities that exist at time \( t_2 \).

At time \( t_3 \) the personal realities all contain the information about the results and choices at both \( R_A \) and \( R_B \). Thus the locality condition can be implemented. However, since these personal realities have received information from \( R_A \) and \( R_B \) by normal slower-than-light methods it is not clear that a failure of the locality condition at \( t_3 \) has the implication of a need for faster-than-light information transfer.

The important point in this connection is that the development between time \( t_2 \) and \( t_3 \) is essentially completely classical. The interaction in region III simply combines the information from the regions I and II in a completely classical way: the observers coming through region I inform those that have come through region II what they observed, and vice versa. Thus each of the \( 2 \times 2^n \) classical worlds in I combines with one of the \( 2 \times 2^n \) classical worlds in II to give one of the \( 4 \times 4^n \) classical worlds in region III. Each combination \( i \) is assigned a weight \( w_i \) by the quantum formalism. However, since the process of combining the two individual classical worlds to make one of the final worlds \( i \) is completely classical (it is via the conversations of scientists) this classical world \( i \) can, at least in the imaginations of the scientists, be extend back to time \( t_2 \). They will believe that the classical world \( i \) was present already at \( t_2 \), and with the assigned weight \( w_i \).

In mathematical terms, the classical world \( i \) can be extended backward in time from \( t_3 \) to \( t_2 \), by using the universally valid Schroedinger equation, and during this interval each of the classical worlds \( i \) behaves in a completely classical manner, with no variation in its weight. Thus the fact that the weight of this world has the quantum mechanical value \( w_i \) is perceived to be a property that is fixed and determined already at time \( t_2 \). And the fact that a transmission of information from the separated regions \( R_A \) and \( R_B \) into region III is required in order to compare the results in one region to the setting in the other is regarded as a perfectly natural consequence of the fact that signals or messages can travel no faster than light.

Adopting this viewpoint we turn to the problem of formulating within the mathematical structure provided by the dual-reality quantum theory the locality condition that corresponds to the intuitive
idea that a change of setting in one region leave unchanged the results in the other region. This formulation must be based, naturally, on the intuitive meaning of the probabilities \( w_i \).

Figure 2 shows four big boxes each of which contains four little boxes. The four big boxes correspond to the four possible settings. The four little boxes correspond to the four possible results. The arrows indicate the directions of the deflections, which are called the spin directions. The numbers in the small boxes are the quantum mechanical probabilities \( w_i \) for the four results in the case \( n = 1 \), subject to the condition that the setting be known.

\[
\begin{array}{cccc}
\uparrow\uparrow & \uparrow\downarrow & \downarrow\uparrow & \downarrow\downarrow \\
0 & \frac{1}{4} & \frac{1}{4} & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 \\
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\
\frac{1}{8} & -\frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\end{array}
\]

The weights occurring in the boxes are the quantum probabilities \( w_i \) for the case \( n = 1 \). However, they pertain equally well to the case of arbitrary \( n \), in the sense that an appropriate measure for the box for general \( n \) is the sum of the measures for the \( n \) individual instances divided by the normalizing factor \( 1/n \). This is seen clearly if one considers cases in which the probabilities for the individual instances vary over the \( n \) instances. Thus the weight appearing in each box is a representation, for any \( n \), of the average probability of the indicated spin combination. When \( n \) becomes large this probability acquires an intuitive meaning as an approximation to the fraction of the instances in which the observer finds himself in a branch with the indicated spin directions, subject to the condition that he find himself on a branch where the settings are those indicated in the large box. The accuracy of this approximation increases with \( n \).

For any particular value of \( n \), and any particular individual reality, each of the integers 1, \ldots, \( n \) is assigned to some particular small box. The number of integers in this box is a fraction of the number \( n \) that should, for large \( n \), according to the quantum theoretical predictions, be approximately equal to the probability shown in the box of Fig. 2. This is the intuitive meaning of this probability.

The locality condition is supposed to represent the idea that a change in the setting in one region leave unchanged the observations (i.e., the spin directions) in the other region. For any particular personal reality, as represented by a particular distribution of the \( n \) integers among the small boxes in one particular one of the large boxes, the meaning of this condition is clear: if one of the two
settings is changed, and the original personal reality is thus forced to change to some personal reality corresponding to one of the two big boxes lying adjacent to the original one, then the integer \( m \), whose position represents what is observed in instance \( m \), must shift into another little box in a way that leaves unchanged the observation (i.e. spin direction) that locality requires be left unchanged. For example, an integer \( m \) appearing little box (a) of Fig. 2 should, if a change of the second setting is made, go into either box (b) or (c). For a shift into box (e) or (f) would change the observation (i.e. spin direction) that the intuitive idea of locality demands be left unchanged.

The probabilities appearing in the boxes of Fig. 2 refer, in the sense already described, to all values of \( n \). They provide a way to formulate the locality condition simultaneously for all values of \( n \), and for all personal realities. To do this one can consider the probability measure assigned to each little box to represent an aggregate of identifiable infinitesimal elements. This corresponds to all the possible collections of integers that it could represent. Then the intuitive locality condition translates into the mathematical condition \( \mathcal{L} \) that the part of the measure residing in any small box when the settings are fixed in some particular way must shift, when \( x_A \) (resp. \( x_B \)) is changed, to one of the neighboring big boxes in such a way as to leave unchanged the observation (i.e. spin direction) associated with the region \( R_B \) (resp. \( R_A \)). This condition \( \mathcal{L} \) is a direct translation into the mathematical structure of dual-reality quantum theory the intuitive requirement of locality. The formulation combines into one simple mathematical condition on the probabilities appearing in Fig. 2 the content of intuitive locality for all values of \( n \).

This locality condition \( \mathcal{L} \) cannot be reconciled with the quantum theoretical probabilities that appear in Fig. 2. This is precisely what was shown in the key step of the proof in Ref. 8 of the result \( L \cap Q = \emptyset \).

The property \( \mathcal{L} \) represents, essentially, just the ensemble of all condition like \( L \cap Q = \emptyset \), formulated directly as a condition on the probabilities. This probabilistic formulation of the locality condition, unlike the probabilistic formulation of Bell, and Clauser and Horne, does not require the introduction of hidden-variables to identify imagined entities or systems for which the probabilities factorize. Nor does it require any stochastic hidden variables to specify particular individual results. Rather, it is formulated directly within the probabilistic quantum theoretical framework.

Although the dual-reality version of quantum theory was used to make this discussion concrete, it played no essential role in the discussion of locality. By speaking of alternative conceivable possibilities the whole analysis becomes applicable also to the pragmatic quantum theory of Bohr.

\section{Nonlocality and Signals}

The failure of the locality property in dual-reality quantum theory, or in Bohr's pragmatic quantum theory, does not contradict Einstein's principle that no signal travels faster than light. For by a signal is meant a controllable transfer of information—a message. Within the structure of these formalisms no such controlled faster-than-light information transfer is possible. This follows immediately
from the fact that whereas within the quantum formalism (or the classical formalism) the probability of a specified result in one region, subject to the condition of a specified result in the other region, depends in general on the latter specification, and hence on the experimental setting in the other region, nevertheless a summation over all possible results of the experiment in the other region, with proper weights gives a result that is independent of the choice of the experimental setting in that region. This entails that there is no predictable dependence of the observations in one region upon the choice of setting in the other.

Within the dual-reality framework it is also clear that no information is transferred faster than light. For the absolute reality is controlled by the Schroedinger equation, which develops according to local information. (I am assuming here that the program of local field theory can, in principle, be completed.)

Viewed differently, the nonlocal connection is a mathematical relationship between different classical worlds that do not communicate. The shifts among these worlds entailed by changes of the settings modifies the probabilities in a way that makes it neccessary to change the results in a way that is not compatible with the locality condition, if one is to stay in accord with the quantum predictions. But there is no real shift and hence no real transfer of information. The various classical worlds merely exist in noncommunicating unison.

For this reason the many-worlds type of theories have always been carefully excluded from those to which the nonlocality result was claimed to apply. It has been shown here that the argument for nonlocality can be naturally extended to the many-worlds case. However, the nonlocality property that emerges in this case is a purely mathematical one, devoid of any implication of either faster-than-light signals or faster-than-light information transfer.

§8. CHOICE AND NONLOCALITY

Two objections can be raised against accepting dual-reality quantum theory as a picture of nature itself. Both regard choice. The first is that we experience directly the fact that we do, to some extent, choose between alternative courses of action: we experience the unfolding of classical worlds not as passive spectators but as active participants. This direct experience of active participation is not explained by the dual-reality picture, and there is no argument that adequately justifies discounting the direct evidence.

The second objection concerns the notion of "intrinsic likelihood." If two different classical worlds really exist then can any meaning logically be given to the idea that in this single existing actual situation "the likelihood that I will find myself in one of the worlds is much larger than the likelihood that I will find myself in the other." Since both possibilities exist in unison how can one give meaning to this notion except by referring to different real situations or ensembles of imaginary systems. Such a reference is certainly justified in a pragmatic context, but in a realistic description all real meanings should inhere in the real situation itself.

To meet these objection, and still retain the many attractive features of dual-reality quantum theory, one can take the dual-reality structure to be the board upon which the game of creation is played, rather than the game itself.
In this context the experiment discussed in §2, but with human experimenters making the choices, can be regarded as a model that illustrates two kinds of choices, one associated with personal realities, the other associated with absolute reality. The choices of setting illustrate choices that are under the sway of localized personal realities, and can be called personal choices, whereas the choices of the results of the experiments illustrate impersonal selections associated with absolute reality. This view of nature, which accords both with quantum theory and common sense, provides for a meaningful dialog between man and nature.

The nonlocality theorem of §2 applies directly to this completed quantum theory. It implies that the impersonal selections cannot be made by a local process if the statistical demands of quantum theory are to be met: the observations selected in one region must depend on the choice of setting made in the other region.

Since man is a resident of ordinary space time his choices can naturally be considered localized in his brain or body. However, the absolute reality does not reside in ordinary spacetime, and it is therefore not unnatural that the absolute or impersonal selections should not respect a simple notion of spacetime localizability, but should possess, instead, a basically global nature.

The personal choices also have a certain nonlocal character in the sense that they refer to a highly correlated portion of the absolute reality that can be naturally assigned a location, but not a point, in ordinary spacetime.

This complete quantum theory gives a prime example of the applicability of the nonlocality theorem of §2 to a case where there is no objective reality in which the observable attributes can become definite independently of observers. In this theory the definite observed attributes arise only in the confluence of the object, as represented by the wave function, which represents all possibilities and potentialities, with the observer. Thus this theory, though fully realistic, conforms to the notion of "contextualism" implicit in Bohr's complementarity principle, rather than to the naive "doctrine of realism."

§9. CONCLUSIONS

The mathematical nonlocality property of quantum theory $L \cap Q = \emptyset$ entails that no deterministic-stochastic or probabilistic local hidden variable theory can give the statistical predictions of quantum theory. This property $L \cap Q = \emptyset$ directly entails also the stronger result that any process that selects observed results that conform to the contingent quantum-theoretical predictions must be nonlocal: it cannot satisfy the locality property that a change in the setting in the spacetime region $R_A$ leave unchanged the observation selected in the spacelike separated region $R_B$, and vice versa.

The locality condition can be formulated directly as a condition on probabilities that does not refer to hidden variables. In this form it can be applied directly to pragmatic quantum theory and to dual-reality quantum theory.
Dual-reality quantum theory reconciles Einstein's demand for a complete description of physical reality with Bohr's contention that quantum theory is complete. Two objections to this theory, as a description of nature itself, can be met by considering it to be the framework for the action of choice. Corresponding to the two levels of reality there are two kinds of choices, the personal choices associated with the personal realities, and the impersonal selections associated with absolute reality. The personal realities are automatic residents of four-dimensional spacetime, in a manner discussed in some detail, and the personal choices can be localized. However, the nonlocality property \( L \cap Q = \emptyset \) then entails that the selections must be nonlocal, in the sense already described.

This model of reality conforms to the statistical laws of quantum theory, to our direct experience of active participation, and to the apparent fact that man alone does not determine the events he observes. It provides a meeting ground for diverse philosophical ideas about the connection of mind to matter, and gives, moreover, a mathematical framework for the quantum theoretical consideration of systems that do not conform to the separation requirement of Bohr's pragmatic interpretation of quantum theory. This requirement is never completely satisfied in nature. But a satisfactory complete physical theory must provide some coherent basis for the estimation of errors introduced by the approximations needed to make precise calculations.

It is in the area of research into the connection between brain processes and conscious experiences that the inadequacy of Bohr's interpretation becomes most severe. For in this research the system under study is not at all isolated from its environment. It is interacting with its environment both to maintain consciousness and to be observed. And it is also being directly experienced from within. Moreover, the system is, in an important sense, not a member of a quantum theoretical ensemble, since it has important genetic roots that make it effectively unique. The prime virtue of the pragmatic interpretation, namely that it circumvents the problem of the connection of mind to body, becomes a failing when this problem is precisely the one under investigation.

The completed quantum theory outlined here, while increasing in this way, at least in principle, the comprehensiveness of quantum theory, provides also, by virtue of the explicitly nonlocal or global character of the process of selection, a basis for the incorporation of certain holistic or global characteristics into the framework provided by the local laws of quantum theory. The occurrence of such global or nonlocal characteristics is a necessary feature of all models or theories in which man has some effective freedom, and the validity of certain contingent statistical predictions of quantum theory is maintained.
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