Binary star statistics: the mass ratio distribution for very wide systems

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The distribution of mass ratios for a sample of common proper motion (CPM) binaries is determined and compared with that of 798 visual binaries (VBs) studied earlier, in hopes of answering the question: Can the member stars of these systems have been drawn at random from the normal initial mass function for single stars? The observed distributions peak strongly toward \( q = 1 \) for both kinds of systems, but less strongly for the CPMs than for the VBs. Due allowance having been made for assorted observational selection effects, it seems quite probable that the CPMs represent the observed part of a population drawn at random from the normal IMF, while the VBs are much more difficult to interpret that way and could, perhaps, result from a formation mechanism that somewhat favors systems with roughly equal components.

Die Verteilung der Massenverhältnisse für eine Auswahl von Doppelsternen mit gemeinsamer Eigenbewegung (CPM) wird bestimmt und mit der von 798 schon früher untersuchten visuellen Doppelsternen (VB) verglichen in der Hoffnung, die Frage beantworten zu können, ob die Mitglieder dieser Systeme nach der normalen ursprünglichen Massefunktion (IMF) für Einzelsterne verteilt sind. Beide Systeme haben eine deutliche Spitze bei \( q = 1.0 \), jedoch weniger ausgeprägt für die CPM als für die VB. Nach Berücksichtigung von Auswahleffekten – durch die Beobachtung scheint es wahrscheinlich, dass die CPM eine zufallsverteilte Teilpopulation der normalen IMF repräsentieren, während die VB auf diese Weise wesentlich schwieriger zu erklären sind und vielleicht das Ergebnis eines Bildungsmechanismus darstellen, der Systeme mit annähernd gleichen Komponenten bevorzugt.

Key words: binaries – mass distribution – initial mass function

1. Introduction: binary star statistics

The processes of binary star formation are still not at all well understood, and the hope of probing or constraining them has provided the primary motivation for study of binary star statistics (BATTEN 1973) from the time of KUIPER’s (1935) pioneering investigations to the present. Studies of the distributions of separations and periods are still capable of yielding surprises in the form, for instance, of a large number of systems with separations of 10-100 AU and periods of 30-300 yr which are not easily found by either spectroscopic or visual methods (McAUSTER et al. 1986; SHARA et al. 1987). It is possible that this range constitutes a preferred scale for binary system formation.

Investigations of the mass ratio distribution normally address the question: can this population have been drawn at random from a Salpeter-type mass distribution (SALPETER 1955; ScALo 1986), or do we need to invoke more complex assumptions? The hypothesis of selection from random at an initial mass function of stars with \( N(M) = N_0 M^{-x} \) automatically predicts a population of binaries with an initial mass ratio distribution \( N(q = M_2/M_1) = N_q - x \).

Observed distributions have been reported both for small samples of stars thoroughly searched (ABT and LEVY 1976, 1978; WOLFF 1978; GARMANY et al. 1980) and for larger samples of binaries catalogued as eclipsing (TRIMBLE and WALKER 1986), spectroscopic (STANIUCHA 1979; KRAJCHEVA et al. 1977; TRIMBLE 1978; LUCY and RICCO 1979; HALBWACHS 1987), or visual (JORISSEN 1984; HALBWACHS 1986; TRIMBLE and WALKER 1986) systems.

Unfortunately, before comparing the prediction with observations, it is necessary to correct one or the other for the effects both of post-formation evolution and of observational selection, which discriminates strongly against small mass ratios, no matter how one thinks of trying to identify the systems. Evolutionary processes in the form of mass, energy, and angular momentum exchange (SHU and LUBOW 1981) entirely dominate the interacting and contact systems that predominate among eclipsing binaries and thereby fully account for their preponderance of mass ratios of 0.5-0.7 (TRIMBLE and WALKER 1986). Mass transfer is also important for at least the closer spectroscopic systems and is, perhaps, responsible for many of the late BV stars with low-mass (white dwarf?) companions (WOLFF 1978). Even for the visual systems, it has been suggested (JORISSEN 1984) that the statistical properties cannot be understood without allowance for mass loss from the individual stars large enough to affect the separations as well as the mass ratios significantly.

Against this background, it is perhaps not surprising that rather similar data bases have suggested very different conclusions to different authors. KUIPER (1935) found the spectroscopic binary population consistent with random selection from a normal initial mass function (though he ignored double line systems in order to do it). KRAJCHEVA et al. (1977) and TRIMBLE (1974, 1978) found a bimodal distribution for the complete spectroscopic sample, with peaks at mass ratios of 1 and 0.3 (the latter being presumably an observational cutoff of a real distribution rising still higher at smaller ratios). Concentrating on very close systems, Lucev and Ricco (1979) and GARMANY et al. (1980) found only the \( q = 1 \) peak, while WOIH (1978) thought it was, at any rate, the dominant feature for unevolved systems. A tendency at least for close systems to favor \( q = 1 \) turns up also in the data of ABT and LEVY (1976, 1978) and TRIMBLE and CHEUNG (1976). But HALBWACHS (1987) has looked yet again at the catalogued spectroscopic systems and found only the low-\( q \) peak. Meanwhile, his 1986 analysis of visual systems concurs with that of TRIMBLE and WALKER (1986) in finding so large an excess of large mass ratios that it is hard to attribute it entirely to observational selection.
The present investigation pertains to a sample of 326 common proper motion pairs catalogued by HALBWACHS (1986a) and is intended to probe whether the large-\(q\) fraction of binaries becomes large or smaller as one goes from visual to still wider systems.

2. \section*{Data}

The sample consists of 326 pairs of AGK 3 stars defined (HALBWACHS 1986a) as having common proper motions greater than 0.05"/yr and ratios of separation to proper motion less than 10^3 yr. The proportion of optical pairs should be only about 1/.

For these 326 pairs, magnitude differences were turned into mass ratios, with due allowance for spectral type, using the algorithm described by TRIMBLE and WALKER (1986). Systems for which one or both components are known visual, spectroscopic, or eclipsing binaries were also treated as described there, and the mass ratio is then that of less- to more-massive component, one or both components consisting of two stars. The sample includes 39 triples and 3 quadruple systems.

Table 1 gives the distribution of mass ratios found in this way for the common proper motion stars and shows the visual binaries from the Fourth Catalogue (WORLEY and HEINTZ 1983) for comparison. Both samples are strongly peaked toward \(q = 1\), but the CPM one much less so than the VB one.

<table>
<thead>
<tr>
<th>(q = \frac{M_2}{M_1})</th>
<th>Common Proper Motion Stars</th>
<th>Visual Binaries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of Systems</td>
<td>Fraction</td>
</tr>
<tr>
<td>0.91 - 1.00</td>
<td>118</td>
<td>0.362</td>
</tr>
<tr>
<td>0.81 - 0.90</td>
<td>79</td>
<td>0.242</td>
</tr>
<tr>
<td>0.71 - 0.80</td>
<td>54</td>
<td>0.166</td>
</tr>
<tr>
<td>0.61 - 0.70</td>
<td>31</td>
<td>0.095</td>
</tr>
<tr>
<td>0.51 - 0.60</td>
<td>18</td>
<td>0.055</td>
</tr>
<tr>
<td>0.41 - 0.50</td>
<td>16</td>
<td>0.049</td>
</tr>
<tr>
<td>0.31 - 0.40</td>
<td>6</td>
<td>0.018</td>
</tr>
<tr>
<td>0.21 - 0.30</td>
<td>4</td>
<td>0.012</td>
</tr>
<tr>
<td>0.11 - 0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01 - 0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>326</td>
<td></td>
</tr>
</tbody>
</table>

3. \section*{Implications}

What can these data tell us about the real initial distribution of mass ratios? Evolutionary effects, in the sense of mass transfer, are clearly irrelevant, and I will assume that mass loss from the individual stars has also been negligible, since the majorities of both populations consist of pairs of late type main sequence stars.

Observational selection, on the other hand, is obviously of overwhelming importance and cannot be properly corrected for since neither sample is complete to well-defined limits in magnitude, separation, or magnitude difference. We can, nevertheless, address in two ways the question of whether the samples could have been drawn from a Salpeter-type mass function.

Consider first the CPM stars. Fig. 1 shows the mass ratio distribution for the brighter (\(V < 8.4\)) and fainter (\(V < 8.5\)) halves of the sample separately, where the magnitudes are those of the primary star. The sample is not complete even for the bright half, in the sense that the number of systems brighter than \(V\) does not increase by a factor of four for each 1 m interval in \(V\) much beyond \(V = 4\). But HEINTZ's (1978) criterion for the resolution of close pairs suggests that, within the bright half, the bins \(q = 0.8-0.9\) and 0.9-1.0 will, at any rate, be about equally incomplete. There are 37 systems with \(q = 0.8-0.9\) and 26 with \(q = 0.9-1.0\), and (37/26) = (0.85/0.95)-3.8. In other words, this small part of the sample could have been drawn at random from an IMF with the not implausible form \(N(M) = N_0 M^{-3.2}\).

A second approach to the CPM stars consists of asking just how many binaries there will be in total if the sample we see reflects only the actions of selection upon the predicted population. It is possible to do this because the 326 systems came from precisely 31 342 AGK 3 stars with proper motions larger than 0.05"/yr. We need to allow for incompleteness in both mass ratio and separation. Taking mass ratio first, 118 systems with \(q = 0.9-1.0\) and an IMF slope of 3.2 imply a total of 5784 systems down to \(q = 0.3\) (at which point, with a typical \(M_1\) near 1.0M_☉, the IMF is beginning to turn over from a power law form; ScALO 1986). This total is only 2454 systems using the flatter IMF, \(x = 2\), suggested by TINSLEY (1980) for solar-type stars.

The fact that the CPM stars include only binaries in a restricted range of separations can also be allowed for. The median separation is 17,5" corresponding to 8575 AU at the median spectroscopic-parallax distance of 490 pc. Just about one-half (in fact 48\% or 155 of 326) of the systems have angular separations within one-half \(<\lex centered around the median value and so, on average, linear separations between 4820 and 15750 AU. Thus 2750 or 1178 of the hypotethical total number of systems should also fall in this range. In principle, it would be desirable both to deproject these separations back into three-dimensional ones and to correct for average orbital eccentricity to get semi-major axes. This cannot be done accurately, because additional selection effects favor face-on orbits and discovery of systems near maximum separation, by unknown amounts. Fortunately,
deprojection and circularization are not absolutely essential. If, as seems to be the case (ABT and LEVY 1976, 1978; MCALISTER et al. 1986; SHARA et al. 1987), numbers of binary systems are roughly equal per logarithmic interval in separation, we need only ask how many half-dexes of semi-major axis are possible between contact systems ($a \sim 0.01$ AU) and those that will be tidally disrupted in less than their main sequence lifetimes ($a \sim 10^5$ AU). The answer is about 14, with the CPM range falling at a somewhat uncertain point (owing to our failure to deproject and circularize) near the largest possible separations. Then the 1178 to 2750 hypothetical systems in the CPM separation range will represent 16500-38500 over all possible separations.

There should also be about 4000 triples and 300 quadruples. Since the parent population consists of 31 342 stars, we are led to conclude that virtually every star must be part of a double or multiple system, which is in accord with much of modern thinking on the subject.

To summarize, the CPM stars could have been drawn at random from a population in which virtually all stars belong to double or multiple systems, distributed uniformly through the possible logarithmic range of semi-major axes, and for which the slope of the IMF is 2 to 3.2. The argument carries through, to order of magnitude, for other reasonable choices of IMF, semi-major axis statistics, and so forth.

The situation is somewhat different for the visual binaries. They are much more sharply peaked in $q$ and somewhat more widely distributed in separation than the common proper motion stars. And they are not drawn from any particular, well-defined number of stars that can be said to constitute the parent population. As a result, the case that they could have been drawn from a normal initial mass function proves rather less persuasive for the VB's than it is for the CPM's.

First, let us attempt to choose a subsample that might be complete in some sense and see whether it shows evidence of an IMF that rises toward small $q$. HEINTZ's (1978) criterion says only that all binaries within certain ranges of $\eta$ and separation will have been identified as double (those for which $C = 0.22 - \log \eta $; $0.5$ and $9.5$), not that they will have had their orbits determined. Fifty-six relatively bright systems with semi-major axes greater than 3" should, in this limited sense, be "complete" down to $q = 0.3$, but as can been seen in Fig. 2, they show only a roughly-flat distribution in $q$, not a rising one. The still-fewer systems within 5 pc of us also yield a flat $N(q)$ according to RUCINSKI (1987).

Second, we can attempt to correct for incompleteness in $q$ and semi-major axis in the way that proved plausible for the CPM systems, though we have no a priori guidance on the right choice of IMF slope for the population. There are 432 systems (Table 1) with $q = 0.9 - 1.0$. Given a slope between 2 and 3.2, these imply 8984 to 21 176 systems down to $q = 0.3$. The correction for semi-major axis range is straightforward, because $a$ in arc-seconds is, of course, one of the orbit parameters and no deprojection is needed. The median value of $a$ is 0.41", and the half-dex centered around this contains 46.7% of the systems. Allowing again for 14 possible half-dexes between contact systems and tidally disrupted ones, the total number of binaries implied is $14 \times 0.467 \times 8984$ or 21 176, that is 58 700 to 138 000 binaries.

Whether this is a plausible number depends on the exceedingly-uncertain number of stars from which the catalogued visual binaries are drawn. One approach to the question is to note that the median apparent magnitude of the WORLEY and HEINTZ (1983) Catalogue systems ($V = 8.45$) and the absolute magnitude corresponding to the median spectral type ($M_v = 4.4$ for
Fig. 2. Number of visual binaries as a function of mass ratio, \( q = M_2 / M_1 \) for the subsample of relatively bright, wide, close systems that ought be about as complete as can be found within the WORLEY and HEINJ (1983) Catalogue. The distribution is, at best, flat in \( q \), not rising toward small values.

GO V) imply a median distance of 65 pc. (This set of numbers is, at any rate, self-consistent: the median semi-major axis of 0.41" then corresponds to 26.5 AU and, introducing the median total spectroscopic mass, \( M_1 + M_2 = 2M_* \), the median period should be about 100 yr, which is quite close to the actual value of about 130 yr). Half the hypothetical systems (29400 to 69 200 of them) should be within the same 65 pc median distance. If we adopt BAHCALL's (1986 and references therein) model of the nearby stellar population, the total number of stars of 0.4M* or more (corresponding to the latest spectral type represented in the catalogue, M4V) should be about 20000 or 52000 for \( X = 2 \) and 3.2 respectively. Thus, in each case, the number of binaries would exceed the number of stars, though only by 30-50%.

To summarize, it is just possible for the catalogued VB's to be drawn at random from a Salpeter-type IMF if essentially every star in the sky is part of a binary or multiple system. The numbers are, however, less certain and less persuasive than in the case of the CPM stars. In addition, we are unable to identify any subsample that convincingly shows a distribution in \( q \) that rises toward small values.

4. Conclusions

A sample of common proper motion stars catalogued by HALBWACHS (1986a) resembles catalogued visual binaries in that mass ratios near 1:0 dominate both samples. For the CPM's, however, it is possible to identify a subsample distributed like \( q - x \), as would be expected for pairs of unevolved stars chosen at random, while likely subsamples of the VB's (the closest, widest, or brightest systems) give, at best, a flat distribution. In addition, if the distribution of semi-major axes is flat in logarithmic intervals in \( a \), then the CPM stars could represent an observational selection from systems with \( N(q) \propto q^{-x} \) and \( x = 2-3 \) if most stars are binary or multiple, while the same set of assumptions applied to the VB's requires the number of binaries and multiples to exceed to total number of stars by 30-50%. This is not a physical impossibility if triple and higher systems are commoner than generally supposed, but it is somewhat unlikely.

These data suggest that the process of binary formation is the same as single star formation only for the very widest systems, represented by common proper motion pairs.

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